CHAPTER 7

FLOW THROUGH PIPES

Introduction

Pipes were introduced in the earliest days of the practice of hydraulics. Their common place use today makes it of great importance that the laws governing the flow in them should be fully understood.

Water is conveyed from its source, normally in pressure pipelines, to water treatment plants where it enters the distribution system & finally arrives at the consumer. In addition oil, gas, irrigation water, sewerage can be conveyed by pipeline system.

Some loss of energy is inevitable in the flow of any real fluid. In the case of flow in a horizontal uniform pipeline, this is evidenced by the fall of pressure in the direction of flow. Predicting the energy loss per unit length is essential to efficient pipeline design.

The prime concern in the analysis of real flows is to account for the effect of friction. The effect of friction is to decrease the pressure, causing a pressure '**loss**' compared to the ideal, frictionless flow case. The loss will be divided into ***major losses*** (due to friction in fully developed flow in constant area portions of the system) & ***minor losses*** (due to flow through valves, elbow fittings & frictional effects in other non-constant –area portions of the system).

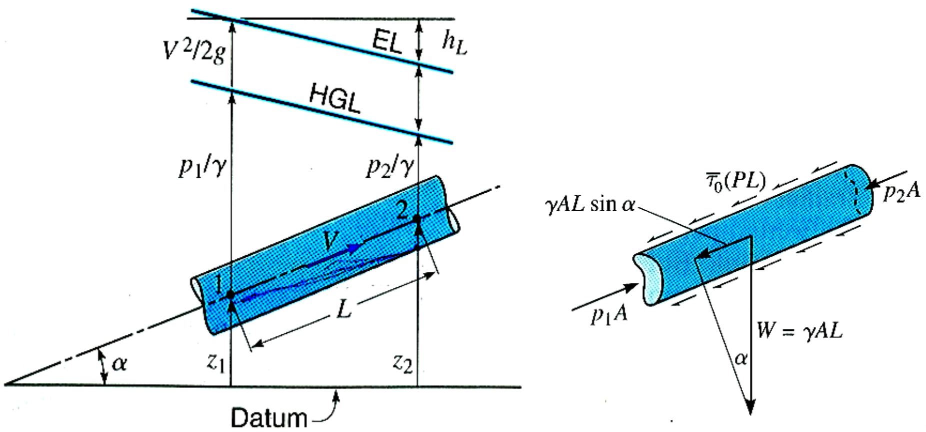

Figure 4.1 Flow in the pipes (circular pipe)

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L \qquad \text{.......................................4.1}$$

h_L = Head loss (major + minor)

7.1 Major Losses *(Head loss in conduits of constant cross-section)*

Referring to Figure 5.1 and for equilibrium in steady flow, the summation of forces acting on any fluid element must be equal to zero, i.e. $\sum F = 0,$

$$p_1A - p_2A + W\sin\alpha - \overline{\tau_o}(pL) = 0$$

$$\sin\alpha = \frac{(z_1 - z_2)}{L}$$

$\overline{\tau}_o$ - average shear stress (average shear force per unit area) at the conduit wall, is defined by:

$$\overline{\tau}_o = \frac{1}{P}\int_0^P \tau_o dP \quad \text{.. (4.2)}$$

τ_o - is the ***local shear stress***[1] acting over a small incremental portion *dP* of the wetted perimeter.

$$p_1A - p_2A - \gamma AL\frac{(z_2 - z_1)}{L} - \overline{\tau}_o PL = 0$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - (z_2 - z_1) - \overline{\tau}_o\frac{PL}{\gamma A} = 0$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \overline{\tau}_o\frac{PL}{\gamma A} \quad \text{.. (4.3)}$$

Form the above equations (4.1) and (4.3)

$$h_L = \overline{\tau}_o\frac{PL}{\gamma A} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

$$h_L = \overline{\tau}_o\frac{L}{R_h\gamma} \quad \text{... } \mathbf{(4.4)}$$

This equation is applicable to any shape of uniform cross-sections, regardless of whether the flow is *laminar or turbulent*. For smooth-walled conduits, where wall roughness may be neglected, it may be assumed that the average shear stress $\overline{\tau_o}$ is a function of ρ, μ, ν & some characteristic linear dimension, which will here be taken as hydraulic radius R. Thus:

$\overline{\tau_o}$ = ϕ(ρ, μ, ν, R)

By dimensional analysis:

$$\overline{\tau}_o = \rho V^2\phi\left(\frac{R_hV\rho}{\mu}\right) = \rho V^2\phi(\text{Re})$$ and let ϕ (Re) = ½ C_f (dimensionless term)

1 The local shear stress varies from point to point around the perimeter of all conduits (irrespective of whether the wall is smooth or rough), except for the case of a circular pipe flowing full where the shear stress at the wall is the same at all points of the perimeter.

$$\bar{\tau}_o = C_f \rho \frac{V^2}{2} \quad \text{...} \quad (4.5)$$

From equation (4.4): $h_L = C_f \dfrac{L}{R_h} \dfrac{V^2}{2g}$ .... (4.6)

(Applied for any shape of smooth walled conduits).

For ***circular conduits (pipe)*** flowing full, R= ¼ D, Therefore,

$$h_L = C_f 4 \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{V^2}{2g} \quad \text{..} \quad (4.7)$$

Where, $f = 4C_f = 8\phi(\text{Re})$ .... (4.8)

Equation (4.7) is applicable for both smooth-walled and rough walled conduits. It is known as ***pipe – friction equation***, and commonly referred to as the ***Darcy-Weisbach*** equation. Friction factor, *f*, is dimensionless and is also some function of Reynolds number. The exact form of $\phi(\text{Re})$ and numerical values for C_f and *f* must be determined by experiments or other means.

For ***laminar flow*** (Recall chapter three)

$$f = 64 \frac{v}{DV} = \frac{64}{\text{Re}} \qquad (for\ laminar\ flow) \quad \text{.......................} \quad (4.9))$$

Head loss: $h_f = \left(\dfrac{64}{\text{Re}}\right) \dfrac{L}{D} \dfrac{V^2}{2g}$ .... (4.10)

Experimental Investigation on friction losses in Turbulent flow

In fully developed turbulent flow, the pressure drop, Δp, due to friction in a horizontal constant area pipe depends upon the diameter, D, the pipe length, L, the pipe roughness, ∈, the average velocity, $\bar{V}$, the fluid density, ρ, and the fluid viscosity, μ.

By dimensional analysis $\Delta p = \phi(V, D, \rho, \mu, \varepsilon)$

$$\frac{\Delta P}{\rho V^2} = \phi\left(\frac{\mu}{\rho v D}, \frac{L}{D}, \frac{\varepsilon}{D}\right)$$

$$\frac{h_L}{V^2/g} = \frac{L}{D} \phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

$$\frac{h_L}{V^2/2g} = \frac{L}{D} \phi_1\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

$$\therefore\ f = \phi_1\left(\mathrm{Re}, \frac{\varepsilon}{D}\right) \quad \text{..} \quad (4.11)$$

Blasius had concluded that there were two types of pipe friction in turbulent flow. The first is the smooth pipes where the viscosity effects predominate so that the friction factor is dependent solely on the Reynolds number ($f=\phi$ (Re). He deduced the following expression for the friction in smooth pipes:

$$f = \frac{0.316}{\sqrt[4]{\mathrm{Re}}} \quad \text{..} \quad (4.12)$$

The second type was relevant to rough pipes where the viscosity & roughness effects influence the flow & the friction factor (f) is dependent both on the Reynolds number & a parameter of relative roughness ($^{\varepsilon}/_{D}$).**L.F Moody** prepared a chart for determining friction factor for rough pipes experimentally by plotting f versus Re curve for each value of $\frac{\varepsilon}{D}$. (**See Moody Chart**)

The moody chart, the various flows it represents, may be divided into four zones: the ***laminar flow zone***; a ***critical zone*** where values are uncertain because the flow might be either laminar or turbulent; a ***transition zone***. Where f is a function of both Reynolds number and relative pipe roughness; and a zone of ***complete turbulence*** (fully rough pipe flow), where the value of f is independent of Reynolds number and depends solely upon the relative roughness.

There is no sharp line of demarcation between the transition zone and the zone of complete turbulence. The dashed line that separates the two zone was suggested by R. J. S. Pigott; the equation of this line is **Re**= $3500 / (\varepsilon / D)$. On the other hand side of the equation of this line is corresponding to the curve and not the grid.

The Colebrook has developed the formula:

$$\frac{1}{\sqrt{f}} = -0.809 \ln\left(\frac{\in / D}{3.7} + \frac{2.523}{\mathrm{Re}\sqrt{f}}\right) \quad \text{.......................................} \quad (4.13)$$

A simplified form of this equation is provided with restriction placed on it:

$$f = \frac{1.325}{\left[\ell n\left(\frac{\in}{3.7D} + \frac{5.74}{R^{0.9}}\right)\right]^2} \Rightarrow \left\{\begin{matrix} 10^{-6} \le \frac{\in}{D} \le 10^{-12} \\ 5000 \le \mathrm{Re} \le 10^{8} \end{matrix}\right\} \quad \text{................................} \quad (4.14)$$

(For Rough pipes)

$\therefore$ Head loss in pipes is given by:

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$ (For all pipes rough, smooth, laminar, & turbulent)

7.2 Minor losses in the pipes

Loss due to the local disturbances of the flow conduits such as changes in cross-section; bend, elbows, valves, joints, etc are called ***minor losses***. In case of a very long pipe, these losses may be insignificant in comparison with the fluid friction in the length considered.

Whenever, the velocity of a flowing stream is altered either *in direction* or *in magnitude* in turbulent flow, ***eddy currents*** are set up and a loss of energy in excess of the pipe friction in that same length is created[2]. Head losses in decelerating (i.e., diverging) flow is much larger than that in accelerating (i.e., converging) flow.

The most common minor losses can be represented in one of two ways. It may be expressed as $kv^2/2g$, where the ***loss coefficient*** k must be determined for each case. Or it may be expressed as an equivalent length of a straight pipe, usually in terms of the number of pipe diameters, N. Since,

$$k\frac{V^2}{2g} = \frac{f(ND)}{D}\frac{V^2}{2g}$$, it follows that $k = Nf$.

i. ***Loss of head at entrance***

A poorly designed inlet to a pipe can cause an appreciable head loss. Referring to Figure 4.2 it may be seen that, a cross section with maximum velocity and minimum pressure at B. This minimum flow area is known as the ***vena contracta***.

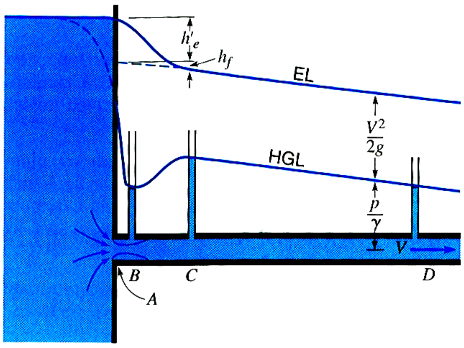

Figure 4.2 Condition at entrance

It is seen that the loss of energy at entrance is distributed along the length AC, a distance of several diameters. The increased turbulence and vortex motion in this portion of the pipe cause the friction loss to be much greater than in a corresponding length where the flow is normal, as it is shown by the drop of the total-energy line. Of this total loss, a small portion h_f would be due to the normal pipe friction (See figure 4.2). Hence, the difference between this and that total, or $h_e^{'}$ is the true value of the extra loss caused at entrance.

The loss of head at entrance may be expressed as

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[2] In laminar flow these losses are insignificant, because irregularities in the flow boundary create a minimal disturbance to the flow and separation is essentially nonexistent.

$$h_e' = k_e \frac{V^2}{2g} \quad \text{...} (4.15)$$

Where V is the mean velocity in the pipe, and k_e is the loss coefficient

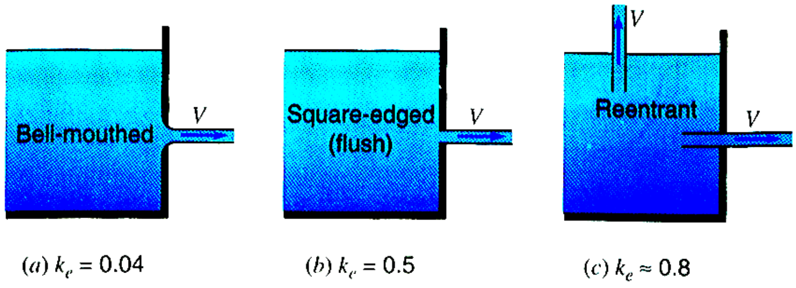

Figure 4.3 Entrance Loss Coefficients

ii. ***Loss of head at submerged discharges: (leave of pipe),*** (**h**$_{d'}$)

When the fluid with a velocity V is discharged from the end of a pipe in to a large reservoir, (v = 0), the entire kinetic energy of the coming flow is dissipated.

This may be shown by writing an energy equation between (a) and (b) in Figure 5.4 Taking the datum plane through (a) and recognizing that the pressure head of the fluid at (a) is *y*, its depth below the surface, $H_a = y + 0 + V^2/2g$ and $H_c = 0 + y + 0$. Therefore,

$$h_d' = H_a - H_c = \frac{V^2}{2g} \quad \text{....................................} (4.16)$$

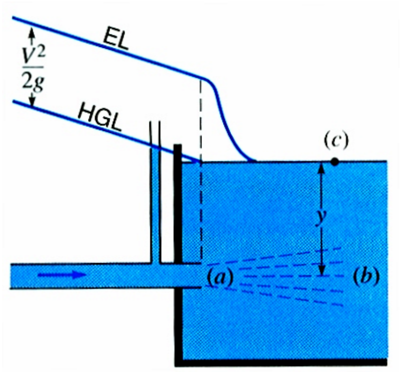

Figure 4.4 Submerged Discharge Loss

iii***. Loss due to contraction*** (h$_c$)

a) Sudden contraction

There is a marked drop in pressure due to increase in velocity and to the loss of energy in turbulence. The loss of head for sudden contraction may be represented by

$$h_c' = k_C \frac{V_2^2}{2g} \qquad (4.17)$$

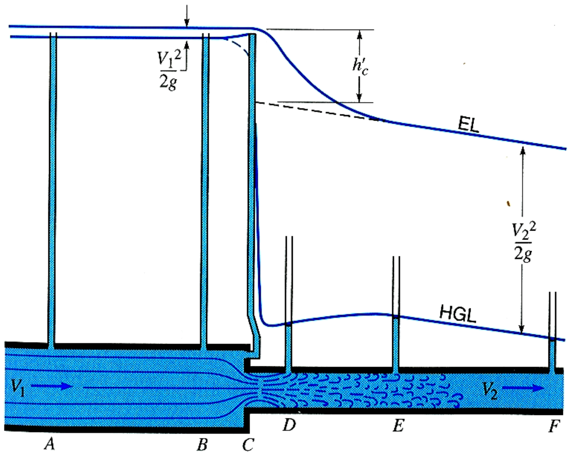

Figure 4.5 Loss due to sudden contraction

Table 4.1 Loss coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K_C	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

b) Gradual contraction

In order to reduces high losses, abrupt changes of cross section should be avoided. This is accomplished by changing from one diameter to the other by means of a smoothly curved transition or by employing the frustum of a cone. With a smoothly curved transition a loss coefficient k_c as small as 0.05 is possible. For conical reducers, a minimum k_c of about 0.10 is obtained, with a total cone angle of 20-40^0. Smaller or larger total cone angle results in higher values of k_c.

A nozzle at the end of a pipe line is a special case of gradual contraction. The head loss through a nozzle at the end of a pipeline is given by equation (5.17), where k_c is the nozzle loss coefficient whose value commonly ranges from 0.04 to 0.20 and v_j is the jet velocity. The head loss through a nozzle cannot be regarded as a minor loss because the jet velocity head is usually quite large.

iv. *Loss due to Expansion* (h_e)

a) Sudden Expansion

Both the figures in Figure 5.6, drawn to scale from test measurements for the same diameter ratios and the same velocities, and show that the loss due to sudden expansion is greater than the loss due to a corresponding contraction. This is so because of the inherent instability of flow in an expansion where the diverging paths of the flow tend to encourage the formation of eddies within the flow. Moreover, separation of the flow from the wall of the conduit induces pockets of eddying turbulence outside the flow region. In converging flow, there is a dampening effect on eddy formation, and the conversion from pressure energy to kinetic energy is quite efficient.

After the flow enters expanded pipe, there is excessive turbulence and formation of eddies which causes loss of energy. The loss due to sudden enlargement in a pipe line system can be calculated with the application of energy and momentum equations by neglecting the small shear force exerted on the walls of between sections 1 and 2 (***figure5.6***) for steady incompressible turbulent flow.

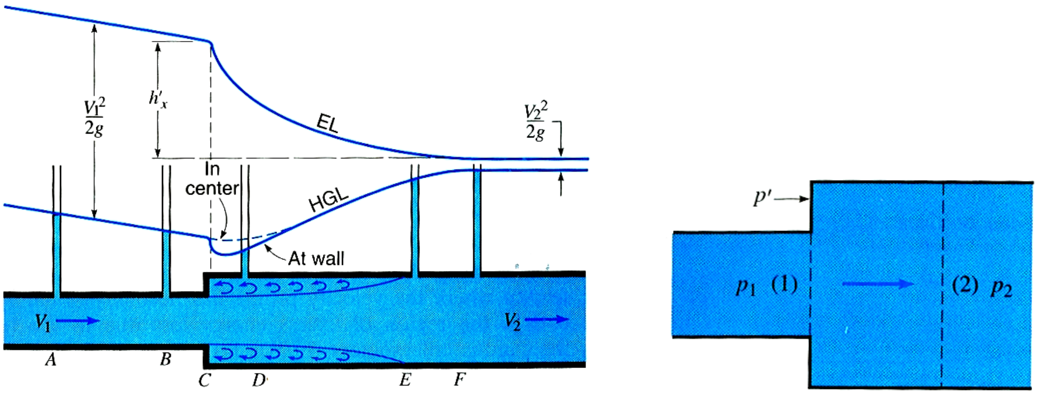

Figure 4.6 Loss due to sudden enlargement

Rate of momentum between section (1) & (2)

$$p_1A_2 - p_2A_2 = \frac{\gamma}{g}(A_2V_2^2 - A_1V_1^2)$$

Energy relation between section (1) & (2)

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_e$$

Solving for $\frac{p_1 - p_2}{\gamma}$ in each equation and equating the results

$$\frac{v_2^2 - v_2v_1}{g} = \frac{v_2^2 - v_1^2}{2g} + h_e$$

And noting that from continuity equation $A_1V_1 = A_2V_2$ and that

$A_1V^2_1 = (A_1V_1)\ V1 = (A_2V_2)V_1$

Substituting in the above equation

$$\therefore \quad h_x' = \frac{(V_1 - V_2)^2}{2g} = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 \frac{V_1^2}{2g} = \left(\frac{D_2^2}{D_1^2} - 1\right)^2 \frac{V_2^2}{2g} \quad \text{.................................. (4.18)}$$

b) Gradual Expansion

To minimize the loss accompanying a reduction in velocity a diffuser may be used. Diffuser is a curved outline, or it may be a frustum of cone. In figure (5.8) the head loss will be some function of the angle of divergence and also of the ratio of two areas, the length of the diffuser being determined by these two variables.

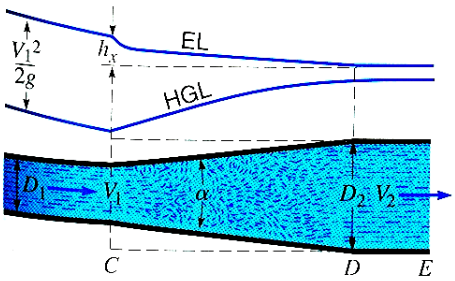

Figure 4.7 Loss due to gradual enlargement

In flow through a diffuser, the total loss may be considered as made up of two components. One is the ordinary pipe-friction loss, which may be represented by

$$h_{\ell e} = \int \frac{f}{D} \cdot \frac{V^2}{2g} dL \ .$$

In order to integrate, it is necessary to express the variables *f,* D, and V as functions of L. For our present purpose, it is sufficient, however, merely to note that the friction loss increases with the length of the cone. Hence, for given values of D_1 and D_2, the larger the angle of the cone, the less its length and the less the pipe friction.

The other is turbulence loss due to divergence. Turbulence loss increase with the degree of divergence, if the rate of divergence is great enough then there may be a separation at the wall and eddies flowing backward along the walls.

The total loss for gradual expansion pipe is the sum of these two losses, marked k'. It has been seen that the loss due to a sudden enlargement is very nearly represented by $(V_1 - V_2)^2 / 2g$. The loss due to a gradual enlargement is expressed as

$$h' = k' \frac{(V_1 - V_2)^2}{2g}$$ .... (4.19)

.

Where K' loss coefficient which is a function of cone angle α

Table 4.2 Loss coefficients for gradual expansion

K'	0.4	0.6	0.95	1.1	1.18	1.09	1.0	1.0
α	20^0	30^0	40^0	50^0	60^0	90^0	120^0	180

v. ***Loss in pipe fittings***

The loss of head in pipefittings is expressed as $h_f = k_f \frac{V^2}{2g}$ where v is the velocity in a pipe of the nominal size of the fitting. Typical values are given below.
Table 4.3 Values of "K_f" based on the type of fittings.

Table 4.3 values of k_f loss for pipe fittings

Fitting	**K**
Globe valve, wide open	10
Angle valve, wide open	5
Close –return bend	2.2
T-through side outlet	1.8
Short-radius elbow	0.9
Medium radius elbow	0.75
Long radius elbow	0.60
Gate valve, wide open	0.19
Half open	2.06
Pump foot value	5.60
Standard branch flow	1.80

vi. ***Losses in bend & Elbow***

In flow around a bend or elbow, because of centrifugal effects, there is an increase in pressure along the outer wall and a decrease in pressure along the inner wall. Most of the loss of head in a sharp bend may be eliminated by the use of a vaned elbow. The vane tends to impede the formation of the secondary flow that would otherwise occur.

The head loss produced by a bend or elbow is:

$$h_b = k_b . \frac{V^2}{2g}$$ ....(4.19i)

k_b - depends on the ratio of curvature r to pipe diameter D.

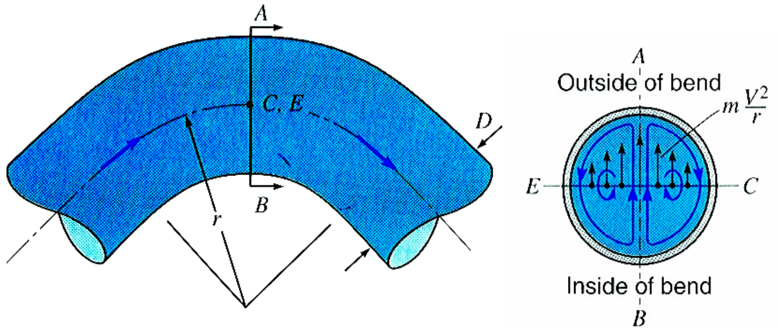

Figure 4.8 secondary flows in bend

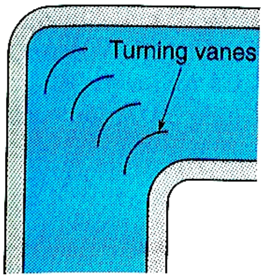

Figure 4.9 Vaned elbow

Solution of single-pipe flow problems

The fundamental fluid mechanics associated with frictional loss of energy in single pipe flow, caused by both the wall roughness of the pipes and by pipe fittings that disturb the flow (minor losses).

It is generally conceded that for pipes of length greater than 1000 diameters, the error incurred by neglecting minor losses is less than that inherent in selecting a value for the friction factor$(f, n, or\ C_{HW})$.

When minor losses are negligible, as they often are, pipe flow problems may be solved by the methods, which are available are **Hazen-Williams** equation, the Manning equation or the Darcy-Weisbach equation. The **Darcy-Weisbach** equation is to be preferred, since it will provide greater accuracy because its application utilizes the basic parameters that influence pipe friction, namely, Reynolds Number **Re** and the relative roughness$(\in/D)$. To get good results with the Hazen-Williams and **Manning's** equations, the user must selected proper values for $\boldsymbol{C_{HW}}$ and $\boldsymbol{n}$, respectively.

The total head losses between two points is the sum of the pipe friction loss plus the minor losses, or

$$h_L = h_{Lf} + \sum h' \qquad (4.20)$$

Where h_L = total head loss h_{Lf} = major head loss $\sum h'$ = total minor losses

In problem where *f* is given, equation (4.18) still has only one unknown, namely, h_L or V or Q or D. In most cases, this equation is explicit in the unknown, and so it is easy to solve. However, for sizing problems, the resulting equation in D is of the fifth degree, requiring trial and error or an equation solver.

The universal turbulent flow equation for use in an equation solver, including minor losses, eliminating h_{Lf} and equation (4.19) with the help of equation (4.7) and (4.11), and by replacing V by $4Q/(\pi D^2)$. Expressing minor losses $\sum h'$ in terms of $\sum kV^2/2g$,

$$\sqrt{\frac{L/D}{\frac{\pi^2 g D^4 h_L}{8Q^2}-\sum k}} = -2\log\left(\frac{\in/D}{3.7}+\frac{2.51\pi\nu}{4Q}\sqrt{\frac{LD}{\frac{\pi^2 g D^4 h_L}{8Q^2}-\sum k}}\right) \quad \text{.......................} \ (4.21)$$

An important reminder when using these equations is to use Reynolds equation to check the Reynolds number and confirm that the flow is turbulent. If **Re** < 2000 the flow is laminar and the problem must instead be solved with equation (4.16).

7.3 Pipeline system

7.3.1 Pipes in Series

When two pipes of different sizes or roughness are so connected that the fluid flows through one pipe& then through the other, they are said to be connected in series. A typical series pipe problem, in which head H may be wanted for a given discharge or the discharge wanted for a given H, is illustrated in figure 4.12 and the continuity equations establish the following two simple relations that must be satisfied.

$$Q = Q_1 = Q_2 = Q_3 = \dots = Q_n.$$
$$h_L = h_{L1} + h_{L2} + h_{L3} + \dots$$

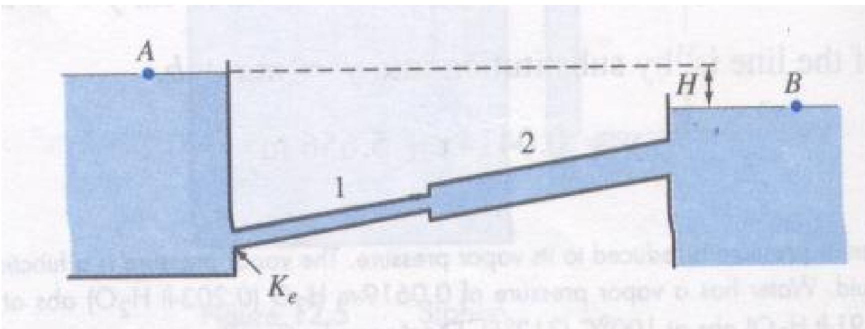

Figure 4.12 Pipes Connected in Series

Applying the energy equation from A to B, including all losses, gives:

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^{\ 2}}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + hi + h_{f1} + he' + h_{f2} + h_{d'}$$

$$H + 0 + 0 = 0 + 0 + 0 + ki\frac{V_1^{\ 2}}{2g} + f_1\frac{L_1}{D_1}\frac{V_1^{\ 2}}{2g} + \frac{(V_1 - V_2)^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^{\ 2}}{2g} + \frac{V_2^{\ 2}}{2g}$$

$$From\ \ continuty\ \ equation\ .: \qquad V_1 D_1^{\ 2} = V_2 D_2^{\ 2}$$

$$H = \frac{V_1^{\ 2}}{2g}\left\{ki + f_1\frac{L_1}{D_1} + \left[1 - \left(D_1/D_2\right)^2\right]^2 + f_2\frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4 + \left(\frac{D_1}{D_2}\right)^4\right\}$$

7.3.2 Equivalent pipes

Series pipes can be solved by the method of equivalent lengths. Two pipe systems are said to be equivalent when the same head loss produces the same discharge in both systems. From Equation (5.7)

$hf_1 = f_1\dfrac{L_1}{D_1^{5}}\dfrac{8Q_1^{\ 2}}{\Pi^2 g}$ For a second pipe $hf_2 = \dfrac{f_2 L_2}{D_2^{5}}\dfrac{8Q_2^{\ 2}}{\Pi^2 g}$

For two pipes to be equivalent,

$h_{f1} = h_{f2}, \quad Q_1 = Q_2$

$$\therefore \frac{f_1 L_1}{D_1^{\ 5}} = \frac{f_2 L_2}{D_2^{\ 5}}$$

$$L_2 = L_1\frac{f_1}{f_2}\left(\frac{D_2}{D_1}\right)^5 \text{ .... (4.24)}$$

7.3.4 Pipes in Parallel

A combination of two or more pipes connected as in **figure** 4.13 so that the flow is divided among the pipes and then is joined again, is a parallel – pipe system. In series pipe system the same fluid flows through all the pipes and the head losses are cumulative, but in parallel pipe – system the head losses are the same in each of the lines the discharge are cumulative.

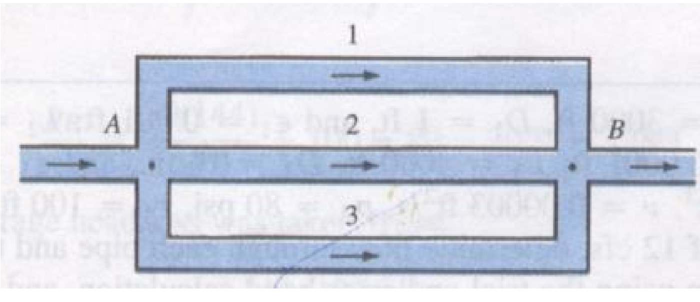

Fig **4.13 Parallel Pipes system**

$h_{f1} = h_{f2} = h_{f3} = \frac{P_A}{\gamma} + Z_A - \left(\frac{P_B}{\gamma} + Z_B\right)$

$Q = Q_1 + Q_2 + Q_3$

Two types of problems occur:

1) If the head loss b/n A & B is given, Q is determined.
2) If the total flow Q is given, then the head loss & distribution of flow are determined.

Size of pipes, properties, and roughness are assumed to be known. Since this type of problem is more complex, as neither the head loss nor the discharge for any one pipe is known. The procedure is:

1) Assume discharge Q'_1 through pipe 1,
2) Solve for $h'f_1$, using assumed discharge,
3) Using $h'f_1$, find Q'_2 & Q'_3
4) With the three discharges for a common head loss, now assume that the given Q is split up among the pipes in the same proportion as Q'_1, Q'_2 & Q'_3, Thus,

$$Q_1 = \frac{Q_1'}{\sum Q'}Q, \qquad Q_2 = \frac{Q_2'}{\sum Q'}Q, \qquad Q_3 = \frac{Q_3'}{\sum Q'}Q$$

5) Check the correctness of these discharges by computing hf_1, hf_2, & hf_3 for the computed Q_1, Q_2 & Q_3

→ $Q - Q_1 - Q_2 - Q_3 = 0$

7.3.5 Branching pipes

Let us consider three pipes connected to three reservoirs as in fig. below & connected together or branching at the common junction point J. We shall assume that all the pipes are sufficiently long that minor losses & velocity heads may be neglected. The continuity & energy eqn. require that the flow entering the junction equal the flow leaving it& that the pressure head at J (with open piezometer tube water at elevation P) be common to all pipes.

There being no pumps, the elevation of p must lie b/n the surfaces of reservoirs A& C. If p is level with the surface of reservoir B then water must flow in to B & $Q_1 = Q_2 + Q_3$

If P is below the surface of reservoir B then the flow must be out of B & $Q_1 + Q_2 = Q_3$

So for the situation of the following fig, we have the following governing conditions:

1) $Q_1 = Q_2 + Q_3$
2) Elevation of p is common to all.

a. Length, diameter, &friction factors are required.
b. The flow is steady & minor lorses reglected
c. Three basic cgns to salue there problems are:-
    i. Continuity sgn
    ii. Bmoulli's egn
    iii. Dorcy- weis bach egn

- Totasl rate of in flow at junction = total rate of out flow (continuity egn)

❖ Pipe 1 | Pipe 2 | Pipe 3

D_1, L_1, V_1, Q1 h+1 | D_2, L_2, V_2, Q_2, h+2 | D_3, L_3, V_3, Q_3, h+3
Elevation , Z1 , Riser ,A | Z2, Riser, B | Z3, Riserv. C

Junction of elevation

Zj, pressure head pj/r = total herd at junction = $(Zj + pj/r)$

❖ Applying Bernoulli's egn b/n the jun citron point & each of reservoirs

$$=> \begin{cases} Z_1 = (pj/r + Zj) + hf_1 \text{ - - - - - - - - - - -------------} & (*) \quad (1) \\ Z2+h+2 = (pj/r + Zj) \text{ --------------------------} & (**) \quad (2) \\ Z3 = h+3 = (pj/r + Zj) \text{ --------------------------} & (***) \quad (3) \end{cases}$$

=> If the head of reseruoir Ais grater than head at junction the flow is in to the junction from A & out of the junction to B&C

=> Q1 = Q2 + Q3 -------------------------- * (4)

$$\frac{\bar{u}}{4} D_1^2 \; V_1 = \frac{\bar{u}}{4} D_2^2 \; V_2 + \frac{\bar{u}}{4} D_3^2 \; V_3 \text{ ------------} \quad (5)$$

$$=> D_1^2 \; V_1 = D_2^2 \; V2 \; V_2 + D_3^2 * V_3 \text{ -------------} \quad (6)$$

❖ Then one three types of problem fouling of branching pies :-

Case 1: Given all pipes data (L, D, E, Z_1 & Z_2 Q_1 or Q_2, find Z_3 ?

=> soin : first hf_1 can be calculated directly (h+1 = $f_1 \frac{L_1}{D_1} \; v_2^1 / ^2 g$)

Then (pj/r + Zj) pizomutnc hard at junction can be determine

- ⇨ From egn (2_o) h +2 & Q_2 can be determined
- ⇨ Q3 can be detrained from egn (4) continuity eng
- ⇨ Then from eng (3) h+3 and finally Z3 (can be determined)

Case 2: Given au pipe data, the surface elevation of two reservoirs (A& C) and the flow to or fro the second, find Z_3 and Q_1 Q_3?

- ⇨ From egn (1) & iii) (h+1 + h+3) = (Z_1- Z_3) (h+1 + h=3) is known & also (Q_1-Q_3) or (Q_3 – Q_1) is known.
- ⇨ Assume frail values of h+1 & h+3 & from these compute the discharge Q_1+Q_3 & compare with (Q_1-Q_3)
- ⇨ Repeat the procedure until the two uues are equal.
- ⇨ From they, pizometric head at junction can be determined

- ⇨ From h +2 & (pj/r Zj) → Z_2 can be determined.

Case:3 cloven au pipe lengths & diameters & the elevation of all the three reservoirs , find Q_1 Q_2, Q_3,

- → In this case the direction of the flow is not known clearly.
- → Assam the elevation of B (z2) is equal to the piezometric head (Z_p) & (je an flow in pipe 2)
- → From Zp the head losses h+1 & h+3 determined, and tlen Q_1 & Q_3 can be obtained
- → If $Q_1 > Q_3$, then Zp must be increased to satisfy continuity egn at J, causing water to flow into reservoir B, and are will hale $Q_1= Q_2+Q_3$
- → If Q1<Q3, then Zp mast be lowered, causing water to flow out of reservoir B, & we will have $Q_1+ Q_2 = Q_3$