Chapter 2

Number systems

Data Representation

- Data in computers is represented in binary form. The represented data can be number, text, movie, colour (picture), sound, or anything else.
- Computers understand and respond to only the flow of electrical charge.
- Data representation using the binary number system results in a large string of 0s and 1s. This makes the represented data large and difficult to read.

two Basic types of number systems.

1.Non-positional number system

- The value of a symbol (digit) in a number does not depend on the position of the digit in number.

2. Positional number system

- The value of a symbol in the number is determined by its position, the symbol and the base of the number system.

Cont…

- **Decimal number system**
  - called the base 10 number system
  - It has 10 different symbols identified as 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
- **Binary number system**
  - known as base 2 number system,
  - has two digits 0 and 1.

Cont…

- **Octal number system**
  - also called base 8 number system,
  - has 8 different symbols: 0, 1, 2, 3, 4, 5, 6, and 7.
- **Hexadecimal number system**
  - also called base 16 number system,
  - has 16 different symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.
  - It is used to write binary numbers in short form.

Converting from one base to another

1. Conversion from Decimal to Base m

Step 1: Divide the given decimal number by m (the desired base).

The result will have a quotient and a remainder.

Step 2: Repeat step 1 until the quotient becomes 0, the quotient is 0 whenever the number < m.

Step 3: Collect and arrange the remainders in such a way that the first remainder is the least significant digit and the last remainder is the most significant digit.

Cont…

Eg: Convert the decimal number 47 into binary, octal, and hexadecimal.

a. Conversion to binary divide by 2.

	Quotient	Remainder
47 ÷ 2	23	1
23 ÷ 2	11	1
11 ÷ 2	5	1
5 ÷ 2	2	1
2 ÷ 2	1	0
1 ÷ 2	0	1

Cont…

Hence the result is 101111_2.

b. Conversion to octal: Here the numbers are divided by 8

	Quotient	Remainder
$47 \div 8$	5	7
$5 \div 8$	0	5

Cont…

Therefore, 47 = 57_8

c. Conversion to hexadecimal

numbers are divided by 16.

	Quotient	Remainder
47 ÷ 16	2	15
2 ÷ 16	0	2

The hexadecimal equivalent for the decimal 15 is F and that of 2 is2.

Therefore, 47 = $2F_{16}$

Cont…

2. Conversion from Base m to Decimal

Step 1: Multiply each digit by its positional value.

Step 2: Calculate the sum of the products you get in step 1.

Example 1: Convert the binary number 110001 into decimal.

$110001_2 = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = (1 \times 32) + (1 \times 16) + (0 \times 8) + (0 \times 4) + (0 \times 2) + (1 \times 1) = 32 + 16 + 0 + 0 + 0 + 1 = 49$

- Therefore, $110001_2 = 49$.

Cont…

Example 2: Convert the octal number 22 into decimal.

$$22_8 = (2 \times 8^1) + (2 \times 8^0) =$$

$$(2 \times 8) + (2 \times 1) = 16 + 2 \quad = 18$$

Therefore, $22_8 = 18$

Example 3: Convert the hexadecimal number D1 into decimal.

$$D1_{16} = (13 \times 16^1) + (1 \times 16^0)$$

Therefore, $D1_{16} = 209$

Cont…

3. Conversion from Binary to Octal

- It is possible to use decimal number system as an intermediate base to convert from any base to any other base. However, for conversion from binary to octal or vice versa, there is a very simple method.

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Cont…

Step 1: Group the binary digits (bits) starting from the rightmost dig into 3 bits.

If the remaining bits at the leftmost position are fewer than 3, add 0s at the front.

Step 2: For each 3-bit binary string, find the corresponding octal number

Example: Convert the binary numbers 110011 and 1101111 to octal.

Cont…

- The bits are grouped in three with the equivalent octal digit given below the three bit group.

  Thus, $110011_2 = 63_8$

  The result is $1101111_2 = 157_8$.

4. **Conversion from Octal to Binary**

Step 1: For each octal digit, find the equivalent three digit binary number.

Step 2: If there are leading 0s for the binary equivalent of the leftmost octal digit, remove them.

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Example: Find the binary equivalent for the octal numbers 73 and 160.

A. 73

7	3
111	011

B. 160

1	6	0
001	110	000

Cont…

- Therefore, $73_8 = 111011_2$ and
- Thus, 1608 = 11100002

5. From Binary to Hexadecimal

- One possible way to convert a binary number to hexadecimal, is first to convert the binary number to decimal and then from decimal to hex.
- The simple steps states are stated below.

Cont…

Step 1: Starting from the rightmost bit, group the bits in 4. If the remaining bits at the leftmost position are fewer than 4, add 0s at the front.

Step 2: For each 4-bit group, find the corresponding hexadecimal number.

Example: Convert the binary numbers 10011110 to hexadecimal.

10011110

$$\underbrace{1001}_{9}\ \underbrace{1110}_{E}$$

Cont…

Therefore, $10011110_2 = 9E_{16}$

6. Conversion from Hexadecimal to Binary

Step 1: For each hexadecimal digit, find the equivalent four digit binary number.

Step 2: If there are leading 0s for the binary equivalent of the leftmost hexadecimal digit, remove them.

Cont…

- **Example:** Find the binary equivalents for the hexadecimal numbers 1C and 823

- Thus, $1C_{16} = 11100_2$ and $823_{16} = 100000100011_2$

B. 823

8	2	3
1000	0010	0011

Cont…

7. Conversion from Octal to Hexadecimal of Vice Versa

- The decimal number system can be used as an intermediate conversion base.
- second alternative is using the binary number system as an intermediate base.

Step 1: Convert the given number into binary.

Step 2: Convert the binary number you got in step 1 into the required base.

Cont…

- **Example :** Convert the octal number 647 to hexadecimal.

Convert 647_8 to binary

6	4	7
110	100	111

Convert 110100111_2 to hexadecimal

0001	1010	0111
1	A	7

Therefore, $647_8 = 1A7_{16}$

Hence, $823_{16} = 100000100011_2$.

Cont...

8. Conversion from Octal to Hexadecimal or Vice Versa

Example 2: Find the octal equivalent for the hexadecimal number 3D5

Convert 3D516 to binary

3	D	5
0011	1101	0101

Convert 1111010101_2 to octal

001	111	010	101
1	7	2	5

Therefore, $3D5_{16} = 1725_8$

Representation of Integers(Fixed point Representation)

- If the numbers we want to represent are only positive (unsigned) integers,
- simply represent the unsigned integer with its binary value.
- Signed integer representations are sign magnitude, 1's complement and 2's complement.

Sign-Magnitude Representation

- treat the most significant bit as a sign bit the remaining bits are used to represent the magnitude of the integer.
- a 0 on the sign bit indicates the integer is positive and a 1 indicates the integer is a negative.
- As example, the sign-magnitude representation of 79 and -79 in 8 bits are 01001111 and 11001111 respectively.

One's Complement Integer Representation

- Every number system has two complement systems. For a given base n the complements are n's complement and (n-1)'s complement.
- The one's complement of a binary integer is found by inverting all 0s to 1s and all 1s to 0s. In one's complement integer representation, the negative of an integer is represented by its complement.

Cont…

For example, the one's complement representation of 16 and -16 in 8 bits are 00010000 and 11101111 respectively.

For 8-bits number system:

- Largest Positive Number: 0 1111111= +(127)10
- Largest Negative Number: 1 0000000 =-(127)10

Two's Complement Integer Representation

- The two's complement of an integer is found by adding 1 to its one's complement
- Example,

two's complement representation of 19 and -19 in 8 bits are 00010011 and 11101101 respectively.

Floating Point Numbers

- Floating Point representation of a number has two parts. The first part represents a signed, fixed-point number called the mantissa.
- The second part designates the position of the decimal (or binary) point and is called the exponent. Floating point is always represented as follows:
- $\boldsymbol{m} \times \mathbf{r}^{\mathbf{e}}$ where, m (mantissa) r(radix) and e(exponent)

Codes

Binary Coded Decimal (BCD)

- The BCD (Binary Coded Decimal), also called packed decimal, in order to have representations for the ten digits of the decimal number system, we need a four bit string.
- Notes: BCD is not equivalent to binary

Characters

- Characters refer to a unit of Information that roughly corresponds to a Grapheme, grapheme-like unit, or symbol, such as in an Alphabet in the Written form of a natural language.
- Examples of characters are, letters of the alphabet, the ten digits (0 through 9), punctuation marks Whitespace.

Cont…

- characters that are used to format the layout of text on pages such as the newline, space, and tab characters, and other characters that are useful for communication.
- Characters are typically combined into String.

American Standard Code for Information Interchange (ASCII)

- each character is represented by 7 bits; hence a total of 128 characters are represented.
- The eighth bit is used for parity (error detection).
- ASCII: 7-bit, plus a *parity bit* for error detection (odd/even parity).

EBCDIC (Extended Binary Coded Decimal for Interchange Code)

- Another character encoding system is the EBCDIC.
- It uses 8 bits per character (and a ninth parity bit), thus represents 256 characters.
- As with IRA, EBCDIC is compatible with BCD.
- In the case of EBCDIC, the codes 11110000 through 11111001 represent the digits 0 through 9.

UTF stands for Unicode Transformation Format.

- In order to be compatible with older systems that didn't support Unicode, Encoding Forms were defined by the Unicode Consortium to be a representation of the character in bits.
- The number indicates the encoding form that is to be used: UTF-8 indicates an 8-bit encoding form, while UTF-16 indicates a 16-bit encoding form.