CHAPTER SIX Open channel Flow

6.1 Introduction

A channel may be defined as a passage which water flows under atmospheric pressure. As such in channels the flow of water takes place with a free surface which is subjected to atmospheric pressure. The figure 6.1 shows a comparison b/n the flow through a pipe and a channel. In the case of flow through a pipe there is no free surface as in the case of flow through a channel. This is so because the water flowing in a pipe is indicated by the level of water column in a piezometric tube installed on the pipe at the section. The water levels on the piezometric tubes installed at different sections of a pipe indicate the hydraulic grade line. On the other hand in the case of channel flow the water surface itself

is the hydraulic grade line. However, the total energy lines in both the cases lie at a distance of $\frac{V^2}{2g}$

above the hydraulics grade line at every section, where V is the mean velocity of flow at the section.





The channels may be classified according to different considerations as indicated below. On the basis of the cross-sectional form of the channel it may be classified as natural channel or artificial channel. A natural channel is one which has irregular sections of varying shapes, which is developed in a natural way.

The examples of natural channels are rivers, streams etc. On the other hand an artificial channel is the one which is built artificially for carrying water for various purposes. Obviously artificial channels have their cross-sections with regular geometrical shapes, which usually remain same throughout the length of the channel. The artificial channels may be further classified according to the shape of the cross-section as, rectangular channel, trapezoidal channel, triangular channel, parabolic channel, and circular channel.

The channels may also be classified as open channels and closed channels. The channels without any cover at the top are known as open channels. On the other hand the channels having cover at the top are known as closed channels.

A channel having the same shape of various sections along its length and laid on a constant bottom slope is known as prismatic channel, otherwise the channel is non-prismatic.

6.2 Types of flow in open channels

The flow in channels can be classified into the following types depending upon the change in the depth of flow with respect to space and time.

a. Steady flow and unsteady flow

Flow in a channel is said to be steady if the flow characteristics at any point do not change with time that is $\frac{\partial v}{\partial t} = 0$, $\frac{\partial y}{\partial t} = 0$, etc. However, in the case of prismatic channels the conditions of steady flow may be obtained if only the depth of flow does not change with time, which is $\frac{\partial y}{\partial t} = 0$. On the other

hand if any of the flow characteristics changes with time the flow is unsteady.

b. Uniform and non-uniform (or varied) flow

Flow in a channel is said to be uniform if the depth, slope, cross-section and velocity remain constant over a given length of the channel. Obviously, a uniform flow can occur only in a prismatic channel in which the flow will be uniform if only the depth of flow y is same at every section of the channel,

which is
$$\frac{\partial y}{\partial s} = 0$$
.

Flow in channels is termed as non-uniform or varied if the depth of flow y, changes from section to section, along the length of the channel, that is $\frac{\partial y}{\partial s}$ is not equal to zero. Varied flow may be further classified as rapidly varied flow (RVF) and gradually varied flow (GVF). If the depth of flow changes abruptly over a comparatively short distance, the flow is characterized as a rapidly varied flow.

c. Laminar flow and turbulent flow

Which depend on the Reynolds number.

d. Sub-critical flow, critical flow and super-critical flow

Based on the value of the Froude number, which is defined as

$$F_r = \frac{V}{\sqrt{gD}}$$
 Where:- V is the mean velocity of flow; g is acceleration due to gravity

D is hydraulic depth of channel section = $\frac{A}{T}$

A is wetted area and T top width

The flow may be sub-critical, critical and supercritical flow. When $F_r = 1$, that is $V = \sqrt{gD}$, the channel flow is said to be in a critical state. If $F_r < 1$, or $V < \sqrt{gD}$, the flow is described as sub-critical or tranquil or streaming. If $F_r > \sqrt{gD}$, the flow is said to be supercritical or rapid or shooting or torrential.

Geometrical properties of open channel flow

Various geometric properties of natural and artificial channels need to be determined for hydraulic purposes. In the case of artificial channels, these may all be expressed algebraically in terms of the depth (y).

The commonly used geometric properties are defined as follows:

- Depth (y): the vertical distance of the lowest point of a channel section from the free surface.
- Stage (h):- the vertical distance of the free surface from an arbitrary datum.
- ♦ Wetted Area (A):- the cross sectional area of flow normal to the flow direction.
- Wetted perimeter (P):- the length of the wetted surface measured normal to the direction of flow.
- \bullet Top width (T):- the width of the channel section at the free surface.
- Hydraulic Radius (R):- the ratio of wetted area to wetted perimeter. R=A/P
- Hydraulic mean depth (D_m):- the ratio of flow area to surface width. $D_m=A/T$

6.2.1 UNIFORM FLOW

It is the flow in which the cross-section through which flow occurs is constant along the channel, and also is the velocity. Thus, $Y_1=Y_2$ and $V_1=V_2$ and the channel bed, water surface, and energy line are

parallel to one another. Also, $S_w = S_0 = S = \frac{-\Delta Z}{\Delta X} = \tan\theta$, while $S = \frac{h_f}{L} = \sin\theta$,

Where θ is the angle the channel makes with the horizontal.



 τ_0 = shear stress

L = length of the channel b/n section-1 and section-2.

• Uniform flow is the result of exact balance between the gravity and frictional force (average boundary shear stress, $\overline{\tau}_{o}$ acting over the area LP) thus:

 $W\sin\theta = \overline{\tau_{\rho}}$.P.L....(1)

 $\gamma A L \sin \theta = \overline{\tau}_{a} P L$

but $\sin \theta = hf/L = S$, solving for τ_o ,

Where γ - unit weight of the water

The shear stress is assumed proportional to the square of the mean velocity,

or $\tau_0 = k_V^2$(3) Therefore, $kv^2 = \gamma RS$

 $V^2 = \frac{\gamma}{k} RS$, Let $\frac{\gamma}{k} = C^2$ -constant (b/c γ &k- are constant) $V = C\sqrt{RS}$(4) This is the Chezy –formula C= chazy coefficient (chezys resistance factor) V= Average velocity of flow Manning Formula

The best as well as most widely used formula for uniformly for uniform flow. n- is the roughness coefficient.

A relation between the Chezy's C and Manning's n may be obtained by comparing eqn (4) & (5)

The value of n ranges from 0.009 (for smooth straight surfaces) to 0.22 (for very dense flood plain forests).

Hydraulically Efficient Channel Cross-Section 6.3.

A channel section is said to be efficient if it gives the maximum discharge for the given shape, area & roughness.

The velocity in an open channel is:

V = f(R,S)	(a)
Q = A * V = A.f(R,S)	(b)

Equation (b) indicates that for the given area of cross-section & slope the discharge Q will be maximum, when R – is maximum.

 \Rightarrow Since, R= A/P, R will be maximum when p- is minimum for a given area.

We can conclude that for most efficient and economical channel section the wetted perimeter should be minimum & also frictional resistance, τ_a is minimum.

For example, a rectangular channel of depth Y and width, B

A= BY.....(i) P= B+2Y.....(ii)

From eqn. (i), B = A/YSubstituting in (ii) P=A/Y+2Y.....(iii) For maximum Q, p- is minimum.

$$\frac{dp}{dY} = 0 \Rightarrow \frac{d}{dY}(A/Y + 2Y) = 0$$

$$\therefore \Rightarrow -\frac{A}{Y^2} + 2 = 0$$

$$\Rightarrow A = 2Y^2, = B * Y,$$

$$So, B = 2Y \quad (orY = B/2)$$

Thus the rectangular channel is most efficient & economical when the depth of water is one half of the width of the channel and the discharge flow will be maximum.

Accordingly, the most efficient channel shape is the semi-circle .The usual shape for new channels & canals is the rectangular or trapezoidal, such that the inscribed semi-circle is tangential to the bed &sides.

6.4. Specific Energy

For any cross-section, shape, the specific energy (E) at a particular section is defined as the energy head to the channel bed as datum, thus,

 $E=Y+\alpha V^2/2g....(1)$

(α - is kinetic energy correction factor $\rightarrow \approx 1.0$).



Fig.6.3

For a rectangular channel, the value of flow per unit width is Q/B=q, and average velocity

$$V = \frac{Q}{A} = \frac{qB}{BY} = \frac{q}{Y}$$

Therefore eqn (A) becomes:
$$E = y + \frac{\left(\frac{q}{y}\right)^2}{2g} = Y + \frac{q^2}{2gy^2}$$
....(2)
(E-Y) $Y^2 = \frac{q^2}{2g}$ (For the case of constant q).....(3)

A plot of E vs. Y is a hyperbola like with asymptotes (E-y) =0 i.e., E=Y and Y=0. Such a curve is known as specific energy diagram.



For a particular q, we see there are two possible values of Y for a given value of E. These are known as Alternative depths (for e.g. $Y_1 \& Y_2$ on fig. above)

The two alternate depths represent two totally different flow regimes: slow & deep on the upper limb of the curve (*sub-critical flow*) & fast & shallow on the lower limb of the curve, (*Super critical flow*)

6.4.1 Critical depth

From fig.1.4 above, at point C for a given q the value of E is a minimum and the flow at this point referred to as critical flow. The depth of flow at that point is the critical depth Y_c & the velocity is the critical velocity V_c .

For example, a relation for critical depth in a wide rectangular channel can be found by differentiating E of eqn. (2) with respect to Y to find the value of Y for which E is a minimum.

$$\frac{dE}{dY} = 1 - \frac{q^2}{gy^3}$$
.....(4)
and when E is a minimum, Y=Y_c and $\frac{dE}{dY} = 0$, so that
 $0 = 1 - \frac{q^2}{gy_c^3} \Rightarrow q^2 = gy_c^3$(5)
Substituting $q = vy = V_c * Y_c$, gives
 $V_c^2 = gy_c$
 $\Rightarrow V_c = \sqrt{gy_c} = \frac{q}{y_c}$(6)

It may be expressed as:

$$y_c = \frac{V_c^2}{g} = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$
....(7)

From eqn. (7)
$$\frac{V_c^2}{2g} = \frac{y_c}{2}$$
, hence,
 $Ec = E_{\min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{1}{2}y_c = \frac{3}{2}y_c$(8)
and $y_c = \frac{2}{3}E_{\min}$(9)
From eqn. (7): $q_{\max} = \sqrt{gy_c^3}$(10)

For non- rectangular cross-section the specific energy eqn.

To find the critical depth,

From fig 1.3 (b) $dA = dy^{*}T$ (at yc, T = Tc)

Therefore the above equation becomes:

$$\frac{Q_{\max}^2 T_c}{g A_c^3} = 1....(13)$$

The critical depth must satisfy this equation

From eqn. (13) $Q^2 = \frac{gA_c^3}{T_c}$ and substituting in eqn. (11) then

eqn. (13) can be solved by trial & error for irregular section by plotting

$$f(y) = \frac{Q^2T}{gA^3}$$
 and critical depth occurs for the value of y which makes $f(Y) = 1$

Sub-critical, critical and super-critical flow

If specific energy curve for Q- constant is redrawn along side a second curve of depth against discharge for constant E, will show the variation of discharge with depth.



a) For a given constant discharge

- i) The specific energy curve has a minimum value Ec at point c with a corresponding depth $Y_{\rm c}$ known as critical depth
- ii) For any other value of E there are two possible depths of flow known as alternate depths, one of which is termed substantial $(Y>Y_c)$ and the other supercritical $(Y<Y_c)$.

b) For a given constant specific energy

- i) The depth discharge curve shows that discharge is a maximum at the critical depth
- ii) For all other discharges there are two possible depths of flow (sub- & super critical) for any particular values of E.

From eqn. (13) above if we substitute

$$Q = AV \text{ (continuity eqn.), we get}$$

$$\frac{Q^2 T}{gA^3} = 1$$

$$\frac{A^2 V^2 T}{gA^3} = 1 \Leftrightarrow \frac{V^2}{g} \frac{T}{A} = 1$$
But $\frac{A}{T} = D (hydraulicdepth), ther[D = Y \text{ for rec tan gular sec tion}]$

$$\frac{V^2}{gy} = 1 \Rightarrow V = \sqrt{gy}.....(*)$$

$$\frac{V}{gy} = 1 = Froude number at critical state.$$

$$F = \frac{V}{\sqrt{gy}}.....(N)$$
Thus, i) F= 1 \Rightarrow critical flow
ii) F < 1 \Rightarrow sub critical flow

iii) F>1 \Rightarrow super critical flow

6.5. Hydraulic Jump

By far the most important of the local non-uniform flow phenomena is that which occurs when supercritical flow has its velocity reduced to sub critical. There is sudden rise in water level at the point where hydraulic jump occurs (Rapidly varied flow). This is an excellent example of the jump serving a useful purpose, for it dissipates much of the destructive energy of the high –velocity water, there by reducing downstream erosion. The turbulence with in hydraulic jumps has also been found to be very useful & effective for mixing fluids, & jumps have been used for this purpose in water treatment plant & sewage treatment plants.



Fig 6.5 hydraulic jump on horizontal bed following over a spillway

Purposes of hydraulic jump:-

- i) To increase the water level on the d/s of the hydraulic structures
- ii) To reduce the net up lift force by increasing the downward force due to the increased depth of water,
- iii) To increase the discharge from a sluice gate by increasing the effective head causing flow,
- iv) For aeration of drinking water
- v) For removing air pockets in a pipe line

> Analysis of hydraulic jump

Assumptions

- 1) The length of the hydraulic jump is small, consequently, the loss of head due to friction is negligible,
- 2) The channel is horizontal as it has a very small longitudinal slope. The weight component in the direction of flow is negligible.
- 3) The portion of channel in which the hydraulic jump occurs is taken as a control volume & it is assumed the just before & after the control volume, the flow is uniform & pressure distribution is hydrostatic.

Let us consider a small reach of a channel in which the hydraulic jump occurs. The momentum of water passing through section (1) per unit time is given as:

Momentum at section (2) per unit time is:

Rate of change of momentum b/n section 1 & 2

The net force in the direction of flow = F1-F2(iv)

$$F_1 = \gamma A_1 \overline{Y_1}, \qquad \qquad F_2 = \gamma A_2 \overline{Y_2}$$

 $\overline{Y}_1 \& \overline{Y}_2$ are the center of pressure at section (1) & (2)

Therefore F_1 - $F_2 = \Delta M = \rho Q (V_2 - V_1)$

$$\gamma A_1 \overline{Y}_1 - \gamma A_2 \overline{Y}_2 = \frac{\gamma Q}{g} (V_2 - V_1) \dots (v)$$

From continuity eqn. Q = A*V, $\square > V = Q/A$, so

$$\gamma A_1 \overline{Y}_1 - \gamma A_2 \overline{Y}_2 = \frac{\gamma Q}{g} \left(\frac{Q}{A_2} - \frac{Q}{A_1} \right)$$
$$A_1 \overline{Y}_1 - A_2 \overline{Y}_2 = \frac{Q^2}{g} \left(\frac{1}{A_2} - \frac{1}{A_1} \right)....(iv)$$

Rearranging this eqn.:

$$\frac{\left[\frac{Q^2}{gA_1} + A_1\overline{Y}_1\right]}{M_1} = \frac{\left[\frac{Q^2}{gA_2} + A_2\overline{Y}_2\right]}{M_2} = \text{Constant.} \dots (\text{vii})$$

 M_1 and M_2 are the specific forces at section (1) & (2) indicates that these forces are equal before & after the jump.

 Y_1 = initial depth Y_2 = sequent depth

Hydraulic jump in a rectangular channel

$$A_1=By_1$$
 the section has uniform width (B)
 $A_2=By_2$

 $\overline{Y}_1 = \frac{Y_1}{2}, \overline{Y}_2 = \frac{Y_2}{2}$

Now from eqn. (Vii) above:

$$\frac{Q^2}{gBy_1} + By_1\left(\frac{y_1}{2}\right) = \frac{Q}{gBy_2} + By_2 * \left(\frac{y_2}{2}\right)$$
$$\frac{Q^2}{gBy_1} + \frac{By_1^2}{2} = \frac{Q^2}{Bgy_2} + \frac{By_2^2}{2} \dots (viii)$$

Flow per unit width of $q = Q/B \implies Q = qB$, then eqn. (viii) becomes

$$\frac{q^2B^2}{Bgy_1} + \frac{By_1^2}{2} = \frac{q^2B^2}{Bgy_2} + \frac{By_2^2}{2}$$

$$\frac{2q^2}{g} = y_1 y_2 \quad \frac{\left(y_2^2 - y_1^2\right)}{\left(y_2 - y_1\right)}$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)...(x)$$

$$y_2 y_1^2 + y_1 y_2^2 - \frac{2q^2}{g} = 0...(xi)$$

$$y_2 y_1^2 + y_1 y_2^2 - \frac{2q}{g} = 0....(xi)$$

This is quadratic eqn. & the solution is given as

$$y_{1} = \frac{-y_{2}}{2} + \sqrt{\left(\frac{y_{2}}{2}\right)^{2} + \frac{2q^{2}}{gy_{2}}}....(xii)(a)$$
$$y_{2} = \frac{-y_{1}}{2} + \sqrt{\left(\frac{y_{1}}{2}\right)^{2} + \frac{2q^{2}}{gy_{2}}}....(b)$$

$$y_{1} = \frac{y_{2}}{2} \left(-1 + \sqrt{1 + \frac{8q^{2}}{gy_{2}^{3}}}\right)....(c)$$

$$y_{2} = \frac{y_{1}}{2} \left(-1 + \sqrt{1 + \frac{8q^{2}}{gy_{1}^{3}}}\right)....(xii)(d)$$

The ratio of conjugate depths;

$$\frac{y_1}{y_2} = \frac{1}{2}(-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}}....(xii)(e)$$

$$\frac{y_2}{y_1} = \frac{1}{2}(-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}....(f)$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} =, F_2 = \frac{V_2}{\sqrt{gy_2}} \frac{q/y_2}{gy_2} = \frac{q}{\sqrt{gy_2^3}}$$

Therefore $\frac{y_1}{y_2} = \frac{1}{2} \left(-1 + \sqrt{1 + 8F_2^2} \right)$(g) $\frac{y_2}{y_1} = \frac{1}{2} \left(-1 + \sqrt{1 + 8F_1^2} \right)$(h)

> Energy dissipation in a Hydraulic Jump

The head loss $h_{l.f}$ caused by the jump is the drop in energy from section (1) to (2) or: $h_{lf}=\Delta E = E_1 - E_2$

$$=\frac{q^2}{2g}\left(\frac{y_2^2-y_1^2}{y_1^2y_2^2}\right)-(y_2-y_1)....(c)$$

From eqn. (x) substituting: $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$ in to this eqn. & by rearranging:

$$h_{lf} = \Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2}.$$
(2)

Therefore power lost = $\gamma Q h_{lf}$ (kw).....(3)

> Types of Hydraulic jump

Hydraulic jumps are classified according to the upstream Froude number and depth ratio.

F ₁	Y_{2}/y_{1}	Classification
<1	1	Jump impossible
1-1.7	1-2	Undular jump (standing wave)
1.7-2.5	2-3.1	Weak jump
2.5-4.5	3.1-5.9	Oscillating jump
4.5-9.0	5.9-12	Steady jump (45-70% energy loss)
>9.0	>12	Strong or chopping jump (=85% energy loss)

Examples

1 .A rectangular channel is to be dug in the rocky portion of a soil. Find its most economical cross-section if its to convey 12 m^3 /s of water with an average velocity of 3 m/s. Take chezy constant C=50

Given

Q=12 m³/s V=3 m/s C=50

Solution

The geometric relations for optimum discharge through a rectangular channel are

$$B = 2Y$$
 and $R = \frac{Y}{2}$

Then area $A = B \times Y = 2Y^2$

When B,Y and R are base width, depth of flow and hydraulic radius respectively

Now $Q = A \times V$ or $12 = 2Y^2 \times 3$

From this equation solve for depth of flow

$$Y = 1.414m$$

Therefore base width of flow $B = 2Y = 2 \times 1.414 = 2.828m$

Hydraulic radius, $R = \frac{Y}{2} = \frac{1.414}{2} = 0.707m$

Also $V = C\sqrt{RS}$ chezy formula

$$S = \frac{V^2}{RC^2} = \frac{3^2}{0.707 \times 50^2} = \frac{1}{196}$$

Hence $B = 2.828, Y = 1.414, S = \frac{1}{196}$