CHAPTER EIGHT1

SLOPE STABILITY

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7 Introduction

The term ***slope*** as used in here refers to any natural or man made earth mass, whose surface forms an angle with the horizontal. Hills and mountains, river banks, etc. are common examples of natural slopes. Examples of man made slopes include ***fills***, such as embankments, earth dams, levees; or ***cuts***, such as highway and railway cuts, canal banks, foundations excavations and trenches. Natural forces (wind, rain, earthquake, etc.) change the natural topography often creating unstable slopes. Failure of natural slopes (landslides) and man made slopes have resulted in much death and destruction.

In assessing the stability of slopes, geotechnical engineers have to pay particular attention to geology, drainage, groundwater, and the shear strength of the soils. The most common slope stability analysis methods are based on simplifying assumptions and the design of a stable slope relies heavily on experience and careful site investigation. In this chapter, we will examine the stability of earth slopes in two dimensional space using limit equilibrium methods.

When you complete this chapter, you should be able to:

- Understand the causes and types of slope failure.
- Estimate the stability of slopes using limit equilibrium methods.

Sample Practical Situation: A reservoir is required to store water for domestic use. Several sites were investigated and the top choice is a site consisting of clay soils (clay is preferred because of its low permeability – it is practically impervious). The soils would be excavated, forming sloping sides. You are required to determine the maximum safe slope of the reservoir.

7.0 Definitions of Key Terms

Slip plane or ***failure plane*** or ***slip surface*** or ***failure surface*** is the surface of sliding.

Sliding mass is the mass of soil within the slip plane and the ground surface.

Slope angle (or simply ***slope***) is the angle of inclination of a slope to the horizontal. The slope angle is usually referred to as a ratio, for example, 2:1 (horizontal: vertical)

7.1 Some Types of Slope Failure

Slope failures depend on the soil type, soil stratification, groundwater, seepage, and the slope geometry. A few types of slope failure are shown in Figure 7.1. Failure of a slope along a weak zone of soil is called a ***translational slide*** (Fig. 7.1 a).

Translational slides are common in coarse-grained soils.

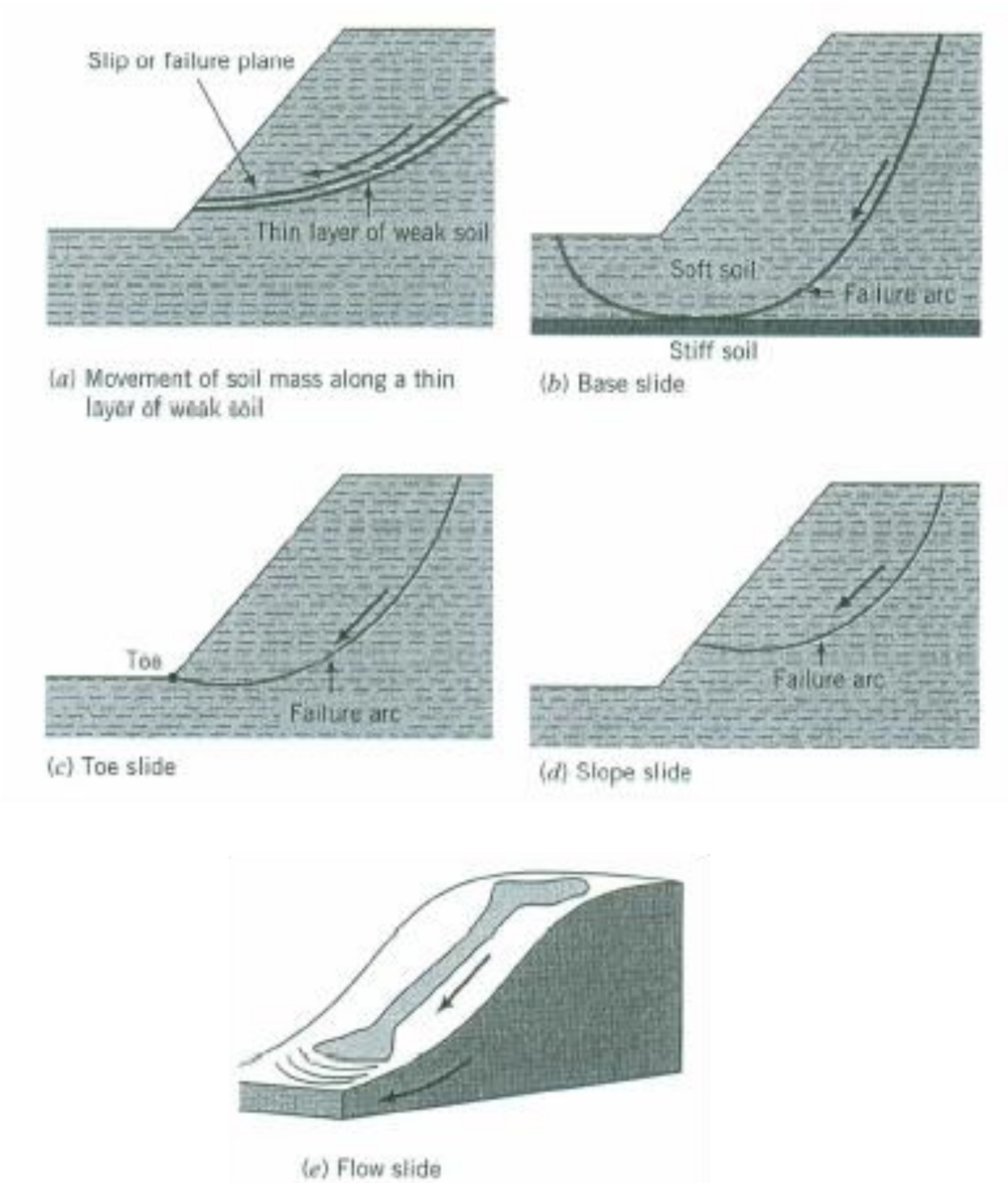

Figure 7.1: Some types of slope failure (Budhu, pp. 524)

A common type of failure in homogeneous fine-grained soils is a ***rotational slide***. Three types of rotational slides often occur. One type, called a ***base slide***, occurs by an arc enclosing the whole slope. A soft soil layer resting on a stiff layer of soil is prone to base failure (Fig. 7.1 b). The second type of rotational failure is the ***toe slide***, whereby the failure surface passes through the toe of the slope (Fig. 7.1 c). The third type of rotational failure is the ***slope slide***, whereby the failure surface passes through the slope (Fig. 7.1 d). ***A flow slide*** occurs when internal and external conditions force a soil to behave like a viscous fluid and flow down even shallow slopes, spreading out in several directions (Fig. 7.1 e).

7.2 Some Causes of Slope Failure

Slope failures are caused in general by natural forces, human mismanagement and activities. Some of the main factors that provoke failure are summarised in Figure 7.2 below.

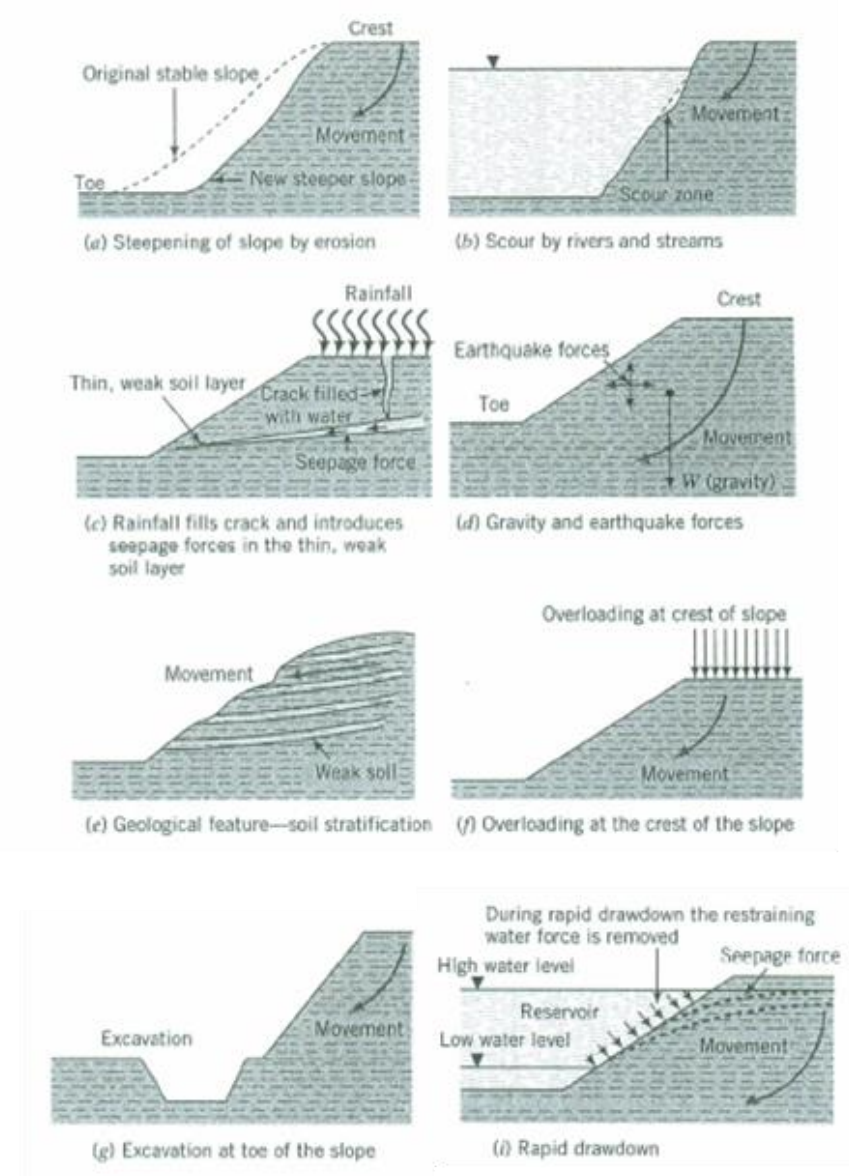

Figure 7.2: Some causes of slope failure (Budhu, pp. 526)

As shown in Fig. 7.2, some of the most common causes of slope failures are erosion, rainfall, earthquake, geological features, external loading, construction activities (ex. excavation & fill), and reservoir rapid drawdown.

7.3 Two-Dimensional Slope Stability Analysis

Slope stability can be analyzed using one or more of the following: the limit equilibrium method, limit analysis, finite difference method, and finite element method.

Limit equilibrium is the most widely used method for stability analysis. In the following sections, we will learn some of the commonly used slope stability analysis methods that are based on the limit equilibrium.

7.4 Stability Analysis of Infinite Slopes

Infinite slopes have dimensions that extend over great distances. In practice, the infinite slope mechanism is applied to the case when a soft material of very long length with constant slope may slide on a hard material (e.g. rock) having the same slope. Let's consider a clean, homogeneous soil of infinite slope α_s as shown in Figure 4.3. ***To use limit equilibrium method, we must first speculate on a failure of slip mechanism***. We will assume the slip would occur on a plane parallel to the slope. If we consider a slice of soil between the surface of the soil and the slip plane, we can draw a free-body diagram of the slice as shown in Figure 7.3.

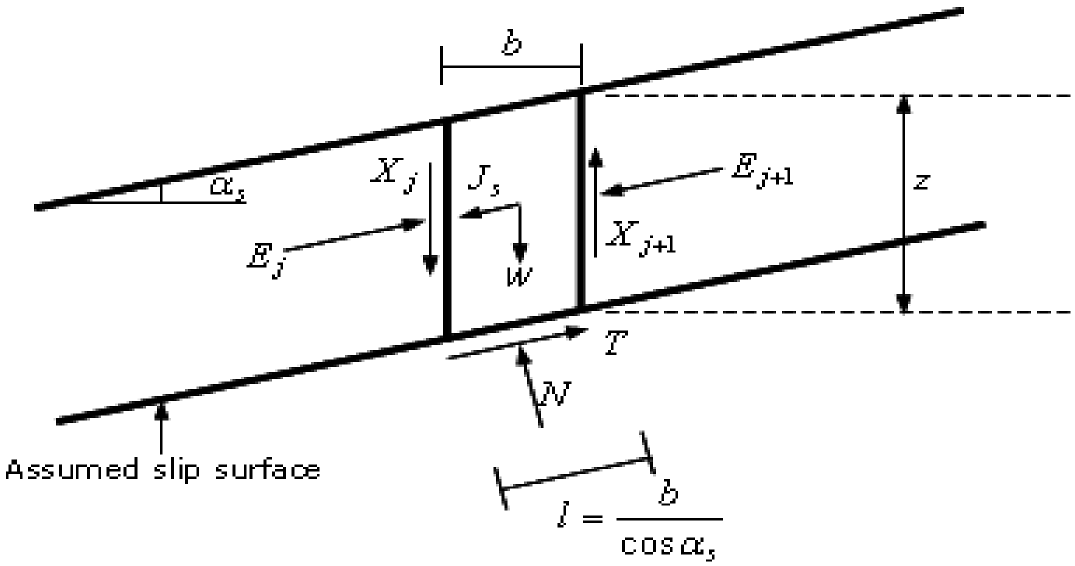

Figure 7.3: Forces on a slice of soil in an infinite slope.

The forces acting on the slice per unit thickness are the weight, $W = \gamma bz$, the shear forces X_j and X_{j+1} on the sides, the normal forces E_j and E_{j+1} on the sides, the normal force N on the slip plane and the mobilized shear resistance of the soil, T, on the slip plane. We will assume that forces that provoke failure are positive. If seepage is present, a seepage force $J_s = i\gamma_w bz$ develops, where i the hydraulic gradient. For a uniform slope of infinite extent, $X_j = X_{j+1}$ and $E_j = E_{j+1}$. To continue with the limit equilibrium method, we must now use the equilibrium equations to solve the problem. But before that we will define the factor of safety (***FS***) of a slope in the following subsection. The general objective of infinite slope stability analysis is to determine either the ***critical slope*** or ***critical height***, or alternatively, the ***factor of safety*** of the slope.

7.4.1 Factor of Safety

The factor of safety of a slope is defined as the ratio of the available shear strength, τ_f , to the minimum shear strength required to maintain stability (which is equal to the mobilized shear stress on the failure surface), τ_m , that is:

$$FS = \frac{\tau_f}{\tau_m} \tag{7.1}$$

The shear strength of the soil is governed by the Mohr-Coulomb failure criterion (Chapter 1).

7.4.2 Stability of Infinite Slopes in ϕ_u =0, c_u soil.

For the ϕ_u =0, c_u soil, the Mohr-Coulomb shear strength is given by:

$$\tau_f = c_u \tag{7.2}$$

From statics and using Figure 4.3,

$$N = W\cos\alpha_s \quad \text{And} \quad T = W\sin\alpha_s \tag{7.3}$$

The shear stress per unit length on the slip plane is given by:

$$\tau_m = \frac{T}{l} = \frac{W\sin\alpha_s\cos\alpha_s}{b} = \frac{\gamma bz}{b}\sin\alpha_s\cos\alpha_s = \gamma z\sin\alpha_s\cos\alpha_s \tag{7.4}$$

The factor of safety is then,

$$FS = \frac{c_u}{\gamma z\sin\alpha_s\cos\alpha_s} = \frac{2c_u}{\gamma z\sin(2\alpha_s)} \tag{7.5}$$

At limit equilibrium, FS = 1. Therefore, the ***critical slope*** is

$$\alpha_c = \tfrac{1}{2}\sin^{-1}\left(\frac{2c_u}{\gamma z}\right) \tag{7.6}$$

And the ***critical depth*** is:

$$z_c = \frac{2c_u}{\gamma\sin(2\alpha_s)} \tag{7.7}$$

7.4.3 Stability of Infinite Slopes in c’, ϕ' soils - with no seepage.

For a c’, ϕ' soil, the Mohr-Coulomb shear strength is given by:

$$\tau_f = c' + \sigma_n' \tan\phi' \tag{7.8}$$

The factor of safety FS is then:

$$FS = \frac{c' + \sigma_n' \tan\phi'}{\tau_m} = \frac{c'}{\tau_m} + \frac{\sigma_n' \tan\phi'}{\tau_m} \tag{7.9}$$

The normal and shear stresses per unit length at the failure plane in reference to figure

7.3 are given by:

$$\sigma'_n = \frac{N}{l} \quad \text{And} \quad \tau_m = \frac{T}{l} \tag{7.10}$$

For a slope without seepage, J_s=0. From Eqns. (7.4, 7.9 and 7.10) we get:

$$FS = \frac{c'}{\gamma z \sin\alpha_s \cos\alpha_s} + \frac{W'\cos\alpha_s \tan\phi'}{W\sin\alpha_s} = \frac{c'}{\gamma z \sin\alpha_s \cos\alpha_s} + \frac{\tan\phi'}{\tan\alpha_s} \tag{7.11}$$

At limit equilibrium FS = 1. Therefore, the ***critical depth*** z_c is given by

$$z_c = \frac{c'}{\gamma}\left(\frac{\sec^2\alpha_s}{\tan\alpha_s - \tan\phi'}\right) \tag{7.12}$$

For the case where, $\alpha_s < \phi'$, the factor of safety is always greater than 1 and is computed from Eqn. (7.6). This means that there is no limiting value for the depth *z*, and at an infinite depth, the factor of safety approaches to $\tan\phi' / \tan\alpha_s$. For a coarse-grained soil with *c*' = 0, Eqn. (4.6) becomes:

$$FS = \frac{\tan\phi'}{\tan\alpha_s} \tag{7.13}$$

At limit equilibrium FS = 1. Therefore, the ***critical slope angle*** is:

$$\alpha_c = \phi' \tag{7.14}$$

The implication of Eqn. (7.8) is that the maximum slope angle of a coarse-grained soil with *c*' = 0, can't exceed ϕ'. In other words, the case *c*' = 0 and $\alpha_s > \phi'$ is always unstable and can not be applied to practical situations.

Example 7.1

An infinitely long slope is resting on a rock formation with the same inclination. The height of the slope is 3.2 m.

Determine a) the factor of safety, b) the shear stress developed on the sliding surface, and c) the critical height. $\alpha_s = 25^0$, $\gamma = 17.5$ kN/m^3, *c*' = 12 kPa and $\phi' = 20^0$.

7.4.4 Stability of Infinite Slopes in c', ϕ' soils – steady state seepage.

We will now consider **groundwater at the ground surface** and assume that **seepage is parallel to the slope**. The seepage force is:

$$J_s = i\gamma_w bz$$

Since seepage is parallel to the slope, $i = \sin\alpha$. From statics,

$$N' = W'\cos\alpha_s = \gamma' bz\cos\alpha_s \tag{7.15}$$

and

$$\begin{aligned} T &= W'\sin\alpha_s + J_s \\ &= \gamma' bz\sin\alpha_s + \gamma_w bz\sin\alpha_s = (\gamma' + \gamma_w) bz\sin\alpha_s \\ &= \gamma_{sat} bz\sin\alpha_s \end{aligned} \tag{7.16}$$

Therefore, the shear stress at the slip plane is:

$$\tau_m = \frac{T}{l} = \frac{\gamma_{sat} bz \sin\alpha_s \cos\alpha_s}{b} = \gamma_{sat} z \sin\alpha_s \cos\alpha_s$$

From the definition of factor of safety (Eqn. 5.3), we get:

$$\begin{aligned} FS &= \frac{c'}{\gamma_{sat} z \sin\alpha_s \cos\alpha_s} + \frac{\gamma' bz \cos\alpha_s \tan\phi'}{\gamma_{sat} zb \cos\alpha_s \tan\alpha_s} \\ &= \frac{c'}{\gamma_{sat} z \sin\alpha_s \cos\alpha_s} + \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi'}{\tan\alpha_s} \end{aligned} \qquad (7.17)$$

At limit equilibrium, FS=1. Therefore, the ***critical height*** is:

$$z_c = \frac{c' \csc^2 \alpha_s}{\gamma \tan\alpha_s - \gamma' \tan\phi'}$$

At infinite depth the factor of safety in Eqn. (7.17) becomes:

$$FS = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi'}{\tan\alpha_s} \qquad (7.19)$$

Eqn. (7.19) can also be used for calculating the factor of safety for a coarse-grained soil with c' = 0. At limit equilibrium FS = 1, and hence, the critical slope for a coarse-grained soil with c' = 0 is given by:

$$\tan\alpha_s = \frac{\gamma'}{\gamma_{sat}} \tan\phi' \qquad (7.18)$$

For most soils, $\gamma'/\gamma_{sat} \approx \frac{1}{2}$. Thus, seepage parallel to the slope reduces the limiting slope of a clean, coarse-grained soil by about one-half.

If the groundwater level is not at the ground surface, weighted average unit weights have to be used in Eqns. (7.17 and 7.18).

Example 7.2

A long slope of 4.5 m deep is to be constructed of material having the following properties: γ_{sat} =20 kN/m^3, γ_{dry} =17.5 kN/m^3, c'=10 kPa, and ϕ' =32^0.

Determine the factor of safety a) when the slope is dry, b) there is steady state seepage parallel to the surface with the water level 2 m above the base and c) the water level is at the ground surface.

7.5 Rotational Slope Failure

The infinite slope failure mechanism is reasonable for infinitely long and homogeneous slopes made of coarse-grained soils, where the failure plane is assumed to be parallel to the ground surface. But in many practical problems slopes have been observed to fail through a rotational mechanism of finite extent. As shown in Fig. (7.1), rotational failure mechanism involves the failure of a soil mass on a circular or

non-circular failure surface. In the following sections, we will continue to use the limit equilibrium method assuming a circular slip surface. Those methods,
Which are based on non-circular slip surface, are beyond the scope of this course

7.5.1 Stability of Slopes in c_u, ϕ_u =0 soils – circular failure surface.

The simplest circular analysis is based on the assumption that a rigid, cylindrical block will fail by rotation about its center and that the shear strength along the failure surface is defined by the undrained strength c_u. Figure 7.4 shows a slope of height *H* and angle α_s. The trial circular failure surface is defined by its center *C*, radius *R* and central angle θ.

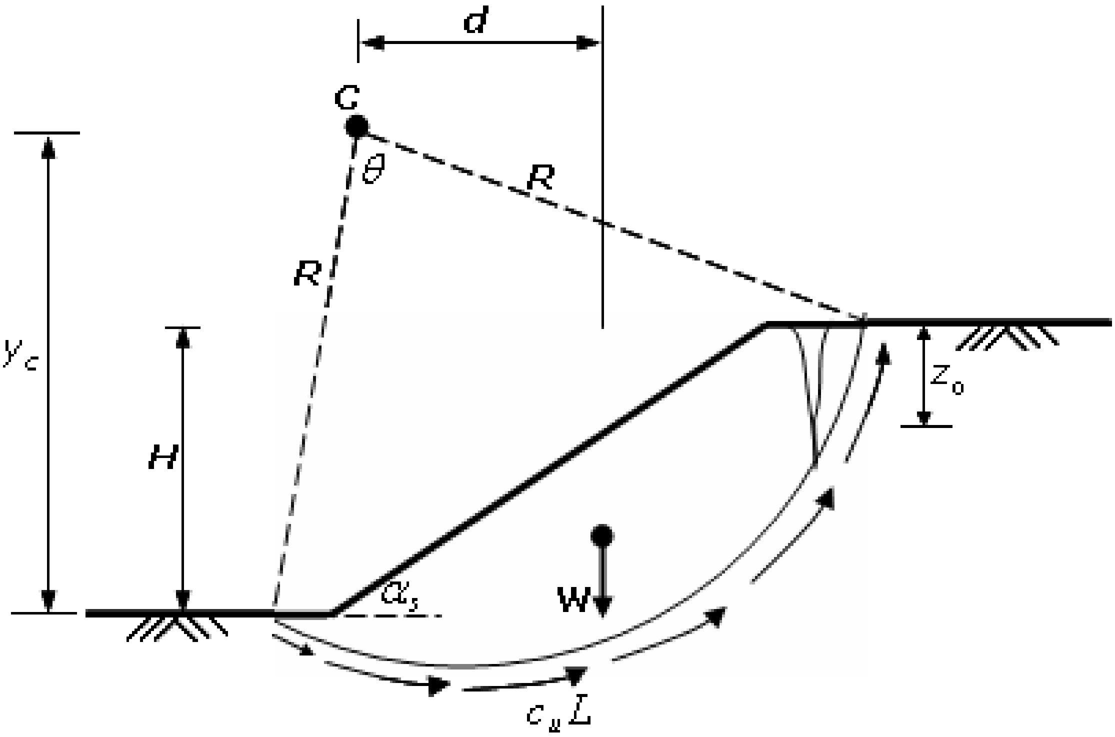

Figure 7.4: Slope failure in c_u, ϕ_u =0.

The weight of the sliding block acts at a distance *d* from the center. Taking moments of the forces about the center of the circular arc, we have:

$$FS = \frac{c_u LR}{Wd} = \frac{c_u R^2 \theta^0}{Wd} \times \frac{\pi}{180^0} \tag{7.19}$$

Where *L* is the length of the circular arc, *W* is the weight of the sliding mass and *d* is the horizontal distance between the circle center, *C*, and the centroid of the sliding mass. If c_u varies along the failure surface, then:

$$FS = \frac{R^2 (c_{u1}\theta_1^{\ 0} + c_{u2}\theta_2^{\ 0} + ... + c_{un}\theta_n^{\ 0})}{Wd} \times \frac{\pi}{180^0} \tag{7.20}$$

The centroid of the sliding mass is obtained using a mathematical procedure based on the geometry or the sub-division of the sliding mass into narrow vertical slices.

Example 7.3

Find the factor of safety of a 1V:1.5H slope that is 6 m high. The center of the trial mass is located 2.5 m to the right and 9.15 m above the toe of the slope. Cu = 25 kPa, and γ =18 kN/m^3. Take d = 3.85 m.

7.5.2 Effect of Tension Cracks

Tension cracks may develop from the upper ground surface to a depth z_0 that can be estimated using Eqn. (7.13). The effect of the tension crack can be taken into account by assuming that the trial failure surface terminates at the depth z_0, thereby reducing the weight W and central angle θ. Any external water pressure in the crack creates a horizontal force that must be included in equilibrium considerations.

Example 7.4

Rework Example 4.3 by taking into account tension cracks. Geometric data are: θ=66.6^0, area of sliding mass = 27.46 m^2 and d = 3.48 m.

7.5.3 Stability of Slopes in c', ϕ' soils – Method of Slices.

The stability of a slope in a c', ϕ' soil is usually analyzed by discretizing the mass of the failure slope into smaller slices and treating each individual slice as a unique sliding block (Fig. 7.5). This technique is called ***the method of slices***.

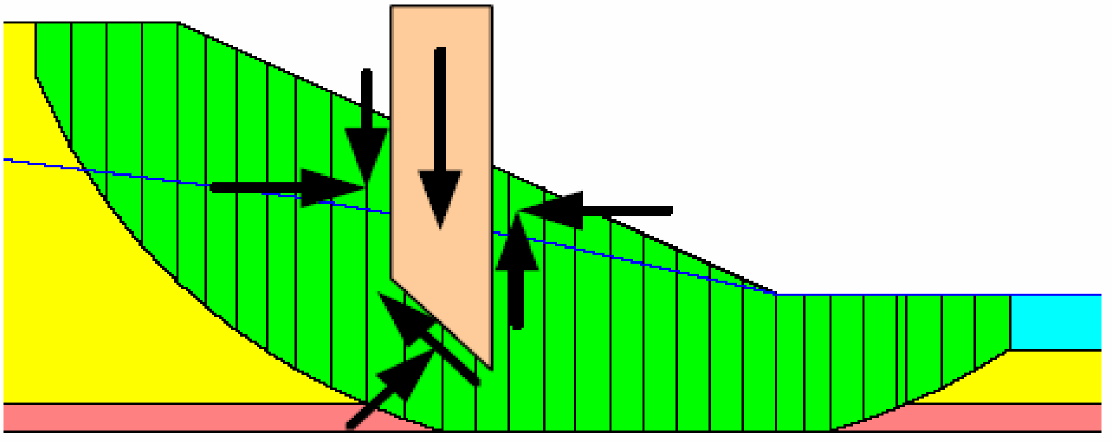

Figure 7.5: Slice discretization and slice forces in a sliding mass.

In the method of slices, the soil mass above a trial failure circle is divided into a series of vertical slices of width b as shown in Fig. 7.6 (a). For each slice, its base is assumed to be a straight line defined by its angle of inclination θ with the horizontal whilst its height h is measured along the centerline of the slice.

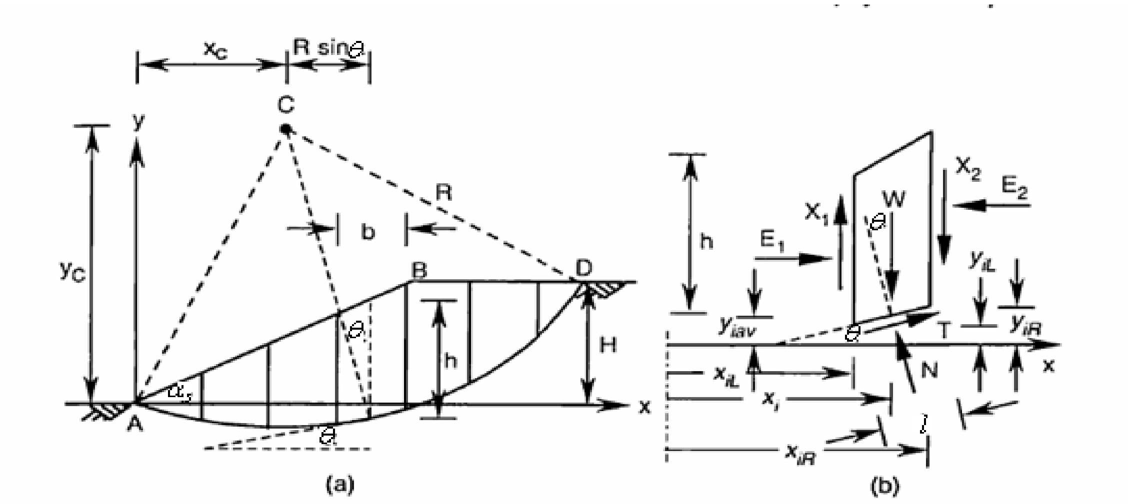

Figure 7.6 a) Method of slices in c', ϕ' soil, b) Forces acting on a slice.

The forces acting on a slice shown in Fig. 7.6 (b) are:

W = total weight of the slice = $\gamma \times h \times b$

N = total normal force at the base = ***N'*** + ***U***, where ***N'*** is the effective total normal force and ***U*** = ***ul*** is the force due to the pore water pressure at the midpoint of the base length ***l***.

T = the mobilized shear force at the base $= \tau_m \times l$, where τ_m is the minimum shear stress required to maintain equilibrium and is equal to the shear strength divided by the factor of safety, $\tau_m = \tau_f / FS$.

$\boldsymbol{X}_1$, $\boldsymbol{X}_2$ = shear forces on sides of the slice and $\boldsymbol{E}_1$, $\boldsymbol{E}_2$ = normal forces on sides the slice. The sum of the moments of the inter slice or side forces about the centre *C* is zero.

Thus, for moment equilibrium about the centre *C* (note the normal forces pass through the centre):

$$\sum_{i=1}^{i=n} T_i R = R\sum_{i=1}^{i=n}(\tau_m l) = R\sum_{i=1}^{i=n}\frac{(\tau_f l)_i}{FS} = \sum_{i=1}^{i=n}(W\sin\theta)_i R \qquad (7.20)$$

Where, *n* is the total number of slices. Replacing τ_f by the Mohr-Coulomb shear strength, we obtain:

$$FS = \frac{\sum_{i=1}^{i=n}\left[(c'+\sigma_n'\tan\phi')l\right]_i}{\sum_{i=1}^{i=n}(W\sin\theta)_i} = \frac{\sum_{i=1}^{i=n}\left[(c'l+N'\tan\phi')\right]_i}{\sum_{i=1}^{i=n}(W\sin\theta)_i} \qquad (7.21)$$

The term *c'l* may be replaced by $c'b/\cos\theta$. For uniform *c*', the algebraic summation of *c'l* is replaced by *c'L*, where *L* is the length of the circular arc. ***The values of N' must be determined from the force equilibrium equations***. ***However, this problem is statically***

indeterminate – because we have six unknown variables for each slice but only three equilibrium equations. Therefore some simplifying assumptions have to be made. In this chapter two common methods that apply different simplifying methods will be discussed. These methods are called the *Fellenius method* and *Bishop simplified method*.

7.5.3.1 Fellenius or Ordinary or Swedish Method

The ordinary or Swedish method of slices was introduced by Fellenius (1936). This method assumes that for each slice, the interslice forces $X_1=X_2$ and $E_1=E_2$. Based on this assumption and from statics, the forces normal to each slice are given by:

$$N = W\cos\theta = N' + ul \quad \Rightarrow \quad N' = W\cos\theta - ul \tag{7.22}$$

Substituting *N*' into Eqn. 5.21, we obtain:

$$FS = \frac{\sum_{i=1}^{i=n}\left[(c'l + (W\cos\theta - ul)\tan\phi')\right]_i}{\sum_{i=1}^{i=n}(W\sin\theta)_i} \tag{7.23}$$

For convenience, the force due to pore water is expressed as a function of *W*:

$$r_u = \frac{u_i b_i}{W_i} \tag{7.24}$$

Where r_u is called the pore water pressure ratio. Consequently, we have:

$$FS = \frac{\sum_{i=1}^{i=n}\left[(c'l + W(\cos\theta - r_u\sec\theta)\tan\phi')\right]_i}{\sum_{i=1}^{i=n}(W\sin\theta)_i} \tag{7.25}$$

The term r_u is dimensionless because the term $ub = \gamma_w \times h_w \times b \times 1$ represents the weight of water with a volume of $h_w \times b \times 1$. Furthermore, r_u can be simplified as follows:

$$r_u = \frac{ub}{W} = \frac{\gamma_w h_w b}{\gamma hb} = \frac{\gamma_w h_w}{\gamma h} \tag{7.26}$$

In the case of the steady state seepage the height of water above the midpoint of the base is obtained by constructing the flow net. Alternatively, an average value of r_u may be assumed for the slope. By doing so it is assumed that the height of water above the base of each slice is a constant fraction of the height of each slice. If the height of the water and the average height of the slice are equal, the maximum value of r_u becomes γ_w/γ, which for most soils, is approximately 0.5. Note that the effective normal force *N*' acting on the base is equal to $N' = W\cos\theta - ul$ or $N' = W(\cos\theta - r_u\sec\theta)$. If the term $(\cos\theta - r_u\sec\theta)$ is negative, *N*' is set to zero because effective stress can not be less than zero (i.e. soil has no tension strength).

The whole procedure explained above must be repeated for a number of trial circles until the ***minimum factor of safety corresponding to the critical circle*** is determined. The accuracy of the predictions depends on the number of slices, position of the critical surface, and the magnitude of r_u. There are several techniques that are used to reduce the number of trial slip surfaces. One simple technique is to draw a grid and selectively use the nodal points as centers of rotation.

Example 7.5

Using Fellenius' method of slices, determine the factor of safety for the slope of example 4.3 for r_u = 0 and 0.4. Take the number of slices as 8, each having 1.5 m width (check the width of the last slice). Soil properties are c' = 10 kPa, $\phi' = 29^0$, and $\gamma = 18$ kN/m^3.

Bishop Simplified Method

This method assumes that for each slice $X_1 = X_2$ but $E_1 \neq E_2$. These assumptions are considered to make this method more accurate than the Swedish method. An increase of 5% to 20% in the factor of safety over the Swedish method is usually obtained. Referring to Figure 7.6 b, and writing the force equilibrium in vertical direction (in order to eliminate E_1 and E_2), the following equation for N' can be found:

$$N' = \frac{W - ul\cos\theta - \dfrac{c'l\sin\theta}{FS}}{\cos\theta + \dfrac{\sin\theta\tan\phi'}{FS}} \qquad (7.27)$$

In addition to the force in the vertical direction, Bishop Simplified method also satisfies the overall moment equilibrium about the center of the circle as expressed in Eqn. (7.21). Putting $l = b/\cos\theta$ and $ub = r_u W$, and substituting Eqn. (7.27) into Eqn. (7.21), we obtain:

$$FS = \frac{1}{\sum_{i=1}^{i=n}(W\sin\theta)_i}\sum_{i=1}^{i=n}\left[\frac{c'b + W(1-r_u)\tan\phi'}{m_\theta}\right]_i \qquad (7.28)$$

where,

$$m_\theta = \cos\theta + \frac{\sin\theta\tan\phi'}{FS} \qquad (7.29)$$

Equation (7.29) is non-linear in *FS* (that is *FS* appears on both sides of the equations) and is solved by iteration. An initial value of *FS* is guessed (slightly greater than FS obtained by Fellenius' method) and substituted to Eqn. (7.29) to compute a new value for *FS*. This procedure is repeated until the difference between the assumed and computed values is negligible. Convergence is normally rapid and only a few iterations are required. ***The procedure is repeated for number of trial circles to locate the critical failure surface with the lowest factor of safety***.

Example 7.6

Re-work Example 7.5 for r_u = 0.4 using Bishop's simplified Method.