4

SOIL MOISTURE, PERMEABILITY SEEPAGE AND STRESSES

4.1 SOIL MOISTURE

Rainwater percolating into the ground continues to travel down wards under the influence of gravity till it meets an impervious stratum above which it collects in the pores of the soil and forms the ground water. Ground water is, therefore, the continuous body of sub-surface water that fills the voids and fissures and is free to move under the influence of gravity. The upper surface of this water is called the water table or phreatic surface. The water table can also be defined as the surface at which the water is at atmospheric pressure. (i.e., it is the level to which the water would rise in a pit or a hole dug in the ground.)

If the water contained in the soil were subjected to no force other than gravity, the soil above the water table would be completely dry. It is well known, however, that soils are almost fully saturated for some height above the water table and partially saturated for some further height. The water that occupies the voids of the soil located above the water tables constitutes soil moisture.

The soil water is broadly classified into two categories: (1) Free water or gravitational water, and (2) Held water. Free water moves in the pores of the soil under the influence of gravity. A soil containing free water may be considered to be saturated under ordinary conditions. The held water is retained in the pores of the soil, and cannot move under the influence of gravitational force. The held water may be sub-divided into *structural water*, *adsorbed water* and *capillary water*.

The structural water is chemically combined water in the crystal structure of the mineral of the soil. This water cannot be removed without breaking the structure of the mineral. A temperature of more than $300^{o}c$ is required for removing the structural water. In soil engineering, the structural water is considered as integral part of the soil solid.

The water held by electrochemical forces existing on the surface is known as adsorbed water. The water held in the interstices of soils due to capillary forces is called capillary water.

4.1.1 Adsorbed Water

In nature every soil particle is surrounded by water. Since the centers of positive and negative charges of water molecules do not coincide, the molecules behave like dipoles. The negative charge on the surface of the soil particle, therefore, attracts the positive (hydrogen) end of water molecules. The water molecules are arranged in a definite pattern in the immediate vicinity of the boundary between solid and water. More than one layer of water molecules sticks on the surface with considerable forces and this attractive force decreases with the increase in the distance of water molecules from the surface. The water located within the zone of influences is known as the adsorbed water as shown in Fig.4.1. The amount of water held by adsorption depends on specific surfaces, which in turn depends on particle size, shape, and gradation. A relatively fine, well-graded material will normally have much greater adsorption power

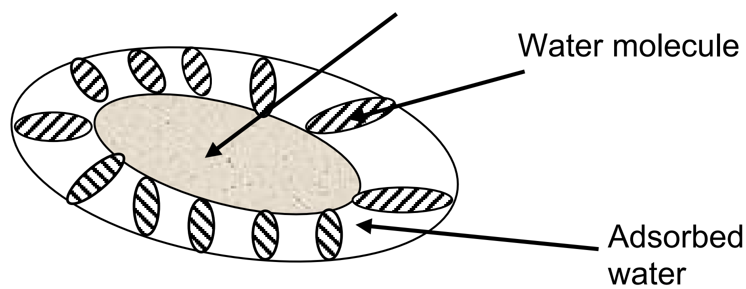


Fig. 4.1 Adsorbed water layer surrounding a soil particle.

Adsorbed water reduces the area available for free flow. In fine-grained soils the pore passages may be small and the thickness of immobilized water films constitutes a significant part of the pore diameter. To drive off the adsorbed water, the soil particle must be heated to more than 200^0c, which would indicate that the bond between the water molecules and the surface is considerably greater than that between normal water molecules.

4.1.2 Capillary Water

If the lower part of a mass of dry soil comes into contact with water, the water rises in the voids to a certain height above the free-water surface. The upward flow into the voids of the soil is attributed to the surface tension of the water. The height to which water rises above the water table against the force of gravity is called capillary rise. The water associated with capillary rise is capillary water.

Rise of Water in Capillary Tubes

The phenomenon of capillary rise can be demonstrated by immersing the lower end of a very small diameter glass tube into water. Such a tube is known as a capillary tube. As soon as the lower end of the tube comes into contact with the water, the attraction between the glass and the water molecules combined with the surface tension of the water pulls the water up into the tube to a height $\mathbf{h_C}$ above the water level as shown in Fig. 4.2(a). The height $\mathbf{h_C}$ is known as the height of capillary rise. The upper surface of the water assumes the shape of a cup, called the meniscus that joins the walls of the tube at an angle α known as the contact angle. The value of α depends on the material that constitutes the wall and on the type of impurities that cover it.

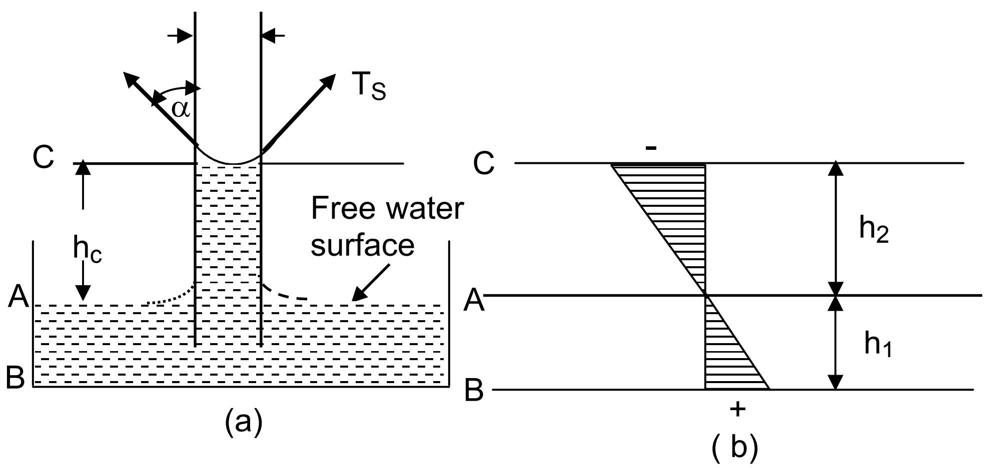


Fig 4.2 (a) Rise of water in capillary tubes; (b) State of stress of water in capillary tube.

Let the surface tension per unit perimeter $= T_s$

Force acting upward $= 2\pi r T_s \cos\alpha$

Force acting downward $= h_c \gamma_\omega \pi r^2$

For equilibrium condition

$$h_c \gamma_\omega r^2 = 2\pi r T_s \cos\alpha$$

$$h_c = \frac{2T_S}{r\gamma_\omega}\cos\alpha$$

For chemically clean water and tube, α=0

$$h_c = \frac{2T_s}{r\gamma_\omega} \quad \ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

From the above equation it is evident that, the height of capillary rise increases as the diameter of tube decreases. It can be deduced from this that capillary is more pronounced in fine grained soils than in coarse-grained soil.

Pressure variation above and below water table:-

At level B-B the water pressure

$$P_w = \gamma_\omega h_1$$

Total pressure at level B-B = $\gamma_\omega h_1 + P_{atm}$.

Suction pressure, negative pressure at level C.C = - $h_2\gamma_\omega$

Total pressure at level C-C= $P_{atm} - h_2\gamma_\omega$

4.1.3 Gravitational Water

This differs from adsorbed and capillary water in that it is completely free to move through or drain from soil under the influence of gravity. That is, the flow of gravitational water in soil is caused by the action of gravity, which tends to pull the water down ward to a lower elevation.

In many respects it is similar to free flow of water in an open channel or conduit. The gravitational pull acts to over come resistance to movement or flow of the water. In soil such resistance is due to viscous drag along the sides walls of the pore spaces.

In the study of gravitational flow in soils we are primarily interested in the laminar type of flow. Laminar flow is said to exist when all particles of water move in parallel paths and lines of flows are not inter-twined as the water moves forward. The quantity of water flowing past a fixed point in a given time is equal to the cross-sectional area of the water multiplied by the average velocity of flow. This relationship may be expressed by the formula

$$Q = AV \quad \ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

In which Q = Volume of flow per unit time

A = Cross- sectional area of flowing water

V = Velocity of flow

According to hydraulics principle for steady state laminar flow condition, the velocity is proportional to the hydraulic gradient,

$$V \alpha i$$

$$V = k\, i \qquad \ldots\ldots\ldots \qquad (4.3)$$

where k = Coefficient of proportionality

i = Hydraulic gradient

Hydraulic Gradient, The driving force which causes water to flow may be represented by a quantity known as hydraulic gradient, i. This is defined as the drop in head divided by the distance in which the drop occurs (Fig. 4.3). It may be expressed by the relation:

$$i = \frac{h_1 - h_2}{L} = \frac{\Delta h}{L} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (4.4)$$

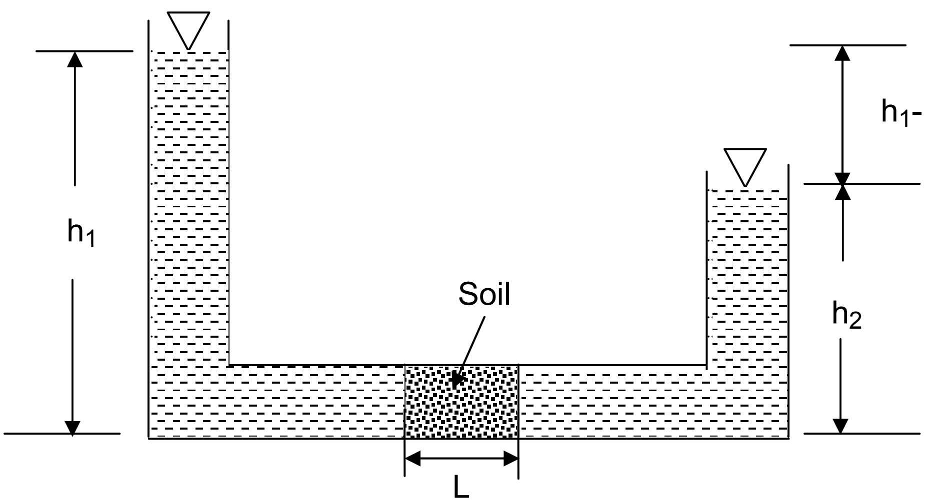


Fig. 4.3 Flow through a sample of soil

Darcy's Law, For laminar flow condition, Darcy's law states that the rate of flow of ground water is proportional to gradient.

$$V = ki$$

but $Q = VA$

Therefore $Q = kiA$ .... (4.5)

where Q = The discharge passing the total cross sectional area of the soil, A, per unit time

i = The hydraulic gradient

k = Darcy's coefficient of permeability which is usually called coefficient of permeability.

4.2 PERMEABILITY

Permeability is a soil property, which indicates the ease which water will flow through the soil. It denotes the capacity of soil to conduct or discharge water under a given hydraulic gradient.

The permeability of soil varies greatly. Coarse sand and gravel are highly pervious and have correspondingly high permeability coefficients. Clays on the other hand are relatively impervious and hence have low permeability coefficients.

4.2.1 Methods for the Determination of Coefficients of permeability of Soils

The flow of water through soils depends upon its permeability coefficient. Greater the value of the coefficient of permeability, greater is the flow. The water retaining capacity and the stability of earth dams, the capacity of pumping installations for the lowering of the ground water level during excavations, the rate of settlement of buildings and many other depend upon the value of coefficient of permeability of soils. Methods that are in common use for determining the coefficient of permeability, k, can be classified under laboratory and field methods.

Laboratory Methods

1. Constant head permeability method
2. Falling head permeability method

Field Method

Pumping tests

4.2.1.1 Laboratory Measurement of Permeability

The various types of apparatus, which are used in soil laboratories for determining coefficients of permeability of soils, are called permeameters. The apparatus used for constant head permeability test is called constant head permeameter and the one used for falling head test as falling (variable) head permeameter.

i) Constant Head Permeameter

The arrangement of the apparatus is shown in Fig. 4.4. The constant-head permeameter test is more suited for coarse-grained soils such as gravely sand and coarse and medium sand.

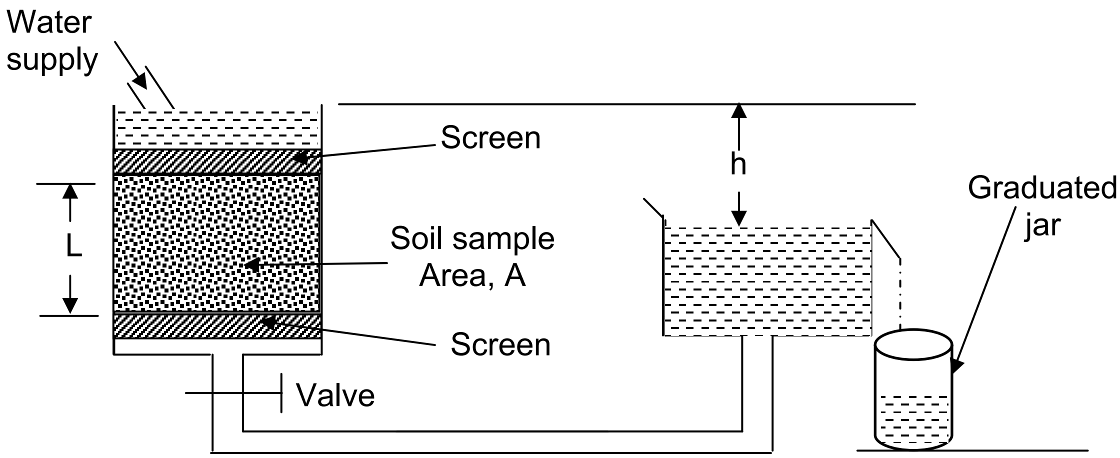


Fig. 4.4 Constant Head permeameter

In constant head permeameter test, the sample of length L and cross- sectional area A is subjected to a head h, which is constant during the progress of a test. A test is performed by allowing water to flow through the sample and measuring the quantity of discharge Q in time t. The coefficient of permeability k can be computed directly from Darcy's law expressed as follows

$$Q = kiAt = k\frac{h}{L}At \qquad \text{……………………} \qquad (4.6)$$

$$k = \frac{QL}{hAt} \qquad \text{………………..} \qquad (4.7)$$

The coefficient of permeability, k, is conventionally reported at a standard temperature of $+20^0$c. Tests carried out at different temperatures should be corrected as follow.

$$K_{20}{}^0c = K_T \frac{\mu_T}{\mu_{20}{}^0c} \qquad \text{…………………} \qquad (4.8)$$

where $K_{20}{}^0c$ = Coefficient of permeability at temperature 20^0c.

K_T = Coefficient of permeability at test temperature.

μ_T = Viscosity of water at test temperature

$\mu_{20}{}^0c$ = Viscosity of water at temperature 20^0c.

ii) Falling (Variable) Head Permeameter

Falling head permeameter type used for the test is shown in Fig. 4.5. In this test, a transparent standpipe of cross-sectional area, a, is attached to the test cylinder which contains the soil sample. Before the commencement of the test the soil is got saturated by allowing the water to flow continuously through the sample from the standpipe. After the saturation is complete, the standpipe is filled with water up to a height of h_o and a stopwatch is started.

Let the initial time be t_o. The time t_1 when the water level drops from h_o to h_1 is noted. The coefficient of permeability k can be determined on the basis of the drop in head (h_o-h_1) and the elapsed time (t_1-t_o) required for the drop as explained below.

Let h be the head of water at any time t. Let in time dt the head drop by an amount dh. The quantity of water flowing through the sample in time dt from Darcy's law is

$$dQ = kiAdt = k\frac{h}{L}A\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (4.9)$$

The quantity of discharge dQ can also be expressed as

$$dQ = -a\,dh \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.10)$$

Since the head decreased as the time increases dh is a negative quantity in Eqn. (4.10).

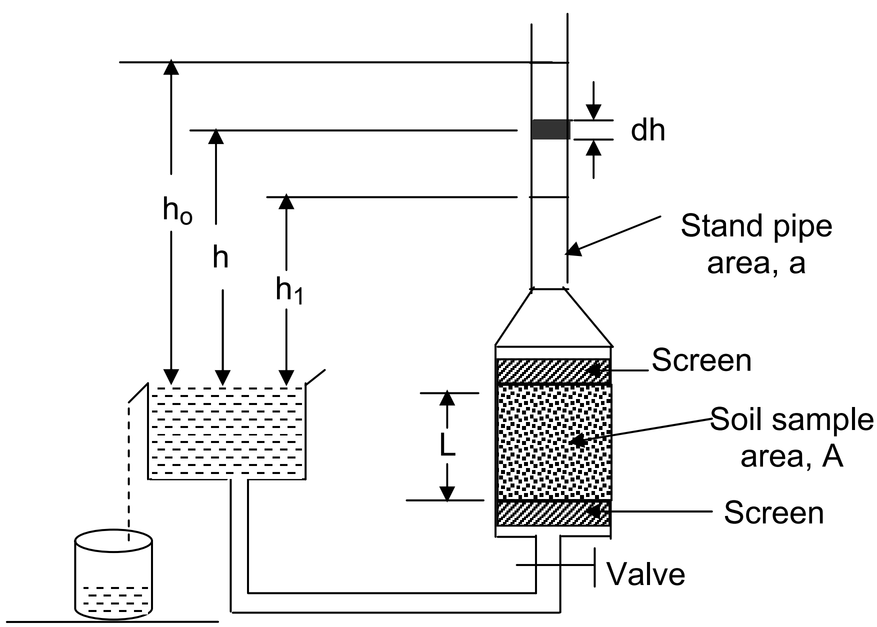


Fig. 4.5 Falling head permeameter

Eqn (4.9) can be equated to Eqn. (4.10)

$$-adh = k\frac{h}{L}A\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.11)$$

The discharge Q in time (t_1-t_o) can be obtained by integrating Eqn (4.9) or (4.10). Therefore, Eqn (4.11) can be rearranged and integrated as follows.

$$-a\int_{h_o}^{h_1}\frac{dh}{h}=\frac{kA}{L}\int_{t_o}^{t_1}dt$$

or $$a \log_e \frac{h_o}{h_1}=\frac{kA}{L}[t_1-t_2]$$

The general expression for k is

$$k = \frac{aL}{A[t_1-t_0]}\log_e\frac{h_0}{h_1} \qquad \dots\dots\dots\dots\dots\dots\dots \qquad (4.12)$$

Changing to common logarithm

$$k= \frac{2.3\,a\,L}{A[t_1-t_o]}\log_{10}\frac{h_o}{h_1} \qquad \dots\dots\dots\dots\dots\dots\dots\dots \qquad (4.13)$$

The falling head permeameter test is more suited for fine sands, silts and clays, where the time required for permeability is relatively long.

4.2.1.2 Field Measurement of Permeability

The most reliable information concerning the permeability of a deposit of coarse-grained material below the water table can usually be obtained by conducting pumping tests in the field. Although such tests have their most extensive application in connection with dam foundations, they may also prove advisable on large bridge or building foundation jobs where the water table must be lowered.

The arrangement consists of a test well and a series of observation wells. The test well is sunk through the permeable stratum up to the impermeable layer. A well sunk into a water-bearing stratum, termed as an aquifer, and tapping free flowing ground water having a free ground water table under atmospheric pressure, is termed as an unconfined well. Whereas a well sunk into an aquifer where the ground water flow is confined between two impermeable soil layers, and is under pressure greater than atmospheric, is termed as an artesian or confined well. The observation wells are drilled at various radial distances from the test or pumping well. A minimum of two observation wells and their distances from the test well are needed.

The test consists in pumping out water continuously at a uniform rate from the test well till the water levels in the test and observation wells become constant. When this condition is achieved the water pumped out of the well is equal to the inflow into the well from the surrounding strata. The water levels in the observation wells and the rate of water pumped out of the well would provide the necessary data for computing the coefficient of permeability of the soil within the zone of influence of the test well.

The draw down resulted due to pumping is called as the cone of depression. The maximum draw down D_o is in the test well. It decreases with the increase in the distance from the test well. The depression dies out gradually and forms theoretically, a circle around the test well called as the circle of influence. The radius of this circle, R_i, is termed as the radius of influence of the depression cone.

Assumptions

The development of equations for the determination of k which is based on Dupuit- Thiem's theory of well hydraulics involves the following assumptions:

1. The soil surrounding the pumped well is assumed as homogeneous.
2. The flow towards the pumped well is assumed as steady, laminar, radial and horizontal
3. The horizontal velocity is independent of depth
4. The ground water table is assumed as horizontal in all directions
5. The hydraulic gradient at any point on the draw down is equal to the slopes of the tangent at the point.

i) Equation for k for an Unconfined Aquifer

Figure 4.6 gives the arrangement of test (pumped) and observation wells for an unconfined aquifer. Only two observation wells at radial distances of r_1 and r_2 from the pumped well are shown. When the inflow of water into the pumped well is steady, the depths of water in these observation wells are h_1 and h_2 respectively.

Table 4.1 Permeability and Drainage Characteristics of Soils*

<table>
<tr><th></th><th colspan="11">Coefficient of permeability k in cm per sec</th></tr>
<tr><td></td><td>10^{2}</td><td>10^{1}</td><td>1.0</td><td>10^{-1}</td><td>10^{-2}</td><td>10^{-3}</td><td>10^{-4}</td><td>10^{-5}</td><td>10^{-6}</td><td>10^{-7}</td><td>10^{-8} 10^{-9}</td></tr>
<tr><td>Drainage</td><td colspan="6">Good</td><td colspan="2">Poor</td><td colspan="3">Practically impervious</td></tr>
<tr><td rowspan="2">Soil types</td><td colspan="2" rowspan="2">Clean sand</td><td colspan="2" rowspan="2">Clean sands, clean sand, and gravel mixtures</td><td></td><td colspan="4">Very fine sands , organic inorganic silts, mixtures of sand, silt and clay , glacial till stratified clay deposited, etc.</td><td colspan="2" rowspan="2">“Impervious” soils, e.g.homogenous clays below zone of weathering.</td></tr>
<tr><td colspan="5">“Impervious” soils modified by effects of vegetation and weathering.</td></tr>
<tr><td rowspan="2">Direct determination of K</td><td colspan="6">Direct testing of soil in its original position- pumping tests. Reliable if properly conducted. Considerable experience required.
.</td><td colspan="5"></td></tr>
<tr><td colspan="5">Constant head permeameter. Little experience required</td><td colspan="6"></td></tr>
<tr><td rowspan="2">Indirect determination of K</td><td colspan="2"></td><td colspan="3">Falling-head permeameter. Reliable. Little experience required.</td><td colspan="3">Falling-head permeameter. Unreliable. Much experience required.</td><td colspan="3">Falling-head permeameter. Fairly reliable. Considerable experience necessary.</td></tr>
<tr><td colspan="5">Computation from -grain size distribution. Applicable only to clean cohesionless sands and gravels.</td><td colspan="4"></td><td colspan="2">Computation based on results of consolidation tests. Reliable. Considerable experience required.</td></tr>
</table>

*After Cassagrande and R.E Fadum

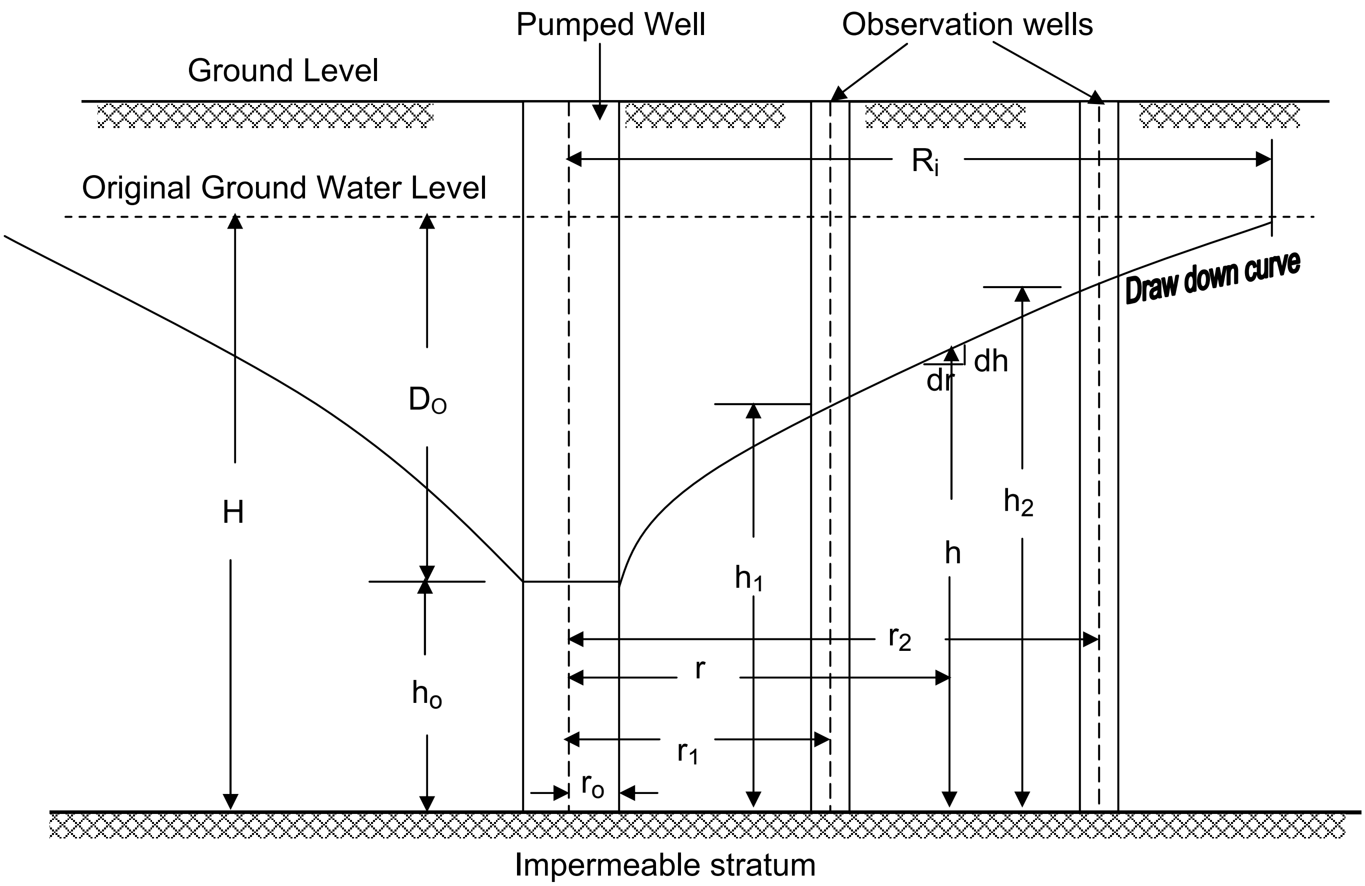


Fig 4.6. Pumping test in an unconfined aquifer

Let h be the depth of water at radial distance r. The area of the vertical cylindrical surface of radius r and depth h through which water flow is

$$A = 2\pi rh \qquad \text{................} \qquad (4.14)$$

The hydraulic gradient is

$$i = \frac{dh}{dr} \qquad \text{...............} \qquad (4.15)$$

As per Darcy's law the rate of inflow into the well when the water levels in the wells remain constant is

$$Q = k\ i\ A \qquad \text{.....................} \qquad 4.16)$$

Substituting for A and i in Eqn. (4.16) from Eqns. (4.14) and (4.15), the rate of inflow across the cylindrical surface is

$$Q = k\,\frac{dh}{dr}\,2\pi rh \qquad \text{..................} \qquad (4.17)$$

Rearranging the terms, we have

$$\frac{dr}{r} = \frac{2\pi\, k\, h\, dh}{Q} \quad \text{……………………} \qquad (4.18)$$

The integral equation of Eqn. (4.18) within the boundary limits is

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{Q} \int_{h_1}^{h_2} h\,dh \quad \text{………………} \qquad (4.19)$$

The integral of Eqn. (4.19) is

$$\log_e {}^{r_2}\!/_{r_1} = \frac{\pi k}{Q}[h_2^{\,2} - h_1^{\,2}] \text{…………} \qquad (4.20)$$

The equation for k is

$$k = \frac{Q}{\pi[h_2^{\,2} - h_1^{\,2}]} \log_e {}^{r_2}\!/_{r_1} \quad \text{………} \qquad (4.21)$$

When reduced to common logarithm,

$$k = \frac{2.3Q}{\pi[h_2^{\,2} - h_1^{\,2}]} \log 10 \; {}^{r_2}\!/_{r_1} \quad \text{………} \qquad (4.22)$$

ii) Equation for k in a Confined Aquifer

Figure 4.7 shows a confined aquifer with the pumped and observation wells. The water in the observation wells rises above the top of the aquifer due to artesian pressure. When pumping from such an artesian well two cases might arise. They are:

Case 1 The water level in the pumped well might remain above the roof level of the aquifer at steady flow condition.

Case 2 The water level in the pumped well might fall below the roof level of the aquifer at steady flow condition.

If H_o is the thickness of the confined aquifer and h_o is the depth of water in the pumped well at the steady flow condition, case 1 and case 2 may be stated as:-

Case 1 when $h_o > H_o$

Case 2 when $h_o < H_o$

Case 1 When $h_o > H_o$

In this case, the area of a vertical cylindrical surface of any radius r does not change as in Eqn. (4.14) , since the depth of the water bearing state is limited to the thickness H_o. Therefore, the discharge surface area is

$$A = 2\pi r H_o \qquad (4.23)$$

Again writing i = dh/dr, the flow equation as per Darcy's law is

$$Q = k\, i\, A$$

or

$$Q = k \frac{dh}{dr} 2\pi r H_o \qquad (4.24)$$

Arranging the terms in Eqn. (4.24), we have

$$\frac{dr}{r} = \frac{2\pi k H_o}{Q} dh \qquad (4.25)$$

Integration of Eqn. (4.25) yields,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k H_o}{Q} \int_{h_1}^{h_2} dh$$

$$\log_e \frac{r_2}{r_1} = \frac{2\pi k H_o}{Q} [h_2 - h_1] \qquad (4.26)$$

The equation for k is

$$k = \frac{Q}{2\pi H_o [h_2 - h_1]} \log_e \frac{r_2}{r_1} \qquad (4.27)$$

When reduced to common logarithm

$$k = \frac{2.3Q}{2\pi H_o [h_2 - h_1]} \log_{10} \frac{r_2}{r_1} \qquad (4.28)$$

Case 2 When $h_o < H_o$

Under the condition when h_o is less than H_o, the flow pattern close to the well is similar to that of an unconfined aquifer whereas at distances, further from the well the flow is artesian. Musket has developed an equation, which would be used to determine the coefficient of permeability.

The equation is

$$k = \frac{2.3Q}{\pi[2HH_o - H_o^2 - h_o^2]}\log_{10}\frac{R_i}{r_o} \quad \ldots\ldots \qquad (4.29)$$

Based on experience, Sichardt gives an equation for estimating the radius of influence, R_i, for stabilized flow condition as;

$$R_i = 3000\, D_o \sqrt{k} \text{ meters} \quad \ldots\ldots\ldots\ldots\ldots \qquad (4.30)$$

where D_o= Maximum draw down in meters

k = Coefficient of permeability in m/sec.

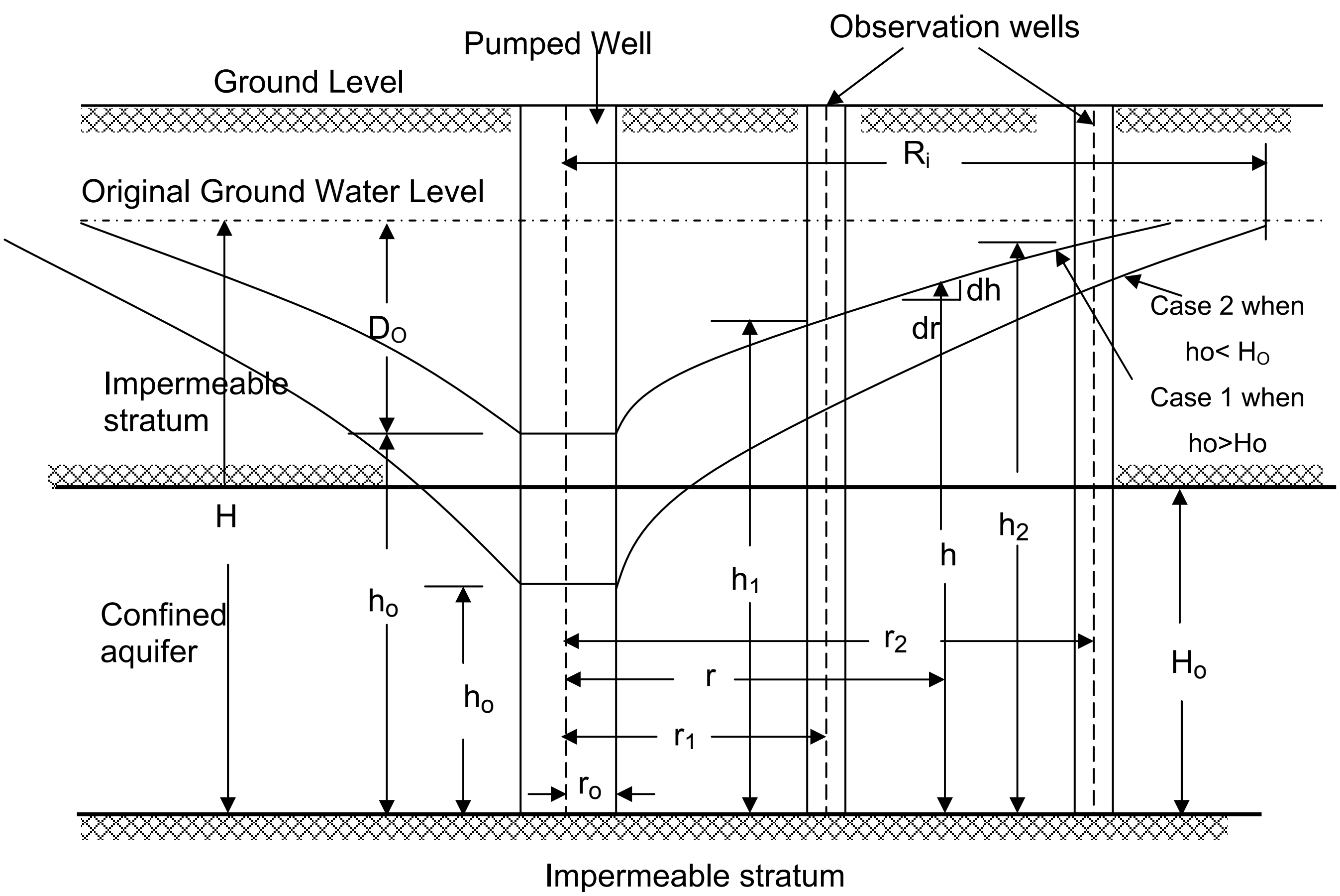


Fig. 4.7 Pumping test in confined aquifer

4.2.2 Permeability of Natural Deposits

Depending on the nature of soil deposit, the coefficient of permeability of a given layer of soil may vary with the direction of flow. Furthermore in a stratified soil deposit where the permeability coefficient for a given directional flow changes from layer to layer, an equivalent permeability determination becomes necessary for simplifying calculations. The following derivations relate to the equivalent permeabilities for flow in vertical and horizontal directions through multi-layered soils with horizontal stratification.

4.2.2.1 Flow Parallel to Bedding Plane

When the flow is in the horizontal direction the hydraulic gradient i remains the same for all layers. Let K_h be the average coefficient of permeability in the horizontal direction.

Then

$$Q = Q_1 + Q_2 + Q_3 + ...Q_n ....\ldots\ldots\ldots \quad 4.31)$$

$$k_h iH = k_{h1} iH_1 + k_{h2} iH_2 + k_{h3} iH_3 + ...kh_n iH_n \ldots \quad (4.32)$$

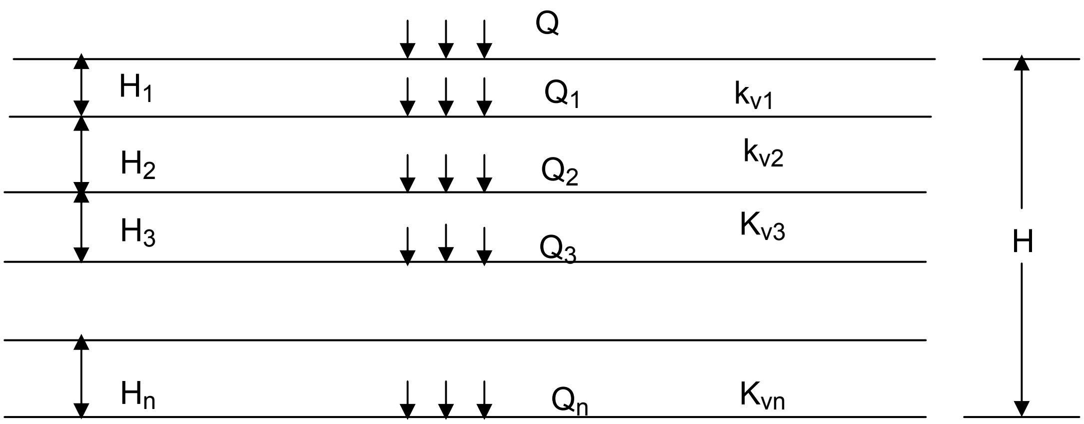
$$k_h H = k_{h1} H_1 + k_{h2} H_2 + k_{h3} H_3 + ...+ kh_n H_n .... \quad (4.33)$$

$$k_h = \frac{kh_1 H_1 + kh_2 H_2 + kh_3 H_3 + ...+ kh_n H_n}{H} \quad \ldots\ldots\ldots \quad (4.34)$$

The general equation may be written as,

$$k_h = \frac{\sum_{i=1}^{n} (k_h H)_i}{\sum_{i=1}^{n} H_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.35)$$

4.2.2.2 Flow Normal to Bedding Plane



When the flow is in the vertical direction, the hydraulic gradients for each of the layer are different. The average coefficient of permeability, K_v, perpendicular to the bedding planes is calculated using the principle of continuity of flow

$$A_1V_1 = A_2V_2 = A_3V_3 = \ldots\ldots\ldots\ldots = A_nV_n. \quad \ldots\ldots\ldots\ldots \tag{4.36}$$

Since the cross- sectional area A is constant throughout, the velocity of flow through all layers is the same

$$V = V_1 = V_2 = V_3 = \ldots\ldots\ldots = V_n \quad \ldots\ldots\ldots\ldots\ldots\ldots \tag{4.37}$$

If h represents the total head loss and h_1, h_2, h_3….h_n represent the head losses through different layers, then, we have

$$h = h_1 + h_2 + h_3 + \ldots . h_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{4.38}$$

but $\quad i = \dfrac{h}{H}$; and $i_1 = \dfrac{h_1}{H_1}$; $i_2 = \dfrac{h_2}{H_2}$; $i_3 = \dfrac{h_3}{H_3}$ ……… $i_n = \dfrac{h_n}{H_n}$

$\therefore \quad h = iH;\ h_1 = i_1H_1;\ h_2 = i_2H_2;\ h_3 = i_3H_3\ldots\ldots.;\ h_n = i_nH_n$

$$\therefore \quad iH = i_1H_1 + i_2H_2 + i_3H_3 + \ldots\ldots\ldots\ldots + i_nH_n \quad \ldots\ldots\ldots\ldots\ldots\ldots \tag{4.39}$$

If k_v represents the average coefficient of permeability in the normal direction, then from Darcy's law $V = k_v i$. Similarly,

$$V_1 = k_{v1}\, i_1,\ V_2 = k_{v2}\, i_2,\ V_3 = k_{v3}\, i_3, \ldots\ \ldots\ldots, V_n = k_{vn}\, i_n$$

$$\therefore \quad k_v i = k_{v1}\, i_1 = k_{v2}\, i_2 = k_{v3}\, i_3 = \ldots\ldots\ldots\ldots = k_{vn}\, i_n$$

$$\therefore \qquad i_1 = \frac{\mathrm{k_v}}{k_{V1}}\, i$$

$$i_2 = \frac{\mathrm{k_v}}{k_{V2}}\, i \qquad \text{.................................} \qquad (4.40)$$

$$i_3 = \frac{\mathrm{k_v}}{k_{V3}}\, i$$

$$\bullet \quad \bullet$$
$$\bullet \quad \bullet$$
$$\bullet \quad \bullet$$

$$i_n = \frac{\mathrm{k_v}}{k_{Vn}}\, i$$

Substituting these values in Eqn.(4.39) gives

$$i\,\mathrm{H} = \frac{\mathrm{k_v}}{k_{V1}} iH_1 + \frac{\mathrm{k_v}}{k_{V2}} iH_2 + \frac{\mathrm{k_v}}{k_{V3}} iH_3 + \ldots\ldots + \frac{\mathrm{k_v}}{k_{V1}} i_n H_n \ldots\ldots \qquad (4.41)$$

or $$\mathrm{H} = k_V \left[\frac{H_1}{k_{V1}} + \frac{H_2}{k_{V2}} + \frac{H_3}{K_{V3}} + \ldots\ldots\ldots + \frac{H_n}{K_{Vn}}\right] \ldots\ldots\ldots\ldots \qquad (4.42)$$

or $$k_V = \frac{H}{\frac{H_1}{k_{V1}} + \frac{H_2}{k_{V2}} + \frac{H_3}{k_{V3}} + \ldots\ldots\ldots\ldots\ldots + \frac{H_n}{k_{Vn}}} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (4.43)$$

The general equation is expressed as

$$k_V = \frac{\sum_{i=1}^{n} (H)_i}{\sum_{i=1}^{n} \left(\frac{H}{k_V}\right)_i} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4.44)$$

It should be noted that in all stratified layers of soils the horizontal permeability is generally greater than the vertical permeability.

4.3 SEEPAGE THROUGH SOILS

Seepage through soils may be defined as the flow of a fluid through the soil porous under a pressure gradient.

We have considered some simple cases for which direct application of Darcy's law is required to calculate the flow of water through soil. Often the flow of water through soil is not in one direction only and it is not uniform over the entire area perpendicular to the flow. In such cases, ground water flow is generally found by use of graphs referred to as flow nets. The concept of flow net is based on Laplace's equation of continuity, which describes the steady flow condition for a given point in the soil mass.

4.3.1 Laplace differential equation of continuity

To derive the equation of continuity of flow, consider an elementary soil prism at point A (Fig.4.8 b) for the hydraulic structure shown in Fig.4.8 (a).

(a) (b)

Fig. 4.8 Derivation of continuity equation

The flows entering the soil prism in the x, y and z directions can be given from Darcy's law as

$$q_x = k_x i_x A_x = k_x \frac{\partial h}{\partial x} dydz \qquad \text{...(4.45)}$$

$$q_y = k_y iyA_y = k_y \frac{\partial h}{\partial y} dxdz \qquad \text{...........................} \qquad (4.46)$$

$$q_z = k_z i_z A_z = k_z \frac{\partial h}{\partial z} dxdy \text{...................................} \qquad (4.47)$$

Where q_x,q_y, q_z =flow entering in directions x,y ,z, respectively

K_x, k_y, k_z = coefficient of permeability in directions x, y, z, respectively

h = hydraulic head at point A

The flows leaving the soil prism in the x,y and z directions are, respectively,

$$q_x + dq_x = k_x(i_x + di_x)A_x = k_x(\frac{\partial h}{\partial x} + \frac{\partial^2 x}{\partial x^2}dx)dydz \cdots\cdots \quad (4.48)$$

$$q_y + dq_y = k_y(i_y + di_y)A_y = k_y(\frac{\partial h}{\partial y} + \frac{\partial^2 y}{\partial y^2}dy)dxdz \cdots\cdots \quad (4.49)$$

$$q_z + dq_z = k_z(i_z + di_z)A_z = k_z(\frac{\partial h}{\partial z} + \frac{\partial^2 z}{\partial z^2}dz)dxdy \cdots\cdots \quad (4.50)$$

For steady flow through an incompressible medium, the flow entering the elementary prism is equal the flow leaving the elementary prism. So,

$$q_x + q_y + q_z = (q_x + dq_x) + (q_y + dq_y) + (q_z + dq_z) \cdots\cdots \quad (4.51)$$

Combining Eqs. (4.45) to (4.51), we obtain

$$k_x \frac{\partial h}{\partial x}dydz + k_y \frac{\partial h}{\partial y}dxdz + k_z \frac{\partial h}{\partial z}dxdy = k_x(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2}dx)dydz + k_y(\frac{\partial h}{\partial y} + \frac{\partial^2 h}{\partial y^2}dy)dxdz +$$

$$k_z(\frac{\partial h}{\partial z} + \frac{\partial^2 h}{\partial z^2}dz)dx$$

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \cdots\cdots \quad (4.52)$$

For two dimensional flow in the x-z plane, the above equation becomes

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \cdots\cdots \quad (4.53)$$

If the soil is isotropic with respect to permeability, k_x,=k_z= k, and the continuity equation simplifies to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \cdots\cdots \quad (4.54)$$

This is generally referred to as Laplace's equation.

4.3.2 Flow Nets

The continuity equation (Eqn. 4.54) in an isotropic medium represents two orthogonal families of curves, i.e., the flow lines and the equipotential lines. The flow line is a line along which a water particle travels from the upstream to the downstream side in a permeable soil medium. The equipotential line is a line along which the potential head is the same at all points. That is, if piezometers are placed at different points along an equipotential line, the height of the water will rise to the same elevation in all of them. A combination of a number of flow lines and equipotential lines is called flow net.

The hydraulic boundary conditions have a great effect on the general shape of the flow net, and hence must be examined before sketching is started. The flow net can be plotted by trial and error by observing the following properties of flow net.

1. The flow lines and equipotential lines meet at right angles to each other
2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.
3. The quantity of water flowing through each flow path (flow channel) is the same. Similarly, the same potential drop occurs between two successive equipotentials.
4. Smaller the dimension of the field, the larger will be the hydraulic gradient and velocity of flow through it.
5. In a homogeneous soil, every transition in the shape of the curves is smooth, being either elliptical or parabolic in shape.

Figure 4.9 shows an example of a flow net for a single row of sheet piles. The permeable layer is isotropic with respect to the coefficient of permeability, i.e., $k_x=k_y= k$. note that the solid lines in Fig.4.9 are the flow lines, and the broken lines are the equipotential lines. For the flow net shown in Fig.4.9, the following boundary conditions apply

1. The upstream and downstream surfaces of the permeable layer (line AB and EF) are equipotential lines.

2. Since AB and EF are equipotential lines, all the flow lines intersect them at right angles.

3. The boundary of the impervious layer, i.e., line GH is a flow line, so also is the surface of the impervious sheet pile, i.e., line BCDE.

4. The equipotential lines intersect BCDE and GH at right angles.

Fig 4.9 Flow net around a single row of sheet piles

4.3.3 Determination of Flow from flow net

In a flow net, the strip between any two adjacent flow lines is called a flow channel. Figure 4.10 shows a flow channel that with the equipotential lines form elements.

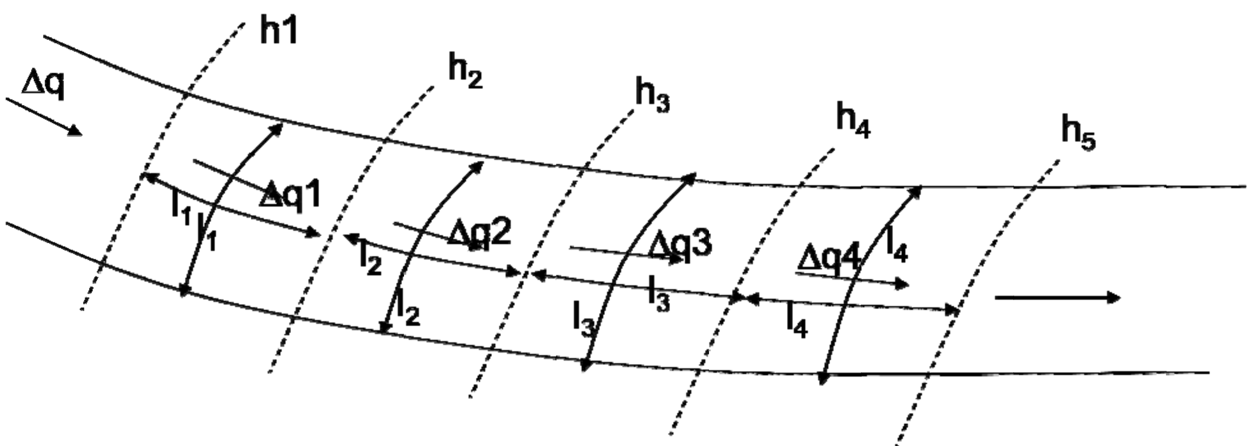


Fig. 4.10 Seepage through flow channel with square elements

Let h_1, h_2,h_3,…. be the peizometric levels corresponding to the equipotential lines.

The rate of seepage through the flow channel per unit width can be calculated as follows.

Since there is no flow across the flow line

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \Delta q_4 = \ldots \Delta q \quad \ldots\ldots\ldots\ldots \qquad (4.55)$$

From Darcy's law, the rate of flow is kiA. Thus, the above equation can be written as

$$\Delta q = k(\frac{h_1 - h_2}{l_1}) = k(\frac{h_2 - h_3}{l_2}) = k(\frac{h_3 - h_4}{l_3}) \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.56)$$

Eqn. (4.56) shows that if the flow elements are drawn as approximate squares, the drop in peizometric level between any two adjacent equipotential lines (the potential drop) is the same. Therefore,

$$h_1 – h_2 = h_2 - h_3 = h_3 - h_4 = \ldots\ldots = H/N_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.57)$$

$$\Delta q = k(H/N_d) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.58)$$

Where

H= difference of head between upstream and downstream sides

N_d = number of potential drops

If the number of flow channels in a flow net is N_f, the total rate of flow through all channels per unit width can be given by

$$q = N_f \Delta q = kH\frac{N_f}{N_d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.59)$$

Although flow nets are usually constructed in such a way that all flow elements are approximately squares, that need not always be the case. We could construct flow nets with all the flow elements drawn as rectangles. In that case, the length-to-width ratio of the flow nets has to be a constant, i.e.,

$$\frac{b_1}{l_1} = \frac{b_2}{l_2} = \frac{b_3}{l_3} = n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.60)$$

For such flow nets, the rate of seepage per unit length of hydraulic structure can be given by

$$q = kH\frac{N_f}{N_d}n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.61)$$

4.3.4 Flow Net in Anisotropic Soil

To account for soil anisotropy with respect to permeability, some modification of the flow net construction is necessary. The D.E. of continuity for two dimensional flow is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.62)$$

where

$k_x = k_{horizontal}$ and $k_z = k_{vertical}$

The solution of this equation will no longer give families of equipotential and flow lines which always intersect at right angles. However, we can rewrite Eqn. (4.62) as

$$\frac{\partial^2 h}{\frac{k_z}{k_x}\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0 \quad \text{……………………………} \quad (4.63)$$

$$Let\ x' = \sqrt{k_z/k_x}\ x\ ;then$$

$$\frac{\partial^2 h}{\frac{k_z}{k_x}\partial x^2}=\frac{\partial^2 h}{\partial x'^2} \quad \text{…………………………..} \quad (4.64)$$

Substituting Eqn. (4.64) into Eqn. (4.63), we obtain

$$\frac{\partial^2 h}{\partial x'^2}+\frac{\partial^2 h}{\partial z^2}=0 \text{…………………………} \quad (4.65)$$

Equation (4.65) is of the same form as Eqn. (4.54), which gives the flow in isotropic soils and should represent two sets of orthogonal lines in the x' – z plane. The steps for construction of a flow net in an anisotropic medium are as follows:

1. To plot the section of the hydraulic structure, adopt a vertical scale

$$2. Deter\,\text{mine}\ \sqrt{k_z/k_x}$$

$$3. Adopt\ a\ horizontal\ scale\ such\ that$$

$$Scale(horizontal)=\sqrt{k_z/k_x}\ \ x\ (vertical\, scale)$$

4. With the scales adopted in steps 1 and 3, plot the cross-section of the structure.
5. Draw the flow net for the transformed section plotted in steps 4 in the same manner as is done for seepage through isotropic soil.
6. Calculate the rate of seepage as

$$q=\sqrt{k_x k_z}H\,\frac{N_f}{N_d} \text{……………………………} \quad (4.66)$$

Where

H = total head loss

N_f and N_d = number of flow channels and potential drops, respectively

4.4 EFFECTIVE AND NEUTRAL STRESSES IN SOILS

The total stress on a horizontal plane through a submerged soil mass is equal to the sum of two components known as effective stress and neutral stress. It is necessary in many engineering problems to distinguish between these two components.

Effective Stress

The stress transmitted through grain to grain at the contact points through a soil mass is termed as effective or intergranular stress. It is known as effective stress since this stress is responsible for the decrease in the void ratio or increase in the frictional resistance of a soil mass.

Neutral Stress

Neutral stress is a unit stress carried by the water in the soil pores. It is sometimes called the pore water pressure. The neutral stress does not press the soil grains against one another and hence does not lead to compression of the soil or an increase in frictional resistance.

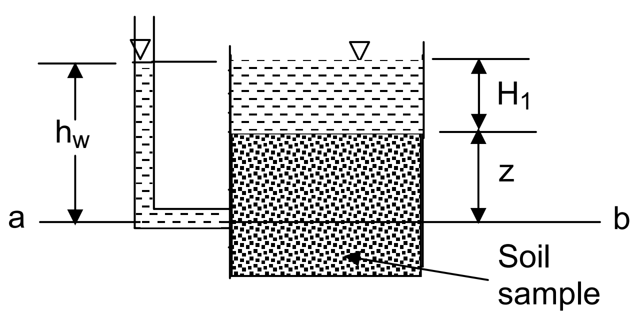


Fig. 4.11 Effective and neutral stresses in a soil

In accordance with the above concept the total normal stress σ on a horizontal plane through a submerged soil = an effective stress, σ' + neutral stress, $u_{w.}$

The lower part of the container shown in Fig. 4.10 is filled with saturated soil having a unit weight γ_{sat}. Water stands to a height H_1 above the surface of the soil. After equilibrium is established, the peizometric head h_w at depth z is H_1 +z, the neutral stress is

$$u_w = (H_1 + z)\gamma_\omega$$

and the total normal stress is

$$\sigma = H_1\gamma_{\omega} + z\gamma_{sat}$$

Hence the effective stress at depth z is

$$\sigma - u_{\omega} = H_1\gamma_{\omega} + z\gamma_{sat} - (H_1+z)\gamma_{\omega}$$

$$= H_1\gamma_{\omega} - H_1\gamma_{\omega} + z(\gamma_{sat} - \gamma_{\omega})$$

$$= z\gamma_b \quad \text{.................} \qquad (4.67)$$

The quantity γ_b is called the submerged unit weight of the soil.

4.5 CRITICAL HYDRAULIC GRADIENT AND QUICK SAND CONDITION

When water flows through a soil mass, the viscous resistance within the pore channels results in seepage forces which being transmitted by the water to the soil particles. In areas where the flow occurs predominantly in an upward direction, these seepage forces tend to reduce the effective stress between the soil particles and hence tend to reduce the shear strength of the soil mass. This may be demonstrated with reference to the simple arrangement of a sand in a container and an adjustable water reservoir, shown in Fig 4.12. On positioning the water reservoir above the top of the sand, water flows upwards through the sand and overflows at the surface. The difference in level between the surface of the reservoir and the surface of the sand defines the drop in total head h during flow through the sand of length L. The hydraulic gradient across the sand is then given by

$$i = \frac{h}{L} \quad \text{............................} \qquad (4.68)$$

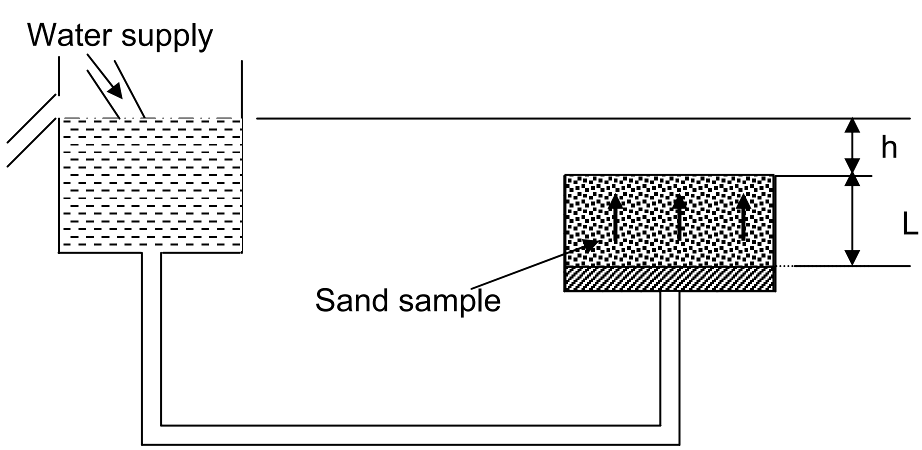


Fig. 4.12 Critical hydraulic gradient

At the bottom plane of the sand the total downward force is equal to the saturated weight of sand

$$\gamma_{sat}\, L\, A = \left(\frac{G_S + e}{1+e}\right)\gamma_\omega LA$$

The upward force at the same plane, is the pressure of water under a head of (h+L) on an area A and this is equal to

$$\gamma_\omega (h+L)A$$

If these two forces happen to be equal, the net downward force on the bottom plane will be nil, and neglecting friction from the sides of the container, there will be no force preventing the outflow of the sand from the container. For this condition to occur

$$\left(\frac{G_S + e}{1+e}\right)\gamma_\omega L\, A = \gamma_\omega (h + L)\, A$$

$$\gamma_\omega L\, A \left(\frac{G_S + e}{1+e} - 1\right) = \gamma_\omega\, h\, A$$

$$\frac{h}{L} = i_C = \frac{G_S - 1}{1+e} = \frac{\gamma_b}{\gamma_\omega} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (4.69)$$

Where i_c is the critical hydraulic gradient. It is a gradient through the soil at which the effective stresses on a horizontal plane in the soil are reduced to zero.

When the hydraulic gradient is i_c, the effective normal stress on any plane will be zero, gravitational forces having been canceled out by upward seepage forces. In the case of sands the contact forces between particles will be zero and the soil will have no strength. The soil is then said to be in a quick condition and if the critical gradient is exceeded the surface will appear to be 'boiling' as the particles are moved around in the upward flow of water. It should be realized that 'quicksand' is not a special type of soil but simply sand through which there is an upward flow of water under a hydraulic gradient equal to or exceeding i_c.

For the simple arrangement shown in Fig. 4.12 a quick sand condition could equally well be produced by maintaining the reservoir level constant and decreasing the thickness of the sand. Thus in practice, if an excavation is made in a deposit of sand below groundwater level, the resulting upward seepage flow through the base of the excavation could result in a quick sand condition being formed. In a similar manner, the base of an excavation in a deposit of clay overlying water-bearing sand may heave if the total weight of clay, decreasing by virtue of excavation, becomes equal to the up thrust on the clay due to the pore water pressure in the sand. To maintain the stability of such excavation it is often necessary to install pumping wells around the perimeter of the excavation and reduce the pressure head in the sand.

Equation (4.69) indicates that the critical hydraulic gradient is independent of particle size and therefore a quick sand condition is possible in all types of soil. However, in practice, a quick sand condition is most likely to arise in silts and fine-to-medium sands. For clay soils the attraction between the clay mineral particles tends to resist separation, and with highly permeable coarse sands and gravels it is unlikely that the large volumes of water needed to maintain a quick sand condition would be encountered.

Example 4.1

A constant head permeameter test has been run on a sand sample 25 cms. in length and 20 cm^2 in area, with a head loss of 50 cms. The discharge was found to be 260 cm^3 in 130 seconds. Determine the coefficient of permeability of the soil. If the specific gravity of the solid particles was 2.62 and the dry weight of the sand 9.16N, find the void ratio of the sample.

Solution

$$k = \frac{QL}{Aht}$$

Q = 260 cm^3

L = 25 cms

A = 20 cm^2

h = 50 cms

t = 130 sec.

Therefore, $k = \frac{260 \text{ X } 25}{20 \text{ X } 50 \text{ X } 130} = 0.05 cm/\sec$

$$e = \frac{V_V}{V_S}$$

$$V_S = \frac{W_S}{G_S \gamma_\omega} = \frac{9.16 \text{ X } 10^{-3}}{2.62 \text{ X } 10} = 3.50 x 10^{-4} m^3 = 350 cm^3$$

$$V_V = V - V_S = (20 \text{ X } 25) - 350 = 150 cm^3$$

$$e = \frac{150}{350} = 0.429$$

Example 4.2

The coefficient of permeability of a soil sample was found out in a soil mechanics laboratory by making use of a falling head permeameter. The data used and the test results obtained were as follows

Diameter of sample	= 6 cms
Height of sample	= 15 cms
Diameter of stand pipe	= 2 cms
Initial head, h_o	= 45 cms
Final head, h_1, after a time of 105 sec.	= 30 cms

Determine the coefficient of permeability in m/day

Solution

$$k = \frac{2.3aL}{At} \log \frac{h_o}{h_1}$$

Area of stand pipe, $a = \frac{\pi \text{ X } 2^2}{4 \text{ X } 100^2} = 3.14 \text{ X } 10^{-4} m^2$

Area of sample $A = \frac{\pi \text{ X } 6^2}{4 \text{ X } 100^2} = 28.26 \text{ X } 10^{-4} m^2$

Height of sample $L = \frac{15}{100} = 15 \text{ X } 10^{-2} m$

Head $h_o = \frac{45}{100} = 45 \text{ X } 10^{-2} m$

$$h_1 = \frac{30}{100} = 30 \text{ X } 10^{-2} m$$

Elapsed time t = $105 = \frac{105}{60 \text{ X } 60 \text{ X } 24} = 12.15 \text{ X } 10^{-2} days$

$$k = \frac{2.3 \text{ x } 3.14 \text{ x } 10^{-4} \text{ x } 15 \text{ x } 10^{-2}}{28.26 \text{ x} 10^{-4} \text{ x } 12.15 \text{ x } 10^{-2}} \log \frac{45}{30}$$

= 5.5 m/day

Example 4.3

A pumping test was carried out in an unconfined aquifer to determine the amount of discharge for the supply of drinking water for the nearby village. Observation wells were established at distances of 5 m and 10 m from the center of the test well. The following data were obtained.

Coefficient of permeability of the aquifer as determined in the laboratory	= 3x10^{-3}cm/sec
Thickness of the aquifer	= 20m
Original ground water level (from the bottom of the aquifer)	= 14 m
Draw down at the test well	= 3m
Draw down at the inner wall	= 1
Draw down at the outer wall	= 0.5 m

How many days will it take to fill a reservoir having a capacity of 1000 m^3 after steady flow is attained?

Solution

$$k = \frac{2.3Q}{\pi\,[h_2^{\ 2} - h_1^{\ 2}]} \log_{10} \frac{r_2}{r_1}$$

$$Q = \frac{k\pi[h_2^{\ 2} - h^2{}_1]}{2.3\log_{10} r_2 / r_1}$$

h_1 = 14 -1 = 13m

h_2 = 14 – 0.5 = 13.5 m

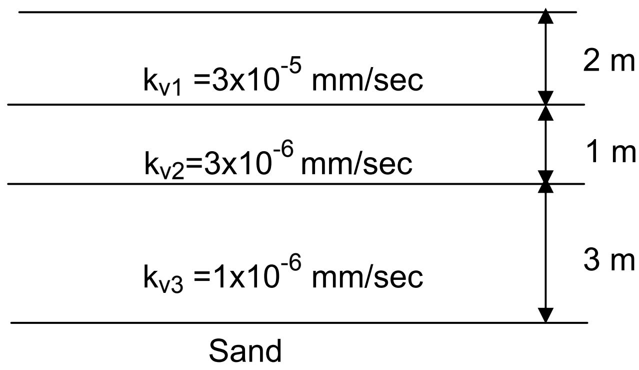
$$Q = \frac{3 \text{ x } 10^{-5} \text{ x } \pi \text{ x } [(13.5)^2 - (13)^2]}{2.3\log_{10} \frac{10}{5}}$$

= 1.80 x 10^{-3} m^3/sec = 155.52 m^3/days

$$\text{time} \ = \frac{1000}{155.52} = 6.43\, days$$

Example 4.4

The figure hereunder represents the soil profile beneath a reservoir with 10 m water depth. A sand layer lies below the profile. Assume vertical flow through the layers shown and compute the water loss in three months from the reservoir having cross sectional area of 3000 m^2.

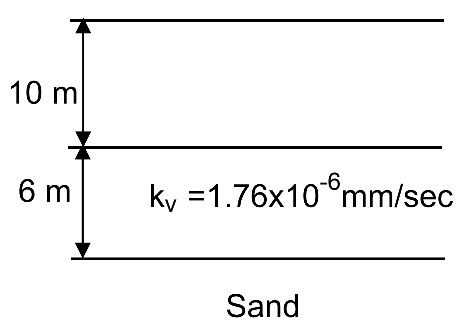


Solution

Average coefficient of Permeability in the vertical direction is given by

$$k_V = \frac{\sum_{i=1}^{n} (H)_i}{\sum_{i=1}^{n} \left(\frac{H}{k_V}\right)_i}$$

$$k_v = \frac{(2+1+3)1000}{\frac{2000}{3\,X10^{-5}} + \frac{1000}{3\,X10^{-6}} + \frac{3000}{1\,X10^{-6}}} = 1.76\,X10^{-6}\,mm/\sec$$



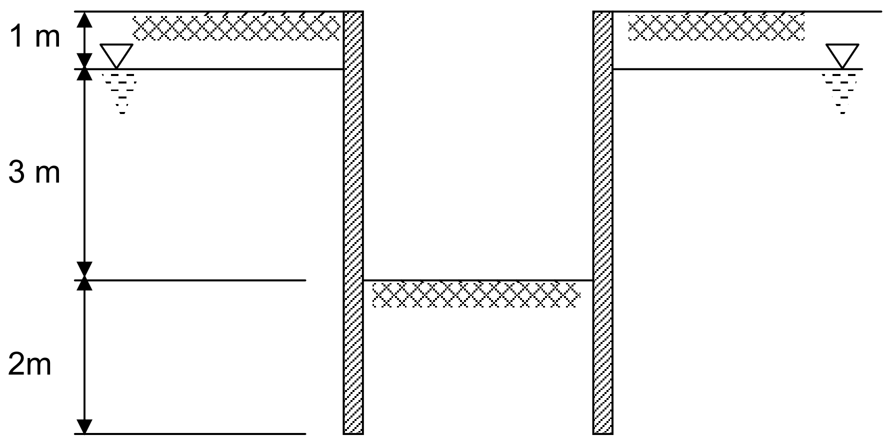
From Darcy law

$$Q = k_V iA = \frac{1.76x10^{-6}}{1000} X \frac{16}{6} X 3000 = 14.08 X 10^{-6} m^3 / \sec$$

In three months = $14.08x10^{-6}x60x60x24x30x3=109\ m^3$

Example 4.5

A trench is excavated in fine sand for a Building foundation, up to a depth of 4m. The excavation was carried out by providing the necessary side supports and pumping water. The water levels at the sides and the bottom of the trench are as given in figure below. Examine whether the bottom of the trench is subjected of quick sand condition if G_s=2.64 and e=0.7. If so what is the remedy?



Solution

$$i_c = \frac{G_s - 1}{1 + e}$$

If the trench is to be stable, the hydraulic gradient, i, prevailing at the bottom should be less than i_c. The hydraulic gradient

$$i = \frac{h}{L}$$

There will be no quick condition if

$$\frac{h}{L} < \frac{G_s - 1}{1 + e}$$

From the given data

$$i_c = \frac{2.64 - 1}{1 + 0.7} = \frac{1.64}{1.7} = 0.96$$

$$i = \frac{h}{L} = \frac{3}{2} = 1.50$$

It is obvious that $i > i_c$. There will be quick condition

Remedy

i) Increase L to at least 4m depth below the bottom of trench or

ii) Keep the water table outside the trench also at a low level by pumping out water.

EXERCISES

4.1 A block of soil is 12cm long and 6 cm^2 in cross section. The water level at one end of the block is 20 cm above a fixed plane, and at the other end, it is 3m above the same plane. The flow rate is 20 cc in 1.5 minutes. Compute the soil permeability.

4.2 A sample of clay soil having cross sectional area of 78.5 cm^2 and a height of 5 cm is placed in a falling head permeameter in which the area of the standpipe is 0.53 cm^2. In a test run, the head on the sample drops from 60cm to 38 cm in 1 hr, 24 min 18 sec. What is the coefficient of permeability of the soil?

4.3 In order to determine the average permeability of a bed of sand 14 m thick covering on impermeable stratum, a well was sunk through the sand and a pumping test was carried out. After a certain interval, the discharge was 12.4 liters per second and draw downs on observation wells at 16 m and 33 m from the pumping wells were found to be 1.787 m and 1.495 m respectively If G.W.L. was originally 2.14 m below ground level, find the coefficient of permeability of the sand.

4.4 A horizontal stratified deposited consists of three layers each uniform in itself. The permeabilities of the layers are $8x10^{-4}$ cm/sec, $50x10^{-4}$ cm/sec, $5x10^{-4}$ cm/sec and their thickness are 6m, 3m and 18m respectively. Find the average coefficient of permeability of the deposit in horizontal and vertical directions.

4.5 Calculate the neutral and effective stress at a depth of 20m below the ground surface. The soil has Gs 2.70, e=0.80, ω=5% for the soil above the water tables. The ground water table is located at 4m below the ground surface.

4.6 A foundation trench is to be excavated in a clay stratum 6 m thick underlain by sandy stratum. The water table is observed in a borehole to be 1m below the ground level. Find the depth to which the excavation can proceed without the danger of blow. Take the specific gravity of clay particles to be 2.65 and the water content of the clay soil in saturated condition equal to 37%.

5

CONSOLIDATION OF SOILS

5.1 INTRODUCTION

Any structure built on the ground causes increase of pressures on the underlying soil layers. The soil layers are unable to spread laterally as they are confined by surrounding soil strata. Hence they must adjust to the new pressures by vertical deformation only. The compression of the soil mass leads to the decrease in the volume of the mass, which results in the settlement of the structure, built on the mass. Settlement is, therefore, the sinking of a structure due to a compressive deformation of the underlying soil mass. The vertical compression of a soil mass under increased pressures is thus made up of the following components.

- Deformation of the soil grains
- Compression of water and air within the voids
- An escape of water and air from the voids

It is quite reasonable and rational to assume that the solid matter and the pore water relatively are incompressible under the loads usually encountered in soil masses. The change in volume of a mass under imposed stresses must be only due to the escape of water if the soil is saturated. But if the soil is partially saturated, the change in volume of the mass is partially due to the compression and escape of air from the voids and partially due to the dissolution of air in the pore water.

A study of compressibility of soils is necessary to be able to forecast the probable settlement of structures on different types of soils. Unequal settlement of different parts of a structure or excessive settlement of the structure as a whole may lead to serious damage to the structure or imparts ability to fulfill its function. It may be noted that the settlement of a structure is not determined only by the layers immediately under its foundations, but may be caused by a soft clay layer buried under relatively incompressible layers on top.

5.2 CONSOLIDATION

When a saturated soil mass is subjected to a load increment, the load is usually carried initially by the water in the pores because the water is incompressible in comparison with the soil structure. The pressure in the water causes it to drain away to the surrounding materials and produces a change in volume. This gradual decrease of the volume at constant load is known as consolidation. The rate at which the water can escape depends upon the permeability of the soil. In a free drained cohesionless soil, such as medium and coarse sand or gravel, the change in volume is generally slight and almost instantaneous so that very soon after its application the load is carried by the soil skeleton. With clay soils, however, the permeability is low and there is a time lag between the application of load and the change in volume. During this period, the soil remains fully saturated and the change in volume corresponds to a change in the amount of pores (the void-ratio) in the soil. As that amount of pores change the soil shrinks and this result in settlement of foundation. Since the period over which consolidation takes place is usually very long, it is important to measure the rate at which the void ratio changes and also the final amount of consolidation. The consolidation test measures the rate and amount of volume change with the application of load on a laboratory specimen and the results thus obtained are used to calculate the settlement of a foundation. The process opposite to consolidation is called swelling, which involves an increase in the water content due to an increase in the volume of the voids.

5.2.1 Principle of Consolidation

The principle of consolidation of clay soil-water system may be explained with the help of a mechanical model as described by Terzaghi and Froehlich. The model consists of a cylinder with a frictionless piston as shown in Fig. 5.1. The piston is supported on one or more helical metallic springs. The space underneath the piston is completely filled with water. This model indicates that the springs represent the mineral skeleton in the actual soil mass and water below the piston is the pore water under saturated condition in the soil mass.

When a load per unit area is placed on the piston, the load is fully transferred to water (as water is assumed to be incompressible) and the water pressure increases. The pressure in water is $u_{\omega} = p$. This is analogous to pore water pressure that would be developed in a clay water system under external pressures. If the whole model is leak proof without any holes in the piston, there is no chance for the water to escape. Such a condition represents a highly impermeable clay-water system in which there is a very high resistance for the flow of water.

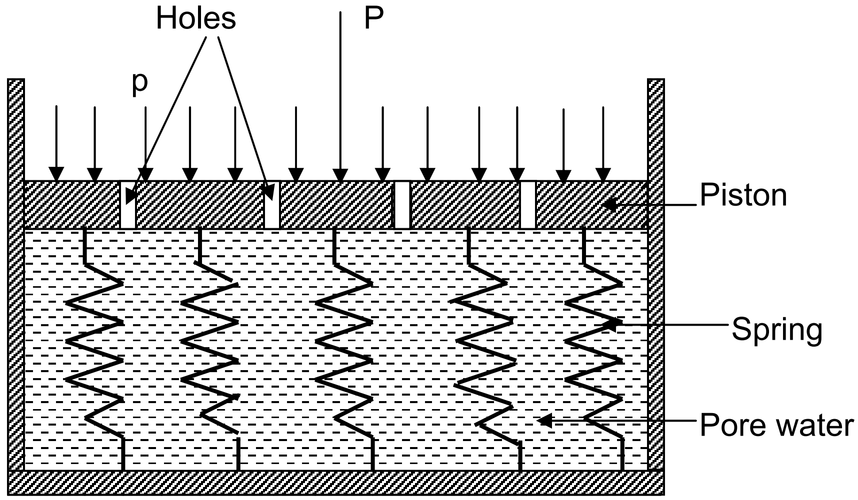


Fig. 5.1 Mechanical model to explain the principle of consolidation

If a few holes are made in the piston, the water will immediately escape through the holes. With the escape of water through the holes a part of the load carried by the water is transferred to the springs. This process of transference of load from water to spring goes on till the flow stops when the entire load will be carried by springs and none by the water. The time required to attain this condition depend upon the number and size of the holes made in the piston. A few small holes represent clay soil with poor drainage characteristics. After the spring water system attains equilibrium condition under the imposed load the settlement of the piston is analogous to the compression of the clay-water system under external pressures.

5.2.2 Consolidation Test

The test is performed in a consolidometer (sometimes referred to as an oedometer). The schematic diagram of a consolidometer is shown in Fig 5.2. In this test a small representative sample of undisturbed soil is carefully trimmed and fitted into a rigid metal ring. The soil sample is mounted on a porous stone base, and a similar stone is placed on top to permit water, which is squeezed out of the sample to escape freely at the top and bottom. Prior to loading, the height of the sample should be accurately measured. Also, a micrometer dial is mounted in such a manner that the vertical strains in the sample can be measured as loads are applied.

The consolidation test apparatus is designed to permit the sample to be submerged in water during the test to simulate the position below a water table of the prototype soil sample from which the test sample was taken.

Loads are applied in steps in such a way that the successive load intensity, p, is twice the preceding one. The load intensities commonly used being ¼, ½, 1,2,4,8 and 16 kg/cm^2.

Each load is allowing to stand until compression has practically ceased. The dial readings are taken at elapsed time of 0, ¼, 1, 2¼, 4, 6¼, 9,12¼, 16, 20¼, 25 minutes …. 24 hours.

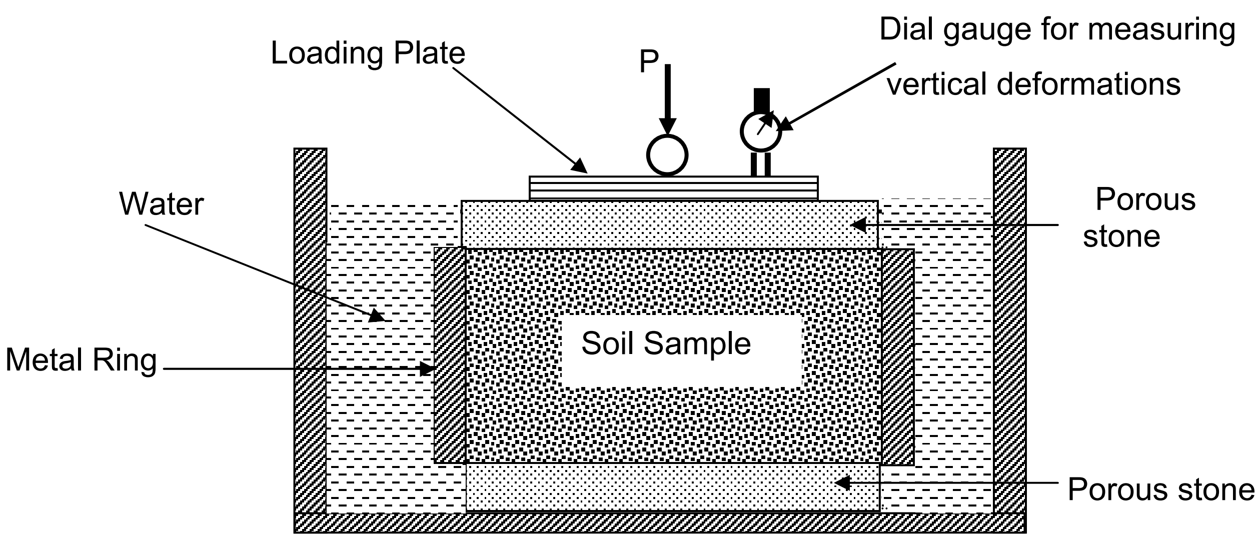


Fig. 5.2 Consolidometer

After the greatest load required for the test has been applied to the soil sample, the load is removed in decrements to provide data for plotting the expansion curves of the soil in order to learn its elastic properties and magnitudes of plastic or permanent deformations.

The following data should also be obtained

1. Moisture content and weight of the soil sample before the commencement of the test.
2. Moisture content and weight of the sample after the completion of the test.
3. The specific gravity of the solids.

5.2.3 Pressure Void Ratio Curves

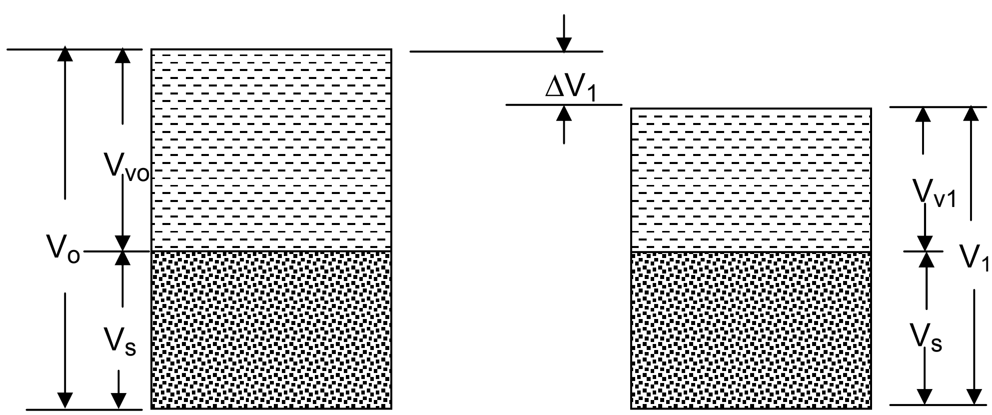
The results of the consolidation test are presented graphically. The void ratio, e, is plotted to a natural scale in a vertical direction. If the intensity of pressure p is plotted to a natural scale in the horizontal direction, the resulting curve is designated as an e-p curve. If the pressure is plotted to a logarithmic scale, the result is called an e-log^p curve.

The equilibrium void ratio at any load increment can be found out by the change of void ratio methods as follows.

In one-dimensional compression the change in height Δh per unit of original height h_o equals the change in volume ΔV per unit of original volume V_o.

$$\frac{\Delta h_1}{h_o} = \frac{\Delta V_1}{V_o} \qquad \text{......................} \qquad (5.1)$$

V may now be expressed in terms of void ratio e



a) Initial condition b) Compressed condition

Fig.5.3 Change of height of sample in consolidation test

From Fig. 5.3

$$V_o = V_s + V_{Vo}$$

$$V_o = V_s(1 + e_o)$$

$$V_1 = V_s + V_{v1}$$

$$V_1 = V_s(1 + e_1)$$

$$\frac{\Delta V}{V_o} = \frac{V_o - V_1}{V_o} = \frac{V_s(1+e_o) - V_s(1+e_1)}{V_s(1+e_o)} = \frac{\Delta e_1}{1+e_o}$$

$$\frac{\Delta h_1}{h_o} = \frac{\Delta V_1}{V_o} = \frac{\Delta e_1}{1+e_o}$$

Therefore

$$\frac{\Delta h_1}{h_o} = \frac{\Delta e_1}{1+e_o}$$

$$\Delta h_1 = \left(\frac{\Delta e_1}{1+e_o}\right) h_o \quad \text{.................} \qquad (5.2)$$

or

$$\Delta e_1 = \frac{\Delta h_1}{h_o}(1+e_o) \quad \text{.................} \qquad (5.3)$$

where Δe_1 = Change in void ratio under load

h_o = Initial height of sample

e_o = Initial void ratio of sample

e_1 = Void ratio after compression under a load

Δh_1 = Compression of sample under the load which may be obtained from dial gauge readings.

The initial void ratio of the sample can be calculated as follows

$$W = W_s + W_w$$

$$W = W_s(1+\omega) \Rightarrow W_s = \frac{W}{1+\omega}$$

$$V_s = \frac{W_s}{\gamma_s} = \frac{W_s}{G_s \gamma_\omega}$$

$$V_{vo} = V_o - V_s = V_o - \frac{W_s}{G_s \gamma_s}$$

$$e_o = \frac{V_{vo}}{V_s} = \frac{V_o - \left(W_s / G_s \gamma_\omega\right)}{W_s / G_s \gamma_\omega}$$

$$e_o = \frac{V_o G_s \gamma_\omega}{W_s} - 1 \quad \text{.................} \qquad (5.4)$$

The new void ratio e_1 after consolidation by the pressure increment P_1:

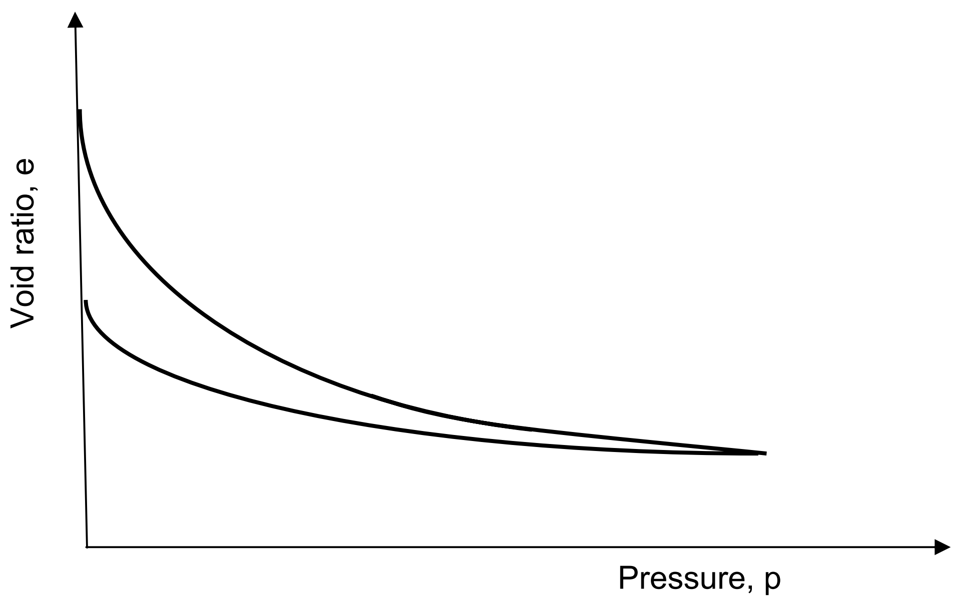
$$e_1 = e_o - \Delta e_1 \text{ ………………….. } \quad (5.5)$$

For the next loading P_2 (the cumulative load per unit area of sample), causing additional deformation of Δh_2, the void ratio e_2 at the end of consolidation can be calculated as

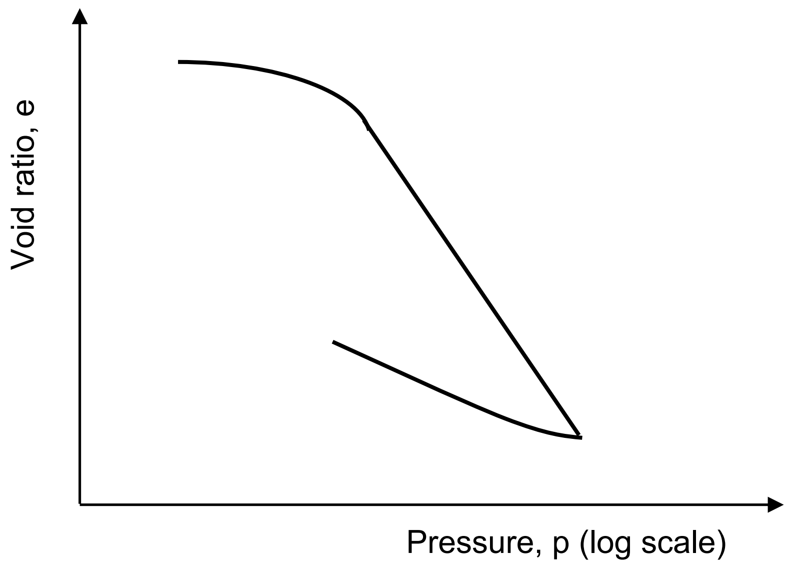
$$e_2 = e_1 - \Delta e_2 = e_1 - \left(\frac{\Delta h_2}{h_1}(1 + e_1) \right) \text{ …………………. } \quad (5.6)$$

Proceeding similarly, the void ratio at the end of the consolidation for each load increment can be obtained.

The total pressures p and the corresponding void ratios e at the end of consolidation are plotted both on arithmetic scale and on semi–log, the general nature of which are shown in Fig 5.4.



a) Typical e-p curve



b) Typical e-log^P curve

Fig 5.4 Pressure-void ratio curves

5.2.4 Normally Consolidated and Over Consolidated Clays

In the semi – log^P plot shown in Fig 5.4 (b), it can be seen that the upper part of the plot is some what curved with flat slope, followed by a linear relationship with a steeper slope. This curve may be explained in the following manner.

A soil in the field at some depth has been subjected to a certain maximum effective overburden pressure in its geologic history. This pressure may be the same as or greater than the existing overburden pressure at the time of sampling. The reduction of pressure in the field may be artificial or due to natural geological processes. During the soil sampling the existing overburden pressure is released, resulting in some expansion. When this sample is subjected to a consolidation test, a small amount of compression (change in void ratio) will occur if the total pressure applied is less than the maximum past effective overburden pressure. If the total pressure applied on the sample is greater than the maximum past effective overburden pressure, the change of void ratio is much larger and the e-log^p relation is practically linear with a steeper slope.

Based on the above explanation, one can define the two conditions of a clay soil:

1. Normally consolidated, whose present effective overburden pressure is the maximum that the soil has been subjected to in the past.

2. Over consolidation, whose present effective overburden pressure is less than the soil has experienced in the past. The maximum past effective overburden pressure is called the preconsolidation pressure.

In some foundation studies it is desirable to know the approximate value of the pressure (i.e. pre-compression pressure). A. Cassagrande has developed a graphical method for estimating this pressure from e-log^{P} plot as shown in Fig. 5.5. The method consists of the following.

1. Select the point of maximum curvature on the plot, say the point C.
2. Draw a tangent and a horizontal line through the point C.
3. Bisect the angle formed by the tangent and the horizontal line
4. Produce the straight portion of the curve to cut the bisector at D.
5. The abscissa of point D gives the pre-consolidation pressure, p_c

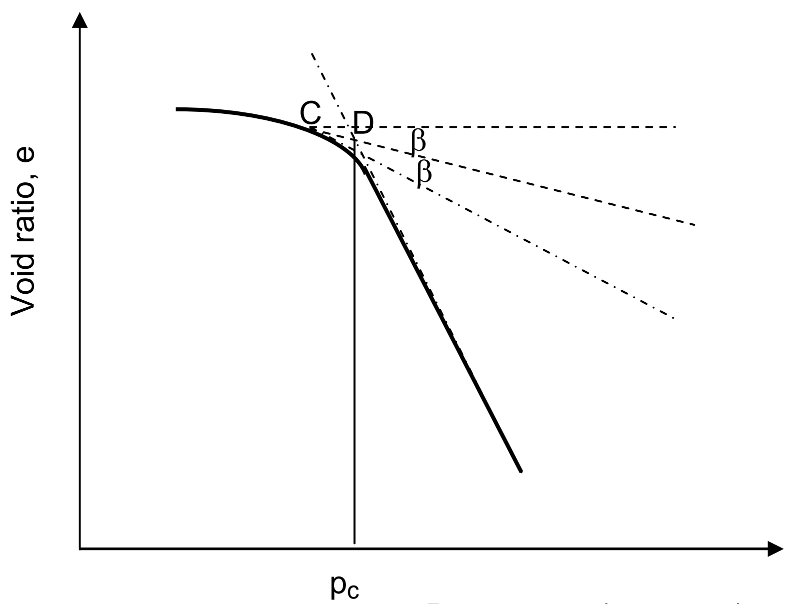


Fig 5.5 Determination of precompression pressure

5.2.5 Coefficient of Compressibility, a_v.

This is the rate of change of void ratio with pressure. It is numerically equal to the slope of pressure versus void ratio curve on a natural scale.

$$a_V = \frac{e_1 - e_2}{p_2 - p_1} = \frac{\Delta e}{\Delta p} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (5.7)$$

a_v has a dimension inverse of pressure.

As the e-p curve is not a straight line, a_v is not constant but decreases with increasing pressure. At any point on the curve, the slope of the tangent with the horizontal gives the value of a_v for that point.

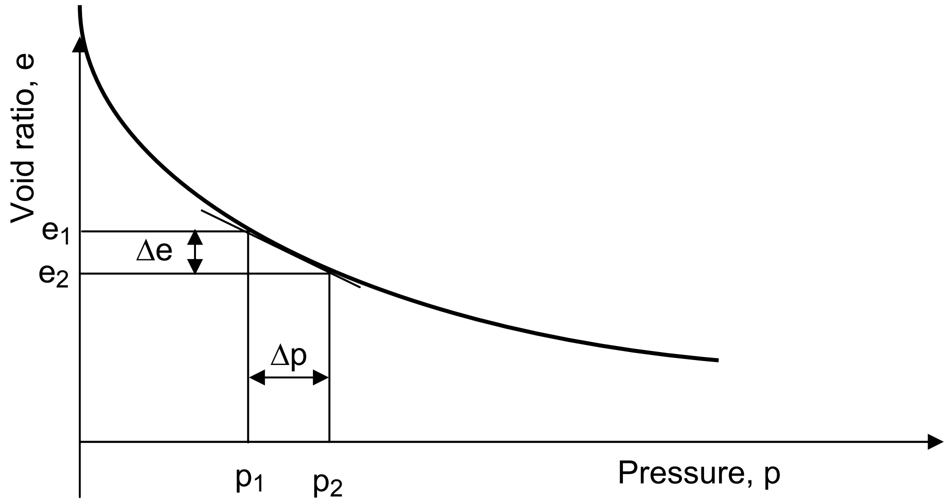


Fig 5.6 Determination of coefficient of compressibility

5.2.6 Compression Index, C_c

Compression index, C_c, is numerically equal to the slops of the straight portion of the e-log^p curve.

$$C_c = \frac{e_1 - e_2}{\log^{P_2} - \log^{P_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.8)$$

$$e_1 - e_2 = C_c \log \frac{P_2}{P_1}$$

$$\Delta e = C_c \log \frac{P_1 + \Delta P}{P_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.9)$$

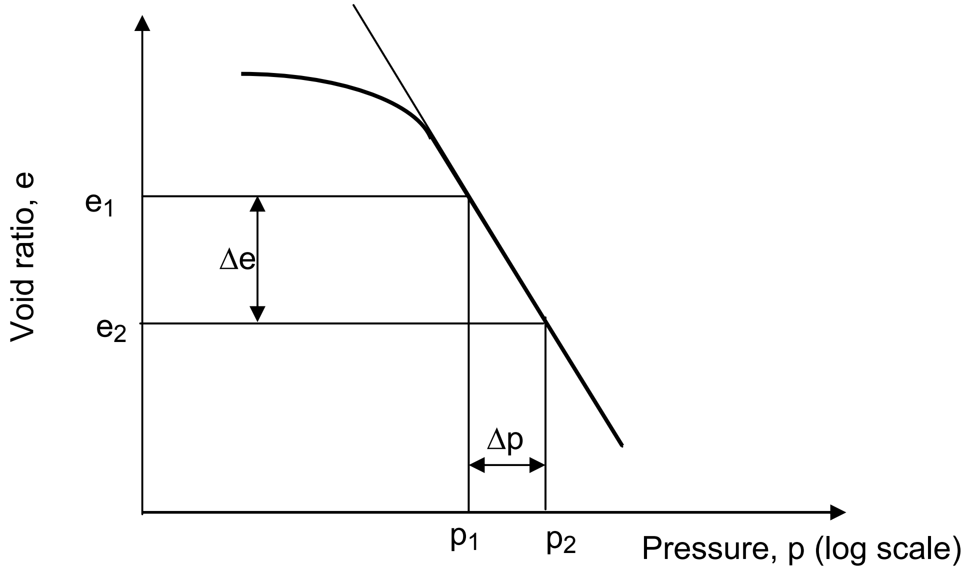


Fig 5.7 Determination of compression index

Terzaghi and peck (1967) suggested the following empirical expressions for compression index

C_c= 0.009 (ω_ℓ -10) for undisturbed clays …………… (5.10)

C_c= 0.007 (ω_ℓ -10) for remolded clays ………….. (5.11)

where ω_ℓ = liquid limit, in percent

In the absence of laboratory consolidation data, Eqn. (5.10) is often used for approximate calculation of a foundation on clay layer.

5.2.7 Swelling Index, C_s

Swelling index, C_s, denotes the slope of an expansion or rebound curve of e-log^P plot.

$$C_s = \frac{e_1 - e_2}{\log^{P_2} - \log^{P_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.12)$$

$$e_1 - e_2 = C_s \log \frac{P_2}{P_1}$$

$$e_2 = e_1 - C_s \log \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.13)$$

The swelling index is appreciably smaller in magnitude than the compression index [approximately 0.1 Cc to 0.2 Cc].

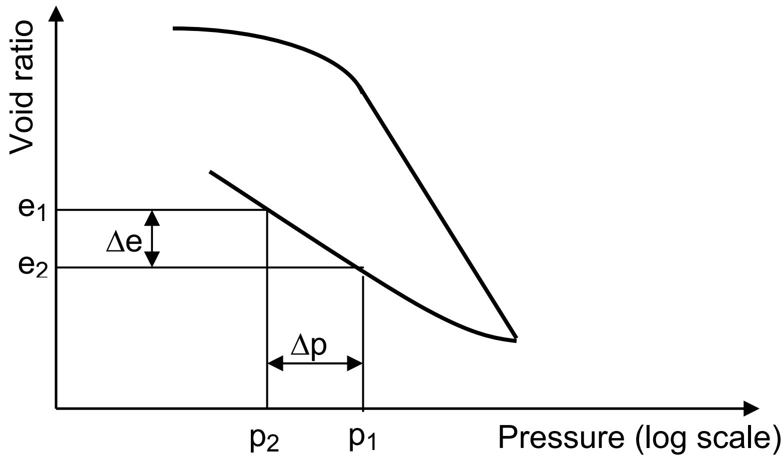


Fig. 5.8 Determination of swelling index

5.2.8 Calculation of One- Dimensional Consolidation Settlement

Consider a saturated clay layer of thickness H and cross–sectional area A under an existing average effective overburden pressure p_o. Let the consolidation settlement due to an increase of pressure Δp be S. Thus the change in volume ΔV (Fig. 5.9) is given by

$$\Delta V = V_o - V_1 = HA - (H - S)A = SA \ldots\ldots\ldots\ldots\ldots \qquad (5.14)$$

Where V_o and V_1 are the initial and final volumes respectively. However, the change in total volume is equal to the change in volume of voids ΔV_v.

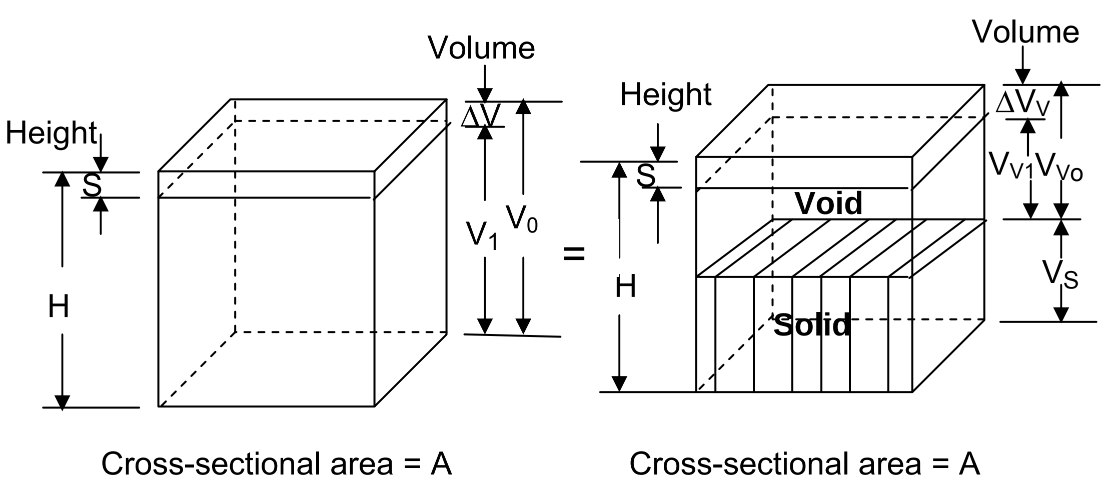


Fig 5.9 Settlement due to one–dimensional consolidation

Thus

$$\Delta V = V_{vo} - V_{v1} = \Delta V_v \quad \ldots\ldots\ldots\ldots. \quad (5.15)$$

where V_{vo} and V_{v1} are the initial and final void volumes.

From the definition of void ratio

$$\Delta V_v = \Delta e \, V_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.16)$$

where Δe= Change of void ratio.

but

$$V_s = \frac{V_o}{1+e_o} = \frac{AH}{1+e_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \quad (5.17)$$

where e_o = Initial void ratio at volume Vo.

Thus, from Eqns. (5.14), (5,15), (5,16) and (5,17)

$$\Delta V = SA = \Delta e \, V_S = \frac{AH}{1+e_o}\Delta e \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.18)$$

$$S = H \frac{\Delta e}{1+e_o}$$

For normally consolidated clays exhibiting a linear e – log^p relation (Fig. 5.7)

$$\Delta e = C_c\left[\log^{(P_o+\Delta P)} - \log^{P_o}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.19)$$

where C_c= Compression index

Substituting of Eqn. (5.19) in Eqn (5.18) gives compressibility of soil

$$S = \frac{C_c H}{1+e_o}\log\frac{(P_o+\Delta P)}{P_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.20)$$

5.2.9 Basic Concepts of Terzaghi's Theory of Consolidation.

To understand the basic concepts of Terzaghi's theory of consolidation, consider a clay layer of thickness 2H located below the ground water level and between two highly permeable sand layers as shown in Fig. 5.10. If a unit load equal to p is applied at the ground surface over a very large area, the clay layer will begin to compress as the excess of the water from its pores is squeezed out towards the two permeable boundaries. If the clay is homogeneous, excess pore water from the upper half of the layer will flow towards the upper sand layer, where as the excess pore water from the lower half of the layer will flow towards the lower sand layer. Such an arrangement is called double drainage. $p = u + \bar{p}$ must remain valid at all times and at all points in the clay layer.

At the instant the pressure, p, is applied (i.e. at time t=0), it is entirely carried by the pore water, i.e. p=u and $\bar{p}=0$. A few instances later, water will start escaping into the sand, so that u at both permeable boundaries will equal to zero. At any time $p=u+\bar{p}$. As time goes by, the variation of hydrostatics excess pressure, u, over the depth will successively be indicated by the curves t_1, t_2, t_3 as shown in Fig. 5.10. After a theoretical time ($t=\infty$) consolidation will be complete and excess pore water pressure will equal zero ($u=0$, $p=\bar{p}$). At any time, the area between the curves pertaining to that time and initial hydrostatic excess pressure diagram gives the load transferred to the soil grains up to that time. For the time interval t_1, this area has been shown shaded in the figure. The ratio of this area to the area of the initial hydrostatic excess pressure diagram ABCD gives the degree of consolidation at that time and is expressed as a percentage.

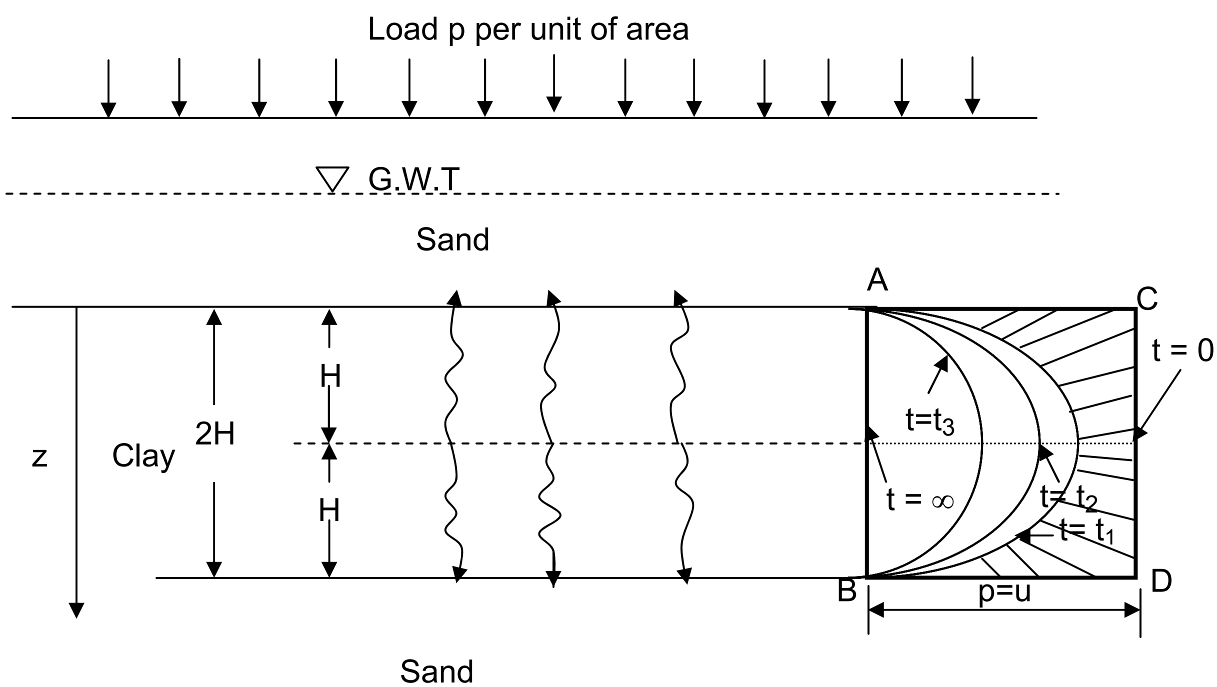


Fig. 5.10 Progress of consolidation

Progress of consolidation process at a given point in the soil is indicated by U_z.

$$U_z = \frac{u_o - u}{u_o} = 1 - \frac{u}{u_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots \tag{5.21}$$

where U_z= Percent of consolidation at a point

u_o = Initial hydrostatic excess pressure

u= Hydrostatic excess pressure at time t

The average percent consolidation of the entire layer at any time is numerically equal to the percentage change in thickness or settlement. Then to estimate the rate of settlement it is necessary to establish the variation of u with time.

Assumptions Used in Terzaghi's Theory

1. The clay layer is homogeneous.
2. The clay layer is saturated.
3. The compression of the soil layer is due to the change in volume, which, in turn, is due to the squeezing out of water from the void spaces.
4. Darcy's law is valid.
5. Deformation of soil occurs only in the direction of the load application.
6. The flow of water is in one direction, i.e. in the direction of compression.
7. Coefficient of permeability, k, is the same everywhere within the layer and remains constant during consolidation.

Coefficient of Consolidation, C_v, is a coefficient containing the physical constants of a soil affecting its rate of volume change. It indicates the combined effects of permeability and compressibility for a given void ratio change

$$C_v = \frac{k}{\gamma_\omega\, m_v} = \frac{k(1+e)}{\gamma_\omega a_v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{5.22}$$

where C_v = Coefficient of consolidation

k = Coefficient of permeability

m_v= Coefficient of volume compressibility = $\dfrac{a_v}{(1+e)}$

a_v= Coefficient of compressibility

γ_ω = Unit weight of water

5.2.10 Mathematical Expression and Solution of Terzaghi's Theory of Consolidation

$$\frac{du}{dt} = C_V \frac{d^2u}{dz^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5.23)$$

Equation (5.23) is the basic differential equation of Terzaghi's consolidation theory, which describes the distribution of hydrostatic excess pressure with time as long as the direction z. A solution of this equation in conformity with the known boundary condition leads to the distribution of hydrostatic excess pressures in the consolidating layer at different time intervals.

Integration of the area under each curve gives the proportion of the load transferred to the soil grains up to that time and hence represents the degree of consolidation for the entire layer. It should be understood that at any instant consolidation has progressed to different stages at different depths inside the layer. The degree of consolidation measures the total effect for the full thickness.

A mathematical description of the boundary conditions for a layer of thickness 2H, drainage at both ends and with a uniform pressure distribution along its depth will be as follows

$u = 0$ at $z = 0$

$u = 0$ at $z = 2H$

at $t = 0$, $u = u_o = p$

Eqn. (5.23) has been solved for different boundary conditions and the solution is obtained in the form

$$u = \sum_{m=1}^{m=\infty} \frac{2u_o}{M} \sin\left(\frac{Mz}{H}\right) e^{-M^2T} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (5.24)$$

where u = excess pore water pressure

m = is an integer

$M = \frac{\pi}{2}(2m+1)$

u_o = initial excess pore water pressure

$T = \frac{C_V t}{H^2_{dr}}$ time factor (dimensionless)

The average degree of consolidation U for the entire depth of the clay layer at any t can be written from Eqn. (5.21)

$$U = \frac{S_t}{S} = 1 - \frac{\frac{1}{2H}\int_o^{2H} u\,dz}{u_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.25)$$

where U = average degree of consolidation

S_t = Settlement of the layer at time t

S = Ultimate settlement due to primary consolidation

Substitution of the expression for excess pore water pressure u [given in Eqn. (5.24)] in Eqn. (5.25) gives

$$U = 1 - \sum_{m=1}^{m=\infty} \frac{2}{M^2} e^{-M^2 T} \quad \ldots\ldots\ldots\ldots\ldots \quad (5.26)$$

Eqn. (5.26) express the relationship between time and the average state of consolidation over the height of the stratum. The variation of the average degree of consolidation with the non-dimensional time factor is given in Table 5.1, which represents the case where u_o is the same for the entire depth of consolidation layer.

Table 5.1 Variation of Time Factor with Degree of Consolidation when u_o is Constant

U (%)	T	U (%)	T
0	0	55	0.238
10	0.008	60	0.287
15	0.018	65	0.342
20	0.031	70	0.405
25	0.049	75	0.477
30	0.071	80	0.565
35	0.096	85	0.684
40	0.126	90	0.848
45	0.159	95	1.127
50	0.197	100	∞

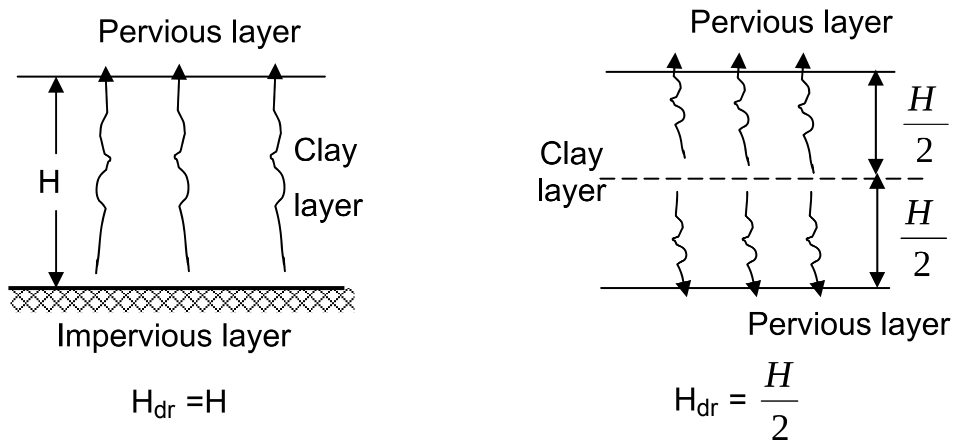
When U is required to be determined after a given time interval t, T can be worked out from T= $\frac{C_v t}{H^2_{dr}}$, C_v is determined from consolidometer test data. Knowing T, U is directly determined from Table 5.1.

The values of the time factor and the corresponding average degree of consolidation presented in Table 5.1 may also be approximated by the following simple relations.

$$T = \frac{\pi}{4}\left(\frac{U}{100}\right)^2 \text{ for } (0 \leq U \leq 60\ \%) \quad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (5.27)$$

$$T = 1.781 - 0.933 \log (100\text{-}U\%) \text{ for } U > 60\% \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5.28)$$

Longest Drainage Path, H_{dr}, represents the longest distance traveled by drop of pore water in reaching outlet.



a) Single drainage b) Double drainage

Fig. 5.11 Drainage paths

Factors Affecting Time Rate of Consolidation

According to Terzaghi's mathematical analysis of consolidations, the time rate of consolidation of clay stratum depends on the,

1. Thickness of clay layer
2. Number of drainage face
3. Permeability of the soil
4. Magnitude of the consolidating pressure acting on the clay layer.

Factors 1 and 2 influence the distance through which the water in the soil voids must travel in order to escape and permit the volume of the voids to decrease. The 3rd factor controls the rate at which the water can escape. The 4th factor influences the hydrostatic excess pressure, which causes the out flow of water.

5.4.11 Calculation of Coefficient of Consolidation from Laboratory Test Results

For a given load increment, the coefficient of consolidation, C_v, can be determined from laboratory observation of time versus dial reading. Two graphical procedures are commonly used for this; the logarithm-of-time fitting method proposed by Cassagrande and Fedum and the square-root-of- time fitting method proposed by Taylor. The general procedures for obtaining C_v follow.

Logarithm- of- Time- Fitting- Method

For the given incremental loading of the laboratory test, the sample deformation dial reading versus log-of-time plot is shown in Fig.5.12. The following constructions are needed to determine C_v.

1. Extend the straight-line portions of primary and secondary consolidation to intersect at A. The ordinate of A is represented by d_{100}, i.e., the deformation at the end of 100% primary consolidation.
2. Select times t_1 and t_2 in the curved portion such that $t_2 = 4t_1$. Let the difference of sample deformation during time (t_2-t_1) be x.
3. Draw a horizontal line DE such that the vertical distance BD equals x. The deformation corresponding to the line DE is equal to d_o (i.e., deformation at 0% consolidation).
4. Determine that point F on the consolidation curve, which corresponds to a deformation dial reading of $\frac{(d_o + d_{100})}{2} = d_{50}$. The time corresponding to the point F is t_{50}. i.e., time for 50% consolidation.
5. Determine C_v from the equation

$$T = \frac{C_V \, t}{H^2{}_{dr}}$$

The value of T for U=50% is 0.197 (Table 5.1).

So

$$C_v = \frac{(0197)(H^2{}_{dr})}{t_{50}} \quad \text{………………} \quad (5.29)$$

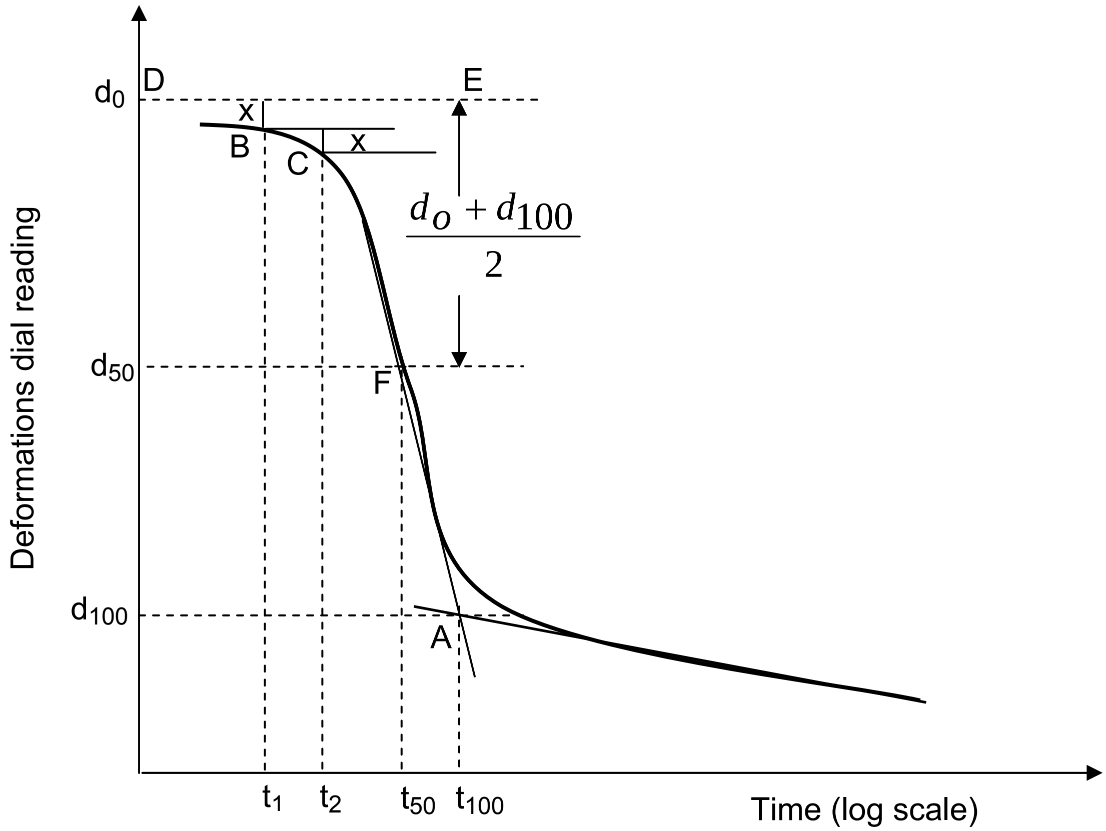


Fig. 5.12 Logarithm- of- time fitting method for determination of C_v

Square – Root – of – Time Fitting Method

In this method a deformation dial reading versus square-root-of- time plot is made for the incremental loading (Fig.5.13). Other graphic constructions required are;

1. Draw a line AB through the early portion of the curve.
2. Draw a line OC = 1.15 (OB). The abscissa of point D, which is the intersection of AC and the consolidation curve, gives the square-root-of-time for 90% consolidation $\left(\sqrt{t_{90}}\right)$.
3. For 90% consolidation, T_{90} = 0.848 (Table 5.1).

Therefore

$$C_v = \frac{(0.848)(H^2{}_{dr})}{t_{90}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (5.30)$$

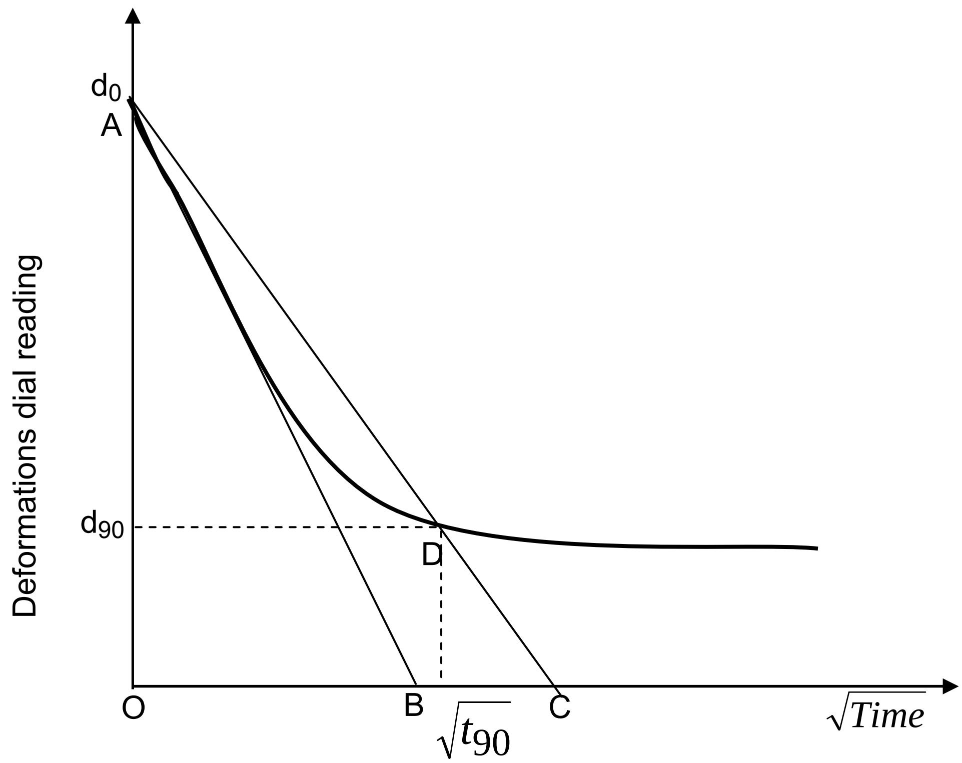


Fig. 5.13 Square- root-of- time fitting method for determination of C_v

Example 5.1

During a consolidation test, a sample of fully saturated clay 3 cm thick is consolidated under a pressure increment of 200kN/m^2. When equilibrium is reached, the sample thickness gets reduced to 2.60 cm. The pressure is then removed and the sample is allowed to expand and takes water. The final thickness is observed as 2.80 cm and the final moisture content is determined as 24%. If the specific gravity of the soil solids is 2.70, find the void ratio of the sample before and after consolidation.

Solution

From Eqn. (5.3)

$$\Delta e = \frac{\Delta h}{h_f}(1 + e_f)$$

where e_f = Final void ratio after the test

$= \omega G_s = 0.24 \times 2.70 = 0.648$

h_f = Final thickness of sample after the test = 2.80

Δh = Change in thickness from final stage to equilibrium stage

$= 2.80 - 2.60 = 0.20$ cm

$$\Delta e = \frac{0.20}{2.80}(1 + 0.648) = 0.118$$

Void ratio after consolidation

$$e_f - \Delta e = 0.648 - 0.118 = 0.530$$

Change in void ratio from the commencement to the end of consolidation is

$$\Delta e = \frac{3.00 - 2.60(1 + 0.530)}{2.60} = 0.235$$

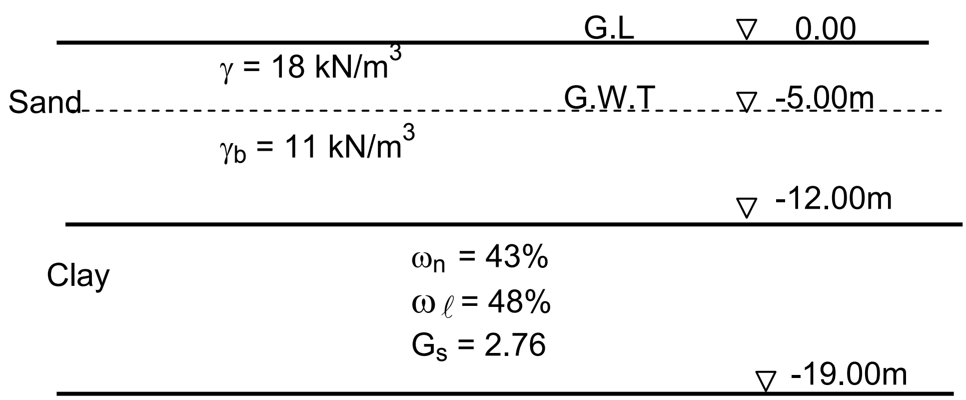
Void ratio before consolidation

$$e = 0.530 + 0.235 = 0.765$$

Example 5.2

A stratum of normally consolidated clay of 7 m thick is located at a depth of 12m below ground level. The natural moisture content of the clay is 43% and its liquid limit is 48%. The specific gravity of the solid particles is 2.76. The water table is located at a depth of 5m below ground surface. The soil is sand above the clay stratum. The submerged unit weight of sand 11kN/m^3 and the same weight 18kN/m^3 above the water table. The average increase in pressure at the center of the clay stratum is 120 kN/m^2 due to the weight of building that will be constructed on the sand above the clay stratum. Estimate the expected settlement of the structure.

Solution



Determination of e and γ_b for clay stratum

$$e = \omega_n G_s = 0.43 \times 2.76 = 1.19$$

$$\gamma_b = \frac{(Gs - 1)}{1 + e}\gamma_\omega = \frac{(2.76 - 1)}{1 + 1.19}10 = 8kN/m^3$$

Effective overburden pressure at the center of the clay layer, p_o

$$p_o = 18 \times 5 + 11 \times 7 + 8 \times 3.5 = 195 \text{ kN/m}^2$$

Compression index

$$C_c = 0.009\,(\omega_\ell - 10\%) = 0.009\,(48 - 10) = 0.34$$

Total settlement

$$S = \frac{C_c}{1+e} H \log_{10} \frac{p_o + p}{p_o}$$

$$= \frac{(0.34)}{1+1.19}(700)\log_{10}\frac{195+120}{195} = 22.6 \text{ cm}$$

Example 5.3

A 2.5 cm thick sample of clay was taken from field for predicting the time of settlement for a proposed building, which was, exerts a uniform pressure of 100 kN/m^2 over the clay stratum. The sample was loaded to 100kN/m^2 and proper drainage was allowed from top and bottom. It was seen 50% of the total settlement occurred in 3 minutes. Find the time required for 50% of the total settlement of the building if it is to stand on 6 m thick layer of clay which extends from ground surface and is underline by sand

Solution

The data applied to sample

$$U = 50\%$$

T for 50% consolidation = 0.197 (Table 5.1)

$$t = 3 \text{ minutes}, \qquad H_{dr} = \frac{2.5}{2}$$

The coefficient of consolidation, C_v, is

$$C_v = \frac{T_{50} H^2{}_{dr}}{t} = (0.197)\,\frac{\left(\frac{2.5}{2}\right)^2}{3} = 0.1026 \text{ cm}^2/\text{min}$$

The date applied to field

$$U = 50\%, \qquad T = 0.197$$

$$H_{dr} = \frac{600}{2}$$

$C_v = 0.1026\ cm^2/min$

The time required for 50% consolidation is,

$$t = \frac{T\ H^2{}_{dr}}{C_v} = \frac{0.197\left(\frac{600}{2}\right)^2}{0.1026} = 172807\ \text{min}$$

$$t = \frac{172807}{60\ \text{X}\ 24} = 120 days$$

Alternative Method

Since C_v is the same, both for the sample and the clay layer, we may write

$$\frac{(H_{dr})^2{}_{sample}}{t_{sample}} = \frac{(H_{dr})^2{}_{field}}{t_{field}}$$

$$t_{filed} = \frac{(H_{dr})^2{}_{field}}{(H_{dr})^2{}_{sample}}\ \text{X}\ t_{sample}$$

$$t_{field} = \frac{\left(\frac{600}{2}\right)^2}{\left(\frac{2.5}{2}\right)^2}\ \text{X}\ \frac{3}{60\ X\ 24}$$

$$t_{filed} = 120\ days$$

Example 5.4

Given, A soil profile shown below

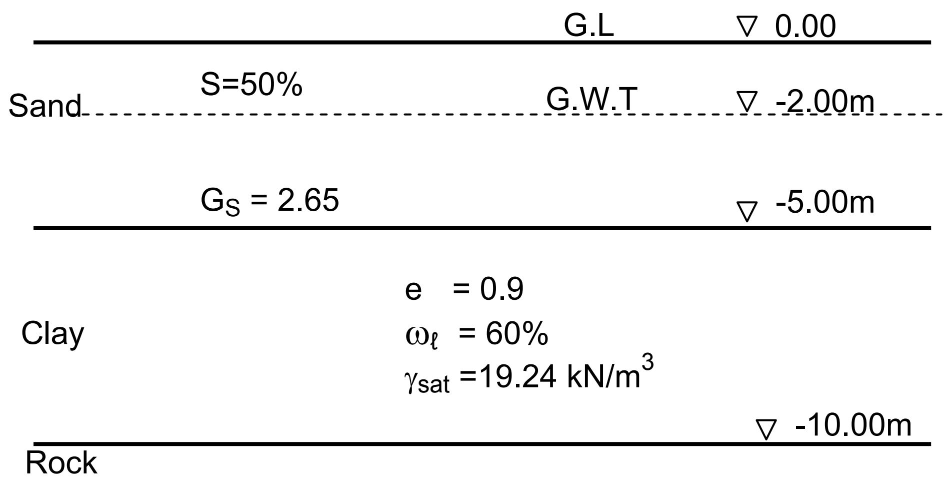
Required, a) The settlement due to primary consolidation for the 5 m clay layer due to a surcharge of 100kN/m^2 applied at ground level. The clay is normally consolidated.

b) The average degree of consolidation for the clay layer when the settlement is 10cm.

c) If the average value of C_v for the pressure range is 0.3 mm^2/sec, how long will it take for 50% settlement?

d) If the 5m clay layer is drained on both sides, how long will it take for 50% consolidation?

If the 5 m clay layer is drained on both sides, how long will it take for 50% consolidation?



Solution

a) Settlement

Determination of effective overburden pressure, p'_o, at the center of the clay layer. The moist unit weight of sand above the ground water table is,

$$\gamma_t = \frac{\gamma_\omega (Gs + eS)}{1+e} = \frac{10[2.65 + 0.7 \text{ X } 0.50]}{1+0.7} \qquad = 17.65 \text{ kN/m}^3$$

The submerged unit weight of sand below the ground water table is

$$\gamma_b = \frac{(G_S - 1)\gamma_\omega}{1+e} = \frac{(2.65-1)\text{X}10}{1+0.7} = 9.71 kN/m^3$$

The submerged unit weight of clay is

$$\gamma_b = \gamma_{sat} - \gamma_\omega = 19.24 - 10 = 9.24 \text{ kN/m}^3$$

$$p'_o = 2 \text{ x } 17.65 + 3 \text{ x } 9.71 + 2.5 \text{ x } 9.24 = 87.53 \text{ kN/m}^2$$

Calculation of compression index, C_c,

$$C_c = 0.009\ (\omega_\ell - 10) = 0.009\ (60 - 10) = 0.45$$

Calculation of settlement

$$S = \frac{C_c H}{1+e} \log 10 \frac{P_o + \Delta P}{P_o}$$

$$= \frac{(0.45)(500)}{1+0.9} \log_{10} \frac{87.53+100}{87.53} = 39.19 cm$$

b) Degree of consolidation, U,

$$U\% = \frac{Settlement\ \ at\ any\ time}{Maximum\ settlement} = \frac{10\ cm}{39.19\ cm} \text{ X } 100 = 25.5\%$$

c) Time for 50% consolidation, with single drainage

From Table 5.1, when U = 50%, T_{50} = 0,197

$$C_v = \frac{T_{50} H^2 dr}{t}$$

$$t = \frac{T_{50} H^2 dr}{Cv} = \frac{0.197 \text{ X} (5 \text{ X } 100 \text{ X } 10)^2}{0.3} \text{X} \frac{1}{60 x 60 x 24} = 190 days$$

d)Time for 50% consolidation, with double drainage

$$t = \frac{T_{50} H^2 dr}{Cv} = \frac{0.197 \text{ X} \left(\frac{5 \text{X} 100 \text{X} 10}{2}\right)^2}{0.3} \text{X} \frac{1}{60 \text{X} 60 \text{X} 24} = 47.50 days$$

EXERCISES

5.1 A bed of sand 10m thick in underlain by a compressible stratum of clay 6 m thick. The water table is at a depth of 4 m below ground level. The bulk densities of sand below and above the ground water table are 20.50 kN/m^3 and 17.70 kN/m^3 respectively. The clay has a natural water content of 42 percent, liquid limit 46 percent and specific gravity 2.76. Assume the clay to be normally consolidated, estimate the probable final settlement under an average excess pressure of 100 kN/m^2**.**

5.2 A 2.5 cm thick sample was tested in a consolidometer under saturated conditions with drainage both sides. 30 percent of consolidation was reached under a load in 15 minutes. For the same conditions of stressing but with only one way drainage, estimate the amount of time in days it would take for a 6m thick layer of the same soil to consolidate in the field to attain the same degree of consolidations.

5.3 A layer of normally loaded clay is 7 m thick and lies under recently constructed building. The weight of sand overlying the clay layer is 300 kN/m^2, and the new construction increases the overburden pressure by 100 KN/m^2. If the compression index is 0.50, compute the final settlement by assuming ω= 45 percent and G_s = 2.70.

5.4 In a laboratory consolidation test a sample of clay with a thickness of 2 cm reached 50 percent consolidation in 8 minutes. The sample was drained top and bottom. The clay layer from which sample was taken is 8 m thick. It is covered by a layer of sand through which water can escape and is underlain by a practically impervious bed of intact shale. How long will the clay layer required to reach 50 percent consolidation.

5.5 Given the following information for a normally loaded clay formation

In place void ratio = 1.40

Effective overburden pressure = 240 kN/m^2

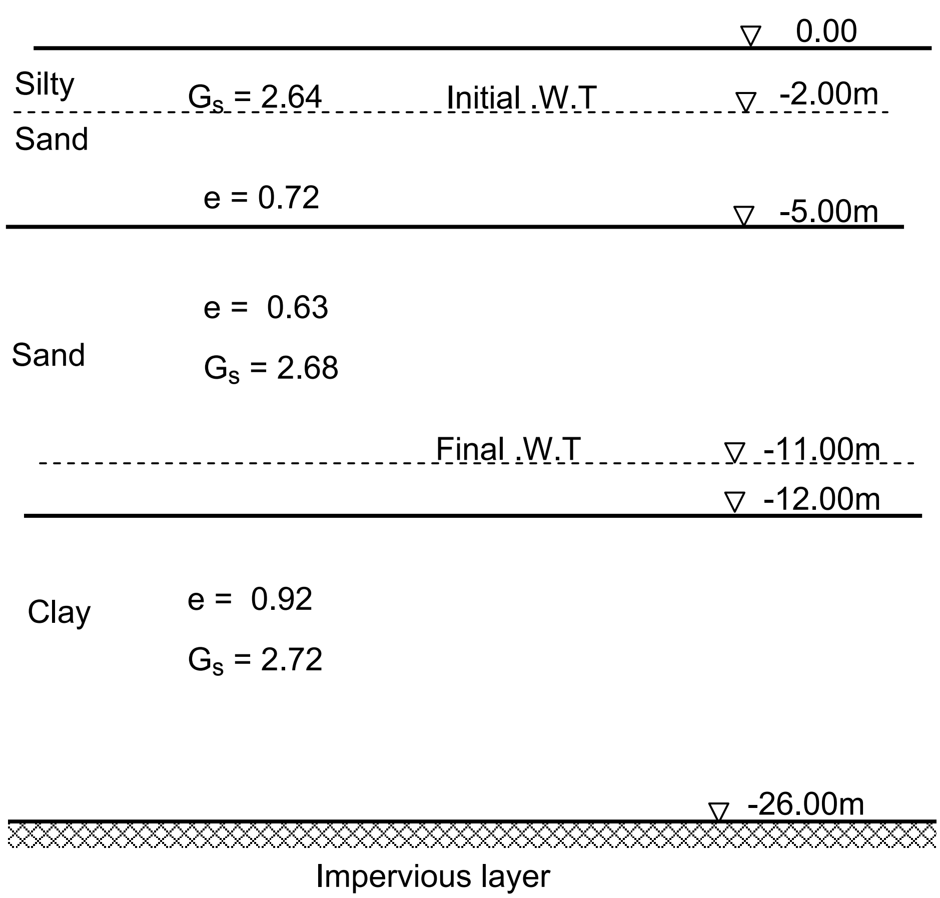
Total thickness = 6m

Compression index = 0.204

Coefficient of consolidation =9000 cm^2/month

A surface loading will create an average stress increment of 120kN/m^2 in the layer. If the clay has double drainage, find the time in months for a settlement of 4.5 cm due to this loading.

5.6 The soil profile at a building construction site is as shown below. If the water table is lowered by pumping from elevation -2.00m to elevation -11.00m and is permanently held there, determine the resulting settlement. The soil above water table (for each position) may be assumed to have water content of 28 percent. Assume C_c = 0.38 and C_v= 40x10^{-4} cm^2/sec. Calculate also the time required in days for 50 percent of the ultimate settlement occur.



6. STRESS DISTRIBUTION IN SOILS AND SETTLEMENT ANALYSIS

6.1 STRESS DISTRIBUTION IN SOILS

When a load is placed on the surface of the ground or below the ground level, some stresses are transmitted to the deeper layers of the soil underneath. These stresses decrease with increasing depth and distance from the loaded area. An adequate knowledge of distribution of these stresses is essential for predicting the settlement of structures due to compression of layers buried beneath the surface. Theoretical solutions for distribution of stresses induced by loads applied to elastic materials are available. The fundamental requirement for application of any of these solutions is that the stresses and corresponding strains are proportional.

Soils, in reality, are not elastic in nature. However, for some problems in soils where it is reasonable to assume that the stresses and corresponding strains are proportional, it may be worthwhile to determine the distribution of stresses by these theoretical relationships. In the consolidation theory, for example, it was assumed that for a load increment, the increase in effective pressure is proportional to the void ratio change and since void ratio change is approximately proportional to the vertical strain in one-dimensional consolidation, this assumption of proportionality between stresses and strains for an increment may be made with reasonable accuracy. In estimating stresses induced by loads for calculating the settlements under foundations in clayey soils, therefore, it may be worthwhile to use the formulae based upon the elastic theory. This chapter, therefore, presents methods based on elastic theory for estimating stresses in soils.

6.1.1 Boussinesq Equation for a Point Load

The state of stress at any point within an elastic, homogeneous and isotropic medium can be described by six components of stress-three of the them being normal stresses and the other being shear stresses. For the purposes of settlement analysis, however, we are concerned with only the vertical normal stress, σ_z the whole of which would finally be taken by the soil grains as intergranular or effective pressure.

Let a point load Q be applied on the ground surface. Also, let the point A, where the stress caused by this load Q is to be determined, be situated at a vertical distance z and horizontal radial distance r from the point of application of this point load. The vertical normal stress σ_z on such a point A, is given by Boussinesq, as

$$\sigma_z = \frac{Q}{2\pi} \frac{3z^3}{(r^2 + z^2)^{\frac{5}{2}}}$$

This can be arranged as

$$\sigma_z = \frac{Q}{z^2} \frac{\frac{3}{2\pi}}{\left[1 + \left(r/z\right)^2\right]^{\frac{5}{2}}} \qquad \ldots\ldots\ldots\ldots \qquad (6.1)$$

The above equation indicates that for a given load, σ_z is a function only of depth, z and the ratio of the radial to vertical distance, $\frac{r}{z}$

$$\sigma_Z = \frac{Q}{z^2} N_B$$

where $$N_B = \frac{\frac{3}{2\pi}}{\left[1 + \left(r/z\right)^2\right]^{\frac{5}{2}}} = \frac{0.477}{\left[1 + \left(r/z\right)^2\right]^{\frac{5}{2}}}$$

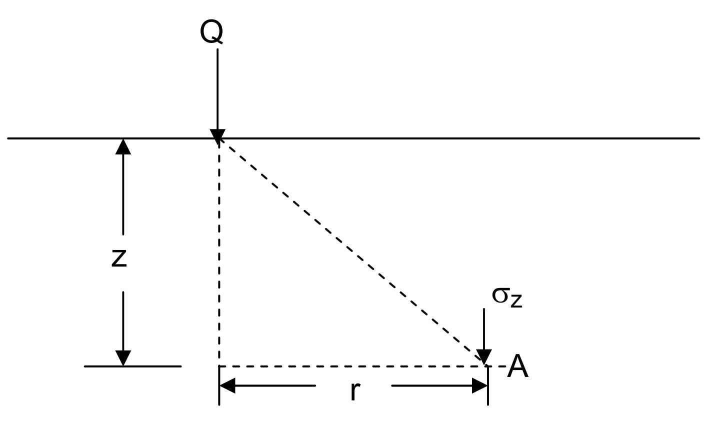
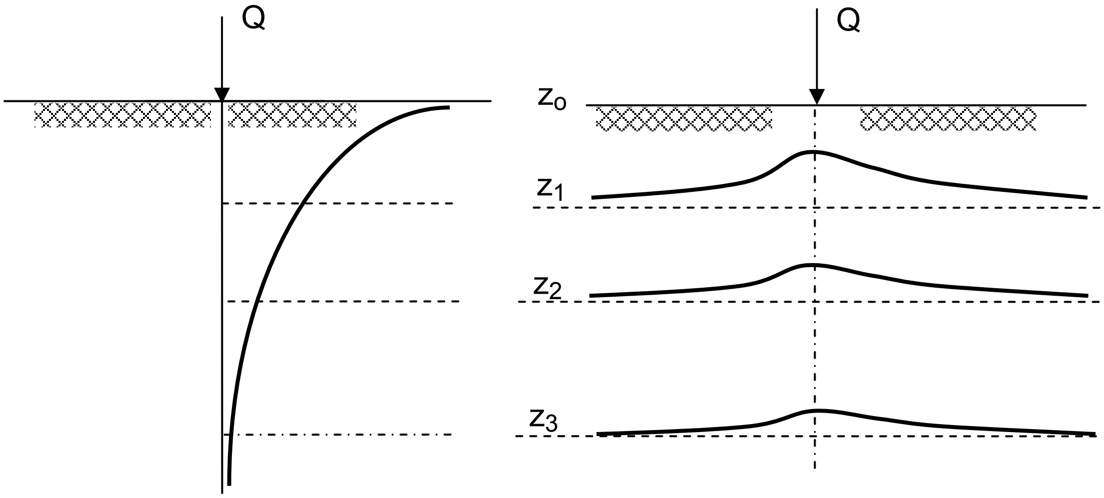


Fig. 6.1 Stress intensity under a point load

The variation of vertical stress due to point load with depth and horizontal distance from the point of application of the load is shown in Fig. 6.2



a) Variation of stress with depth b) Stress distribution on horizontal planes

Fig. 6.2 Vertical stresses in soil mass Due to point load applied at the ground surface.

6.1.2 Stress Due to Distributed Load

In actual structures, load is never applied at a point; it is spread over a certain area by the foundation structure. It is, therefore, necessary to determine the stress distribution in soil beneath loaded area. On basis of Boussinesq's equation for point loads, there are two possible approaches to the problem.

6.1.2.1 Application of Point Load Formula

Eqn. (6.1) may be used for the computation of stresses in a soil mass due to point loads acting at the surface. Since loads occupy finite areas, point load formula may still be used if the loaded areas divided into smaller areas as shown in Fig. 6.3 and a series of concentrated loads are assumed to act at the center of each smaller area. The only principle to be followed in dividing a bigger area into smaller blocks is that the width of the smaller block should be less than one-third the depth z of the point at which the stress is required to be compute. The loads acting at the centers of each smaller area may be considered as point loads and Boussinesq formula may then be applied.

$$\frac{a}{z} \le \frac{1}{3}, \frac{b}{z} \le \frac{1}{3}$$

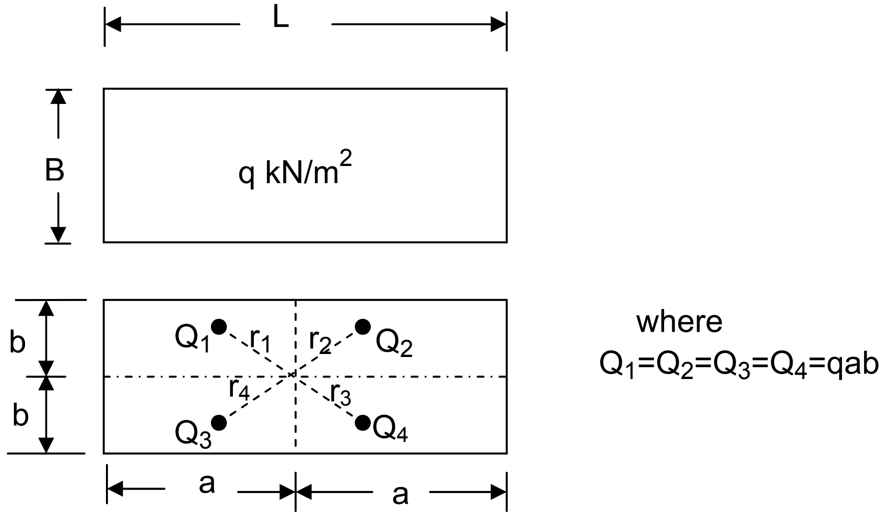


Fig. 6.3 Application of point load formula for rectangular load areas

1.1.2.2 Newmark's Influence Diagram

Newmark's charts are based on subdivision of the loaded area into infinitesimal units and on integration of the effects of simulated point loads. That is, if an area on the surface of a very large mass carries a uniformly distributed load of intensity q, the intensity of the vertical stress at any point within the mass may be computed by dividing the loaded area into small parts dA, each of which supports a load dQ = qdA. This load is considered to be concentrated at the centroid of the elementary area dA. According to Boussinesq's equation, each concentrated load produces at point N a vertical stress

$$d\sigma_Z = \frac{3q}{2\pi z^2}\left(\frac{1}{\left[1+\left(r/z\right)^2\right]^{5/2}}\right)dA \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (6.2)$$

The intensity of the vertical stress at N due to the load is computed by integrating the above equation over the loaded area

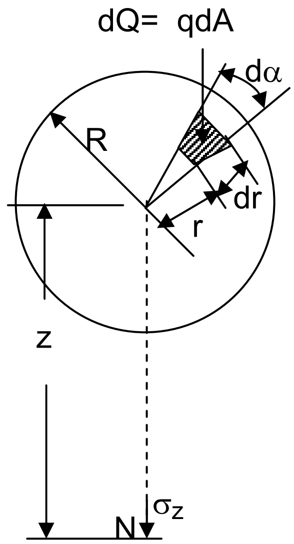


Fig 6.4 Analysis of vertical stress below the center of uniformly loaded circular area

$$\int d\sigma_z = \int \frac{3q}{2\pi\, z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} dA$$

$$\sigma_z = \int d\sigma_z = \int_0^{2\pi}\int_o^R \frac{3q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} r\,d\alpha\,dr \quad\cdots\cdots\cdots\cdots \qquad (6.3)$$

After performing the integration and inserting the limits one can obtain the following equation

$$\sigma_z = q\left[1-\left(\frac{1}{1+\left(R/z\right)^2}\right)^{3/2}\right] \quad\cdots\cdots\cdots\cdots \qquad (6.4)$$

or $$\frac{\sigma_z}{q} = 1-\left(\frac{1}{1+\left(R/z\right)^2}\right)^{3/2} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad (6.5)$$

It may be seen from Eqn. (6.5) that when $\frac{R}{z} = \infty, \frac{\sigma_Z}{q} = 1$, that is σ_z = q. This indicates that if the loaded extends to infinity the vertical stress in the semi-infinity solid at any depth z is the same as the unity load q at the surface. If the loaded area is limited to any given radius R it is possible to determine from Eqn. (6.5) the ratios $\frac{R}{z}$ for which the ratio of $\frac{\sigma_Z}{q}$ may have any specified value. Table 6.1 gives the ratio of $\frac{R}{z}$ for different values of $\frac{\sigma_Z}{q}$

Table 6.1 Values of $\frac{R}{Z}$ for Different Values of $\frac{\sigma_Z}{q}$

Circle No.	$\frac{\sigma_Z}{q}$	$\frac{R}{Z}$
0	0.0	0.000
1	0.1	0.270
2	0.2	0.401
3	0.3	0.518
4	0.4	0.637
5	0.5	0.766
6	0.6	0.917
7	0.7	1.110
8	0.8	1.387
9	0.9	1.908
	1	∞

Table 6.1 helps to compute the vertical stress σ_Z at any depth z below the center of a circular loaded area of radius R. For example at any depth z, the vertical stress σ_Z =0.8q if the radius of the loaded area at surface is R= 1.387z. At the same depth, the vertical stress is σ_Z =0.7q if R = 1.110z. If instead of loading the whole area, if only the annular space between the circles of radii 1.387 z and 1.110 z are loaded, vertical stress at z at the center of the circle is $\Delta\sigma_Z$ = 0.8 q – 0.7q = 0.1q. Similarly if the annular space between circles of radii 1.110 z and 0.917 z are loaded the vertical stress at the same depth z is $\Delta\sigma_Z$ = 0.7q - 0.6q= 0.1 q. We may therefore draw a series of

concentric circles on the surface of the ground in such a way that when the annular space between any two consecutive circles are loaded with a load q per unit area, the vertical stress $\Delta\sigma_z$ produced at any depth z below the center remains a constant fraction of q. we may write, therefore,

$$\Delta\sigma_z = C\,q$$

where C is a constant. If an annular space between any two consecutive circles is divided into n equal blocks and if any one of the blocks is loaded with a distributed load q, the vertical stress produced at the center is, therefore,

$$\frac{\Delta\sigma_z}{n} = \frac{C}{n} q = C_i q$$

$$\frac{\Delta\sigma_z}{n} = C_i \qquad \text{when } q = 1$$

That is, a load q = 1 covering one of the blocks will produce a vertical stress C_i. In other words, the "influence value" of each loaded block is C_i. If the number of loaded blocks are N, and if the intensity of load is q per unit area, the total vertical stress at depth z below the center of the circles is

$$\sigma_z = C_i N q \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (6.6)$$

Figure 6.5 shows an influence chart obtained by drawing concentric circles. The radii of the circles are equal to the $\frac{R}{z}$ values corresponding to $\frac{\Delta\sigma}{q}$ = 0, 0,1,0,2 …, 0.9. The unit length for plotting the circles is AB. The circles are divided in to several equally spaced radial lines. The influence value (C_i) of the chart is given $\frac{1}{n}$, where n is the number of elements in the chart.

The procedure for obtaining vertical pressure at any point below a loaded area is as follows.

1. Draw the foundation plan on a tracing paper to such a scale that the depth z at which the stress σ_Z is to be computed will be equal to distance AB of the chart.
2. Lay the tracing of the foundation plan over the chart in such a way that the surface point O beneath which the stress σ_Z is to be computed coincides with the center of the chart.

3. Count the number of blocks covered by the foundation area.
4. Multiply the number found by counting by the influence value of the blocks and distributed load of the product obtained gives the value σ_Z for that particular point.

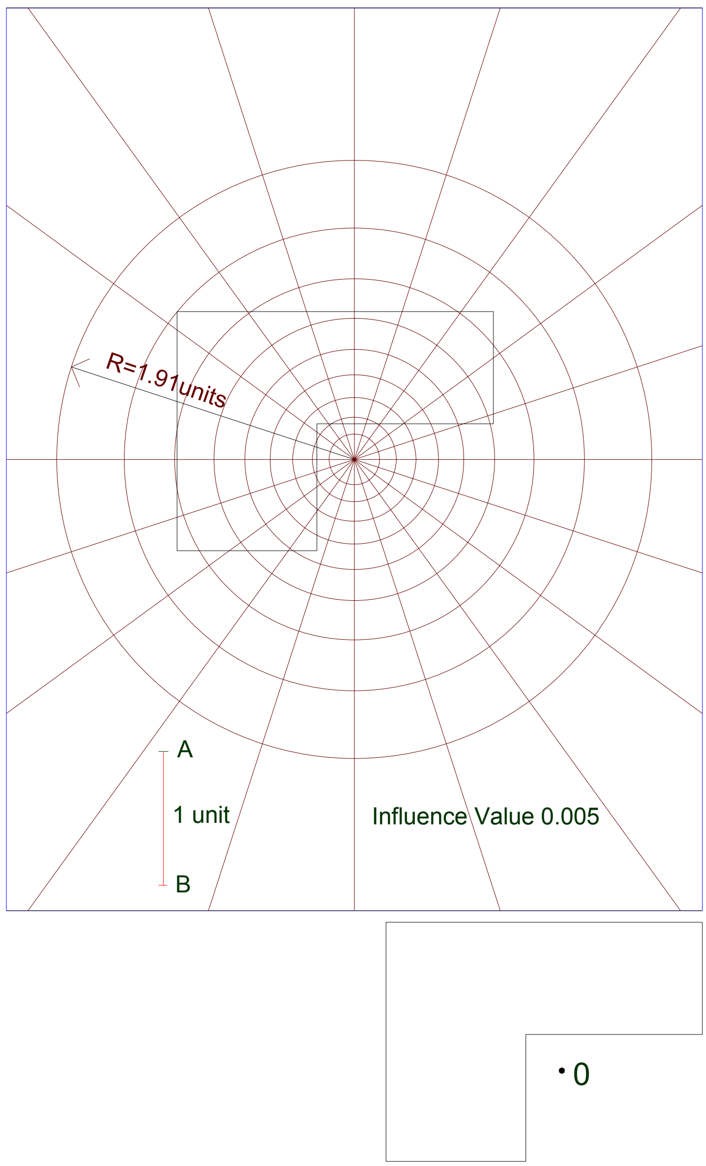


Fig.6.5 Newmark's influence chart

6.1.2.3 Vertical stress under a corner of a rectangular area

The Vertical stress under a corner of a rectangular area with a uniformly distributed load of intensity q can be obtained from Boussinesq's solution. Consider an infinitely small unit of area of size dx x dy, shown in Fig. 6.6. The pressure acting on the small area may be replaced by a concentrated load dQ applied to the center of the area.

Hence

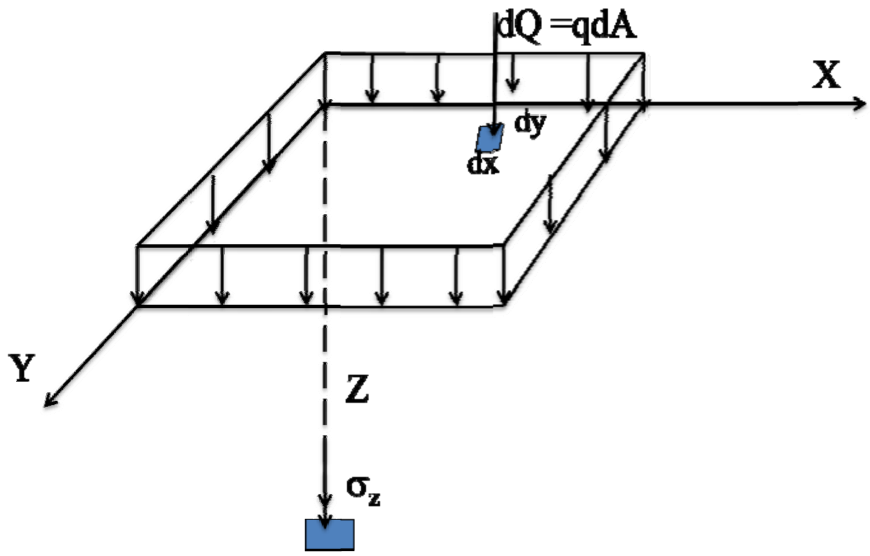


Fig.1.6 Vertical stress under a corner of a rectangular area

Hence

dQ= qdA= qdxdy

The increase of the vertical stress σ_z due to the load dQ can be expressed as per Eqn. (1.1) as

$$\Delta\sigma_z = \frac{3(qdxdy)z^3}{2\pi}\frac{1}{(x^2+y^2+z^2)^{5/2}}$$

By integration

$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^L\int_0^B \frac{qdxdy}{(x^2+y^2+z^2)^{5/2}}$$

Although the integral is quite complicated, Newmark was able to perform it. The results were presented as follows:

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1+m^2n^2} * \left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\frac{2mn((m^2+n^2+1)^{1/2}}{m^2+n^2+1-m^2n^2}\right] \ldots\ldots \quad (6.7)$$

Where m= B/z and n = L/z

The value of m and n can be interchanged without any effect on the values of σ_z. The above equation can be expressed as

$$\sigma_z = I_\sigma q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6.8)$$

Fadum gave charts for determination of the influence factor I_σ . The charts can be used for the determination of the vertical stress under a strip load, in which case the length tends to infinity and the curve for m ≅∞ can be used.

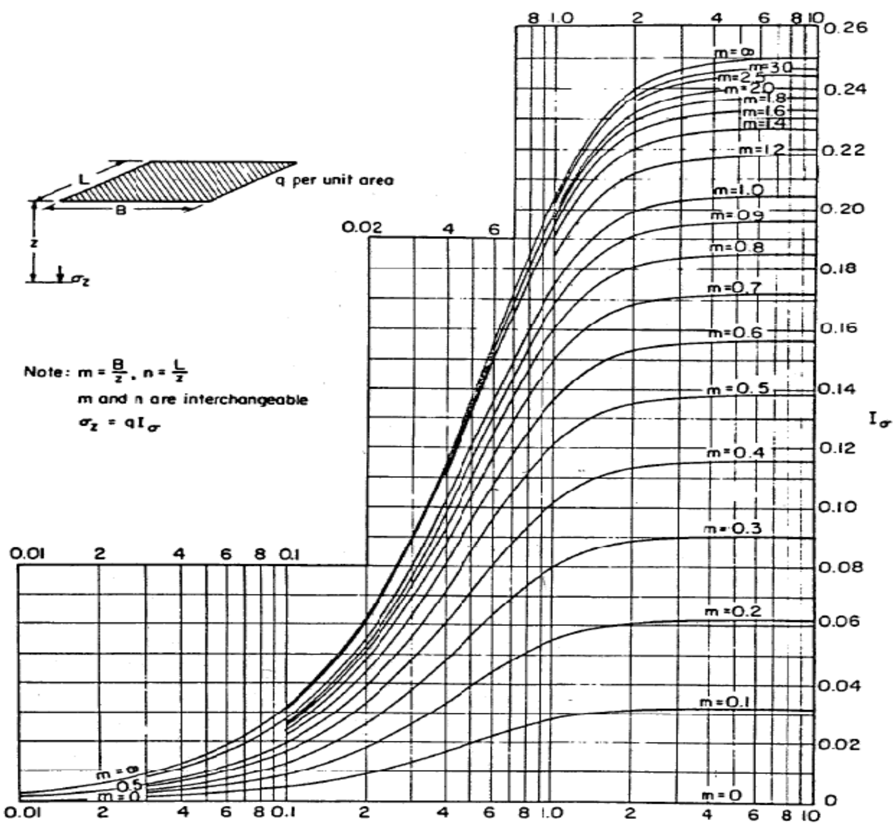


Fig.6.7 Fadum's chart

6.1.2.4 Vertical Stress at Any Point under a Rectangular Area

The equations developed in the preceding section can also be used for finding the vertical stress at a point which is not located below the corner. The rectangular area is subdivided into rectangles such that each rectangle has a corner at the point where the vertical stress is required. The vertical stress is determined using the principle of superposition. The following three cases can occur.

1) Point anywhere below the rectangular area. Fig. 6.8(a) shows the location of the point P below the rectangular area ABCD. The given rectangle is subdivided into 4 small rectangles AEPH, EBFP, HPGD and PFCG, each having one corner at P. The vertical stress at P due to the given rectangular load is equal to that from the four small rectangles. Therefore, using Eqn. 6.8,

$$\sigma_z = q[(I_\sigma)_1 + (I_\sigma)_2 + (I_\sigma)_3 + (I_\sigma)_4] \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.9)$$

where $(I_\sigma)_1$, $(I_\sigma)_2$,$(I_\sigma)_3$ and $(I_\sigma)_4$ are influence factors obtained from Fig. 6.7 for the four rectangles marked (1), (2), (3) and (4).

For the special case, when the point P is at the center of the rectangle ABCD, all the four small rectangles are equal, and Eqn. 6.9 becomes

$$\sigma_z = 4qI_\sigma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6.10)$$

where I_σ is the influence factor for the small rectangle

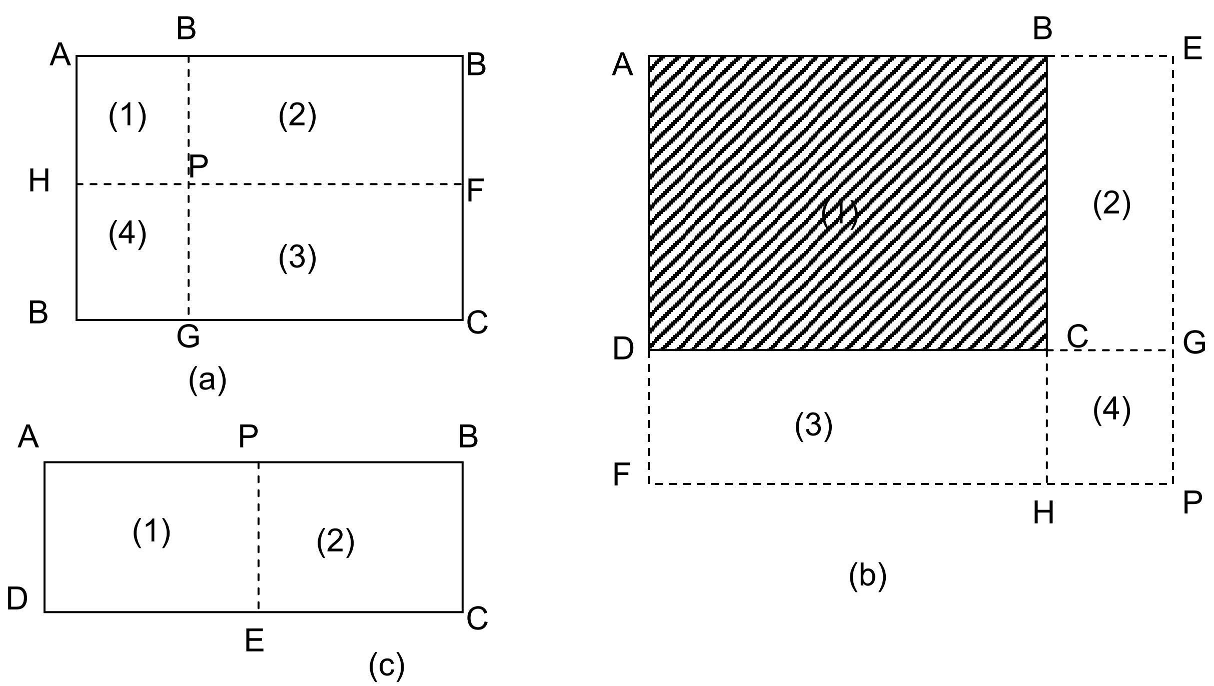


Fig . 6.8 Vertical stress under a rectangular area

2) Point outside the loaded area. Fig. 6.8(b) shows the point P outside the loaded area ABCD. In this case, a large rectangle AEPF is drawn with its one corner at P.

Now rectangle ABCD = rectangle AEPF –rectangle BEPH –rectangle DGPF + rectangle CGPH

The last rectangle CGPH is given plus sign because this area has been deducted twice, once in rectangle BEPH and once in DGPPF.

Therefore, the stress at P due to a load on rectangle ABCD is given by

$$\sigma_z = q[(I_\sigma)_1 - (I_\sigma)_2 - I_\sigma)_3 + (I_\sigma)_4] \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.11)$$

where $(I_\sigma)_1$, $(I_\sigma)_2$,$(I_\sigma)_3$ and $(I_\sigma)_4$ are the influence factors for the rectangles AEPF, BEPH, DGPF and CGPH, respectively.

3) Point below the edge of the loaded area. If the point P is below the edge of the loaded area ABCD (Fig. 6.8 c), the give rectangle is divided into two small rectangles APED and PBCE. In this case,

$$\sigma_z = q[(I_\sigma)_1 + (I_\sigma)_2] \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.12)$$

where $(I_\sigma)_1$ and $(I_\sigma)_2$ are the influence factors for the rectangles APED and PBCE, respectively

6.1.2.5 Two- to-One Load Distribution Method

This method assumes that stress increment at successive depths beneath the footing is distributed uniformly over a finite area. The finite area is defined by planes descending at a slope of 2:1 (2 vertical and 1 horizontal) from the edges of the footing. The planes descending from the edges of the footing (area A_1), at each depth define the area A_2 over which the stress is uniformly distributed. Thus the stress increment at any depth is assumed equal to the total load P on the footing divided by the area A_2 defined by the planes.

The above method is satisfactory for individual spread footing of relatively small area.

Stress at depth z, $$\sigma_z = \frac{P}{A_2}$$

For square footing $A_2 = (B + z)^2$

$A_1 = B^2$

For rectangular footing $A_2 = (B+z)\ (L+z)$

$A_1 = (B \times L)$

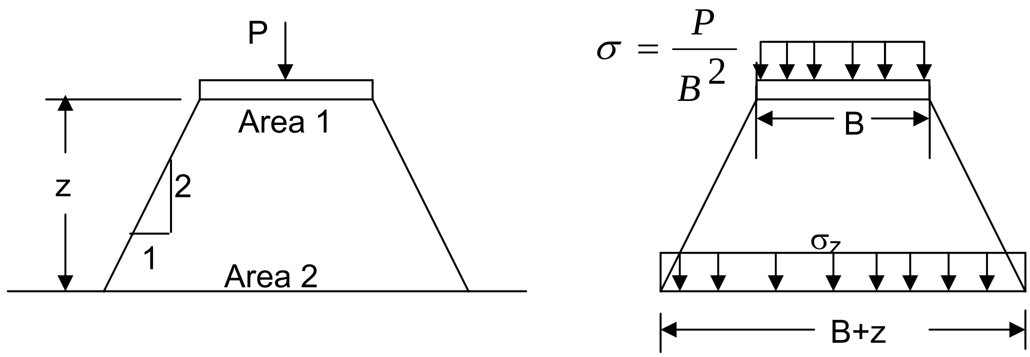


Fig.1.6 Two-to-one method for computing vertical stress

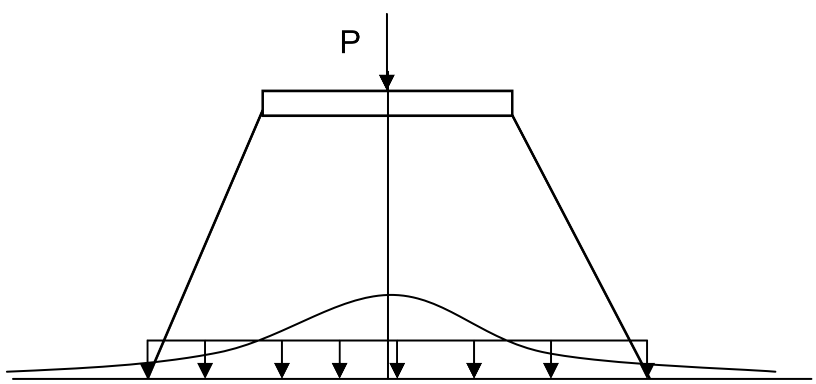


Fig 6.9 Comparison of vertical stress distribution by Boussinesq's and two-to-one method

6.2 SETTLEMENT ANALYSIS

6.2.1 Introduction

The increase of stress in soil layers due to the load imposed by various structures at the foundation level will always be accompanied by some strain, which will result in the settlement of the structures. The type of analysis that is used to predict the magnitudes of settlements and the times required for their occurrence is called a settlement analysis. The following are the necessary steps that one has to follow to predict the total settlement or to calculate the rate of settlement.

i) Soil Profile

The soil profile may vary within the foundation area. To establish the profile a minimum of five bore holes, four at the four corners and one in the center are needed. The location and thickness of the different soil strata, moisture content, specific gravity of soil grains, and void ratio of each are necessary information for computation of settlement. It is also essential to establish the position of the water table in order to compute the existing effective pressure in the soil mass.

ii) Consolidation Test

Consolidation test run on undisturbed sample from the compressible layer will provide such date as compression index, C_c, obtained from e-$\log^P$ curve, and coefficient of consolidation, C_v, for each loading increment and presented in the from of C_v versus $\log^P$ curves.

iii) Settlement Calculations

The following steps may be adopted for settlement calculation

a) Determine the pre-loading pressure, these are vertical pressures exerted by soil weight. Of practical interest, in this regard, is the intergranular pressure. For thick clay layers, the pressures are usually computed at the top, middle, and bottom of clay layer and their average is determined as follows

$$P_{avg} = \frac{1}{6}[P_t + 4P_m + P_b]$$

b) Determine stresses induced by column load: These are usually computed by Boussinesq’s equation. For thick layers, it is worthwhile determining the pressures at the top, middle, and bottom and taking a mean by Simpson’s Rule.

$$\Delta P_{avg} = \frac{1}{6}[\Delta P_t + 4\Delta P_m + \Delta P_b]$$

c) Tabulate final pressure analysis data

d) Computation of settlement

Initial intergranular pressure,

$p_{avg} = p_1$

Increased pressure due to column load,

$p_{avg} = \Delta p$

$$S = C_c \frac{H}{1+e_o} \log_{10} {P_2}/{P_1}$$

where $p_2 = p_1 + \Delta p$

e) Time settlement predictions:- Time settlement predictions are usually presented in the form of curves showing settlement in cm versus time in years.

Data needed to draw the curves

a) Ultimate settlement
b) Theoretical consolidation curve
c) The values of coefficient of consolidation, C_v
d) The thickness of the compressible stratum
e) Information on the drainage condition

If S_t is the settlement at any time t after the imposition of load on the compressible layer, the degree of consolidation of the layer in time t may also be expressed as

$$U\ \% = \frac{S_t}{S} \times 100$$

or $$S_t = (S)\,(U)$$

The rate of settlement curve of a structure built on a compressible layer may be obtained by the following procedure.

I. For different degrees of consolidation, compute the corresponding settlements using the above relationship.
II. From the theoretical curve giving the relation between U and T. Find T for different degrees of consolidation.
III. Compute from equation $t = \frac{TH^2 dr}{C_v}$, the values of t for different values of T.

IV. Now a curve can be plotted giving the relation between t and S_t as shown in Fig. 6.10

U	**T**	**t**	**S_t**
U_1	T_1	t_1	S_1
U_2	T_2	t_2	S_2
U_3	T_3	t_3	S_3
.	.	.	.
.	.	.	.

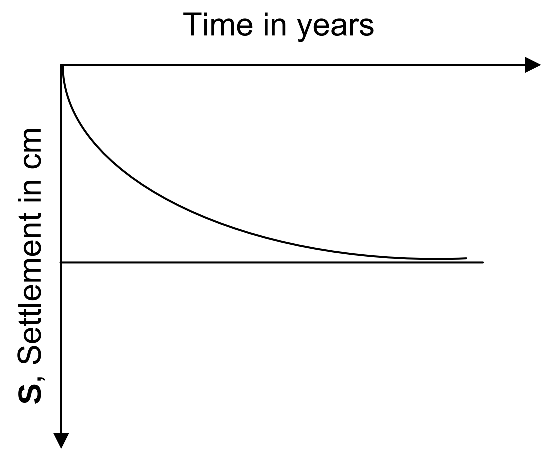


Fig. 6.10 Time settlements curve

6.2.2 Immediate Settlement

Immediate or elastic settlement of foundation occurs immediately after the application of load without change of moisture content. The magnitude of the contact settlement depends on the flexibility of foundation and material on which it is resting.

Immediate settlement for foundations resting on elastic material can be calculated from equations derived by using the principle of the theory of elasticity, which is of the form

$$S_i = qB\frac{(1-\mu^2)}{E_s} I\omega$$

where S_i = Immediate (elastic) settlement

q = Intensity of contact pressure

μ = Poisson's ratio

E_s = Modulus of elasticity of soil

B = Least lateral dimension of foundation

I_ω = Non dimensional influence factor

For computation of I_ω, for corners of flexible footing Terzaghi used the following relationships

$$I_\omega = \frac{1}{\pi}\left[m\log_e\left(\frac{1+\sqrt{m^2+1}}{m}\right) + \log_e\left(m+\sqrt{m^2+10}\right)\right]$$

where m = $\frac{(length\,of\;\;foundation)}{(width\,of\;\;foundation)}$

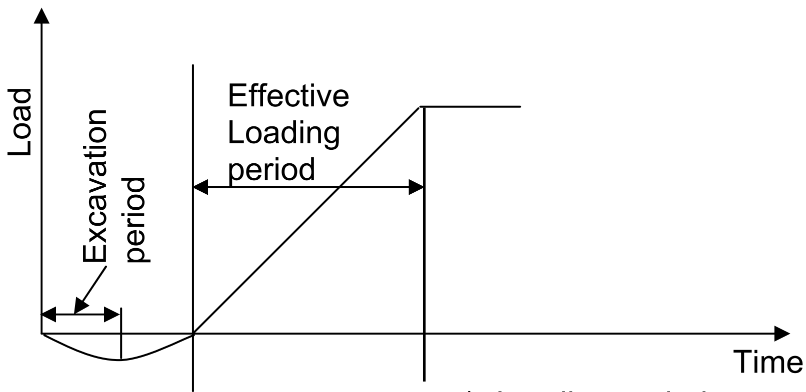
6.3 CONSTRUCTION PERIOD CORRECTION

The load due to a building or other structures is not applied instantaneously, but is spread over a certain period of time. First there is excavation for foundation that results in relief of pressure. After the excavation, the foundation is constructed and the load of the super structure is applied gradually, as shown in Fig. 6.11(a).

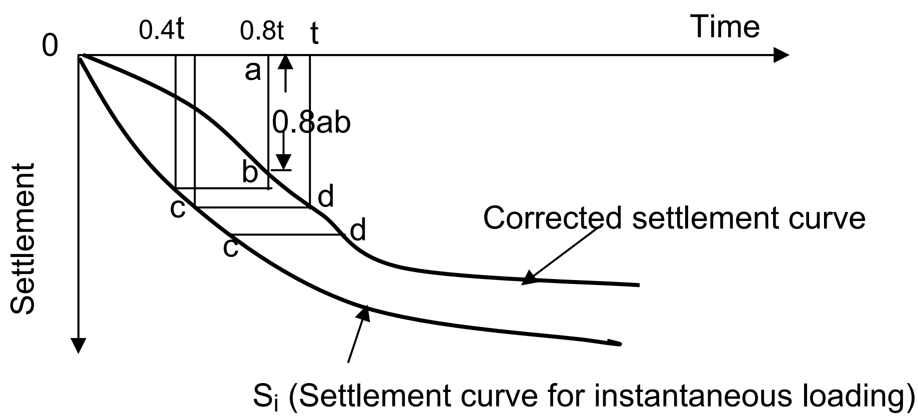
The following are the necessary procedures used to correct settlement for the construction period (refer to Fig. 6.11 (b)).

1. S_i is the instantaneous load settlement curve.
2. The correct settlement at time t is assumed to be the same as if the load is applied at time $\frac{t}{2}$. This is found by drawing a vertical line from $\frac{t}{2}$ to S_i and moving horizontally to time t.
3. The same procedure may be applied for points between 0 and t. For example take point 0.8t. At point 0.8t the corrected settlement is found by drawing a vertical line from 0.4t to curve S_i and then horizontally to the line 0.8 t to give ab. Since the load added at this time is 80% of the total load, the settlement S_i multiplied by a factor 0.8. This follows the same reasoning as noted above in that load applied gradually over the time 0.8t will have the same effect as the same load applied instantaneously at time 0.4 t.

4. Points beyond the time t are determined by displacing the curve S_i a horizontal distance equal to $\frac{t}{2}$ or cd as shown in Fig. 6.11 (b).



a) Loading period



b) Time settlement curve

Fig. 6.11 Construction period correction

Example 6.1

A concentrated load of 20 kN acts on the surface of a homogeneous soil mass of large extent. Find the stress intensity at a depth of 10m.

a) Directly under the load
b) At a horizontal distance of 5m

Given

a) Q = 20 kN

z = 10m, r = 0

Required σ_z

Solution

$$\sigma_z = \frac{Q}{z^2}\frac{\frac{3}{2\pi}}{\left[1+\left(r/z\right)^2\right]^{\frac{5}{2}}}$$

$$\sigma_z = \frac{20}{10^2}\frac{\frac{3}{2\pi}}{\left[1+\left(0/10\right)^2\right]^{\frac{5}{2}}} \qquad = 0.0955 \text{ kN/m}^2$$

Given

b) Q = 20 kN

z = 10m, r= 5m

Required σ_z

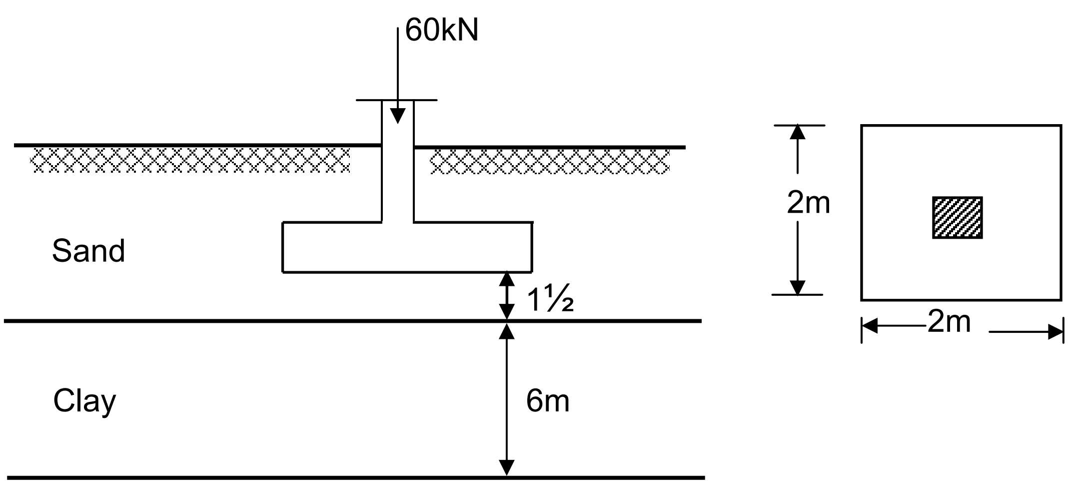
Solution

$$\sigma_z = \frac{Q}{Z^2}\frac{\frac{3}{2\pi}}{\left[1+\left(r/z\right)^2\right]^{\frac{5}{2}}} = \frac{20}{10^2}\frac{\frac{3}{2\pi}}{\left[1+\left(5/10\right)^2\right]^{\frac{5}{2}}} = 0.0547 \text{ kN/m}^2$$

Example 6.2

For the 2 m by 2m square footing shown below compute the intensity of vertical pressure below center of footing at the center of clay layer using.

a. Point load equation for uniformly distributed load and
b. Two-to-One distribution method. Compare the results obtained by the two methods.



a) Point load equation for uniformly distributed load

Given

$Q = 60$ KN

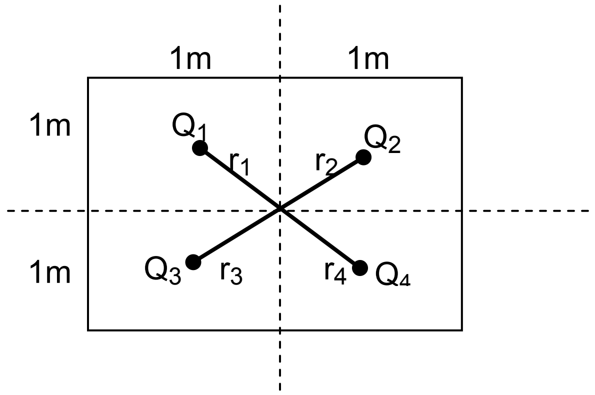
$z = 4.5$m

$a = b = 2$m

Required σ_z

Solution

$$\frac{a}{z} = \frac{2}{4.5} = 0.44 > \frac{1}{3}$$, divide the area into smaller areas



$$\frac{a}{z} = \frac{1}{4.5} = 0.22 < \frac{1}{3}$$

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{60}{2^2} \times 1^2 = 15\text{kN}$$

$$r_1 = r_2 = r_3 = r_4 = \sqrt{(0.5)^2 + (0.5)^2}$$

$$= 0.71 \text{ m}$$

$$\sigma_z = \sigma_{z1} + \sigma_{z2} + \sigma_{z3} + \sigma_{z4}$$

$$= 4\,\sigma_{z1}$$

$$\sigma_z = 4\left[\frac{Q}{z^2}\frac{\frac{3}{2\pi}}{\left[1+\left(r/z\right)^2\right]^{\frac{5}{2}}}\right]$$

$$= 4\left[\frac{15}{(4.5)^2}\frac{\frac{3}{2\pi}}{\left[1+\left(0.71/4.5\right)^2\right]^{\frac{5}{2}}}\right] = 1.33 \text{ kN/m}^2$$

b) Two-to-One distribution method

Given

$Q = 60\ kN$

$z = 4.5\ m,\ B = 2m$

Required σ_z

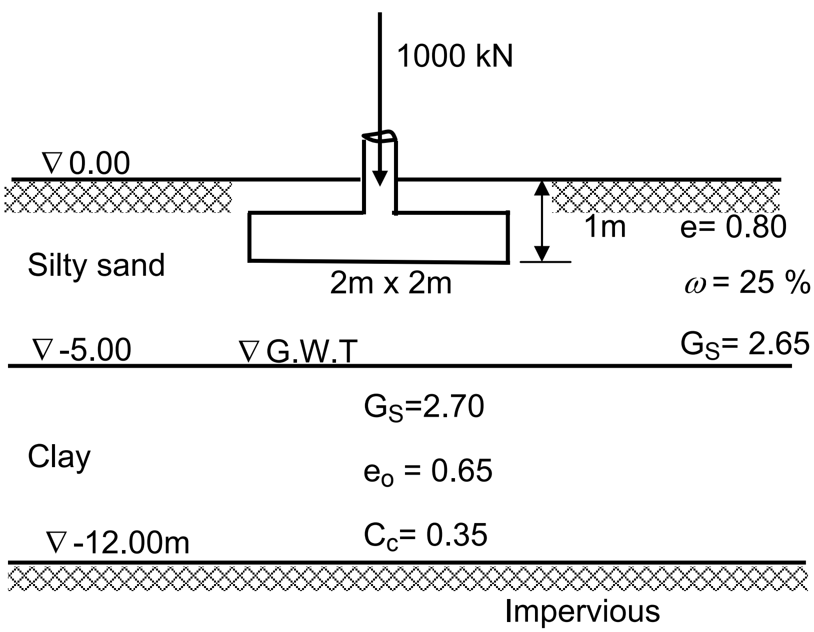
Solution

$$\sigma_z = \frac{Q}{(B+z)^2}$$

$$= \frac{60}{(2+4.5)^2} = 1.42\ kN/m^2$$

Example 6.3

The soil stratification of a building site with the proposed depth and width of a foundation element are shown in Figure below. Determine the total settlement of the footing due to the compression of the clay layer.



Solution

- Effective overburden pressure
- Unit weight of silty sand

$$\gamma_t = \frac{G_S \gamma_\omega (1+\omega)}{(1+e)}$$

$$= \frac{(2.65)(10)(1+0.25)}{(1+0.8)}$$

$$= 18.\ 40\ kN/m^3$$

- Submerged unit weight of clay

$$\gamma_b = \frac{\gamma_\omega (G_S - 1)}{(1+e)}$$

$$= \frac{10(2.70-1)}{(1+0.65)}$$

$$= 10.30\ kN/m^3$$

Effective overburden pressure at the top of the clay layer

$$p_t = (\gamma_t)(5) = (18.40)(5) = 92\ kN/m^2$$

Effective overburden pressure at the middle of the clay layer

$$p_m = (\gamma_t)(5) + (\gamma_b)(3.5)$$

$$= 92\ kN/m^2 + (10.30)\ (3.5)$$

$$= 128.05\ kN/m^2$$

Effective overburden pressure at the bottom of the clay layer

$$p_b = (\gamma_t)(5) + (\gamma_b)(7)$$

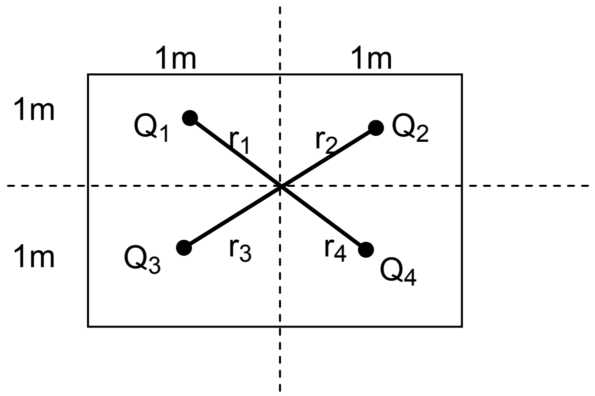
$$= 92\ kN/m^2 + (10.30)(7)$$

$$= 164.10\ kN/m^2$$

Stress due to column load

- At the top of the clay layer

$\frac{a}{z} = \frac{2}{4} = 0.5 > \frac{1}{3}$, divide the area into smaller areas



$\frac{a}{z}$ = ¼ = 0.25 < $\frac{1}{3}$

$Q_1 = Q_2 = Q_3 = Q_4 = \frac{1000}{2^2}(1)^2$ = 250 kN

$r_1 = r_2 = r_3 = r_4 =$ 0.707

$$\Delta p_t = \sigma_z = 4\left[\frac{Q}{z^2}\frac{\frac{3}{2\pi}}{\left[1+\left(r/z\right)^2\right]^{\frac{5}{2}}}\right]$$

$$= 4\left[\frac{250}{4^2}\frac{\frac{3}{2\pi}}{\left[1+\left(0.707/4\right)^2\right]^{\frac{5}{2}}}\right] = 27.63\ \text{kN/m}^2$$

- At the middle of the clay layer

$$\frac{a}{z} = \frac{2}{7.5} = 0.267 < \frac{1}{3}$$

$$\Delta p_m = \frac{1000}{(7.5)^2}(0.477) = 8.48 \text{ kN/m}^2$$

- At the bottom of the clay layer.

$$\Delta p_b = \frac{1000}{(11)^2}(0.477) = 3.94 \text{ kN/m}^2$$

- Average pressure

Due to effective overburden pressure

$$P_{avg} = \frac{1}{6}(p_t + 4p_m + p_b)$$

$$= \frac{1}{6}(92 + 4(128.05) + 164.10)$$

$$= 128.\ 05 \text{ kN/m}^2$$

$$\Delta P_{avg} = \frac{1}{6}(P_t + 4P_m + P_b) = \frac{1}{6}(27.63 + 4(8.48) + 3.94)$$

$$= 10.915 \text{ kN/m}^2$$

Settlement

$$S = \frac{C_C H}{1 + e_o} \log \frac{p_{avg.} + \Delta p_{avg.}}{P_{avg.}}$$

$$= \frac{(0.35)(700)}{1 + 0.65} \log \frac{128.05. + 10.915}{128.05.} = 5.28 \text{ cm}$$

EXERCISES

6.1 A load of 4500 kN is applied as a point concentration at the ground surface. Determine by the use of Boussinesq formula, the vertical pressure at a point 12 m below the load and 5 away horizontally.

6.2 The center of a rectangular area at ground surface has Cartesian coordinates (0,0) and the corners have coordinates (±2, ±5), all dimensions being in meters. The area carries a uniform pressure 15 kN/m^2. Estimate the stresses at a depth of 8 m below ground surface at each of the following locations. (0,0), (0,5), (2,0), (2,5) and (3,8), obtain by Boussinesq method.

6.3 a) Using the two-to-one distribution method calculate the average intensity of stress exerted by a concentrated load of 100 kN on a 2 m square footing at a depth of 6 m beneath the footing.

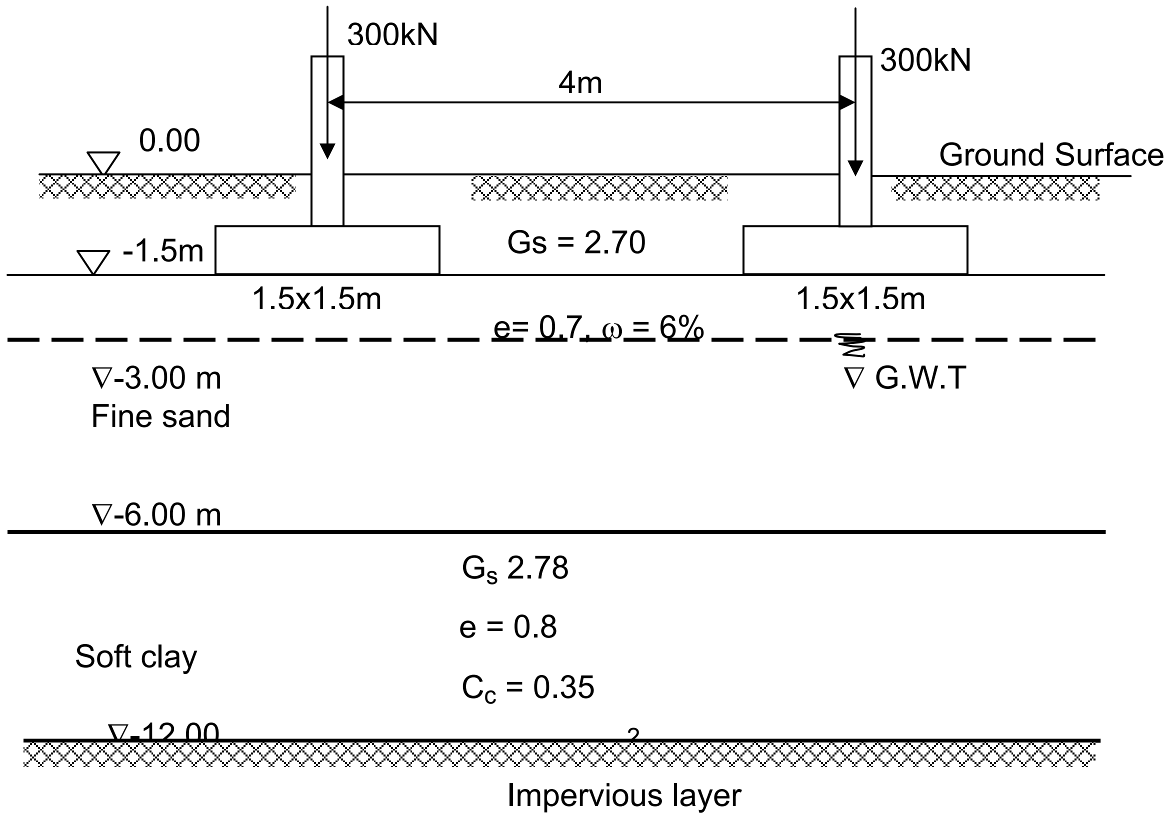
b) Make the same calculation using Boussinesq method.

6.4 A square footing is to be constructed at a distance of 1.5 m above a relatively weak soil layer. The average stress increments on the weak layer not exceed 5 kN/m^2. Under these conditions, what size of footing would be required to support a 50 kN column load.

6.5 The average soil profile at a proposed building site is as shown below. Two column footings 1.5m x 1.5m each, spaced at 4m apart center to center transmit building load of 300kN each as shown in the figure.

Determine;

a. The ultimate settlement of the columns
b. The time required for 50% consolidation


300kN
300kN
4m
0.00
Ground Surface
-1.5m
Gs = 2.70
1.5x1.5m
1.5x1.5m
e= 0.7, ω = 6%
∇-3.00 m
∇ G.W.T
Fine sand
∇-6.00 m
Gs 2.78
e = 0.8
Soft clay
Cc = 0.35
∇-12.00
Impervious layer