Module
12
Yield Line Analysis for Slabs

Lesson
33
Numerical Examples

Instructional Objectives:

At the end of this lesson, the student should be able to:

- apply the theory of segmental equilibrium for the solution of different types of slab problems,
- apply the theory of virtual work for the solution of different types of slab problems.

12.33.1 Introduction

The theoretical formulations of the yield line analysis of slabs are explained in earlier Lessons 30 to 32, considering (i) the method of segmental equilibrium and (ii) the method of virtual work. Illustrative examples are solved in Lesson 30. This lesson includes different types of illustrative examples of slabs with different combinations of support conditions. Two types of loadings, viz., (i) point loads and (ii) uniformly distributed loads, are considered. In some cases, two or three possible yield patterns are examined to select the correct one and the corresponding loads are determined. The problems include different types of rectangular or square two-way slabs, triangular, quadrantal and circular slabs. Circular slabs also include one practical example where it is supported by a central column in addition to clamped support along the periphery. Some practice problems and test problem are also included. Understanding the numerical problems and solving practice and test problems will give a better understanding of the theories of yield line analysis and their applications.

12.33.2 Illustrative Examples

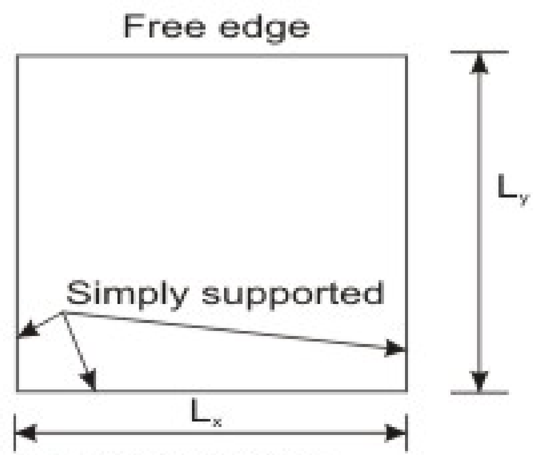

Fig. 12.33.1 (a): Slab

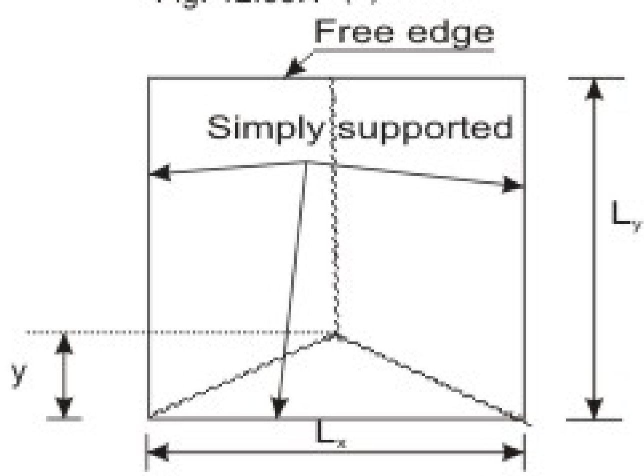

Fig. 12.33.1(b): Yield pattern 1

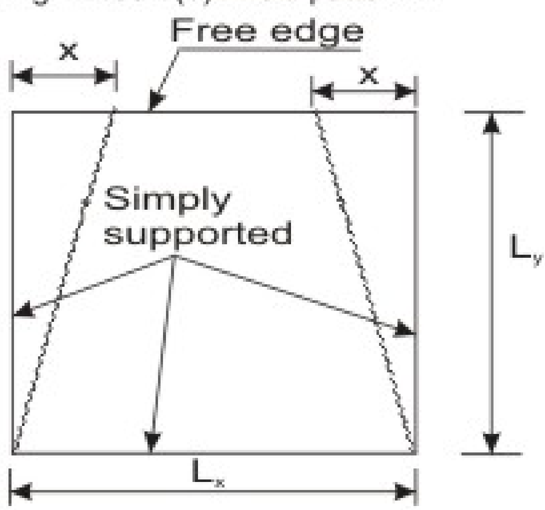

Fig. 12.33.1 ©: Yield pattern 2

Fig. 12.33.1: Problems 1, 2, 3, 4 and TQ.1

Problem 1. Determine the uniformly distributed collapse load w kN/m^2 of a square slab ($L\ m \times L\ m$), simply supported at three edges and the other edge is free. Assume $M_y = 2M_x = 2M$.

Solution 1. The slab is shown in Fig.12.33.1a. Given data are:
$L_x = L_y = L$ and $M_y = 2M_x = 2M$.

Step 1. To examine the possibility of yield patterns 1, 2 or both

Equation12.45 of Lesson 32 stipulates that $(M_y/M_x) \langle\ 4\ (L_y/L_x)^2$ for yield pattern 1 and Eq.12.54 of Lesson 32 gives the condition that $(M_y/M_x) \rangle\ (4/3)\ (L_y/L_x)^2$ for yield pattern 2.

Here, $L_y/L_x = 1$ and $M_y/M_x = 2$. So, $(M_y/M_x) \langle\ 4\ (L_y/L_x)^2$ and $(M_y/M_x) \rangle$ $(4/3)\ (L_y/L_x)^2$. Thus, both the yield patterns are to be examined.

Step 2. Value of *y* for yield pattern 1

The value of y is obtained from Eq.12.43 of Lesson 32, which is

$$4\ M_x L_y y^2 + 2\ M_y L_x^2\ y - 3\ M_y L_x^2\ L_y = 0 \qquad (12.43)$$

or

$$2y^2 + 2L\ y - 3L^2 = 0 \qquad (1)$$

which gives $y = 0.823L$ (2)

Step 3. Determination of collapse load *w* kN/m^2

Equations 12.41 and 12.42 of the method of segmental equilibrium of Lesson 32 and Eq.12.48 of the method of virtual work of Lesson 32 shall be used to determine w. The results are given below.

(i) From Eq. 12.41: $w = (24ML)/\{3L^2\ (L - y) + L^2y\} = 17.722\ (M/L^2)$ (3)

(ii) From Eq. 12.42: $w = 12M/y^2 = 17.722\ (M/L^2)$ (3)

(iii) From Eq. 12.48: $w = \{12\ M\ (2y + L)\}\ /\ \{L(3yL - y^2)\} = 17.722\ (M/L^2)$ (3)

Step 4. Value of *x* for yield pattern 2

The value of x is obtained from Eq. 12.52 of Lesson 32, which is

$$6x^2 + 4Lx - 3L^2 = 0 \quad (4)$$

which gives: $x = 0.448L$ (5)

Step 5. Determination of collapse load w kN/m^2

Equations 12.50 and 12.51 of the method of segmental equilibrium of Lesson 32 and Eq. 12.57 for the method of virtual work of Lesson 32 shall be used to determine w. The results are given below:

(i) From Eq. 12.50: $w = \{6M(L^2 - 2x^2)\} / x^2L^2 = 17.841\ (M/L^2)$ (6)

(ii) From Eq.12.51: $w = 24\ Mx / \{L^2\ (3L\text{-}4x)\} = 17.841\ (M/L^2)$ (6)

(iii) From Eq.12.57: $w = (M/L^2)\ \{12\ (L^2 + 2x^2)\} / \{x\ (3L - 2x)\} = 17.841\ (M/L^2)$

(6)

Step 6: Correct yield pattern and collapse load

Comparison of results of steps 3 and 5 reveals that yield pattern 1 is the correct one giving the lower value of w. For this yield pattern, $y = 0.823\ L$ and $w = 17.722\ (M/L^2)$.

Problem 2: Determine the uniformly distributed collapse load w kN/m^2 of a rectangular slab whose $L_y/L_x = 3/4$, simply supported at three edges and free at the other edge. Assume $M_y = 2M_x$.

Solution 2: The slab is shown in Fig.12.33.1a. Given data are: $L_y/L_x = 3/4$ and $M_y/M_x = 2$.

Step 1: To examine the possibility of yield patterns 1, 2 or both

Equation 12.45 of Lesson 32 stipulates that $(M_y/M_x) \langle\ 4\ (L_y/L_n)^2$ for yield pattern 1 and Eq. 12.54 of Lesson 32 gives the condition that $(M_y/M_x) \rangle\ (4/3)\ (L_y/L_x)^2$ for yield pattern 2.

Here, $L_y/L_x = 3/4$ and $M_y/M_x = 2$ give $(M_y/M_x) \langle\ 4\ (L_y/L_x)^2$ and $(M_y/M_x) \rangle\ (4/3)\ (L_y/L_x)^2$. Thus, both the yield patterns are to be examined.

Step 2: Value of y for yield pattern 1

The value of y is obtained from Eq.12.43 of Lesson 32, which gives:

$$6y^2 + 8L_x - 9L_x^2 = 0 \qquad (7)$$

The solution of Eq. 7 is: $y = 0.728\ L_x$ (8)

Step 3: Determination of collapse load *w* kN/m²

Equations 12.41 and 12.42 of the method of segmental equilibrium of Lesson 32 and Eq. 12.48 of the method of virtual work of Lesson 32 shall be used to determine w. The results are given below:

(i) From Eq. 12.41: $w = 24M_x\ L_y/\{3L_x^2(L_y - y) + L_x^2 y\} = 22.657(M_x/L_x^2)$ (9)

(ii) From Eq. 12.42: $w = 6M_y\ /\ y^2 = 22.657\ (M_x/L_x^2)$ (9)

(iii) From Eq. 12.48: $w = (24M_xL_y\ y + 6M_y\ L_x^2)\ /\ \{L_x^2\ (3y\ L_y - y^2)\}$
$= 22.657\ (M_x/L_x^2)$ (9)

Step 4: Value of *x* for yield pattern 2

The value of x is obtained from Eq. 12.52 of Lesson 32, which is

$$6x^2 + 2.25\ L_x\ x - 1.6875\ L_x^2 = 0 \qquad (10)$$

The solution of the above equation is

$$x = 0.375\ L_x \qquad (11)$$

Step 5: Determination of collapse load *w* kN/m²

Equations 12.50 and 12.51 of the method of segmental equilibrium of Lesson 32 and Eq. 12.57 of the method of virtual work of Lesson 32 shall be used to determine w. The results are:

(i) From Eq. 12.50: $w = 6(M_x\ L_y^2 - M_y\ x^2)\ /(x^2L_y^2) = 21.334\ (M_x/L_x^2)$ (12)

(ii) From Eq. 12.51: $w = 24\ M_y\ x\ /\{L_y^2\ (3L_x - 4x)\} = 21.334\ (M_x/L_x^2)$ (12)

(iii) From Eq. 12.57: $w = 12\{M_x\ L_y^2 + M_y x^2\}/\{L_y^2\ x(3L_x - 2x)\}$
$= 21.334\ (M_x/L_x^2)$ (12)

Step 6: Correct yield pattern and collapse load

Comparison of results of steps 3 and 5 reveals that yield pattern 2 is the correct one for which w is lower than that of yield pattern 1. For yield pattern 2, $x = 0.375\ L_x$ and $w = 21.334\ (M_x\ /\ L_x^2)$.

Problem 3: Determine the uniformly distributed collapse load w kN/m^2 of a square slab (L m $\times$ L m), simply supported at three edges and free at the other edge. Assume $M_y = M_x = M$.

Solution 3: The slab is shown in Fig. 12.33.1a. Given data are:

$$L_x = L_y = L \text{ and } \quad M_y = M_x = M.$$

Step 1. To examine the possibility of yield patterns 1, 2 or both

From Eq. 12.45 of Lesson 32, it is seen that $(M_y/M_x) < 4\,(L_y/L_n)^2$. So, yield pattern 1 has to be considered. But, (M_y/M_x) is not greater than $(4/3)\,(L_y/L_x)^2$ (vide Eq. 12.54 of Lesson 32). So, yield pattern 2 is not to be considered.

Step 2. Value of *y* for yield pattern 1

Equation 12.43 of Lesson 32 gives: $4y^2 + 2Ly - 3L^2 = 0$ (13)

The solution of Eq.13 is: $y = 0.651\,L$ (14)

Step 3. Collapse load *w* kN/m^2

(i) From Eq.12.41 of Lesson 32: $w = (M/L^2)\{(24)/(3 - 2y)\} = 14.141(M/L^2)$ (15)

(ii) From Eq. 12.42 of Lesson 32: $w = 6M/y^2 = 14.141\,(M/L^2)$ (15)

(iii) From Eq. 12.48 of Lesson 32: $w = (M/L^2)\,\{(24y + 6)/\{3y - y^2)\} = 14.141(M/L^2)$ (15)

Problem 4: Determine the uniformly distributed collapse load w kN/m^2 of a rectangular slab whose $L_y/L_x = 0.4$, simply supported at three edges and free at the other edge. Assume $M_y/M_x = 1$.

Solution 4: The slab is shown in Fig.12.33.1a. Given data are: $L_y/L_x = 0.4$ and $M_y/M_x = 1$.

Step 1: To examine the possibility of yield patterns 1, 2 or both

Here, (M_y/M_x) is not less than 4 $(L_y/L_x)^2$. So, yield pattern 1 is not to be considered. However, (M_y/M_x) is 〉 (4/3) $(L_y/L_x)^2$. So, yield pattern 2 has to be considered (vide Eqs. 12.45 and 12.54 of Lesson 32).

Step 2: Value of *x* for yield pattern 2

Equation 12.52 (Lesson 32) gives: $3x^2 + 0.64\,L_x^2 - 0.48\,L_x^2 = 0$ (16)

The solution of Eq. 16 is: $x = 0.3073\,L_x$ (17)

Step 3: Collapse load *w* kN/m^2

(i) Eq.12.50 gives: $w = (M_x/L_x^2)\,\{6(0.16-x^2\}/0.16x^2\} = 26.032\,(M_x/L_x^2)$ (18)

(ii) Eq.12.51 gives: $w = (M_x/L_x^2)\{24x\,/\,0.16(3-4x)\}= 26.032\,(M_x\,/\,L_x^2)$ (18)

(iii) Eq. 12.57 gives: $w = (M_x/L_x^2)\{12\,(0.16+x^2)/0.16(3x - 2x^2)\}$

$= 26.032\,(M_x\,/\,L_x^2)$ (18)

(Equations 12.50, 12.51 and 12.57 are from Lesson 32.)

Problem 5: Determine the correct yield pattern and the collapse point load P of the isosceles triangular slab of Q.4b of sec.12.31.7 of Lesson 31 (Fig.12.31.10), if the load is applied at the centroid of the slab. The slab is also shown in Fig. 12.33.2a having B = 6 m and 2L = 12 m. Assume M_p = 9 kNm/m and M_n = 12 kNm/m. Use the method of virtual work.

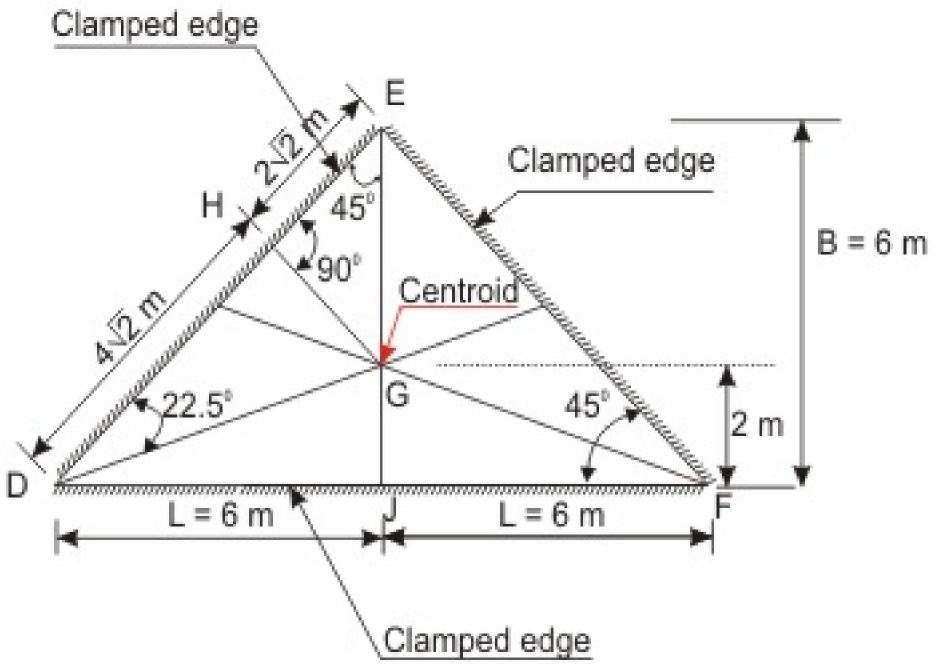

Fig. 12.33.2(a): Problem 5

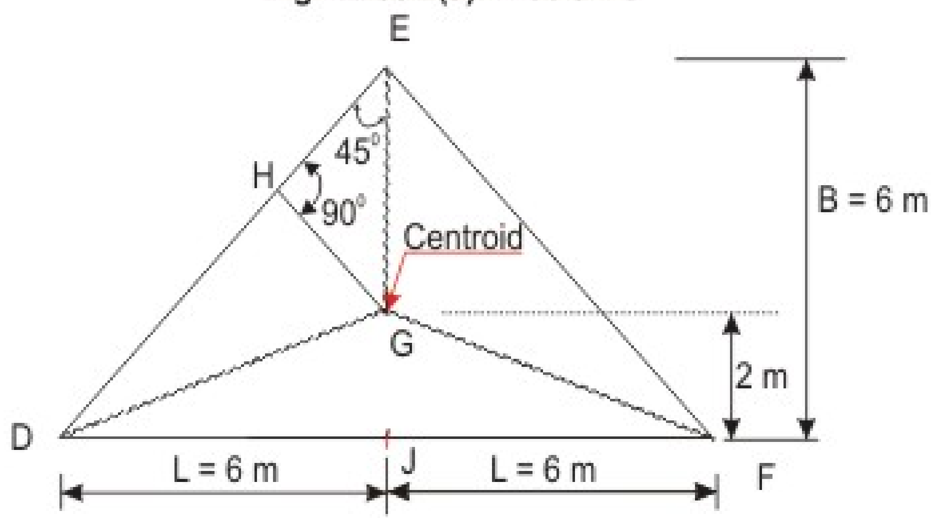

Fig. 12.33.2(b): Yield pattern 1

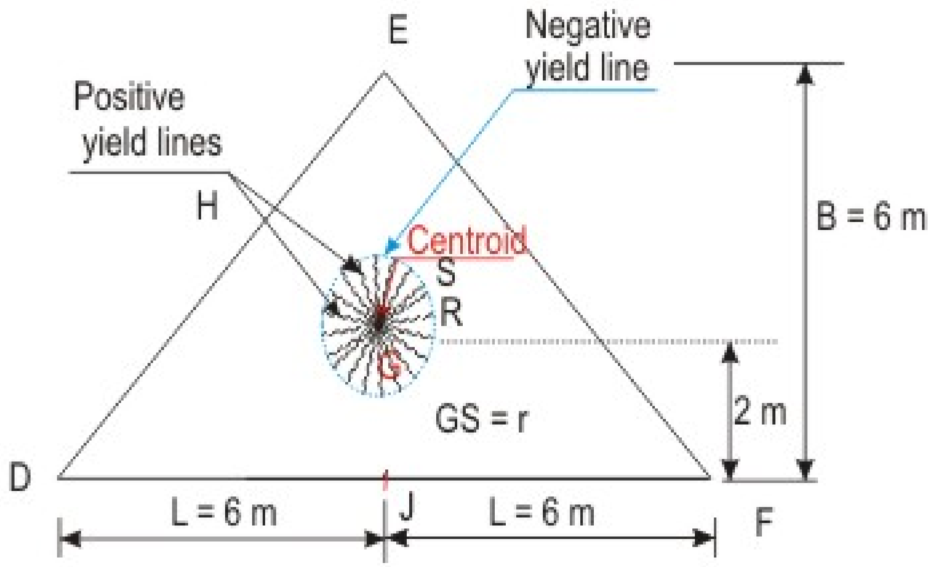

Fig. 12.33.2(c): Yield pattern 2

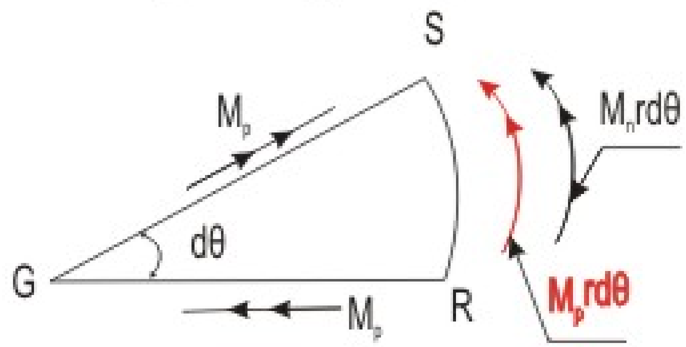

Fig. 12.33.2(d): Free body diagram of segment GSR

Fig. 12.33.2: Problem 5

Solution 5: The slab is shown in Fig. 12.32.2a. Given data are: $2L$=12 m, B = 6 m, M_p = 9 kNm/m and M_n = 12 kNm/m. In this problem, two possible yield patterns, as shown in Figs.12.33.2b and c, are to be considered. The lower of the two loads shall be taken, as the correct collapse load and the corresponding yield pattern is the correct one.

Step 1: **Yield pattern 1**

Yield pattern 1, shown in Fig. 12.33.2b, divides the slab into three segments. Assuming the deflection of the slab at the centroid G = Δ, the rotation θ_1 of segment DFG = $\theta_1 = \Delta/2$ (as the distance GJ = 2 m). The length of the side DE = $6\sqrt{2}$ m and the perpendicular distance from G to ED is GH = EG sin 45^0 = $2\sqrt{2}$ m. The rotation of the segment DEG = rotation of the segment FEG = Δ/GH = $\Delta/2\sqrt{2}$.

Step 2: *TIW* and *TEW* due to moments and loads

$TIW = (M_n + M_p)$ {DF (θ_1) + 2DE (θ_2)} = $21\{12(\Delta/2) + 2(6\sqrt{2})\,(\Delta/2\sqrt{2})\}$ = $252\,\Delta$

So, $TIW = 252\,\Delta$ (19)

$TIW = P\,\Delta$ (20)

Step 3: Collapse load *P*

Equating *TIW* and *TEW* from Eqs. 19 and 20, we have, P = 252 kN (21)

Thus, the collapse load for the yield pattern 1 is 252 kN.

Step 4: Yield pattern 2

Yield pattern 2 divides the segment into a large number of sub-segments as shown in Fig. 12.33.2c. The free body diagram of a typical segment GSR is shown in Fig. 12.33.2d. The internal work done by moments M_p and M_n of the segment GSR is given below.

Internal work done by moments

$$= M_p\, r\, d\theta\,(\Delta/r) + M_n\, r\, d\theta\,(\Delta/r) = (M_n + M_p)\, d\theta\,\Delta \qquad (22)$$

Total number of such segment = $2\pi/d\theta$ (23)

So, $TIW = (M_n + M_p)\, d\theta\,\Delta\,(2\pi/d\theta) = (M_n + M_p)\, 2\pi\,(\Delta)$ (24)

Total external work done by the load:

$$TEW = P\Delta \qquad (25)$$

Equating *TIW* and *TEW* from Eqs. 24 and 25, we have

$$(M_n + M_p)\, 2\pi\ (\Delta) = P\Delta$$

which gives

$$P = (M_n + M_p)\, 2\pi = 42\,\pi = 119.428 \text{ kN} \qquad (26)$$

Step 5: Correct yield pattern and the collapse load

The correct yield pattern is the second one as it gives the lower collapse load P =119.428 kN.

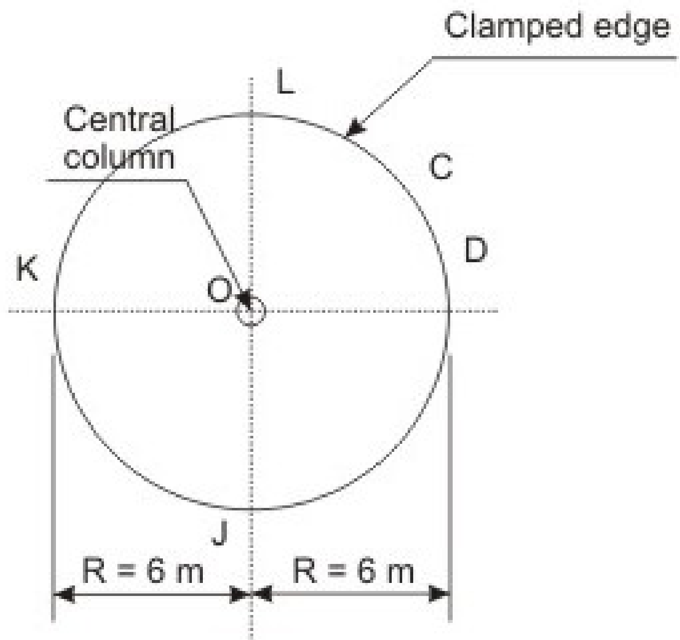

Fig. 12.33.3: Problem 6

Problem 6: A circular slab of 6 m radius is shown in Fig. 12.33.3 which is clamped along the circumference and has a column support at the centre. Determine the yield pattern and uniformly distributed collapse load of the slab if the positive and negative moments of resistance M_p and M_n are 30 kNm/m. Use the method of virtual work.

Solution 6: Figure 12.33.3 shows the slab. Given data are R = 6 m and $M_n = M_p$ = 30 kNm/m.

The analysis of this type of slab is explained in sec. 12.32.6D of Lesson 32 by the method of virtual work. We have Eqs.12.89 and 12.90 to determine the values of r and w (Fig. 12.32.7a to c). Using those equations with reference to Fig. 12.32.7 of Lesson 32, we have

(i) From Eq. 12.89: $r = 0.2679\ R = 1.6074$ m (27)

(ii) From Eq. 12.90: $w = 22.39\ (M/R^2) = 18.658$ kN/m^2 (28)

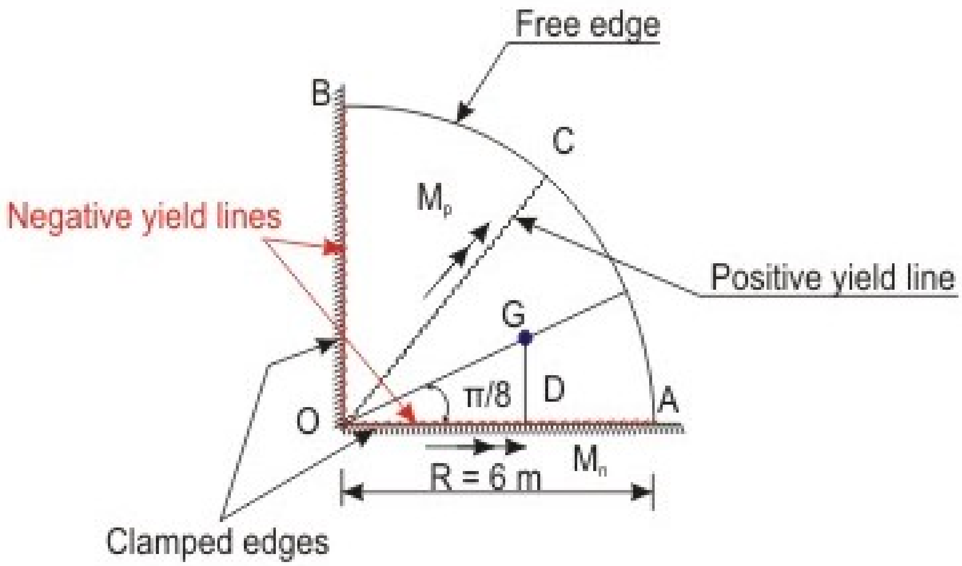

Fig. 12.33.4: Problem 7

Problem 7: Determine the yield pattern and uniformly distributed collapse load of the quadrantal slab of radius R = 6 m as shown in Fig.12.33.4, which is clamped along the straight edges OA and OB, and free along the curved edge ACB. Assume $M_n = M_p$= 30 kNm/m. Use the method of segmental equilibrium for this problem.

Solution 7:

Step 1: Yield pattern

The slab and the yield pattern are shown in Fig. 12.33.4. Here, the negative yield lines are along OA and OB, and the positive yield line OC divides the slab into two symmetrical segments. The point G is the centroid of the segment COA. The angle GOA = $\pi/8$. We know,

$OG = (2R/3)\ (\sin \pi/8\ /\ \pi/8)$, $GD = OG \sin \pi/8$ and area of the segment COA = $\pi R^2/8$
(29)

Step 2: Segmental equilibrium equation

Taking moment of the load of segment COA about OA and considering M_n and M_p, we have

$$M_n\ (OA) + M_p\ (OC \cos 45^0) - w\ (\pi R^2/8)\ (GD) = 0$$

or $M_n (R) + M_p (R/\sqrt{2}) – w (\pi R^2/8) (2R/3) (sin\ \pi/8 / \pi/8)\ sin\ \pi/8 = 0$

or $M_n + M_p/\sqrt{2} – (2\ w\ R^2/3)\ \ sin^2 (\pi/8) = 0$ (30)

The value of the collapse load w can be determined from known values of M_n , M_p and R from Eq.30.

Let us assume:

$$M_n = k\ M_p \quad (31)$$

Equation 30 then gives

$$w = \{3\ (\sqrt{2}\ k +1)\ M_p\} /\{R^2(\sqrt{2} –1)\} \quad (32)$$

For this particular problem k = 1 and R = 6 m.
Thus, from Eq. 32, we have:

w = 14.571 kN/m^2 (33)

12.33.3 Practice Questions and Problems with Answers

Q.1: Determine the uniformly distributed collapse load w kN/m^2 of a circular slab of radius = 6 m simply supported along the periphery, if M_p = 30 kNm/m. Use the method of segmental equilibrium.

A.1: The analysis of such slabs is explained in sec. 12.32.6B of Lesson 32 employing the method of segmental equilibrium with reference to Fig. 12.32.5a and b. Equation 12.82 gives the value of the uniformly distributed load, which is

$w = 6M_p/R^2$ = 6(30)/(36) = 5 kN/m^2 (34)

Therefore, the collapse load of this slab is 5 kN/m^2.

Q.2: Determine the uniformly distributed collapse load w kN/m^2 of a circular slab of radius = 6 m and clamped along the periphery, if M_p = M_n = 30 kNm/m. Use the method of segmental equilibrium.

A.2: The analysis of such slabs is explained in sec. 12.32.6C of Lesson 32 employing the method of segmental equilibrium with reference to Figs.12.32.6a and b. Equation 12.83 gives the value of the uniformly distributed load, which is

$w = 6\ (M_n + M_p)\ /\ R^2 = 60(60)/36 = 10\ \text{kN/m}^2$ (35)

Therefore, the collapse load of this slab is 10 kN/m^2.

Q.3: Determine the uniformly distributed collapse load of a 6 m $\times$ 6 m square slab clamped along four edges and has the moment of resistance $M_{px} = M_{py} = M_{nx} = M_{ny} = M_p = M_n = 30$ kNm/m.

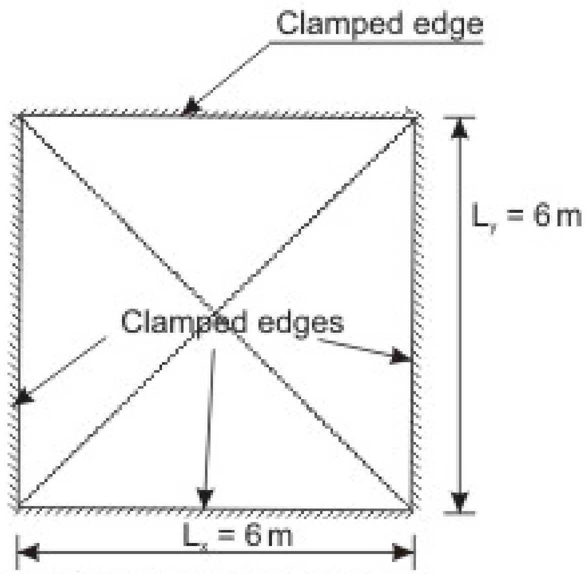

Fig. 12.33.5(a): Yield pattern 1

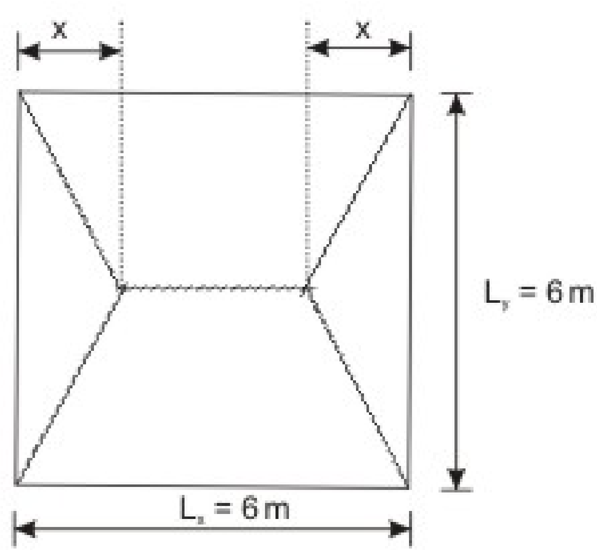

Fig. 12.33.5(b): Yield pattern 2

Fig. 12.33.5: Q.3

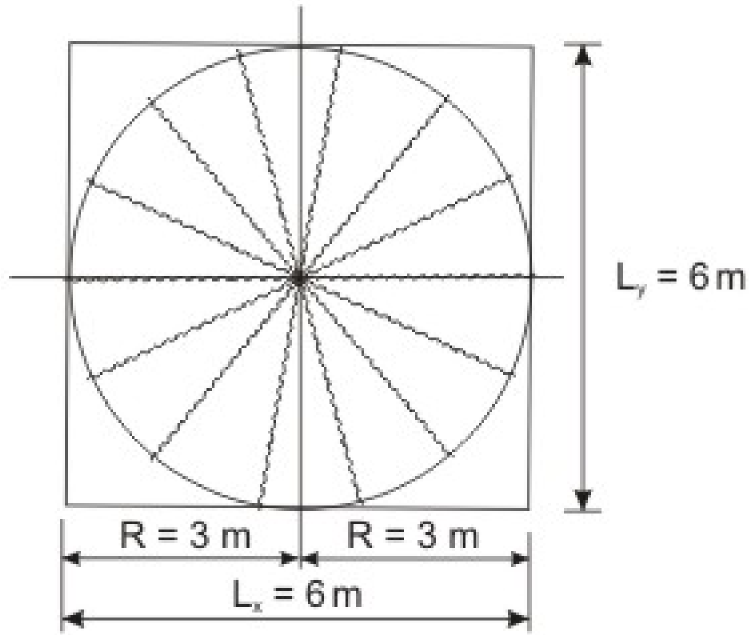

Fig. 12.33.5(c): Yield pattern 3

A.3: For this problem, we have to examine three yield patterns 1, 2 and 3, shown in Figs.12.33.5a, b and c, respectively.

Step 1: Selection of yield patterns

In this problem, $(L_y / L_x)^2 = 1$ and $(M_{py} + M_{ny})/ (M_{px} + M_{nx}) = 1$. So, yield pattern 1 has to be considered. Since, $(M_{py} + M_{ny}) / (M_{px} + M_{nx})$ is not greater than $(4/3) (L_y/L_x)^2$, yield pattern 2 is not possible. Yield pattern 3 is also possible. So, we have to examine yield patterns 1 and 3.

Step 2: Yield pattern 1

Equation 12.26 of Lesson 31 gives the value of *w* for yield pattern 1, which is

$$w = 12\{(M_{px} + M_{nx})/(L_x)^2 + (M_{py} + M_{ny})/(L_y)^2 \quad (12.26)$$

$= 12 \{60/36 + 60/36\} = 40$ kN/m^2

Thus, $w = 40$ kN/m^2 for yield pattern 1

Step 3: Yield pattern 3

Equation 12.83 of Lesson 32 gives the value of load w for yield pattern 3, which is:

$$w = 6(M_n + M_p) / R^2 \qquad (12.83)$$

Using $M_n = M_p$ = 30 kNm/m as R = 3 m, we have w = 40 kN/m^2.

Therefore, the collapse load of this slab is 40 kN/m^2 either for the yield pattern 1 or for the yield pattern 3.

12.33.4 Reference

1. Reinforced Concrete Limit State Design, 6th Edition, by Ashok K. Jain, Nem Chand & Bros, Roorkee, 2002.
2. Limit State Design of Reinforced Concrete, 2nd Edition, by P.C. Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
3. Advanced Reinforced Concrete Design, by P.C. Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.
4. Reinforced Concrete Design, 2nd Edition, by S. Unnikrishna Pillai and Devdas Menon, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2003.
5. Limit State Design of Reinforced Concrete Structures, by P. Dayaratnam, Oxford & I.B.H. Publishing Company Pvt. Ltd., New Delhi, 2004.
6. Reinforced Concrete Design, 1st Revised Edition, by S.N. Sinha, Tata McGraw-Hill Publishing Company. New Delhi, 1990.
7. Reinforced Concrete, 6th Edition, by S.K. Mallick and A.P. Gupta, Oxford & IBH Publishing Co. Pvt. Ltd. New Delhi, 1996.
8. Behaviour, Analysis & Design of Reinforced Concrete Structural Elements, by I.C.Syal and R.K.Ummat, A.H.Wheeler & Co. Ltd., Allahabad, 1989.
9. Reinforced Concrete Structures, 3rd Edition, by I.C.Syal and A.K.Goel, A.H.Wheeler & Co. Ltd., Allahabad, 1992.
10. Textbook of R.C.C, by G.S.Birdie and J.S.Birdie, Wiley Eastern Limited, New Delhi, 1993.
11. Design of Concrete Structures, 13th Edition, by Arthur H. Nilson, David Darwin and Charles W. Dolan, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2004.
12. Concrete Technology, by A.M.Neville and J.J.Brooks, ELBS with Longman, 1994.
13. Properties of Concrete, 4th Edition, 1st Indian reprint, by A.M.Neville, Longman, 2000.
14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C. Steedman, E & FN SPON, London, 1997.

15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4th Revision), IS 456: 2000, BIS, New Delhi.
16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

12.33.5 Test 33 with Solutions

Maximum Marks = 50,   Maximum Time = 30 minutes

Answer all questions.

TQ.1: Determine the uniformly distributed collapse load of a rectangular slab (Figs. 12.33.1a, b and c) having $L_y/L_x = 0.5$, simply supported at three edges and free at the other edge. Assume $M_y = 0.5\ M_x$.

A.TQ.1: The slab is shown in Figs. 12.33.1a, b and c. Given data are: $L_y/L_x = 0.5$ and $M_y/M_x = 0.5$.

Step 1: To examine the possibility of yield patterns 1, 2 or both

Here, $(M_y/M_x) \langle\ 4(L_y/L_x)^2$. So, yield pattern 1 is possible. Again $(M_y/M_x) \rangle\ (4/3)\ (L_y/L_x)^2$. So, yield pattern 2 is also possible. Thus, both yield patterns 1 and 2 are to be examined.

Step 2: Value of *y* for yield pattern 1

Equation 12.43 of Lesson 32 gives: $2y^2 + L_x\, y - 0.75\ L_x^2 = 0$, which gives,

$$y = 0.4114\ L_x \tag{36}$$

Step 3: Determination of collapse load *w* kN/m²

(i) Eq. 12.41 gives: $w = (M_x/L_x^2)\ \{12/(1.5\text{-}2y)\} = 17.722(M_x/L_x^2)$ (37)

(ii) Eq. 12.42 gives: $w = (M_x/L_x^2)\ (3/y^2) = 17.722\ (M_x\ /\ L_x^2)$ (37)

(iii) Eq. 12.48 gives: $w = (M_x/L_x^2)\ \{(12y + 3)\ /\ y(1.5\text{-}y)\}$

$$= 17.722\ (M_x/L_x^2) \tag{37}$$

(Equations 12.41, 12.42 and 12.48 are from Lesson 32.)

Step 4: Value of *x* for yield pattern 2

The value of x is obtained from Eq. 12.52 of Lesson 32, which is,

$$1.5\ x^2 + L_x\, x - 0.75\ L_x^2 = 0 \tag{38}$$

The solution of Eq. 38 is:

$$x = 0.448\, L_x \tag{39}$$

Step 5: Determination of collapse load w kN/m^2

(i) Eq. 12.50 of Lesson 32 gives: w = 17.841 (M_x/L_x^2)

(40)

(ii) Eq. 12.51 of Lesson 32 gives: w = 17.841 (M_x/L_x^2)

(40)

(iii) Eq. 12.57 of Lesson 32 gives: w = 17.841 (M_x/L_x^2) (40)

Step 6: Correct yield pattern and collapse load

Comparison of results of Steps 3 and 5 reveals that the yield pattern 1 is the correct one giving lower value of w = 17.722 (M_x/L_x^2).

12.33.6 Summary of this Lesson

This lesson explains the different types of yield pattern of triangular, rectangular, quadrantal and circular slabs through several numerical problems. Both the methods of segmental equilibrium and virtual work are employed. The loadings considered are either point load or uniformly distributed load. All possible yield patterns of a particular problem are examined to select the correct one and the corresponding lower/lowest load for the slab problems. The moments of resistance of the slab in the two directions are also varied in the problems. Different types of support conditions are taken up in the numerical problems. In addition to illustrative examples, practice problems and test problem will help in understanding the theories and their applications in analysing slab problems with different types of support conditions and loadings.