Chapter Two: Flat Slabs

2.1 Introduction

Concrete two-way slabs may in some cases be supported by relatively shallow, flexible beams, or directly by columns with out the use of beams or girders. Such slabs are generally referred as column supported two-way slabs. Beams may also be used where the slab is interrupted as around stair, walls or at discontinuous edges.

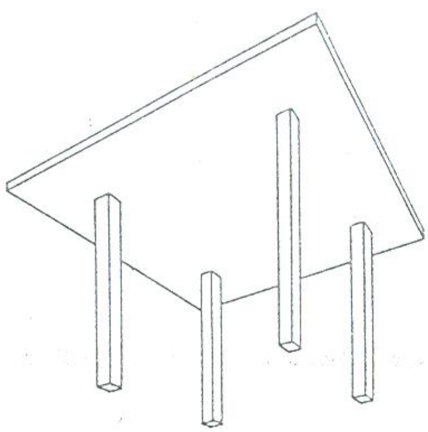
In practice column supported two-way slabs take various forms:

a) **Flat Plate**: They are flat slabs with flat soffit. Such slabs have uniform thickness supported on columns. They are used for relatively light loads, as experienced in apartments or similar buildings. Flat plats are most economical for spans from 4.5m to 6m (see Fig. 2.1a).

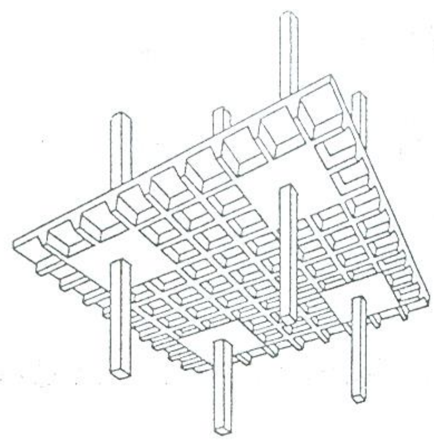
b) **Flat slab**: They are slab systems with the load transfer to the column is accomplished by thickening the slab near the column, ***using drop panels*** and/or by flaring the top of the column to form ***a column capital***. They may be used for heavy industrial loads and for spans of 6m to 9m (see Fig. 2.1c)

c) **Waffle slabs**: They are two-way joist systems with reduced self weights. They are used for spans from 7.5m to 12m. (Note: for large spans, the thickness required to transmit the vertical loads to the columns exceeds that required for bending. As a result the concrete at the middle of the panel is not efficiently used. To lighten the slab, reduce the slab moments, and save material, the slab at mid span can be replaced by intersecting ribs. Near the columns the full depth is retained to transmit loads from the slab to the columns (see Fig. 2.1b)

In this chapter, consideration will be given to flat slabs with or with out drop panels or column capitals.



(a) Flat plate.



(b) Waffle slab.

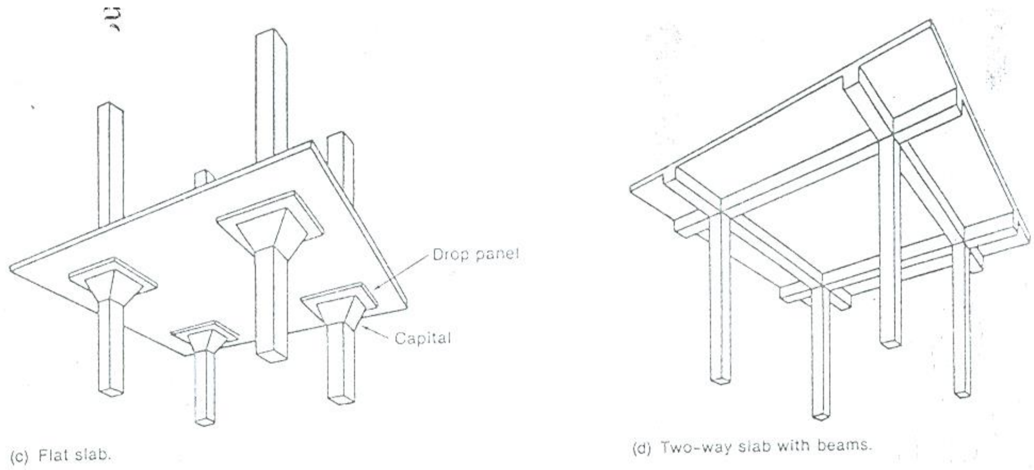


Fig. 2.1 Types of two way slabs

For analysis and design purpose the panel in flat slab is divided in to column strips and middle strips as shown below (EBSC 2)

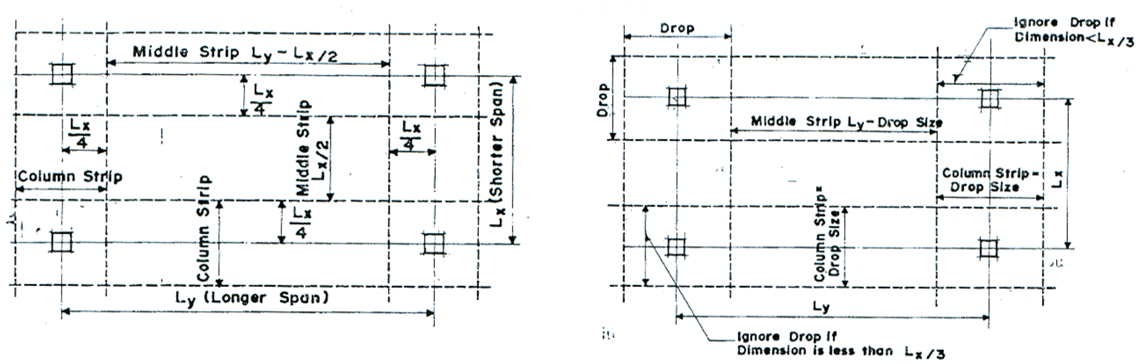


Fig.2.2 Division of panels in Flat slabs

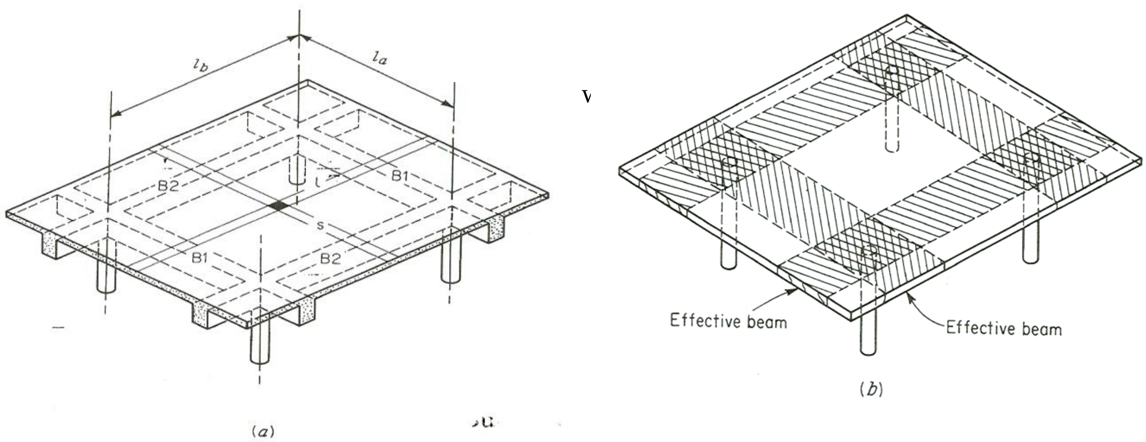
- A column strip is a design strip with a width on each side of a column centerline equal to 0.25 Lx or if drops with dimension not less than Lx/3 are used, a width equal to the drop dimension.
- A middle strip is a design strip bounded by two column strips.

The drop panels are rectangular (may be square) and influence the distribution of moments in the slab. The smaller dimension of the drop is at least one third of the smaller dimension of the surrounding panels, Lx/3 and the drop may be 25 to 50 percent thicker than the rest of the slab.

2.2 Load Transfer in Flat Slabs

Consider the following column supported two way slabs. If a surface load w is applied (see Fig. 2.3a), it is shared between imaginary slab strips l_a in the short direction and l_b in the longer direction. Note that the portion of the load that is carried by the long strips l_b is delivered to the beams B_1 which in turn carried in the short direction plus that directly carried in the short direction by the slab strips l_a, sums up to 100 percent of the load applied to the panel. The same is true in the other direction.

A similar situation is obtained in the flat plate floor (see Fig. 2.3b) where broad strips of the slab centered on the column lines in each direction serve the same function as the beams. Therefore; for column supported construction, 100 percent of the applied load must be carried in each direction, jointly by the slab and its supporting beams.



2.3 Moments in Flat slab Floors

Consider the flat slab floor supported by columns at A, B, C, and D as shown in Fig. 2.4a

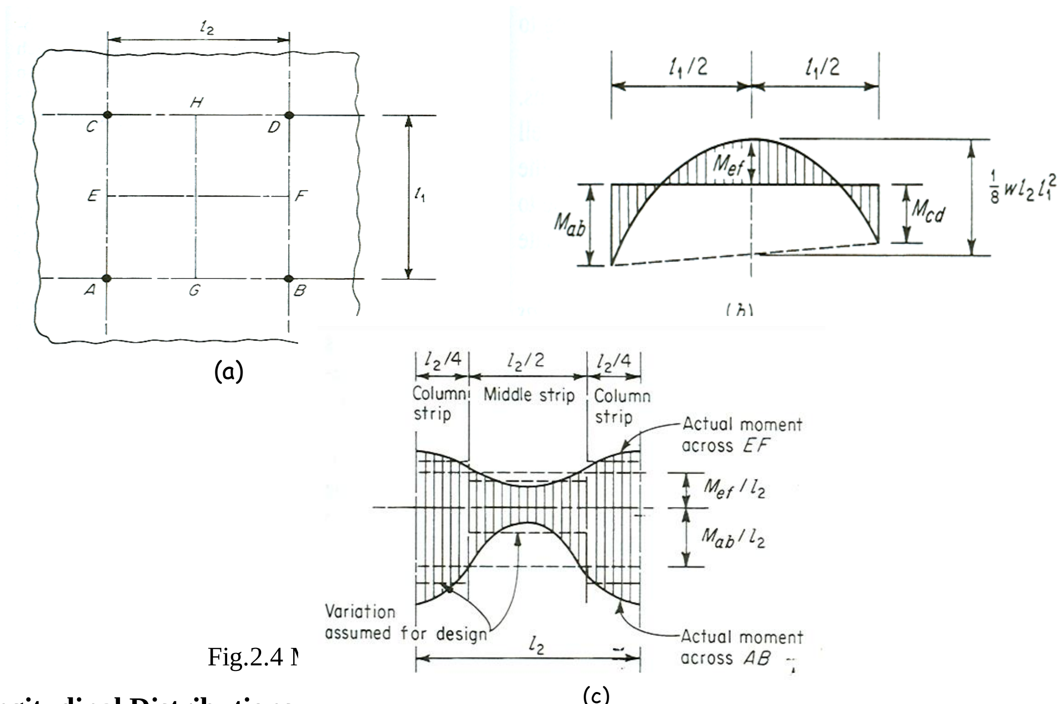


Fig.2.4

2.4 Longitudinal Distributions

For the determination of moment in the direction of span l_1, the slab may be considered as a broad, flat beam of width l_2.

The load, $P_2 = wl_2$ per m length of span.

From the requirement of statics:

a) In the longitudinal direction (see Fig. 2.4b)

$$\frac{1}{2}(M_{ab} + M_{cd}) + M_{ef} = \frac{1}{8} w l_2 l_1^2$$

b) In the perpendicular direction

$$\frac{1}{2}(M_{ac} + M_{bd}) + M_{gh} = \frac{1}{8} w l_1 l_2^2$$

From the above static moment in each direction, the moment in the long direction is larger than those in the short direction unlike to the situation for the slab with stiff edge beams.

Lateral Distributions of moments

The moments across the width of critical sections such as AB or EF are not constant as shown qualitatively (see Fig.2.4 c). For design purpose, moments may be considered constant within the bounds of a middle strip or column strip, unless beams are present in column lines.

2.5 Practical Analysis of Flat slabs

The two methods for the analysis of flat slabs are:

a) Direct Design method
b) Equivalent Frame Method

Generally, for both methods of analysis, the negative moments greater than those at a distance $h_c/2$ from the center-line of the column may be ignored provided the moment *Mo* obtained as the sum of the maximum positive design moment and the average of the negative design moments in anyone span of the slab for the whole panel width is such that:

$$M_o \geq \frac{(g_d + q_d)L_2}{8}(L_1 - \frac{2h_c}{3})^2$$

Where L_1 is the panel length parallel to span, measured from centers of columns.

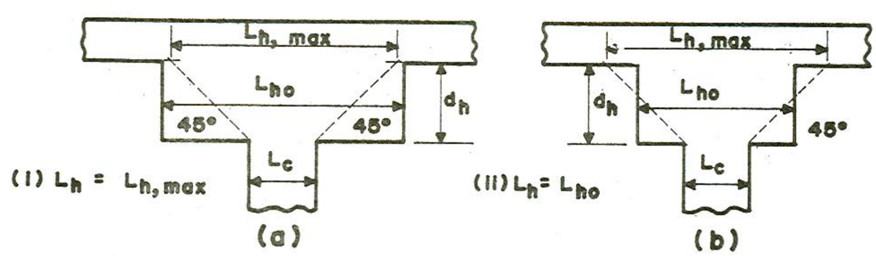
L_2 is the panel width, measured from centers of columns
h_c is the effective diameter of a column or column head (see below)

When the above condition is not satisfied, the negative design moments shall be increased.

The effective diameter of a column or column head h_c is the diameter of a circle whose area equals the cross-sectional area of the column or, if column heads are used, the area of the column head based on the effective dimensions as defined below. In no case shall h_c be taken as greater than one-quarter of the shortest span framing in to the column.
The effective dimensions of a column head for use in calculation of h_c are limited according to the depth of the head. In any direction, the effective dimension of a head L_h shall be taken as the lesser of the actual dimension L_{ho} or $L_{h,max}$, where $L_{h,max}$ is given by:

$$L_{h,max} = L_c + 2d_h$$

For a flared head, the actual dimension L_{ho} is that measured to the center of the reinforcing steel (see Fig. 2.5)



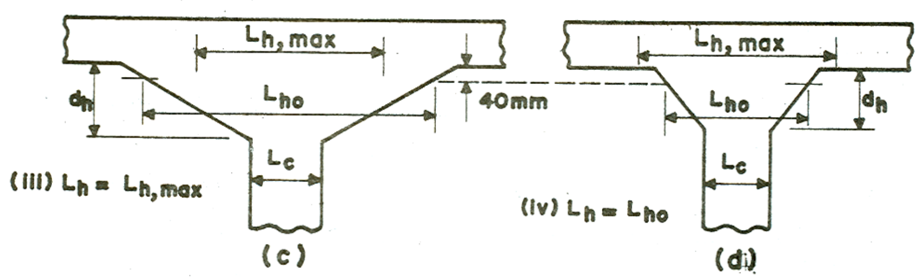


Fig. 2.5 Types of Column Head

2.6 Direct Design Method as per EBCS 2, 1995

According to the EBCS 2 specification, the direct design method of analysis is subjected to the following restrictions.

1. Design is based on the single load case of all spans loaded with the maximum design ultimate load.
2. There are at least three rows of panels of approximately equal span in the direction being considered.
3. Successive span length in each direction shall not differ by more than one-third of the longer span
4. Maximum offsets of columns from either axis between center lines of successive columns shall not exceed 10% of the span (in the direction of the offset)

Longitudinal Distribution

The distribution of design span and support moments depends on the relative stiffness of the different sections which in turn depends on the restraint provided for the slab by the supports. Accordingly, the distribution factors are given in the following table.

Table 2.1 Bending Moment and Shear Force Coefficients for Flat slabs of Three or More Equal Spans.

	Outer support		Near center of first span	First interior support	Center of interior span	Interior support
	Column	Wall				
Moment	-0.040FL	-0.020FL	0.083FL	-0.063FL	0.071FL	-0.055FL
Shear	0.45F	0.40F	-	0.60F	-	0.50F
Total Column moments	0.040FL	-	-	0.022FL	-	0.022FL

NOTE:

1. **F** is the total design ultimate load on the strip of slab between adjacent columns considered.
2. **L** is the effective span = $\mathbf{L_1\text{-}2h_c/3}$
3. The limitations of **Section A.4.3.1(2) of EBCS 2**, need not be checked
4. The moments shall not be redistributed

Lateral Distribution

The design moment obtained from the above (or equivalent frame analysis) shall be divided b/n the column and middle strips according to the following table.

Table 2.2 Distribution of Design Moments in Panels of Flat Slabs

	Apportionment been column and middle strip expressed as percentages of the total negative or positive design moment	
	Column strip (%)	Middle. strip (%)
Negative	75	25
Positive	55	45

NOTE: For the case where the width of the column strip is taken as equal to that of the drop and the middle strip is thereby increased in width, the design moments to be resisted by the middle strip shall be increased in proportion to its increased width. The design moments to be resisted by the column strip may be decreased by an amount such that the total positive and the total negative design moments resisted by the column strip and middle strip together are unchanged.

2.7 Equivalent Frame Method

The direct design method is applicable when the proposed structures satisfy the restrictions on geometry and loading. If the structure does not satisfy the criteria, the more general method of elastic analysis is the equivalent frame method.

In the equivalent frame method, the structure is divided in to continuous frames centered on the column lines on either side of the columns, extending both longitudinally and transversely. Each frame is composed of abroad continuous beam and a row of columns.

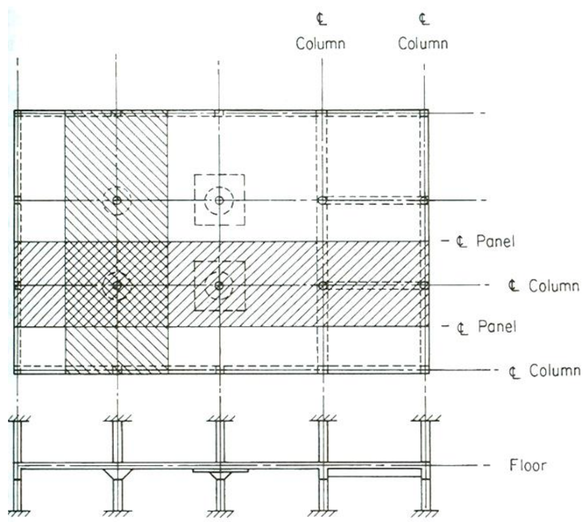


Fig. 2.6 Building idealization for equivalent frame analysis

Equivalent Frame Method as per EBCS 2, 1995

According to the EBCS 2 specification, Equivalent Frame Method of analysis is treated as follows:

(1) The width of slab used to define the effective stiffness of the slab will depend upon the aspect ratio of the panels and the type of loading, but the following provisions may be applied in the absence of more accurate methods:
   - In the case of vertical loading, the full width of the Panel, and
   - For lateral loading, half the width of the panel may be used to calculate the stiffness of the slab.

(2) The moment of inertia of any section of slab or column used in calculating the relative stiffness of members may be assumed to be that of the cross section of the concrete alone.

(3) Moments and forces within a system of flat slab panels may be obtained from analysis of the structure under the single load case of maximum design load on all spans or panels simultaneously, provided:

- The ratio of the characteristic imposed load to the characteristic dead load does not exceed 1.25.
- The characteristic imposed load does not exceed 5.0 kN/m2 excluding partitions.

(4) Where it is not appropriate to analyze for the single load case of maximum design load on all spans, it will be sufficient to consider following arrangement of vertical loads:

- All spans loaded with the maximum design ultimate load, and
- Alternate spans with the maximum design ultimate load and all other spans loaded with the minimum design ultimate load ($1.0G_k$).

(5) Each frame may be analyzed in its entirety by any elastic method. Alternatively, for vertical loads only, each strip of floor and roof may be analyzed as a separate frame with the columns above and below fixed in position and direction at their extremities. In either case, the analysis shall be carried out for the "appropriate design ultimate loads on each span calculated for a strip of slab of width equal to the distance between center lines of the panels on each side of the columns.

Equivalent Frame Method as per ACI Code

According to the ACI Code specification, the Equivalent Frame method was developed with the assumption that the analysis would be done using the moment distribution method.

a) Basis of Analysis

The equivalent Frame method was developed with the assumption that the analysis would be done using the moment distribution method. For vertical loading, each floor with its columns may be analyzed separately by assuming the columns to be fixed at the floors above and below.

b) Moment of Inertia of Slab Beam

The slab beam includes the portion of then slab bounded by panel centerlines on each side of the columns, together with column line beams or drop panels (if used).

The moment of inertia used for analysis may be based n the concrete cross-section, neglecting reinforcement, but variations in cross section along the member axis should be accounted for (see below).

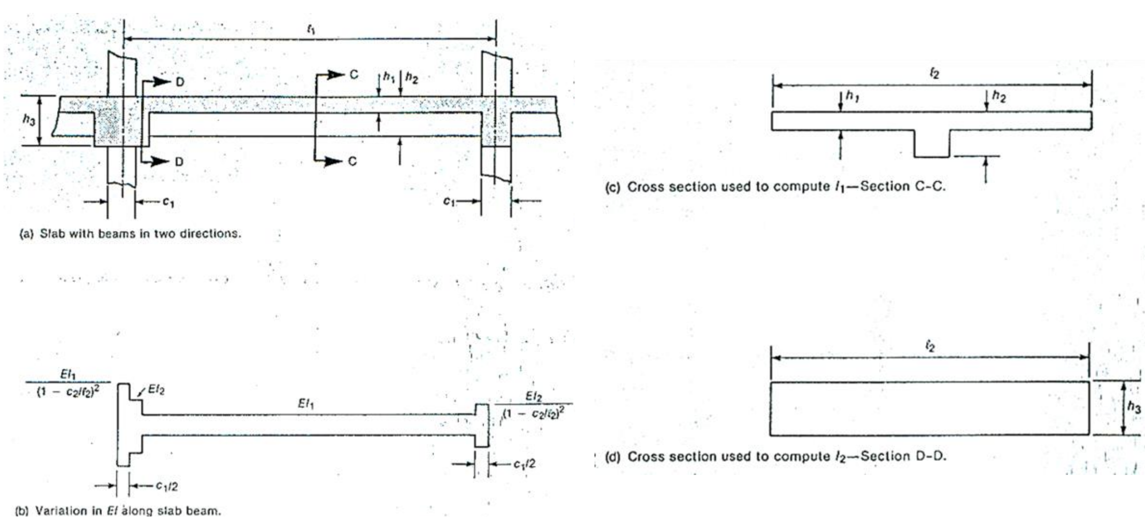


Fig. 2.7 EI values for slab with drop

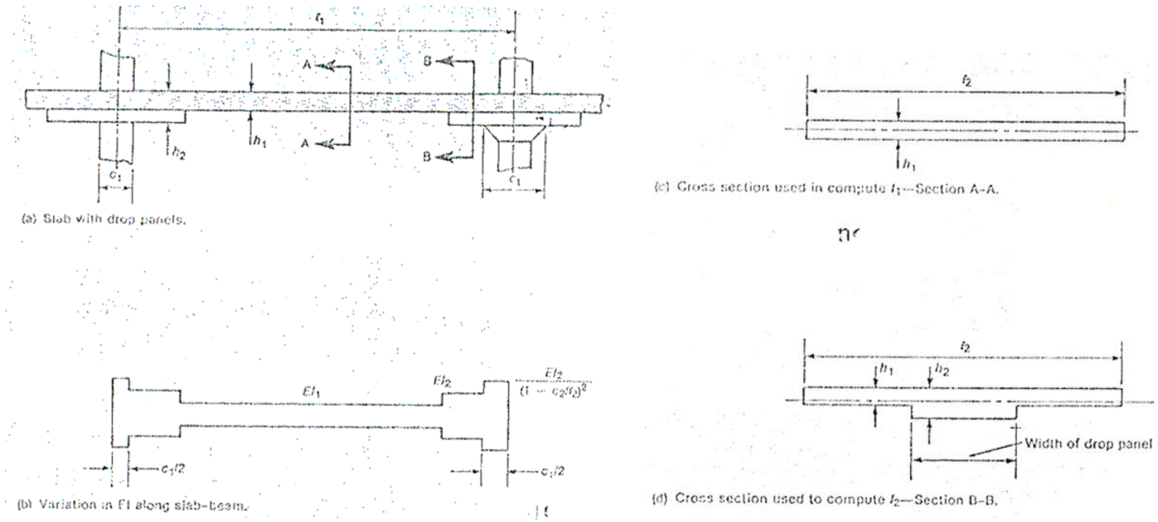


Fig.2.8 EI values for slab and beam

c) The equivalent Column

In the equivalent frame method of analysis, the columns are considered to be attached to the continuous slab beam by torsional members transverse to the direction of the span for which moments are being found. Torsional deformation of these transverse supporting members reduces the effective flexural stiffness provided by the actual column at the support.

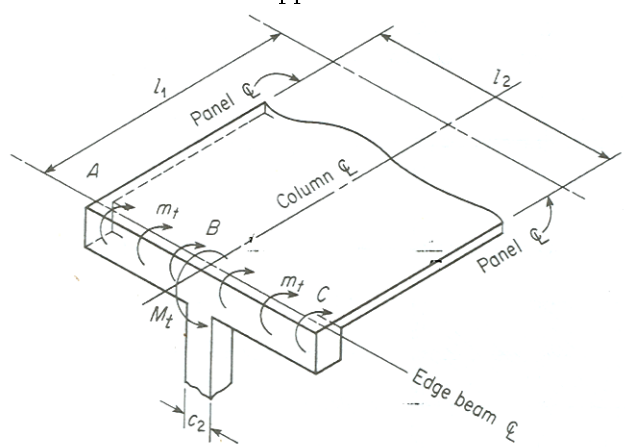


Fig. 2.9 Torsion at a transverse supporting member

The above effects can be considered by replacing the actual beam and columns with an equivalent column having the following stiffness:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

Where K_{ec} = Flexural stiffness of equivalent column
K_c = flexural stiffness of actual column
K_t = torsional stiffness of edge beam

The torsional Stiffness K_t can be calculated by:

$$K_t = \Sigma \frac{9E_{cs}C}{l_2\left(1 - c_2/l_2\right)^3}$$

Where E_{cs} = modulus of elasticity of slab concrete
c_2 = size of rectangular column, capital, or bracket in the direction of l2.
C = cross sectional constant (roughly equivalent to polar moment of inertia)

The torsional constant C can be calculated by:

$$C = \Sigma (1 - 0.63\frac{x}{y})\frac{x^3y}{3}$$

Where **x** is the shorter side of a rectangle and **y** is the longer side.
C is calculated by sub-dividing the cross section of torsional members in to component rectangles and the sub-division is to maximize the value of C.

The torsional members according to ACI Code are as follows:

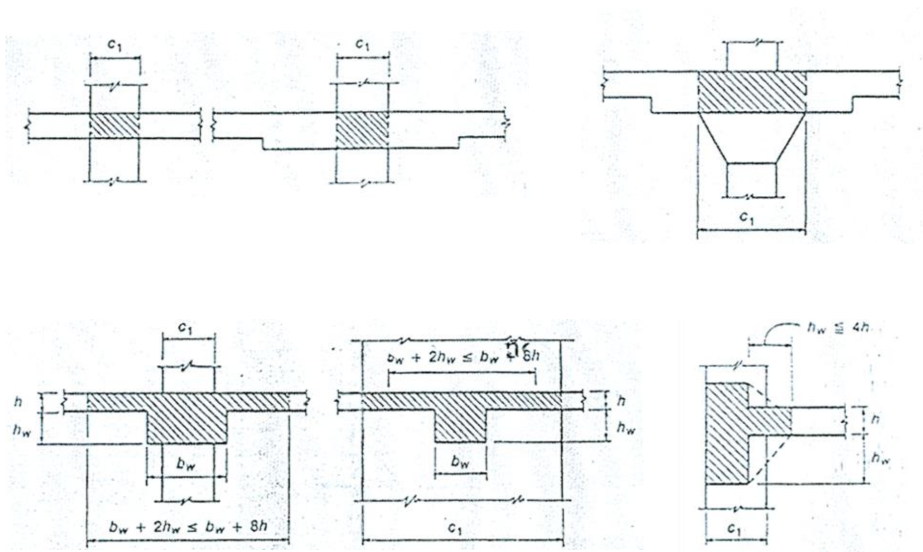


Fig. 2.10 Torsional members

d) Arrangement of Live Load for Analysis

1. If the unfactored live load does not exceed 0.75 times the unfactored dead load, it is not necessary to consider pattern loadings, and only the case of full factored live load and dead load on all spans need to be analyzed
2. If the unfactored live load exceeds 0.75 times the unfactored dead load the following pattern loadings need to be considered.
   a) For maximum positive moment, factored dead load on all spans and 0.75 times the full factored live load on the panel in question and on alternate panels
   b) For maximum negative moment at an interior support, factored dead load on all panels and 0.75 times the full factored live load on the two adjacent panels.

The final design moments shall not be less than for the case of full factored dead and live load on all panels.

2.8 Shear in Flat Slabs, as per EBCS 2

The concrete section (thickness of the slab) must be adequate to sustain the shear force, since stirrups are not convenient.

Two types of shear are considered

i) Beam type Shear: Diagonal tension Failure and critical section is considered at **d** distance from the face of the column or capital and V_c is the same expression given earlier for beams or solid slabs.

   i.e. $V_c = 0.25 f_{ctd}\, k_1\, k_2\, b_w\, d$

ii) Punching Shear: perimeter shear which occurs in slabs with out beams around columns. It is characterized by formation of a truncated punching cone or pyramid around concentrated loads or reactions. The outline of the critical section is shown in Fig. below.

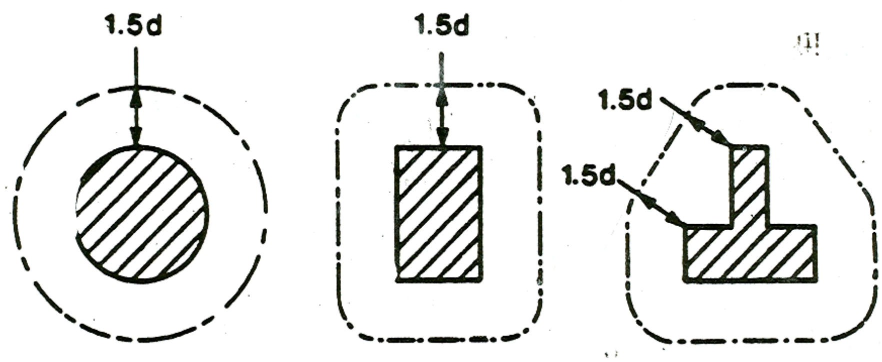


Fig. 2.11 Critical section remote from a free edge

The shear force to be resisted can be calculated as the total design load on the area bounded by the panel centerlines around the column less the load applied with in the area defined by the critical shear perimeter.
The punching shear resistance with out shear reinforcement is:

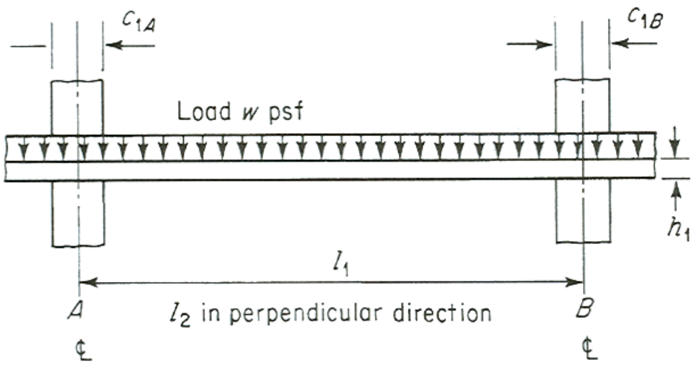
$$V_{cp} = 0.5\, f_{ctd}\, k_1\, k_2\, u\, d$$
$$K_1 = (1+50\rho) \leq 2.0$$
$$\rho_e = (\rho_{ex}\, \rho_{ey})^2 \leq 0.015$$

u = perimeter of critical section

$d = ½(d_x+d_y)$, average effective depth

Table 2.4a

Coefficients for slabs with variable moment of inertia[a]



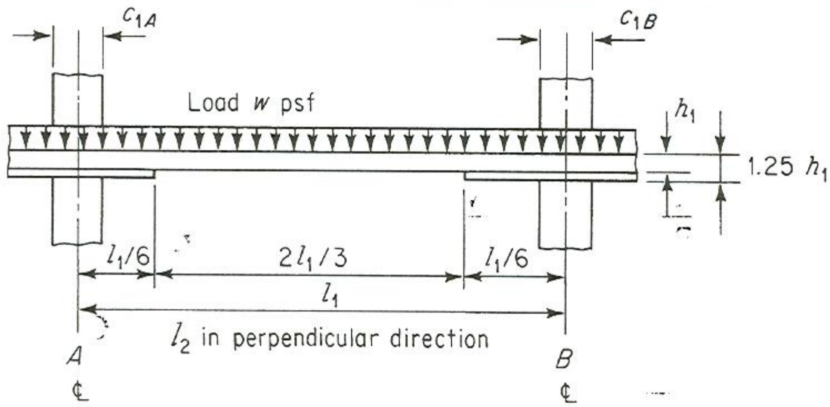
Column dimension		Uniform load FEM = coeff. $(wl_2l_1^2)$		Stiffness factor[b]		Carryover factor	
c_{1A}/l_1	c_{1B}/l_1	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.00	0.083	0.083	4.00	4.00	0.500	0.500
	0.05	0.083	0.084	4.01	4.04	0.504	0.500
	0.10	0.082	0.086	4.03	4.15	0.513	0.499
	0.15	0.081	0.089	4.07	4.32	0.528	0.498
	0.20	0.079	0.093	4.12	4.56	0.548	0.495
	0.25	0.077	0.097	4.18	4.88	0.573	0.491
0.05	0.05	0.084	0.084	4.05	4.05	0.503	0.503
	0.10	0.083	0.086	4.07	4.15	0.513	0.503
	0.15	0.081	0.089	4.11	4.33	0.528	0.501
	0.20	0.080	0.092	4.16	4.58	0.548	0.499
	0.25	0.078	0.096	4.22	4.89	0.573	0.494
0.10	0.10	0.085	0.085	4.18	4.18	0.513	0.513
	0.15	0.083	0.088	4.22	4.36	0.528	0.511
	0.20	0.082	0.091	4.27	4.61	0.548	0.508
	0.25	0.080	0.095	4.34	4.93	0.573	0.504
0.15	0.15	0.086	0.086	4.40	4.40	0.526	0.526
	0.20	0.084	0.090	4.46	4.65	0.546	0.523
	0.25	0.083	0.094	4.53	4.98	0.571	0.519
0.20	0.20	0.088	0.088	4.72	4.72	0.543	0.543
	0.25	0.086	0.092	4.79	5.05	0.568	0.539
0.25	0.25	0.090	0.090	5.14	5.14	0.563	0.563

[a] Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

[b] Stiffness is $K_{AB} = k_{AB}E(l_2h_1^3/12l_1)$ and $K_{BA} = k_{BA}E(l_2h_1^3/12l_1)$.

Table 2.4b

Coefficients for slabs with variable moment of inertia[a]



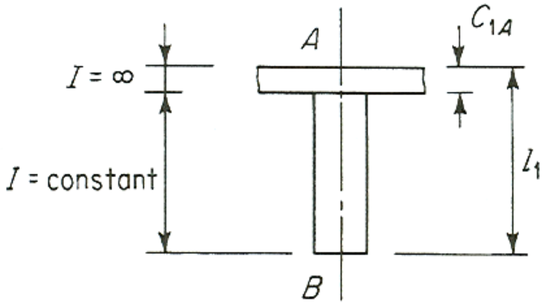
Column dimension		Uniform load FEM = coeff. ($wl_2l_1^2$)		Stiffness factor[b]		Carryover factor	
c_{1A}/l_1	c_{1B}/l_1	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.00	0.088	0.088	4.78	4.78	0.541	0.541
	0.05	0.087	0.089	4.80	4.82	0.545	0.541
	0.10	0.087	0.090	4.83	4.94	0.553	0.541
	0.15	0.085	0.093	4.87	5.12	0.567	0.540
	0.20	0.084	0.096	4.93	5.36	0.585	0.537
	0.25	0.082	0.100	5.00	5.68	0.606	0.534
0.05	0.05	0.088	0.088	4.84	4.84	0.545	0.545
	0.10	0.087	0.090	4.87	4.95	0.553	0.544
	0.15	0.085	0.093	4.91	5.13	0.567	0.543
	0.20	0.084	0.096	4.97	5.38	0.584	0.541
	0.25	0.082	0.100	5.05	5.70	0.606	0.537
0.10	0.10	0.089	0.089	4.98	4.98	0.553	0.553
	0.15	0.088	0.092	5.03	5.16	0.566	0.551
	0.20	0.086	0.094	5.09	5.42	0.584	0.549
	0.25	0.084	0.099	5.17	5.74	0.606	0.546
0.15	0.15	0.090	0.090	5.22	5.22	0.565	0.565
	0.20	0.089	0.094	5.28	5.47	0.583	0.563
	0.25	0.087	0.097	5.37	5.80	0.604	0.559
0.20	0.20	0.092	0.092	5.55	5.55	0.580	0.580
	0.25	0.090	0.096	5.64	5.88	0.602	0.577
0.25	0.25	0.094	0.094	5.98	5.98	0.598	0.598

[a] Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

[b] Stiffness is $K_{AB} = k_{AB}E(l_2h_1^3/12l_1)$ and $K_{BA} = k_{BA}E(l_2h_1^3/12l_1)$.

Table 2.4c

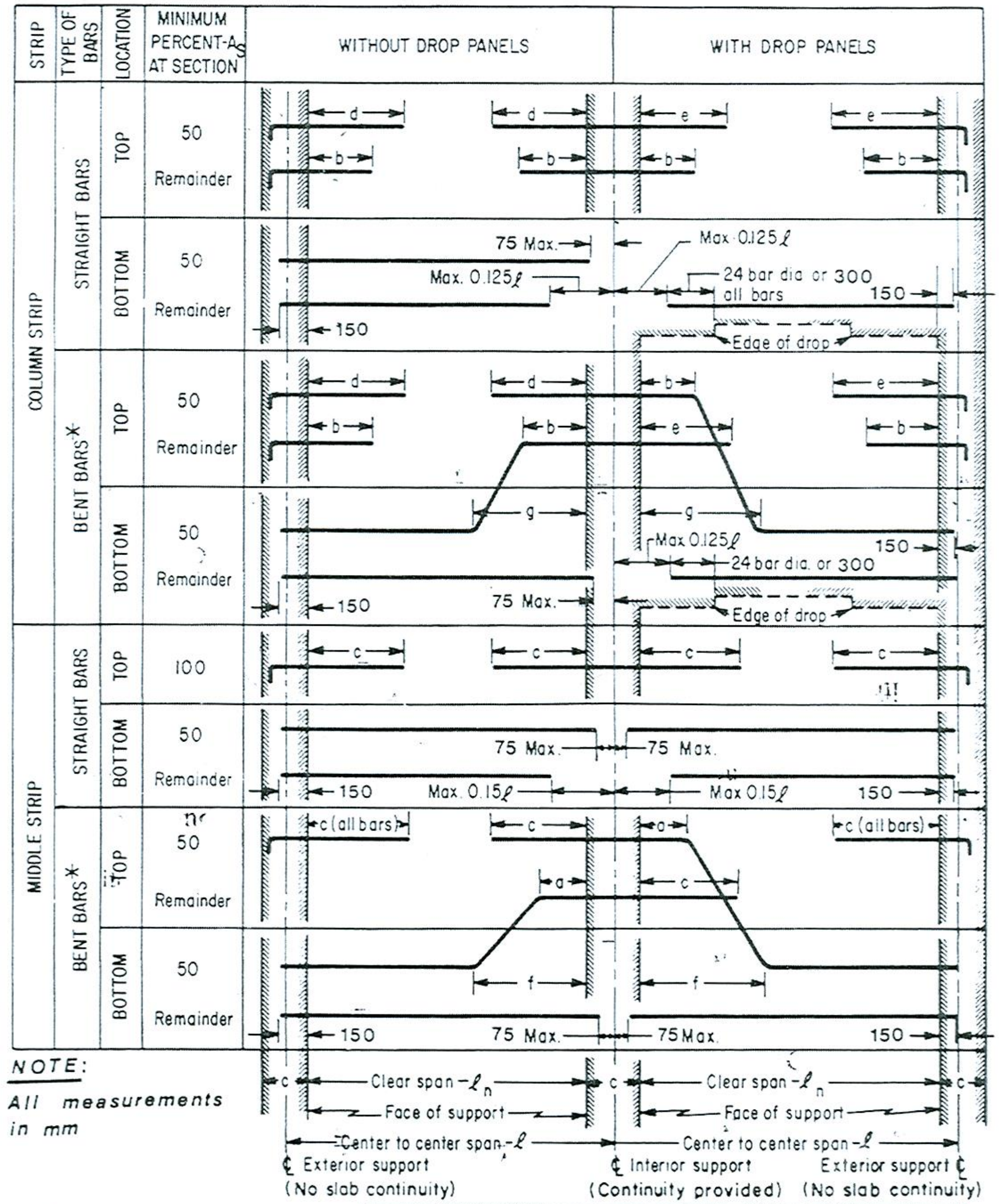
Coefficients for columns with variable moment of inertia



Slab depth c_{1A}/l_1	Uniform load FEM = coeff. $(wl_2l_1^2)$ M_{AB}	M_{BA}	Stiffness factor k_{AB}	k_{BA}	Carryover factor COF_{AB}	COF_{BA}
0.00	0.083	0.083	4.00	4.00	0.500	0.500
0.05	0.100	0.075	4.91	4.21	0.496	0.579
0.10	0.118	0.068	6.09	4.44	0.486	0.667
0.15	0.135	0.060	7.64	4.71	0.471	0.765
0.20	0.153	0.053	9.69	5.00	0.452	0.875
0.25	0.172	0.047	12.44	5.33	0.429	1.000

Table 2.5

Minimum Bend Point Locations and Extensions for reinforcement in Flat Slabs



NOTE:

All measurements in mm

* Bent bars at exterior supports may be used if a general analysis is made

	BAR LENGTH FROM FACE OF SUPPORT						
	MINIMUM LENGTH					MAXIMUM LENGTH	
MARK	a	b	c	d	e	f	g
LENGTH	$0.14\ell_n$	$0.20\ell_n$	$0.22\ell_n$	$0.30\ell_n$	$0.33\ell_n$	$0.20\ell_n$	$0.24\ell_n$

General Procedue in the design Of Flat Slab

1-Design Constants: Fcd ,fyd, fctd,etc.

2. Depth required for deflection

   d≥(0.4+0.6*fyk/400)*Le/βa,

   Le- longer span

3. Loading, Pd=1.3DL+1.6LL in KN/m2

4. Analysis: using simplified direct method

   M=αFL

   F=Pd*(Lx*Ly), L=Lx-2hc/3 and L= Ly-2hc/3 in both directions

   Division in to strips (long direction and short direction)

5. Check depth for Punching

   Punching shear force at the critical section Pp

   Pp=Pd*A where A=[Lx*Ly-(b+d)*(h+d)], d-average depth

   Calculate punching shear area Ap

   Ap={(b+d)*2+(h+d)*2}*d

   Then calculate acting Punching Shea Vp

   Vp=PP/Ap

   Punching shear resistance of Concrete Vcp

   Vcp=0.5fctd*K1*k2

   If Vcp>Vp , safe if not provide drop panels or other measures.

6. Calculate new design load and effective depth and Re –check the punching shear when drop panel is applied
   - Around the Column head or drop panels
   - Around the panel on the slab
7. Design for flexure using design table and distribute the moment in to strips using design Table
8. Design of Reinforcement using design table
9. Reinforcement detailing and bar cut off schedule as per Table 3.5 above