CHAPTER-3

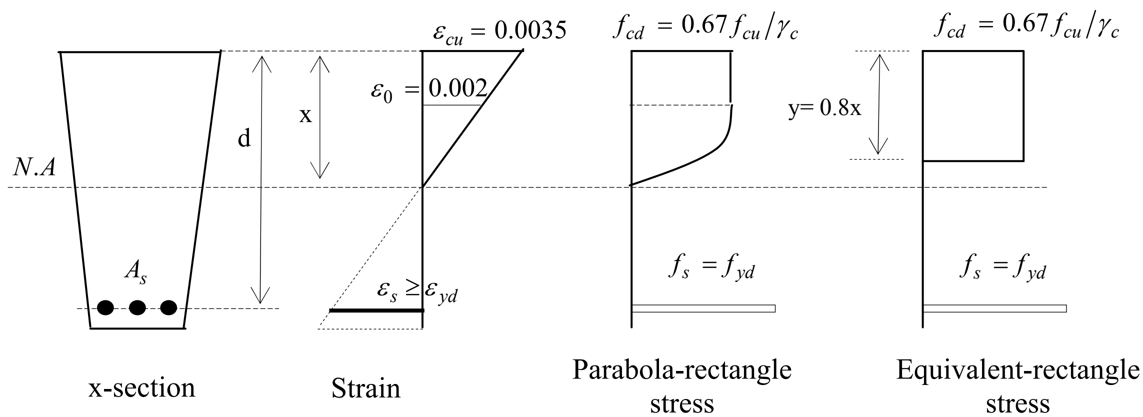
LIMIT STATE DESIGN FOR FLEXURE AND SERVICEABILITY LIMIT STATE

3.1. Basic Assumptions:

Assumption made for determining ultimate resistance of a member for flexure and axial force according to EBCS-2/95 are,

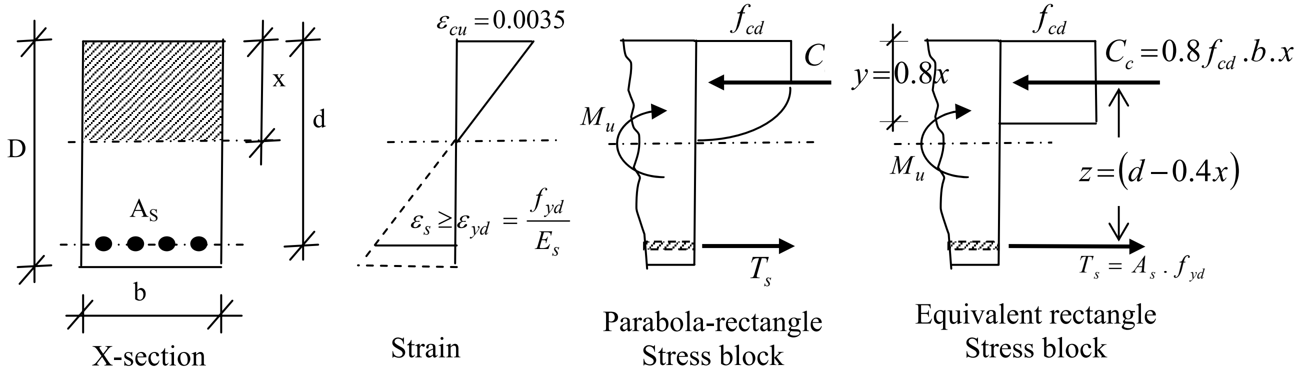
1. A section which is plane before bending remains plane after bending. This implies strains across section are linearly varying. This is true for most section of flexural member except deep beam where shear deformation is significant.
2. The reinforcement is subjected to the same variations in the strain as the adjacent concrete. This implies there is no slip between steel bars and the adjacent concrete. This is possible if adequate development length of bars and concrete cover are provided.
3. Tensile strength of concrete is ignored. The reinforcement assumed to takes all the tension due to flexure.
4. The maximum compressive stain in concrete when a section complete plastic deformation is taken to be $\varepsilon_{cu} = 0.0035$ in bending (simple or compound) $\varepsilon_{cu} = 0.002$ in axial compression
5. The maximum tensile strain in the reinforcement is taken to 0.01. This limit assumed to limit crack-width with in tension zone of section to the acceptable limit.
6. Either idealized parabola-rectangle stress distribution or equivalent rectangle stress distribution for concrete in compression zone given by code as shown below shall be used in derivation of design equation.

The ultimate resistance of section may be determined using equilibrium of both internal and external forces based on the stress block obtained from the basic assumption.



3.2. Design Equations for Singly Reinforced Rectangular Section:

Consider a singly reinforced rectangular section subjected to a factored load moment, M_u as shown below.



-Equilibrium of both internal and external forces,

i) $\left[\sum F_H = 0\right] \quad \Rightarrow \quad C_c = T_s$

$$\Leftrightarrow \quad 0.8 f_{cd} \,.\, b \,.\, x = A_s \,.\, f_{yd} \qquad \text{Let } \rho = \frac{A_s}{b \,.\, d} \quad \text{--steel ratio of section}$$

$$\Leftrightarrow \quad 0.8 f_{cd} \,.\, b \,.\, x = \rho \,.\, b \,.\, d \,.\, f_{yd}$$

Simplifying, depth of neutral axis obtained as,

$$x = \left(\frac{\rho \,.\, f_{yd}}{0.8 f_{cd}}\right) . \, d \tag{1}$$

ii) $\left[\sum M = 0\right] \quad \Rightarrow \quad M_u = C_c \,.\, z = T_s \,.\, z \qquad$ Where $z = \left(d - 0.4x\right)$ --lever arm

-taking moment about T_s :

$$M_u = C_c \,.\, z$$

$$\Leftrightarrow \quad M_u = \left(0.8 f_{cd} \,.\, b \,.\, x\right) . \left(d - 0.4x\right)$$

Substituting x from Eq.(1),

$$\Rightarrow \quad M_u = 0.8 f_{cd} \,.\, b \,.\left(\frac{\rho \,.\, f_{yd}}{0.8 f_{cd}} \,.\, d\right) . \left(d - \frac{0.4\rho \,.\, f_{yd}}{0.8 f_{cd}} \,.\, d\right)$$

Simplifying, ultimate moment of resistance of section is obtained as,

$$M_u = \rho \,.\, f_{yd} \,.\, b \,.\, d^2 \,.\left(1 - \frac{\rho \,.\, f_{yd}}{2 f_{cd}}\right) \tag{2}$$

The same equation of ultimate moment of resistance of section can be obtained if moment center is taken at C_c.

-Defining the ultimate moment and relative steel-area using the following dimension-less parameters:

$$\mu = \frac{M_u}{f_{cd} \,.\, b \,.\, d^2} \qquad \text{--relative ultimate moment}$$

And $$\omega = \rho \,.\, \frac{f_{yd}}{f_{cd}} \qquad \text{--mechanical reinforcement ratio}$$

Then, neutral-axis depth obtained in Eq.(1) can be written as,

$$x = \frac{\omega \,.\, d}{0.8} \tag{1a}$$

Therefore, depth of equivalent stress-block is obtained as,

$$y = 0.8x = \omega \,.\, d$$

Writing equation of moment of resistance of section in the form as shown below by rearranging Eq.(2),

$$\frac{M_u}{f_{cd} \,.\, b \,.\, d^2} = \frac{\rho \,.\, f_{yd}}{f_{cd}} \,.\left(1 - \frac{\rho \,.\, f_{yd}}{2 f_{cd}} \right)$$

Writing the above equation in terms of dimension less parameters,

$$\Rightarrow \qquad \mu = \omega \,.\left(1 - \frac{\omega}{2} \right) = \omega - \frac{\omega^2}{2} \tag{2a}$$

Rearranging Eq.(2a), $\Rightarrow \quad \omega^2 - 2\omega + 2\mu = 0$

Solving for ω ,

$$\omega = 1 - \sqrt{1 - 2\mu} \tag{3}$$

Therefore, area of tension steel required to resist the ultimate moment, M_u is obtained by taking moment about C_c as,

$$M_u = T_s \,.\, z$$

$$\Leftrightarrow \qquad M_u = A_s \,.\, f_{yd} \,.\, z$$

Where $z = (d - 0.4x)$ substituting x from Eq.(1a) and ω from Eq.(3)

$$z = \left(1 - \frac{\omega}{2} \right) . \, d = \frac{d}{2} \,.\left(1 + \sqrt{1 - 2\mu} \right)$$

Rearranging, the required area of tension steel is obtained by,

$$A_s = \frac{M_u}{f_{yd} \cdot z} \tag{4}$$

3.2.1. Balanced Singly Reinforced Section

In balanced section, yielding of tension steel and crushing of concrete takes place at same time when the section complete plastic deformation. That is, the maximum compressive strain in concrete reaches the ultimate strain, $\varepsilon_c = \varepsilon_{cu} = 0.0035$ and the strain in tension steel is just yielded, $\varepsilon_s = \varepsilon_{yd} = f_{yd}/E_s$.

From strain distribution, using similarity of triangles,

$$\frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_s}$$

Substituting $x = x_b$ & $\varepsilon_s = \varepsilon_{yd} = f_{yd}/E_s$, the balanced neutral-axis depth is obtained as,

$$x_b = \frac{\varepsilon_{cu}}{\left(\varepsilon_{cu} + f_{yd}/E_s\right)} \cdot d \tag{5}$$

Where $\varepsilon_{cu} = 0.0035$ --ultimate compressive strain of concrete

Equating x_b with equation of neutral-axis depth obtained in Eq.(1) and Eq.(1a), the balanced reinforcement ratio and the balanced mechanical reinforcement ratio are obtained as,

$$\rho_b = \frac{0.8\varepsilon_{cu}}{\left(\varepsilon_{cu} + f_{yd}/E_s\right)} \cdot \frac{f_{cd}}{f_{yd}} \tag{6}$$

And $$\omega_b = \frac{0.8\varepsilon_{cu}}{\left(\varepsilon_{cu} + f_{yd}/E_s\right)} \tag{7}$$

If $\rho < \rho_b$, the steel yields first at the load near collapse (a case of under-reinforced section and ductile-type failure).

If $\rho > \rho_b$, crushing of concrete takes place first prior to yielding of tension steel at the load near collapse (a case of over-reinforced section and brittle-type failure).

To ensure ductility, in practice the maximum amount of tension steel is fairly below the amount corresponding to the balanced-one.

ACI:318 code recommend: maximum reinforcement ratio ensuring ductility as $\rho_{\max} = 0.75\rho_b$. For seismic load resisting member, the same code recommends, $\rho_{\max} = 0.5\rho_b$.

Based on ACI recommendation ($\rho_{\max} = 0.75\rho_b$), maximum design constants of singly reinforced section are obtained as shown in table below.

Table: Maximum design constants of singly reinforced section (ACI-code)

Steel Grade	$\omega_{\max}$	$\mu_{\max}$
S-300 MPa	0.437	0.341
S-400 MPa	0.401	0.320
S-460 MPa	0.382	0.309

EBCS:2/95 recommend: the maximum amount of tension steel used to ensure ductility is based on limiting the neutral-axis depth at,

$x_{\max} = 0.448d$ --for no redistribution of elastic moments

$x_{\max} = 0.368d$ --for 10% redistribution of elastic moments

$x_{\max} = 0.288d$ --for 20% redistribution of elastic moments

$x_{\max} = 0.208d$ --for 30% redistribution of elastic moments

Based on EBCS-2/95 recommendation, maximum design constants of singly reinforced section are obtained as shown in table below.

Table: Maximum design constants of singly reinforced section (EBCS-2/95 code)

% Redistribution of elastic moments	$\omega_{\max}$	$\mu_{\max}$
0%	0.3584	0.294
10%	0.2944	0.251
20%	0.2304	0.204
30%	0.1664	0.152

Better approach as follows:

In accordance with LSD method, at ULS of collapse:-

- ε_c approaches $\varepsilon_{cu} = 0.0035$
- The reinforcing steel shall yield first ($\varepsilon_{yd} = \frac{f_{yd}}{E_s}$)

  $\Rightarrow$ Ductility is ensured by means of under reinforcement.
- At balanced failure simultaneous failure of the two materials (Concrete & Steel) occurs.

Let x_b be the depth to the NA at balanced failure. From the strain relation,

$$\frac{x_b}{\varepsilon_{cu}} = \frac{d - x_b}{\varepsilon_{yd}} \quad \Rightarrow \quad x_b = \frac{\varepsilon_{cu} * d}{\varepsilon_{cu} + \varepsilon_{yd}}$$

- If $x < x_b \Rightarrow$ Steel yields first
- If $x > x_b \Rightarrow$ Crushing of concrete takes place first.

$$\Sigma F_H = 0 \quad \Rightarrow \quad T_s = C_C \quad \Rightarrow \quad A_s f_{yd} = 0.8\, x_b\, b\, f_{cd}$$

Substituting for x_b and simplifying, $\rho_b = \frac{0.8 * \varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} * \frac{f_{cd}}{f_{yd}}$

(a steel ratio for balanced case)

However, for ductility purpose the steel ratio ρ may range b/n 0.75 ρ_b to 0.9 ρ_b, and in some cases as low as $0.5\,\rho_b$ in ACI code, but in EBCS-2 ductility is ensured by keeping $k_{x\,max}$ = 0.448 for 0% redistribution or even less for redistribution > 0%.

Rewriting the force equilibrium

$$byf_{cd} = A_s f_{yd} \quad \Rightarrow b * 0.8x\, f_{cd} = \rho bd\, f_{yd}$$

$$k_x = \frac{x}{d} = \frac{\rho * f_{yd}}{0.8 * f_{cd}} = \rho m, \quad \text{Where } m = \frac{f_{yd}}{0.8 * f_{cd}}$$

$$\Sigma M_c = 0 \Rightarrow M_d = A_s f_{yd} (d - 0.4x)$$

Substituting the value of x and simplifying

$$M_d = 0.8\, bd^2 f_{cd}\, k_x\, (1\text{-}0.4\, k_x)$$

When the above equation is solved for k_x,

$$k_x = 0.5\left\{c_1 - \sqrt{c_1^2 - \frac{4M_d}{bd^2 c_2}}\right\} \leq k_{x\,\max}$$

Where $c_1 = 2.5/m$, $c_2 = 0.32m^2 f_{cd}$, $m = f_{yd}/(0.8f_{cd})$ $k_{x\,\max} = 0.448$ for 0% redistribution.

The section capacity for single reinforcement case may be computed from $\Sigma M_t = 0$, when $k_x < k_{x\,\max}$

$$\Rightarrow \quad M_u = 0.8bx\, f_{cd}\,(d - 0.4x) \qquad x = k_{x\,max}\, d$$

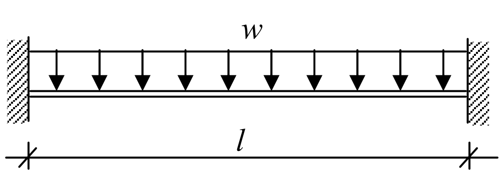
$$= 0.8bd^2 f_{cd}\, k_{x\,\max}\,(1 - 0.4\, k_{x\,\max})$$

3.2.2. Inelastic Redistribution of Moments in Continuous-beams and Frames

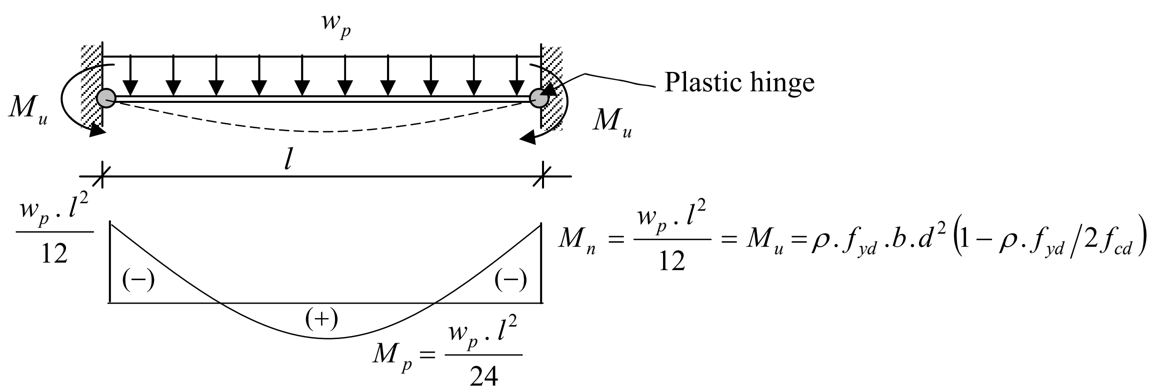
When statically indeterminate beam is loaded beyond the working loads, plastic hinges forms at the location of maximum bending moment. On further loading the beam, the maximum moment do not increase beyond the ultimate moment capacity of section of beam, however, rotation at plastic hinges keep on increasing until the ultimate rotation capacity is reached. A redistribution of moment takes place with the changes in the moment elsewhere in the beam as if a real hinges are existing. With further increase of additional plastic hinge, redistribution moments continue until a collapse mechanism is produced.

Plastic analysis can be applied in analysis of steel structures. However, its use for analysis of reinforced concrete structures is limited. A limited redistribution of moments obtained from elastic analysis of indeterminate structures is permitted by most codes if members are designed under-reinforced section provided equilibrium is maintained under each combination of ultimate loads.

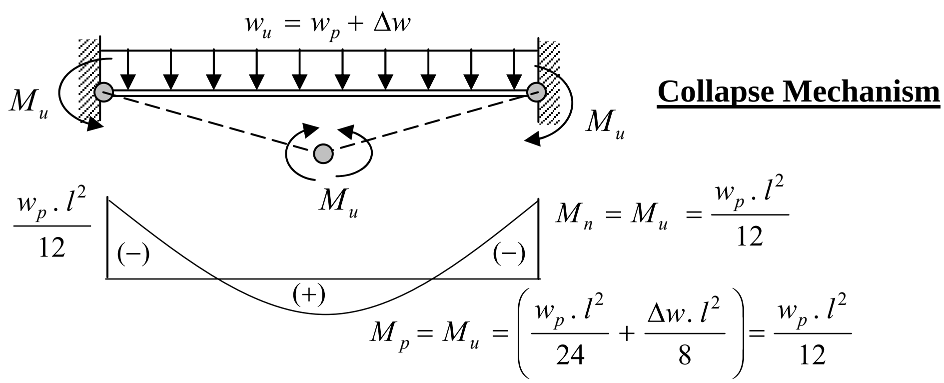
For illustration of plastic analysis of structure, consider a fixed-beam, which is statically indeterminate, subjected to increasing uniform load shown below.



Let the beam subjected to the load 'w_p' that cause the plastic hinges at the ends when the maximum moment at supports equal to the ultimate resistance of beam section. But, with the formation of plastic hinges, the beam is still able to support additional load without complete collapse. After formation of plastic hinges at supports, the beam behaves as if simply supported.



On further loading, the moment at center of span increases proportionally with the change of loading. Additional load Δw is slowly applied until it causes the beam to transform into a collapse mechanism with the formation of one or more hinges at the middle.



At collapse, mid-span moment equal to the ultimate resistance of beam section,

$$M_u = \frac{w_p \cdot l^2}{24} + \frac{\Delta w . \, l^2}{8} = \frac{w_p \cdot l^2}{12}$$

Equating negative and positive collapse moment, additional load that causes collapse mechanism in terms of the load 'w_p' that causes the plastic hinges at the ends is,

$\Delta w = w_p / 3$ And, collapse load in terms of 'w_p'

$$w_u = w_p + \Delta w = w_p + w_p / 3 = \frac{4}{3} w_p$$

These shows, the beam may carry a load of $4/3\, w_p$ with redistribution. The ultimate moment in terms of ultimate load is:

$$M_u = \frac{w_p \,.\, l^2}{12} \qquad\qquad \text{Substituting } w_p = \frac{3}{4} w_u$$

$$\rightarrow \qquad M_u = \frac{(3/4\, w_u)\,.\, l^2}{12} = \frac{w_u \,.\, l^2}{16}$$

If elastic analysis is made using the ultimate load $'w_u'$, the maximum moment at support is $w_u .l^2/12$. The percentage reduction in bending is:

$$\frac{w_u.l^2/12 - w_u.l^2/16}{w_u.l^2/12} x100 = 25\%$$

Plastic analysis of continuous beams and frames also can be done using virtual work method. Assume any reasonable collapse mechanism, equating internal work done by ultimate moments at plastic hinges with external work done by collapse load on deflecting collapsed span of continuous beam and frame, the location of plastic hinges and the minimum collapse load can be determined.

According to EBCS-2/95, elastic moments of continuous beams and frames are redistributed using the following reduction coefficient, δ

1) For continuous beams and rigid jointed braced frames with span/effective depth ratio not greater than 20,

$$\delta = 0.44 + 1.25\left(\frac{x}{d}\right) \qquad \text{Where x—is calculated at ultimate limit state}$$

Based on the above equation, the limiting maximum neutral axis depth ratio used for proportioning of sections of continuous beams and rigid jointed braced frames are obtained as follow:

For 30% redistribution of elastic moment, $x/d = 0.208$

For 20% redistribution of elastic moment, $x/d = 0.288$

For 10% redistribution of elastic moment, $x/d = 0.368$

For no reduction of elastic moment, $x/d = 0.448$

2) For other continuous beams and rigid braced frames

$\delta \geq 0.75$

3) For sway frames with slenderness ratio λ of columns less than 25

$\delta \geq 0.90$

Examples on Design of Singly Reinforced Beams using Limit State Design Method

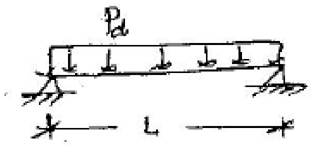
1. Design a Singly Supported Beam of Span and Width of 7m and 300mm Respectively. The loads acting on the beam are live load of 24 KN/m in addition to Self - Weight.

Material used are C-25, S-300 and class I Work

Soln :-

Step 1: Depth from deflection requirement

From EBCS 2, 1995,



$$d \geq \left(0.4 + \frac{0.6 f_{yk}}{400}\right)\frac{L_e}{\beta_a} \quad ; \quad \beta_a = 20 \text{ for Simply Support Beam}$$

$$= \left(0.4 + 0.6 * \frac{300}{400}\right)\left(\frac{7000}{20}\right)$$

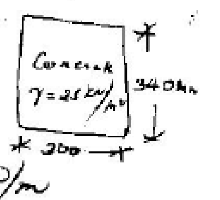
$$= 297.5 \text{ mm}$$

Assuming $\phi 20$mm is used, $\phi 6$ for Stirrup

$$D = d + d' = 297.5 + 25 + 6 + \frac{20}{2} = 338.5 \text{mm}$$

$\Rightarrow$ use $D = 340$mm

Step 2: Design load and Moment;



$$DL = G_k = 0.3 * 0.34 * 25 = 2.55 \text{ KN/m}$$

$$LL = Q_k = 24 \text{ KN/m}$$

Design load;

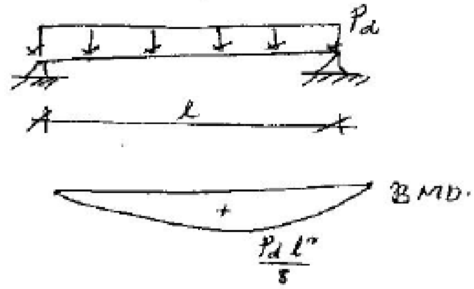
$$P_d = 1.3\, G_k + 1.6\, Q_k$$

$$= 1.3\,(2.55) + 1.6\,(24)$$

$$= 41.72 \text{ KN/m}$$

~ The Mean Modulus of elasticity, E_s, may be assumed as 200 Gpa From EBCS 2, 1995.

$$\Rightarrow M_{max} = \frac{P_d l^2}{8}$$



$$= \frac{41.72 * 7^2}{8}$$

$$= \underline{255.5 \text{ KNm}}$$

Step 3: Determine design Constants:

$$f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} \quad \text{Where } f_{ck} = 0.8 f_{cu}$$

& for class I work, $\gamma_c = 1.5$

$$\Rightarrow f_{cd} = 0.85 * 0.8 * \frac{25}{1.5} = \underline{11.33 \text{ Mpa}}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \quad \text{For class I work } \gamma_s = 1.15$$

$$\Rightarrow f_{yd} = \frac{300}{1.15} = \underline{260.87 \text{ Mpa}}$$

$$m = \frac{f_{yd}}{0.8 f_{cd}} = \frac{260.87}{0.8 * 11.33} = 28.78$$

$$C_1 = \frac{2.5}{m} = \frac{2.5}{28.78} = 0.0869.$$

$$C_2 = 0.32 m^2 f_{cd} = 0.32 (28.78)^2 (11.33) = 3003 \text{Mpa}$$

$$\rho_{max} = 0.75 \rho_b = 0.75 \left(\frac{0.8\, \epsilon_{cu}}{\epsilon_{cu} + \epsilon_{yd}}\right)\left(\frac{f_{cd}}{f_{yd}}\right)$$

$$= \left(\frac{0.75 * 0.8 * 0.0035}{0.0035 + \frac{260.87}{2 * 10^5}}\right)\left(\frac{11.33}{260.87}\right)$$

$$= \underline{0.019}$$

Step 4: Check depth for flexure:

$$d \geq \sqrt{\frac{M_{max}}{0.8\, b\, f_{cd}\, \rho m\,(1-0.4\rho m)}}$$

$$\geq \sqrt{\frac{255.54 * 10^6}{0.8 * 300 * 11.33 * 0.019 * 28.7\,(1-0.4 * 0.019 * 28.7)}}$$

$$\geq 469.01\, mm$$

$$\Rightarrow D = d + d' = 469.01 + 25 + 6 + \frac{20}{2} = \underline{510.01\, mm} > D_{used}$$

No!

Revise the depth;

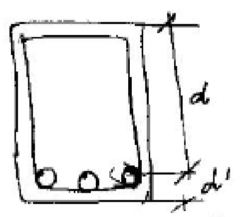
→ use $D = 520mm$

$$P_d = 41.72 + 1.3\,(0.52 - 0.34) * 0.3 * 25 = 43.48\, KN/m$$

$$M_{max} = \frac{P_d\, l^2}{8} = \underline{266.32\, KNm}$$

Check the depth again;

$$d \geq \sqrt{\frac{266.32 * 10^6}{0.8 * 11.33 * 300 * 0.019 * 28.78\,(1-0.4 * 0.019 * 28.78)}}$$



$$= 478.8\, mm$$

$$\Rightarrow D = 478.8 + 25 + 6 + \frac{20}{2} = 519.8\, mm < D_{used}!\ ok!$$

$d' = 25 + 6 + \frac{20}{2} = 41mm$

$$\therefore\ d = D - d' = 520 - (25 + 6 + \frac{20}{2}) = \underline{479\, mm}$$

Step 5: Reinforcement:

$$\rho = \frac{1}{2}\left\{ C_1 - \sqrt{C_1^2 - \frac{4 M_{max}}{b d^2 C_2}} \right\}$$

$$= \frac{1}{2}\left\{ 0.0569 - \sqrt{0.0569^2 - \frac{4 * 266.32 * 10^6}{300 * 479^2 * 3003}} \right\}$$

$$= 0.01897 < \rho_{max} = 0.019$$

$$A_{st} = \rho b d = 0.01897 * 300 * 479 = 2725.36 \, mm^2$$

$$No\ of\ \phi 20mm = \frac{2725.36}{\frac{\pi}{4}(20)^2} = 8.68 \Rightarrow Use\ 9\ \phi 20mm$$

If then, No of ϕ 20mm bar which can be placed in a row; 'n'

min. spacing b/n bars;

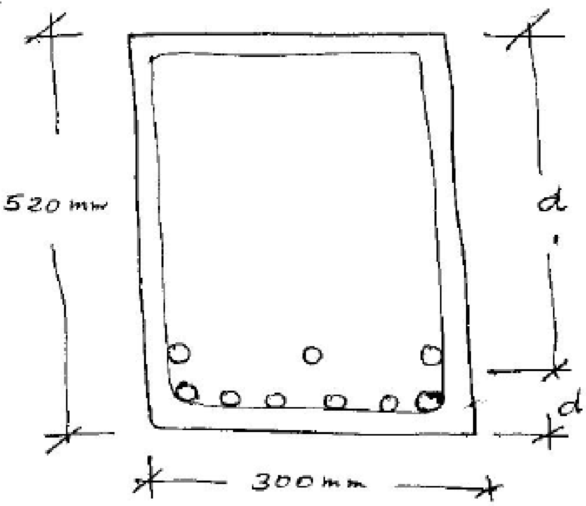
$$2n - 1 = \frac{b - 50}{\phi}$$

$$\Rightarrow n = \frac{1}{2}\left(\frac{300 - 50}{20} + 1\right) = 6.75$$

∴ place 6 ϕ20mm in the 1st row and 3 ϕ20mm in the second row.

→ NB. If bars of different diameters are used, the minimum spacing is the diameter of the larger bar.

Sketch:



$$d' = \left(\frac{6*314*41 + 3*314*81}{314*9}\right) = \underline{54.33\,mm}$$

$D = 479 + 54.33 = 533.33\,mm > 520\,mm$ No!

Revise the design for the depth of the beam; by letting $D = 540\,mm$

$$\Rightarrow d = D - d' = 540 - 54.33 = \underline{485.67\,mm}$$

$$P_d = 41.72 + 1.3\,(0.54 - 0.34) * 0.3 * 25 = 43.67\,KN/m$$

$$M_{max} = \frac{P_d\, l^2}{8} = \underline{267.48\,KNm}$$

$$\rho = \frac{1}{2}\left\{0.0869 - \sqrt{0.0869^2 - \frac{4*267.48*10^6}{300*485.67^2*3003}}\right\}$$

$= \underline{0.0184} < \rho_{max} = 0.019$ ok!

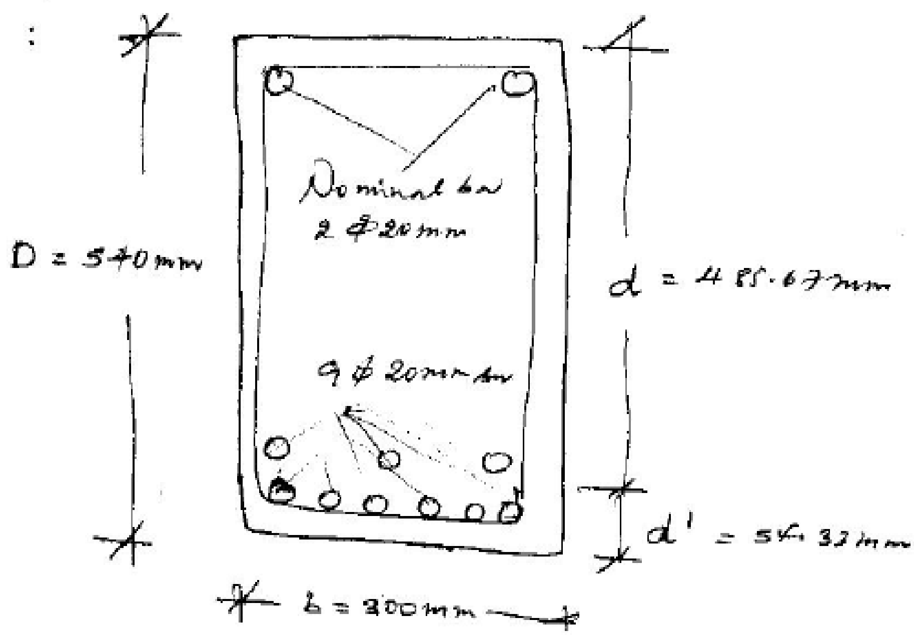
$$A_{st} = \rho bd = 0.0184 * 300 * 485.67 = \underline{2676.04\ mm^2}$$

$$No\ of\ \phi 20mm = \frac{2676.04}{314} = 8.52 \Rightarrow use\ 9\ \phi 20mm\ bars$$

in Two Rows.

Therefore, provide 6 ϕ 20mm Bars in the first row & 2 ϕ 20mm bars in the Second row.

<u>Sketch</u> :



2. Design a Simple Beam Spanning 6m and having a Width of 300mm to carry a dead load of 10KN/m and a live load of 15KN/m, in addition to its own weight. The Concrete is C25, the steel S300 & Class I Work.

Sol^{n}:-

Step 1: Depth from deflection requirement,

From EBCS 2, 1995

$$d \geq \left(0.4 + \frac{0.6\, f_{yk}}{400}\right)\frac{L_e}{\beta_a} \quad \Rightarrow \beta_a = 20$$

$$= \left(0.4 + \frac{0.6\,(300)}{400}\right)\left(\frac{6000}{20}\right)$$

$$= \underline{255\,mm}$$

Assuming Φ20mm Reinforcement & Stirrups of Φ6mm because EBCS 2 Requires the Cover to be provided to Stirrups.

$$D = d + d' = 255 + 25 + 6 + \frac{20}{2} = \underline{296\,mm}$$

Try D = 300mm.

Step 2: Design load and Moment;

$$G_k = 0.3 * 0.3 * 25 + 10 = 12.25\ KN/m$$

$$Q_k = 15\ KN/m$$

$$\Rightarrow P_d = 1.3\,G_k + 1.6\,Q_k$$

$$= 1.3\,(12.25) + 1.6\,(15)$$

$$= \underline{39.93\ KN/m}$$

$$\& \quad M_{max} = \frac{P_d\, l^2}{8} = \frac{39.93 * 6^2}{8} = \underline{179.69\ KNm}$$

Step 3: Determine design Constants:

$$f_{cd} = \frac{0.85\ f_{ck}}{\gamma_c} = \frac{0.85 * 0.8 * 20}{1.5} = \underline{11.33\ mpa}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = \underline{260.87\ mpa}$$

$$m = \frac{f_{yd}}{0.8\ f_{cd}} = \frac{260.87}{0.8 * 11.33} = \underline{28.78}$$

$$C_1 = \frac{2.5}{m} = \frac{2.5}{28.78} = 0.0869$$

$$C_2 = 0.32\ m^2 f_{cd} = 0.32\ (0.0869)^2\ (11.33) = 3003\ Mp$$

$$\rho_{max} = 0.75 \rho_b = 0.75 \left(\frac{0.8\ \epsilon_{cu}}{\epsilon_{cu} + \epsilon_{yd}}\right)\left(\frac{f_{cd}}{f_{yd}}\right) = \underline{0.019}$$

Step 4: Check depth for flexure:

$$d \geq \sqrt{\frac{M_{max}}{0.8\ b\ f_{cd}\ \rho\ m\ (1 - 0.4\ \rho m)}}$$

$$\geq \sqrt{\frac{179.69 * 10^6}{0.8 * 300 * 11.33 * 0.019 * 28.78\ (1 - 0.4 * 0.019 * 28.78)}}$$

$$\geq \underline{393.29\ mm}$$

$\Rightarrow$ $D = 393.29 + 41 = \underline{434.29\ mm} > D_{used}$. No!

$\Rightarrow$ Try $D = 470\ mm$

$\Rightarrow$ $P_d = 39.83 + 1.3\ (0.47 - 0.3) * 0.3 * 25 = \underline{41.39\ KN/m}$

$\therefore\ P_d = 41.59\ kN/m$

$$\Rightarrow M_{max} = \frac{P_d l^2}{8} = \frac{41.59 * 6^2}{8} = \underline{187.14\ kNm}$$

Check the depth again;

$$d \geq \sqrt{\frac{187.14*10^6}{0.8*11.33*300*0.019*28.78\,(1-0.4*0.019*28.78)}}$$

$$= \underline{401.36\,mm}$$

$$\Rightarrow D = 401.36 + 41 = 442.36\ mm < D_{used} = 470mm,\ OK!$$

$$\therefore\ d = D - d' = 470 - 41 = \underline{429\ mm}$$

Step 5: Reinforcement:

$$\rho = \frac{1}{2}\left\{C_1 - \sqrt{C_1^2 - \frac{4 M_{max}}{bd^2 C_2}}\right\}$$

$$= \frac{1}{2}\left\{0.0869 - \sqrt{0.0869^2 - \frac{4*187.14*10^6}{(300)(429.00)^2*3003}}\right\}$$

$$= \underline{0.0159} < \rho_{max} = 0.019$$

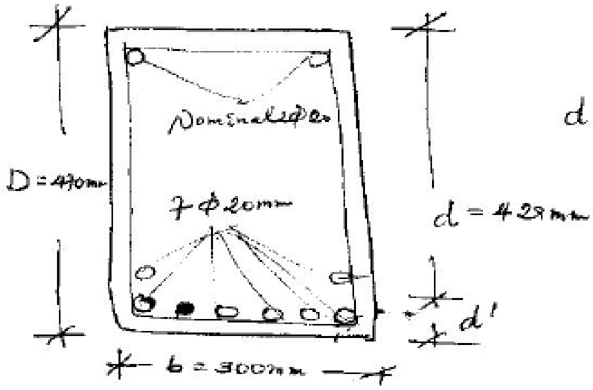
$$A_{st} = \rho b d = 0.0159*300*429.00 = \underline{2046.33\ mm^2}$$

$$No\ of\ \phi 20mm = \frac{2046.33}{314} = 6.517 \Rightarrow use\ 7\phi 20mm$$

& The No of bars in a single row would be;

$$(2n-1) = \left(\frac{b-50}{\phi}\right) \Leftrightarrow n = \frac{1}{2}\left\{\frac{300-50}{20}+1\right\} = 6.75$$

place 5 ϕ20mm in the 1st row & 2 ϕ in 2nd row.



$$d' = \frac{\overset{25+6+\frac{20}{2}}{5*41} + \overset{25+6+20+25+10}{2*86}}{7} = 53.86\,mm$$

$D = 429 + 53.86 = 482.86\,mm > D_{used} = 470\,mm$, No!

Revise the design for the depth of the beam;

$D = 500\,mm$

$\Rightarrow d = D - d' = 500 - 53.86 = 446.14\,mm$

$P_d = 41.59 + 1.3\,(0.5 - 0.47)*0.3*25 = 41.88\,KN/m$

$M_{max} = \frac{P_d\,l^2}{8} = \frac{41.88*6^2}{8} = 188.46\,KN.m$

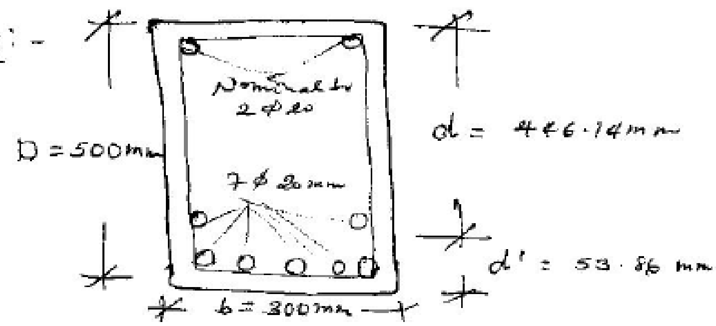
$$\rho = \frac{1}{2}\left\{0.0869 - \sqrt{0.0869^2 - \frac{4*188.46*10^6}{300*446.14^2*3003}}\right\}$$

$= 0.0145 < \rho_{max} = 0.019$, ok!

$A_{st} = \rho bd = 0.0145*300*446.14 = 1943.47\,mm^2$

No of ϕ 20mm $= \frac{1943.47}{314} = 6.19 \Rightarrow$ use $7\,\phi\,20mm$ bars in Two rows;

Sketching:-



3. Redo Example-2, One using design tables and chart

Soln:- $M_d = 188.46$ kNm, $b = 300$mm, $d = 446.14$mm

$f_{cd} = 11.33$ Mpa, $f_{yd} = 260.87$ Mpa

A). Using General Design table 1a:

$$K_m = \frac{\sqrt{\frac{M_{sd,s}}{b}}}{d} = \frac{\sqrt{\frac{188.46}{0.3}}}{0.446} = 56.20 < K_m^* = 57.8$$

⇒ Compression reinforcement not required.

$$\Rightarrow K_s = \underline{\underline{4.63}}$$

$$\Rightarrow A_s = K_s \frac{M_{sd,s}}{d} = 4.63 * \frac{188.46}{0.446} = \underline{\underline{1956.43 mm^2}}$$

(Very close to that found using equations)

B). Using General Design chart No 1.

$$\mu_{sd,s} = \frac{M_{sd,s}}{f_{cd} b d^2} = \frac{188.46 * 10^6}{11.33 * 300 * 446.14^2} = 0.2786$$

For 0% moment redistribution;

$$\mu_{u,s}^* = 0.295$$

Since $\mu_{sd,s} = 0.2786 \leq \mu_{u,s}^*$ ⇒ Compression reinforcement is not required, i.e, the section is singly reinforced

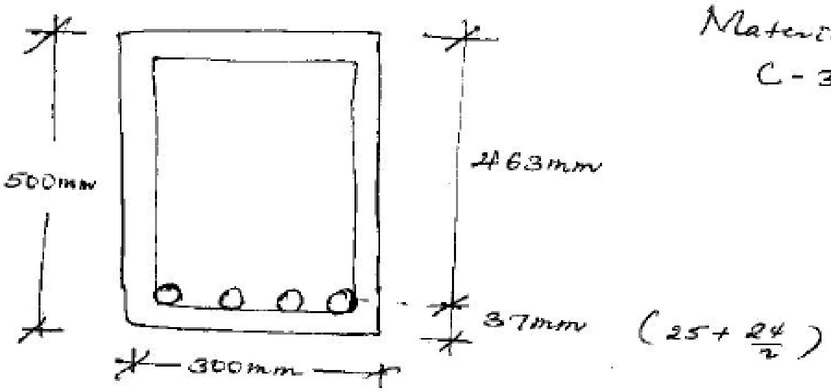
$$\therefore A_{s1} = \frac{M_{sd,s}}{Z f_{yd}}$$

Then, $K_z = \frac{z}{d} = 0.83 \Leftrightarrow z = 0.83 * 446.14 = 370.3$ mm

$$\Rightarrow A_{s1} = \frac{188.46 * 10^6}{370.3 * 260.87} = \underline{\underline{1950.93 mm^2}}$$

4. A simply supported beam spans 8m and is subjected to a live load of 30KN/m in addition to self weight. Material used are C-25, S-300 and class II work. If width $b = 250mm$, determine the depth required to satisfy section at mid-span and the corresponding flexural reinforcement.
Finally sketch it. Available bars are $\phi 16$ & $\phi 10mm$

5. Compute the maximum moment sustained by the beam using the longitudinal bars indicated below:



Material used:
C-30 & S-300

Sol:-

Step 1:- Compute design constants

$f_{cd} = 0.85 * 0.8 * \frac{30}{1.5} = 13.6$ Mpa

$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = 260.87$ Mpa

$m = \frac{f_{yd}}{0.8 f_{cd}} = \frac{260.87}{0.8 * 13.6} = 23.98$

$C_1 = \frac{2.5}{m} = \frac{2.5}{23.98} = 0.1043$

$C_2 = 0.32\, m^2 f_{cd} = 0.32\,(23.98)^2\,(13.6) = 2502.58$ Mpa

$\rho_{max} = 0.75 \left(\frac{0.8\ \varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}}\right)\left(\frac{f_{cd}}{f_{yd}}\right) = 0.0228$

$\rho = \frac{A_s}{bd} = \frac{4 * 452}{300 * 463} = 0.0130 < \rho_{max}$ .. ok!

NB. If $\rho > \rho_{max}$, we use ρ_{max} in computing maximum moment.

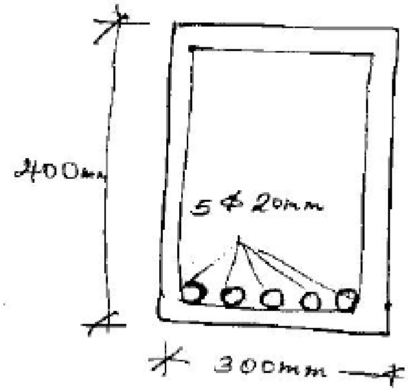
Step 2:- Determine maximum moment.

$M_{max} = 0.8\, b d^2 f_{cd}\, \rho m\, (1 - 0.4 \rho m)$

$= 0.8\,(300)\,(463)^2\,(13.6)\,(0.013)\,(23.98)\,(1 - 0.4\,(0.013)\,(23.98))$

$= 190.93$ kNm

6. The Longitudinal reinforcement bars for a beam is indicated in the figure below. Determine the maximum moment using C-20, S-360 and class II work.



7. A Simply Supported rectangular beam of span 5m Carries a live load of 20 kN/m. There is an additional dead load from a wall to the amount of 13.5 kN/m. Design the section of the beam for flexure using ultimate LSD method.

Use : C-30 Mpa $\Rightarrow f_{cd} = \frac{0.67 f_{cu}}{\gamma_c} = \frac{0.67(30)}{1.5} = 13.4\ N/mm^2$

$S-300\ Mpa \Rightarrow f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = 260.87\ N/mm^2$

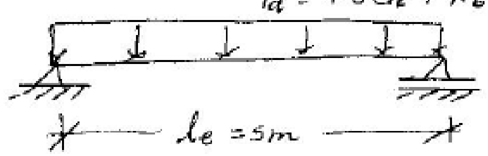
$\gamma_{RC} = 24\ kN/m^3$

Design constants using $\rho_{max} = 0.75\rho_b$,

$\omega_{max} = 0.437$ and $\mu_{max} = 0.34$

Soln :-

$P_d = 1.3 G_k + 1.6 Q_k$



Step 1: a Minimum effective depth required by deflection from serviceability limit;

$$d_{min} = \left(0.4 + 0.6 \frac{f_{yk}}{400}\right) \frac{L_e}{\beta_a}$$

where $\beta_a = 20$ for simply supported beam

$l_e = 5000\ mm$

$$\Rightarrow d_{min} = \left(0.4 + 0.6\left(\frac{300}{400}\right)\right)\left(\frac{5000}{20}\right) = 212.5mm$$

Assuming One layer of $\phi 25$ bars, Cover = 25mm.

$$D_{min} = 212.5 + 25 + 6 + 25/2 = 256\ mm$$

Step 2: 1st Trial :

$$\left.\begin{array}{l} D = 260mm \\ b = 300\ mm \end{array}\right\} \Rightarrow \frac{D}{b} \ngtr 3$$

Load on the beam:

DL (Own Wt.) $= 0.3 * 0.26 * 24 = 1.872$ kN/m

DL (from the Wall) $= 13.8$ kN/m

$\Rightarrow G_k = 1.872 + 13.2 = 15.672$ kN/m
$Q_k = \qquad = 20$ kN/m
} Service loads.

→ Design load, $P_d = 1.3\, G_k + 1.6\, Q_k = 1.3(15.672) + 1.6(20)$
$= \underline{52.374 \text{ kN/m}}$

Design Moment,

$$M_d = M_{max} = \frac{P_d\, l_e^2}{8} = \frac{52.374 * 5^2}{8} = \underline{163.669 \text{ kNm}}$$

~ Check trial depth for Singly reinforcements.

$$d_{req} = \sqrt{\frac{M_d}{\mu_{max} f_{cd}\, b}} = \sqrt{\frac{163.669 * 10^6}{0.34 * 13.4 * 300}} = \underline{346 \text{ mm}}$$

Using One layer of $\Phi 25$ bars:

$D_{req} = 346 + 25 + 6 + 25/2 = 382.5$ mm $\gg D_{assumed} = 260$ mm

∴ Depth is not adequate for Single reinforcement!

<u>2nd Trial</u>:

$$\left.\begin{matrix} D = 410 \text{ mm} \\ b = 300 \text{ mm} \end{matrix}\right\} \Rightarrow D/b \ngtr 3$$

Loads On the beam,

DL (Own Wt.) $= 0.3 * 0.41 * 24 = 2.95$ kN/m

DL (from Wall) $= 13.8$ kN/m

$G_k = 16.72$ kN/m

& $Q_k = \underline{20 \text{ kN/m}}$

Design load on the beam:

$P_d = 1.3\, G_k + 1.6\, Q_k = 1.3(16.72) + 1.6(20) = \underline{53.778 \text{ kN/m}}$

∴ Design Moment,

$$M_d = M_{max} = \frac{P_d l^2}{8} = \frac{53.778 * 5^2}{8} = \underline{168.056\,kNm}$$

Check trial depth for single reinforcement,

$$d_{req} = \sqrt{\frac{M_d}{\mu_{max} f_{cd} b}} = \sqrt{\frac{168.056 * 10^6}{0.34 * 13.4 * 300}} = \underline{350.7\,mm}$$

Using One layer of $\phi 25$ bars;

$$D_{req} = 350.7 + 25 + 6 + 25/2 = 394.2\,mm < D_{Assumed} = 410\,mm$$

ok!

$$\Rightarrow \text{Use} \begin{cases} D = 410\,mm \\ b = 300\,mm \\ d = 410 - 25 - 6 - 25/2 = 366.5\,mm \end{cases}$$

Step 4: Reinforcement:

- <u>At Mid-Span</u>: $M_d = 168.056\,kNm$ (positive)

$$\mu = \frac{M_d}{f_{cd} b d^2} = \frac{168.056 * 10^6}{13.4 * 300 * 366.5^2} = \underline{0.311}$$

- Lever Arm, $Z = \frac{d}{2}\left(1 + \sqrt{1 - 2\mu}\right)$

$$= \frac{366.5}{2}\left(1 + \sqrt{1 - 2(0.311)}\right)$$

$$= \underline{295.9\,mm}$$

Therefore, Area of Tension Steel,

$$(+ve)\quad A_s = \frac{M_d}{f_{yd} Z} = \frac{168.056 * 10^6}{260.87 * 295.9}$$

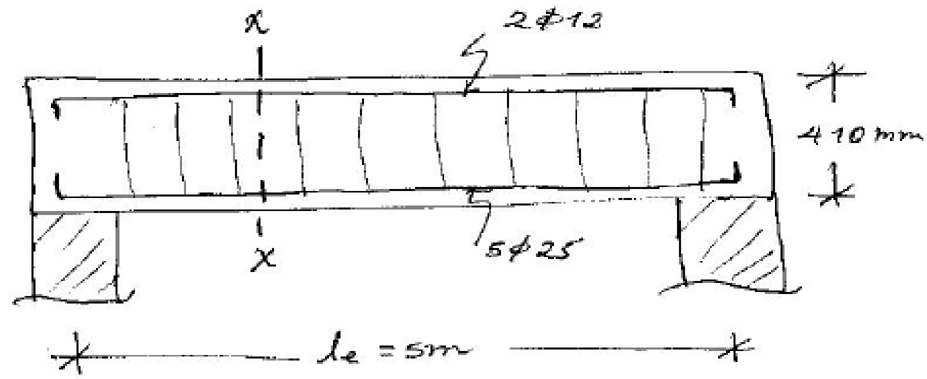
$$= \underline{2177.1\,mm^2} > A_{s\,min} = \frac{0.6}{f_{yk}} b d$$

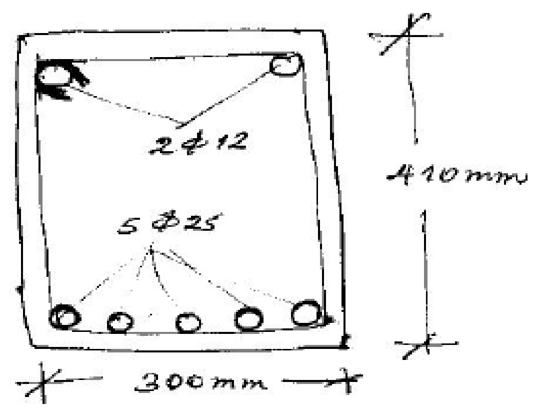
$$\text{No of } \phi 25 \text{ bars} = \frac{A_s}{a_s} = \frac{2177.1}{490.5} = 4.4 \approx 5\ \phi 25 \text{ bars}$$

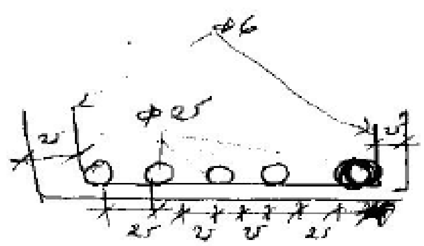
provide $5\phi 25$ bars in One layer.

provide $5\phi 25$ bars in One layer placed at bottom.

Sketch:







$2 * 25 = 50$ mm

$2 * 6 = 12$ mm

$5 * 25 = 125$ mm

$4 * 25 = 100$ mm

$\underline{287 \text{ mm}} < 300\text{mm}$ OK!

8. A Cantilever beam 4m Span Carries a live load of 150 kN/m in addition to its own weight.

A). Design the beam Section for ultimate limit State for flexure using uniform depth.

B). Redesign the Same beam for ultimate limit State for flexure using linearly varying depth (depth at free end is 1/3 of depth at fixed end).

Use: C-30 Mpa ($f_{cd} = 13.4$ Mpa)

S-300 Mpa ($f_{yd} = 260.87$ Mpa)

$\gamma_{RC} = 24\ kN/m^3$

Design constants using $\rho_{max} = 0.75\ \rho_b$ are:

$\omega_{max} = 0.437$ and $\mu_{max} = 0.34$.

Assignment-1:

Question No. 4

Question No. 6

Question No. 8

Exercise-1

1. Given the following information, Calculate the amount of reinforcement required to Support $M_u = 500$ kNm, $f_{cu} = 25\ N/mm^2$, $f_{yk} = 300\ N/mm^2$, $b = 300mm$, $d = 700mm$. Use (A). Design equations
   (B). Design Tables or Chart.

2. Design a Beam of the Smallest depth to resist a moment of 500 kNm. C-30 Concrete and S-300 Steel, Class I Works will be used. $b = 300mm$. Use (a) Design equations, (b) Design tables or Chart

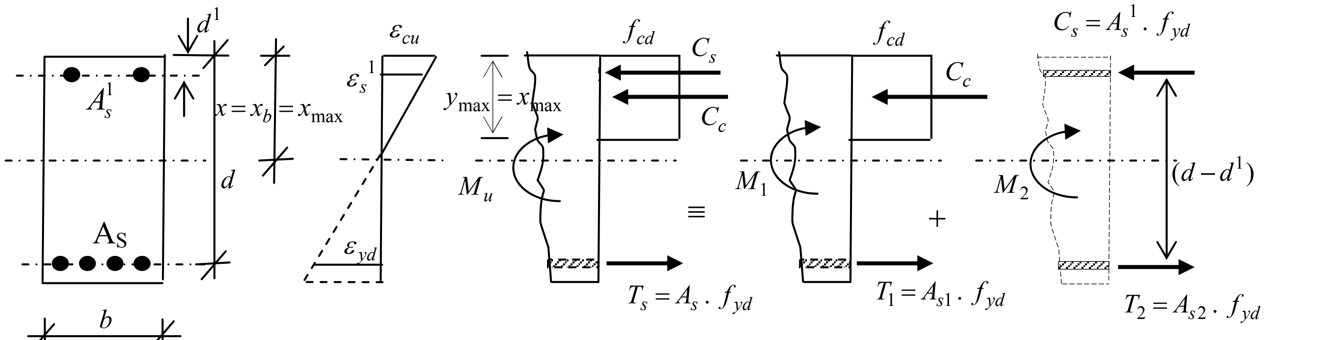
3. Determine the ultimate moment of resistance a rectangular RC section with $b = 250mm$, $d = 700mm$, C-25 Concrete and S-300 Steel, class I Works will be used. $A_s = 1365 mm^2$.

4. Calculate the amount of reinforcement required for a Beam of section 250 X 450mm to Carry a moment of 80 kNm using (a). Design equations
   (b). Design tables or Chart

   $f_{ck} = 20\ N/mm^2$, $f_{yk} = 400\ N/mm^2$, class I Works.

3.3. Doubly Reinforced Rectangular Section

Consider a doubly reinforced rectangular section subjected to an ultimate moment, M_u as shown below. Design equations are derived by dividing the section into two parts: *Balanced singly reinforced section* and *excess tension steel plus compression steel*. It is assumed that both tension and compression steels are yielded. The excess tension steel and compression steel are proportioned in such a way that the neutral axis is maintained at balanced position.



a) x-section   b) Strain   c) stresses, doubly reinforced section, M_u d) Stresses, balanced section, M_1 e) stresses, excess tension-steel plus comp. steel, M_2

Let M_1 --moment capacity of balanced singly reinforced section

M_2 --moment resistance provided by excess tension steel plus compression steel

Thus, the total ultimate moment of resistance of doubly reinforced section is the sum of the two parts: moment capacity of balanced singly reinforced section, M_1 and ultimate moment resisted by excess tension steel plus compressive steel, M_2.

i.e $$M_u = (M_1 + M_2)$$

Moment capacity of balanced singly reinforced section,

$$M_1 = \mu_{max} \cdot f_{cd} \cdot b \cdot d^2$$

And, the corresponding area of tension steel balancing M_1 is,

$$A_{s1} = \frac{M_1}{f_{yd} \cdot z_{min}}$$

Where $$z_{min} = (d - 0.4x_{max}) = d \cdot \left(1 - \frac{\omega_{max}}{2}\right)$$

Excess moment to be resisted by excess tension steel plus compression steel is,

$$M_2 = (M_u - M_1)$$

Equating excess moment with the couple made by internal forces in excess tension steel and compression steel as shown in Fig.(e), area of excess tension steel and compression steel are obtained as (if compression steel yielding)

$$A_{s2} = \frac{M_2}{f_{yd} \cdot (d - d^1)} \qquad \text{And,} \qquad A_s^{\,1} = \frac{M_2}{f_{yd} \cdot (d - d^1)}$$

Therefore, the total area of tension steel required by doubly reinforced section,

$$A_s = A_{s1} + A_{s2}$$

To check yielding of compression steel, referring to stain diagram in Fig.(b), the strain in compression steel is determined and compared with the yield strain of a given steel as obtained below.

$$\frac{x_{\max}}{d^1} = \frac{\varepsilon_{cu}}{(\varepsilon_{cu} - \varepsilon_s^{\,1})}$$

$$\Rightarrow \qquad \varepsilon_s^{\,1} = \varepsilon_{cu} \cdot \frac{(x_{\max} - d^1)}{x_{\max}}$$

Where $\varepsilon_{cu} = 0.0035$

$$x_{\max} = y_{\max}/0.8 = (\omega_{\max} \cdot d)/0.8$$

If compression steel is yielding,

$$\varepsilon_s^{\,1} \geq \varepsilon_{yd} = \frac{f_{yd}}{E_s} \qquad \& \qquad f_s^{\,1} = f_{yd} \qquad \text{(as assumed)}$$

Or, if compression steel is not yielding,

$$\varepsilon_s^{\,1} < \varepsilon_{yd} = \frac{f_{yd}}{E_s} \qquad \& \qquad f_s^{\,1} = E_s \cdot \varepsilon_s^{\,1} < f_{yd}$$

Then, area of compression steel is re-determined using,

$$A_s^{\,1} = \frac{M_2}{f_s^{\,1} \cdot (d - d^1)} = \frac{M_2}{E_s \cdot \varepsilon_s^{\,1} \cdot (d - d^1)}$$

Another Similar approach:

Assume that A_s & A_{s1} are stressed to f_{yd}.

$$M_u = M_{uc} + M_{usc}$$

Where M_{uc} is the BM carried by the concrete and partial area of tensile steal.

$$\Rightarrow \quad M_{uc} = 0.8bd^2 f_{cd}\, k_1\, (1\text{-}0.4\, k_1)$$

In which $k_1 = k_{x\,\max}$, the maximum steel ratio corresponding to single reinforcement section in case of ***design*** and

$$k_1 = \frac{A_s - A_{s1}}{bd} \le k_{x\,\max} \quad \text{for the case of } \textbf{\textit{analysis}}.$$

M_{usc} is the BM carried by compressive steel and the corresponding tensile steel.

$$M_{usc} = A_{s1} f_{yd} (d - d_{c'})$$

The yielding of the compressive steel may be checked from the strain relation as

$$\varepsilon_{sc} = \frac{x - d_{c'}}{x} * \varepsilon_{cu} \ge \varepsilon_{yd}$$

Examples on Design of Doubly Reinforced Beams using Limit State Design Method

1. Design a beam having x-sectional dimension of 300mm x 400mm of One end is simply supported and the other end fixed. The span of the beam is 6m and the loads are 25 kN/m live load in addition to its weight.

Materials used are C-30, S-360 & class I work. Use ϕ20mm bar.

Sol$\underline{n}$:-

Step 1: Check depth for deflection:

$$d \geq \left(0.4 + 0.6\frac{f_{yk}}{400}\right)\frac{L_e}{\beta_a}$$

$$= \left(0.4 + 0.6\left(\frac{360}{400}\right)\right)\left(\frac{6000}{24}\right)$$

$$= \underline{235\,mm}$$

Assuming ϕ20mm in a row,

$$D = 235 + 35 = 270\,mm$$

$$D = 270\,mm < D_{used} = 400\,mm \quad ok!$$

Step 2: Compute design load & moments;

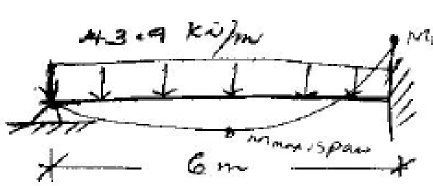
$$G_k = 0.3 * 0.4 * 25 = 3\,kN/m$$

$$Q_k = 25\,kN/m$$

Design load, $P_d = 1.3\,G_k + 1.6\,Q_k$

$$= 1.3 * 3 + 1.6 * 25$$

$$= \underline{43.9\,kN/m}$$



$$M_{max,\,Support} = \frac{P_d\,l^2}{8} = \frac{43.9 * 6^2}{8}$$

$$= \underline{197.55\,kNm}$$

$$M_{max,\,span} = \frac{9}{128}P_d\,l^2 = \frac{9}{128}(43.9)(36) = \underline{111.12\,kNm}$$

Step 3: Determine design Constants:

$f_{cd} = 13.6$ Mpa, $f_{yd} = 313.04$ Mpa, $m = 28.77$

$C_1 = 0.0809$, $C_2 = 3602.21$ Mpa, $\rho_{max} = 0.018$

Step 4: Check depth for maximum Support moment;

$$d \geq \sqrt{\frac{M_{max,supp}}{0.8\, b\, f_{cd}\, \rho m\, (1 - 0.4 \rho m)}}$$

$$= \sqrt{\frac{197.56 * 10^{6}}{0.8 * 300 * 13.6 * 28.77 * 0.018\, (1 - 0.4 (28.77) * 0.018)}}$$

$= 383.94$ mm

$\Rightarrow D = d + d' = 383.94 + 35 = 418.94\text{mm} > D_{prov'd}$

∴ The Beam has to be doubly reinforced, at the support.

Step 5: Reinforcement:

(A). Support:

The Moment Carried by Concrete Section

$$M_{dc} = 0.8\, b\, d^2 f_{cd}\, \rho m\, (1 - 0.4 \rho m)$$

$$= 0.8\,(300)\,(365)\,(13.6)\,(0.018)\,(28.77) * (1 - 0.4 * 0.018 * 28.77)$$

$= 178.54$ kNm

i). Tension Bars

$$A_{s1} = \rho_{max}\, b\, d = 0.018 * 300 * 365 = 1971\, mm^2$$

$$A_{s2} = \frac{M_{dsc}}{f_{yd}(d-d_c')} = \frac{M_{max,supp} - M_{dc}}{f_{yd}(d-d_c')}$$

$$= \frac{(197.86 - 178.54) * 10^6}{313.04\,(365-35)}$$

$$= \underline{184.02\, mm^2}$$

$$\Rightarrow A_s = A_{s1} + A_{s2} = 1971 + 184.02 = \underline{2155.02\, mm^2}$$

No of ϕ 20mm $= \frac{2155.02}{314} = 6.86 \Rightarrow$ Use 7 ϕ 20mm

ii) Compression Bars:

$$X_{max} = \rho_{max}\, m\, d = 0.018 * 28.77 * 365 = \underline{189.02}$$

$$\epsilon_{sc} = \frac{\epsilon_{cu}(X_{max} - d_c')}{X_{max}} = \frac{0.0035\,(189.02 - 35)}{189.02}$$

$$= \underline{2.85 * 10^{-3}}$$

$$f_s' = \epsilon_{sc} E_s = 2.85 * 10^{-3} * 2 * 10^5 = 576\ Mpa$$

Since $f_s' = 576$ Mpa $>$ $f_{yd} = 260.87$ Mpa

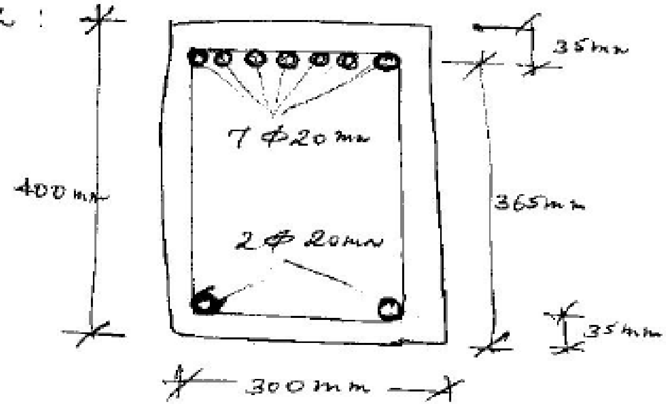
$\Leftrightarrow$ Compression steel is yielding $\Leftrightarrow f_s' = f_{yd}$

$A_s' = A_{s2}$ No of ϕ 20mm $= \frac{184.02}{314} = 0.586 \Rightarrow 1\ \phi$ 20mm

For practical purposes $\Rightarrow$ Use 2 ϕ 20mm

i.e, $A_s' = A_{s2}$

Sketch:



B) At the Span

Since $M_{max, Span} < M_{dc}$ ⇒ It is Singly Reinforced in the Span.

$$\rho = \frac{1}{2}\left\{C_1 - \sqrt{C_1^2 - \frac{4M}{bd^2 C_2}}\right\}$$

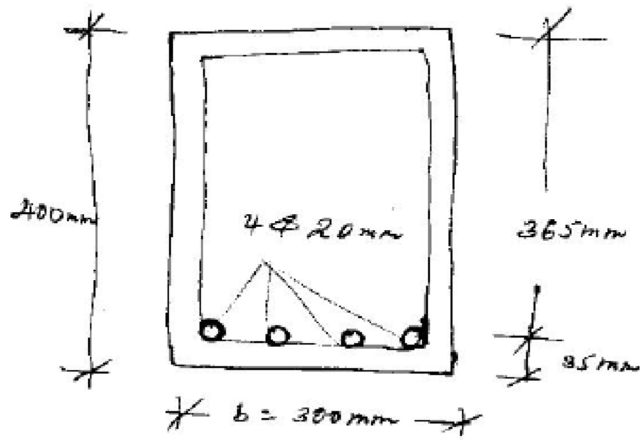
$$= \frac{1}{2}\left\{0.0869 - \sqrt{0.0869^2 - \frac{4 * 111.86 * 10^6}{300 * 365^2 * 3602.21}}\right\}$$

$$= 0.01 < \rho_{max} = 0.018$$

$$\Rightarrow A_s = \rho b d = 0.01 * 300 * 365 = \underline{1095\, mm^2}$$

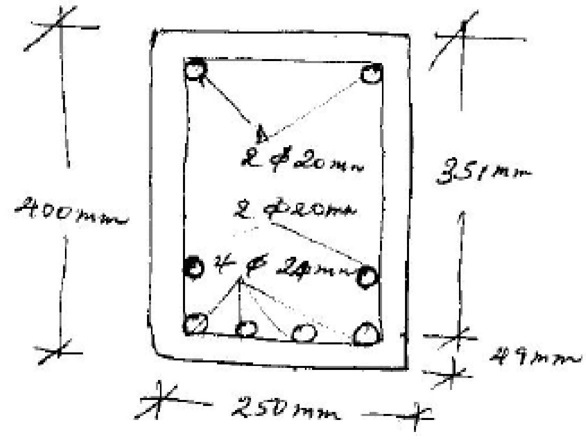
No of φ20mm $= \frac{1095}{314} = 3.48$ ⇒ Use 4 φ20mm in a row.

Sketch:



2. Design a Simply Supported Beam of Span 6m which is subjected to a uniformly distributed live load of 20 kN/m in addition to its weight. The X-sectional dimension of the beam is 200mm X 300mm (restricted). Material used are C-20, S-260 and class II work, Available bars are ϕ14, ϕ16 and ϕ20mm.

3. The Reinforcement bar for a doubly reinforced beam are ($4\phi24 + 2\phi20mm$) for Tension zone and ($2\phi20mm$) for compression zone. materials used are C-25, S-280 and class II work. Compute the maximum moment carried.



Sol:-

Step 1: Determine Design Constants:

$f_{cd} = 11.33$ Mpa, $f_{yd} = 243.48$ Mpa

$m = 26.86$, $\rho_{max} = 0.021$

$$\rho_1 = \frac{A_s - A_s'}{bd} = \frac{(4*450 + 2*314 - 2*314)}{250 * 351}$$

$= 0.0205$

Step 2: Compute the maximum Moment:

$$M_{dc} = 0.8 b d^2 f_{cd} \rho_1 m \, (1 - 0.4 \rho_1 m)$$

$= 0.8\,(250)\,(351)^2\,(11.33)\,(0.0205)\,(26.86)\,(1 - 0.4 * 0.0205 * 26.86)$

$= \underline{119.86 \text{ KNm}}$

Checking the yielding of Steel ;

$$x_{max} = \rho_{max}\, m\, d = 0.021 * 26.86 * 351 = 197.99\ mm$$

$$\epsilon_{sc} = 2.88 * 10^{-3}$$

$\Rightarrow f_s' = 576 \nleq f_{yd}$, Thus , Compression Steel is yielding $\Rightarrow f_s' = f_{yd} \Leftrightarrow A_s' = A_{s2}$

$$M_{dsc} = A_s' f_{yd} (d - d_c') = 2 * 314 * 243.48 (351-35)$$

$$= 48.32\ kNm$$

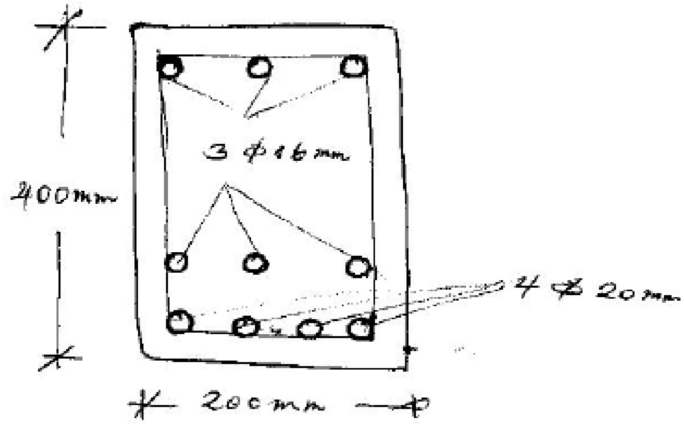
Total Moment ;

$$M_{max} = M_{dc} + M_{dsc}$$

$$= 119.86 + 48.32$$

$$= 168.18\ kNm$$

4. The Reinforcement bars are indicated for a beam having a dimension of 200 X 400mm. Determine the maximum moment using C-20, S-260 and Class II work.



5. The Rectangular Beam with X-Sectional area of 250mm * 450mm supports a service live load of 18 kN/m and a service dead load of 2.81 kN/m on a simple span of 8m. Determine the required reinforcement.

Use :- C-25 Mpa ($f_{cd} = 11.17 \, N/mm^2$)

S-300 Mpa ($f_{yd} = 260.87 \, N/mm^2$)

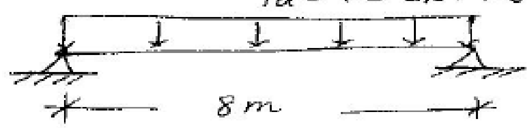
Design Constants using $\rho_{max} = 0.75 \rho_b$ are

$\mu_{max} = 0.34$ and $\omega_{max} = 0.437$.

Soln :-

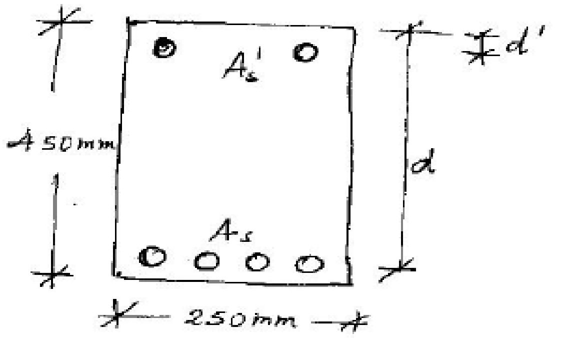
$P_d = 1.3 G_K + 1.6 Q_K = 1.3(2.81) + 1.6(18)$

$= \underline{32.45 \, kN/m}$



Design Moment; $M_d = M_{max} = \frac{P_d l^2}{8} = \frac{32.45 * 8^2}{8}$

$= \underline{259.6 \, kNm}$



Assuming One layer of $\phi 30$ bars for both tension and compression bars;

$d = D - Cover - \phi_{stirrup} - \frac{\phi_L}{2} = 450 - 25 - 6 - \frac{30}{2} = \underline{404mm}$

$d' = Cover + \phi_s + \frac{\phi_L}{2} = 25 + 6 + 30/2 = \underline{46mm}$

Check the section for singly or doubly reinforced.

$$\mu_{req} = \frac{M_d}{f_{cd} b d^2} = \frac{259.6 * 10^6}{11.17 * 250 * 404^2} = 0.5696 > \underset{=0.34}{\mu_{max}}$$

Therefore, the section should be designed as doubly reinforced.

Then, Capacity of singly reinforced balanced section,

$$M_1 = \mu_{max} f_{cd} b d^2 = 0.34 * 11.17 * 250 * 404^2$$
$$= 154.965 * 10^6 \, Nmm$$
$$= \underline{154.965 \, KNm}$$

and, area of tension steel balancing M_1;

$$A_{s1} = \frac{M_1}{f_{yd} Z_{min}} \quad \text{where } Z_{min} = d\left(1 - \frac{\omega_{max}}{2}\right)$$
$$= 404\left(1 - \frac{0.437}{2}\right)$$
$$= \underline{315.7 mm}$$

$$= \frac{154.965 * 10^6}{260.87 * 315.7}$$

$$\Longleftrightarrow A_{s1} = \underline{1881.6 \, mm^2}$$

~ Excess moment;

$$M_2 = M_d - M_1 = 259.6 - 154.965 = \underline{104.635 \, KNm}$$

Then, area of excess tension steel of Compression steel;

$$A_{s2} = A_s' = \frac{M_2}{f_{yd}(d - d')} = \frac{104.635 * 10^6}{260.87(404 - 46)}$$

$$= \underline{1120.4 \, mm^2}$$

Therefore, Total area of Tension,

$$A_s = A_{s1} + A_{s2}$$
$$= 1881.6 + 1120.4$$
$$= \underline{3010.2 \, mm^2}$$

Check, $\rho = \frac{A_s}{bd} = \frac{3010.2}{250 * 404} = 0.03 \le \rho_{max} = 0.04$

Ok!

No of $\phi 30$ bars $= \frac{A_s}{a_s} = \frac{3010.2}{706.8} = 4.26 \approx 5\phi 30$ bars

$\Rightarrow$ provide $5\phi 30$ bars in One row placed at the bottom of the beam.

Check yielding of Compression steel!

$$\epsilon_s' = \epsilon_{cu} * \frac{(X_{max} - d')}{X_{max}}$$

Substitute $\epsilon_{cu} = 0.0035$ & $X_{max} = \frac{\omega_{max}}{0.8} d$

$d' = 46mm$ $\Rightarrow X_{max} = \underline{218.5mm}$

$$\Rightarrow \epsilon_s' = (0.0035)\frac{(218.5 - 46)}{218.5} = 0.00277$$

$$\Rightarrow \epsilon_s' = 0.00277 \nleq \epsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{260.87}{200,000} = 0.0013$$

$\Rightarrow \therefore$ Comp. steel is yielding!

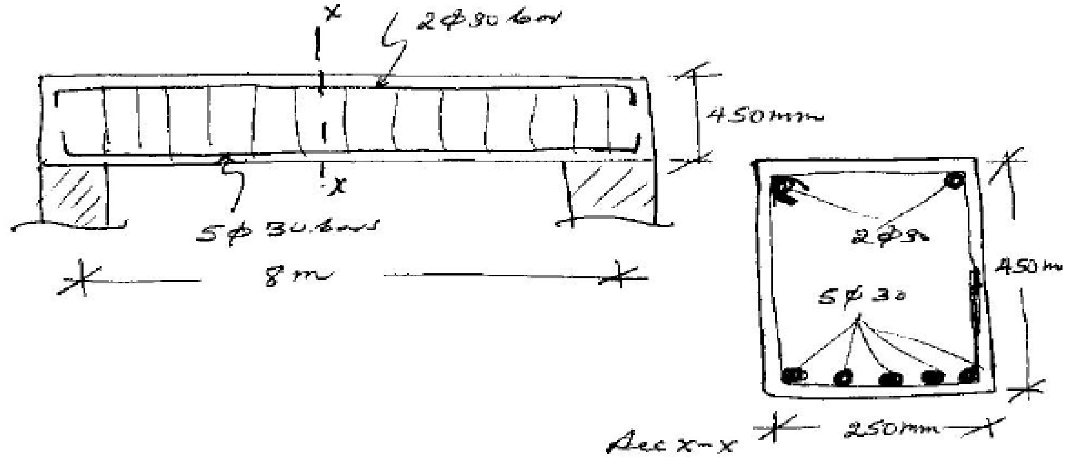
i.e, $f_s' = f_{yd}$

then, $A_s' = A_{s2} = \underline{1120.4 mm^2}$

No of $\phi 30$ bars $= \frac{A_s'}{a_s} = \frac{1120.4}{706.8} = 1.59 \approx 2\phi 30$ bars

$\Rightarrow$ provide: $2\phi 30$ bars in One row placed at the Top of the beam.

Sketch:-



3.4. Flanged Section (T- or L-section) under Flexure

The general discussion with respect to flanged section, effective width of flange in working stress method holds for strength limit state method as well. In treating flanged section using strength limit state method, it is convenient to adopt the same equivalent rectangle stress-block that is used for rectangular cross section.

i) If depth of equivalent rectangle stress-block, $'y'$ is equal to or less than the flanged thickness, $'h_f'$ (i.e $y \leq h_f$), a flanged section may be treated as a rectangular section of width equal to an effective width of flange, $'b_e'$ provided the flange of section is on compression side when the section subjected a moment.

For trial purpose initially, it can be assumed the stress block is with-in the flange (or assume flanged section rectangular with width equal to effective width of flange).

-calculate relative ultimate moment and relative mechanical steel ratio of assumed rectangular section using,

$$\mu = \frac{M_u}{f_{cd} . b_e . d^2}$$

And $$\omega = 1 - \sqrt{1 - 2\mu}$$

-then, compute depth of equivalent rectangle stress-block for assumed section and compare with thickness of the flange of the section,

$$y = \omega \, . \, d$$

-If $y \leq h_f$, the section is designed as rectangular section with width equal to effective width of flange, $'b_e'$. Therefore, area of tension steel required by the section for such case is given by

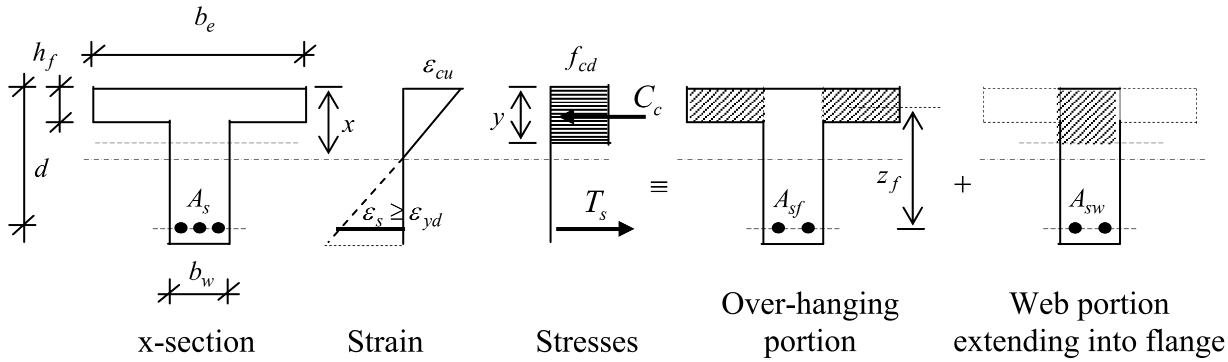
$$A_s = \frac{M_u}{f_{yd} \, . \, z}$$

Where $$z = \frac{d}{2}\left(1 + \sqrt{1 - 2\mu}\right)$$

ii) If the depth of equivalent rectangle stress-block of assumed rectangular section is greater than thickness of the flange of the section (i.e $y > h_f$), a flanged section is treated as T-beam section provided the flange of section is on compression side when the section subjected a moment. To derive design equation of T-beam, it is convenient to divide the compression area of T-beam section into two parts as shown below:

a) the over-hanging portion of the compressive flange

b) the web portion extending into the compressive flange



Let A_{sf} --area of tension steel balancing over-hanging portion of the flange

A_{sw} --area of tension steel balancing web portion extending into the flange

The total ultimate moment of resistance of T-beam section is obtained by taking moment of the internal compressive forces about the center of tension steel; and it is given as the sum of moments produced by over-hanging portion of the flange and the web portion extending into the flange. i.e $M_u = M_{uf} + M_{uw}$

-The moment produced by over-hanging portion of the flange is obtained as

$$M_{uf} = (b_e - b_w).\, h_f \,.\, f_{cd} \,.\, z_f$$

Where $z_f = \left(d - h_f/2\right)$

Then, the corresponding area of tension steel balancing the over-hanging portion of the flange is obtained as $A_{sf} = \dfrac{M_{uf}}{z_f \,.\, f_{yd}}$

-The moment produced by the web portion extending into the flange is obtained by subtracting moment of over-hanging portion from the total ultimate moment of T-beam.

i.e $M_{uw} = (M_u - M_{uf})$

To determine the corresponding area of tension steel balancing web potion extending into the flange, the web portion is treated as rectangular section with width equal to the width of the web, b_w . Therefore, calculate the relative ultimate moment the web portion using

$$\mu_w = \frac{M_{uw}}{f_{cd} \cdot b_w \cdot d^2}$$

Then, the required area of tension steel balancing web potion is obtained as

$$A_{sw} = \frac{M_{uw}}{f_{yd} \cdot z_w}$$

Where $$z_w = \frac{d}{2} \cdot \left(1 + \sqrt{1 - 2\mu_w}\right)$$

Therefore, the total area of tension steel is obtained as

$$A_s = A_{sf} + A_{sw}$$

Check flanged section for single reinforcement using $\mu_w \leq \mu_{\max}$. If the flanged section requires compression reinforcement ($\mu_w > \mu_{\max}$), area of compressive steel and excess tension steel required by web portion is obtained using (if compression steel is yielding)

$$A_s^1 = A_{s2} = \frac{\left(M_{uw} - M_1\right)}{f_{yd} \cdot \left(d - d^1\right)}$$

and, area of tension steel balancing web portion is re-determined using

$$A_{sw} = \frac{M_{uw}}{f_{yd} \cdot z_{\min}}$$

Where $$M_1 = \mu_{\max} \cdot f_{cd} \cdot b_w \cdot d^2 \qquad \& \qquad z_{\min} = \left(d - 0.4x_{\max}\right) = d \cdot \left(1 - \frac{\omega_{\max}}{2}\right)$$

iii) If the flange of the section is on the tension side when subjected to a moment, flanged section is designed as if it were a rectangular section with width equal to the width of the web, b_w.

Another similar approach:

Reinforced concrete floors or roofs are monolithic and hence, a part of the slab will act with the upper part of the beam to resist longitudinal compression. The resulting beam cross-section is, then, *T-shaped* (inverted L), rather than *rectangular* with the slab forming the beam flange where as part of the beam projecting below the slab forms the web or stem.

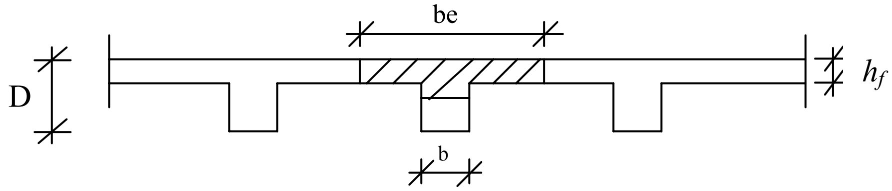


Fig. 3.3.1

The T-sections provide a large concrete cross-sectional area of the flange to resist the compressive force. Hence, T-sections are very advantageous in simply supported spans to resist large positive bending moment, where as the *inverted T-sections* have the added advantage in *cantilever beam* to resist negative moment.

As the longitudinal compressive stress varies across the flange width of same level, it is convenient in design to make use of an *effective flange width* (may be smaller than the actual width) which is considered to be uniformly stressed.

Effective flange width (according to EBCS 2, 1995)

For ***interior beams*** $\Rightarrow$ *T-sections*

$$b_e \leq \begin{cases} b_w + \dfrac{l_e}{5} \\ C/C \text{ beam spacing} \end{cases}$$

For ***edge beams*** $\Rightarrow$ *inverted L- sections*

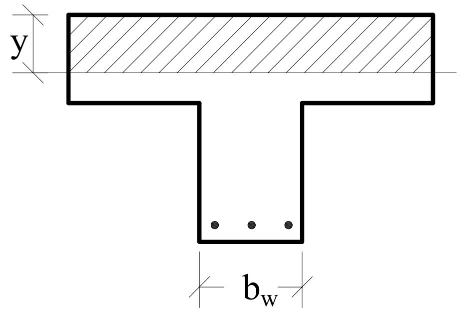
$$b_e \leq \begin{cases} b_w + \dfrac{l_e}{10} \\ b_w + \text{half the clear distance to adjacent beam} \end{cases}$$

Where l_e – is the effective span length & b_w is the width of the web.

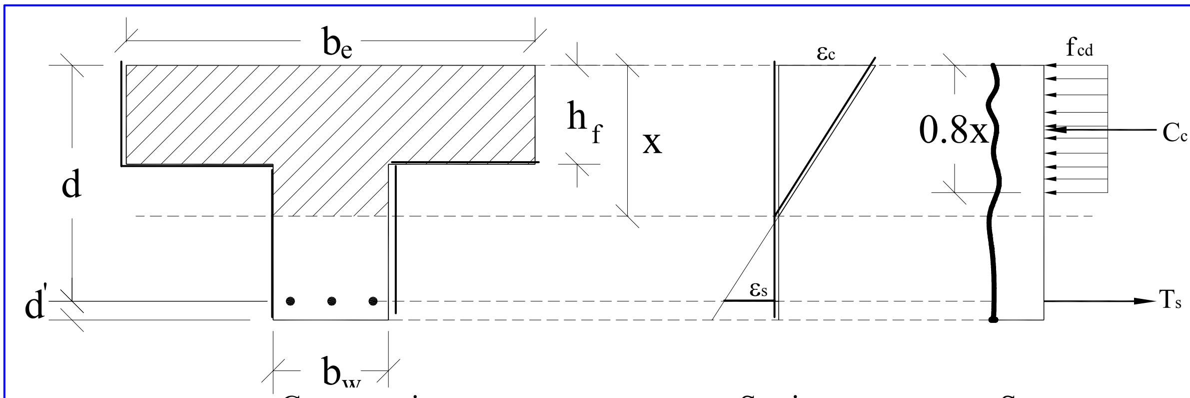
There are three distribution type of flexural behavior of T-sections.

- When the T-section is subjected to BM, and tension is produced on the flange portion, it is treated as a rectangular section with b = b_w.
- When the T-section is subjected to +ve bending moment and the equivalent compressive stress block lies within the flange as shown below ($y < h_f$), the section can be analysed as rectangular with effective width b_e.

– This case is a case of under reinforced condition or large flange thickness, which can be confirmed first by computing ρ (with $b = b_e$, $\rho = A_s/(b_e d)$) using relation established for rectangular beams and evaluate the NA depth, $x = \rho m d$. Compare $y = 0.8x$ with h_f.



- When $y > h_f$, the section acts as T-beam and hence analysis accounting the T-geometry becomes essential which is shown in the figure below.



Cross section Strain Stress

Cross-section Design and Analysis

Design

- Assuming $b = b_e$ *compute* $k_x = 0.5\left\{c_1 - \sqrt{c_1 - \dfrac{4M_d}{b_e d^2 c_2}}\right\}$ *and* $x = k_x d$

i) If $y = 0.8x < h_f$, section is rectangular as assumed.

$$\Rightarrow A_s = \frac{k_x}{m} b_e d$$

ii) If $y > h_f \Rightarrow$ *T beam* analysis is required.

$$A_s = A_{Sf} + A_{sw} = \frac{M_{uf}}{Z_f * f_{yd}} + \rho_w b_w d \quad \text{in which,}$$

$$M_{uf} = (b_e - b_w)\, h_f f_{cd} z_f$$

$$Z_f = d - \frac{h_f}{2}$$

$$\rho_w = \frac{k_w}{m} = \frac{0.5}{m}\left\{c_1 - \sqrt{c_1 - \frac{4M_{uw}}{b_w d^2 c_2}}\right\}$$

$$M_{uw} = M_u - M_{uf}$$

iii) When the flange is on the tension side, then the cross- section is designed as if it were rectangular with $b = b_w$

Analysis: $\rho = \frac{A_s}{b_e * d}$, $\boldsymbol{X = \rho m d}$

i) If $y = 0.8X <= h_f \Rightarrow$ the section is analyzed as rectangular with $b = b_e$.

$$Mu = 0.8 b_e d^2 f_{cd}\, \rho m\, (1 - 0.4\, \rho m)$$

ii) If $y = 0.8X < h_f \Rightarrow$ the section is analyzed as T-beam.

$$M_{uf} = (b_e - b_w)\, h_f f_{cd} z_f\,, \qquad A_{Sf} = \frac{M_{uf}}{Z_f * f_{yd}}\,, A_{sw} = A_s - A_{Sf}$$

$$\rho_w = \frac{A_{sw}}{b_w * d} \qquad M_{uw} = 0.8 b_w d^2 f_{cd}\, \rho_w m (1 - 0.4 \rho_w m)$$

$$M_u = M_{uf} + M_{uw}$$

Examples on Design of T-Section Beams using Limit State Design Method

1. Floor Systems are Sustained by parallel T-beams of Span 6m and 1.8m On Center to Center Spacing. The Total design moment is 450 kNm.

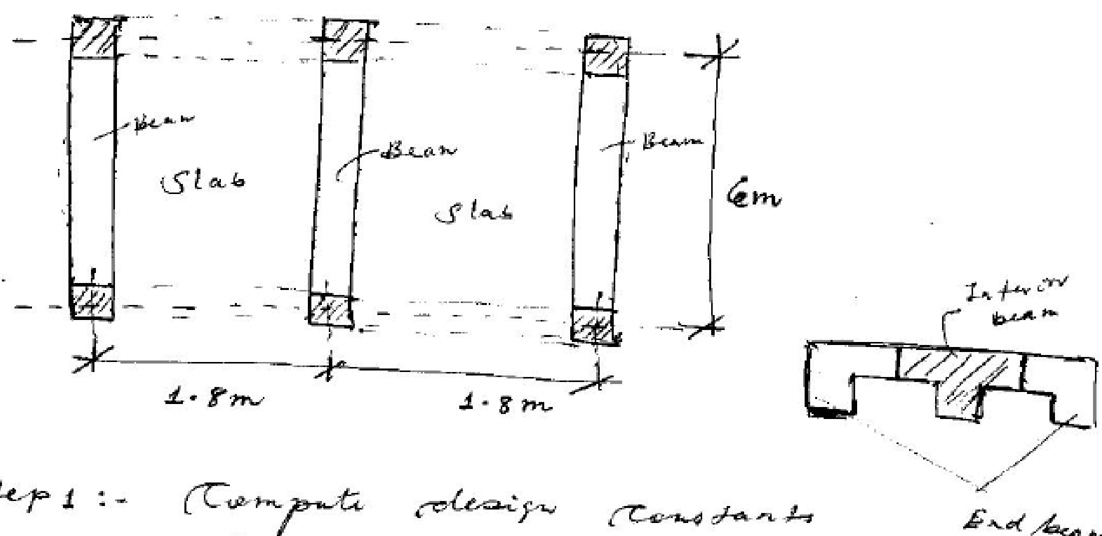
Dimensions for the beam: $D = 400mm$, $b_w = 300mm$

Flange depth, $h_f = 80mm$.

Material used are: $f_{cu} = 25 Mpa$, $f_{yk} = 300 Mpa$ & Class I Works

Design the Interior beam.

Soln:-



Step 1 :- Compute design Constants

$$f_{cd} = 0.8 * 0.85 * \frac{f_{cu}}{\gamma_c} = 11.33 \text{ Mpa}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = 260.87 \text{ Mpa}$$

$m = 28.78$

$c_1 = 0.0869$

$c_2 = 3003 \text{ Mpa}$

$\rho_{max} = 0.019$

Step 2 :- Check depth for deflection:

(i). Slab :

$$d \geq \left(0.4 + 0.6 \frac{f_{yk}}{400}\right) \frac{L_e}{\beta_a}$$

$$= 0.85 \left(\frac{1800}{30}\right)$$

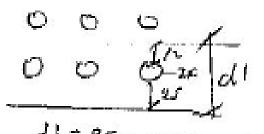
$$= \underline{51 mm}$$

Assuming $\phi 12$ mm used ; $h_f = 80mm$

$$D = 51 + \underbrace{15}_{Cover} + \underbrace{6}_{\frac{\phi 12}{2}} = 72mm < D_{used} \text{ Ok!}$$

(ii). Beam :

$$d \geq 0.85 \left(\frac{6000}{20}\right) = 255 mm$$



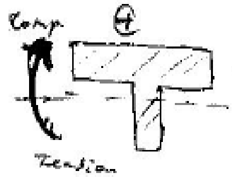
$d' = 25 + 24 + 12 = 61mm$

Assuming two rows of $\phi 24$ mm are used:

$$D = 255 + 25 + 24 + 12 = 316 mm < D_{used} \text{ (400mm)}$$

Ok!

Step 3 : Compute Effective Width:



⇒ Since positive bending moment is given, the Top is on Compression Side

$$b_e \leq \begin{cases} b_w + L_e/5 = 300 + \frac{6000}{5} = 1500mm \\ c/c = 1800 mm \end{cases}$$

$$\Rightarrow \underline{b_e = 1500 mm}$$

Step 4: Check for T-Beam:

$$\rho = \frac{1}{2}\left\{ C_1 - \sqrt{C_1^2 - \frac{4M}{b_e d^2 C_2}} \right\} \quad \text{where } d = 400-61 = 339\,mm$$

$$= \frac{1}{2}\left\{ 0.0869 - \sqrt{0.0869^2 - \frac{4*450*10^6}{1500*339^2*3003}} \right\}$$

$$= \underline{0.0115}$$

$\Rightarrow x = \rho m d = 0.0115 * 28.78 * 339 = 112.2mm$

$\Rightarrow y = 0.8x = 0.8 * 112.2 = \underline{89.76mm} > h_f = 80mm$

$\therefore$ The Beam is treated as T- beam!

Step 5: Reinforcement Bars:

(i). Compute moment Carried by Flange;

$$M_{df} = (b_e - b_w)\, h_f * f_{cd}\, (d - h_f/2)$$

$$= (1500-300)(80)(11.33)(339 - 80/2) * 10^{-6}$$

$$= \underline{325.22\ kNm}$$

$$\therefore A_{sf} = \frac{M_{df}}{f_{yd}\, Z_f} = \frac{325.22*10^6}{260.87\,(339-80/2)}$$

$$= \underline{4169.48\ mm^2} \qquad \text{where } Z_f = d - h_f/2$$

(ii). Compute M_{dw}

$\Rightarrow M_{dw} = M_d - M_{df} = 450 - 325.22 = 124.78$ kNm

$\Rightarrow \rho_w = \frac{1}{2}\left\{ C_1 - \sqrt{C_1^2 - \frac{4 M_{dw}}{b_w d^2 C_2}} \right\} = \underline{0.0173} < \rho_{max}$

NB. If $\rho_w > \rho_{max} \Rightarrow$ Doubly reinforced!

$A_{sw} = \rho_w b_w d = 0.0173 * 300 * 339 = \underline{1759.41 mm^2}$

$\therefore A_s = A_{sw} + A_{sf} = 1759.41 + 4169.48$

$= \underline{5928.89 mm^2}$

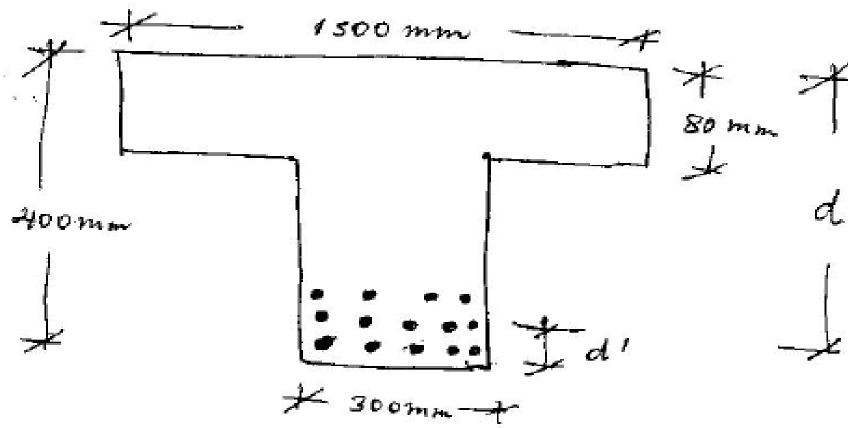
No of $\phi 24mm = \frac{5928.89}{452} = 13.11$

$\Rightarrow$ <u>use 14 ϕ 24 mm bar</u>

No of bars which can be placed in a row,

$n = \frac{1}{2}\left(\frac{250^{\leftarrow 300-50}}{24} + 1\right) = 5.7 \Rightarrow$ use $\underline{5}$

Sketch :-



Since No of rows are changed from assumed, Thus, Revise the section ; / Exercise /

2. Design the Edge beam using 2/3 of the bending moment and with the same data in example-1.

3. Design a T-beam with $b_e = 1000\,mm$, $h_f = 100\,mm$, $b_w = 250\,mm$, $d = 450\,mm$, Use C-25 Concrete & S-460 steel, class I works & $M = 470\,kNm$.

Assignment-2:

Question No. 2

Question No. 3

Question No. 5

4. A Floor System is Supported by Beams Spaced at 3m centerlines which are Simply Supported at One end and Fixed at the Other end. The beams are 8m in Span, Web width $b_w = 250mm$ The Overall depth is limited to 500mm and the Slab thickness, $h_f = 100mm$. The Floor is subjected to a Superimposed Service load of $4 KN/m^2$. Design a typical interior beam for flexure. Use C-30, S-300 & Class I work. Assume Column dimension is to be 250 X 250mm.

Sol:-

Step 1:- Check the depth for deflection requirement,

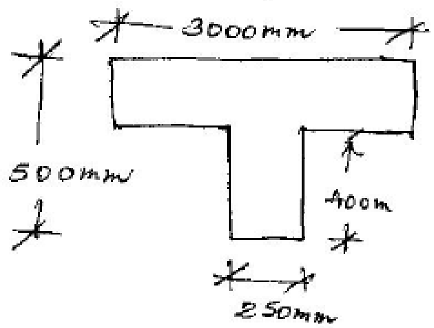
$$d \geq \left(0.4 + 0.6 \frac{f_{yk}}{400}\right) \frac{L_e}{\beta_a}$$

$$= \left(0.4 + 0.6 * \frac{300}{400}\right) \left(\frac{3000}{24}\right)$$

$$= \underline{283 \text{ mm}}$$

$\Rightarrow D = 283 + 61 = 344 mm < D_{used}$ (= 500mm) OK!

Step 2:- Loading:



↳ For loading, use C/C distance always.

$G_K = [3*0.1 + 0.4*0.25]*25 + [0.05*3]*20 = \underline{13 KN/m}$

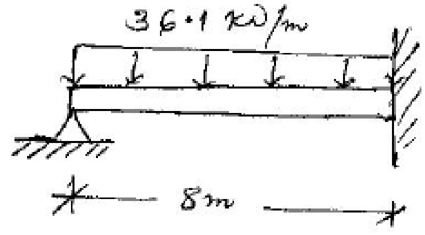
$Q_K = 4*3 = \underline{12 KN/m}$

$P_d = 1.3 G_K + 1.6 Q_K$

$= 1.3(13) + 1.6(12) = 36.1 KN/m$

$= \underline{3 6.1 KN/m}$

Step 3 :- Analysis :



$M_{sup} = -\frac{P_d l^2}{8}$ &

$M_{span} = +\frac{9}{128} P_d l^2$

$\Rightarrow M_{sup} = -\frac{36.1 (8)^2}{8} = \underline{-288.8 \text{ KNm}}$.

$M_{span} = +9 \frac{* 36.1 * 8^2}{128} = \underline{+162.48 \text{ KNm}}$

Since M_{sup} is larger as compared with M_{span} and the flange in the span is subjected to compression stress, so redistribute 20% of the support moment to the span.

$\therefore M_{sup,d} = 0.8 * 288.8 = 231.04 \text{ KNm}$

$M_{span,d} = 162.48 + 0.2 * 288.8 = 220.21 \text{ KNm}$

Step 4 : Check the depth for flexure since the flange around the support is subjected to hogging moment, $b = b_w$;

$$d = \frac{M_{sup,d}}{0.8 b_w d^2 f_{cd} \rho_{max} m (1 - 0.4 \rho_{max} m)}$$

Where $\rho_{max} m = K_{x\,max} = \frac{x}{d} = 0.~~288~~$ for 20% redistribution of elastic moment !

$\Rightarrow d = 613 \text{mm} > d_{used} (= 439 \text{mm})$

$\therefore \Rightarrow$ Doubly Reinforced !!

$$M_{uc} = 0.8\, b_w\, d^2 f_{cd}\, k_{x,max} (1 - 0.4\, k_{x,max})$$

$$= 0.8 * 0.25 * 0.439^2 * 13.6 * 10^3 * 0.288 (1 - 0.4 * 0.288)$$

$$= \underline{133.58 \text{ kNm}}$$

$$M_{usc} = M_u - M_{uc} = 231.04 - 133.58 = \underline{97.46 \text{ kNm}}$$

$As_1 = \dfrac{M_{uc}}{f_{yd}(d - 0.4\, k_x d)}$

$$\Rightarrow A_{s1} = \frac{133.58 * 10^3}{260.87 (0.439 - 0.4 (0.288)) (0.439)}$$

For Tension Bar

$$= \underline{1318.28 \text{ mm}^2}$$

$$A_{s2} = \frac{M_{usc}}{f_{yd} (d - d_c')} = \frac{97.46 * 10^6}{260.87 (439 - 41)} = \underline{938.684 \text{ mm}^2}$$

$$\Rightarrow A_s = A_{s1} + A_{s2} = 1318.28 + 938.684 = \underline{2257 \text{ mm}^2}$$

$$\Rightarrow \text{use } \underline{4\phi 20 + 5\phi 16}$$

For Compression Bar

$$x_{max} = k_{x,max}\, d = 0.288 * 439 = \underline{126.43 \text{mm}}$$

$$\epsilon_{sc} = \frac{\epsilon_{cu} (x_{max} - d_c')}{x_{max}} = \frac{0.0035 (126.43 - 41)}{126.43}$$

$$= \underline{2.365 * 10^{-3}}$$

$$f_s' = \epsilon_{sc} E_s = 2.365 * 10^{-3} * 2 * 10^5 = \underline{473 \text{ Mpa}}$$

Since $f_s' = 473 \text{ Mpa} > f_{yd} = 260.87 \text{ Mpa}$

$\Rightarrow$ Compression Steel is yielding

$\therefore f_s' = f_{yd}$

Thus, $A_s' = A_{s2} = 938.684 \text{ mm}^2$

Then, $\Rightarrow$ use $\underline{5\phi 16}$

Step 5: Design the Span section

~ Let's use Design tables & Charts ; (Exercise)

→ Design of Span for flexure ;

$$M_{max} = M_{span,d} = 220.21\ kNm$$

$$b_e = \begin{cases} b_w + \frac{L_e}{5} = 250 + \frac{3000}{5} = 1850\ mm \\ C/C = 3000\,mm \end{cases}$$

$$\Longrightarrow b_e = \underline{1850\,mm}$$

Assume $d_c' = 61\,mm$ & $d = 439\,mm$;

Check for T-beam ;

$$k_x = 0.5\left[C_1 - \sqrt{C_1^2 - \frac{4M_{span}}{bd^2C_2}}\right] \leq k_{x,max}$$

$$\Rightarrow k_x = 0.058 < k_{x,max} = 0.448$$

$$x = 0.058 * 439 = 25.5\ mm < h_f = 100\,mm$$

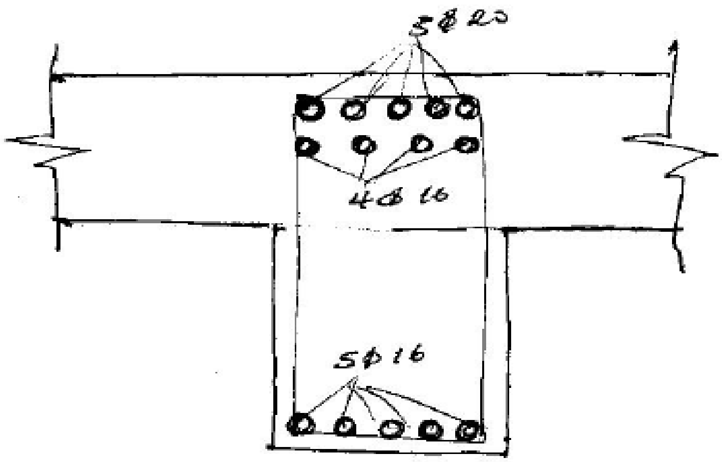
$\therefore$ $b = b_e$ (the assumption is ok!)

$$A_s = \frac{M_{span}}{f_{yd}(d - 0.4x)} = \frac{220.21\ kN}{(260.87)(0.439 - 0.4*0.025)}$$

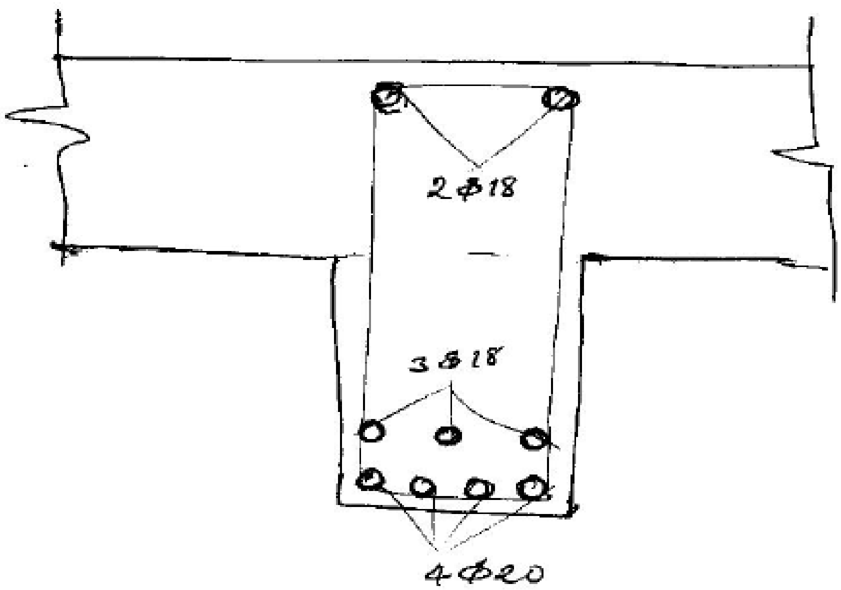
$$= \underline{1968\ mm^2}$$

⇒ provide 4 Φ20 + 3 Φ18 on the bottom of the Tension zone.

Sketch:



X-section around the Support subjected to M_{sup}.



X-section at the span su
to M_{span}.

5. A T-beam has an Effective Flange Width of 960 mm, b_w = 350 mm, h_f = 100 mm and effective depth, d = 500 mm if S-400 steel, f_{ck} = 20 Mpa, Class II works are used. What will be the design moment Capacity for this beam when A_s = 3000 mm^2 is used?

Alternative method using design tables (singly reinforced Sections)

1-USING DESIGN TABLES

Derivation

$$M_d = 0.8bd^2 f_{cd}\, \rho_m (1-0.4\,\rho_m)$$

$$\frac{M_d}{bd^2} = 0.8 f_{cd}\, \rho m (1 - 0.4 \rho m)$$

$$\text{Let } k_m = \sqrt{\frac{M_d}{bd^2}} = 0.8 f_{cd}\, \rho m (1 - 0.4 \rho m)$$

$$\sum M_c = 0 \Rightarrow A_s = \frac{M_d}{f_{yd}(d - 0.4x)} = \frac{M_d}{d} * \frac{1}{f_{yd}(1 - 0.4\frac{x}{d})}$$

$$\text{Let } k_s = \frac{1}{f_{yd}(1 - 0.4\frac{x}{d})} \quad \Rightarrow A_s = \frac{k_s * M_d}{d}$$

From table 1a there are different K_m values and the max. Value of K_m for different moment redistribution is given and represented by Km*.

- If $K_m \leq K_m$*, the section is singly reinforced.
- If $K_m > K_m$*, it is doubly reinforced.

STEPS:

a) For Singly Reinforced Sections

1. Evaluate $k_m = \dfrac{\sqrt{\dfrac{M_d}{b}}}{d}$
2. Enter the general design Table No.1a using k_m and concrete grade.
3. Read k_s from the same Table corresponding to steel grade and k_m.
4. Evaluate $A_s = \dfrac{k_s * M_d}{d}$

b) For Doubly Reinforced Sections

1. This is so, when Km>Km*(is the value of Km shown shaded in general design table 1a , corresponding to the concrete grade)
2. compute Km/Km*
3. Read Ks & Ks* corresponding to Km/Km* & the steel grade from general design table 1a
4. Assume dc, (d2) & read ρ (correction factor) from the same table corresponding to Km/Km* & dc'/d.
5. Read ρ' corresponding to dc'/d ,then

As = KsMd ρ/d $\qquad\qquad$ A_s' = Ks'Md ρ'/d

Note: - In all cases

- M_d is in KN-m
- b " " m
- d " " m

2- USING DESIGN CHARTS

►Compute $\gamma_{u,s} = \dfrac{Mu,s}{f_{cd}bd^2}$ & Kx, max = 0.8(δ-0.44), where δ=1, 0.9, 0.8 & 0.7 for 0%, 10%, 20% & 30% moment redistribution.

►Compare $\gamma_{u,s}$ or Kx with the corresponding values of $\gamma_{u,s}$* Kx,max

Where: $\gamma_{u,s}$* = 0.143, 0.205, 0.252 & 0.295 for 30%, 20%, 10%, and 0% respectively.

► If $\gamma_{u,s} \le \gamma_{u,s}$* then the section is singly reinforced and As1:

$$\text{As1} = \frac{Msd,s}{zf_{yd}} + \frac{Nsd}{fyd}$$

► If $\gamma_{u,s} > \gamma_{u,s}$*,then the section is doubly reinforced and As1 ,As2:

As2 = $\dfrac{Msd,s - Mu,s*}{(d-d_2)\sigma_{s2}}$ - area of compression reinforcement,

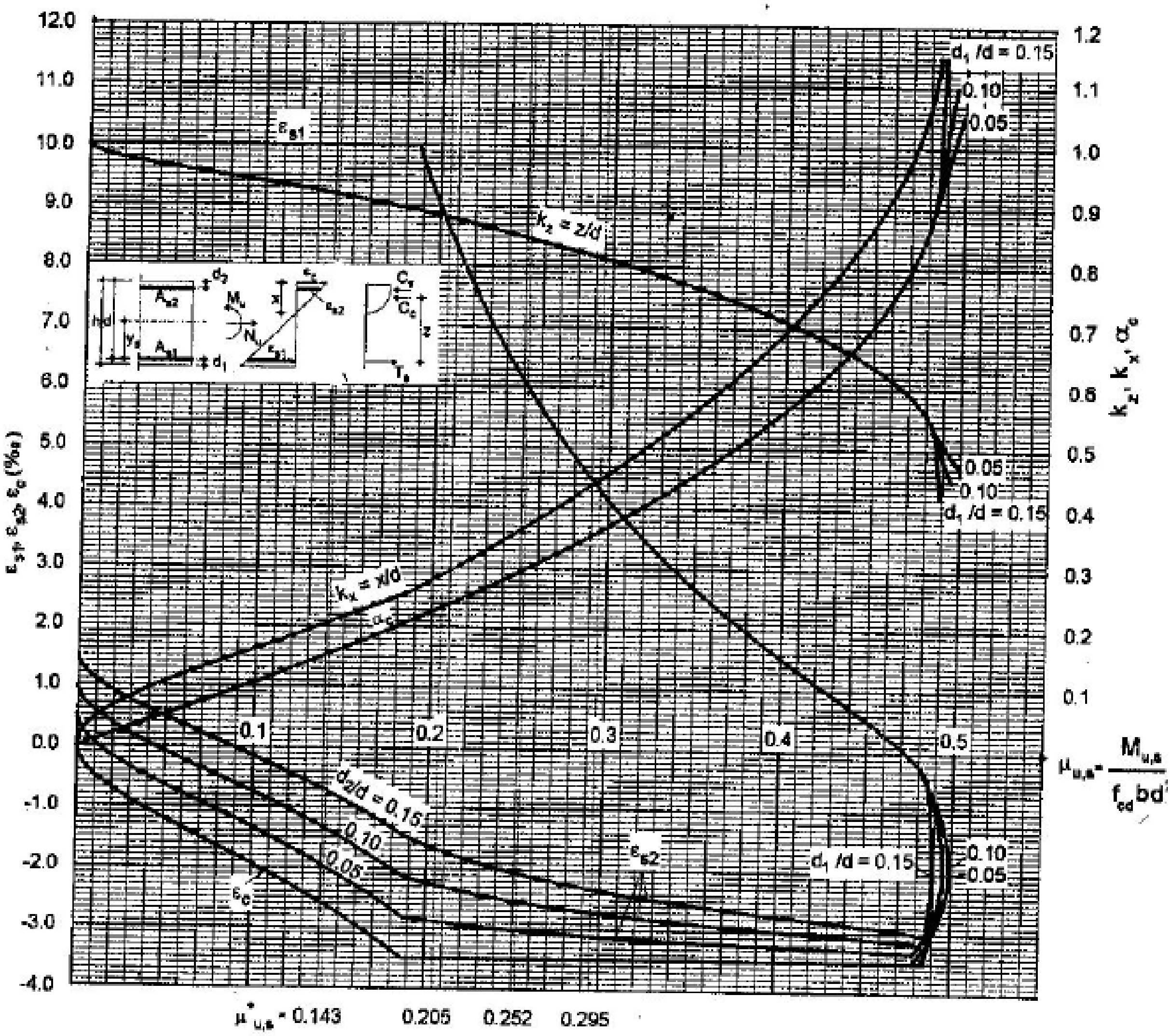
Where: Mu, s* = $\gamma_{u,s}$* fcd bd^2 & $\gamma_{u,s}$* is the value given above.

σ_{s2} - is actual compressive stress on compression steel & is Es*εsc

$$As1 = \frac{Mu,s*}{Zf_{yd}} + \frac{Msd,s - Mu,s*}{(d - d_2)\sigma_{s2}} + \frac{N_{sd}}{f_{yd}}$$ -area of tensile reinforcement

Using $\gamma_{u,s}$ read Z/d, X/d etc & compute A_{s1} and A_{s2}.

General Design Chart No. 1



(1) If $\mu_{Sd,s} \leq \mu^*_{u,s}$ compression reinforcement is not required:

$$A_{s1} = \frac{M_{Sd,s}}{zf_{yd}} + \frac{N_{Sd}}{f_{yd}}$$

(2) If $\mu_{Sd,s} > \mu^*_{u,s}$ compression reinforcement is required:

$$A_{s2} = \frac{M_{Sd,s} - M^*_{u,s}}{(d - d_2)\sigma_{s2}}$$

$$A_{s1} = \frac{M^*_{u,s}}{zf_{yd}} + \frac{M_{Sd,s} - M^*_{u,s}}{(d - d_2)f_{yd}} + \frac{N_{Sd}}{f_{yd}}$$

Bending with or without Normal Load, with or without Compression Reinforcement

General Design Table No. 1a
(no moment redistribution)

k_m					k_s							Redist-ribution
C15	C20	C25	C30	C40	S300	S400	S460	ε_c	ε_{s1}	k_x	k_z	
15	17	19	21	24	3.95	2.96	2.58	0.94	10	0.09	0.97	
19	22	25	27	31	3.99	2.99	2.60	1.27	10	0.11	0.96	
23	26	29	32	37	4.04	3.03	2.63	1.59	10	0.14	0.95	
26	30	33	37	42	4.08	3.06	2.66	1.92	10	0.16	0.94	
29	33	37	40	47	4.12	3.09	2.69	2.24	10	0.18	0.93	
31	35	40	43	50	4.17	3.13	2.72	2.56	10	0.20	0.92	
31.12	35.94	40.18	44.02	50.83	4.18	3.13	2.72	2.63	10	0.208	0.918	30%
33	38	42	46	53	4.21	3.16	2.75	2.89	10	0.22	0.91	
34	40	44	49	56	4.26	3.19	2.78	3.22	10	0.24	0.90	
36	42	46	51	59	4.31	3.23	2.81	3.50	9.7	0.26	0.89	
37.36	43.13	48.23	52.83	61	4.36	3.27	2.84	3.50	8.7	0.288	0.880	20%
37	43	48	53	61	4.36	3.27	2.84	3.50	8.6	0.29	0.88	
39	45	50	55	63	4.41	3.30	2.87	3.50	7.7	0.31	0.87	
40	46	52	56	65	4.46	3.34	2.91	3.50	6.9	0.34	0.86	
41	47	53	58	67	4.51	3.38	2.94	3.50	6.2	0.36	0.85	
41.42	47.83	53.47	58.58	67.64	4.53	3.39	2.95	3.50	6.0	0.368	0.847	10%
42	49	54	60	69	4.56	3.42	2.98	3.50	5.6	0.38	0.84	
43	50	56	61	71	4.62	3.46	3.01	3.50	5.1	0.41	0.83	
44	51	57	63	72	4.67	3.51	3.05	3.50	4.6	0.43	0.82	
44.78	51.72	57.83	63.35	73.16	4.71	3.53	3.07	3.50	4.3	0.448	0.814	

No redistribution ($k_x = 0.448$)

k_m/k^*_m	k_m					k_s		
	C15	C20	C25	C30	C40	S300	S400	S460
	44.79	51.72	57.83	63.35	73.16	4.71	3.53	3.07
1.03	46	53	59	65	75	4.68	3.51	3.05
1.05	47	54	61	67	77	4.65	3.49	3.04
1.08	48	56	63	69	79	4.63	3.47	3.02
1.11	50	58	64	71	81	4.60	3.45	3.00
1.15	51	59	66	73	84	4.57	3.43	2.98
1.19	53	61	69	75	87	4.54	3.40	2.96
1.23	55	64	71	78	90	4.51	3.38	2.94
1.28	57	66	74	81	94	4.48	3.36	2.92
1.33	60	69	77	84	97	4.45	3.34	2.90
1.39	62	72	81	88	102	4.43	3.32	2.89
1.46	66	76	85	93	107	4.40	3.30	2.87

Notes:

1 See Table 1b for Design Equations

2 Initial values for 0%,10%,20%&30% Moment Redistribution are shown shaded in Tables No.1a & 1b

Factors ρ and k_s' are applicable to all values of moment redistribution

Correction factor ρ

k_m/k^*_m	d_2/d						
	0.07	0.08	0.1	0.12	0.14	0.16	0.18
1	1	1	1	1	1	1	1
1.03	1	1.00	1.00	1.00	1.00	1.00	1.01
1.05	1	1.00	1.00	1.00	1.01	1.01	1.01
1.08	1	1.00	1.00	1.01	1.01	1.01	1.02
1.11	1	1.00	1.01	1.01	1.01	1.02	1.02
1.15	1	1.00	1.01	1.01	1.02	1.02	1.03
1.19	1	1.00	1.01	1.02	1.02	1.03	1.04
1.23	1	1.00	1.01	1.02	1.03	1.03	1.04
1.28	1	1.00	1.01	1.02	1.03	1.04	1.05
1.33	1	1.00	1.01	1.02	1.03	1.04	1.05
1.39	1	1.00	1.02	1.03	1.04	1.05	1.06
1.46	1	1.01	1.02	1.03	1.04	1.05	1.07

k_m/k^*_m	k_s'		
	S300	S400	S460
1	0	0	0
1.03	0.20	0.15	0.13
1.05	0.40	0.30	0.26
1.08	0.60	0.45	0.39
1.11	0.80	0.60	0.52
1.15	1.00	0.75	0.65
1.19	1.20	0.90	0.78
1.23	1.40	1.05	0.91
1.28	1.60	1.20	1.04
1.33	1.80	1.35	1.17
1.39	2.00	1.50	1.30
1.46	2.20	1.65	1.43

Correction factor ρ'

All K_m/K^*_m	1	1.01	1.03	1.06	1.08	1.11	1.13

Rectangular Section ... Moment with or without Axial Load,

General Design Table No. 1b
(with moment redistribution)

10% redistribution ($k_x = 0.368$)

k_m/k^*_m	k_m C15	C20	C25	C30	C40	k_s S300	S400	S460
1	41.42	47.83	53.47	58.58	67.64	4.53	3.39	2.96
1.03	42	49	55	60	69	4.52	3.39	2.95
1.05	44	50	56	62	71	4.52	3.39	2.95
1.08	45	52	58	63	73	4.52	3.39	2.95
1.11	46	53	60	65	75	4.51	3.38	2.94
1.15	48	55	61	67	78	4.51	3.38	2.94
1.19	49	57	63	70	80	4.51	3.38	2.94
1.23	51	59	66	72	83	4.50	3.38	2.94
1.28	53	61	68	75	86	4.50	3.37	2.93
1.33	55	64	71	78	90	4.50	3.37	2.93
1.39	58	67	74	82	94	4.49	3.37	2.93
1.46	61	70	76	86	99	4.49	3.37	2.93

Notes:

1 k^*_m is k_m value corresponding to $\mu^*_{u,s}$ (shown shaded in Tables No.1a &1b)

2 Units:

M (kNm)
b,d (m)
A_s (mm²)

Correction factor ρ'

All	d_2/d 0.07	0.08	0.1	0.12	0.14	0.16	0.18
K_m/K^*_m	1	1.01	1.03	1.06	1.08	1.11	1.13

20% redistribution ($k_x = 0.288$)

k_m/k^*_m	k_m C15	C20	C25	C30	C40	k_s S300	S400	S460
1	37.36	43.13	48.23	52.83	61	4.36	3.27	2.84
1.03	38	44	49	54	63	4.34	3.26	2.83
1.05	39	45	51	56	64	4.33	3.25	2.83
1.08	40	47	52	57	66	4.32	3.24	2.82
1.11	42	48	54	59	68	4.31	3.23	2.81
1.15	43	50	55	61	70	4.30	3.22	2.80
1.19	44	51	57	63	72	4.29	3.22	2.80
1.23	46	53	59	65	75	4.28	3.21	2.79
1.28	48	55	62	68	78	4.26	3.20	2.78
1.33	50	57	64	70	81	4.25	3.19	2.77
1.39	52	60	67	74	85	4.24	3.18	2.77
1.46	55	63	71	77	89	4.23	3.17	2.76

Correction factor ρ'

All	d_2/d 0.07	0.08	0.1	0.12
K_m/K^*_m	1	1.01	1.03	1.06

30% redistribution ($k_x = 0.208$)

k_m/k^*_m	k_m C15	C20	C25	C30	C40	k_s S300	S400	S460
1	31.07	35.88	40.12	43.98	50.74	4.18	3.13	2.72
1.03	32	37	41	45	52	4.17	3.13	2.72
1.05	33	38	42	46	53	4.17	3.13	2.72
1.08	34	39	43	48	56	4.17	3.13	2.72
1.11	3[illegible]	40	45	49	57	4.17	3.12	2.72
1.15	36	41	46	50	58	4.16	3.12	2.71
1.19	37	43	48	52	60	4.16	3.12	2.71
1.23	38	44	49	54	62	4.16	3.12	2.71
1.28	40	46	51	56	65	4.16	3.12	2.71
1.33	41	48	53	59	68	4.15	3.11	2.71
1.39	43	50	56	61	71	4.15	3.11	2.71
1.46	46	53	59	64	74	4.15	3.11	2.70

Correction factor $\rho' = 1$ for $d_2/d \le 0.07$
(higher values of d_2/d not recommended)

$$k_m = \frac{\sqrt{M_{Sd,s}/b}}{d}$$

$$M_{Sd,s} = M_{Sd} - N_{Sd}\, y_s$$

$$A_{s1} = \frac{M_{Sd,s}}{d} k_s \rho + \frac{N_{Sd}}{f_{yd}}$$

$$A_{s2} = \frac{M_{Sd,s}}{d} k'_s \, \rho'$$

Rectangular Section ... Moment with or without Axial load, with or without Compression Reinforcement

Cover to Reinforcements

- The concrete cover is the distance between the outermost surface of reinforcement (usually stirrups) and the nearest concrete surface.
- The thickness of cover required depends both upon the exposure conditions and on the concrete quality.
- To transmit bond forces safely, and to ensure adequate compaction, the concrete cover should never be less than:

  (a) ϕ or ϕ_n ($\leq$ 40mm), or

  (b) (ϕ + 5mm) or (ϕ_n + 5mm) if d_g > 32mm

  Where ϕ = the diameter of the bar.

  ϕ_n = the equivalent diameter for a bundle.

  d_g = the largest nominal aggregate size.

Minimum cover

Type of exposure	Mild	Moderate	Sever
Min. cover (mm)	15	25	50

Durability and control of crack width is related with finishing and provision of adequate cover to reinforcement. Nominal cover for structural elements located in the interior of the building with dry environment and mild condition is 15 mm, example slab; humid environment with moderate exposure is 25 mm, example beam; severe environment is 50 mm, example footing.

Spacing of Reinforcements

- The clear horizontal and vertical distance between bars shall be at least equal to the largest of the following values.

  (a) 20 mm

  (b) The diameter of the largest bar or effective diameter of the bundle

  (c) The maximum size of the aggregate d_g plus 5mm.

- Where bars are positioned in separate horizontal layers, the bars in each layer should be located vertically above each other and the space between the resulting columns of the bars should permit the passage of an internal vibrator.

Effective Span Length

- The effective span of a simply supported member shall be taken as the lower of the following two values:
  (a) The distance between the center lines of supports.
  (b) The clear distance between the faces of supports plus the effective depth.
- The effective span of a continuous element shall normally be taken as the distance between the center lines of the supports.
- For a cantilever, the effective span is taken to be its length, measured from.
  (a) The face of the supports, for an isolated, fixed ended cantilever.
  (b) The center line of the support for a cantilever which forms the end of a continuous beam.

Deflection limits are assumed to be satisfied when the minimum effective depth for a particular member is

$$d \geq \left(0.4 + \frac{0.6 * f_{yk}}{400}\right) * \frac{L_e}{\beta_a}$$

where f_{yk} is equal to character strength of reinforcement, L_e is the effective span (the shorter span in case of two way slab), is constant, a function of restraints given below).

Table – values of β_a

Member	Simple	End span	Interior span	cantilever
Beams	20	24	28	10
Slabs: Span ratio 2:1	25	30	35	12
Span ratio 1:1	35	40	45	10

* For intermediate values – *interpolation.*

Preliminary Sizing of Beam Sections

Ideal values of span effective depth ratios, recommended in the ISE manual for the preliminary sizing of reinforced concrete beams are given in table below.

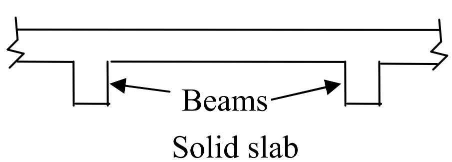
Support conditions	Cantilever	Simple Support	Continuous	End spans
ISE manual	6	12	15	13.5

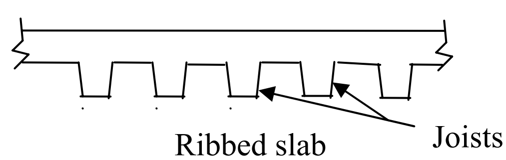
3.6. One-way RC Slabs

A reinforced concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so. It is used to provide flat surfaces mainly for roofs and floors of buildings, parking lots, air fields, roadways …etc. It may be supported by reinforced concrete beams (and is poured monolithically with such beams), by masonry or reinforced concrete walls, by structural steel members, directly by columns, or continuously by the ground.

Classification: - Beam supported slabs may be classified as:-

1. *One-way slabs* – main reinforcement in each element runs in one direction only. (Ly/Lx >2). There are two types- one way solid slabs and one way ribbed slabs.



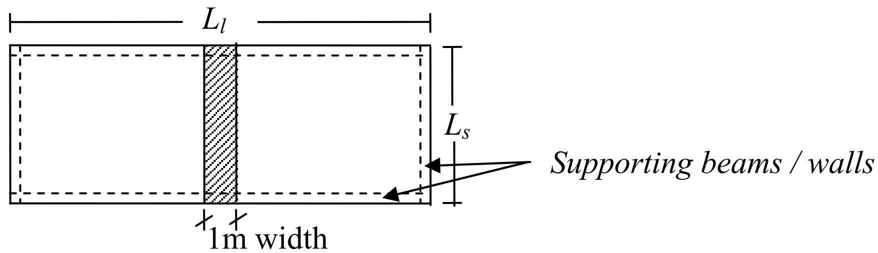


2. *Two – way slabs* – main reinforcement runs in both direction where ratio of long to short span is less than two. (Ly/Lx ≤ 2)

Others include flat slabs, flat plates, two way ribbed or grid slabs etc.

3.6.1. Analysis of one-way solid slabs

They are considered as rectangular beams of comparatively large ratio of width to depth and ratio of longer span to width (short span) is greater than two.



When $L_l / L_s > 2$, about 90% or more of the total load is carried by the short span, i.e., bending takes place in the direction of the shorter span.

The analysis is than carried out by assuming a beam of unit width with a depth equal to the thickness of the slab and span equal to the distance between supports (in the short direction). The strip may be analyzed in the same way as singly reinforced rectangular sections.

- Load per unit area on the slab would be the load per unit length on this imaginary beam of unit width.
- As the loads being transmitted to the supporting beams, all reinforcement shall be placed at right angles to these beams. However some additional bars may be placed in the other direction to carry temperature and shrinkage stresses.

Generally the design consists of selecting a slab thickness for deflection requirement and flexural design is carried out by considering the slab as series of rectangular beams side by side

Remark:-

- The ratio of steel in a slab can be determined by dividing the sectional area of one bar by the area of concrete between two successive bars, the latter area being the product of the depth to the center of the bars and the distance between them, center to center.

- Unless condition warrant some change, cover to reinforcement is 15 mm.
- The following minimum slab thicknesses shall be adopted in design:
  a) 60mm for slabs not exposed to concentrated loads (eg. Inaccessible roofs).
  b) 80mm for slabs exposed mainly to distributed loads.
  c) 100mm for slabs exposed to light moving concentrated loads (eg. slabs accessible to light moving vehicles).
  d) 120mm for slabs exposed to heavy dynamic moving loads (eg. slabs accessible to heavy vehicles).
  e) 150mm for slabs on point supports (eg. flat slabs).
- Flexural reinforcements should fulfill the following minimum criteria:
  a) The ratio of the secondary reinforcement to the main reinforcement shall be at least equal to 0.2.
  b) The geometrical ratio of main reinforcement in a slab shall not be less than:

$$\rho_{\min} = \frac{0.5}{f_{yk}} \quad where\ f_{yk}\ in\ MPa$$

  c) The spacing between main bars for slabs shall not exceed the smaller of 2h or 350mm.
  d) The spacing between secondary bars (in a direction ⊥ to the main bars) shall not exceed 400mm.

3.6.2. Analysis and Design of one way Ribbed Slab

In one way ribbed slab, the supporting beams called joists or ribs are closely spaced. The ribbed floor is formed using temporary or permanent shuttering (formwork) while the hollow block floor is generally constructed with blocks made of clay tile or with concrete containing a light weight aggregate. This type of floor is economical for buildings where there are long spans and light or moderate live loads such as in hospitals and apartment buildings.

General Requirements:

Minimum slab thickness

To ensure adequate stiffness against bending and torsion and to allow ribbed slabs to be treated as solid slabs for the purpose of analysis, EBCS-2 recommends that the following restrictions on size are satisfied:

- Ribs shall not be less than 70mm in width; and shall have a depth, excluding any topping of not more than 4 times the minimum width of the rib. The rib spacing shall not exceed 1.0m
- Thickness of topping shall not be less than 50mm, nor less than $1/10$ the clear distance between ribs. In the case of ribbed slabs incorporating permanent blocks, the lower limit of 50mm may be reduced to 40mm.

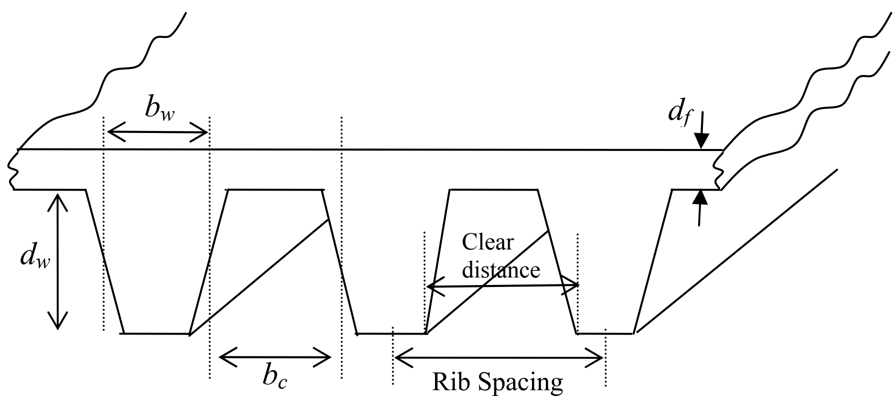


Fig. Ribbed slab

Minimum Reinforcement

- The topping shall be provided with a reinforcement mesh providing in each direction a cross-sectional area not less than 0.001 of the section of the slab.
- The breadth of ribs may be governed by shear strength requirements. The method proposed in the ISE manual for the estimation of rib breadths limits the shear stress in the rib to 0.6 N/mm^2 for concretes with characteristic cylinder strength of 25 N/mm^2 or more. The required breadth is given by:

$$b = \frac{V}{0.6d} \quad [mm]$$

Where V is the maximum shear force in Newton's on the rib considered as simply supported and d is the effective depth in millimeters. For characteristic cylinder strengths less than 25 MPa, the breadth should be increased in proportion.

- If the rib spacing exceeds 1.0m, the topping shall be designed as a slab resting on ribs, considering load concentrations, if any.
- The function of the flanges with the web shall be checked for longitudinal shear.
- The ultimate limit state in longitudinal shear is governed either by the effect of inclined flange compression (acting parallel to its middle plane) or by tension in the transverse reinforcement.
- The longitudinal shear per unit length v_{sd}, which may be obtained as a function of the applied transverse shear V_{sd} :

  (a) For flange in compression :

$$v_{sd} = \left(\frac{b_e - b_w}{2b_e}\right)\frac{V_{sd}}{z}$$

  (b) For flange in tension.

$$v_{\mathrm{sd}} = \left(\frac{A_s - A_{sw}}{2A_s}\right)\frac{V_{sd}}{z}$$

Where: V_{sd} – applied transverse shear.

Vsd - longitudinal shear per unit length

b_e – effective width of a T-section.

z - Internal lever arm.

A_s – area of the longitudinal steel in the effective flanges outside the projection of Web into the slab.

A_{sw} – area of the longitudinal steel inside the slab within the projection of the web into the slab.

- Resistance to longitudinal shear.

  (a) Resistance to inclined compression per unit length v_{Rd1}

$$v_{Rd1} = 0.25 f_{cd}\, h_f$$

  Where : h_f = total thickness of the flange.

  (b) Resistance to diagonal tension per unit length v_{Rd2}

$$v_{Rd2} = 0.50\,f_{ctd}\,h_f + \frac{A_{sf}\,f_{yd}}{s_f}$$

Where : A_{sf} = area of transverse reinforcement per unit length , perpendicular to the web-flange interface.

- If, at the section with M = M_{max} , the flange is subjected to a tensile force, the concrete contribution $0.50\,f_{ctd}\,h_f$ (in the above equation) should be neglected.

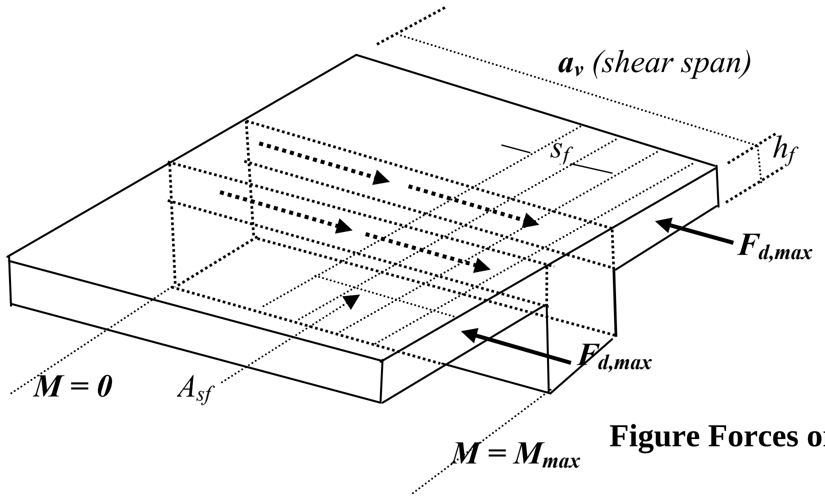


Figure Forces on ribbed slab

- Because joists are closely spaced, thickness of slab (topping),

$$D \geq \begin{cases} 40\ mm \\ \dfrac{1}{10} clear\ \ distance\ between\ \ joists \end{cases}$$

- Unless calculation requires for rib spacing larger than 1m, toppings or slabs are provided with mesh reinforcement of *0.001 bD* in both directions for temperature and shrinkage problem.
- Unless calculation requires, min reinforcement to be provided for joists includes two bars, where one is bent near the support and the other straight.
- Rib with $b_w \geq 70mm$, and overall depth $D_j \leq 4\, b_{w,\,joist} + t_{slab}$
- Rib spacing is generally less than 1m.
- In case of ribbed spacing larger than 1m, the topping (slab) need to be design as if supported on ribs. (i.e. As one way solid slab between the ribs).

- If the span of the ribs exceeds 6m, transverse ribs may be provided, as the thickness of the topping will be larger.
- The girder supporting the joist may be rectangular or T-beam with the flange thickness equal to the floor thickness.

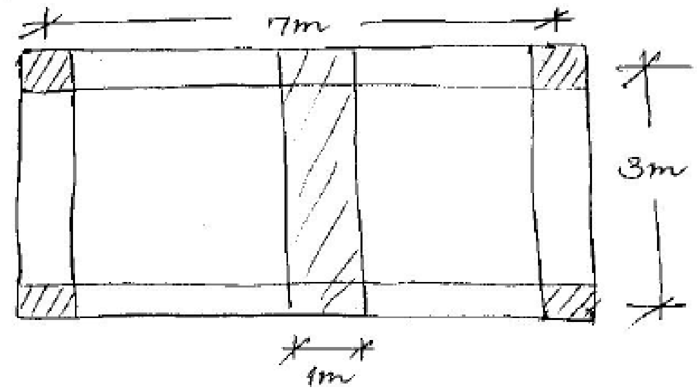
Procedure of Design of a floor system of ribbed Slab

1. Thickness of toppings and ribs assumed based on min requirement.
2. Loads may be computed on the basis of center line of the spacing of joists.
3. The joists are analyzed as regular continuous T-beams supported by girders.
4. Shear reinforcement shall not be provided in the narrow web of joist thus a check for the section capacity against shear is carried out. The shear capacity may be approximated as: $1.1\ V_c$ of regular rectangular sections.
5. Determine flexural reinforcement and consider min provision in the final solution.
6. Provide the topping or slab with reinforcement as per temp and shrinkage requirement.
7. Design the girder as a beam.

Examples on Design of One Way slabs and Continuous Beams using Limit State Design Method

1. Design a Floor System which is used as a lobby area (lobbies) is 5 kN/m² as given by EBCS-1, (1995). The materials used are: C-25, S-360 and Class I work.

   Use ϕ12mm bars and 48mm Cement Screed 20kN/m³ and 2mm pvc tile 16kN/m³ as finishing.



Sol^{n}:-

Step 1: Check whether the Slab is treated as One way or two way:

$$\frac{L_e}{L_s} = \frac{7}{3} = 2.33 > 2 \Rightarrow \text{Design as One Way}$$

Step 2: Determine depth from deflection requirement:

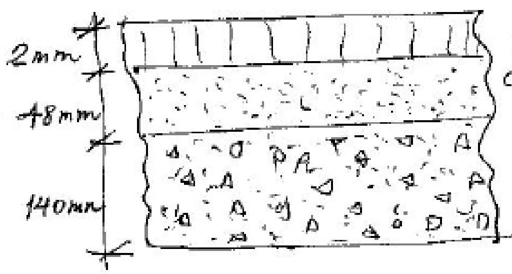
$$d \geq \left(0.4 + 0.6\frac{f_{yk}}{400}\right)\frac{L_e}{\beta_a}$$

$$= \left(0.4 + 0.6\left(\frac{360}{400}\right)\right)\left(\frac{3000}{25}\right) = 112.8\,mm$$

$$\Rightarrow D = 112.8 + 15 + 6 = 133.8\,mm$$

Take: $D = 140\,mm$

Step 3: Determine design load and Moment:



pvc, $\gamma = 16\ kN/m^3$

Cement screed, $\gamma = 20\ kN/m^3$

Slab concrete, $\gamma = 25\ kN/m^3$, from EBCS-1

Dead Load of Slab $= 0.14 * 1 * 25 = 3.5\ kN/m$

Dead load of finishing
$= 0.048 * 20 + 0.002 * 16 = 0.99\ kN/m$

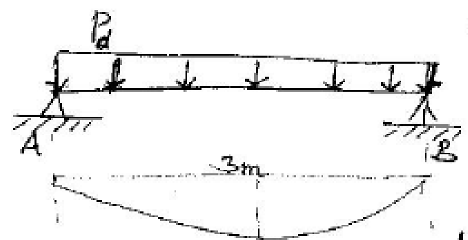
$\Rightarrow G_k = 3.55 + 0.99 = \underline{4.49\ kN/m}$

Live load, $Q_k = 5 * 1 = \underline{5\ kN/m}$

$\Rightarrow$ Design load, $P_d = 1.3\, G_k + 1.6\, Q_k$

$= 1.3\,(4.49) + 1.6\,(5)$

$= \underline{13.84\ kN/m}$



$$M_{max} = \frac{P_d l^2}{8} = \frac{13.84\ (3)^2}{8} = \underline{15.57\ kNm}$$

Step 4: Design Constants:

$f_{cd} = 11.33\ Mpa$, $f_{yd} = 313.04\ Mpa$, $m = 34.54$

$C_1 = 0.0724$, $C_2 = 4325.38\ Mpa$, $\rho_{max} = 0.015$

$\rho_{min} = 0.00139$

$$\rho_{min} = \frac{0.5}{f_{yk}}$$

Step 5: Check depth for maximum moment:

$$d \geq \sqrt{\frac{M_{max}}{0.8\, f_{cd}\, b\, \rho_{max}\,(1 - 0.4\rho m)}}$$

$$= \sqrt{\frac{15.57 * 10^6}{0.8\,(11.33)(1000)(0.015)(34.54)\left(1 - 0.4 * 0.015 * 34.54\right)}}$$

$$= \underline{64.67\, mm}$$

$\Rightarrow D = 64.67 + 15 + 6 = 85.67\, mm < D_{used} = 140\, mm$ Ok!

Step 6: Reinforcement:

Minimum Requirements:

$$d = D - d' = 140 - 15 - 6 = \underline{119\, mm}$$

$$A_{s\,min} = \rho_{min}\, b d = 0.00139\,(1000)(119)$$

$$= \underline{165.28\, mm^2}.$$

$$S_{max} \leq \begin{cases} 2D = 2(140) = 280\, mm \\ 350\, mm \end{cases}$$

$k_x = \frac{x}{d} = \rho m$
$\rho = 0.5[c_1 - \sqrt{\ }\ ]$
$A_s = \rho b d$

$\Rightarrow$ use $\underline{S_{max} = 280\, mm}$.

$$A_s = \frac{bd}{2}\left[C_1 - \sqrt{C_1^2 - \frac{4M}{bd^2 C_2}}\right]$$

$$= \frac{1000\,(119)}{2}\left[0.0724 - \sqrt{0.0724^2 - \frac{4 * 15.57 * 10^6}{10^3 * 119^2 * 4325.38}}\right]$$

$$= \underline{440.31\, mm^2} > A_{s,min}$$

$\Rightarrow$ Spacing, $S = \frac{b\, a_s}{A_s} = \frac{1000\,(113)}{440.31} = \underline{256.64\, mm} < S_{max}$ ok!

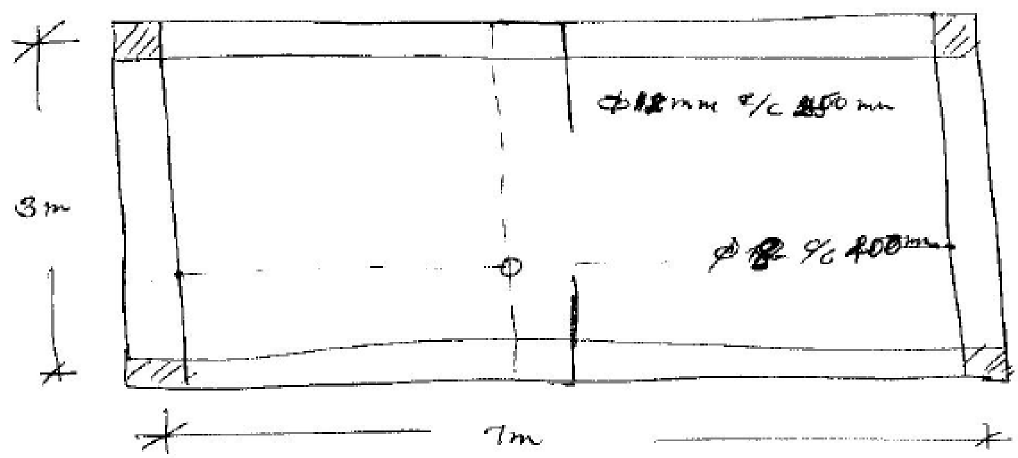
$\Rightarrow$ use $\Phi 12$ @ 250mm c/c in the Shorter dxn.

$$S_{max} \leq \begin{cases} 5D = 5(140) = 700mm \\ 400mm \end{cases} \Rightarrow S = 400mm$$

[4]

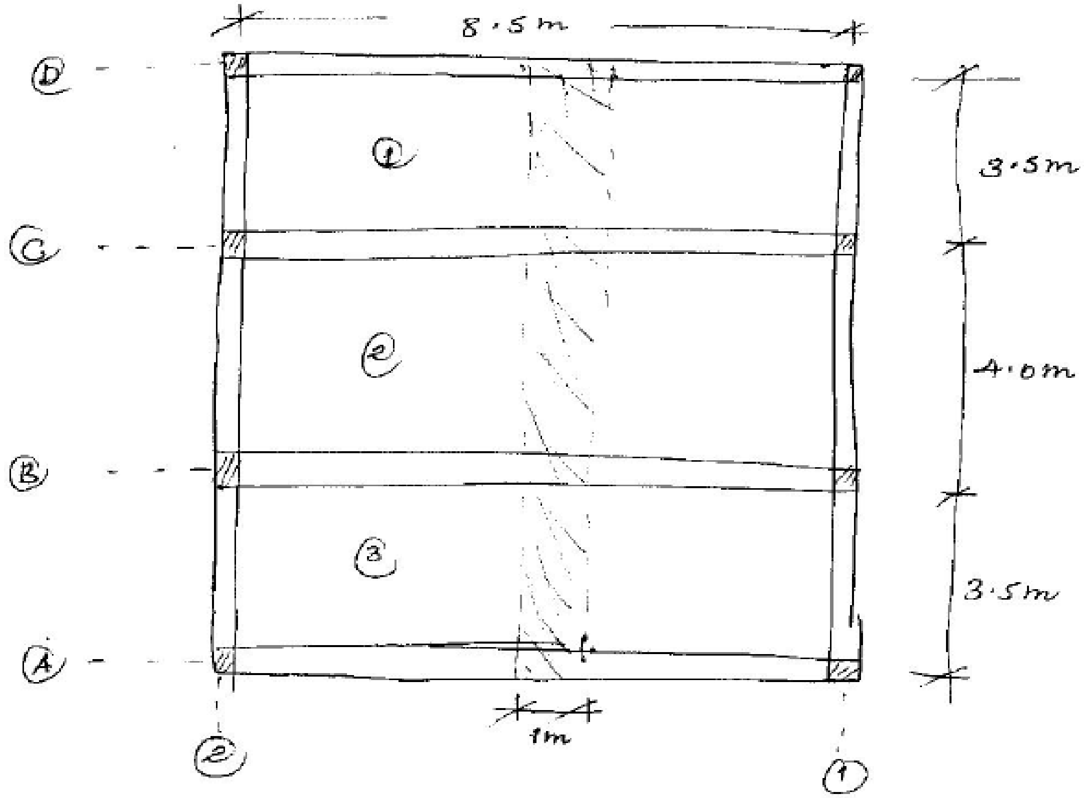
∴ provide $\phi 8mm$ @ 400mm c/c in the transverse dxn.

Sketch:-



2. Design the slab in Example-1 above using WSD method and compare the results.

3. Design the floor slab system which consists of One-way solid slab supported by beams. The load consists of $Q_k = 5\,kN/m^2$, load from the partition wall, $G_k = 3\,kN/m^2$. Materials used are C-25, S-300 & class I works.
Take $b_w = 250mm$ & 5cm thick finishing.



Sol$\underline{n}$:-

Step 1: Since $\frac{L_y}{L_x} > 2$ for all panels, they can be designed as One way solid slab by taking a strip of unit meter width.

$$d \geq \left(0.4 + 0.6\frac{f_{yk}}{400}\right)\frac{L_e}{\beta_a} = 0.85\frac{L_e}{\beta_a}$$

for S-1 & S-3, $\Rightarrow d = 0.85\left(\frac{3500}{30}\right) = 99.2mm$

& for S-2, $\Rightarrow d = 0.85\left(\frac{4000}{35}\right) = 97.2mm$

$\beta_a = 30$ for end span

$\beta_a = 35$ for interior span

$$D = 99.2 + 15 + 12/2 = \underline{120.2\,mm}$$

$\Rightarrow$ Use $D = 150mm$

Step 2: Design load:

G_K: Concrete $= 1 * 0.15 * 25 = 3.75\ kN/m$
Finishing $= 0.05 * 1 * 20 = 1.00\ kN/m$
Partition $= 1 * 3 = 3.00\ kN/m$

$\Rightarrow G_K = \underline{7.75\ kN/m}$

$Q_K = 1 * 5 = 5.00\ kN/m$

$\therefore P_d = 1.3\ G_K + 1.6\ Q_K$
$= 1.3\ (7.75) + 1.6\ (5.00)$
$= \underline{18.075\ kN/m}$

Step 3: Analyse the strip as Continuous beam & Compute the design moments envelope diagram for each support and span moments.

- Due to symmetry, the support moment at B and C are the same and span moments of AB and CD are equal.
- The support moment at B is maximum when spans AB and BC are fully loaded and span CD partially loaded.

(*) Sign Convention (M)

- CCW = -ve
- CW = +ve

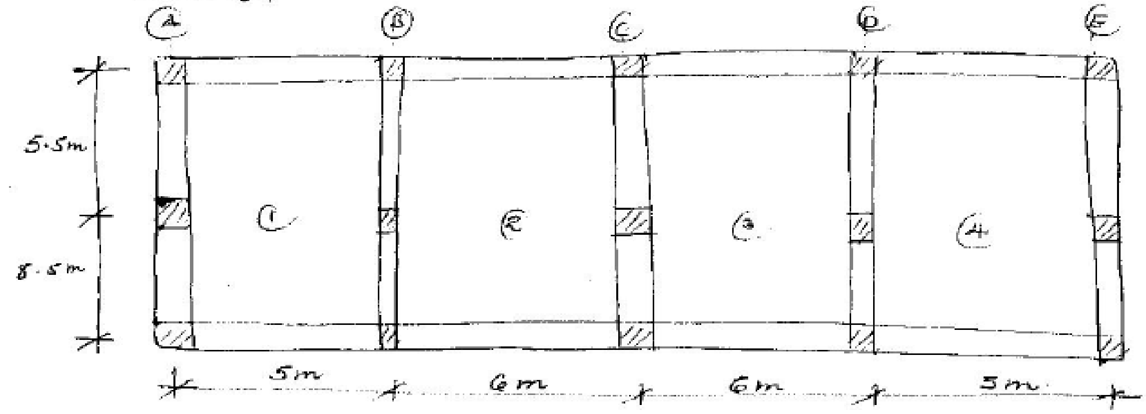
$$FEM = \frac{Wl_e^2}{8} \text{ for end span}$$

$$FEM = \frac{Wl_e^2}{12} \text{ for interior span}$$

4. Design the One Way Solid Slab and Continuous beams Shown in figure below.

Given $Q_K = 5 kN/m^2$, floor finish $1 kN/m^2$.

Use C-25 Concrete and S-300 Steel, Class I works.



Sol^{n}:-

Step 1: Slab → Beam → Column (No Girders)

Span ratio $\Rightarrow \frac{L_y}{L_x} = \frac{14}{5} = 2.8$ for S-1 & S-4

$\Rightarrow \frac{L_y}{L_x} = \frac{14}{6} = 2.3$ for S-2 & S-3

↳ One way Continuous Slab.

Step 2: d from serviceability limit state

$$d = \left(0.4 + 0.6\frac{f_{yk}}{400}\right)\frac{L_e}{\beta_a} = 0.85\frac{L_e}{\beta_a}$$

Calculate d for all spans

Span	l_e	β_a	d
S-1, S-4	500 mm	24	177.1 mm
S-2, S-3	600 mm	28	182.14 mm

For Construction purpose, using the Same depth for all Slabs, thus $d = \underline{182.14 mm}$

$\therefore \quad D = 182.14 + 15 + 10/2 = 202.14 mm$

Take $D = \underline{210 mm}$

Step 3: Loadings:

$G_k = 0.21 * 1 * 25 + 1 * 1 = 6.25\ kN/m$

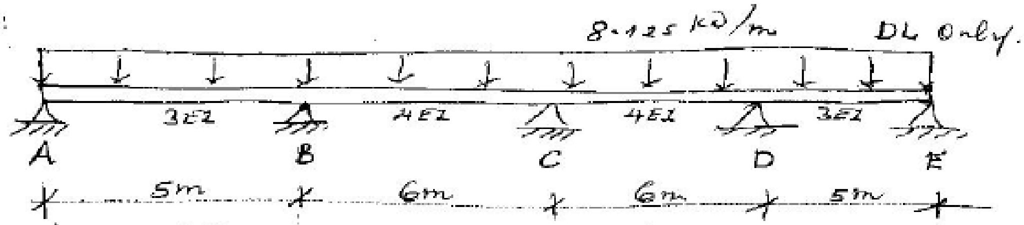
$Q_k = 5 * 1 = 5\ kN/m$

Factored Dead load, $g_k = 1.3\ G_k = 1.3\ (6.25) = 8.125\ kN/m$

Factored Live load, $q_k = 1.6\ Q_k = 1.6 * 5 = 8\ kN/m$

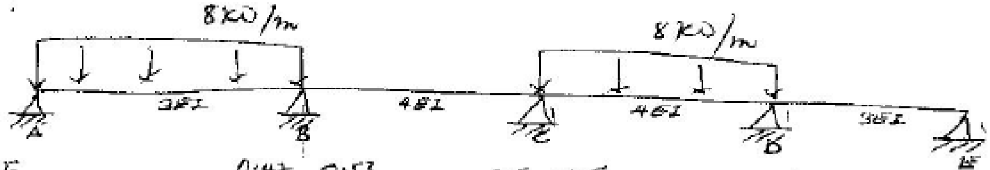
Step 4: Using Moment distribution methods.

Case a:



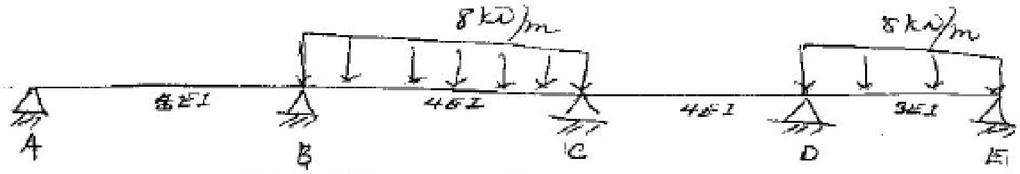
	A		B		C		D		E
K		$\frac{3EI}{5}$		$\frac{4EI}{6}$		$\frac{4EI}{6}$		$\frac{3EI}{5}$	
DF			0.47, 0.53		0.5 0.5		0.53 0.47		
FEM									
Final moment	0		-24.91 +24.91		-24.11 +24.11		-24.91 +24.91		0

Case b :



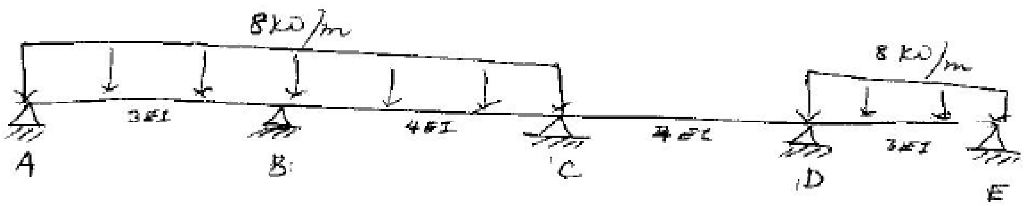
	A	B		C		D		E
DF		0.47	0.53	0.5	0.5	0.53	0.47	
FEM								
Final moment	0	−8.25	+8.25	−11.87	+11.87	−16.28	+16.28	0

Case c



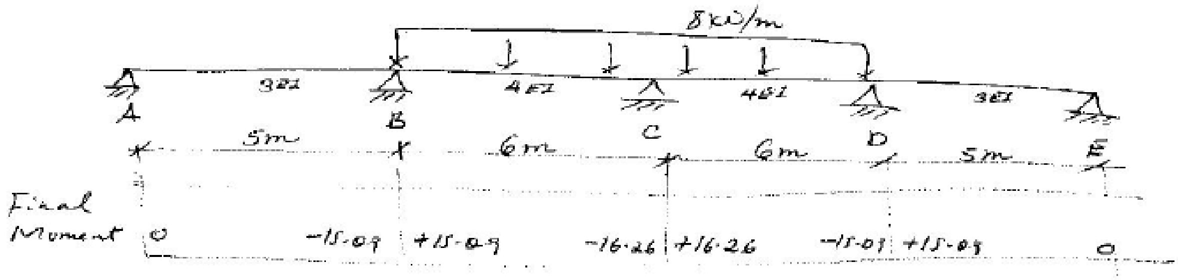
	A	B		C		D		E
DF		0.47	0.53	0.5	0.5	0.53	0.47	
Final Moment	0	−16.28	+16.28	−11.87	+11.87	−8.25	+8.25	0

Case D

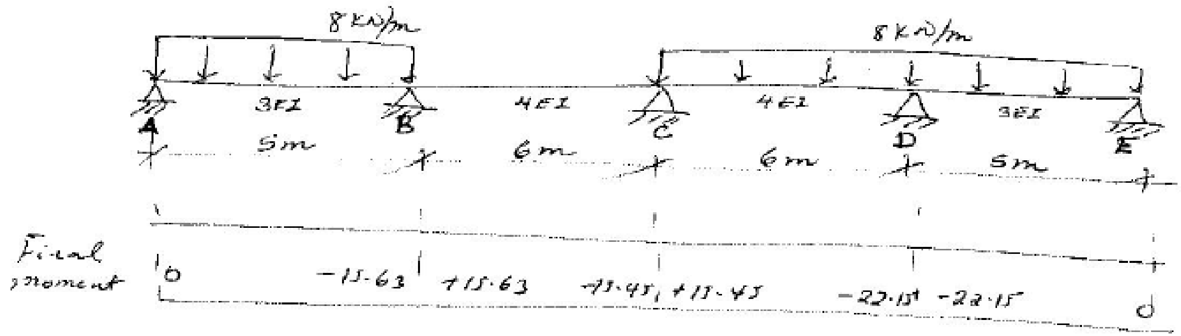


	A	B		C		D		E
Final moment	0	−22.75	+22.75	−15.45	+15.45	−15.63	+15.63	0

Case E

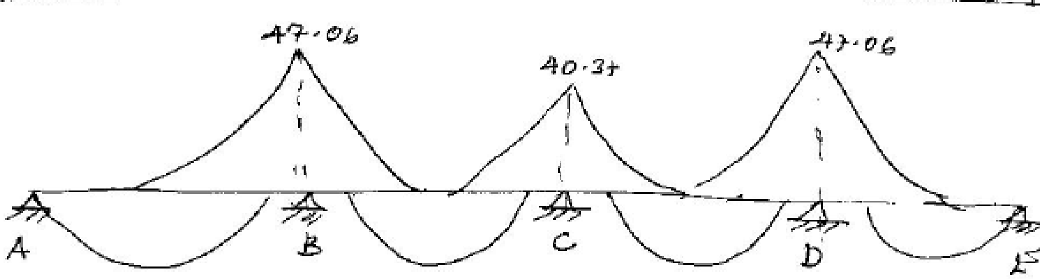


Case F



Summary of Moments:

Support	Cases										
	a	b	C	D	E	F	a+b	a+c	a+d	a+e	a+F
B	24.91	8.25	16.28	22.15	15.09	15.63	33.16	41.19	47.06	40.00	40.54
C	24.91	11.87	11.87	15.45	16.26	15.45	35.78	35.78	39.56	40.37	39.56
D	24.91	16.28	8.25	15.63	15.09	22.15	41.14	33.16	40.54	40.00	47.06



Step 5: Check the depth for flexure:

$$M_{max} = 47.06\ kNm$$

$$d_{check} = \sqrt{\frac{M_{max}}{0.2952 f_{cd} b}} = \sqrt{\frac{47.06 * 10^6}{0.2952 * 11.33 * 1000}}$$

$$= 118.67\ mm < d_{used} = 210 - 15 - 10/2 = 190\ mm$$

OK!

Step 6: Reinforcements:

At Support B and D:

$$\rho = \left[1 - \sqrt{1 - \frac{2M_{max}}{bd^2 f_{cd}}}\right] \frac{f_{cd}}{f_{yd}}$$

$$= \left[1 - \sqrt{1 - \frac{2 * 47.06 * 10^6}{1000 * 190^2 * 11.33}}\right] \left(\frac{11.33}{260.87}\right)$$

$$= 0.0058$$

$$\Rightarrow A_s = \rho b d = 0.0058 * 1000 * 190 = 1102\ mm^2$$

Using ϕ 10mm $\Rightarrow a_s = \pi (10/2)^2 = 78.54\ mm^2$

Spacing $\Rightarrow S = \dfrac{a_s * 1000}{A_s} = \dfrac{78.54 * 10^3}{1102} = 71.27\ mm$

OR Using ϕ 12mm $\Rightarrow S = \dfrac{113.1 * 1000}{1102} = 102.6\ mm$

$\Rightarrow$ provide ϕ 12 bars @ 100mm c/c

At Support C :

$M_{max} = 40.37$ kN.m.

$$\Rightarrow \rho = \left[1 - \sqrt{1 - \frac{2 * 40.37 * 10^6}{1000 * 190^2 * 11.33}}\right]\left(\frac{11.33}{260.87}\right) = 0.0045$$

$$\Rightarrow A_s = \rho b d = 0.0045 * 10^3 * 190 = 859.21\ mm^2$$

Using ϕ 12 mm bars :

$$\Rightarrow S = \frac{a_s * 1000}{A_s} = \frac{113.1 * 10^3}{859.21} = 131.63\ mm$$

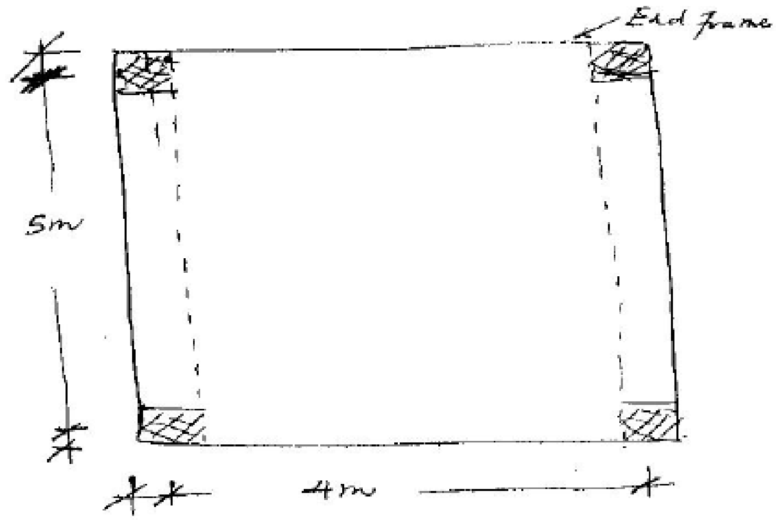
$\Rightarrow$ provide ϕ 12 mm bars @ 120 mm c/c

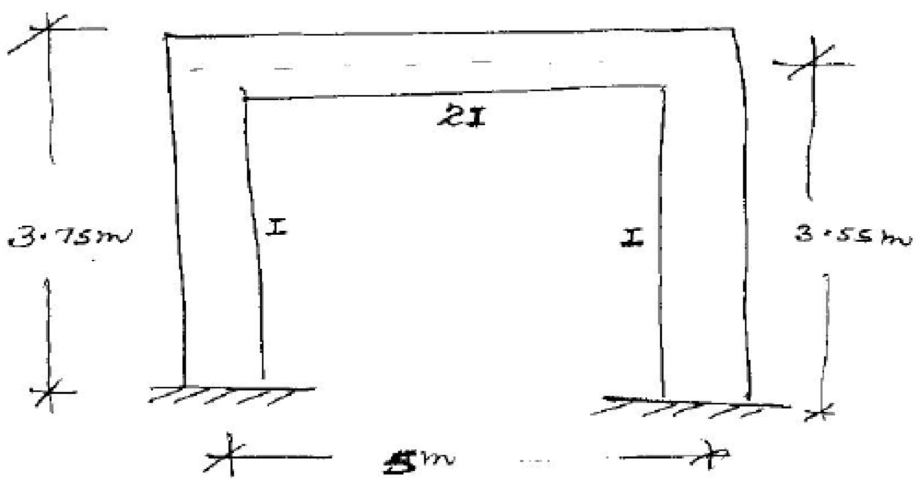
EXERCISE :- provide Reinforcement on the span :

1st find span moments for each cases & select the maximum value.

2nd Design the slab for such max. moment

5. A Building of Single Storey with height from footing to Top of roof is 3.75m as shown below

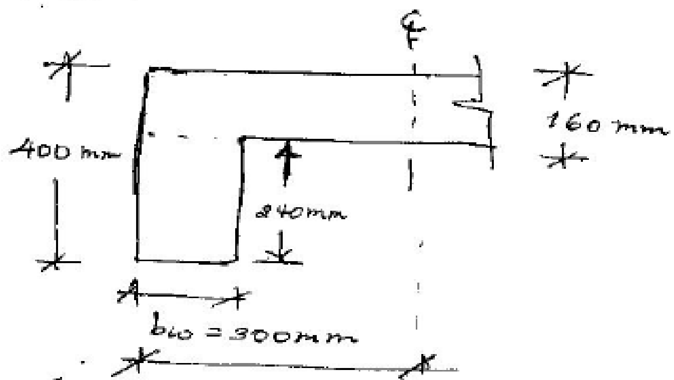




Sol^n :- Minimum depth required by deflection (ACI), $D_{min} = \frac{l}{26}$ (interior span)

$$\Rightarrow D_{min} = \frac{5000}{26} = 192.3 \text{ mm}$$

1^{st} Trial : $D = 400\,mm$



$\left(\frac{b_w}{2} + \frac{l_e}{2}\right) = 0.15 + \frac{4}{2} = 2.15\,m$

Load On Edge Beam:

DL (Wt. of Slab) $= 0.16 * 2.15 * 24 = 8.256\,kN/m$

DL (Floor finish) $= 2.15 * 1\,kN/m^2 = 2.15\,kN/m$

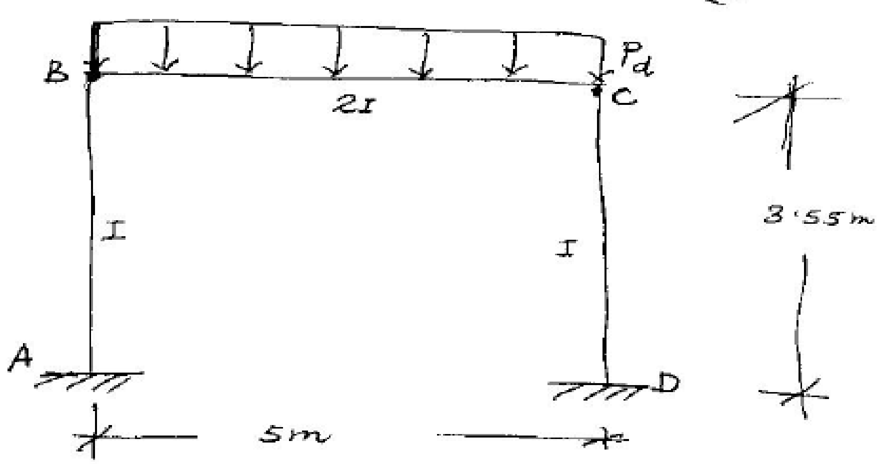
DL (Own Wt. of Beam) $= 0.3 * 0.24 * 24 = 1.728\,kN/m$

Total DL $= 12.134\,kN/m = G_k$

& $Q_k = 2.15\,m * 1.5\,kN/m^2 = 3.225\,kN/m$

Design load,

$P_d = 1.3DL + 1.6LL = 20.934\,kN/m$



Let $I = 3.55$ & $E = 1$

$$K_{AB} = K_{CD} = \frac{3.55}{3.55} = 1.0$$
$$K_{BC} = \frac{2 * 3.55}{5} = 1.42$$

} Stiffness factor.

~ Distribution factors:

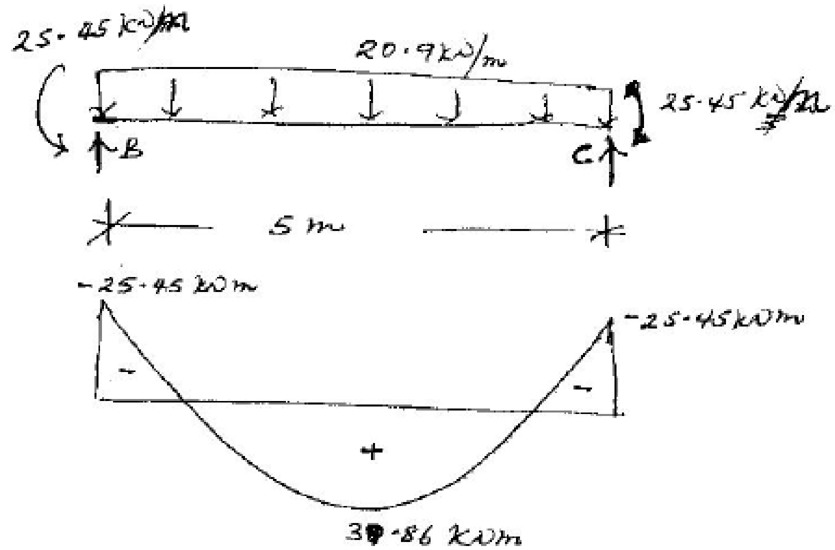
$$(DF)_{AB} = (DF)_{DC} = 0.0 \qquad \text{(fixed end)}$$
$$(DF)_{BA} = \frac{1}{(1+1.42)} = 0.413 = (DF)_{CD}$$
$$(DF)_{BC} = \frac{1.42}{(1+1.42)} = 0.587 = (DF)_{CB}$$

~ Fixed End Moment (FEM)

$$M_{BC}^{F} = \frac{P_d l^2}{12} = \frac{20.9 * 5^2}{12} = 43.54 \text{ kNm}$$
$$M_{CB}^{F} = \frac{-P_d l^2}{12} = -43.54 \text{ kNm}$$
$$M_{AB}^{F} = M_{BA}^{F} = M_{CD}^{F} = M_{DC}^{F} = 0.0$$

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
DF	0.0	0.413	0.587	0.587	0.413	0.0
FEM	0	0	43.54	−43.54	0	0
		−17.98	−25.56			
	−8.99			−12.78		
				33.06	23.26	
			16.53			11.63
		−6.83	−9.70			
	−3.41			−4.85		
				2.85	2.00	
			1.42			1.00
		−0.59	−0.83			
				−0.42		
				0.25	0.17	
			0.12			0.09
		−0.05	−0.07			
	−0.03			−0.04		

Member	AB	BA	BC	CB	CD	DC
				0.02	0.02	
Final moment	-12.72	-25.45	25.45	-25.45	25.45	12.72



Therefore, Design moments are:

$M_B = M_C = -25.45$ kNm (Support) → Rectangular ⇒ $b = b_w$.

$M_{BC} = 39.86$ kNm (Span) → Rectangular, $b = b_e$ T-Beam.

~ Effective Flange Width Smaller of (Edge) beam

(1) $b_w + l_e/10 = 300 + \frac{3000}{10} = 800$ mm

(2) $b_w + \frac{1}{2}$ (Clear distance to the next beam)

$= 300 + \frac{1}{2}(3700) = 2150$ mm

$\therefore \underline{b_e = 800 \text{ mm}}$

Reinforcement :

A). At Supports :

using ϕ 16 bars

$$\left\{\begin{array}{l} M_d = -25.45\ kNm \\ b = b_w = 300mm \\ d = 400 - 25 - 6 - 16/2 = 361mm \end{array}\right.$$

For $\mu = \dfrac{M_d}{f_{cd} bd^2} = 0.0486 < \mu_{max} = 0.34 \Rightarrow$ Singly !

$\Rightarrow Z = \dfrac{d}{2}\left(1 + \sqrt{1 - 2\mu}\right) = 352\ mm.$

$\Rightarrow$ (-ve) $A_s = \dfrac{M_d}{f_{yd} Z} = \underline{278\ mm^2} > A_{smin} = \dfrac{0.5}{f_{yk}} bd$

$\Rightarrow$ No of $\phi 16 = \dfrac{A_s}{a_s} = \dfrac{278}{201} = 1.32 \approx \underline{2\ \phi 16\ bars}$

$\therefore$ provide : $\underline{2\ \phi\ 16 \text{ bars at top of beam}}$

B). Span BC :

$$\left\{\begin{array}{l} M_d = 39.86\ kNm \\ d = 361mm\ (\text{using } \phi\ 16 \text{ bars}) \\ b = b_e = 800mm. \end{array}\right.$$

Assume it is Rectangular !

$\mu = \dfrac{M_d}{f_{cd} b_e d^2} = 0.0285 < \mu_{max} = 0.34$

$\hookrightarrow \omega = 1 - \sqrt{1 - 2\mu} = 0.029$

Check : $y = \omega d = 0.029 * 361 = \underline{10.47mm} < t = 160mm$

Assumption is correct !

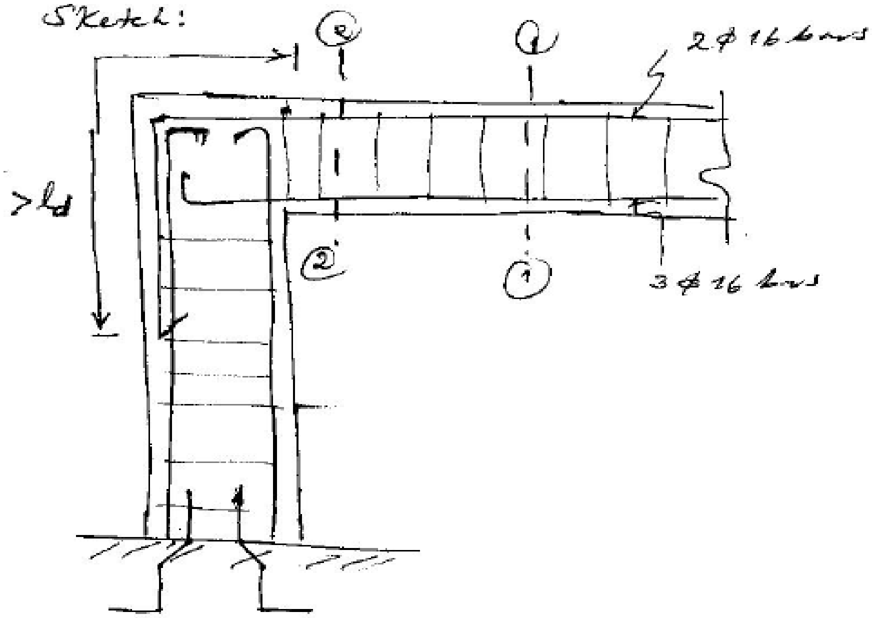
Then, $Z = \dfrac{d}{2}\left(1 + \sqrt{1 - 2\mu}\right) =$ ——

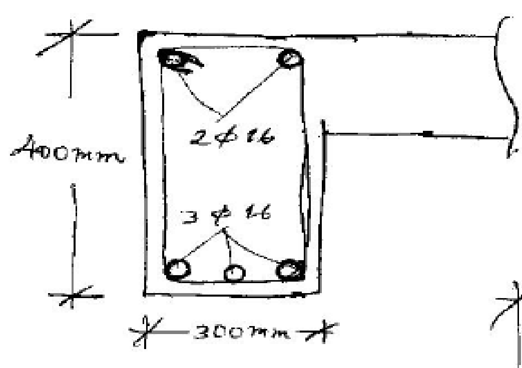
(+ve) $A_s = \dfrac{M_d}{f_{yd} Z} = 430.2mm^2 > A_{smin} = \dfrac{0.5 b_w d}{f_{yk}}$

No of $\phi 16$ bars $= \frac{A_s}{a_s} = \frac{430.2}{201} = 2.14 \approx 3\ \phi 16$ bars

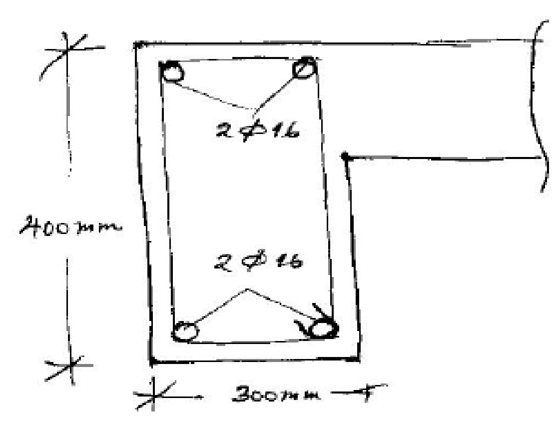
$\Rightarrow$ provide : 3 $\phi 16$ bars @ bottom of beam:

Sketch:





Sec 1-1.



Sec 2-2.

3.7. Serviceability limits states of deflection and crack width

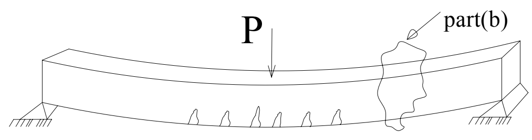
It is important that member performance in normal service be satisfactory, when loads are those actually expected to act i.e. when load factors are 1.0. This is not guaranteed simply by providing adequate strengths. Service load deflections under full load may be excessively large or long-term deflections due to sustained loads may cause damage .Tension cracks in beams may be wide enough to be visually disturbing or may even permit serious corrosion of reinforcing bars. These and other questions such as vibration or fatigue, require consideration

Serviceability studies are carried out based on elastic theory, with stresses in both concrete and steel assumed to be proportional to strain. The concrete on the tension side of the neutral axis may be assumed uncracked, partially cracked, or fully cracked depending on the loads and material strengths.

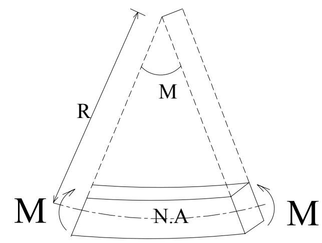
Reinforced concrete members carrying lateral loads respond to these loads by bending. The moment curvature relationship for a segment of the simply supported reinforced concrete member of fig.3.7.1 (a) is illustrated in fig.3.7.1 (c). It can be seen that the segment remains uncracked and has a large stiffness EIu, , until the moment reaches the cracking moment, Mcr, (Point A) .When this happens, the member cracks and the stiffness at the cracked section reduces to EIc.

As the load (and hence the moment) is increased further, more cracks occur and existing cracks increase in size .Eventually ,the reinforcement yields at the point of maximum moment corresponding to point C on the diagram. Above this point the member displays large increases in deflection for small increases in moment .The service load range is between the origin and point C on the diagram and it is in this range that deflections are checked and stresses calculated.

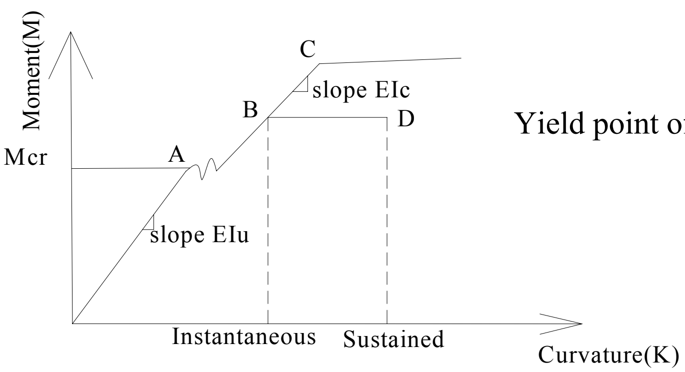
Consider a point B within the service load range. This curvature represents the instantaneous (short term) curvature under an applied moment, M. If the moment is sustained, however, the curvature increases with time to point D owing to the creep of the concrete. The curvature at this point is known as the long term or sustained curvature. As deflection results, from curvature, there are both instantaneous and sustained deflections which must be considered in the design of members with bending.



a) deflected shape



b) Curvature of segment of beam
Curvature (K)=1/R



Yield point of reinforcement

c) Moment/ curvature plot for segment of part (b)

Fig.3.7.1 Moment / Curvature relationship for beam segment

3.7.1. Deflections

The deflections which result from bending must be limited such that they do not adversely affect the function and appearance of the member or the entire structure.

a) Limits on Deflections

The final deflection (including the effects of temperature, creep and shrinkage) of all horizontal members shall not, in general, exceed the value.

$$\delta = \frac{Le}{200}$$ Where: Le effective span

For roof or floor construction supporting or attached to nonstructural elements (e.g. partitions and finishes) likely to be damaged by large deflections, that part of the deflection which occurs after the attachment of the non-structural elements shall not exceed the value .

$$\delta = \frac{Le}{350} \leq 20mm$$

b) Calculation of Deflections

Effect of creep and shrinkage strains on the curvature, and there by on the deflection shall be considered.

Immediate deflections shall be computed by the usual elastic methods as the sum of the two parts δ_i and δ_{ii} given by Eqs. 1 and 2, but not more than $\delta_{\max}$ given by eqs. 3

$$\delta_i = \beta L^2 \frac{M_{cr}}{E_{cm} I_c} \qquad \text{------} \quad (1)$$

$$\delta_{ii} = \beta L^2 \frac{M_k - M_{cr}}{0.75 E_s A_s Z(d-X)} \qquad \text{---------------} \quad (2)$$

$$\delta_{\max} = \beta L^2 \frac{M_k}{E_s A_s Z(d-X)} \qquad \text{----------------------} \quad (3)$$

$$M_{cr} = 1.70 f_{ctk} S \qquad \text{------------------------------------} \ (4)$$

δ_i = deflection due to the theoretical cracking moment (Mcr) acting on the uncracked transformed section

δ_{ii} =deflection due to the balance of the applied moment over and above the cracking value and acting on a section with an equivalent stiffness of 75% of the cracked value. $\delta_{\max}$ = deflection of fully cracked section

A_s = area of the tension reinforcement

Ecm = short term elastic modulus (secant modulus) of the concrete

$E_{cm} = 9.5\ (f_{ck} + 8)^{\frac{1}{3}}$ fck-mpa, Ecm-Gpa

Grade of concrete	C15	C20	C25	C30	C40	C50	C60
Ecm	26	27	29	32	35	37	39

Es-modulus of elasticity of steel, **Iu**-moment of inertia of the uncraked transformed section

M$_k$-Maximum applied, moment at mid span due to sustained characteristic loads; for cantilevers it is the moment at the face of the support

S- Section modulus, **d**-effective depth of the section,

X-neutral axis depth at the section of max. moment,

Z-internal lever arm at the section of max moment.

β -deflection coefficient depending on the loading and support conditions.

(e.g β =5/48 for simply supported span subjected to uniformly distributed load)

Note: The value of X & Z may be determined for the service load condition using a modular ratio of 10, or for the ultimate load condition.

Long term deflection of flexural members shall be obtained by multiplying the immediate deflection caused by the sustained load considered, by the factor,

(2-1.2As'/As) ≥0.6--------------- (5)

Where: As'-area of compression reinforcement, As-area of tension reinforcement.

3.7.2. Limits on cracking

Flexural cracks are inevitably formed in reinforced concrete members. For structures in aggressive environments, corrosion is a problem and stringent limits are imposed on the width of cracks that are allowed to develop. Environment in the interior of the building is usually non-sever, corrosion does not generally pose a problem and limits on crack widths will be governed by their appearance.

a) Crack Formation

- The max. tensile stresses in the concrete are calculated under the action of design loads appropriate to a serviceability limit state and on the basis of the geometrical properties of the transformed uncracked concrete X-section.
- The calculated stresses shall not exceed the following values:

  a) Flexure, ($\delta_{ct} = 1.70 f_{ctk}$) b) direct(axial) tension ($\delta_{ct} = f_{ctk}$)
- Minimum flexural reinforcement in beams for the control of cracking is given by:

$$\rho_{\min} = \frac{0.6}{f_{yk}}$$

b) Crack widths

Crack widths are calculated using the quasi permanent service load combination. Specifically crack widths can be assumed not to exceed the limiting values if the limits on the bar spacing or bar diameter (Table 1) are satisfied, and if min. areas of reinforcement, also specified are provided.

Table 1 Maximum size and spacing of high bond bars for control of cracking.

Steel stress*	Max. bar spacing (mm)	Max. bar diameter(mm)
160	300	32
200	250	25
240	200	20
280	150	16
320	100	12
360	50	10
400	-	8
450	-	6

*steel stresses are determined using quasi –permanent loads.

Table 2 Characteristic crack widths for concrete Members

Type of exposure	Dry environment: Interior of buildings of normal habitation or office (mild)	Humid environment: Interior components(e.g. laundries), exterior components; components in non-aggressive soil and /or water (Moderate)	Sea water and/or aggressive chemicals environments completely or partially submergeed in seawater ,components in saturated salt air ,aggressive industrial atmospheres (sever)
Characteristic crack width,wk(mm)	0.4	0.2	0.1

In specific cases where a crack width Calculation is considered necessary

Wk= $\beta s_{rm} \varepsilon_{sm}$ Where: wk=characteristic crack width, s_{rm} =average final crack width

ε_{sm} =mean strain in the tension steel allowing for tension stiffening and time dependent effects

β =coefficient relating the average crack width to the design value

$\beta = 1.7$ for sections in bending under applied loads.

The mean strain is simply the strain in the steel adjusted by the distribution factor, ξ

$\varepsilon_{sm} = \xi \dfrac{f_s}{E_s}$, Where: **fs**-stress in the tension reinforcement, Es-elastic modulus of steel $\xi = 1 - \beta_1 \beta_2 \ (\dfrac{f_{sr}}{f_s})$

β_1 =coefficient which accounts for the bond properties of the reinforcement

β_1 =1.0 for high bond bars (normally used or deformed) and 0.5 for plain bars

β_2 = coefficient which accounts for the duration of loading or of repeated loading

β_2 =1.0 for single short term loading & 0.5 for sustained loading or repeated loading

f_s= stress in tension steel assuming a cracked section

f_{sr}= stress in tension steel assuming a cracked section due to loading which causes initial cracking

The average final crack spacing in (mm) is calculated using the equation

$S_{rm} = 50 + 0.25 K_1 K_2 \dfrac{\phi}{\rho_r}$ - 152 -

Where: k_1= coefficient which accounts for the bond properties of the reinforcement: k_1=0.8 for high bond bars:k_1=1.6 for plain bars.

K_2= coefficient which takes account of the form of strain distribution for bending it is 0.5

ϕ = bar diameter, ρ_r = effective reinforcement ratio $A_s/A_{c,eff.}$

Where: $A_{c,eff}$= effective tension area of he concrete , as illustrated below

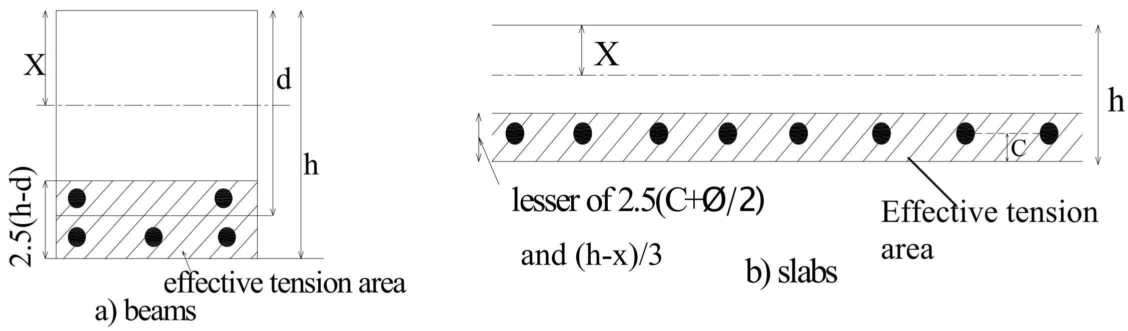


Fig.3.7.2. Effective tension area of concrete

Exercise-2

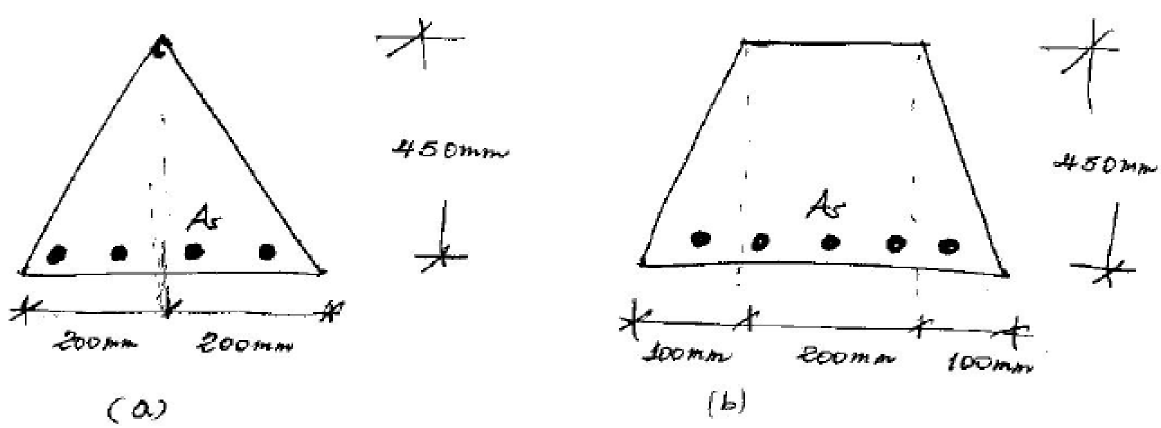
1. Design a rectangular RC Section to resist an ultimate moment of 360 kNm. The dimensions of the beams are limited to $b/d = 350/500$ mm for architectural reasons. Use C-25 Concrete and S-400 Steel, Class I works. Use (a). Equation (b) Design Aids.

2. A doubly reinforced Section has $A_s = 2581\ mm^2$, $A_{s2} = 645\ mm^2$, $b = 280$ mm, $d = 580$ mm, $d_2 = 50$ mm, C-25 Concrete, S-300 Steel, Class I works. Calculate the flexural strength capacity of the section.

3. Calculate the Moment Capacity of the above section if C-40 Concrete is used and $A_s = 1804\ mm^2$

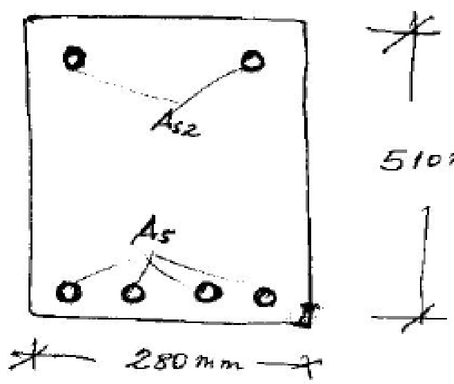
4. A 100 mm Concrete Floor Slab is monolithically Cast with Continuous beams of Span 5m Spaced at 1.2m on Centers. Beam Sections are $b_w = 250$ mm, $h = 500$ mm. Determine the area of reinforcement at mid-span to resist an ultimate moment of 250 kNm. C-25 Concrete, S-300 Steel, Class I works are used.

5. Calculate the area of reinforcement for a positive span moment of 15 kNm for the Sections Shown in figure below. C-25 Concrete and S-300 Steel, Class I works are to be used.



6. A Rectangular Section with limited dimensions of $b = 280mm$, $d = 500mm$, $d_2 = 60mm$ is made of C-25 Concrete and S-300 Steel, Class I Works is to Carry a Service load moment of 170 kNm due to permanent action and 215 kNm due to Variable action. Calculate the area of Steel required.

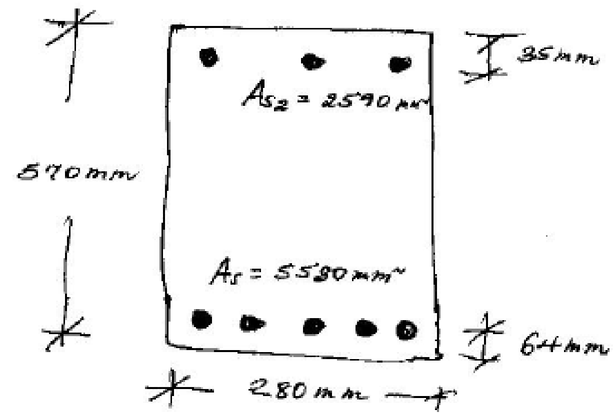
7. Determine the ultimate moment of resistance M_u, of the Cross-Section Shown in Figure below, given that the Concrete is C30, the Steel S 425, Class I Works, $A_s = 2410mm^2$, $A_{s2} = 628mm^2$.



8. Determine the Flexural Strength of the rectangular reinforced Concrete Section Shown in Figure below for (a). a positive bending moment

(b). Negative bending moment

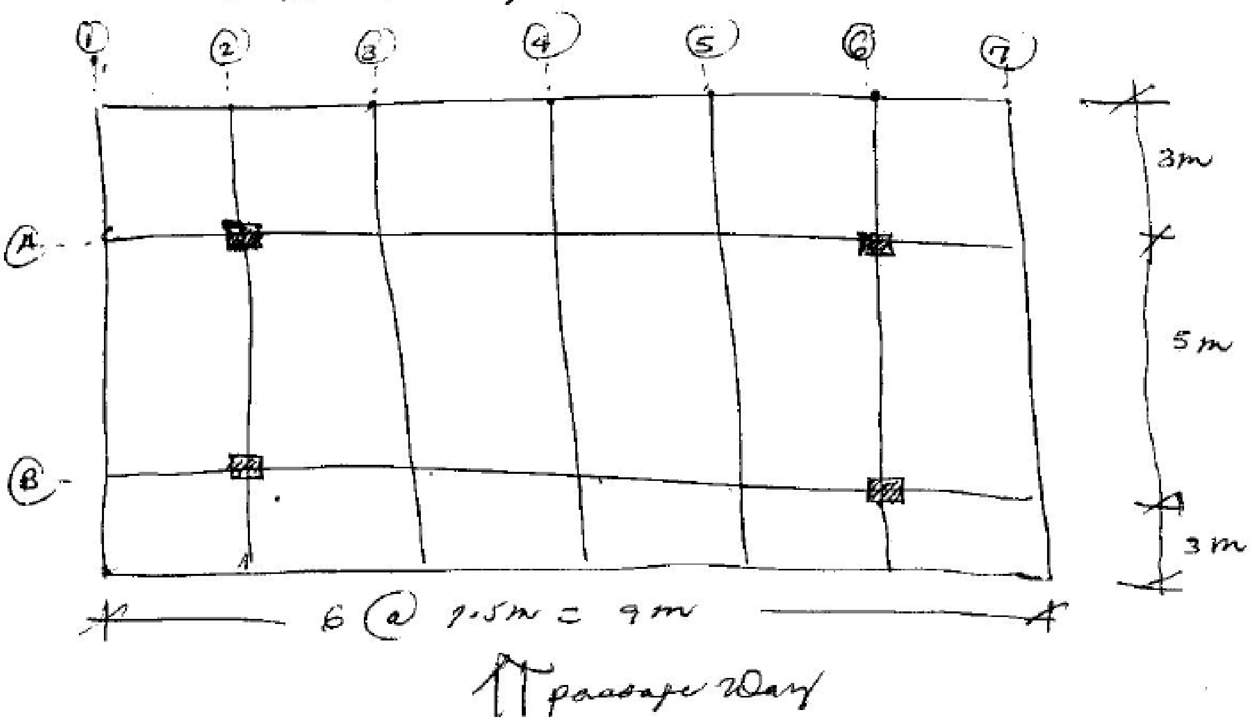
C-25 Concrete and S-300 Steel, Class I Works are to be used.



12. In a Residential Building, a passage way must be provided for cars; One side of columns could not be placed within a width of 6m as shown in figure below. The first floor slab-beam system shown in the figure separated from the remaining portion of the building by expansion joint. Use the LSD method:

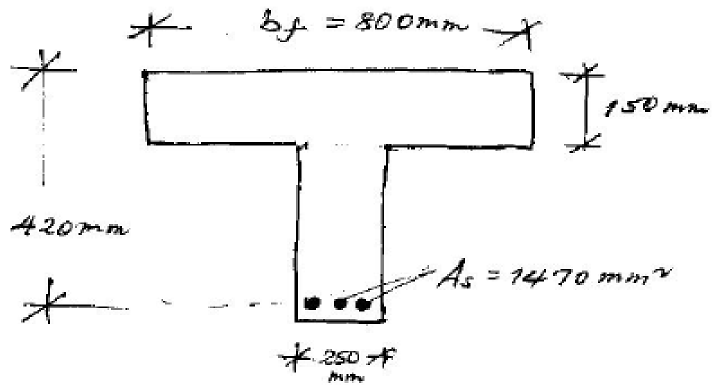
(A). to design an interior beam on either of axes 2 to 6 and give its depth for prefabrication to prefab factory. Breadth of the beam is to be 250 mm.

(B). Determine the amount of reinforcement for the girders on axes A and B both at the supports and at the span; Breadth of the Girders is 300mm, depth below slab is 270mm. The thickness of the slab is 80 mm. C-30 Concrete and S-300 Steel, class I works is to be used for the prefabrication; partition load is 1.2 KN/m². Consider live load variation in the design.

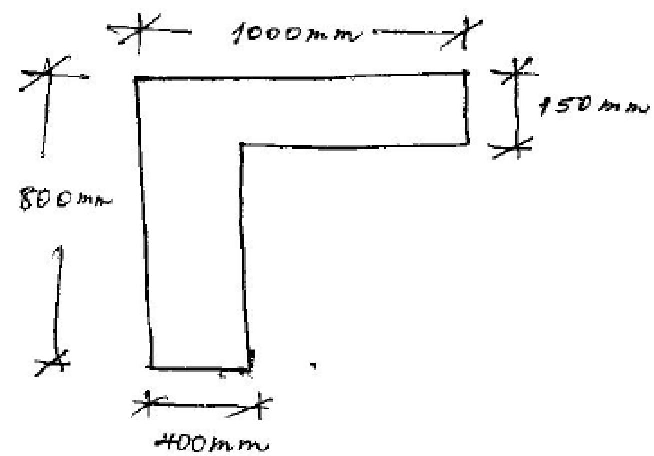


9. Determine the area of Steel required at mid-Span of a Continuous beam of Spans 6m, Spaced at 2m on Centers that are monolithically cast with 150mm Slab to Support a factored moment of 1080 KNm. Concrete is C-25, Steel S-425, Class I Works. b_w = 300mm, D = 600mm.

10. Determine the ultimate moment of resistance of the T-beam Shown in Figure below if C-25 Concrete and S-425 Steel, Class I Works are used.



11. Determine the Maximum moment of resistance and area of Steel required for the L-beam Shown in Figure below. C-30 Concrete & S-425 Steel, Class I Works are used



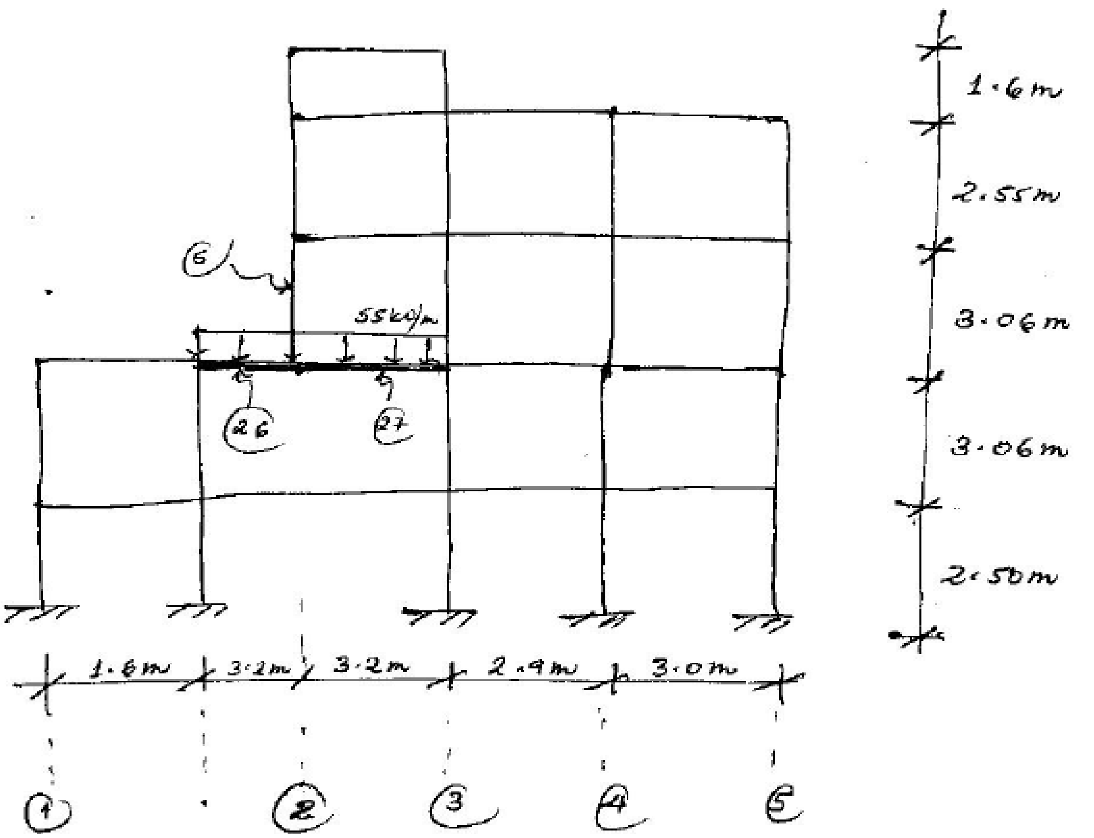
13. Check deflection, and if necessary crack width for the beam 26-27 for ultimate gravity loads (Combination 1 in the computer output). The ratio of ultimate load to service load, and of ultimate external actions to service load actions may be taken as 1.33. Model is as shown in Fig(b). Output for column members forces are given below (memb. 5).

Sap2000

ELT ID	Load Comb.	Axial Force	DIST	END Shear	Moment
(26)					
	1	-26.86			
			0.0	188.17	-185.80
			3.2	12.17	134.74
	2.	1.80			
			0.0	112.42	-95.49
			2.7	0.00	57.69
			3.2	-19.58	53.04
	3.	-42.69			
			0.0	169.84	-183.21
			3.2	37.84	149.07
(27)					
	1.	-31.06			
			0.0	-7.76	126.32
			3.2	-183.76	-180.10
	2.	-26.42			
			0.0	-8.66	91.71
			3.2	-140.66	-147.20
	3.	-20.17			
			0.0	-2.98	97.77
			3.2	-134.98	-122.95

(5) - - - - - - - - - - - - - -

1	-19·93 kN		
	0·0m	4·20 kN	8·42 kNm
	3·2m	4·20 kN	21·27 kNm
2.	10·92 kN		
	0·0m	28·22	-38·67 kNm
	3·1 m	28·22	47·68 kNm
3.	-40·81 kN		
	0·0m	-21·92 kN	51·30 kNm
	3·1 m	-21·92 kN	-15·77 kNm



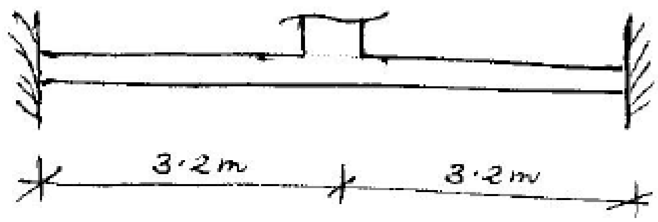


Fig (6)

NB. Comb 1 : $1.3 G_k + 1.6 Q_k$

2. $0.75 (1.3 G_k + 1.6 Q_k)$

3. $G_k + Q_k$ (Serviceability)

Sign Convention: