CHAPTER FOUR

4. KINEMATICS OF FLUID FLOW

INTRODUCTION: -Kinematics of fluid deals with the geometry of motion, i.e. space – time relation ships of fluids only without regards to the forces causing the motion.

They are generally deals with velocity & acceleration of fluid, and the description and visualization of motion.
The concept of a free body diagram, as used in static of rigid bodies in a fluid static is usually inadequate for the analysis of moving fluids. Instead we frequently find the concepts of system & control volume to be useful in the analysis of fluid mechanics.

A fluid system refers to a specific mass of fluid within the boundaries defined by a closed surface. The shape of the system, & so the boundaries, may change with time, as when liquid flows through a constriction, as a fluid moves& deforms, so the system containing it moves & deforms.
In contrast, ***a control volume*** refers to a fixed region in space, which doesn't move or change shape. It is usually chosen as a region that fluid flows in to & out of it.
The control volume approach is also called the Eulerian approach.

In the Eulerian method the observers concern is to know what happens at any given point in the space, which is filled by fluid in motion, what are the velocities, acceleration, pressure, etc at various parts at a given time.

Therefore, Eulerian method is mostly used because it is more useful in the analysis of the majority of engineering problems.

4.1. DIMENSION OF FLOW

A Fluid flow said to be one, two or three-dimensional flow depending up on the number of independent space coordinate & required to describe the flow.

When the dependent variables (example, velocity, pressure density etc) are a function of one space co-ordinate say x- coordinate) it is known as one-dimensional flow.
Example of one –dimensional flow (1D): flow through pipes & channels, between boundaries, etc if the velocity distribution is considered constant at each cross-section.

‘’ One-dimension’’ is taken along the central streamline of the flow dependent variables vary only with x- direction (or s- direction).

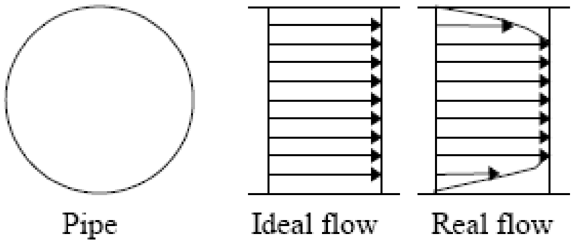


Fig 4.1 One-dimensional flow in a pipe

When the dependent variables vary only with two-space coordinates, the flow is known as two-dimensional flow (2D).

Example: Flow over a weir

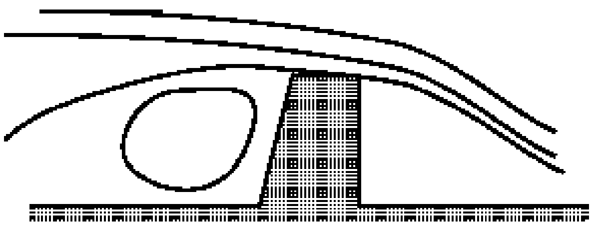


Fig 4.2 Two-dimensional flow over a weir

Generally a, fluid is a rather complex three- dimensional, time dependent phenomenon, i.e., V= V(x, y, z, t). In almost any flow situation, the velocity field actually contains all three-velocity components (u, v, w) & each is a function of all three-space coordinates (x, y, z).

Example of a 3D flow: the flow of air past an airplane wing provides a complex three-dimensional flow.

4. 1.1 Velocity & Acceleration in a fluid flow

In general, fluids flow from one point in space to another point as a function of time. This motion of fluid is described in terms of the velocity & acceleration of the fluid particles. At a given time instant, a description of any fluid property (such as density, pressure, Velocity, & acceleration) may be given as a function of the fluids location.

i.e. V = u (x, y, z, t)i +v (x, y, z, t)j + w (x, y, z, t)k

An infinitesimal change in velocity (δu) is given by:

$$\delta u = \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z + \frac{\partial u}{\partial t}\delta t$$

The acceleration components are given by:

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + w\frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial t}$$

a_y, a_y & a_z are called total or substantial acceleration in the x, y & z direction, the components are called convective acceleration excluding the last expression $\left(\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \& \frac{\partial \omega}{\partial t}\right)$ Which are called local acceleration

Total acceleration $\Rightarrow \bar{a} = a_x\ i + a_y\ j + a_z k$

- Convective acceleration – it is instantaneous space rate of change of velocity,
- Local acceleration: - it is the local time rate of change of velocity,

Example1: A fluid flow is described by the velocity field:

V = $5x^3$ j - 15 x^2 y j + t k. Evaluate the velocity & acceleration components at pints (1, 2, 3, 1)

4.2. Describing the pattern of flow

Although fluid motion is complicated, there are various concepts that can be used to help in the visualization & analysis of flow fields. This pattern of flow may be described by mean of streamlines, stream tubes, path lines and streamlines.

Stream lines: - it is an imaginary curve drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of the velocity of flow at those points.

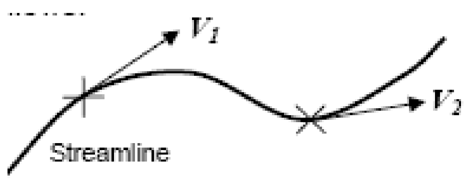
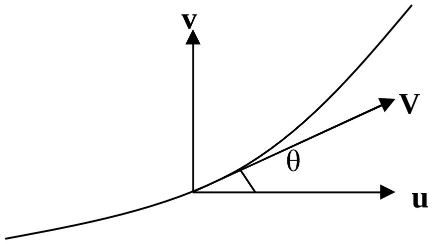


Fig. 4.4 streamlines

Since the velocity vector is everywhere tangent to the streamlines, there can be no component of velocity at right angles to the streamlines and hence there is no flow across the streamlines.
Since the instantaneous velocity at a point in a fluid must be unique in magnitude & direction, the same point can't pass more than one streamlines. Therefore, streamlines don't cross or intersect each other.

The velocity vector at point p must be tangent to the streamline at that point.



Therefore, $\frac{dy}{dx} = \tan\theta = \frac{v}{u}$

$u\,dy - v dx = 0$ …………Equation of streamlines

Example: - Given the velocity field:

$\mathbf{V} = 5x^3\,i - 15x^2\,y\,j$
Obtain the equation of the streamlines.

Stream tube: - is a tube imaginated to be formed by a group of streamlines passing through a small closed curve.

- A fluid can enter or leave a stream tube only at its ends

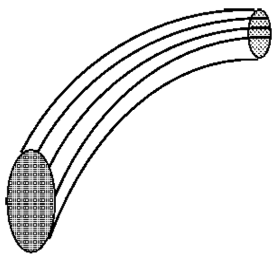


Fig. 4.4 A stream tube

Path line: - a path line is a line traced out by a given single fluid particle as it moves from one point to another over a period of time.
In steady flow path lines & streamlines are identical.

Streak lines: - A streak line consists of all particles in flows that have previously passed through a common point. They can be obtained by taking instantaneous

photographs of marked particles that all passed through a given location in the flow field at some earlier time.
In experimental work often a color or a dye is injected in the flowing fluid, in order to trace the motion of the fluid particles. The resulting trail of color is known as streak lines. For steady flow, each successively injected particle follows precisely behind the previous one, forming a steady streak line that is exactly behind the previous one, forming a steady streak line that is exactly the same as the streamline through the injection point.
Hence, path line, streamlines & streak lines are the same for steady flows.

4.3 Types of flow

A. ***Classification according to type of fluid***

(i) **Ideal fluid flow** – the fluid is assumed to have no viscosity. The velocity distribution is thus assumed uniform ---- (idealized)

(ii) **Real fluid flow:** viscosity is taken in to consideration, which leads to the development of shear stress b/n moving layers. However, some fluids such as water are near to an ideal fluid, and this simplifying assumption enables mathematical methods to be adopted in the solution of certain flow problems.

(iii) **Compressible fluid flow: -** if variation of pressure results in considerable changes in volume & density. Gases are generally treated as compressible.

(iv) **Incompatible fluid flow** - if extremely large variation in pressure is required to affect very small changes in volume. Liquids are generally treated as incompressible.

B. ***Classification according to variation of velocity, displacement and etc***

(i) **Steady flow: -** A flow is said to be steady if at any point in the flowing fluid characteristic such as velocity, pressure, density etc don't change with time. However this characteristic may be different at different points in the flowing fluid.

$$\therefore \frac{\partial v}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \, etc$$

(ii) **Unsteady flow**: - if at any point in the flowing fluid any one of all of the characteristics, which describes the behavior of fluids in motion changes with time.

$$\therefore \frac{\partial v}{\partial t} \neq 0, \frac{\partial p}{\partial t} \neq 0, etc$$

(iii) **Uniform flow**: - this occurs when the velocity both in magnitude & direction remains constant with respect to distance, i. e it doesn't change from point to point

$$\therefore \partial v / \partial s = 0$$

Example: flow of fluid under pressure through long tube of constant diameter.

(iv) **Non- uniform flow**: - if there is a change in velocity either in magnitude or direction with respect to distance , then:

$$\frac{\partial v}{\partial s} \neq 0$$

(v) **Laminar flow**: - in laminar flow the particles of fluid move in orderly manners & the steam lines retain the same relative position in successive cross section. Laminar flow is associated with low velocity of flow and viscous fluids.

(vi) **Turbulent flow**: - Here the fluid particles flow in a disorder manner occupying different relative positions in successive cross section. Turbulent flow is associated with high velocity flows.

Around 1883, Reynolds established the boundary between the laminar and turbulent flow, using the dimensionless number called Reynolds's number, Re.

$$R_e = \frac{VD}{\upsilon}$$

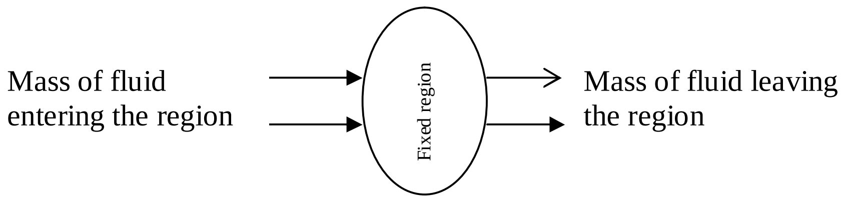
Where V- mean velocity
D- Diameter
ν- Kinematics viscosity

Reynolds showed that if
$R_e < 2000$ ---- laminar flow
$R_e > 4000$ ----- Turbulent

In b/n 2000 & 4000 it is transition flow.

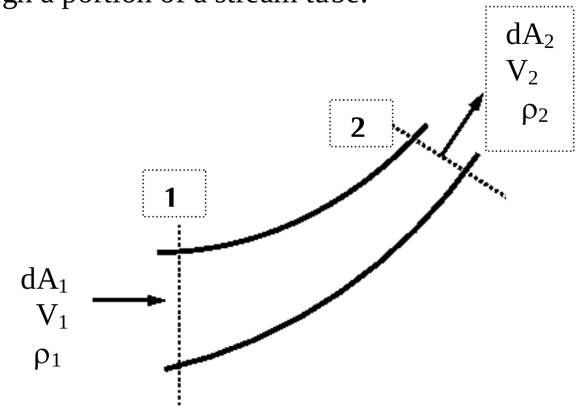
4.3. Continuity Equation

The continuity equation is a mathematical statement of the principle of conservation of mass. Consider the following fixed region with flowing fluid. Since fluid is neither created nor destroyed with in the region it may be stored that the rate of increase of mass contained with in the region must be equal to the differences b/n the rate at which the fluid mass enters the region & the rate of which it leaves the region.



However, if the flow is steady, the rate of increase of the fluid mass with in the region is equal to zero; then the rate at which fluid mass enters the region is equal to the rate at which the fluid mass leaves the region.

Considers flow through a portion of a stream tube:



<u>At section-1</u>

Area of elementary tube = dA_1
Average velocity = V_1
Density = ρ_1

- ☞ Mass of fluid per unit time flowing past section-1 = $\rho_1 * dA_1 * V1$ [kg/s]

<u>At section-2</u>

Area of elementary tube= dA_2
Average velocity = V_2
Density = ρ_2

- ☞ Mass of fluid flowing per unit of time past section 2 = $\rho_2 * dA_2 * V_2$ [kg/s]

For steady flow, by the principle of conservation of mass

$$\rho_1\, dA_1\, V_1 = \rho_2\, d\, A_2\, V_2$$

For the entire area of the stream tube:

$$\int_{A1} \rho_1 \; dA_1 V_1 \; = \int_{A2} \rho_2 \; dA_2 V_2 \; = \text{constant}$$

If ρ_1 and ρ_2 are average densities at section (1) and (2), then

$$\rho_1 \int_{A1} V_1 dA_1 \; = \rho_2 \int_{A2} V_2 dA_2 \; = \rho\, VA \; = \text{constant}$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho VA = \text{constant}$$

This is equation of continuity applicable to steady, one-dimensional flow of compressible as well as incompressible ($\rho_1 = \rho_2$) flow.

For incompressible flow, ρ = constant and doesn't vary form point to point, $\rho_1 = \rho_2$

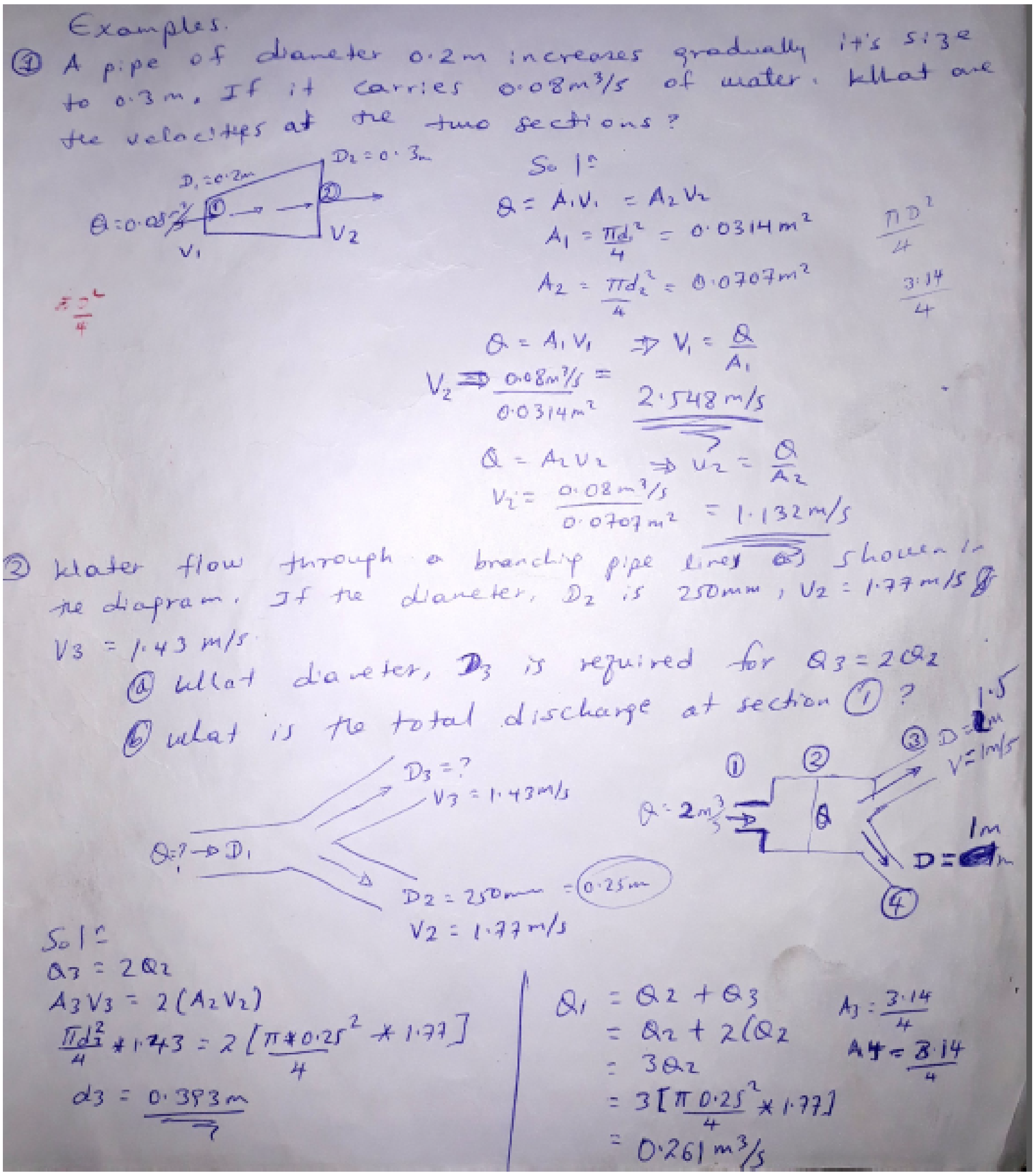
$A_1V_1 = A_2V_2 = Q = \text{constant}$

This is continuity equation for steady incompressible flow.
Q is the discharge (or volumetric flow rate or flow) defined as

$$Q = AV\ [m^2 m/s = m^3/s = \text{Volume/time}]$$

$$Q = A_1V_2 = A_2V_2 \ \text{---}\ V_1 = \frac{Q}{A_1}\ ,\ V_2 = \frac{Q}{A_2}$$

Hence, the velocity of flow is inversely proportional to the area of flow section. This is useful for most engineering application.



Examples.

① A pipe of diameter 0.2 m increases gradually it's size to 0.3 m. If it carries 0.08 m³/s of water. What are the velocities at the two sections?

Sol:

$Q = A_1V_1 = A_2V_2$

$A_1 = \frac{\pi d_1^2}{4} = 0.0314\,m^2$

$A_2 = \frac{\pi d_2^2}{4} = 0.0707\,m^2$

$Q = A_1V_1 \Rightarrow V_1 = \frac{Q}{A_1}$

$V_2 \Rightarrow \frac{0.08\,m^3/s}{0.0314\,m^2} = 2.548\,m/s$

$Q = A_2V_2 \Rightarrow V_2 = \frac{Q}{A_2}$

$V_2 = \frac{0.08\,m^3/s}{0.0707\,m^2} = 1.132\,m/s$

② Water flow through a branching pipe lines as shown in the diagram. If the diameter, D_2 is 250 mm, $V_2 = 1.77$ m/s & $V_3 = 1.43$ m/s

(a) What diameter, D_3 is required for $Q_3 = 2Q_2$

(b) What is the total discharge at section ①?

Sol:

$Q_3 = 2Q_2$

$A_3V_3 = 2(A_2V_2)$

$\frac{\pi d_3^2}{4} * 1.43 = 2\left[\frac{\pi * 0.25^2}{4} * 1.77\right]$

$d_3 = 0.393\,m$

$Q_1 = Q_2 + Q_3$

$= Q_2 + 2Q_2$

$= 3Q_2$

$= 3\left[\frac{\pi\, 0.25^2}{4} * 1.77\right]$

$= 0.261\,m^3/s$

$A_3 = \frac{3.14}{4}$ $A_4 = \frac{3.14}{4}$

The general equation of continuity for three dimensional (3D) flow can be derived as follows.
Consider a flow through a rectangular parallelepiped of dimensions: $\delta x,\ \delta y, \delta z$

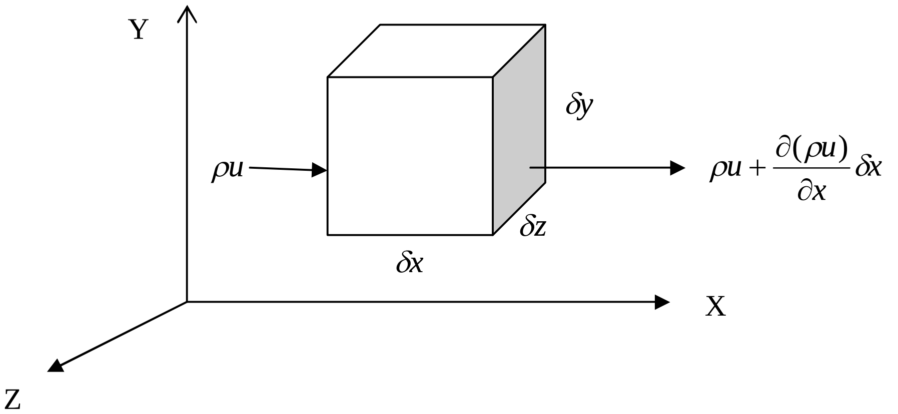


Fig. 4.5 Derivation of a differential equation of continuity

The mass of fluid flowing per unit time through the left face ABCD:

$$= \rho u (\delta y \delta z)$$

The mass of fluid flowing out of the parallelepiped through face A'B'C'D':

$$= (\rho u + \frac{\partial(\rho u)}{\partial x}\delta x)\delta y \delta z$$

- The **net mass of fluid** that remain in the parallelepiped per unit time:

$$= \rho u\ \delta y\ \delta z - \left[\rho u\ \delta y \delta z + (\frac{\partial(\rho u)}{\partial x}\delta x)\delta y \delta z\right]$$

$$= -\frac{\partial(\rho u)}{\partial x}\delta x \delta y \delta z$$

By similar procedure the mass of fluid remaining in the others two pairs of faces (Y, Z – directions)

Y- direction = $-\frac{\partial}{\partial y}(\rho v)\ \delta x \delta y \delta z$

Z- direction = $-\frac{\partial}{\partial z}(\rho w)\delta x \delta y \delta z$

- The net total mass of fluid that remains in the parallelepiped per unit time is :

$$= -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]\delta x \delta y \delta z ------------1$$

The mass of fluid in the parallelepiped is:

$$= \left(\rho\, \delta x\, \delta y\, \delta z\ \right)$$

its rate of increase with time is:

$$= \partial\!\!\diagup_{\partial t} \left(\rho\ \delta x\, \delta y\, \delta z\ \right) = \frac{\partial \rho}{\partial z}\ \left(\delta x\, \delta y\, \delta z\ \right) - - - - - - - - - - 2$$

Equating 1 & 2 we get:

$$-\left[\left[\frac{\partial}{\partial x}(\rho u)\right] + \frac{\partial}{\partial y}\left(\frac{\rho v}{}\right) + \frac{\partial}{\partial z}\left(\frac{\rho w}{}\right)\ \right] = \frac{\partial \rho}{\partial t}$$

$$\frac{\partial \rho}{\partial t} + \partial\left(\frac{\rho u}{\partial x}\right) + \partial\left(\frac{\rho v}{\partial y}\right) + \partial\left(\frac{\rho w}{\partial z}\right) = 0$$

(General continuity equation in 3D Flow)

$$\frac{\partial \rho}{\partial t} + \partial\left(\frac{\rho v_t\ r}{r\,\partial r}\right) + \partial\left(\frac{\rho V_\theta}{r\,\partial \theta}\right) + \partial\left(\frac{\rho V_z}{\partial z}\right) = 0$$ In Cylindrical coordinate system

For steady flow, $\frac{\partial \rho}{\partial t} = 0$

$$\therefore\ \ \partial\left(\frac{\rho u}{\partial x}\right) + \partial\left(\frac{\rho v}{\partial y}\right) + \partial\left(\frac{\rho w}{\partial z}\right) = 0$$ (Steady compressible fluid)

For incompressible flow, ρ doesn't change with x, y, z, and t

ρ = constant

$$\therefore\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ - (Continuity equation for incompressible, steady flow in 3D)

4.4 Stream function (ψ) and velocity potential (ϕ)

Stream function (ψ)

Stream function (ψ) (psi) is the mathematical postulation such that its differentiation with respect to x gives the velocity in y-direction (generally taken as –ve) and its differentiation with respect to y gives the velocity in x-direction.

$$u = \frac{\partial \psi}{\partial y} \quad ; \quad v = -\frac{\partial \psi}{\partial x}$$

Some of the salience characteristics of a stream function are enumerated below:

- Since ψ is a function of x and y, its total differential is

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = -vdx + udy \text{-------------------------}1$$

Further, for a two-dimensional motion parallel to x-y plane the streamline for an incompressible fluid is prescribed as:

$$\frac{v}{u} = \frac{dy}{dx} \; ; \; udy - vdx = 0 \text{-------------------------------------}2$$

From equation 1 and 2 it follows that

$$d\psi = 0 \quad \text{or} \quad \psi = constant$$

i.e., the stream function is constant along a streamline. The streamlines are thus lines of constant stream function. However, the stream function varies from one streamline to another. Each streamline of flow pattern can be represented as:

$$\psi_1 = C_1 \quad ; \quad \psi_2 = C_2$$

- Substitute for u and v in terms of stream function in the continuity equation for an incompressible fluid:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0$$

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

This is true. And hence, the stream function satisfies the equation of continuity.

- Let A and B be the two points lying on the streamlines prescribed by stream function ψ and $\psi + d\psi$, respectively. From the figure below, the velocity vector V perpendicular to line AB has components u and v in the direction of x and y-axis respectively. From continuity consideration:

  Flow across AB = flow across AO + flow across OB

$$Vds = -vdx + udy$$

The minus sign indicates that the velocity v is acting in the downward direction.

$$Vds = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = d\psi$$

$$i.e. \quad dq = d\psi$$

Evidently, the stream function can also be defined as the flux or flow rate between two streamlines. The units of ψ are m^2/s; discharge per unit thickness of flow.

- The line joining the points A and B may be AMB or ANB but the discharge between the two streamlines will remain the same. The quantity of fluid flowing past the line AB' would also remain same provided no fluid enters or leaves between the points BB'. Apparently, the fluid flow is unaffected by the shape of the line between A and B.

Velocity potential (ϕ)

A fluid element in the shape of cube, which is initially at one position, will move to another position during a short time interval dt. because of generally complex velocity variation within the field, we expect the element to not only translate from one position but change in shape (angular deformation). The form of movement may be in the form of:

a) Translation or rotation and
b) Volume dilation or angular deformation

Translation: - means simply picking up the element and moving a distance during a small time dt.

Rotation: - is defined as the average angular velocity of two elements originally at right angles to each other.

Flow is described rotational if every fluid element rotates about its axis which is perpendicular to the plane of motion.

Consider a rectangular fluid element occupying the position ABCD at a certain time in a two-dimensional x-y plane in the figure below. The velocity components in the x-direction at points A and D are u and $u + \frac{\delta u}{\delta y} dy$ respectively. Since these velocities are different, there will be an angular velocity developed for the linear element AD. Similarly the velocity components in y-direction at points A and B are v and $v + \frac{\delta v}{\delta x} dx$ respectively, and these different velocities will result in to an angular velocity of the linear element AB. Apparently during the time interval dt the element AB and AD would move relative to point A, the fluid element gets displaced and occupies the dotted position AB'C'D'.

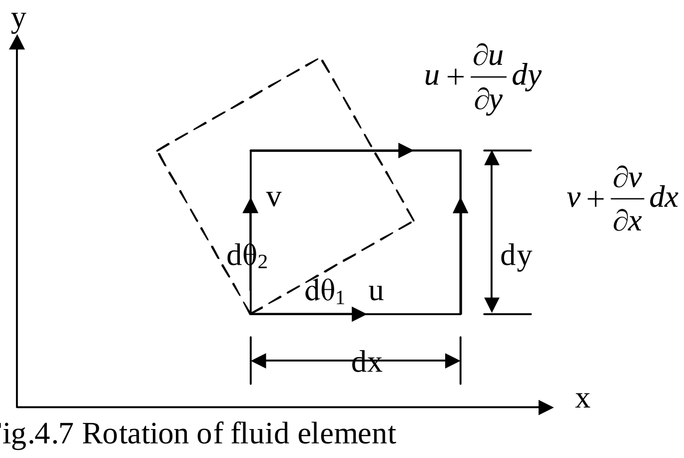


Fig.4.7 Rotation of fluid element

Taking the counter clockwise direction as positive:

- The angular velocity of element AB about the z-axis is

ω_{AB} = angular displacement of element AB per unit time

$$= \frac{d\theta_1}{dt}$$

From the figure, $\tan d\theta_1 \cong d\theta_1 = \dfrac{\left(\partial v/\partial x\, dx.dt\right)}{dx}$ (for the horizontal element dx).

$$\therefore \omega_{AB} = \frac{d\theta_1}{dt} = \frac{\partial v}{\partial x}$$

- The angular velocity of element AD about the z-axis is

ω_{AD} = angular displacement of element AD per unit time

$$= \frac{d\theta_2}{dt}$$

$$\tan(d\theta_2) \cong d\theta_2 = \frac{\left(-\partial u/\partial y\, dy.dt\right)}{dy} = -\frac{\partial u}{\partial y} dt$$

$$\omega_{AD} = \frac{d\theta_2}{dt} = -\frac{\partial u}{\partial y}$$

- Average of the angular velocity of line AB (dx element) and line AD (dy element) gives the rotation ω_z of the element ABCD about z-axis.

i.e.,

$$\omega_z = 1/2\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Rotations about the other two axes are defined as:

$$\omega_x = 1/2\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = 1/2\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

Angular velocity is a vector quantity:

$$\vec{\Omega} = \omega_x i + \omega_y j + \omega z\, k$$

Where ω_x, ω_y and ω_z - are rotation components

Irrotational flow: occurs when the cross-gradient of the velocity (or shear) are zero or cancel each other. i.e., the fluid element has a zero angular velocity about its own mass centre.

i.e., $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ = 0 (for two-dimensional flow)

Velocity potential ϕ (phi) is a function such that its derivative in any direction gives the velocity in that direction.

i.e., $u = \frac{\partial \phi}{\partial x}$; $v = -\frac{\partial \phi}{\partial y}$

Lines of constant potential function are termed as **equipotential lines**.

ϕ is a function of x and y alone, its total differential is

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = udx + vdy$$

for an equipotential line, the potential function ϕ is constant

i.e.,

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = udx + vdy = 0$$

$$\frac{dy}{dx} = -\frac{v}{u}$$

Which prescribes the slope of equipotential line at any point.

For two dimensional(x-y plane), Irrotational flow:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial y \partial x} - \frac{\partial^2 \phi}{\partial x \partial y} = 0$$

For continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

This equation is known as ***Laplac's equation***.

The in viscid, incompressible, Irrotational flow fields are governed by Laplace's equation.

The above discussion with reference to stream function and velocity potential function leads us to conclude that:

- Stream function applies to both rotational and irrotational flows. The flow has only to be steady and incompressible,
- Potential function exists only for irrotational flow,
- For irrotational flow, both stream function and the velocity potential function satisfies Laplace equation; consequently they are interchangeable. We have the following important relationships between the stream function and potential function for a steady, irrotational and incompressible fluid flow.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}\ ;\ v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$ -----------------Cauchy-Riemann equation

☞ **Orthogonality of streamlines and equipotential lines**

Stream function of x and y, its total differential is

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$

But the stream function $\psi(x, y)$ is a constant along a streamline,
i.e.,

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0$$

$$\frac{dy}{dx} = -\frac{\partial \psi}{\partial x}\bigg/\frac{\partial \psi}{\partial y} = \frac{v}{u}$$

The slope of a streamline for constant ψ is

$$\frac{dy}{dx} = \frac{v}{u}$$ ---------------------i

Further ϕ is a function of x and y, and therefore its total differential is

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$$

But the potential function $\phi(x, y)$ is constant along an equipotential line
i.e.,

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = 0$$

$$\frac{dy}{dx} = -\frac{\partial\phi}{\partial x}\Big/\frac{\partial\phi}{\partial y} = -\frac{u}{v}$$

The slope of an equipotential line for constant ϕ is

$$\frac{dy}{dx} = -\frac{v}{u} \text{--------------------------ii}$$

Combining expressions i and ii

Slope of streamline * slope of equipotential line

$$= \frac{v}{u} * \left(-\frac{u}{v}\right) = -1 \text{--------------------------------*}$$

This equation is a mathematical statement of the fact that equipotential lines are normal to the streamlines. The orthognality between the streamlines and equipotential lines serves to draw a flow net.

Flow Net

Flow net is a graphical representation of streamlines and equipotential lines for a potential flow. Flow nets are drawn to indicate flow patterns in case of two-dimensional floe, or even three-dimensional flow. The flow net consists of

- a system of streamlines so spaced that the rate of flow q is the same between each successive pair of lines, and
- another system of lines normal to the streamlines and so spaced that the distance between the normal lines equals the distance between adjacent streamlines.

Streamlines and equipotential lines are drawn between the flow boundaries with the requirements that they form small squares.