1.1 A reservoir of glycerin (glyc) has a mass of 1200 kg and a volume of 0.952 m^3. Find the glycerin's weight (W), mass density (ρ), specific weight (γ), and specific gravity (s.g.).

I

$$F = W = ma = (1200)(9.81) = 11\,770\ \text{N} \quad \text{or} \quad 11.77\ \text{kN}$$

$$\rho = m/V = 1200/0.952 = 1261\ \text{kg/m}^3$$

$$\gamma = W/V = 11.77/0.952 = 12.36\ \text{kN/m}^3$$

$$\text{s.g.} = \gamma_{\text{glyc}}/\gamma_{\text{H}_2\text{O at 4°C}} = 12.36/9.81 = 1.26$$

1.2 A body requires a force of 100 N to accelerate it at a rate of 0.20 m/s^2. Determine the mass of the body in kilograms and in slugs.

I

$$F = ma$$

$$100 = (m)(0.20)$$

$$m = 500\ \text{kg} = 500/14.59 = 34.3\ \text{slugs}$$

1.3 A reservoir of carbon tetrachloride (CCl_4) has a mass of 500 kg and a volume of 0.315 m^3. Find the carbon tetrachloride's weight, mass density, specific weight, and specific gravity.

I

$$F = W = ma = (500)(9.81) = 4905\ \text{N} \quad \text{or} \quad 4.905\ \text{kN}$$

$$\rho = m/V = 500/0.315 = 1587\ \text{kg/m}^3$$

$$\gamma = W/V = 4.905/0.315 = 15.57\ \text{kN/m}^3$$

$$\text{s.g.} = \gamma_{\text{CCl}_4}/\gamma_{\text{H}_2\text{O at 4°C}} = 15.57/9.81 = 1.59$$

1.4 The weight of a body is 100 lb. Determine (*a*) its weight in newtons, (*b*) its mass in kilograms, and (*c*) the rate of acceleration [in both feet per second per second (ft/s^2) and meters per second per second (m/s^2)] if a net force of 50 lb is applied to the body.

I (*a*)

$$W = (100)(4.448) = 444.8\ \text{N}$$

(*b*)

$$F = W = ma \qquad 444.8 = (m)(9.81) \qquad m = 45.34\ \text{kg}$$

(*c*)

$$m = 45.34/14.59 = 3.108\ \text{slugs}$$

$$F = ma \qquad 50 = 3.108a \qquad a = 16.09\ \text{ft/s}^2 = (16.09)(0.3048) = 4.904\ \text{m/s}^2$$

1.5 The specific gravity of ethyl alcohol is 0.79. Calculate its specific weight (in both pounds per cubic foot and kilonewtons per cubic meter) and mass density (in both slugs per cubic foot and kilograms per cubic meter).

I

$$\gamma = (0.79)(62.4) = 49.3\ \text{lb/ft}^3 \qquad \gamma = (0.79)(9.79) = 7.73\ \text{kN/m}^3$$

$$\rho = (0.79)(1.94) = 1.53\ \text{slugs/ft}^3 \qquad \rho = (0.79)(1000) = 790\ \text{kg/m}^3$$

1.10 A certain gasoline weighs 46.5 lb/ft³. What are its mass density, specific volume, and specific gravity?

$$\rho = \gamma/g = 46.5/32.2 = 1.44 \text{ slugs/ft}^3 \qquad V_s = 1/\rho = 1/1.44 = 0.694 \text{ ft}^3\text{/slug}$$

$$\text{s.g.} = 1.44/1.94 = 0.742$$

1.11 If the specific weight of a substance is 8.2 kN/m³, what is its mass density?

$$\rho = \gamma/g = 8200/9.81 = 836 \text{ kg/m}^3$$

1.12 An object at a certain location has a mass of 2.0 kg and weighs 19.0 N on a spring balance. What is the acceleration due to gravity at this location?

$$F = W = ma \qquad 19.0 = 2.0a \qquad a = 9.50 \text{ m/s}^2$$

1.25 A liquid compressed in a cylinder has a volume of 1000 cm³ at 1 MN/m² and a volume of 995 cm³ at 2 MN/m². What is its bulk modulus of elasticity (K)?

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{2-1}{(995-1000)/1000} = 200 \text{ MPa}$$

1.26 Find the bulk modulus of elasticity of a liquid if a pressure of 150 psi applied to 10 ft³ of the liquid causes a volume reduction of 0.02 ft³.

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{(150-0)(144)}{-0.02/10} = 10\,800\,000 \text{ lb/ft}^2 \quad \text{or} \quad 75\,000 \text{ psi}$$

1.27 If $K = 2.2$ GPa is the bulk modulus of elasticity for water, what pressure is required to reduce a volume by 0.6 percent?

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2.2 = -\frac{p_2 - 0}{-0.006} \qquad p_2 = 0.0132 \text{ GPa} \quad \text{or} \quad 13.2 \text{ MPa}$$

1.28 Find the change in volume of 1.00000 ft³ of water at 80 °F when subjected to a pressure increase of 300 psi. Water's bulk modulus of elasticity at this temperature is 325 000 psi.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 325\,000 = -\frac{300-0}{\Delta V/1.00000} \qquad \Delta V = -0.00092 \text{ ft}^3$$

1.29 From the following test data, determine the bulk modulus of elasticity of water: at 500 psi the volume was 1.000 ft³, and at 3500 psi the volume was 0.990 ft³.

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{500-3500}{(1.000-0.990)/1.000} = 300\,000 \text{ psi}$$

1.30 A rigid steel container is partially filled with a liquid at 15 atm. The volume of the liquid is 1.23200 L. At a pressure of 30 atm, the volume of the liquid is 1.23100 L. Find the average bulk modulus of elasticity of the liquid over the given range of pressure if the temperature after compression is allowed to return to its initial value. What is the coefficient of compressibility (β)?

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{(30-15)(101.3)}{(1.23100-1.23200)/1.23200} = 1.872 \times 10^6 \text{ kPa} \quad \text{or} \quad 1.872 \text{ GPa}$$

$$\beta = 1/K = 1/1.872 = 0.534 \text{ GPa}^{-1}$$

1.34 At a depth of 7 km in the ocean, the pressure is 71.6 MPa. Assume a specific weight at the surface of 10.05 kN/m³ and an average bulk modulus of elasticity of 2.34 GPa for that pressure range. Find (*a*) the change in specific volume between the surface and 7 km; (*b*) the specific volume at 7 km; (*c*) the specific weight at 7 km.

I (*a*)

$$(V_s)_1 = 1/\rho_1 = g/\gamma_1 = 9.81/10\,050 = 0.0009761 \text{ m}^3/\text{kg}$$

$$K = -\frac{\Delta p}{\Delta V_s/V_s} \qquad 2.34 \times 10^9 = -\frac{71.6 \times 10^6 - 0}{\Delta V_s/0.0009761} \qquad \Delta V_s = -0.0000299 \text{ m}^3/\text{kg}$$

(*b*)

$$(V_s)_2 = (V_s)_1 + \Delta V_s = 0.0009761 - 0.0000299 = 0.000946 \text{ m}^3/\text{kg}$$

(*c*)

$$\gamma_2 = g/V_2 = 9.81/0.000946 = 10\,370 \text{ N/m}^3$$

1.117 A 1-in-diameter soap bubble has an internal pressure 0.0045 lb/in² greater than that of the outside atmosphere. Compute the surface tension of the soap–air interface. Note that a soap bubble has two interfaces with air, an inner and outer surface of nearly the same radius.

I

$$p = 4\sigma/r \qquad (0.0045)(144) = (4)(\sigma)/[(\tfrac{1}{2})/12] \qquad \sigma = 0.00675 \text{ lb/ft}$$

1.118 What force is required to lift a thin wire ring 6 cm in diameter from a water surface at 20 °C?

I Neglecting the weight of the wire, $F = \sigma L$. Since there is resistance on the inside and outside of the ring, $F = (2)(\sigma)(\pi d) = (2)(0.0728)[(\pi)(0.06)] = 0.0274$ N.

1.119 The glass tube in Fig. 1-28 is used to measure pressure p_1 in the water tank. The tube diameter is 1 mm and the water is at 30 °C. After correcting for surface tension, what is the true water height in the tube?

I

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.0712)(\cos 0°)}{(996)(9.81)[(\tfrac{1}{2})/1000]} = 0.029 \text{ m} \quad \text{or} \quad 2.9 \text{ cm}$$

True water height in the tube = 17 − 2.9 = 14.1 cm.

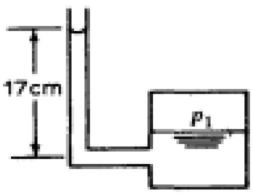


Fig. 1-28

1.123 Find the angle the surface tension film leaves the glass for a vertical tube immersed in water if the diameter is 0.25 in and the capillary rise is 0.08 in. Use $\sigma = 0.005$ lb/ft.

I

$$h = \frac{2\sigma \cos\theta}{\rho g r} \qquad \frac{0.08}{12} = \frac{(2)(0.005)(\cos\theta)}{(1.94)(32.2)[(0.25/2)/12]} \qquad \cos\theta = 0.433806 \qquad \theta = 64.3°$$

Conversion Factors

$0.00001667\ m^3/s = 1\ L/min$
$0.002228\ ft^3/s = 1\ gal/min$
$0.0145\ lb/in^2 = 1\ mbar$
0.3048 m = 1 ft
2.54 cm = 1 in
3.281 ft = 1 m
4 qt = 1 gal
4.184 kJ = 1 kcal
4.448 N = 1 lb
$6.894\ kN/m^2 = 1\ lb/in^2$
$7.48\ gal = 1\ ft^3$
12 in = 1 ft
14.59 kg = 1 slug
25.4 mm = 1 in
60 min = 1 h
60 s = 1 min
100 cm = 1 m
100 kPa = 1 bar
101.3 kPa = 1 atm
$144\ in^2 = 1\ ft^2$
550 ft-lb/s = 1 hp
778 ft-lb = 1 Btu
1000 N = 1 kN
$1000\ L = 1\ m^3$
1000 mm = 1 m
1000 Pa = 1 kPa
$1728\ in^3 = 1\ ft^3$
2000 lb = 1 ton
3600 s = 1 h
4184 J = 1 kcal
5280 ft = 1 mile
86 400 s = 1 day
1 000 000 N = 1 MN
1 000 000 Pa = 1 MPa
1 000 000 000 N = 1 GN
1 000 000 000 Pa = 1 GPa

A differential manometer is shown in Fig. 2-33. Calculate the pressure difference between points A and B.

$$p_A + [(0.92)(62.4)][(x+12)/12] - [(13.6)(62.4)](\tfrac{12}{12}) - [(0.92)(62.4)][(x+24)/12] = p_B$$

$$p_A - p_B = 906\ \text{lb/ft}^2$$

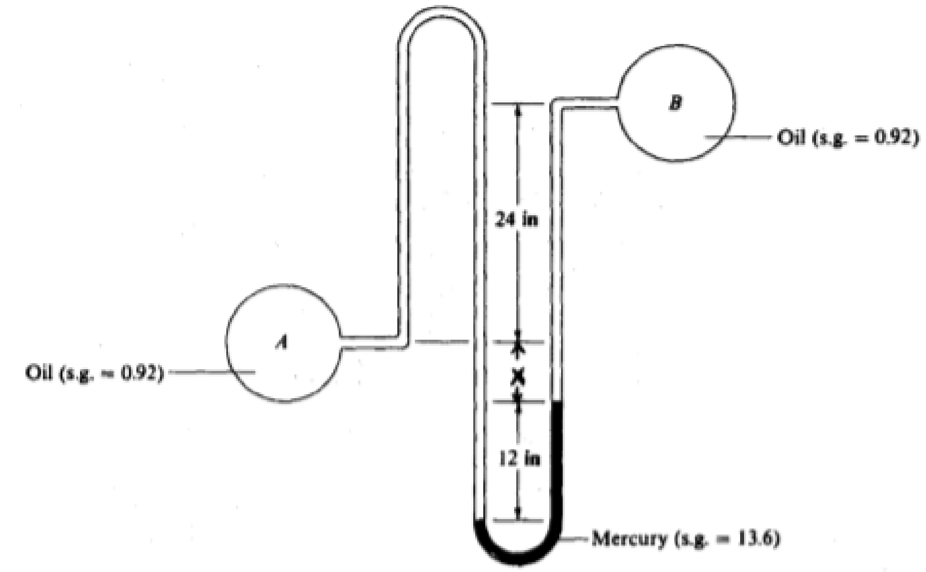


Fig. 2-33

Calculate the pressure difference between A and B for the setup shown in Fig. 2-42.

$$p_A + (62.4)(66.6/12) - [(13.6)(62.4)](40.3/12) + (62.4)(22.2/12) - [(13.6)(62.4)](30.0/12) - (62.4)(10.0/12) = p_B$$

$$p_A - p_B = 4562\ \text{lb/ft}^2 \quad \text{or} \quad 31.7\ \text{lb/in}^2$$

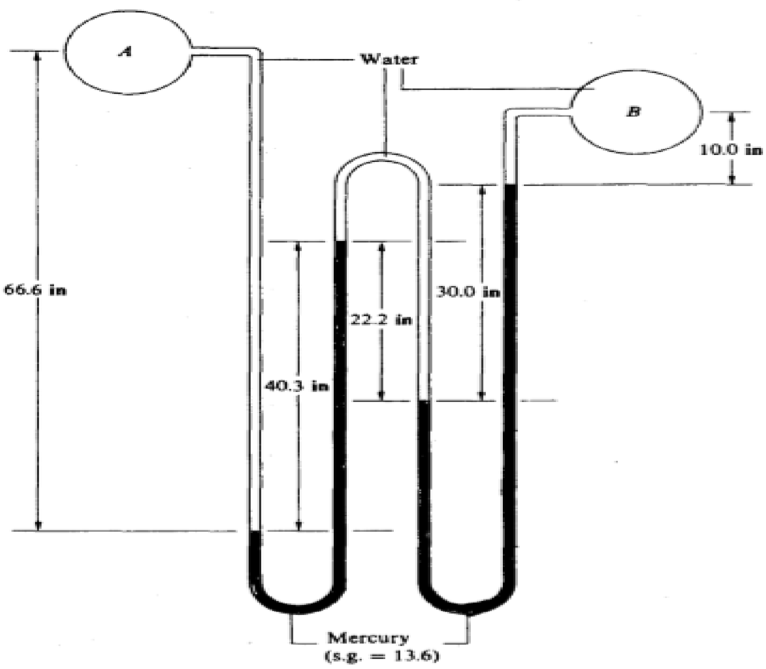


Fig. 2-42

Gate AB in Fig. 3-15 is 1.0 m long and 0.9 m wide. Calculate force F on the gate and the position X of its center of pressure.

$$F = \gamma h_{cg} A = [(0.81)(9.79)][3 + (1 + 1.0/2)(\sin 50^\circ)][(0.9)(1.0)] = 29.61\ \text{kN}$$

$$y_{cp} = \frac{-I_{xx} \sin \theta}{h_{cg} A} = \frac{-[(0.9)(1.0)^3/12](\sin 50^\circ)}{[3 + (1 + 1.0/2)(\sin 50^\circ)][(0.9)(1.0)]}$$

$$= -0.015\ \text{m from the centroid}$$

$$X = 1.0/2 + 0.015 = 0.515\ \text{m from point } A$$

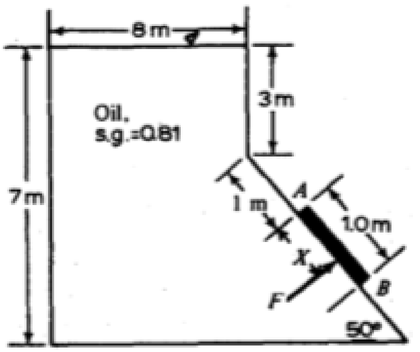


Fig. 3-15

An inclined, circular gate with water on one side is shown in Fig. 3-10. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)[1.5 + \tfrac{1}{2}(1.0 \sin 60^\circ)][\pi(1.0)^2/4] = 14.86\ \text{kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg} A} = \left[\frac{1.5}{\sin 60^\circ} + \frac{1}{2}(1.0)\right] + \frac{\pi(1.0)^4/64}{[1.5/\sin 60^\circ + \frac{1}{2}(1.0)][\pi(1.0)^2/4]} = 2.260\ \text{m}$$

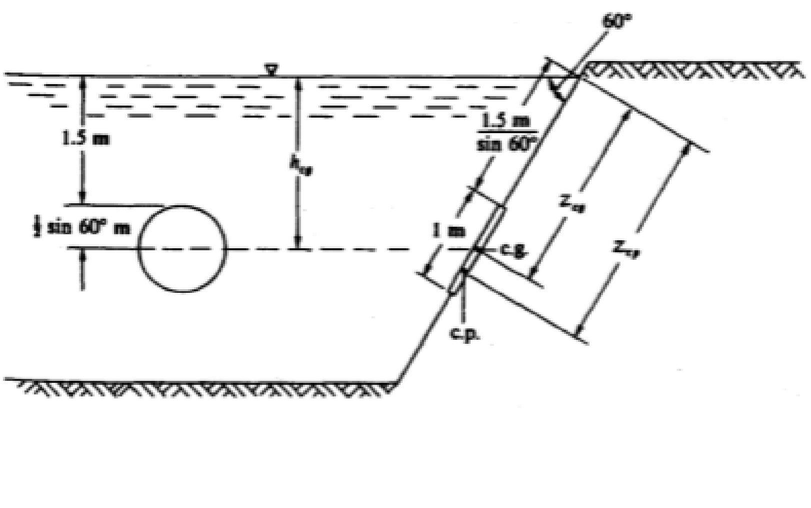
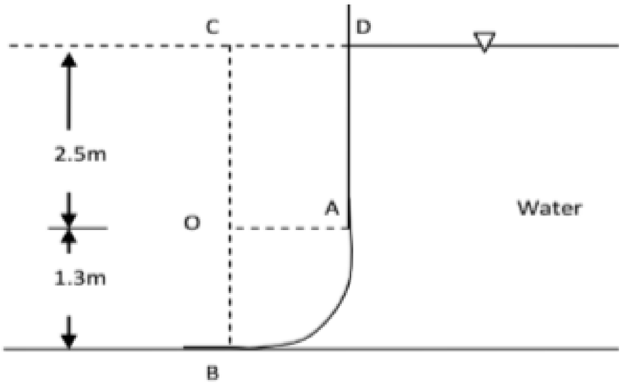


Fig. 3-10

Example 1: The water is on the right side of the curved surface AB, which is one quarter of a circle of radius 1.3m. The tank's length is 2.1m. Find the horizontal and vertical component of the hydrostatic acting on the curved surface.



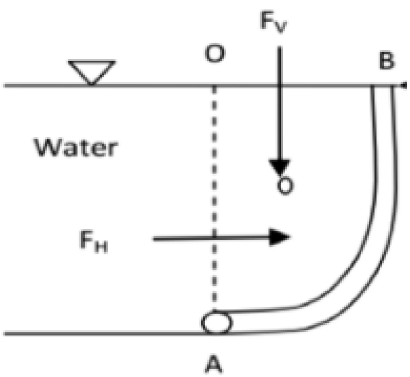
Solution:

Horizontal force, $F_H = \gamma . A . h_c \rightarrow F_H = 1000 * 9.81 * \left(2.5 + \frac{1.3}{2}\right) 1.3 * 2.1 = 84.4\ kN$

Vertical force, $F_V = \gamma . volume$

$$F_V = 1000 * 9.81 * \left(\frac{\pi}{4} * 1.3^2 * 2.1 + 1.3 * 2.1 * 2.5\right) = 94.297 kN$$

Example 2: The gate AB shown is hinged at A and is in the form of quarter-circle wall of radius 12m. If the width of the gate is 30m, calculate the horizontal and vertical force.



Solution:

Horizontal force, $F_H = \gamma . A . h_c \rightarrow F_H = 1000 * 9.81 * \left(\frac{12}{2}\right) 12 * 30 = 21189.6\ kN$

Vertical force, $F_V = \gamma . volume$

$$F_V = 1000 * 9.81 * \left(\frac{\pi}{4} * 12^2 * 30\right) = 33284.546 kN$$

A vertical, triangular gate with water on one side is shown in Fig. 3-8. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (62.4)(6 + 3/3)[(2)(3)/2] = 1310 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(6 + \frac{3}{3}\right) + \frac{(2)(3)^3/36}{(6+3/3)[(2)(3)/2]} = 7.07 \text{ ft}$$

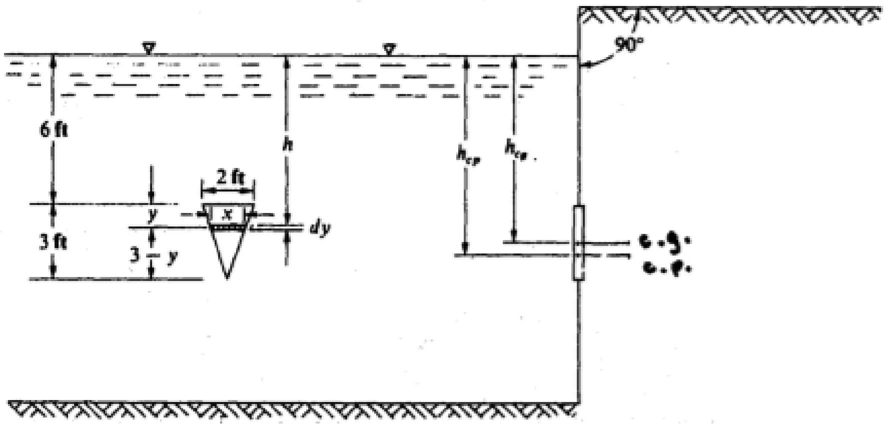


Fig. 3-8

A fishpond gate 6 ft wide and 9 ft high is hinged at the top and held closed by water pressure as shown in Fig. 3-16. What horizontal force applied at the bottom of the gate is required to open it?

$$F = \gamma h_{cg} A = (62.4)(8 + 4.5)[(6)(9)] = 42\,120 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (8 + 4.5) + \frac{(6)(9)^3/12}{(8+4.5)[(6)(9)]} = 13.04 \text{ ft}$$

$$\sum M_A = 0 \qquad (P)(9) - (42\,120)(13.04 - 8) = 0 \qquad P = 23\,587 \text{ lb}$$

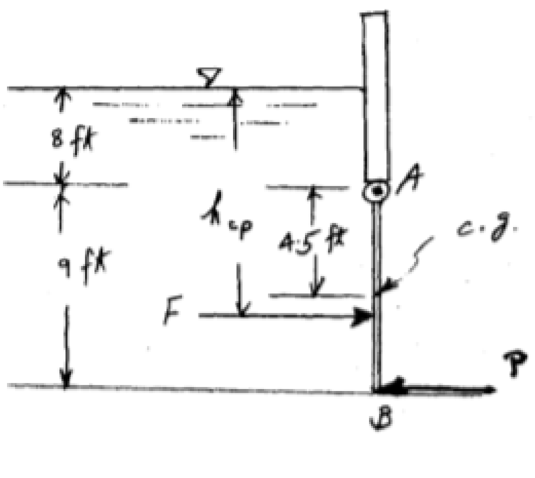


Fig. 3-16

4.17

4.17 The velocity of air in the diverging pipe shown in Fig. P4.17 is given by $V_1 = 4t$ ft/s and $V_2 = 2t$ ft/s, where t is in seconds. **(a)** Determine the local acceleration at points (1) and (2). **(b)** Is the average convective acceleration between these two points negative, zero, or positive? Explain.

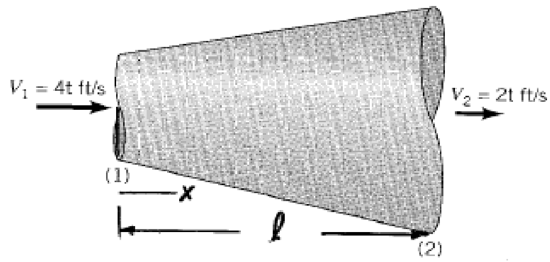


FIGURE P4.17

a) $\left.\frac{\partial u}{\partial t}\right|_{(1)} = \underline{\underline{4\,\frac{ft}{s^2}}}$ and $\left.\frac{\partial u}{\partial t}\right|_{(2)} = \underline{\underline{2\,\frac{ft}{s^2}}}$

b) convective acceleration along the pipe $= u\frac{\partial u}{\partial x}$ where $u > 0$. At any time, t, $V_2 < V_1$. Thus, between (1) and (2)

$$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} < 0$$

Hence, $u\frac{\partial u}{\partial x} < 0$ or the average convective acceleration is $\underline{\underline{\text{negative}}}$.

4.21 The fluid velocity along the x axis shown in Fig. P4.21 changes from 6 m/s at point A to 18 m/s at point B. It is also known that the velocity is a linear function of distance along the streamline. Determine the acceleration at points A, B, and C. Assume steady flow.

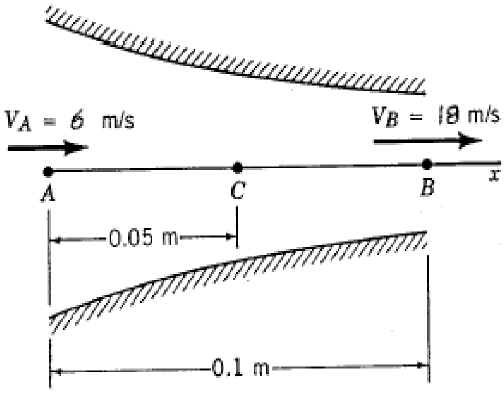


FIGURE P4.21

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x)$, $v = 0$, and $w = 0$

this becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = u\frac{\partial u}{\partial x}\hat{i} \qquad (1)$$

Since u is a linear function of x, $u = c_1 x + c_2$ where the constants c_1, c_2 are given as: $u_A = 6 = c_2$

and $u_B = 18 = 0.1c_1 + c_2$

or $c_1 = 120$, $c_2 = 6.$

Thus, $u = (120x + 6)\frac{m}{s}$ with $x \sim m$

From Eq. (1)

$$\vec{a} = u\frac{\partial u}{\partial x}\hat{i} = (120x + 6)\frac{m}{s}\left(120\frac{m}{m\,s}\right)\hat{i}$$

or

for $x_A = 0$, $\vec{a}_A = \underline{\underline{720\,\hat{i}\,\frac{m}{s^2}}}$

for $x_B = 0.05\,m$, $\vec{a}_B = \underline{\underline{1440\,\hat{i}\,\frac{m}{s^2}}}$

and

for $x_C = 0.1\,m$, $\vec{a}_C = \underline{\underline{2160\,\hat{i}\,\frac{m}{s^2}}}$

4.27 A nozzle is designed to accelerate the fluid from V_1 to V_2 in a linear fashion. That is, $V = ax + b$, where a and b are constants. If the flow is constant with $V_1 = 10$ m/s at $x_1 = 0$ and $V_2 = 25$ m/s at $x_2 = 1$ m, determine the local acceleration, the convective acceleration, and the acceleration of the fluid at points (1) and (2).

With $u = ax + b$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$$\vec{a} = a_x \hat{i} \quad \text{where} \quad a_x = u\frac{\partial u}{\partial x}. \qquad (1)$$

Since $u = V_1 = 10\,\frac{m}{s}$ at $x = 0$ and $u = V_2 = 25\,\frac{m}{s}$ at $x = 1$ we obtain

$$10 = 0 + b$$

$$25 = a + b \quad \text{so that} \quad a = 15 \text{ and } b = 10$$

That is, $u = (15x + 10)\,\frac{m}{s}$, where $x \sim m$, so that from Eq.(1)

$$a_x = (15x+10)\frac{m}{s}\left(15\,\frac{1}{s}\right) = \underline{\underline{(225x+150)\,\frac{m}{s^2}}}$$

Note: The local acceleration is zero, $\underline{\underline{\frac{\partial \vec{V}}{\partial t} = 0}}$, and the convective acceleration is $u\frac{\partial u}{\partial x}\hat{i} = \underline{\underline{(225x+150)\,\hat{i}\,\frac{m}{s^2}}}$

At $x = 0$, $\vec{a} = \underline{\underline{150\,\hat{i}\,\frac{m}{s^2}}}$; at $x = 1\,m$, $\vec{a} = \underline{\underline{375\,\hat{i}\,\frac{m}{s^2}}}$

4.29

4.29 Repeat Problem 4.27 with the assumption that the flow is not steady, but at the time when $V_1 = 10$ m/s and $V_2 = 25$ m/s, it is known that $\partial V_1/\partial t = 20\ \text{m/s}^2$ and $\partial V_2/\partial t = 60\ \text{m/s}^2$.

With $u = u(x,t)$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$\vec{a} = a_x\,\hat{i}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$, with $u = a(t)x + b(t)$. (1)

At the given time $(t = t_o)$ $u = V_1 = 10\frac{m}{s}$ at $x = 0$ and $u = V_2 = 25\frac{m}{s}$ at $x = 1m$

Thus, $10 = 0 + b(t_o)$

$25 = a(t_o) + b(t_o)$ so that $a(t_o) = 15$ and $b(t_o) = 10$

Also at $t = t_o$, $\frac{\partial u}{\partial t} = \frac{\partial V_1}{\partial t} = \underline{\underline{20\ \frac{m}{s^2}}}$ at $x = 0$

and $\frac{\partial u}{\partial t} = \frac{\partial V_2}{\partial t} = \underline{\underline{60\ \frac{m}{s^2}}}$ at $x = 1m$ Note: These are local accelerations at time $t = t_o$

The convective acceleration at $x = 0$ (Eq. (1)) is

$$u\frac{\partial u}{\partial x} = (ax + b)(a) = (15(0) + 10)\frac{m}{s}(15\ \frac{1}{s}) = \underline{\underline{150\ \frac{m}{s^2}}}$$

while at $x = 1$ it is

$$u\frac{\partial u}{\partial x} = (15(1) + 10)\frac{m}{s}(15\ \frac{1}{s}) = \underline{\underline{375\ \frac{m}{s^2}}}$$

The fluid acceleration at $t = t_o$ is

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = (20 + 150)\hat{i}\ \frac{m}{s^2} = \underline{\underline{170\,\hat{i}\ \frac{m}{s^2}\ \text{at}\ x = 0}}$$

and

$$\vec{a} = (60 + 375)\hat{i}\ \frac{m}{s^2} = \underline{\underline{435\,\hat{i}\ \frac{m}{s^2}\ \text{at}\ x = 1m}}$$

6.10

6.10 Some velocity measurements in a three-dimensional incompressible flow field indicate that $u = 6xy^2$ and $v = -4y^2z$. There is some conflicting data for the velocity component in the z direction. One set of data indicates that $w = 4yz^2$ and the other set indicates that $w = 4yz^2 - 6y^2z$. Which set do you think is correct? Explain.

To satisfy the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

Since $\frac{\partial u}{\partial x} = 6y^2$ and $\frac{\partial v}{\partial y} = -8yz$

then from Eq. (1)

$$6y^2 - 8yz + \frac{\partial w}{\partial z} = 0$$

Thus,

$$\frac{\partial w}{\partial z} = 8yz - 6y^2 \qquad (2)$$

Equation (2) can be integrated with respect to z to obtain

$$\int dw = \int 8yz\,dz - \int 6y^2 dz + f(x,y)$$

or

$$w = 4yz^2 - 6y^2z + f(x,y)$$

The set of data (with $f(x,y) = 0$)

$$\underline{\underline{w = 4yz^2 - 6y^2z}}$$

would appear to be the correct set.

6.27

6.27 In a certain two-dimensional flow field the velocity is constant with components $u = -4$ ft/s and $v = -2$ ft/s. Determine the corresponding stream function and velocity potential for this flow field. Sketch the equipotential line $\phi = 0$ which passes through the origin of the coordinate system.

From the definition of the stream function

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

so that for the velocity components given

$$\frac{\partial \psi}{\partial y} = -4 \qquad (1)$$

$$\frac{\partial \psi}{\partial x} = 2 \qquad (2)$$

Integrate Eq.(1) with respect to y to obtain

$$\int d\psi = \int -4\, dy + f_1(x)$$

or

$$\psi = -4y + f_1(x) \qquad (3)$$

Similarly, integrate Eq.(2) with respect to x to obtain

$$\int d\psi = \int 2\, dx + f_2(y)$$

or

$$\psi = 2x + f_2(y) \qquad (4)$$

Thus, to satisfy both Eqs.(3) and (4)

$$\psi = \underline{\underline{2x - 4y + C}}$$

where C is an arbitrary constant.

From the definition of the velocity potential

$$u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y} \qquad \text{(Eqs. 6.64)}$$

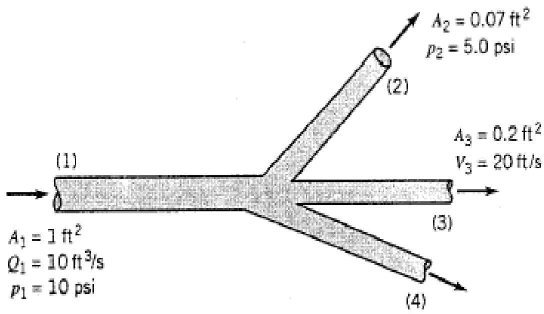
so that for the velocity components given

$$\frac{\partial \phi}{\partial x} = -4 \qquad (5)$$

$$\frac{\partial \phi}{\partial y} = -2 \qquad (6)$$

3.82

3.82 Water flows through the horizontal branching pipe shown in Fig. P3.82 at a rate of 10 ft^3/s. If viscous effects are negligible, determine the water speed at section (2), the pressure at section (3), and the flowrate at section (4).



■ FIGURE P3.82

From (1) to (2): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $z_1 = z_2$, $p_1 = 10\,psi$, $p_2 = 5\,psi$, and $V_1 = \frac{Q_1}{A_1}$ or

$V_1 = (10\,\frac{ft^3}{s})/(1\,ft^2) = 10\,\frac{ft}{s}$

Thus, with $\gamma = \rho g$

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{(10\,\frac{ft}{s})^2}{2} = \frac{(5\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{V_2^2}{2} \text{ or } \underline{\underline{V_2 = 29.0\,\frac{ft}{s}}}$$

From (1) to (3): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ where $z_1 = z_3$, $p_1 = 10\,psi$, $V_1 = 10\,\frac{ft}{s}$ and $V_3 = 20\,\frac{ft}{s}$

Thus,

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + \frac{(10\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = \frac{p_3}{62.4\,\frac{lb}{ft^3}} + \frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$p_3 = 1150\,\frac{lb}{ft^2} = \underline{\underline{7.98\,psi}}$$

Also,

$$Q_4 = Q_1 - Q_2 - Q_3 = Q_1 - A_2V_2 - A_3V_3$$

or

$$Q_4 = 10\,\frac{ft^3}{s} - 0.07\,ft^2(29.0\,\frac{ft}{s}) - 0.2\,ft^2(20\,\frac{ft}{s}) = \underline{\underline{3.97\,\frac{ft^3}{s}}}$$

3.10R

3.10R (Bernoulli/continuity) Water flows steadily through a diverging tube as shown in Fig. P3.10R. Determine the velocity, V, at the exit of the tube if frictional effects are negligible.

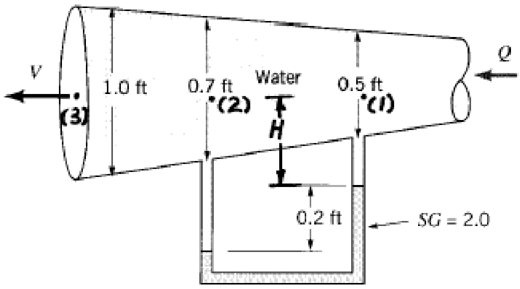


FIGURE P3.10R

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (z_1 = z_2) \qquad (1)$$

where $A_1 V_1 = A_2 V_2$, or

$$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{0.7\,m}{0.5\,m}\right)^2 V_2 = 1.96\, V_2 \qquad (2)$$

Also, $p_2 + \gamma(H + 0.2\text{ft}) - SG\,\gamma\,(0.2\text{ft}) - \gamma H = p_1$

or

$$p_2 = p_1 + 2\gamma(0.2) - \gamma(0.2) = p_1 + 62.4\frac{lb}{ft^3}\left[2-1\right](0.2\,ft)$$

$$= p_1 + 12.48\,\frac{lb}{ft^2} \qquad (3)$$

By combining (1), (2), and (3):

$$\frac{p_1}{\gamma} + \frac{(1.96\,V_2)^2\,ft^2/s^2}{2(32.2\,ft/s^2)} = \frac{p_1}{\gamma} + \frac{12.48\,lb/ft^2}{62.4\,lb/ft^3} + \frac{V_2^{\,2}\,ft^2/s^2}{2(32.2\,ft/s^2)}$$

or $V_2 = 2.13\ ft/s$ and $V_1 = 1.96(2.13\,ft/s) = 4.17\,ft/s$

Thus, since $V_3 A_3 = V_2 A_2$, then

$$V_3 = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{0.7\,ft}{1\,ft}\right)^2 (2.13\,ft/s) = \underline{\underline{1.04\,ft/s}}$$