CEng-3154: ENGINEERING HYDROLOGY FOR CIVIL ENGINEERING STUDENTS

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MAIN COURSE SYLLABUS

CEng-3154-ENGINEERING HYDROLOGY
PREREQUISITE: **CEng2151, Hydraulics II**
Rainfall-runoff relationships, application of different models; hydrology of ungauged catchments; flood routing: reservoir routing, channel routing; frequency analysis: rainfall, low flow and floods; basic concepts of stochastic hydrology; reservoirs: determination of reservoir capacity, reservoir simulation, reservoir sedimentation; basic concepts of urban hydrology.

1. Meteorological and Hydrological Data Analysis for The Purpose of Hydrological Studies

1.1 General

Hydrological studies require extensive analysis of meteorological, hydrological and spatial data to represent the actual processes taking place on the environment and better estimation of quantities out of it. Precipitation is the source of all waters which enters the land. Hydrologists need to understand how the amount, rate, duration, and quality of precipitation are distributed in space and time in order to assess, predict, and forecast hydrologic responses of a catchment.

Estimates of regional precipitation are critical inputs to water-balance and other types of models used in water-resource management. Sound interpretation of the prediction of such models requires an assessment of the uncertainty associated with their output, which in turn depends in large measure on the uncertainty of the input values.

The uncertainties associated with a value of regional precipitation consist of:
1. Errors due to point measurement
2. Errors due to uncertainty in converting point measurement data into estimates of regional precipitation

It is therefore, necessary to first check the data for its quality, continuity and consistency before it is considered as input. The continuity of a record may be broken with missing data due to many reasons such as damage or fault in recording gauges during a period. The missing data can be estimated by using the data of the neighboring stations correlating the physical, meteorological and hydrological parameters of the catchment and gauging stations. To estimate and correlate a data for a station demands a long time series record of the neighboring stations with reliable quality, continuity and consistency.

1.2 Meteorological data

1.2.1 Principles of Data Analysis

a) Corrections to Point Measurements

Because precipitation is the input to the land phase of the hydrologic cycle, its accurate measurement is the essential foundation for quantitative hydrologic analysis. There are many reasons for concern about the accuracy of precipitation data, and these reasons must be understood and accounted for in both scientific and applied hydrological analyses.

Rain gages that project above the ground surface causes wind eddies affecting the catch of the smaller raindrops and snowflakes. These effects are the most common causes of point precipitation-measurement. Studies from World Meteorological Organization (WMO) indicate that deficiencies of 10% for rain and well over 50% for snow are common in unshielded gages. The daily measured values need to be updated by applying a correction factor K after corrections for evaporation, wetting losses, and other factors have been applied. The following equations are recommended for U.S. standard 8-Inch gauges with and without Alter wind shields.

Correction factor for unshielded rain gauges:

$$K_{ru} = 100 \exp(-4.605 + 0.062\, V_a^{0.58}) \qquad (1.1)$$

Correction factor for Alter wind shielded rain gauges:

$$K_{ru} = 100 \exp(-4.605 + 0.041\, V_a^{0.69}) \qquad (1.2)$$

Where:

V_a = Wind speed at the gage orifice in m/s (Yang et al. 1998)

Errors due to splashing and evaporation usually are small and can be neglected. However, evaporation losses can be significant in low-intensity precipitations where a considerable amount could be lost. Correction for wetting losses can be made by adding a certain amount (in the order of 0.03 – 0.10 mm) depending on the type precipitation.

Systematic errors often associated with recording type rain gauges due to the mechanics of operation of the instrument can be minimized by installing a non recording type gauge adjacent to each recording gauge to assure that at least the total precipitation is measured. Instrument errors are typically estimated as 1 – 5% of the total catch (Winter (1981)).

Although difficult to quantify and often undetected, errors in measurement and in the recording and publishing (personal errors) of precipitation observations are common. To correct the error some subjectivity is involved by comparing the record with stream flow records of the region.

b) Estimation of Missing Data

When undertaking an analysis of precipitation data from gauges where daily observations are made, it is often to find days when no observations are recorded at one or more gauges. These missing days may be isolated occurrences or extended over long periods. In order to compute precipitation totals and averages, one must estimate the missing values.

Several approaches are used to estimate the missing values. Station Average, Normal Ratio, Inverse Distance Weighting, and Regression methods are commonly used to fill the missing records. In Station Average Method, the missing record is computed as the simple average of the values at the nearby gauges. Mc Cuen (1998) recommends using this method only when the annual precipitation value at each of the neighboring gauges differs by less than 10% from that for the gauge with missing data.

$$P_x = \frac{1}{M}\left[P_1 + P_2 + \ldots + P_n\right] \qquad (1.3)$$

Where:

P_x = The missing precipitation record

P_1, P_2 , …, P_m = Precipitation records at the neighboring stations

M = Number of neighboring stations

If the annual precipitations vary considerably by more than 10 %, the missing record is estimated by the Normal Ratio Method, by weighing the precipitation at the neighboring stations by the ratios of normal annual precipitations.

$$P_x = \frac{N_x}{M}\left(\frac{P_1}{N_1} + \frac{P_2}{N_2} + \ldots + \frac{P_m}{N_m}\right) \tag{1.4}$$

Where:
N_x = Annual-average precipitation at the gage with missing values
N_1 , N_2 , …, N_m = Annual average precipitation at neighboring gauges

The Inverse Distance Method weights the annual average values only by their distances, d_m, from the gauge with the missing data and so does not require information about average annual precipitation at the gauges.

$$D = \sum_{m=1}^{m} d_m^{-b} \tag{1.5}$$

The missing value is estimated as:

$$P_x = \frac{1}{D}\sum_{m=1}^{m} d_m^{-b} N_m \tag{1.6}$$

The value of b can be 1 if the weights are inversely proportional to distance or 2, if the weights are proportional to distance squared.

If relatively few values are missing at the gauge of interest, it is possible to estimate the missing value by regression method.

c) Checking the Consistency of Point Measurements

If the conditions relevant to the recording of rain gauge station have undergone a significant change during the period of record, inconsistency would arise in the rainfall data of that station. This inconsistency would be felt from the time the significant change took place. Some of the common causes for inconsistency of record are:
1. Shifting of a rain gauge station to a new location
2. The neighbor hood of the station may have undergoing a marked change
3. Change in the immediate environment due to damages due to deforestation, obstruction, etc.
4. Occurrence of observational error from a certain date both personal and instrumental

The most common method of checking for inconsistency of a record is the Double-Mass Curve analysis (DMC). The curve is a plot on arithmetic graph paper, of cumulative precipitation collected at a gauge where measurement conditions may have changed significantly against the average of the cumulative precipitation for the same period of record collected at several gauges in the same region. The data is arranged in the reverse order, i.e., the latest record as the first entry and the oldest record as the last entry in the list. A change in proportionality between the measurements at the suspect station and those in the region is reflected in a change in the slope of the trend of the plotted points.
If a Double Mass Curve reveals a change in slope that is significant and is due to changed measurement conditions at a particular station, the values of the earlier period of the record should be adjusted to be consistent with latter period records before computation of areal averages. The adjustment is done by applying a correction factor K, on the records before the slope change given by the following relationship.

$$K = \frac{Slope\ for\ period\ AFTER\ slopechange}{Slope\ for\ period\ Before\ slopechange} \quad \textbf{(1.7)}$$

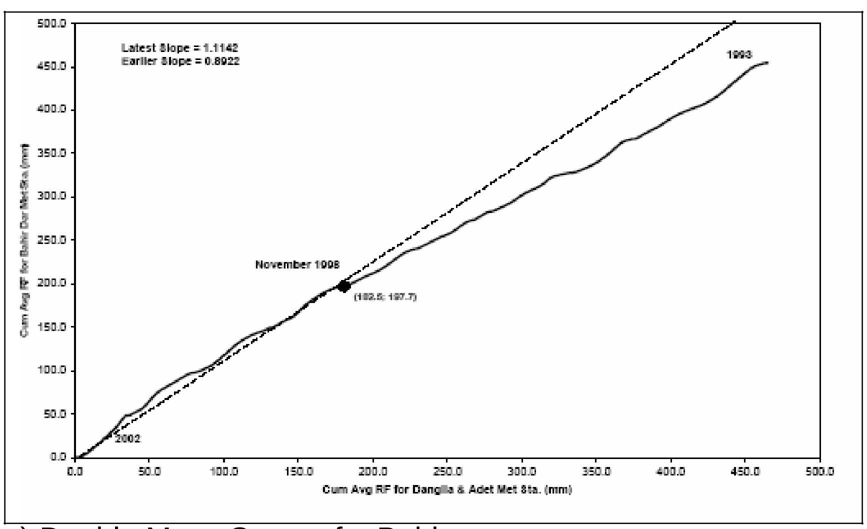
1.3 Areal Estimation

Rain gauges represent only point measurements. in practice however, hydrological analysis requires knowledge of the precipitation over an area. Several approaches have been devised for estimating areal precipitation from point measurements. The Arithmetic mean, the Thiessen polygon and the Isohyetal method are some the approaches.

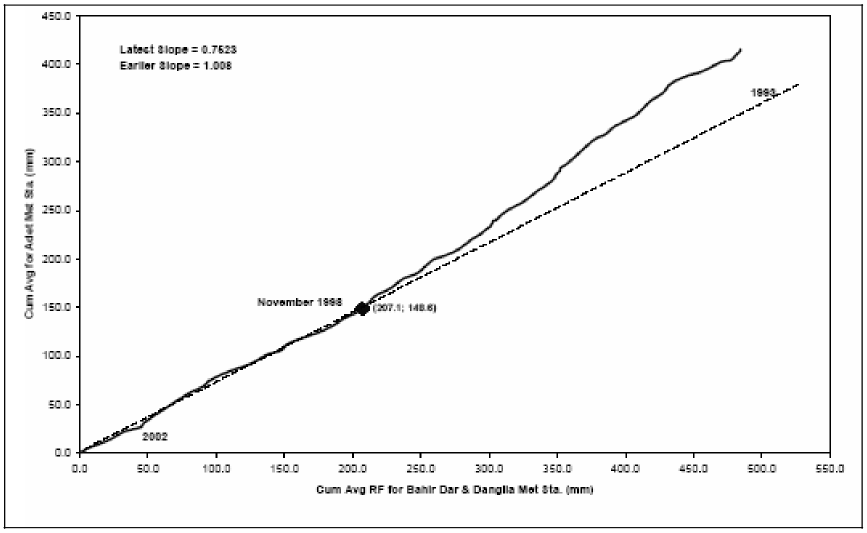
The arithmetic mean method uses the mean of precipitation record from all gauges in a catchment. The method is simple and give good results if the precipitation measured at the various stations in a catchment show little variation.

In the Thiessen polygon method, the rainfall recorded at each station is given a weightage on the basis of an area closest to the station. The average rainfall over the catchment is computed by considering the precipitation from each gauge multiplied by the percentage of enclosed area by the Thiessen polygon. The total average areal rainfall is the summation averages from all the stations. The Thiessen polygon method gives more accurate estimation than the simple arithmetic mean estimation as the method introduces a weighting factor on rational basis. Furthermore, rain gauge stations outside the catchment area can be considered effectively by this method.

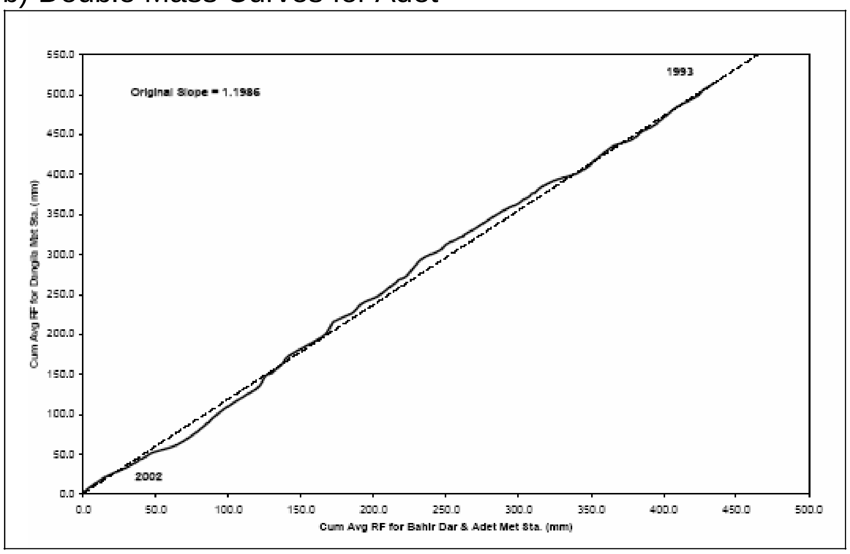
The Isohyetal method is the most accurate method of estimating areal rainfall. The method requires the preparation of the isohyetal map of the catchment from a network of gauging stations. Areas between the isohyets and the catchment boundary are measured. The areal rainfall is calculated from the product of the inter-isohyetal areas and the corresponding mean rainfall between the isohyets divided by the total catchment area.



a) Double Mass Curves for Bahir



b) Double Mass Curves for Adet



c) Double Mass Curves for Dangila
Figure 1.1: Double Mass Analysis

The updated records are computed using equation as given below:

$$P_{cx} = P_x K \qquad (1.8)$$

Where the factor K is computed by equation (g)

Table 1: Slopes of the DMC and correction factor K

Stations	Average slopes		
	Slope for period after slope change	Slope for period before slope change	K
Bahir Dar	1.114	0.892	1.249
Adet	0.752	1.008	0.746
Dangila	1.1986	1.1986	1.000

Precipitation records at Bahir Dar and Adet meteorological station beyond November 1998 should be updated by applying the correction factors 1.25 and 0.75 respectively.

1.4 Hydrological Data

The availability of stream flow data is important for the model calibration process in catchment modelling. Measured hydrograph reflects all the complexity of flow processes occurring in the catchment. It is usually difficult to infer the nature of those processes directly from the measured hydrograph, with the exception of some general characteristics such as mean times of response in particular events. Moreover, discharge data are generally available at only a small number of sites in any region where different characteristics of the catchment are lumped together.

1.4.1 Missing Data and Comparison with the Precipitation Records

The data so far collected do not indicate any missing data. The potential errors in the discharge records would affect the ability of the model to represent the actual condition of the catchment and calibrating the model parameters. If a model is calibrated using data that are in error, then the model parameter values will be affected and the prediction for other periods, which depend on the calibrated parameter values, will be affected.

Prior to using any data to a model it should be checked for consistency. In data where there is no information about missing values check for any signs that infilling of missing data has taken place is important. A common indication of such obvious signs is apparently constant value for several periods suggesting the data has been filled. Hydrographs with long flat tops also often as sign of that there has been a problem with the measurement. Outlier data could also indicate the problem.

Even though there is a danger of rejecting periods of data on the basis on these simple checks, at least some periods of data with apparently unusual behavior need to be carefully checked or eliminated from the analysis.

The available stream flow data for this analysis generally has corresponding match with the precipitation records in the area. The high flows correspond to the rainy seasons. In some of the years there are remarkably high flow records, for instance in the month of august 2000 and 2001 the flow records are as high as 100 and 89 m3/s compared to normal rainy season records which is between 30 and 65 m3/s. These data might be real or erroneous. On the other hand the values match to the days of the peak rainfall records in the area in both the cases.

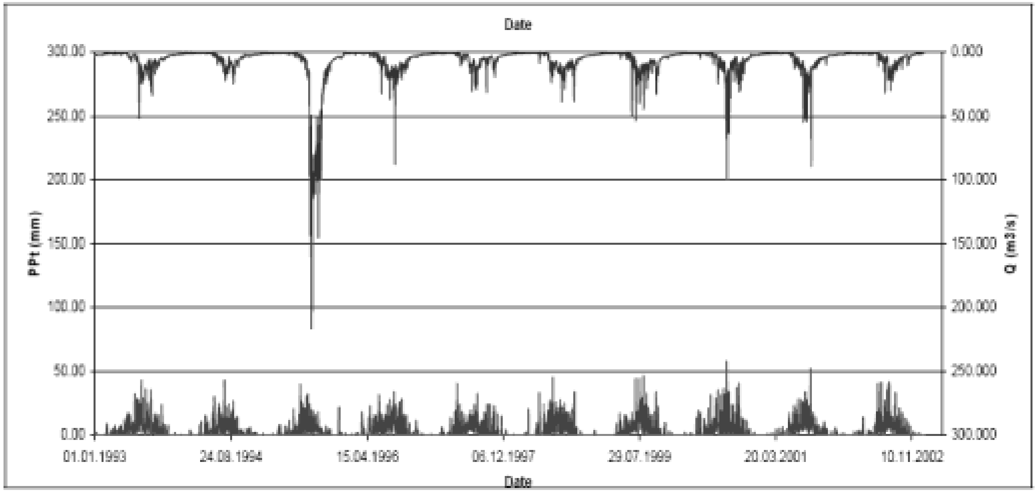


Figure 1.2: Koga stream flow record compared with the precipitation record.

However, the stream flow records of 1995 are exceptionally higher and different from flow magnitudes that had been records for long period of time at Koga River. It is not only the magnitude which is different from the normal flow record, but also it contradicts with the magnitude of the precipitation recorded during the year. These records might be modeled or transferred flows. Hence, the flow records of this year are excluded from being the part of the analysis.

2 Rainfall-Runoff Relationships (Application of Different Rainfall-Runoff Models)

2.1 *Hydrological Models*

The two classical types of hydrological models are the deterministic and the stochastic types.

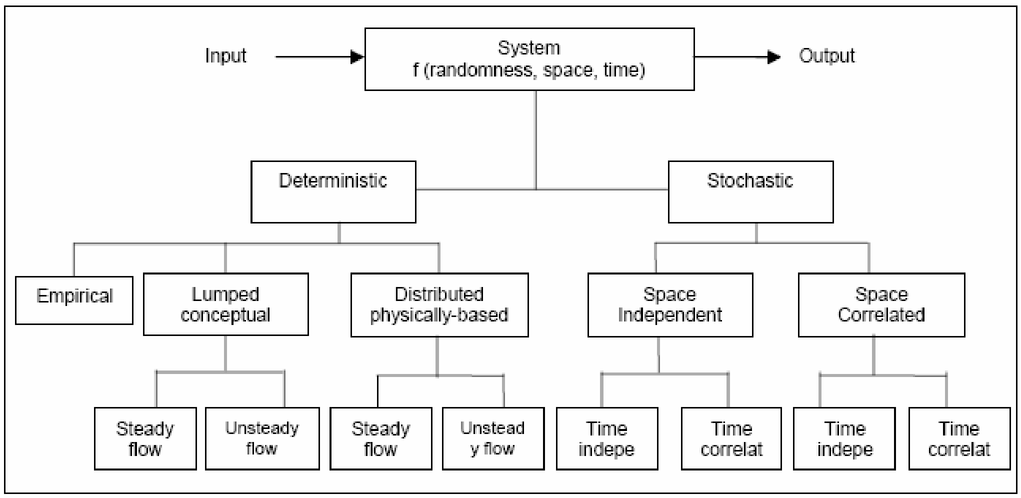


Figure 2.1: Classification of hydrological models according to process description

2.2 *Deterministic Hydrological Models*

Deterministic models permit only one outcome from a simulation with one set of inputs and parameter values. Deterministic models can be classified to whether the model gives a lumped or distributed description of the considered area, and whether the description of the hydrological processes is empirical, conceptual, or more physically-based. As most conceptual models are also lumped and as most physically based models are also distributed. The three main groups of deterministic models:

• Empirical Models (black box)
• Lumped Conceptual Models (grey box)
• Distributed Process (Physically) Description Based Models (white box)

2.2.1 Empirical (Black Box) Models

Black box models are empirical, involving mathematical equations that have been assessed, not from the physical processes in the catchment, but from analysis of concurrent input and output time series.

The first of this kind of model was the ***Rational Method*** published by the Irish engineer Thomas James Mulvaney (1822-1892) in 1851. The model was a single simple equation often used for drainage design for small suburban and urban watersheds. The equation assumes the proportionality between peak discharge, q_{pk}, and the maximum average rainfall intensity, i_{eff}:

$$q_{pk} = C_R * i_{eff} * A_D$$

Where A_D is drainage area and C_R is the runoff coefficient, which depends on watershed land use.

The equation was derived from a simplified conceptual model of travel times on basins with negligible surface storage. *The duration of the rainfall to be used in the equation is the mean intensity of precipitation for duration equal to the time of concentration and an exceedence probability of P.*

The model reflects the way in which discharges are expected to increase with area, land use and rainfall intensity in a rational way and hence its name *Rational Method*.

The scaling parameter C reflects the fact that not all the rainfall becomes discharge. The method does not attempt to separate the different effects of runoff production and runoff routing that controls the relationship between the volume of rainfall falling on the catchment in a storm and the discharge at the hydrograph peak. In addition, the constant C is required to take account of the nonlinear relationship between antecedent conditions and the profile of storm rainfall and the resulting runoff production. Thus, C is not a constant parameter, but varies from storm to storm on the same catchment, and from catchment to catchment for similar storms.

The other best known among the black box models is the unit hydrograph model which was published by Sherman (1932), who used the idea that the various time delays for runoff produced on the catchment to reach the outlet could be represented as a time distribution without any direct link to the areas involved. Because the routing procedure was linear, this distribution could be normalized to represent the response to a unit of runoff production, or effective rainfall, generated over the catchment in one time step. The method is one of the most commonly used hydrograph modelling techniques in hydrology, simple to understand and easy to apply. The unit hydrograph represents a discrete transfer function for effective rainfall to reach the basin outlet, lumped to the scale of the catchment.

Other empirical models are developed using linear regression and correlation methods used to determine functional relationships between different data sets. The relation ships are characterized by correlation coefficients and standard deviation and the parameter estimation is carried out using rigorous statistical methods involving tests for significance and validity of the chosen model.

2.2.2 Lumped Conceptual Models

Lumped models treat the catchment as a single unit, with state variables that represent average values over the catchment area, such as storage in the saturated zone. Due to the lumped description, the description of the hydrological processes cannot be based directly on the equations that are supposed to be valid for the individual soil columns. Hence, the equations are semi-empirical, but still with a physical basis. Therefore, the model parameters cannot usually be assessed from field data alone, but have to be obtained through the help of calibration.
One of the first and most successful lumped digital computer models was the Stanford Watershed model developed by Norman Crawford and Ray Linsley at Stanford University. The Stanford model had up to 35 parameters, although it was suggested that many of these could be fixed on the basis of the physical characteristics of the catchment and only a much smaller number needed to be calibrated.

2.2.3 Distributed Process Description Based Models

Another approach to hydrological processes modelling was the attempt to produce models based on the governing equations describing all the surface and subsurface flow processes in the catchment. A first attempt to outline the potentials and some of the elements in a distributed process description based model on a catchment scale was made by Freeze and Harlan (1969). The calculations require larger computers to solve the flow domain and points at the elements of the catchment.

Distributed models of this type have the possibility of defining parameter values for every element in the solution mesh. They give a detailed and potentially more correct description of the hydrological processes in the catchment than do the other model types. The process equations require many different parameters to be specified for each element and made the calibration difficult in comparison with the observed responses of the catchment.

In principle parameter adjustment of this type of model is not necessary if the process equations used are valid and if the parameters are strongly related to the physical characteristics of the surface, soil and rock. In practice the model requires effective values at the scale of the elements. Because of the heterogeneity of soil, surface vegetation establishing a link between measurements and element values is difficult. The Distributed Process Description Based Models can in principle be applied to almost any kind of hydrological problem. The development is increased over the recent years for the fact that the increase in computer power, programming tools and digital databases and the need to handle processes and predictions of runoff, sediment transport and/or contaminants.

Another reason is the need of the models for impact assessment. Changes in land use, such as deforestation or urbanization often affect only part of a catchment area.
With a distributed model it is possible to examine the effects of such land use changes in their correct spatial context by understanding the physical meaning between the parameter values and the land use changes.

Recent examples of distributed process based models include the SHE model (Abbott et al., 1986), MIKE SHE (Refsgaard and Storm, 1995), IHDM (Institute of Hydrology Distributed Model; Calver and Wood 1995), and THALES (Grayson et al. 1992), etc.

2.3 Stochastic Time Series Models

Stochastic models allow for some randomness or uncertainty in the possible outcomes due to uncertainty in input variables, boundary conditions or model parameters. Traditionally, a stochastic model is derived from a time series analysis of the historical record. The stochastic model can then be used for the generation of long hypothetical sequences of events with the same statistical properties as the historical record. In this technique several synthetic series with identical statistical properties are generated. These generated sequences of data can then be used in the analysis of design variables and their uncertainties, for example, when estimating reservoir storage requirements.

With regard to process description, the classical stochastic simulation models are comparable to the empirical, black box models. Hence, stochastic time series models are in reality composed of a simple deterministic core (the black box model) contained within a comprehensive stochastic methodology.

So, these are the broad generic classes of rainfall-runoff models, lumped or distributed; deterministic or stochastic.

The vast majority of models used in rainfall-runoff modelling are deterministic. Simpler models still offer so wide applicability and flexibility. If the interest is in simulating and predicting a one time series, for instance, run-off prediction, simple lumped parameter models can provide just as good simulation as complex process description based models.

2.4 Rational Method

One of the most commonly used for the calculation of peak flow from small areas is the rational formula given as:

$$Q_p \quad = \frac{1}{3.6} C(i_{tc,\,p})\, A \qquad (2.1)$$

Where, Q_p = peak flow (m^3/s)

C = dimensionless runoff coefficient

$i_{(tc,p)}$ = the mean intensity of precipitation (mm/h) for a duration equal to t_c and an exceedence probability p

A = drainage area in Km^2

Assumptions inherent in the Rational Formula are as follows:

- The peak flow occurs when the entire watershed is contributing to the flow
- The rainfall intensity is the same over the entire drainage area
- The rainfall intensity is uniform over a time duration equal to the time of concentration, t_c . the time of concentration is the time required for water to travel from the hydraulically most remote point of the basin to the point of interest
- The frequency of the computed peak flow is the same as that of the rainfall intensity, i.e., the 10-yr rainfall intensity is assumed to produce the 10-yr peak flow
- The coefficient of runoff is the same for all storms of all recurrence probabilities

Because of these inherent assumptions, the Rational Formula should only be applied to drainage areas smaller than 80 ha.

2.4.1 Runoff Coefficient

The ground cover and a host of other hydrologic abstractions considerably affect the coefficient. The rational equation in general relates the estimated peak discharge to a theoretical maximum of 100% runoff. The Values of C vary from 0.05 for flat sandy areas to 0.95 for impervious urban surfaces, and considerable knowledge of the catchment is needed in order to estimate an acceptable value. The coefficient of runoff also varies for different storms on the same catchment, and thus, using an average value for C, gives only a rough estimate of Q_p in small uniform urban areas. On top of this the Rational Formula has been used for many years as a basis for engineering design for small land drainage schemes and storm-water channels.

If the basin contains varying amount of different land cover or other abstractions, a coefficient can be calculated through areal weighing as shown in equation (2.2). Typical values are given in table 2.1 below.

$$Weighted\ C = \frac{\sum (C_x\, A_x)}{A_{total}} \qquad (2.2)$$

Where x = subscript designating values for incremental areas with consistent land cover

Table 2.1: Runoff coefficients for Rational formula

Type of Drainage Area	Runoff Coefficient, C*
Business: • Downtown areas Neighbor hood areas	 0.70 – 0.95 0.50 – 0.70
Residential: • Single –family areas • Multi- units, detached • Multi-units, attached • Suburban • Apartment dwelling areas	 0.30 – 0.50 0.40 – 0.60 0.60 -0.75 0.25 – 0.40 0.50 – 0.70
Industrial • Light areas • Heavy areas	 0.50 – 0.80 0.60 – 0.90
Parks, cemeteries	0.10 – 0.25
Playgrounds	0.20 – 0.40
Railroad yard areas	0.20 – 0.40
Unimproved areas	0.10 – 0.30
Lawns: • Sandy soil, flat 2% • Sandy soil, average, 2-7% • Sandy soil, steep, 7% • Heavy soil, flat, 2% • Heavy soil, average 2-7% • Heavy soil, steep, 7%	 0.05 – 0.10 0.10 – 0.15 0.15 – 0.20 0.13 – 0.17 0.18 – 0.22 0.25 – 0.35
Streets: • Asphalted • Concrete • Brick	 0.70 – 0.95 0.80 – 0.95 0.70 – 0.85
Drives and walks	0.75 – 0.85
Roofs	0.75 – 0.95

2.4.2 Rainfall intensity

Rainfall intensity, duration curve and frequency curves are necessary to use the Rational method. Regional IDF curves need to be developed for the catchment in question.

* Higher values are usually appropriate for steeply sloped areas and longer return periods because infiltration and other losses have a proportionally smaller effect on runoff in these cases

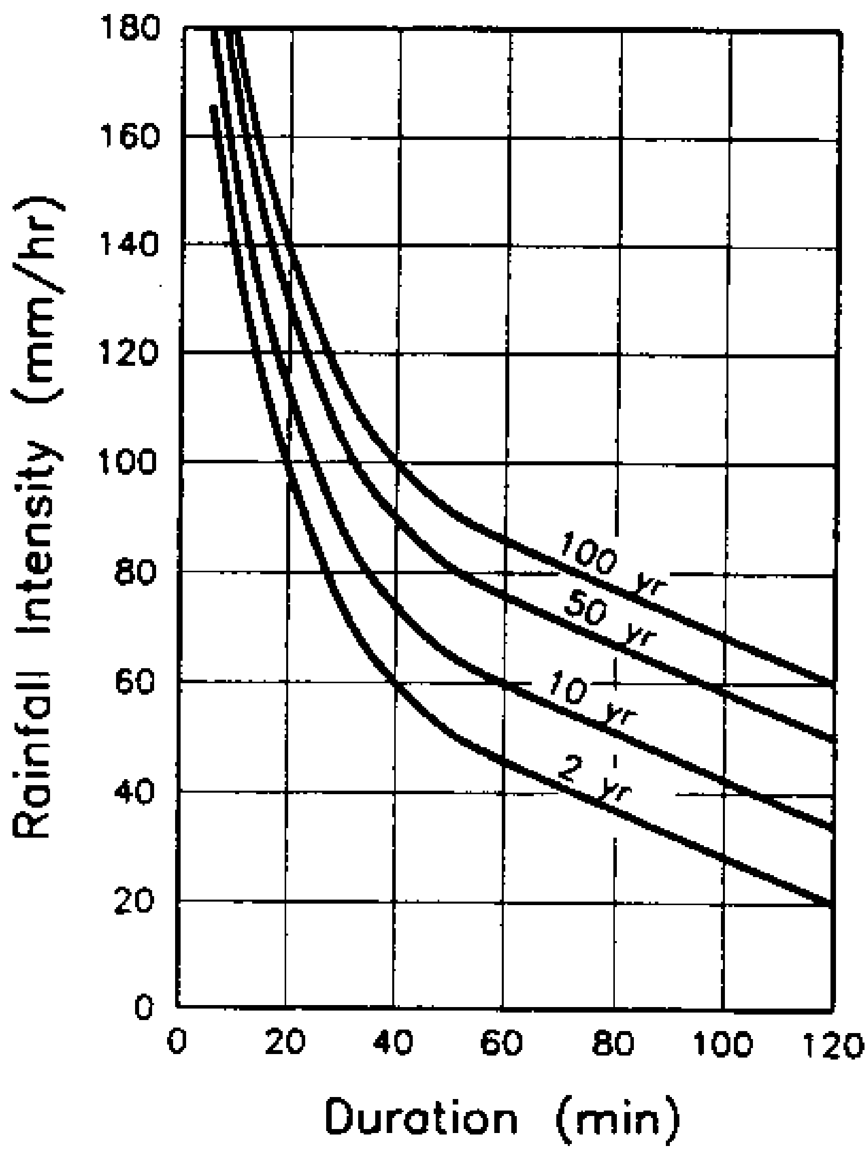


Figure 2.2: Example of IDF Curve

2.4.3 Time of Concentration

t_c is the time of concentration, the time required for rain falling at the farthest point of the catchment to flow to the measuring point of the river. Thus, after time t_c from the commencement of rain, the whole of the catchment is taken to be contributing to the flow. The value of i, the mean intensity, assumed that the rate of rainfall is constant during t_c, and that all the measured rainfall over the catchment area contributes to the peak flow. The peak flow Q_p occurs after the period t_c.

There are a number of methods that can be used to estimate time of concentration (t_c), some of which are intended to calculate the flow velocity within individual segments of the flow path (e.g. shallow concentrated flow, open channel flow, etc.) the time of concentration can be calculated as the sum of the travel times within the various consecutive flow segments.

Sheet Flow Travel Time. Sheet flow is the shallow mass of runoff on a planar surface with a uniform depth across the sloping surface. This usually occurs at the headwater of streams over relatively short distances, rarely more than about 90 m (300 ft), and possibly less than 25 m (80 ft). Sheet flow is commonly estimated with a version of the kinematic wave equation, a derivative of Manning's equation, as follows : [6]

$$T_{ti} = \frac{K_c}{I^{0.4}} \left(\frac{n\ L}{\sqrt{S}}\right)^{0.6} \tag{2.3}$$

Where:

T_{ti} = sheet flow travel time, min
n = roughness coefficient.
L = flow length, m (ft)
I = rainfall intensity, mm/hr (in/hr)
S = surface slope, m/m (ft/ft)
K_c = empirical coefficient equal to 6.943 (0.933 in English units)

Shallow Concentrated Flow Velocity. After short distances of at most 90 m (300 ft), sheet flow tends to concentrate in rills and then gullies of increasing proportions. Such flow is usually referred to as shallow concentrated flow. The velocity of such flow can be estimated using a relationship between velocity and slope as follows: [6]

$$V = k\ S_p^{0.5} \tag{2.4}$$

V = velocity, m/s (ft/s)
k = intercept coefficient
S_p = slope, percent

Open Channel and pipe flow velocity: Flow in gullies empties in to channels or pipes. Open channel flow is assumed to begin where the stream follows and defined path and becomes visible/significant. Manning's equation can be used to estimate average flow velocities in pipe and open channels.

$$V = \frac{K_c}{n} R^{2/3}\ S^{1/2} \tag{2.5}$$

Table 2.2: Intercept coefficients for velocity versus slope relationship of equation (2.5)

Land cover/flow regime	k
Forest with heavy ground litter; hay meadow (overland flow).	0.076
Trash fallow or minimum tillage cultivation; contour or strip cropped; woodland (overland flow).	0.152
Short grass pasture (overland flow).	0.213
Cultivated straight row (overland flow).	0.274
Nearly bare and untilled (overland flow); alluvial fans in western mountain regions.	0.305
Grassed waterway (shallow concentrated flow).	0.457
Unpaved (shallow concentrated flow).	0.491
Paved area (shallow concentrated flow); small upland gullies.	0.619

Table 2.3: Manning's Roughness coefficient (n) for overland flow

Surface Description	n
Smooth asphalt	0.011
Smooth concrete	0.012
Ordinary concrete lining	0.013
Good wood	0.014
Brick with cement mortar	0.014
Vitrified clay	0.015
Cast iron	0.015
Corrugated metal pipe	0.024
Cement rubble surface	0.024
Fallow (no residue)	0.05
Cultivated soils	
Residue cover $\leq$ 20%	0.06
Residue cover > 20%	0.17
Range (natural)	0.13
Grass	
Short grass prairie	0.15
Dense grasses	0.24
Bermuda grass	0.41
Woods*	
Light underbrush	0.40
Dense underbrush	0.80

* When selecting n, consider cover to a height of about 30 mm. This is the only part of the plant cover that will obstruct sheet flow

For a circular pipe flowing full, the hydraulic radius is one-fourth of the diameter. For a wide rectangular channel (w > 10 d), the hydraulic radius is approximately equal to the depth. The travel time is then calculated as follows:

$$T_{ti} = \frac{L}{60V} \tag{2.6}$$

Where:

T_{ti} = travel time for segment i, min

L = flow length for segment i, m (ft)

V = velocity for segment i, m/s (ft/s)

For small natural catchments, a formula derived from data published by Kirprich for agricultural areas could be used to give t_c in hours by the following relationship:

$$t_c\,(Hr) = 0.00025\left(\frac{L}{\sqrt{S}}\right)^{0.8} \tag{2.7}$$

Where: L = the length of the catchment along the longest river channel (in m)

S = overall catchment slope (in m/m)

Example 2.1:

Given : *The following existing and proposed land uses:*

Existing conditions (unimproved):

Land Use	*Area, ha (ac)*	*Runoff Coefficient, C*
Unimproved Grass	8.95 (22.1)	0.25
Grass	8.60 (21.2)	0.22
Total =	17.55 (43.3)	

Proposed conditions (improved):

Land Use	*Area, ha (ac)*	*Runoff Coefficient, C*
Paved	*2.20 (5.4)*	*0.90*
Lawn	*0.66 (1.6)*	*0.15*
Unimproved Grass	*7.52 (18.6)*	*0.25*
Grass	*7.17 (17.7)*	*0.22*
Total =	*17.55 (43.3)*	

Find: *Weighted runoff coefficient, C, for existing and proposed conditions.*

Solution:

Step 1: Determine Weighted C for existing (unimproved) conditions

Weighted C $= \sum (C_x A_x)/A$ $= [(8.95)(0.25) + (8.60)(0.22)] / (17.55)$

Weighted C $= 0.235$

Step 2: Determine Weighted C for proposed (improved) conditions

Weighted C = $[(2.2)(0.90) + (0.66)(0.15) + (7.52)(0.25) + (7.17)(0.22)] / (17.55)$
Weighted C = 0.315

Example 2.2:

Given: *The following flow path characteristics:*

Flow Segment	Length (m)	Slope (m/m)	Segment Description
1	25	0.005	Short grass pasture
2	43	0.005	Short grass pasture
3	79	0.006	Grassed waterway
4	146	0.008	380 mm concrete pipe

Find: *Railfall intensity,* I, = 60 mm/hr. *The time of concentration,* t_c *for the area.*

Solution:

Step 1. Calculate time of concentration for each segment.

Segment 1

Obtain Manning's n roughness coefficient

$n = 0.15$

Determine the sheet flow travel time using equation 3-3:

$T_{t1} = (6.943/I^{0.4})\ (nL/S^{0.5})^{0.6} = [6.943/(60)^{0.4}]\ [(0.15)(25)/(0.005)^{0.5}]^{0.6} = 14.6$ min.

Segment 2

Obtain intercept coefficient, k,

$k = 0.213$

Determine the concentrated flow velocity

$V = k\ S_p^{0.5} = (0.213)(0.5)^{0.5} = 0.15$ m/s

Determine the travel time

$T_{t2} = L/(60\ V) = 43/[(60)(0.15)] = 4.8$ min

Segment 3

Obtain intercept coefficient, k, **= 0.457**

Determine the concentrated flow velocity

$V = k\ S_p^{0.5} = (0.457)(0.6)^{0.5} = 0.35$ m/s

Determine the travel time

$T_{t3} = L/(60\ V) = 79/[(60)(0.35)] = 3.7$ min

Segment 4

Obtain Manning's n roughness coefficient

$n = 0.011$

Determine the pipe flow velocity

$V = (1.0/0.011)(0.38/4)^{0.67}\ (0.008)^{0.5} = 1.7$ m/s

Determine the travel time

$T_{t4} = L/(60\ V) = 146/[(60)(1.7)] = 1.4$ min

Step 2.

Determine the total travel time by summing the individual travel times:

$t_c = T_{t1} + T_{t2} + T_{t3} + T_{t4} = 14.6 + 4.8 + 3.7 + 1.4 = 24.5$ *min; use 25 minutes*

Example 2.3:

Given: *Land use conditions from example 3-1 and the following times of concentration:*

	Time of concentration, t_c (min)	*Weighted C (from example 3-1)*
Existing condition (unimproved)	*88*	*0.235*
Proposed condition (improved)	*66*	*0.315*

Find: ***The 10 - year peak flow using the Rational Formula and the IDF Curve.***

Solution:

Step 1. Determine rainfall intensity, I, from the 10-year IDF curve for each time of concentration

	Rainfall intensity, I
Existing condition (unimproved)	*48 mm/hr (1.9 in/hr)*
Proposed condition (improved)	*58 mm/hr (2.3 in/hr)*

Step 2. Determine peak flow rate, Q.

Existing condition (unimproved): $Q = CIA / K_c = (0.235)(48)(17.55)/360 = 0.55\ m^3/s\ (19.4\ ft^3/s)$

Proposed condition (improved): $Q = CIA / K_c = (0.315)(58)(17.55)/360 = 0.88\ m^3/s\ (31.2\ ft^3/s)$

2.5 SCS Curve Number Method

The SCS (now known as NRCS) peak flow method calculates peak flow as a function of drainage basin area, potential watershed storage, and the time of concentration. The graphical approach to this method can be found in TR-55. This rainfall-runoff relationship separates total rainfall into direct runoff, retention, and initial abstraction to yield the following equation for rainfall runoff:

$$Q_D = \frac{(P - 0.2S_R)^2}{P + 0.8S_R} \tag{2.8}$$

where:
- Q_D = depth of direct runoff, mm (in)
- P = depth of 24 hour precipitation, mm (in). This information is available in most highway agency drainage manuals by multiplying the 24 hour rainfall intensity by 24 hours.
- S_R = retention, mm (in)

Empirical studies found that S_R is related to soil type, land cover, and the antecedent moisture condition of the basin. These are represented by the runoff curve number, CN, which is used to estimate S_R with the following equation:

$$S_R = 25.4\left[\frac{1000}{CN} - 10\right] \tag{2.9}$$

where: CN = curve number, listed in table 3-6 for different land uses and hydrologic soil types. This table assumes average antecedent moisture conditions. For multiple land use/soil type combinations within a basin, use areal weighing (see example 3-1). Soil maps are generally available through the local jurisdiction or the NRCS.

Table 2.4: Runoff Curve Numbers for Urban areas (Average watershed conditions, $I_a = 0.2\ S_R$)

Land Use Description		Curve Numbers for Hydrologic Soil Group A	B	C	D
Fully developed urban areas (vegetation established)					
Lawns, open spaces, parks, golf courses, cemeteries, etc.					
Good condition; grass cover on 75% or more of the area		39	61	74	80
Fair condition; grass cover on 50 to 75% of the area		49	69	79	84
Poor condition; grass cover on 50% or less of the area		68	79	86	89
Paved parking lots, roofs, driveways, etc. (excl. right-of-way)					
Streets and roads		98	98	98	98
Paved with curbs and storm sewers (excl. right-of-way)		98	98	98	98
Gravel (incl. right-of-way)		76	85	89	91
Dirt (incl. right-of-way)		72	82	87	89
Paved with open ditches (incl. right-of-way)		83	89	92	93
	Average % impervious				
Commercial and business areas	85	89	92	94	95
Industrial districts	72	81	88	91	93
Row houses, town houses, and residential with lots sizes 1/8 acre or less	65	77	85	90	92
Residential: average lot size					
1/4 acre	38	61	75	83	87
1/3 acre	30	57	72	81	86
1/2 acre	25	54	70	80	85
1 acre	20	51	68	79	84
2 acre	12	46	65	77	82
Developing urban areas (no vegetation established)					
Newly graded area		77	86	91	94
Western desert urban areas:					
Natural desert landscaping (pervious areas only)		63	77	85	88
Artificial desert landscaping (impervious weed barrier, desert shrub with 1- to 2-in sand or gravel mulch and basin borders).		96	96	96	96
Cultivated agricultural land					
Fallow					
Straight row or bare soil		77	86	91	94
Conservation tillage Poor		76	85	90	93
Conservation tillage Good		74	83	88	90

Peak flow is then estimated with the following equation:

$$q_p = q_u\ A_k\ Q_D \tag{2.10}$$

where:
q_p = peak flow, m^3/s (ft^3/s)
q_u = unit peak flow, $m^3/s/km^2/mm$ ($ft^3/s/mi^2/in$)
A_k = basin area, km^2 (mi^2)
Q_D = runoff depth, mm (in)

$$q_u = 0.000431 \times 10^{C_0 + C_1 \log t_c + C_2 [\log(t_c)]^2} \tag{2.11}$$

where: C_0, C_1, C_2 = coefficients, listed in table 3-7. These are a function of the 24 hour rainfall distribution type and I_a/P. I_a/P ratios are listed in table 3-8.
t_c = time of concentration, hr
I_a = initial abstraction, mm (in)

When ponding or swampy areas occur in a basin, considerable runoff may be retained in temporary storage. The peak flow should be reduced to reflect the storage with the following equation:

$$q_a = q_p F_p \tag{2.12}$$

where: q_a = adjusted peak flow, m^3/s (ft^3/s)
F_p = adjustment factor, listed in table 2.6

This method has a number of limitations which can have an impact on the accuracy of estimated peak flows:

- Basin should have fairly homogeneous CN values.
- CN should be 40 or greater.
- t_c should be between 0.1 and 10 hr.
- I_a/P should be between 0.1 and 0.5.
- Basin should have one main channel or branches with nearly equal times of concentration.
- Neither channel nor reservoir routing can be incorporated.
- F_p factor is applied only for ponds and swamps that are not in the t_c flow path.

Example 2.4:

Given: The following physical and hydrologic conditions.

- *3.3 sq km of fair condition open space and 2.8 sq km of large lot residential.*
- *Negligible pond and swamp land.*
- *Hydrologic soil type C.*
- *Average antecedent moisture conditions.*
- *Time of concentration is 0.8 hr.*
- *24-hr, type II rainfall distribution, 10-yr rainfall of 150 mm.*

Find:* *The 10-yr peak flow using the SCS peak flow method.

Solution:

Step 1: *Calculate the composite curve number using table 3-6 and equation 3-2.*

$$CN = \Sigma (CN_x A_x)/A = [3.3(79) + 2.8(77)]/(3.3 + 2.8) = 78$$

Step 2: *Calculate the retention, S_R, using equation 3-16.*

$$S_R = 25.4(1000/CN - 10) = 25.4\,[(1000/78) - 10] = 72 \text{ mm}$$

Step 3: *Calculate the depth of direct runoff using equation 3-15.*

$$Q_D = (P-0.2S_R)^2 / (P+0.8S_R) = [150 - 0.2(72)]^2/[[150 + 0.8(72)] = 89 \text{ mm}$$

Table 2.5: Coefficients for SCS peak Discharge Method (equation 2.11)

Rainfall type	I_a/P	C_0	C_1	C_2
I	0.10	2.30550	-0.51429	-0.11750
	0.20	2.23537	-0.50387	-0.08929
	0.25	2.18219	-0.48488	-0.06589
	0.30	2.10624	-0.45695	-0.02835
	0.35	2.00303	-0.40769	-0.01983
	0.40	1.87733	-0.32274	0.05754
	0.45	1.76312	-0.15644	0.00453
	0.50	1.67889	-0.06930	0.0
IA	0.10	2.03250	-0.31583	-0.13748
	0.20	1.91978	-0.28215	-0.07020
	0.25	1.83842	-0.25543	-0.02597
	0.30	1.72657	-0.19826	0.02633
	0.50	1.63417	-0.09100	0.0
II	0.10	2.55323	-0.61512	-0.16403
	0.30	2.46532	-0.62257	-0.11657
	0.35	2.41896	-0.61594	-0.08820
	0.40	2.36409	-0.59857	-0.05621
	0.45	2.29238	-0.57005	-0.02281
	0.50	2.20282	-0.51599	-0.01259
III	0.10	2.47317	-0.51848	-0.17083
	0.30	2.39628	-0.51202	-0.13245
	0.35	2.35477	-0.49735	-0.11985
	0.40	2.30726	-0.46541	-0.11094
	0.45	2.24876	-0.41314	-0.11508
	0.50	2.17772	-0.36803	-0.09525

Table 2.6: Adjustment factor (F_p) for pond and swamp areas that are spread throughout the watershed

Area of pond or swamp (%)	F_p
0.0	1.00
0.2	0.97
1.0	0.87
3.0	0.75
5.0	0.72

Step 4: Determine I_a/P from table 3-8.

$I_a/P = 0.10$

Step 5: Determine coefficients from table 3-7.

$C_0 = 2.55323$
$C_1 = -0.61512$
$C_2 = -0.16403$

Table 2.7: I_a/P for selected rainfall depths and Curve Numbers

Rainfall	Runoff Curve Number (CN)											
(mm)	40	45	50	55	60	65	70	75	80	85	90	95
10	*	*	*	*	*	*	*	*	*	*	*	0.27
20	*	*	*	*	*	*	*	*	*	0.45	0.28	0.13
30	*	*	*	*	*	*	*	*	0.42	0.30	0.19	+
40	*	*	*	*	*	*	*	0.42	0.32	0.22	0.14	+
50	*	*	*	*	*	*	0.44	0.34	0.25	0.18	0.11	+
60	*	*	*	*	*	0.46	0.36	0.28	0.21	0.15	+	+
70	*	*	*	*	0.48	0.39	0.31	0.24	0.18	0.13	+	+
80	*	*	*	*	0.42	0.34	0.27	0.21	0.16	0.11	+	+
90	*	*	*	0.46	0.38	0.30	0.24	0.19	0.14	0.10	+	+
100	*	*	*	0.42	0.34	0.27	0.22	0.17	0.13	+	+	+
110	*	*	0.46	0.38	0.31	0.25	0.20	0.15	0.12	+	+	+
120	*	*	0.42	0.35	0.28	0.23	0.18	0.14	0.11	+	+	+
130	*	0.48	0.39	0.32	0.26	0.21	0.17	0.13	0.10	+	+	+
140	*	0.44	0.36	0.30	0.24	0.20	0.16	0.12	+	+	+	+
150	*	0.41	0.34	0.28	0.23	0.18	0.15	0.11	+	+	+	+
160	0.48	0.39	0.32	0.26	0.21	0.17	0.14	0.11	+	+	+	+
170	0.45	0.37	0.30	0.24	0.20	0.16	0.13	0.10	+	+	+	+
180	0.42	0.34	0.28	0.23	0.19	0.15	0.12	+	+	+	+	+
190	0.40	0.33	0.27	0.22	0.18	0.14	0.11	+	+	+	+	+
200	0.38	0.31	0.25	0.21	0.17	0.14	0.11	+	+	+	+	+
210	0.36	0.30	0.24	0.20	0.16	0.13	0.10	+	+	+	+	+
220	0.35	0.28	0.23	0.19	0.15	0.12	0.10	+	+	+	+	+
230	0.33	0.27	0.22	0.18	0.15	0.12	+	+	+	+	+	+
240	0.32	0.26	0.21	0.17	0.14	0.11	+	+	+	+	+	+
250	0.30	0.25	0.20	0.17	0.14	0.11	+	+	+	+	+	+
260	0.29	0.24	0.20	0.16	0.13	0.11	+	+	+	+	+	+
270	0.28	0.23	0.19	0.15	0.13	0.10	+	+	+	+	+	+
280	0.27	0.22	0.18	0.15	0.12	0.10	+	+	+	+	+	+
290	0.26	0.21	0.18	0.14	0.12	+	+	+	+	+	+	+
300	0.25	0.21	0.17	0.14	0.11	+	+	+	+	+	+	+
310	0.25	0.20	0.16	0.13	0.11	+	+	+	+	+	+	+
320	0.24	0.19	0.16	0.13	0.11	+	+	+	+	+	+	+
330	0.23	0.19	0.15	0.13	0.10	+	+	+	+	+	+	+
340	0.22	0.18	0.15	0.12	0.10	+	+	+	+	+	+	+
350	0.22	0.18	0.15	0.12	0.10	+	+	+	+	+	+	+
360	0.21	0.17	0.14	0.12	+	+	+	+	+	+	+	+
370	0.21	0.17	0.14	0.11	+	+	+	+	+	+	+	+
380	0.20	0.16	0.13	0.11	+	+	+	+	+	+	+	+
390	0.20	0.16	0.13	0.11	+	+	+	+	+	+	+	+
400	0.19	0.16	0.13	0.10	+	+	+	+	+	+	+	+

* signifies that $I_a/P = 0.50$ should be used.

\+ signifies that $I_a/P = 0.10$ should be used.

Step 6: *Calculate unit peak flow using equation 3-18.*

$$q_u = (0.000431)\left(10^{C_0 + C_1 \log t_c + C_2 (\log t_c)^2}\right)$$

$$q_u = (0.000431)\left(10^{[2.55323 + (-0.61512) \log (0.8) + (-0.16403) [\log (0.8)]^2]}\right)$$

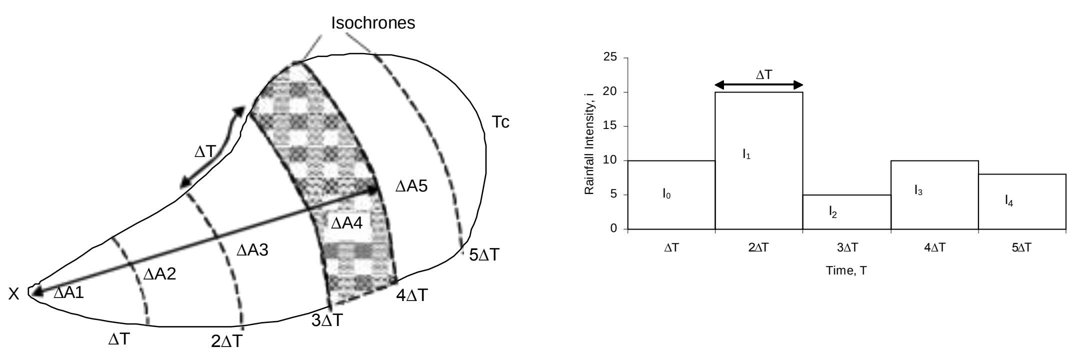
$$q_u = 0.176 \ m^3/s/km^2/mm$$

Step 7: *Calculate peak flow using equation 3-17.*

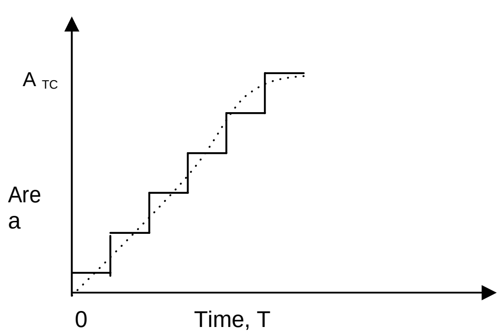
$$q_p = q_u A_t Q_D = (0.176)(3.3 + 2.8)(89) = 96 \ m^3/s$$

2.6 Time-Area Method

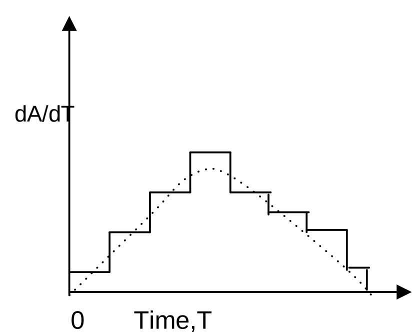
The time – area method of obtaining runoff or discharge from rainfall can be considered as an extension and improvement of the rational method. The peak discharge Q_p is the sum of flow – contributions from subdivisions of the catchment defined by time contours (called ***isochrones***), which are lines of ***equal flow – time*** to the river section where Q_p is required. The method is illustrated in Figure 2.2(a).



a) Rainfall bar graph and Catchment showing isochrones of travel time



b) Time-area curve



c) Time-area concentration curve

Figure 2.3: Time-area method

The flow from each contributing area bounded by two isochrones *(T - ΔT, T)* is obtained from the product of the mean intensity of effective rainfall (i) from time (*T - ΔT, T*) is obtained from the product of the mean intensity of effective rainfall (i) from time *T-ΔT* to time *T* and the area (*ΔA*). Thus Q_4, the flow at X at time 4h is given by:

$Q_4 = I_3\ \Delta A_1 + i_2\Delta A_2 + i_1\Delta A_3 + i_0\ \Delta A_4$

i.e.

$$Q_T = \sum_{k=1}^{T} i_{(T-K)}\ \Delta A_{(k)} \qquad (2.13)$$

As the assumption for the rational method, the whole catchment is taken to be contributing to the flow after T equals to Tc.
Hence the peak flow contributed from the whole catchment after T_c of the commencement of rain is:

$$Q_p = \sum_{k-1}^{n} i_{(n-k)} \Delta A_{(k)} \qquad (2.14)$$

Where n, the number of incremental areas between successive isochrones, is given by Tc/ΔT, and k is a counter.

The unrealistic assumption made in the rational method of uniform rainfall intensity over the whole catchment and during the whole of T_c is avoided in the time – area method, where the catchment contributions are subdivided in time. The varying intensities within a storm are averaged over discrete periods according to the isochrones time interval selected. Hence, in deriving a flood peak for design purposes, a design storm with a critical sequence of intensities can be used for the maximum intensities applied to the contributing areas of the catchment that have most rapid runoff. However, when such differences within a catchment are considered, there arises difficulty in determining T_c, the time after the commencement of the storm when, by definition, Q_p occurs.

Example 2.5: Time-Area Method

2.7 STREAM FLOW HYDROGRAPH

A hydrograph is a graphical plot of discharge of a natural stream or river versus time. The hydrograph is a result of a particular effective rainfall hyetograph as modified by basin flow characteristics. By definition, the volume of water under an effective rainfall hyetograph is equal to the volume of surface runoff.

It has three characteristic parts: *the rising limb*, *the crest segment* and *the falling limb or depletion curve.*

With reference to figure 2.4 the effective rainfall hyetograph consisting of a single block of rainfall with duration *D (T is also used in the lecture note alternatively)* shown in the upper left part of the figure produced the runoff hydrograph. The areas enclosed by the hyetograph and the hydrograph each represent the same volume, V, of water from the catchment. The maximum flow rate on the hydrograph is the peak flow, q_p, while the time from the start of the hydrograph to q_p is the time to peak, $t_{p.}$ The total duration of the hydrograph known as the base time, t_b.

The lag time, t_L is the time from the center of mass of effective rainfall to the peak of runoff hydrograph. It is apparent that $\mathbf{t_p = t_L + D/2}$, using this definition. Some define lag time as the time from center of mass of effective rainfall to the center of the runoff hydrograph.

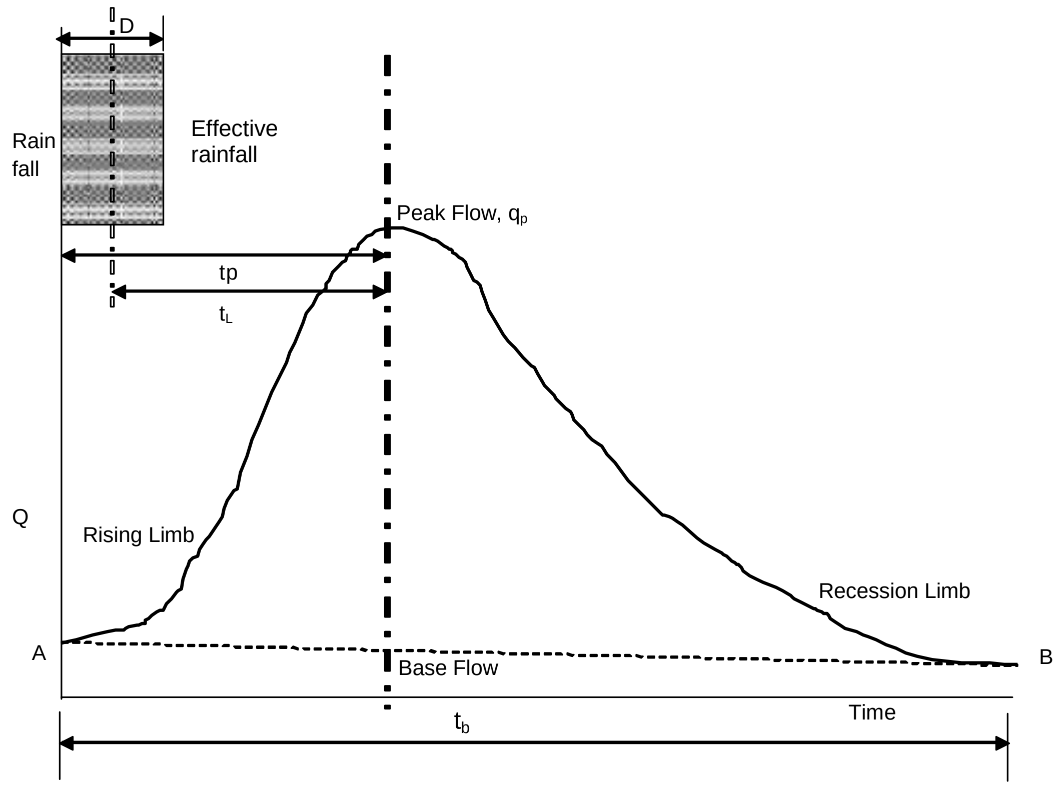


Figure 2.4: Runoff Hydrograph

2.7.1 Hydrograph Analysis

One of the major tasks of the hydrograph analysis is to produce rainfall-runoff relationships for a catchment area, for predicting runoffs as a result of certain rains which does not involve the direct measurement of runoff.

Hydrograph describes the whole time history of the changing rate of flow from a catchment due to rainfall event rather than predicting only the peak flow (Rational Method). A natural hydrograph would be the result of continuous measurements of discharge (with a recording device) producing the required relationship for any times interval, e.g. for a single flood event related to a single storm.

Hydrograph may also show mean values of events observed over a long period (of several years) as daily, monthly or annual averages in their temporal distribution over a year (or the rainy season or any other defined period of interest) giving the solution of specific problems (average storage behavior, average available discharge, etc).

Depending upon the unit of time involved, we have:

1. Annual hydrographs showing the variation of daily or weekly or 10 days daily mean flows over a year.
2. Monthly hydrographs showing the variation of daily mean flows over a month

3. Seasonal hydrographs representing the variation of the discharge in a particular season such as the monsoon season or dry season
4. Flood hydrographs or hydrographs due to storm representing stream flow due to a storm over a catchment

The hydrograph of stream flow against time has two main components, the area under the hump, labeled ***surface runoff*** (which is produced by volume of water derived from the storm event), and the broad band near the time axis, representing ***base flow*** contributed from groundwater.

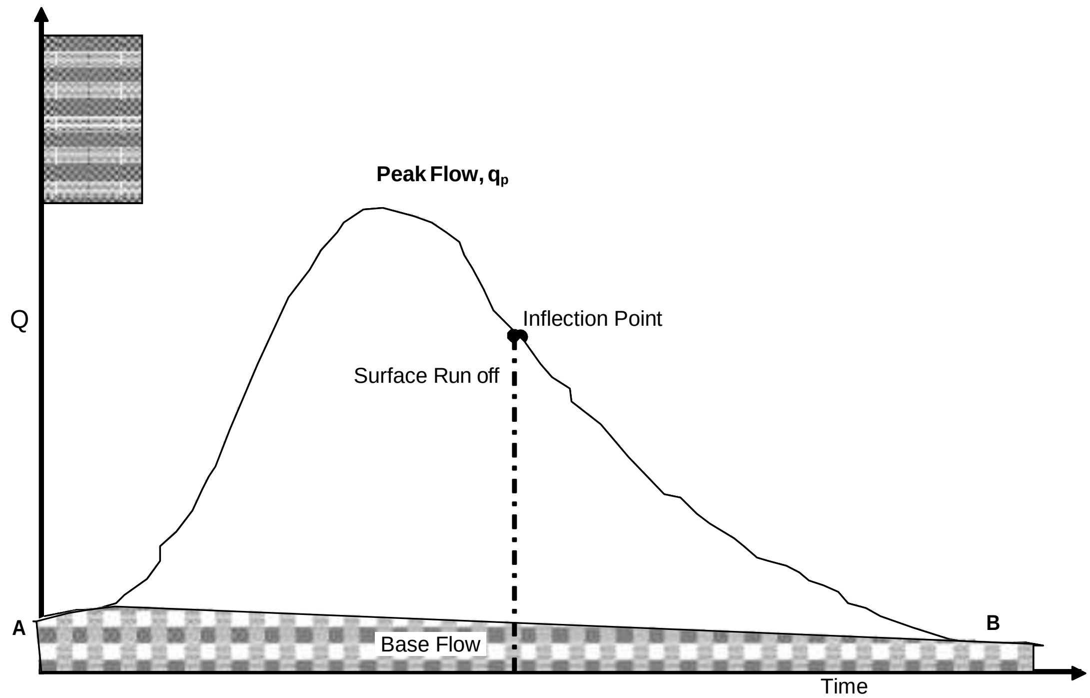


Figure 2.5: Runoff Hydrograph

At the beginning of the rainfall, the river level (and hence the discharge) is low and a period of time elapses before the river begins to rise. During this period the rainfall is being intercepted by vegetation or is soaking into the ground and making up soil-moisture deficits. The length of the delay before the river rises depends on the wetness of the catchment before the storm and on the intensity of the rainfall itself.

When the rainfall has satisfied catchment deficits and when surfaces and soils are saturated, the rain begins to contribute to the stream flow. The proportion of rainfall that finds its way into a river is being the ***effective rainfall***, the rest being lost as in the form of evaporation, detention on the ground and vegetation surface or retention in the soil. As the storm proceeds, the proportion of effective rainfall increases and that of lost rainfall decreases.

The volume of surface runoff, represented by the area under the hydrograph minus the base flow, can be considered in two main subdivisions to simplify the complex water movements over the surface and in the ground. The effective rainfall makes the immediate contribution to the rising limb from **A** to the peak of the hydrograph and, even when the rainfall stops, continue until the inflection point (condition of maximum storage). Beyond this point, it is generally considered that the flow comes from the water temporarily stored in the soil. This so-called interflow continues to provide the flow of the recession curve until the water from the whole of the effective rainfall is completely depleted at **B**.

The boundary between surface runoff and base flow is difficult to define and depends very much on the geological structure and composition of the catchment. Permeable aquifers, such as limestone and sandstone strata, sustain high base flow contributions, but impervious clays and built-up areas provide little or no base flow to a river. The base flow levels are also affected by the general climatic state of the area: they tend to be high after period of wet weather and can be very low after prolonged drought. Groundwater provides the total flow of the recession curve until the next period of wet weather.

The main aims of the engineering hydrologist are to quantify the various components of the hydrograph, by analyzing past events, in order to relate effective rainfall to surface runoff, and thereby to be able to estimate and design for future events. As a result of the complexity of the processes that create stream flow from rainfall, many simplifications and assumptions have to be made.

2.7.2 Factors affecting flood hydrograph

Table 2.8: Factors affecting flood hydrograph

Physiographic factors		Climatic factors	
1	Basin characteristics • shape • size • slope • nature of the valley • elevation • drainage density	1	Storm characteristics: precipitation, intensity, duration, magnitude and movement of storms
2	Infiltration characteristics • land use and cover • soil type and geological conditions • lakes, swamps and other storages	2	Initial losses
3	Channel characteristics: cross-section, roughness and storage capacity	3	Evapotranspiration

2.7.3 Effective Rainfall

At the start of a hydrograph analysis it is advisable to begin with a hydrograph produced by a single rain event to identify the runoff characteristics of the catchment area. Such a single event hydrograph is produced by the **net or effective rain** forming a flood wave or the direct runoff which will be super imposed on the existing flow (base flow) of the river.

The portion of rainfall that finds its way into a river is known as the ***effective rainfall***, the rest being lost in evaporation, detention on the vegetation and ground surface or retention in the soil. As the storm proceeds, the portion of effective rainfall increases and that of lost rainfall decreases.

For the purposes of correlating direct runoff hydrograph (DRH) with the rainfall, which produces the flow, it is necessary to obtain the effective rainfall hydrograph (hyetograph) (ERH) which can be obtained by deducting the losses from the total rain. At the beginning of a storm there could be considerable interception of the rainfall and initial wetting of surfaces before the rainfall become 'effective' to form surface runoff.

The loss-rate is dependent on the state of the catchment before the storm and is difficult to assess quantitatively. The two simplified methods of determining the effective rainfall are:

I. The ϕ-index method
II. The initial and continuing loss method.

I. **The ϕ-index method:** this method assumes a constant loss rate of ϕ-mm from the beginning of the rainfall event. This amount accounts for interception, evaporation loss and surface detention in pools and hollows.
II. **Initial and continuing loss rate method:** In this method all the rainfall up to the time of rise of the hydrograph is considered lost, and there is a continuing loss-rate at same level after words.

A choice between the two methods depends on knowledge of the catchment but, as the timing of the extent of initial loss is arbitrary, the fixing of the beginning of effective rainfall at the beginning of runoff in the stream neglects any lag time in the drainage process and thus somewhat unrealistic. A constant loss-rate, the ϕ-index, would therefore seem to be more readily applicable.

2.7.4 Separation of Base Flow and Runoff

The total runoff consists of direct runoff and the base flow. For hydrograph analysis the base flow has to be separated from the total runoff.

There are several methods of base flow separation. Some of them that are that are in common use are:

Straight-line method (Method-I)

The separation of the base flow is achieved by joining with a straight-line beginning of the direct runoff to a point on the recession limb representing the end of the direct runoff. Point **B** the end of the recession limb may be located by an empirical equation for the time interval N (days) from the peak to the point **B** is

$$\mathbf{N = 0.83A^{0.2}} \qquad (2.15)$$

Where A = drainage in km^2 and N in days

Method-II

In this method the base flow curve existing prior to the commencement of the surface runoff is extended till it intersects the ordinate drawn at the peak Point **C**. This point is joined to point **B** by a straight line. Segment **AC** and **CB** separate the base flow and surface runoff.

Method-III

In this method the base flow recession curve after the depletion of the floodwater is extended backwards till it intersects the ordinate at the point of inflection (**line EF**). Points **A** and **F** are joined by an arbitrary smooth curve. This method of base-flow is realistic in situations where the groundwater

contributions are significant and reach the stream quickly.
The surface runoff obtained after the base-flow separation is known as *direct runoff hydrograph* (DRH).

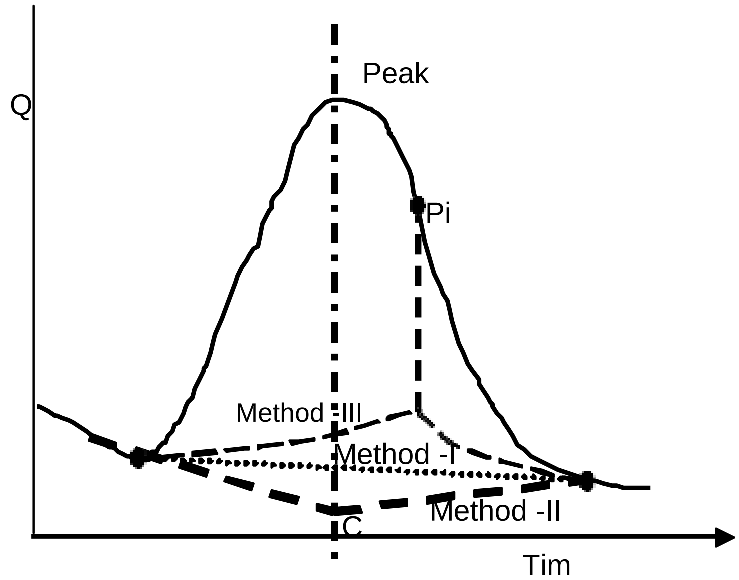


Figure 2.6: Base flow separation

2.8 The Unit Hydrograph (UH)

A major step forward in hydrological analysis was the concept of the unit hydrograph introduced by the American engineer Sherman in 1932.

The unit hydrograph (UH) of duration T is defined as the storm runoff due to unit depth (e.g. 1 mm rain depth) of effective rainfall, generated uniformly in space and time on the catchment in time T. The duration can be chosen arbitrarily so that we can have a 1h UH, a 6h UH, etc. in general a D-h hour unit hydrograph applicable to a given catchment. The definition of unit hydrograph implies the following.

1. The unit hydrograph represents the lumped response of the catchment to a unit rainfall excess of T-h duration to produce a direct-runoff hydrograph. It relates only the direct runoff to the rainfall excess. Hence the volume of water contained in the unit hydrograph must be equal to the rainfall excess. As 1 mm depth of rainfall excess is considered the area of the unit hydrograph is equal to a volume given by 1 mm over the catchment.
2. The rainfall is considered to have an average intensity of *excess rainfall* (ER) of 1/T mm/h for the durationT-h of the storm.
3. The distribution of the storm is considered to be all over the catchment.

The requirement of uniformity in areal distribution of the effective rainfall is rarely met and indeed unless the non-uniformity is pronounced, its effect is neglected.

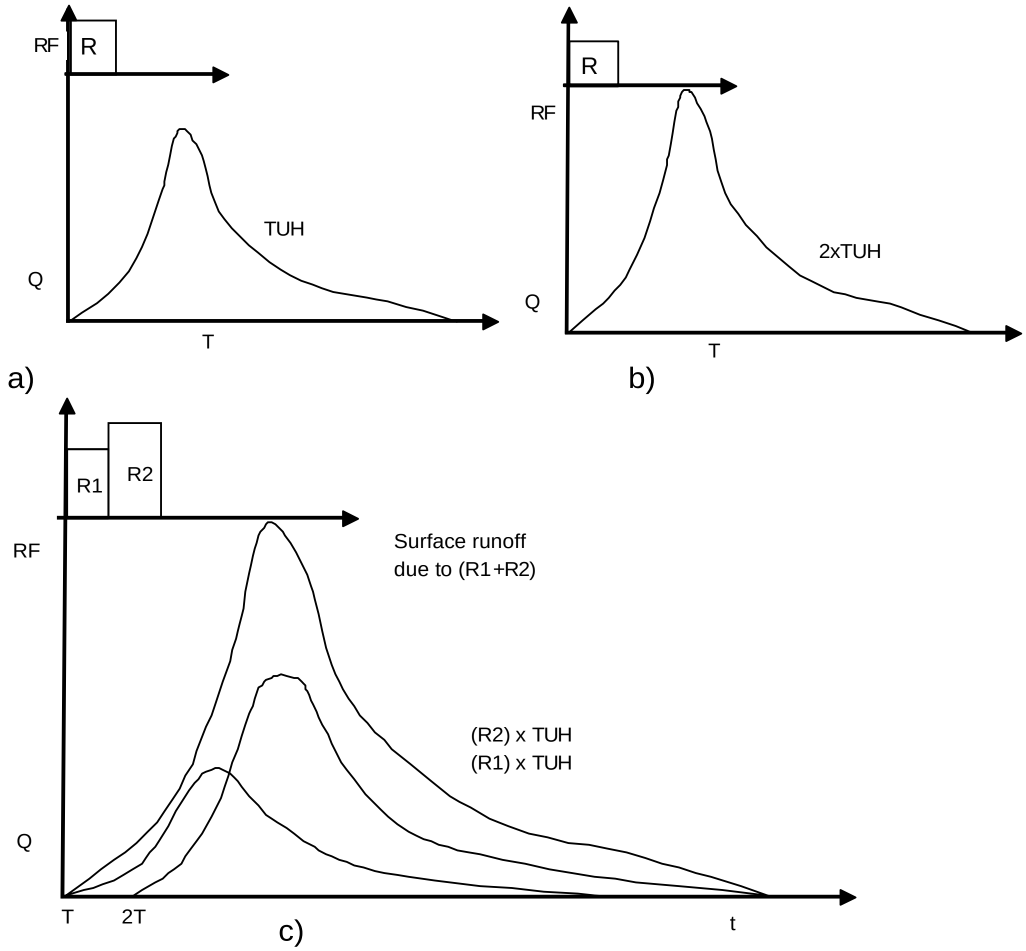


Figure 2.7: The unit hydrograph produced by 1 mm of effective rainfall

The figure shows the definition of rainfall-runoff relationship with 1mm of uniform effective rainfall occurring over a time T producing the hydrograph labeled TUH. The units of the ordinates of the t-hour unit hydrograph are m^3/s per mm of rain. The volume of water in the surface runoff is given by the area under the hydrograph and is equivalent to the 1mm depth of effective rainfall over the catchment area.

The unit hydrograph method makes several assumptions that give it simple properties assisting in its application.

1. There is a direct proportional relationship between the effective rainfall and the storm runoff. This is known as **Law of proportionality**.

Figure 1.6 b) above shows that two units of effective rainfall falling in time T produce a surface runoff hydrograph that has its ordinates twice the TUH ordinates, and similarly for any proportional value. For example, if 6.5 mm of effective rainfall fall on a catchment area in T h, then the hydrograph resulting from that effective rainfall is obtained by multiplying the ordinates of the TUH by 6.5From this law it can be seen that different rain intensities with the same duration of the rain will produce hydrographs with different magnitudes but the same base length;

however, there will be only one unit hydrograph for the same duration.

If the UH for a certain duration T is known then the runoff of any other rain of the duration T may be computed by multiplying the UH ordinates with the ratio of the given rain intensity with unit rain. i.e.:

$$Q_t = \alpha UH \text{ ; } where\ \alpha = \frac{net\ rainfall}{unit\ rainfall}$$

2. The total hydrograph of direct runoff due to **n** successive amounts of effective rainfall (for instance R_1 and R_2) is equal to the sum of the **n** successive hydrographs produced by the effective rainfall (the latter lagged by T h on the former). This is known as **Law of Superposition.** Once a TUH is available, it can be used to estimate design flood hydrographs from design storms. The law of superposition is demonstrated in Figure 1.6 c above.

3. The third property of TUH assumes that the effective rainfall-surface runoff relationship does not change with time, i.e., the same TUH always occurs whenever the unit of effective rainfall in T h is applied on the catchment. Using this **time invariance assumption,** once a TUH has been derived for a catchment area, it could be used to represent the response of the catchment whenever required.

2.8.1 Derivation of the Unit Hydrograph from single storms

The derivation of the unit hydrograph of a catchment from single storms proceeds in the following stages:

1. The rainfall records are scanned to find a storm of desired duration that gives a fairly uniform distribution in time and space. The hyetograph of this storm is constructed using a convenient uniform interval of time.
2. The base flow is separated from the hydrograph using one of the methods presented in section 1.7.3.
3. The surface runoff volume is determined as a depth of flow by numerical integration:

$$d = \frac{3.6\ \Delta t\ \Sigma Q}{A} \quad (2.16)$$

Where,

d = depth of surface runoff in mm

Δt = uniform time interval in hours at which the ordinates of the surface runoff are measured

ΣQ = sum of all ordinates of surface runoff hydrograph in m^3/s

A = catchment area in Km^2

4. The ordinates of the surface runoff hydrograph are divided by the runoff depth **d** due to the ordinates of the unit hydrograph.
5. The unit hydrograph for effective rainfall of duration T, the TUH, is plotted, and the area under the curve is checked to see if the enclosed volume is equivalent to unit effective rainfall over the area of catchment.

Example 2.6 Derivation of Unit Hydrograph

2.8.2 Changing of the Duration of the UH

There are two methods to change the duration of unit hydrograph: ***(i) by superposition from $u(T_1,t)$ to $u(T_2,t)$, where $T_2 = n*T_1$***, with n an integer > 1, hence only enlargements of multiple of T_1 are possible, ***(ii) by S-curve technique from $u(T_1,t)$ to $u(T_2,t)$ where $T_2 = \alpha * T_1$, with α a real > 0,*** hence, T_2 may be larger than T_1 but also smaller than T_1. It should be remembered that a unit hydrograph refers to unit depth of rainfall excess so if the duration is T hours the excess rainfall intensity is 1/T.

Enlargement of T by superposition:

Say, it is required to derive from $U(T_1,t)$ a unit hydrograph of $U(T_2,t)$ with $T_2 = 3T_1$. The $U(T_1,t)$ refers to a rainfall intensity of $1/T_1$ to give a unit depth, whereas $U(T_2.t)$ should refer to $1/T_2$ to give unit depth of effective rainfall. The $U(T_2.t)$ is obtained by superposition of three $U(T_1,t)$ shifted T_1 hours apart. By adding the ordinates of $U(T_1,t)$ $u(T_1, t-T_1)$ and $u(T_1,t-2T_1)$ at the corresponding times the resulting hydrograph Q(t) will refer to an effective rainfall of $3T_1 * 1/T_1 = 3$ units. Hence to get $U(T_2,t)$ all Q(t) ordinates have to be multiplied by $(i_2/i_1) = (1/T_2)/(1/T_1) = T_1/T_2 = 1/3$, to let it refer to unit depth of rainfall.

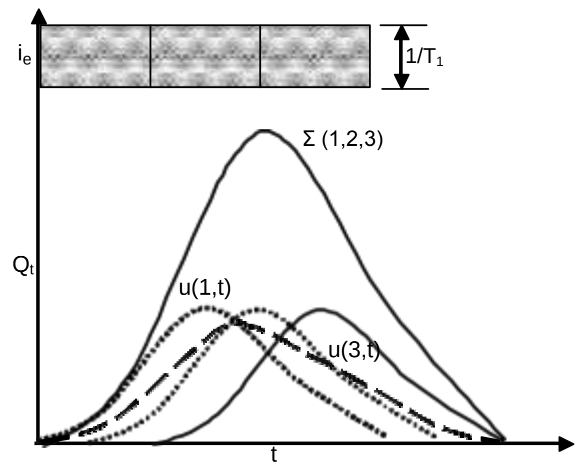


Figure 2.8: Conversion from u(1,t) to u(3,t)

Example 2.7:

S-curve:

The S-curve is the hydrograph of runoff of continuous rainfall of intensity i.e. $=1/T_1$. To derive the S-curve assume a T-hour unit hydrograph with non-zero ordinates: u_1, u_2, $u_3 \ldots, u_n$. The base length is (n+1) T. the S-curve is obtained by superposition of n T-hour UHs as shown in figure 2.9. The maximum is reached after n time of T hours. This maximum is equal to Q_s, i.e. the equilibrium discharge:

$$Q_s = 2.778 \frac{A}{T_1} \qquad (2.17)$$

Where, Q_s = the maximum rate at which an ER intensity of 1/T can drain out of the catchment of area, A (km^2)

T_1 = unit storm in hours

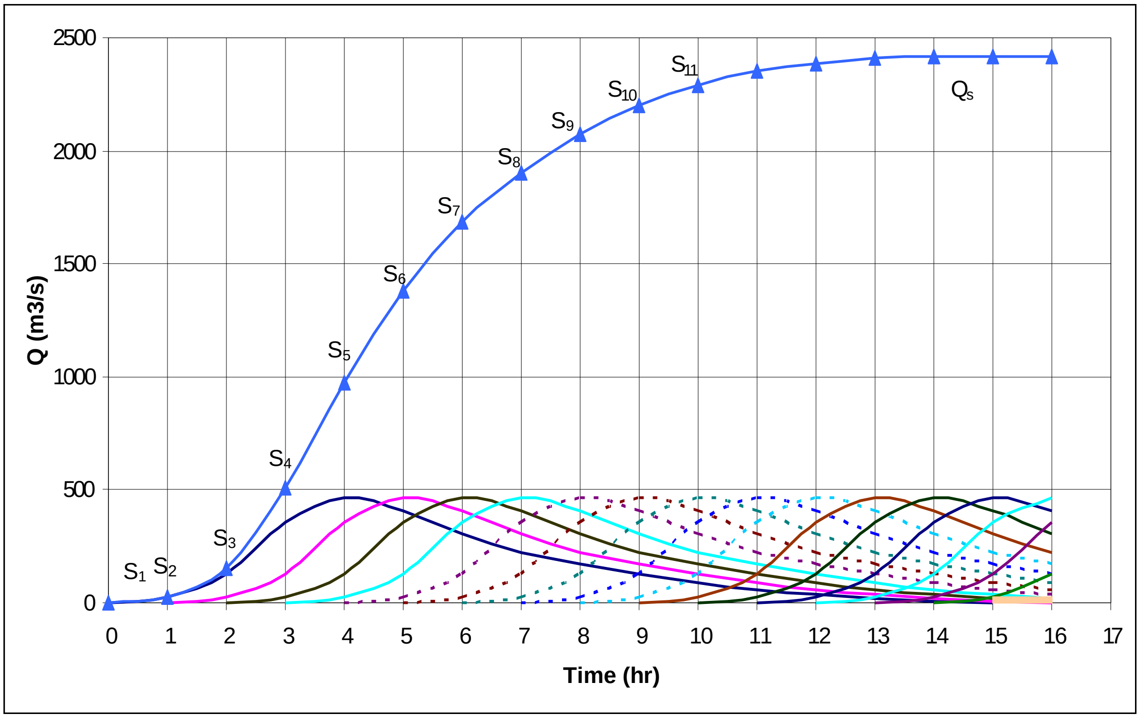


Figure 2.9: S-curve derivation

The S-curve is computed using the following scheme:

$S_1 = u_1$

$S_2 = u_1+u_2 = u_2+S_1$

$S_3 = u_1+u_2+u_3 = u_3 +S_2$

.

.

.

$S_n = u_1+u_2+u_3+\ldots+u_n = u_n+S_{n-1}$ (2.18)

So, generally;

$S_i = u_i+S_{i-1}$ for i = 1,…,n

$S_i = S_{i-1}$ for i >n

The T_2-hour UH is obtained from the difference between two S-curves distanced T_2-hours apart, corrected for the effective intensity as follows. Since the S-curve refers to continuous rain of $1/T_1$ units, the difference between the S-curves displaced by T_2 hours represents surface runoff from $(1/T_1)xT_2$. A rainfall with duration T_2 requires an intensity $i_2 = 1/T_2$ to give unit depth. Hence, the S-curve difference has to be multiplied with the ratio $i_2/i_1= (1/T_2)/(1/T_1) = T_1/T_2$ to get a unit depth in T_2 hours. Hence, $u(T_2,t)$ follows from:

$$u(T_2, t) = \frac{T_1}{T_2}(S(T_1, t) - S(T_1, t - T_2)) \quad (2.19)$$

Note that the base length follows from $T_{b2} = T_{b1} - T_1 + T_2$. The procedure is shown in figure 2.10 below.

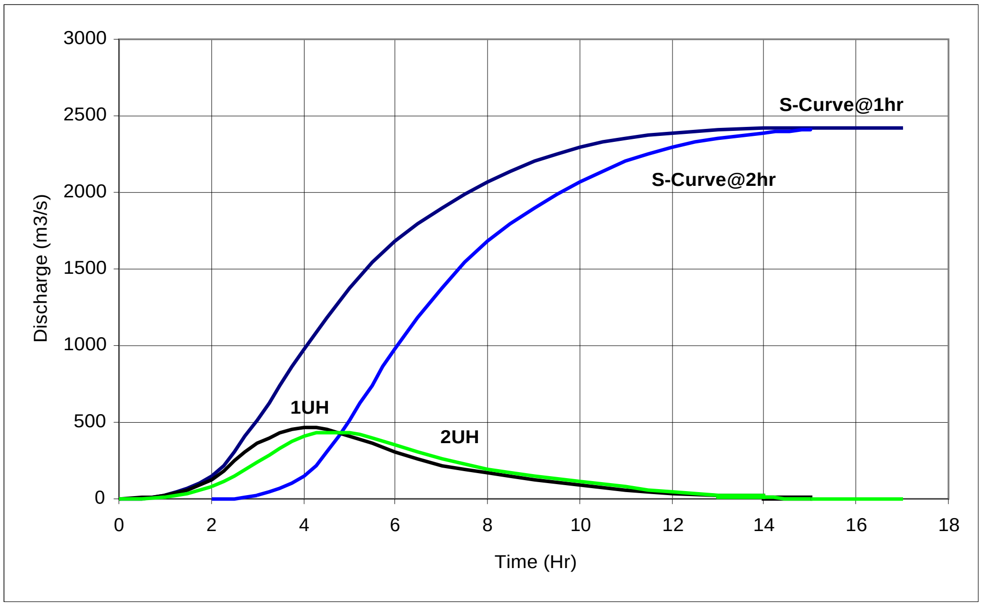


Figure 2.10:1-Hr and 2-Hr UHs from S-Curve of 1 and 2 Hrs

The errors in interpolation of UH ordinates often result in oscillation of S-curve at the equilibrium value, Q_s. This results in the derived T-h UH having an abnormal sequence of discharges (sometimes even negative values) at the tail end. The S-curve and the resulting T h UH is adjusted by smoothening the curves.

2.9 Applications of Unit Hydrograph

As the UH establishes a relationship between the DRH and ERH for a catchment, they are of immense value in the study of the hydrology of a catchment.

They are of great use in:

- The development of flood hydrograph for extreme rainfall magnitudes for use in design of hydraulic structures
- Extension of flood-flow records based on rainfall records
- The development of flood forecasting and warning systems based on rainfall.

2.10 Synthetic Unit Hydrographs

To develop unit hydrographs to a catchment, detailed information about the rainfall and the resulting flood hydrograph are needed. However, such information might be available only at a few locations and in a majority of catchments, especially those, which are at remote locations; the data could normally be scarce. In order to construct UH for such areas, empirical equations of regional validity, which relate the important hydrograph characteristics to the basin characteristics are of most important. Unit hydrographs derived from such relationships are known as synthetic unit hydrographs.

2.10.1 Snyder's method

Snyder (1938), based on a study of a large number of catchments in the Appalachian highlands of eastern United States developed a set of empirical equations for synthetic-unit hydrographs in those areas.

The most important characteristics of a basin affecting a hydrograph due to a given storm is basin lag. Actually basin lag (also known as lag time) is the time difference between the cancroids of the input (rainfall excess) and the out put (surface runoff) i.e. T_L. Physically, it represents the main time of travel of water particles from all parts of the catchment to the outlet during a given storm. Its value is determined essentially on the physical features of the catchment, such as size, length, stream density and vegetation. For its determination, however, only a few important catchment characteristics are considered. For simplicity, Snyder has used a somewhat different definition of basin lag (denoted by t_p) in his methodology. This t_p is practically of the same order of magnitude as T_L and in this section the term basin lag is used to denote Snyder's t_p.

The first of the Snyder's equation relates the basin lag t_p. Defined as the time interval from the mid point of the unit rainfall excess to the peak of the unit hydrograph (Figure 2.12 below) to the basin characteristics as:

$$\mathbf{t_p = C_t (LL_c)^{0.3}} \qquad (2.20)$$

Where,

t_p in hours

L = basin length measured along the watercourse from the basin divide to the gauging station in km.

L_c = distance along the main watercourse from the gauging station to the point opposite (or nearest) the watershed centroid in km

C_t = a regional constant representing watershed slope and storage

The value of C_t in Snyder's study ranged from 1.35 to 1.65. However, studies by many investigators have shown that C_t depends upon the region under study and wide variations with the value of Ct ranging from 0.3 to 6.0 have been reported.

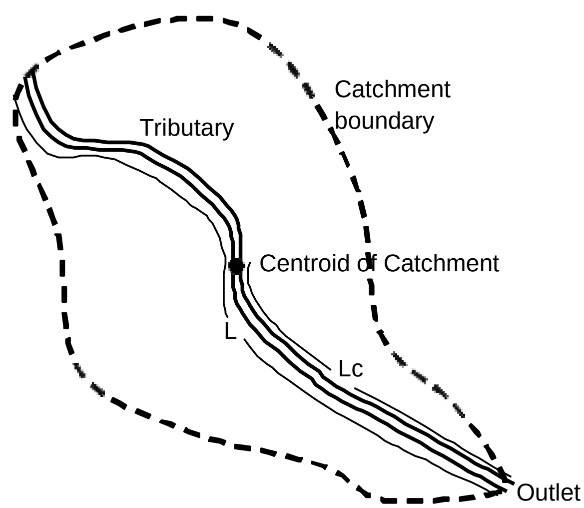


Figure 2.11: Basin characteristics

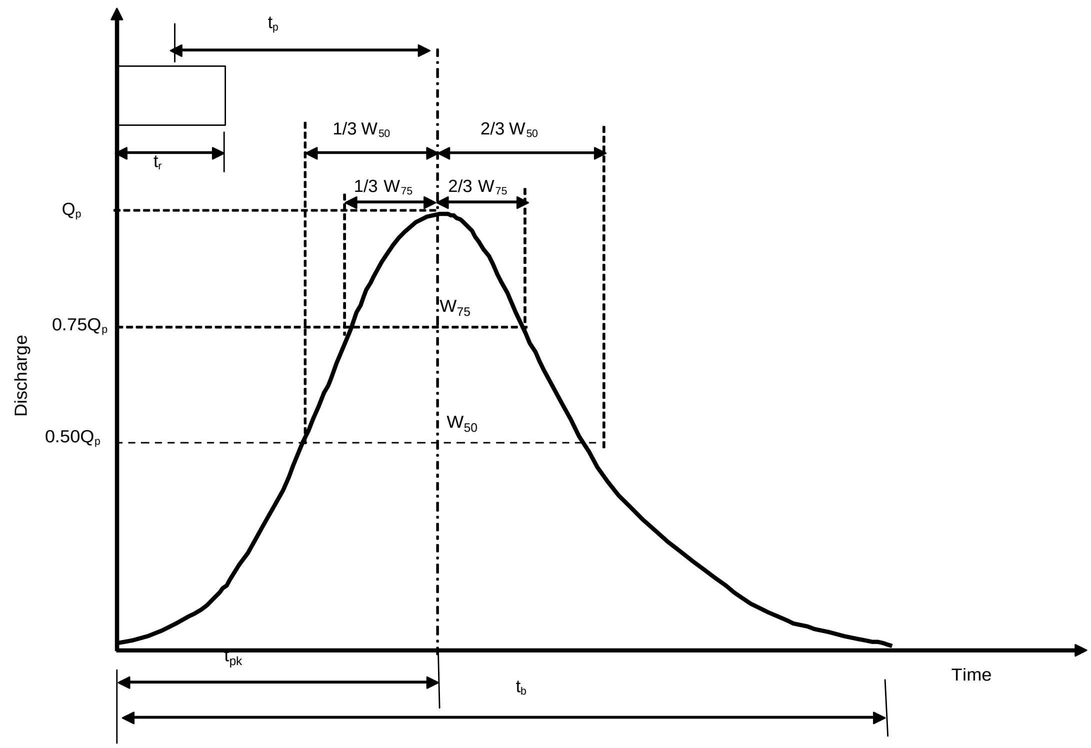


Figure 2.12: Elements of synthetic unit hydrograph

Important relationships:

Basin lag tp

$$t_p = C_{tL}\left(\frac{LL_{ca}}{\sqrt{S}}\right)^n \qquad (2.21)$$

C_{tL} and n are basin constants. (n= 0.38 and C_{tL} = 1.715, 1.03, 0.50 for mountainous, foot-hill and valley drainages of USA)

Standard duration of effective rainfall, t_r (in hours)

$$t_r = \frac{t_p}{5.5} \qquad (2.22)$$

Peak discharge Q_p (m^3/s) of unit hydrograph of standard duration t_r

$$Q_{ps} = \frac{2.78C_p A}{t_p} \qquad (2.23)$$

Where A = $km^{2,}$ C_p = regional constant

If a non-standard rainfall duration t_R h is adopted, instead of the value t_r to derive a unit hydrograph the value of the basin lag is affected. The modified basin lag is given by:

$$t'_p = t_p + \frac{t_R - t_r}{4} \qquad (2.24)$$

$$= \frac{21}{22}t_p + \frac{t_R}{4}$$

Where t'_p = basin lag in hours for an effective duration of t_R. Therefore Q_p,

$$Q_p = \frac{2.78 C_p A}{t'_p} \tag{2.25}$$

Note that when $t_R = t_r$ implies $Q_P = Q_{ps}$

The time base of unit hydrograph is given by Snyder as:

$$t_b = \left(3 + \frac{t'_p}{8}\right) days = (72 + 3t'_p)\ hours \tag{2.26}$$

This equation gives reasonable estimates of time base for large catchments; it may give excessively large values of time base for small catchments.

Taylor and Schwartz recommend

$$t_b = 5\left(t'_p + \frac{t_R}{2}\right) hours \tag{2.27}$$

With t_b taken as the next larger integer value divisible by t_R i.e. t_b is about five times the time to peak.

To assist in the sketching of unit hydrographs,

$$W_{50} = \frac{5.87}{q^{1.08}}$$

$$and\ W_{75} = \frac{W_{50}}{1.75} \tag{2.28}$$

Where W_{50} = width of unit hydrograph in hour at 50% peak discharge
W_{75} = width of unit hydrograph in hour at 75% peak discharge
$q = Q_p/A$ = peak discharge per unit catchment area in $m^3/s/km^2$

Since the coefficients C_t and C_p vary from region to region, in practical applications it is advisable that the value of these coefficients are determined from known unit hydrographs of meteorologically homogeneous catchments and then used in the basin under study. This way Snyder's equations are of use in scaling the hydrograph information from one catchment to another similar catchment.

2.11 UH from a complex storm

In nature storms are most likely occurring with changing intensities over their total duration. Natural hydrograph related to such complex storms may be considered as several superimposed hydrographs related to single storms of constant intensity forming the total given storm.

The resulting storm from the complex storm is divided into sub storms of equal duration and constant intensity. After defining the effective rain from the individual storm and computing the direct runoff hydrograph, the composite DRH is obtained.

At various time intervals 1D, 2D, 3D, … from the start of the ERH, let the ordinates of the unit hydrograph be u_1, u_2, u_3, … and the ordinates of the composite DRH be Q_1, Q_2, Q_3,….

Then;

$$Q_1 = R_1u_1$$
$$Q_2 = R_1u_2 + R_2u_1$$
$$Q_3 = R_1u_3 + R_2u_2 + R_3u_1$$
$$Q_4 = R_1u_4 + R_2u_3 + R_3u_2$$
$$Q_5 = R_1u_5 + R_2u_4 + R_3u_3 \qquad (2.29)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$

And so on.

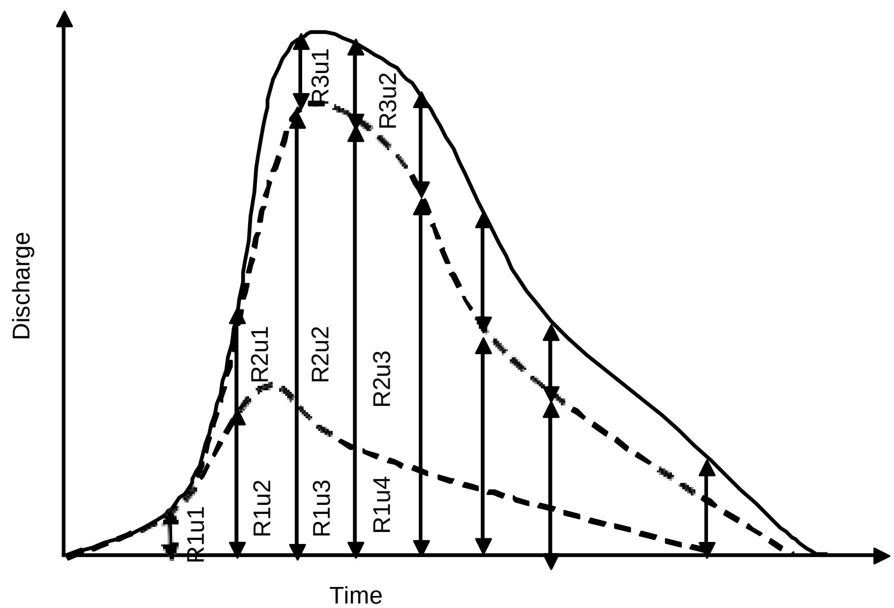


Figure 2.13: Unit hydrograph from complex storm

From equation (2.29) the values of $u_{3,}$ $u_{2,}$ and $u_{1\ ...}$ can be determined. However this method suffers from the disadvantage that the errors propagate and increases as the calculations proceeds.

The Un at higher n values (towards the end of the recession limb) can contain oscillations, if so, the final values may be smoothened to find a reasonable curvature. The reason for such behavior is the accumulation of small errors through the whole process of calculation. Matrix methods with optimization schemes are useful to reduce the number of unknown variables.

The other approach is to fit a suitable shape of UH to an average profile of the individual UH. An arithmetic mean of superimposed ordinates may be lower than the individual peaks. The proper procedure is to compute average peak flow and time to peak. The average unit hydrograph is then sketched to conform to the shape of other graphs, passing through the computed average peak and having the required unit volume.

2.12 Instantaneous unit Hydrograph (IUH)

For a given catchment a number of unit hydrographs of different durations are possible. The shape of these different UHs depends upon the value of D. As the value of D is reduced, the intensity of rainfall excess being equal to 1/D increases and the unit hydrograph becomes more skewed. A finite UH is indicated as the duration D→o. The limiting case of a unit hydrograph of zero duration is known as ***instantaneous unit hydrograph (IUH)***. This IUH is a fictitious, conceptual UH which represent the direct runoff from the catchment due to an instantaneous precipitation of the rainfall excess volume of 1 unit (cm). IUH is represented by U(t) or sometimes by U(0,t). It is a single-peaked hydrograph with a finite base width and its important properties being:

1. $0 \le u \le u(t)$ a positive value, for $t > o$;
2. $u(t) = 0$ for $t \le 0$;
3. $u(t) \rightarrow = 0$ for $t \rightarrow \infty$;
4. $\int_0^\infty u(t)dt = unit\ depth over\ the catchment;\ and$
5. Time to peak = time to the centroid of the curve.

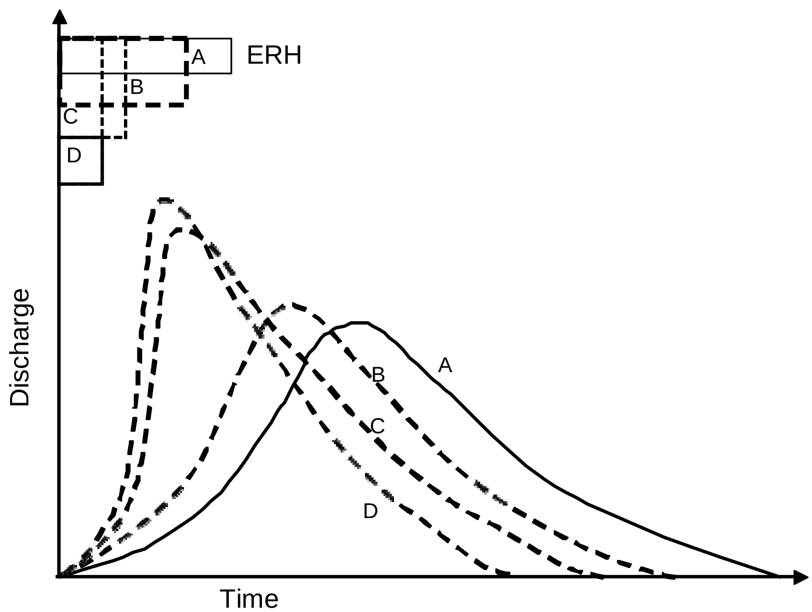


Figure 2.14: Unit hydrograph of different duration

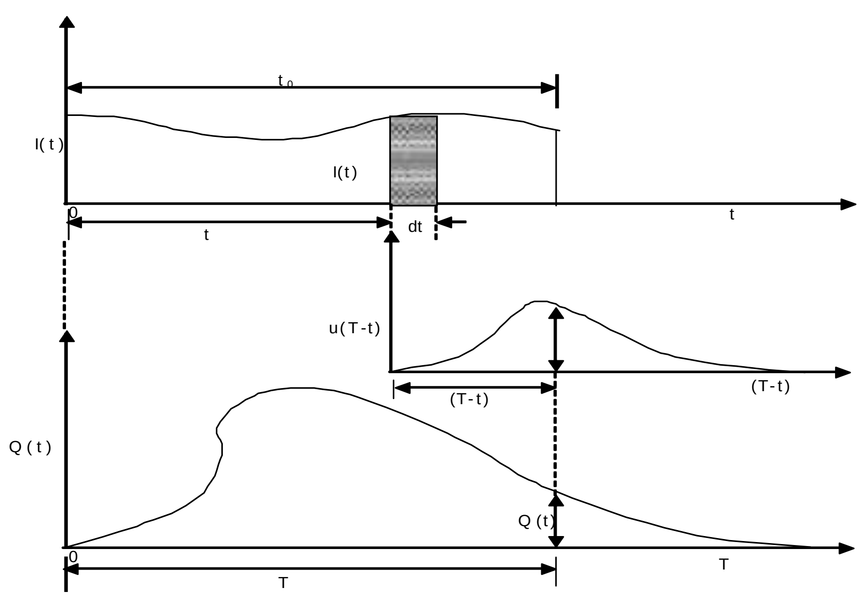


Figure 2.15: Convolution of I(t) of IUH

Consider an effective rainfall I(t) of duration t_0 applied to a catchment as shown in figure 2.15;
Each infinitesimal element of the ERH will operate on the IUH to produce a DRH whose discharge at time t is given by:

$$Q(t) = \int_0^t u(T-t)\; I\,(t)dt \qquad (2.30)$$

t' = t when t< t_0
t' = t_0 when t≥ t_0
Equation (2.30) is called the convolution integral. The main advantage of IUH is that, it is independent of the duration of ERH and thus has one parameter less than a D-h unit hydrograph. This fact and the definition of IUH make it eminently suitable for theoretical analysis of excess-runoff relationship of a catchment. For a given catchment IUH, being independent of rainfall characteristics, is indicative of the catchment storage characteristics.
Derivation of IUH
As dt is made smaller and smaller, i.e., as dt→0 an IUH results.

$$\Rightarrow u(t) = \lim_{dt\to 0}\left(\frac{S_2 - S_1}{idt}\right) = \frac{1}{i}\frac{ds}{dt} \qquad (2.31)$$

if I = 1, then u(t) = ds'/dt = slope of S-curve of intensity 1 and 2 of unit depth
Derivation of D-hour UH from IUH
From ds' = u(t)dt, integrating between two points 1 and 2

$$S'_2 - S'_1 \int_{t_1}^{t_2} u(t)\,dt \qquad 2.32)$$

If u(t) is linear within the range 1 to 2, then for small values of Δt (t_2 –t_1) by taking:

$$u(t) = \bar{u}(t) = \frac{1}{2}\left[u(t_1) + u(t_2)\right]$$
$$S'_2 - S'_1 = \frac{1}{2}\left[\left(u(t_1) - u(t_2)\right)\right](t_2 - t_1) \qquad (2.33)$$

But (S'$_2$-S'$_1$)/(t_2-t_1) = ordinate of a unit hydrograph of duration D_1 = (t_2 –t_1). Thus, in general terms, for small values of D_1, the ordinates of a D_1, the ordinates of a D_1-hour UH are obtained by the equation:

$$D_1 - hour\ UH)_t = \frac{1}{2}\left[\left(IUH\right)_t - (IUH)_{t-D_1}\right] \qquad (2.34)$$

2.13 Dimensionless Unit Hydrograph

Dimensionless unit hydrograph is used to develop a synthetic Uh in place of Synder's equations. A typical UH developed by SCS has ordinates expressed as a ratio to the peak discharge (Q/Q_p) and the abscissa as ratio of time to peak time (t/t_{pk}).
⇒ Q/Q_p = 1.0, when t/t_{pk} = 1.0

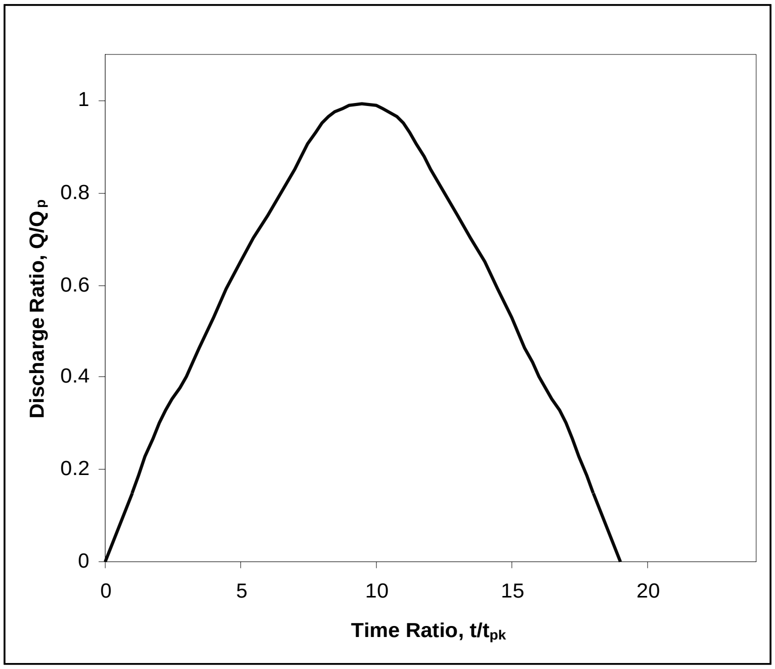


Figure 2.16: Dimensionless SCS unit hydrograph

2.14 Hydrology of Ungauged Catchments

Extrapolation of flow data to ungauged sites:

All too often the stream flow data that are available from measured gauging stations are not from location for which a project site analysis is to be made. Methods are required to develop extrapolation of measured flow data which will be representative of a given site on a stream.

In regions where stream flow does not vary with respect to the contributing drainage area flow duration curves can be plotted for the gauged sites. From these developed flow duration curves, a family of parametric flow duration curves can be developed, in which flow is plotted against the average annual runoff ($\overline{R}$) or annual discharge, $\overline{Q}$ at the respective gages for several exceedence interval percentages. A separate curve is developed for each exceedence interval used. A correlation analysis is then performed to obtain the best-fitting curve for the data taken from the measured records of stream flow.

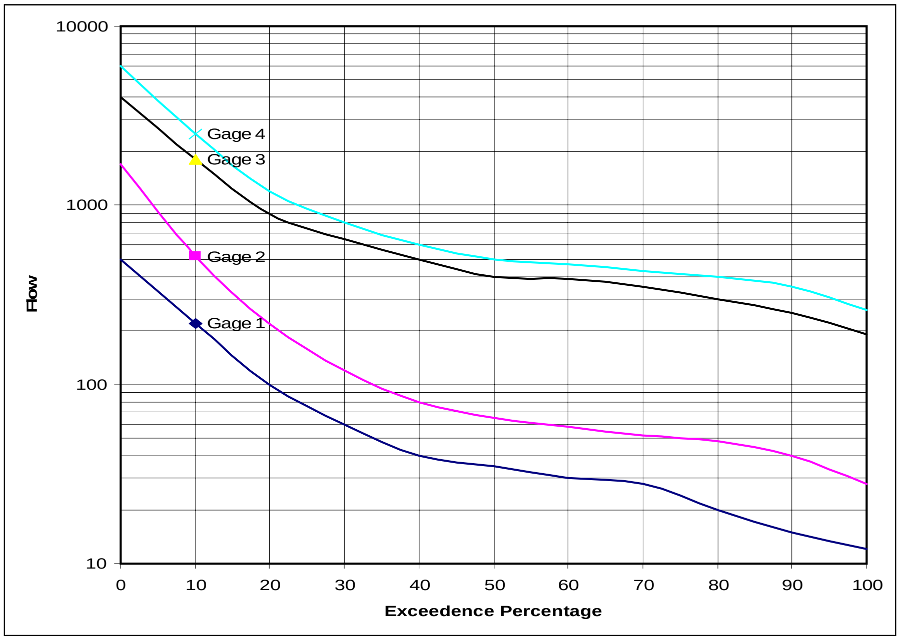


Figure 2.17: FDC for gauging stations in a homogeneous drainage basin

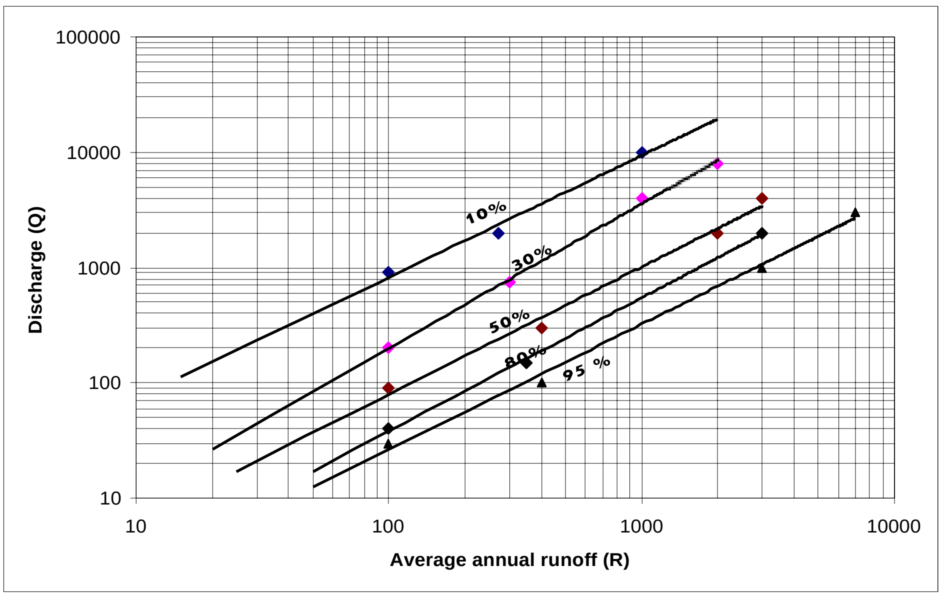


Figure 2.18: Parametric flow duration curves

Determination of average annual discharge:

To use the parametric flow duration curves effectively, it is necessary to determine the average annual discharge, $\overline{Q}$, at the point or location on the stream for which a flow analysis is to be made. Isohytal maps developed for normal annual precipitation in a river basin are helpful for determining the annual discharge. The records of precipitation and stream flow data should represent the same period of record.

Utilizing the records of average annual precipitation input to the basins at measured streams nearby or having similar hydrologic characteristics, a runoff coefficient is estimated for the drainage basin being studied. The product of this coefficient and the computed normal annual precipitation input to the basin and the basin area can be used to calculate the average annual discharge as:

$$\overline{Q} = \frac{k\,\overline{P}\,A}{T} \qquad (2.35)$$

With the average runoff annual discharge estimate it is possible to enter the parametric flow duration curve and determine values of flow for different exceedence percentages for which the parametric flow duration curve has been developed.

Example 2.7: Hydrology of Ungauged Catchment

A drainage basin has a power plant site located at the mouth of the catchment. An upstream reservoir regulates the flow at the upper portions of the drainage. The area of the hydrologic map representative of the drainage basin below the reservoir has been planimetered and given in table A below. A runoff coefficient for the basin on the annual basis is 0.65. The historic monthly flows of a nearby stream gauge on the downstream side of the stream are presented in table B. The gauge records are considered to be a good representation of seasonal variation of runoff for the ungauged portion of the river drainage basin. The outflows from the reservoir are given in table C. Using the information provided compute the river flow at its mouth that would be useful for the hydropower study. Scale of the isohytal map is 1:400,000.

Table a: Values of planimetered areas downstream of the reservoir

Avg value of precipitation between Isohytal lines (mm)	Planimetered Area (mm^2)
762	11.94
889	26.13
1016	14.45

Table b: Monthly flows for an average year in a representative gauged stream

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
No. of Days	31	28	31	30	31	30	31	31	30	31	30	31
Discharge (m3/s)	7.11	7.14	9.88	33.13	80.02	64.31	22.57	11.84	9.40	9.40	9.51	8.44

Table c: Out flow from the upper reservoir

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Discharge (m3/s)	1.42	1.27	2.27	2.83	5.66	7.08	7.08	5.66	1.98	1.84	1.70	1.56

Solution:

Step 1: Compute the average annual runoff using NAP

$$\bar{Q} = k\bar{P}A$$

$$\bar{P} = \frac{\sum PA}{\sum A} = \frac{762 * 11.94 + 889 * 26.13 + 1016 * 14.45}{11.94 + 26.13 + 14.45} = 895.07mm$$

$$\bar{Q} = 0.65 * 895.07/1000 * 52.52/(1000*1000) * 400{,}000^2 = 4888941.2\ m^3/year$$

$$\bar{Q} = \frac{4888941.2}{24*60*60} = 56.58 m^3/\sec/day$$

Step 2: Compute yearly runoff from the representative gauge

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
No. of Days	31	28	31	30	31	30	31	31	30	31	30	31
Discharge (m3/s)	7.11	7.14	9.88	33.13	80.02	64.31	22.57	11.84	9.40	9.40	9.51	8.44
Run off (m3/s)/(day)	220.41	199.92	306.28	993.9	2480.62	1929.3	699.67	367.04	282	291.4	285.3	261.64
Yearly Total (m3/s)(days)	**8317.48**											

Step 3: Compute monthly fraction of runoff

$$q_i = \frac{Runoff \;\; for\, the month}{Total\; runoff \;\; for\, the\, \mathrm{Re}cord\; period}, \; q_i(Jan) = \frac{220.41}{8317.48} = 0.026$$

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Monthly fraction	0.026	0.024	0.037	0.119	0.298	0.232	0.084	0.044	0.034	0.035	0.034	0.031

Step 4: Compute flow for the downstream portion

$$Q\,(Jan) = \frac{0.026 * 56.58}{31} = 0.05\; m^3/\sec$$

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Monthly flow (m3/s)	0.05	0.05	0.07	0.23	0.54	0.44	0.15	0.08	0.06	0.06	0.06	0.06

Step 5: Compute the total flow at the outlet

Step 6: Compute the flow duration curve

	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Monthly flow (m3/s)	0.05	0.05	0.07	0.23	0.54	0.44	0.15	0.08	0.06	0.06	0.06	0.06
Flow from upper Res(m3/s)	1.42	1.27	2.27	2.83	5.66	7.08	7.08	5.66	1.98	1.84	1.70	1.56
Total Flow	1.47	1.32	2.34	3.06	6.20	7.52	7.23	5.74	2.04	1.90	1.76	1.62

Flow	Flow Descending order	Rank	%Exceeded or Equaled
7.52	7.52	1	8.33%
7.23	7.23	2	16.67%
6.20	6.20	3	25.00%
5.74	5.74	4	33.33%
3.06	3.06	5	41.67%
2.34	2.34	6	50.00%
2.04	2.04	7	58.33%
1.90	1.90	8	66.67%
1.76	1.76	9	75.00%
1.62	1.62	10	83.33%
1.47	1.47	11	91.67%
1.32	1.32	12	100.00%

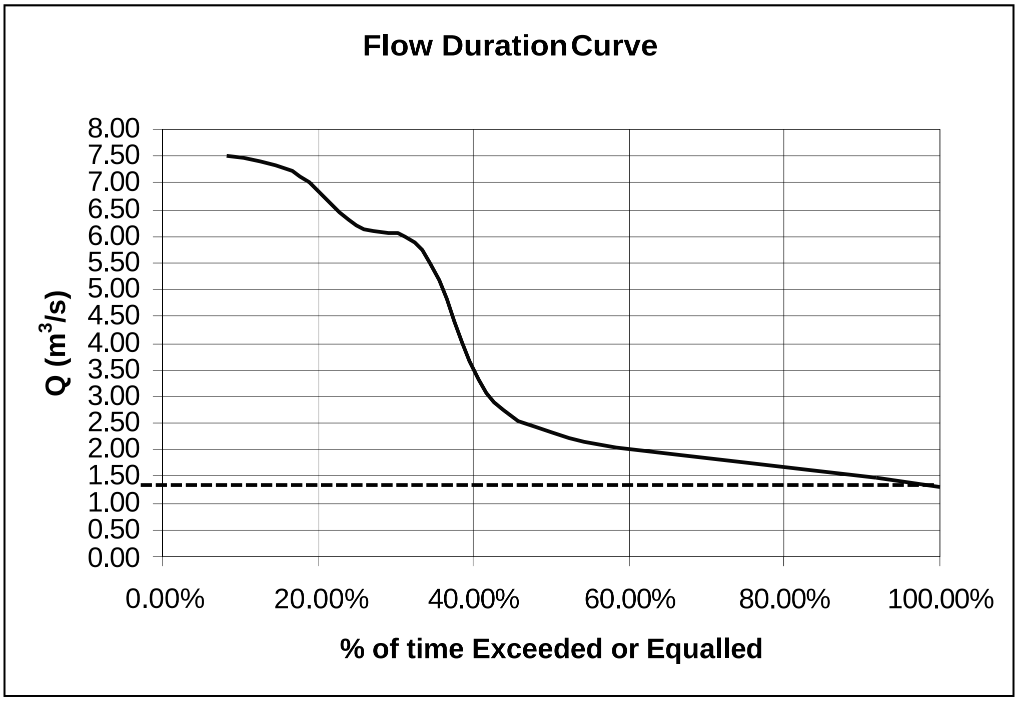


Figure 2.19: Flow duration curves

The firm flow = 1.32 m^3/sec

Tutorial Problem Set 1 (Taken from Chapter 13 – Rainfall-Runoff Relation ship, Elisabeth Shaw)

For G-2HA&B

1. The principal dimensions of a catchment are shown in Figure 1 below. If all isochrones at intervals of 1 min are arcs of circles centered at the outfall and the velocity of flow is assumed constant at 1 ms-1, draw the time area diagram for the catchment. Ignore times of entry. A design storm has the following sequence of 1-min intensities:

5.1, 6.4, 8.1, 10. 7, 14.1, 19.6, 30.3, 43.8, 28.0, 20.8, 12.3 and 4. 5 mm h-1

Assuming a runoff coefficient of 0.25, estimate the peak rate of flow at the outfall.

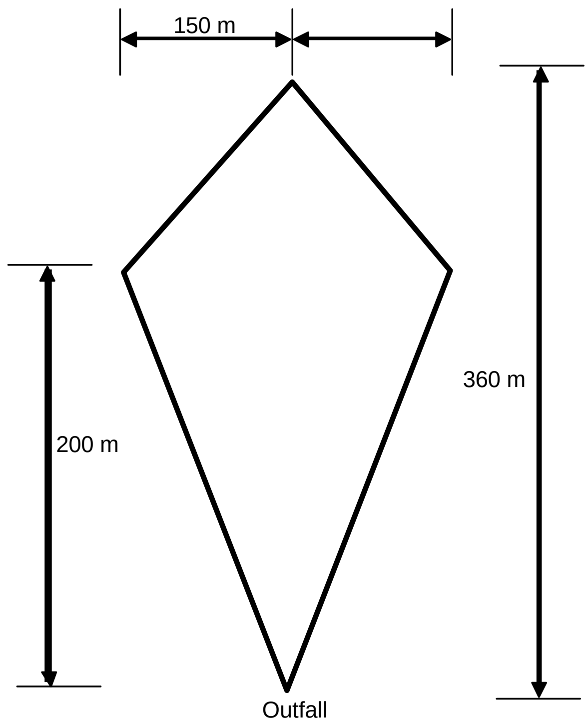


Figure 1

2. The 1-h 1 mm unit hydrograph for a small catchment is given in the Table 1 below. Determine the peak of the hydrograph that should result from the following storm: 5 mm in the first hour, no rain in the next 30 min, and 8 mm of rain in the next final hour. Assume a loss rate of 3 mmh-1 in the first hour and 2 mmh-1 for the remainder of the storm

Table 1

Time (h)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Flow(m3/s)	0	0.9	3.0	5.5	4.3	3.0	1.9	1.0	0.4	0

3. During a notable storm, rainfall measurements were made at five stations in a particular river catchment. Given the ordinates of the 5-h (1mm) unit hydrograph shown in Table 2 below, derive the outflow hydrograph of the storm for the gauging station at the river outlet, assuming 80% of the total precipitation is lost at a constant rate.

(Note: Refer Figure P4 on Page 543 from Elisabeth Shaw)

Table 2

Time(h)	0	5	10	15	20	25	30	35	40
U(m^3s^{-1})	0	0.8	5.0	7.5	5.0	2.7	2.0	1.6	1.3
Time (h)	45	50	55	60	65	70	75	80	
U(m^3s^{-1}	1.1	0.9	0.8	0.6	0.4	0.3	0.1	0	

4. An acceptable 1-h unit hydrograph (10 mm) has been derived for a catchment. Its ordinates are shown in Table 3. What is the approximate area of the catchment? Determine the peak flow that would result from a storm whose effective rainfall, assumed over the whole catchment, is given in Table 4.

Table 3

Time (h)	0	1	2	3	4	5	6	7
u(t) (m^3s^{-1})	0	12	35	24	16	8	3	0

Table 4

Time (h)	0	1	2	30
Total accumulated effective rainfall (mm)	0	6	26	35

5. The ordinates of the 1-h unit hydrograph of a catchment area are summarized in Table 5 below.

Table 5

Time(h)	0	1	2	3	4	5	6	7
TUH Ordinate (m^3s^{-1})	0	0.8	5.0	7.5	5.0	2.7	2.0	1.6
Time (h)	8	9	10	11	12	13	14	15
TUH Ordinate (m^3s^{-1}	1.1	0.9	0.8	0.6	0.4	0.3	0.1	0

a) Derive the S-Curve for the catchment
b) Use the S-Curve to obtain the 2-h unit hydrograph; and
c) Forecast the peak runoff that would result from storm in which the effective rainfall totals in two consecutive 2-h periods were 20 mm and 5 mm.

Suggested solution for Tutorial Problem Set 1 (Taken from Chapter 13 – Rainfall-Runoff Relation ship, Elisabeth Shaw)

1. Time Area Method

	Intensity					
Area	I1	I2	I3	I4	I5	I6
A1	A1I1					
A2	A2I1	A1I2				
A3	A3I1	A2I2	A1I3			
A4	A4I1	A3I2	A2I3	A1I4		
A5	A5I1	A4I2	A3I3	A2I4	A1I5	
A6	A6I1	A5I2	A4I3	A3I4	A2I5	A1I6
		A6I2	A5I3	A4I4	A3I5	A2I6
			A6I3	A5I4	A4I5	A3I6
				A6I4	A5I5	A4I6
					A6I5	A5I6
						A6I6

Intensity	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12
mm/hr	5.1	6.4	8.1	10.7	14.1	19.6	30.3	43.8	28.0	20.8	12.3	4.5

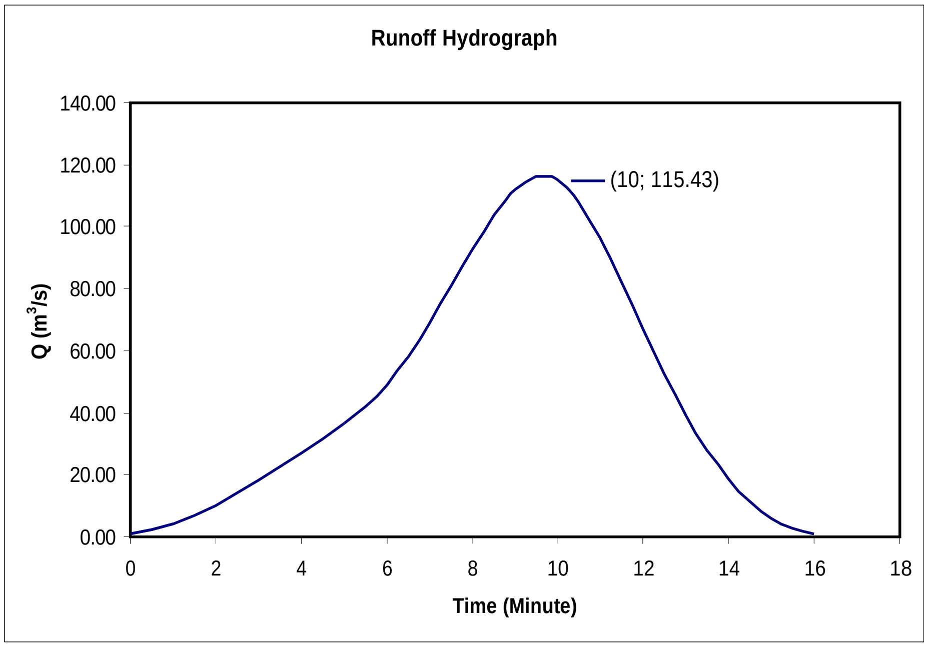
Option-I:

Max Sum	64.000	89.200	126.600	146.500	**156.600**	154.800	139.700

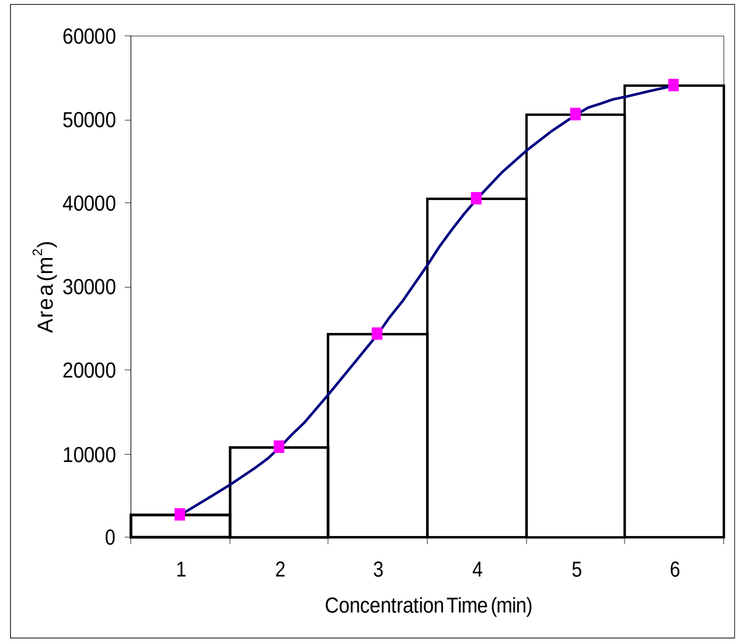
AiIi	AiIi	AiIi	AiIi	AiIi	AiIi	Q_p (l/s)
38070	0	0	0	0	0	2.64
114210	52920	0	0	0	0	11.61
190350	158760	81810	0	0	0	29.93
228420	264600	245430	118260	0	0	59.49
142762.5	317520	409050	354780	75600	0	90.26
47587.5	198450	490860	591300	226800	56160	111.89
0	66150	306787.5	709560	378000	168480	**113.12**
0	0	102262.5	443475	453600	280800	88.90
0	0	0	147825	283500	336960	53.35
0	0	0	0	94500	210600	21.19
0	0	0	0	0	70200	4.88
0	0	0	0	0	0	0.00

Option –II:

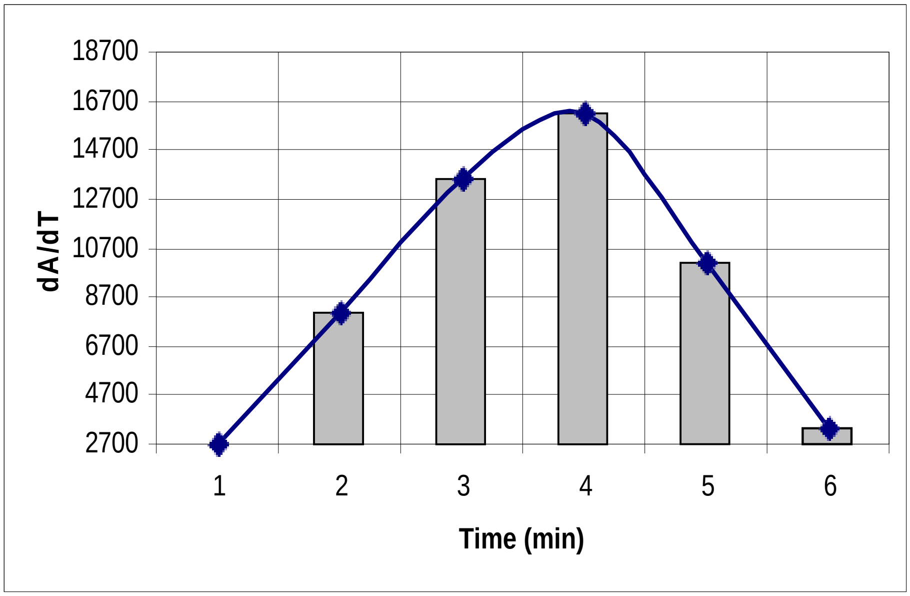
A*I	A*I	A*I	A*I	A*I	A*I	A*I	A*I	A*I	A*I	A*I	A*I	**Q_p (l/s)**
13770.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.96
41310.0	17280.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.07
68850.0	51840.0	21870.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	9.90
82620.0	86400.0	65610.0	28890.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.30
51637.5	103680.0	109350.0	86670.0	38070.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	27.04
17212.5	64800.0	131220.0	144450.0	114210.0	52920.0	0.0	0.0	0.0	0.0	0.0	0.0	36.45
0.0	21600.0	82012.5	173340.0	190350.0	158760.0	81810.0	0.0	0.0	0.0	0.0	0.0	49.16
0.0	0.0	27337.5	108337.5	228420.0	264600.0	245430.0	118260.0	0.0	0.0	0.0	0.0	68.92
0.0	0.0	0.0	36112.5	142762.5	317520.0	409050.0	354780.0	75600.0	0.0	0.0	0.0	92.77
0.0	0.0	0.0	0.0	47587.5	198450.0	490860.0	591300.0	226800.0	56160.0	0.0	0.0	111.89
0.0	0.0	0.0	0.0	0.0	66150.0	306787.5	709560.0	378000.0	168480.0	33210.0	0.0	**115.43**
0.0	0.0	0.0	0.0	0.0	0.0	102262.5	443475.0	453600.0	280800.0	99630.0	12150.0	96.66
0.0	0.0	0.0	0.0	0.0	0.0	0.0	147825.0	283500.0	336960.0	166050.0	36450.0	67.42
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	94500.0	210600.0	199260.0	60750.0	39.24
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	70200.0	124537.5	72900.0	18.59
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	41512.5	45562.5	6.05
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15187.5	1.05



Runoff Hydrograph
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
Q (m3/s)
0
2
4
6
8
10
12
14
16
18
Time (Minute)
(10; 115.43)

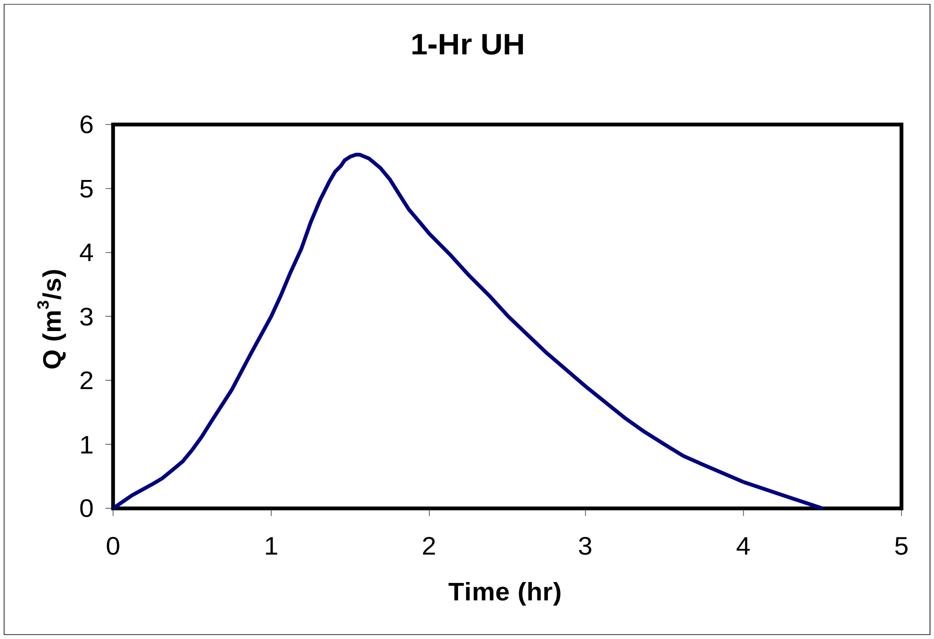


60000
50000
40000
30000
20000
10000
0
Area (m2)
1
2
3
4
5
6
Concentration Time (min)



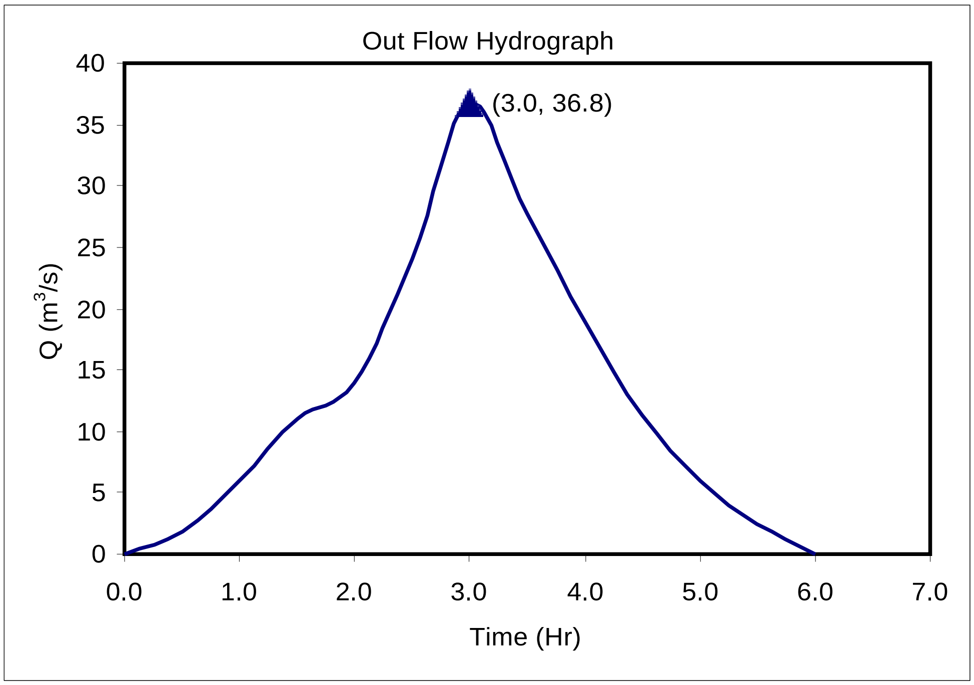
2. Determination of Peak of the hydrograph

Time (h)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
Flow(m3/s)	0	0.9	3	5.5	4.3	3	1.9	1	0.4	0



Time (Minute)	RF (mm)	Loss Rate (mm/hr)	Loss (mm)	Effective RF
60	5	3	3	2
30	0	2	1	0
60	8	2	2	6

Time (h)	U(m^3/s)	2 mm	0 mm	6 mm				Outflow
0.0	0.0	2			0			0
0.5	0.9	2	0		1.8	0		1.8
1.0	3.0	2	0		6	0		6
1.5	5.5	2	0	6	11	0	0	11
2.0	4.3	2	0	6	8.6	0	5.4	14
2.5	3.0	2	0	6	6	0	18	24
3.0	1.9	2	0	6	3.8	0	33	**36.8**
3.5	1.0	2	0	6	2	0	25.8	27.8
4.0	0.4	2	0	6	0.8	0	18	18.8
4.5	0.0	2	0	6	0	0	11.4	11.4
5.0			0	6		0	6	6
5.5				6			2.4	2.4
6.0				6			0	0

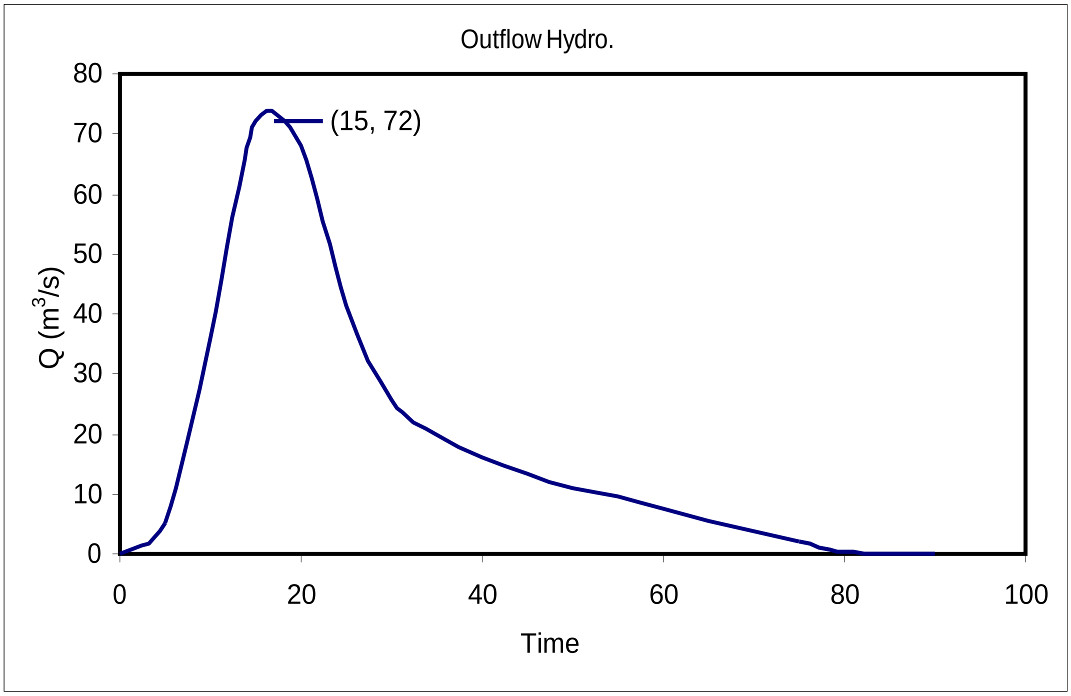


3. Derive the outflow hydrograph of the storm for the gauging station at the river outlet, assuming 80% of the total precipitation is lost at a constant rate

Time (Hr)	Average R (mm)	Loss	Effec. RF (mm)	By TheissonPolygon Method	
5	32	25.6	6.4	32.1875	6.4375
5	24	19.2	4.8	24.0625	4.8125

Time(h)	0	5	10	15	20	25	30	35	40
U(m^3s^{-1})	0	0.8	5.0	7.5	5.0	2.7	2.0	1.6	1.3
Time (h)	45	50	55	60	65	70	75	80	
U(m^3s^{-1}	1.1	0.9	0.8	0.6	0.4	0.3	0.1	0	

Time(h)	U(m^3s^{-1})	R1 = 6.4 mm	R2 = 4.8 mm	Outflow Hydro.	R1 = 6.4375 mm	R2 = 4.8125 mm	Outflow Hydro.
0	0	0		0	0.00		0.00
5	0.8	5.12	0	5.12	5.15	0.00	5.15
10	5	32	3.84	35.84	32.19	3.85	36.04
15	7.5	48	24	**72**	48.28	24.06	**72.34**
20	5	32	36	68	32.19	36.09	68.28
25	2.7	17.28	24	41.28	17.38	24.06	41.44
30	2	12.8	12.96	25.76	12.88	12.99	25.87
35	1.6	10.24	9.6	19.84	10.30	9.63	19.93
40	1.3	8.32	7.68	16	8.37	7.70	16.07
45	1.1	7.04	6.24	13.28	7.08	6.26	13.34
50	0.9	5.76	5.28	11.04	5.79	5.29	11.09
55	0.8	5.12	4.32	9.44	5.15	4.33	9.48
60	0.6	3.84	3.84	7.68	3.86	3.85	7.71
65	0.4	2.56	2.88	5.44	2.58	2.89	5.46
70	0.3	1.92	1.92	3.84	1.93	1.93	3.86
75	0.1	0.64	1.44	2.08	0.64	1.44	2.09
80	0	0	0.48	0.48	0.00	0.48	0.48
90			0	0		0.00	0.00



4. Determine the peak flow that would result from a storm whose effective rainfall, assumed over the whole catchment

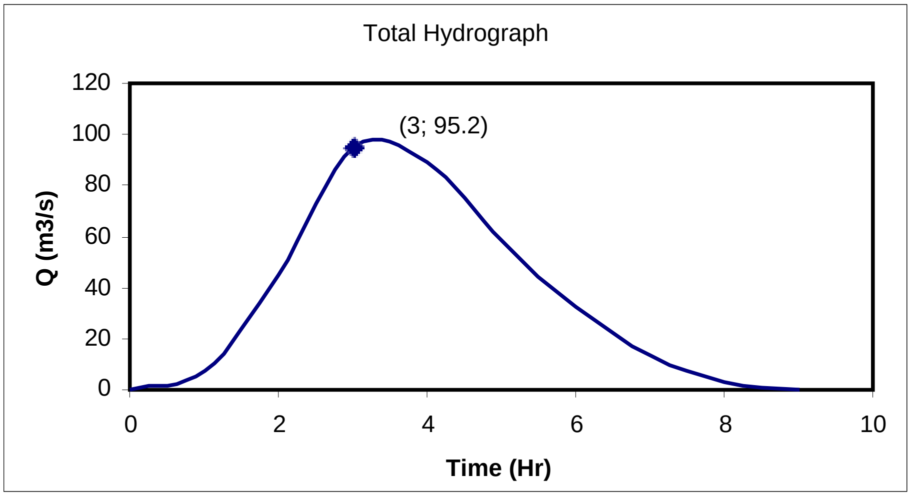
Time (h)	0	1	2	3	4	5	6	7
u(t) (m^3s^{-1})	0	12	35	24	16	8	3	0

Time (h)	0	1	2	3	Time	0	1	2	3
Total acc. Eff.RF(mm)	0	6	26	35	RF (mm)	-	6	20	9

Time (h)	u(t) (m^3s^{-1}) (10 mm)	u(t) (m^3s^{-1}) (1 mm)				Total Hydrograph
0	0	0	0			0
1	12	1.2	7.2	0		7.2
2	35	3.5	21	24	0	45
3	24	2.4	14.4	70	10.8	**95.2**
4	16	1.6	9.6	48	31.5	89.1
5	8	0.8	4.8	32	21.6	58.4
6	3	0.3	1.8	16	14.4	32.2
7	0	0	0	6	7.2	13.2
8	98			0	2.7	2.7
9					0	0

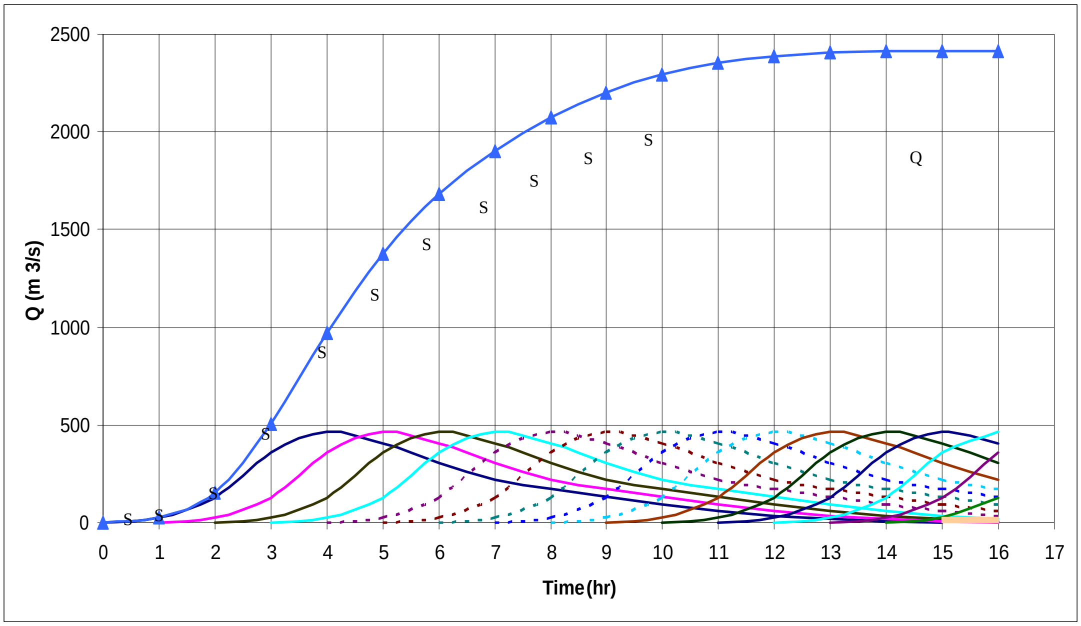
$$d = \frac{3.6\ \Delta t\ \Sigma Q}{A}$$

From this the approximate area of the catchment = 35.28 sq. km.



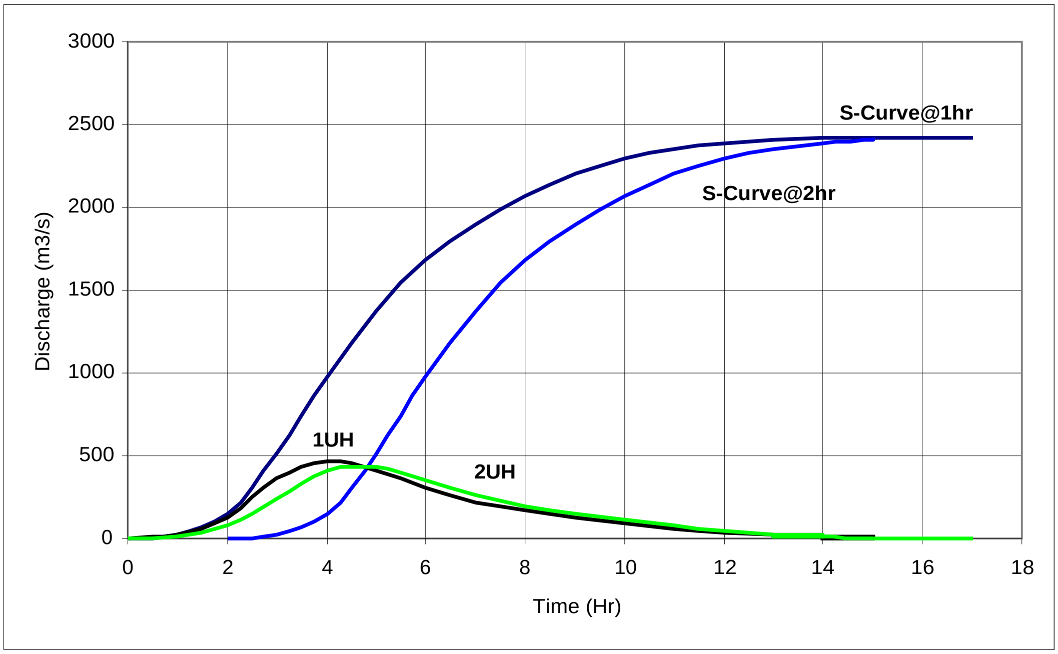
5. a) Derive the S-Curve for the catchment

Time (Hr)	TUH Ord.															S-Curve
0	0															0
1	25	0														25
2	125	25	0													150
3	358	125	25	0												508
4	465	358	125	25	0											973
5	405	465	358	125	25	0										1378
6	305	405	465	358	125	25	0									1683
7	220	305	405	465	358	125	25	0								1903
8	170	220	305	405	465	358	125	25	0							2073
9	130	170	220	305	405	465	358	125	25	0						2203
10	90	130	170	220	305	405	465	358	125	25	0					2293
11	60	90	130	170	220	305	405	465	358	125	25	0				2353
12	35	60	90	130	170	220	305	405	465	358	125	25	0			2388
13	20	35	60	90	130	170	220	305	405	465	358	125	25	0		2408
14	8	20	35	60	90	130	170	220	305	405	465	358	125	25	0	2416
15	0	8	20	35	60	90	130	170	220	305	405	465	358	125	25	2416
16		0	8	20	35	60	90	130	170	220	305	405	465	358	125	2416

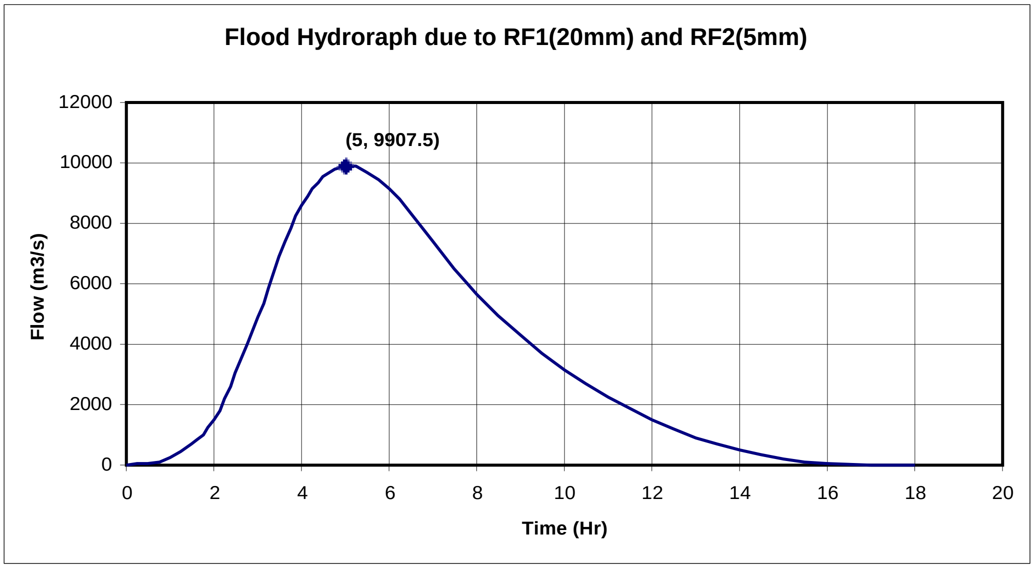


b) Use the S-Curve to obtain the 2-h unit hydrograph

Time (Hr)	S-Curve	**Lagged by 2 hr**	**S1 – S2**	**2TUH**	**RF1=20mm**	**RF2=5mm**	**Total**
0	0		0	0	0		0
1	25		25	12.5	250		250
2	150	**0**	150	75	1500	0	1500
3	508	**25**	483	241.5	4830	62.5	4892.5
4	973	**150**	823	411.5	8230	375	8605
5	1378	**508**	870	435	8700	1207.5	9907.5
6	1683	**973**	710	355	7100	2057.5	9157.5
7	1903	**1378**	525	262.5	5250	2175	7425
8	2073	**1683**	390	195	3900	1775	5675
9	2203	**1903**	300	150	3000	1312.5	4312.5
10	2293	**2073**	220	110	2200	975	3175
11	2353	**2203**	150	75	1500	750	2250
12	2388	**2293**	95	47.5	950	550	1500
13	2408	**2353**	55	27.5	550	375	925
14	2416	**2388**	28	14	280	237.5	517.5
15	2416	**2408**	8	4	80	137.5	217.5
16	2416	**2416**	0	0	0	70	70



c) Forecast the peak runoff that would result from storm in which the effective rainfall totals in two consecutive 2-h periods were 20 mm and 5 mm.



3 FLOOD ROUTING

3.1 General

At a river gauging station, the stage and discharge hydrographs represent the passage of waves of river depth and stream flow during flood, respectively. As this wave moves down the river, the shape of the wave gets modified due to various factors, such as channel storage, resistance, lateral addition or withdrawal of flows etc. when a flood wave passes through a reservoir, its peak is attenuated and the time base is enlarged (translated) due to the effect of storage. Flood waves passing down a river have their peaks attenuated due to friction if there is no lateral inflow. In both reservoir and channel conditions the time to peak is delayed, and hence the peak discharge is translated.

Flood routing is the technique of determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. The hydrologic analysis of problems such as flood forecasting, flood protection, reservoir and spillway design invariably include flood routing. In these applications two broad categories of routing can be recognized. These are:

i) Reservoir routing and
ii) Channel routing

In reservoir routing the effect of a flood wave entering a reservoir is studied. Knowing the volume-elevation characteristics of the reservoir and the out flow elevation relationship for spillways and other outlet structures in the reservoir; the effect of a flood wave entering the reservoir is studied to predict the variation of reservoir elevation and out flow discharge with time. This form of routing is essential (i) in the design of the capacity of spillways and other reservoir outlet structures and (ii) in the location and sizing of the capacity of reservoirs to meet specific requirements.

In channel routing the changes in the shape of a hydrograph as it travels down a channel is studied. By considering a channel reach and an input hydrograph at the upstream end, this form of routing aims to predict the flood hydrograph at a various sections of the reach. Information on the flood-peak attenuation and the duration of high-water levels obtained by channel routing is utmost importance in flood forecasting operations and flood protection works.

A variety of flood routing methods are available and they can be broadly classified in to two categories as: (i) hydraulic routing and (ii) hydrologic routing. Hydrologic routing methods employ essentially the equation of continuity and a storage function, indicated as lumped routing. Hydraulic methods, on the other hand, employ the continuity equation together with the equation of motion of unsteady flow. The basic differential equations used in the hydraulic routing, known as St. Venant equations afford a better description of unsteady flow than hydrologic methods.

A flood hydrograph is modified in two ways as the storm water flows downstream. Firstly, and obviously, the time of the peak rate of flow occurs later at downstream points. This is known as **translation.** Secondly, the magnitude of the peak rate of flow is diminished at downstream points, the shape of the hydrograph flattens out, and the volume at the floodwater takes longer to pass a lower section. This modification of the hydrograph is called **attenuation.**

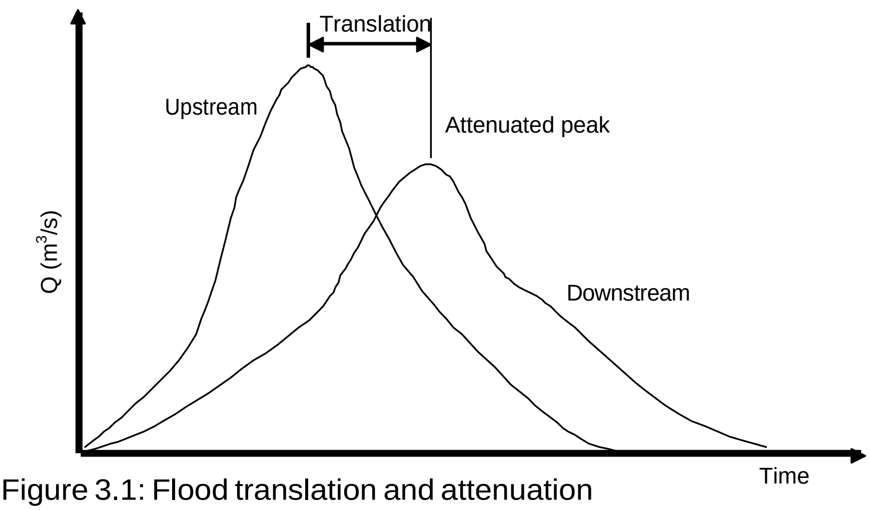


Figure 3.1: Flood translation and attenuation

3.2 Simple Non-storage Routing

Relationship between flood events and stages at upstream and downstream points in a single river reach can be established by correlating known floods and stages at certain conditions. The information could be obtained from flood marks on river banks and bridge sides. Measurements/estimates of floods can then be related to known the level of the flood at the upstream and downstream locations. With such curves it is possible to give satisfactory forecasts of the downstream peak stage from an upstream peak stage measurement.

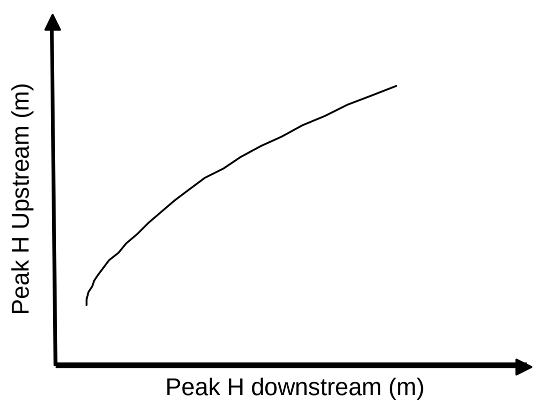


Figure 3.2: Peak stage relationship

The time of travel of the hydrograph crest (peak flow) also need to be determined to know the complete trace of modification of the wave. Curves of upstream stage plotted against time travel to the required downstream point can be compiled from the experience of several flood events.

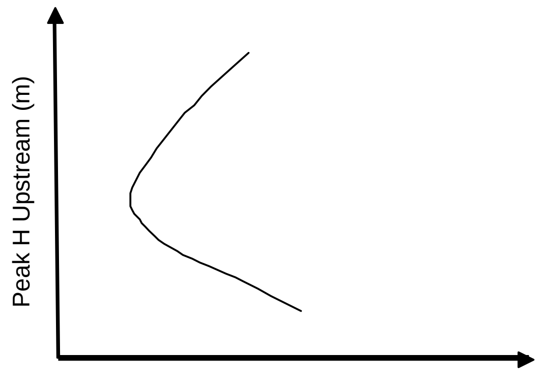


Figure 3.3: Flood Peak travel time

The complexities of rainfall-runoff relationships are such that these simple methods allow only for average conditions. Flood events can have very many different causes that produce flood hydrographs of different shapes.

The principal advantages of these simple methods are that they can be developed for stations with only stage measurements and no rating curve, and they are quick and easy to apply especially for warning of impending flood inundations when the required answers are immediately given in stage heights.

3.3 Storage Routing

When a storm event occurs, an increased amount of water flows down the river and in any one short reach of the channel there is a greater volume of water than usual contained in temporary storage. If at the beginning of the reach the flood hydrograph is (above normal flow) is given as I, the inflow, then during the period of the flood, T1, the channel reach has received the flood volume given by the area under the inflow hydrograph. Similarly, at the lower end of the reach, with an outflow hydrograph O, the flood is given by the area under the curve. In a flood situation relative quantities may be such that lateral and tributary inflows can be neglected, and thus by the principle of continuity, the volume of inflow equals the volume of outflow, i.e. the flood volume $V = \int_0^{T1} Idt = \int_{T2}^{T3} Odt$. At intermediate time T, an amount $\int_0^{T} Idt$ has entered the reach and an amount $\int_0^{T} Odt$ has left the reach. The difference must be stored within the reach, so the amount of storage, S, within the reach at time t = T is given by $S = \int_0^{T} (I - O)\, dt$.

The principle of hydrologic flood routings (both reservoir and channel) uses the continuity equation in the form of "Inflow minus outflow equals rate of change of storage".

i.e. $$\frac{ds}{dt} = I_t - O_t \qquad (3.1)$$

Where:

I_t = Inflow in to the reach
O_t= Outflow from the reach

dS/dt =Rate of change of storage within the reach.

Alternatively, the continuity (storage) equation can be stated as in a small time interval Δt the difference between the total inflow volume and total outflow volume in a reach is equal to the change in storage in that reach, i.e.,

$$\bar{I}\,\Delta t - \bar{O}\,\Delta t = \Delta S \qquad (3.2)$$

Where, $\bar{I}$ = average inflow in time Δt

$\bar{O}$ = average outflow in time Δt

ΔS = change in storage

Δt = routing period.

OR equation 3.2 can be rewritten as:

$$\frac{S_{i+1} - S_i}{\Delta t} = \frac{1}{2}(I_{i+1} + I_i) - \frac{1}{2}(O_{i+1} + O_i) \qquad (3.3)$$

The time interval Δt should be sufficiently short so that the inflow and out flow hydrographs can be assumed to be straight line in that interval. As a rule of thumb Δt ≤ 1/6 of the time to peak of the inflow hydrograph is required.

The continuity equation (I-Q = dS/dt), forms basis for all the storage routing methods. The routing problem consists of finding Q as a function of time, given I as a function of time, and having information or making assumptions about storage, S.

3.4 Reservoir or level pool routing

A flood wave I(t) enters a reservoir provided with an outlet such as a spillway. The outflow is a function of the reservoir elevation only, i.e., O = O (h). The storage in the reservoir is a function of the flow reservoir elevation, S = S(h). Further, the water level in the reservoir changes with time, h = h(t) and hence the storage and discharge change with time. It is required to find the variation of S, h and O with time, i.e., find S=S (t), O = O (t) and h = h (t), given I =I (t)

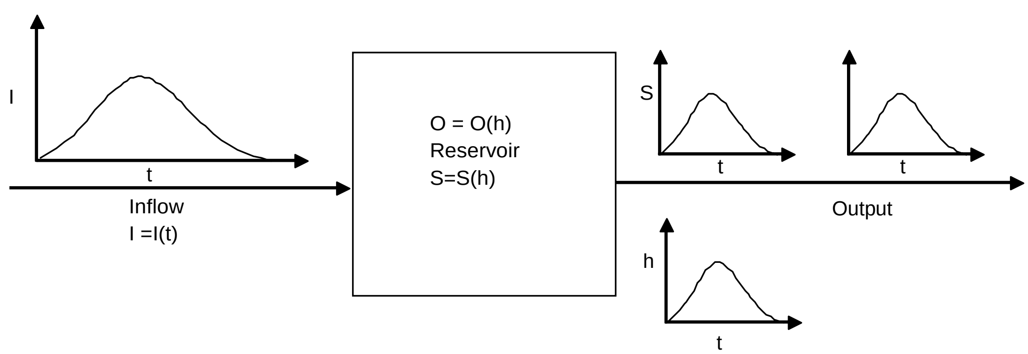


Figure 3.4: Storage routing (schematic)

Depending on the forms of the outlet relations for O (h) will be available.

For reservoir routing, the following data have to be known:

1. Storage volume versus elevation for the reservoir
2. Water surface elevation versus out flow and hence storage versus outflow discharge
3. Inflow hydrograph, I= I(t); and
4. Initial values of S, I and O at time t = 0

The finite difference form of the continuity equation (Equation. 3.4) can be rewritten as:

$$\frac{(I_1+I_2)\Delta t}{2} - \frac{(O_1+O_2)\Delta t}{2} = S_2 - S_1 \qquad (3.4)$$

Where, $(I_1+I_2)/2=\bar{I}$; $(O_{1+}O_2)/2=\bar{O}$ and $S_2-S_1=\Delta S$ and suffixes 1 and 2 to denote the beginning and end of the time interval Δt

Rearranging Equation (3.4) to get the unknowns S_2 and O_2 on one side of the equation and to adjust the O_1 term to produce:

$$\left(\frac{S_2}{\Delta t}+\frac{O_2}{2}\right) = \left(\frac{S_1}{\Delta t}+\frac{O_1}{2}\right) + \frac{I_1+I_2}{2} - O_1 \qquad (3.5)$$

Since S is a function of O, $[(S/\Delta t) + (O/2)]$ is also a specific function of O (for a given Δt). Replacing $\{(S/\Delta t) + (O/2)\}$ by **G**, for simplification, equation (3.5) can be written:

$\mathbf{G_2 = G_1 + I_m - O_1}$ or more generally

$G_{i+1} = G_i + I_{m,i} - O_i$ (3.6)

Where:

$I_m = (I_1 + I_2)/2$

To apply this method we need beside I_t also the G-O relation. The latter is easily established from S-H and O-H relations, where for equal values of H, S and O are determined; after which the proper interval Δt the G-O relation is established. Note that G is dependent on the chosen routing interval Δt.

The routing period, Δt, has to be chosen small enough such that the assumption of a linear change of flow rates, I and O, during Δt is acceptable (as a guide, Δt should be less than 1/6 of the time of rise of the inflow hydrograph).

So, in short, the method consists of three steps:

1. Inspect the inflow hydrograph and select the routing interval: $\Delta t \leq 1/6$ time to peak
2. Establish the G-O relation
3. Carry out the routing according to equation (3.6)

A useful check on the validity of any level pool routing calculation is that the peak of the outflow hydrograph should occur at the intersection of the inflow and out flow hydrograph on the same plot. At that point, I = O, so ds/dt = 0, i.e. storage is a maximum and therefore O is a maximum. Therefore, the temporary storage is depleted.

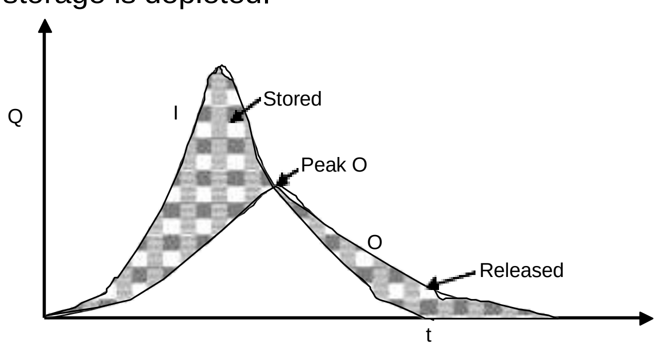


Figure 3.5: Storage routing (schematic)

Table 3.1: Tabular computation of level pool routing

T	a I	b Im	c O	d (Im - O)	e G
1	I1		O1		G1
		(I1+I2)/2		{(I1 + I2)} - O1	
2	I2		O2		G2
.	.		.		.
.	.		.		.
.	.		.		.
i	Ii		Oi		Gi
		(Ii + Ij)/2		{(Ii + Ij)} - Oi	
j	Ij		Oj		Gj

Example 2.1 (Reservoir routing)

3.5 Channel routing

In reservoir routing presented in the previous section, the storage was a unique function of the outflow discharge, S=f(O). However in channel routing the storage is a function of both outflow and inflow discharges and hence a different routing method is needed. The flow in a river during a flood belongs to the category of gradually varied unsteady flow.

For a river reach where the water surface cannot be assumed horizontal to the river bottom during the passage of a flood wave, the storage in the reach may be split up in two parts: (i) prism storage and (ii) wedge storage

Prism Storage is the volume that would exist if uniform flow occurred at the downstream depth, i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at a direct function of the **stage** at the downstream end of the reach. The surface is taken parallel to the river bottom ignoring the variation in the surface in the reach relative to the bottom. Both this storage and the outflow can be described as a single function of the downstream water level and the storage is a single function of the out flow alone.

Wedge Storage is the wedge-like volume formed b/n the actual water surface profile and the top surface of the prism storage. It exists because the inflow, I, differs from O (out flow) and so may be assumed to be a function of the difference between inflow and outflow, (I-O).

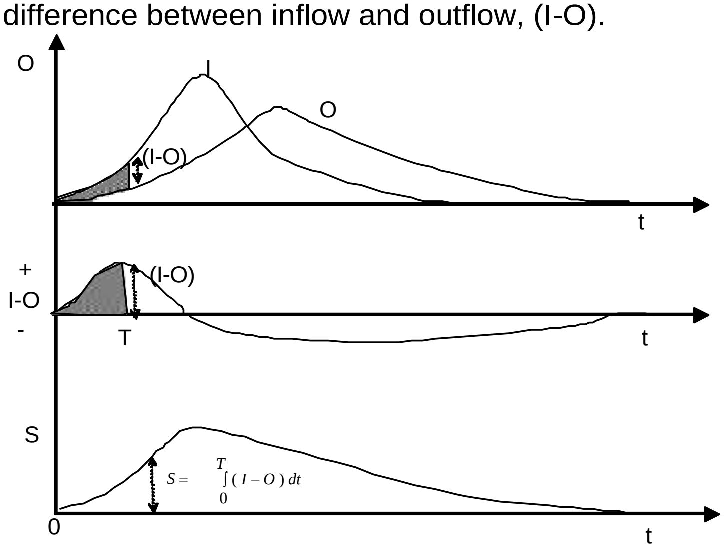


Figure 3.6: Determining storage in a river reach

At a fixed depth at a downstream section of river reach, prism storage is constant while the wedge storage changes from a positive value at the advancing flood wave to a negative value during a receding flood.

The total storage in the channel reach can be generally represented by:

$$S = f_1(O)+f_2(I-O) \qquad (3.7)$$

And this can then be expressed as:

$$S = K\,(x\,I^m + (1-x)O^m) \qquad (3.8)$$

Where **K** and **x** are coefficients and **m** is a constant exponent. It has been found that the value of m varies from 0.6 for rectangular channels to value of about 1.0 for natural channels.

3.5.1 Muskingum Method of Routing

Using m =1 for natural channels, equation (2.8), reduces to a linear relationship for S in terms of I and Q as

$$S= K\,(x\,I+ (1-x)O \qquad (3.9)$$

This relationship is known as the **Muskingum Equation**. In this the parameter **x** is known as weighing factor and take a value between 0 and 0.5. When x=0, obviously the storage is a function of discharge only and equation (3.9) reduces to:

$$S = KQ \qquad (3.10)$$

Such storage is known as linear storage or **linear reservoir.** When x= 0.5 both the inflow and out flow are equally important in determining the storage.

The coefficient K is known as **storage-time constant** and has dimensions of time. K is approximately equal to the time of travel of a flood wave through the channel reach.

As before, writing the continuity equation in finite difference form, we can write

$$S_2 - S_1 = \{(I_1+I_2)\Delta t\}/2 - \{(O_1+O_2)\,\Delta t\}/2 \qquad (3.11)$$

For a given channel reach by selecting a routing interval Δt and using the Muskingum equation, the change in storage can be determined.

$$S_1 = K(xI_1 + (1-x)\,O_1) \qquad (3.12)$$

$$S_2 = K(xI_2 + (1-x)\,O_2) \qquad (3.13)$$

Substituting equations (3.12) and (3.13) in equation (3.11) and after rearrangements gives:

$O_2 = c_1I_1 + c_2I_2 + c_3O_1$ and more generally as

$$O_{i+1} = c_1 I_i + c_2 I_{i+1} + c_3 O_i \qquad (3.14)$$

Where

$$C_1 = \frac{\Delta t + 2Kx}{\Delta t + 2K - 2Kx}$$

$$C_2 = \frac{\Delta t - 2Kx}{\Delta t + 2K - 2Kx}$$

$$C_3 = \frac{-\Delta t + 2k - 2KX}{\Delta t + 2k - 2kx}$$

Note that $\Sigma C=1$ and thus when C_1 and C_2 have been found $C_3=1-C_1-C_2$. Thus the outflow at the end of a time step is the weighted sum of the starting inflow and outflow and the ending inflow. It has been found that best results will be obtained when routing interval should be so chosen that **$K>\Delta t>2kx$.** If **$\Delta t < 2kx$**, the coefficient C_2 will be negative.

3.5.2 Application of the Muskingum Method:

In order to use equation (2.14) for O_{i+1}, it is necessary to know K and x for calculating the coefficients, C. Using recorded hydrographs of a flood at the beginning and end of the river reach, trial values of x are taken, and for each trial the weighted flows in the reach, [xI+(1-x)O], are plotted against the actual storages determined from the inflow and out flow hydrographs as indicated in the following figure.

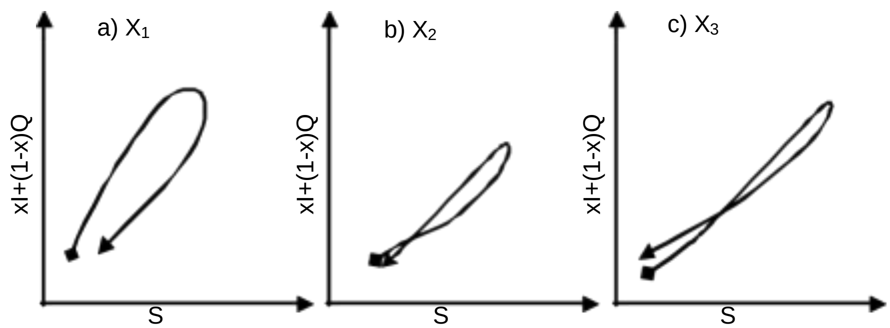


Figure 3.7: Trial plots for Muskingum **x** values

When the looping plots of the weighted discharge against storages have been narrowed down so that the values for the rising stage and the falling stage for a particular value of x merge together to form the best approximation to a straight line, then that x value is used, and the slope of the straight line gives the required value of K. for natural channels, the best plot is often curved, making a straight line slope difficult to estimate.

Example 3.2 (Channel routing)

3.6 Hydraulic Routing

Tutorial Problem Set 2 (Taken from Chapter 16 – Flood Routing, Elisabeth Shaw)

For G-2HA&B

1. The Muskingum constants, K and x, are estimated for a given river reach to be 12 hr and 0.2. Assuming an initial steady flow, determine the peak discharge at the downstream end of the reach for the inflow hydrograph shown in the table 1 below.

Table 1

Time from start Of rise (hr)	0	6	12	18	24	30	36	42
Inflow (m^3s^{-1})	25.0	32.5	58.0	65.0	59.4	49.5	42.5	35.0

2. The crest of a 20 m wide reservoir spillway consists of two 10 m wide gates, one of which is kept 0.5m lower than the other. The flow Q (m^3s^{-1}), over each gate is given by $Q = C_d b h^{3/2}$ where b (m) is the gate width and h (m) is the head on the gates. Position, can be taken as 2.0 ($m^{1/2}s^{-1}$) for each gate. Initial flows into and out of the 1 ha reservoir are equal at 10 m^3s^{-1}). The inflow into the reservoir is to be increased steadily to 20 m^3s^{-1} over 1h. Find the outflow at the end of that time, assuming the gate positions remain unchanged (1 ha = $10^4 m^2$)

3. A lake, having steep banks and a surface area of 6 km^2, discharges into a steep channel which is approximately rectangular in section, with a width of 50 m. Initially, conditions are steady with a flow of 170 m^3s^{-1} passing through the lake; then a flood comes down the river feeding the lake, giving rise to the inflow hydrograph shown in table 2. Compute the outflow hydrograph and plot it on the same graph with the inflow hydrograph. Note the difference in magnitude and time between the two peaks. (Critical flow exists at the lake outlet, g = 9.81 ms^{-1}.)

Table 2

Time from start (hr)	0	1	2	3	4	5	6
Inflow (m^3s^{-1})	170	230	340	290	220	180	170

4. Discharge measurements at two gauging stations for a flood flow on the Macquarie River in Australia are given in Table 3. It is assumed that there are no tributaries to the river between the upstream stations B and the downstream station W. Apply the Muskingum method of flood routing to derive a model for

calculating sequential outflows at the downstream end of the reach from measured flows at B. From the given inflows, derive the computed peak outflow discharge and its time of arrival at W.

Table 3

Time(h)	0	4	8	12	16	20	24
B(m^3s^{-1})	595	1699	3837	5636	4305	3059	2271
W(m^3s^{-1})	130	496	1189	2209	3087	3823	3781
Time (h)	28	32	36	40	44	48	
B (m^3s^{-1})	1756	1359	1062	830	637	504	
W (m^3s^{-1})	3285	2393	1841	1416	1147	850	

5. Analysis of past records for a certain river reach yielded the values of Muskingum coefficient x = 0.22 and K = 1.5 days. Route the flood shown in Table 4 through the reach. The inflow continues at 14.2 m^3s^{-1} after day 12.

Table 4

Day	1	2	3	4	5	6
Inflow (m^3s^{-1})	14.2	76.7	129.4	166.8	171.9	113.8
Day	7	8	9	10	11	12
Inflow(m^3s^{-1})	187.8	264.2	152.1	75.0	45.0	14.2

4 FREQUENCY ANALYSIS (PROBABILITY IN HYDROLOGY)

4.1 General

Water resource systems must be planned for future events for which no exact time of occurrence can be forecasted. Hence, the hydrologist must give a statement of the probability of the stream flows (or other hydrologic factors) will equal or exceed (or be less than) a specified value. These probabilities are important to the economic and social evaluation of a project. In most cases, absolute control of the floods or droughts is impossible. Planning to control a flood of a specific probability recognizes that a project will be overtaxed occasionally and damages will be incurred. However, repair of the damages should be less costly in the long run than building initially to protect against the worst possible event. The planning goal is not to eliminate all floods but to reduce the frequency of flooding, and hence the resulting damages. If the socio-economic analysis is to be correct, the probability of flooding must be eliminated accurately. For major projects, the failure of which seriously threatens human life, a more extreme event, the probable maximum flood, has become the standard for designing the spillway.

This chapter deals with techniques for defining probability from a given set of data and with special methods employed for determining design flood for major hydraulic structures.

Frequency analysis is the hydrologic term used to describe the probability of occurrence of a particular hydrologic event (e.g. rainfall, flood, drought, etc.). Therefore, basic knowledge about probability (e.g. distribution functions) and statistics (e.g. measure of location, measure of spread, measure of skewness, etc) is essential. Frequency analysis usually requires recorded hydrological data.

Hydrological data are recorded either as a *continuous record* (e.g. water level or stage, rainfall, etc.) or in *discrete series* form (e.g. mean daily/monthly/annual flows or rainfall, annual series, partial series, etc.).

For planning and designing of water resources development projects, the important parameters are river discharges and related questions on the *frequency* & *duration* of *normal flows* (e.g. for hydropower production or for water availability) and *extreme flows* (floods and droughts).

4.2 Flow Frequency

The question a planner or decision maker would ask a hydrologist concerning normal flows is the length of time (duration) that a certain river flow is expected to be exceeded. An answer to this question is provided by the flow duration curve (FDC) that is the relationship between any given discharge and the percentage of time that the discharge is exceeded. Taking the n-years of flow records from a river gauging stations, there are 365(6)n daily mean discharges. The discharge is compiled, starting with the highest values. If N number of data points are used for analysis, the plotting position of any discharge (or class value) Q is:

$$P_p = m / (N+1) \qquad (4.1)$$

Where, m = is the order number of the discharge (or class value)

P_p = percentage probability of the flow magnitude being equalled or exceeded.

The FDC only applies for the period for which it was derived. If this is a long period, say more than 10 to 20 years, the FDC may be regarded as a *probability curve or flow frequency curve*, which may be used to estimate the percentage of time that a specified discharge will be equalled or exceeded in the future. An example is demonstrated in table 4.1 below.

Table 4.1: Flow records

Discharge (m3/s)	Descending Order	Rank	%Exceeded or Equaled (m / (N+1))
(a)	(b)	(c)	(d)
106.70	1200	1	8.33%
107.10	964.7	2	16.67%
148.20	497	3	25.00%
497.00	338.6	4	33.33%
1200.00	177.6	5	41.67%
964.70	148.2	6	50.00%
338.60	142.7	7	58.33%
177.60	141	8	66.67%
141.00	141	9	75.00%
141.00	126.6	10	83.33%
142.70	107.1	11	91.67%
126.60	106.7	12	100.00%

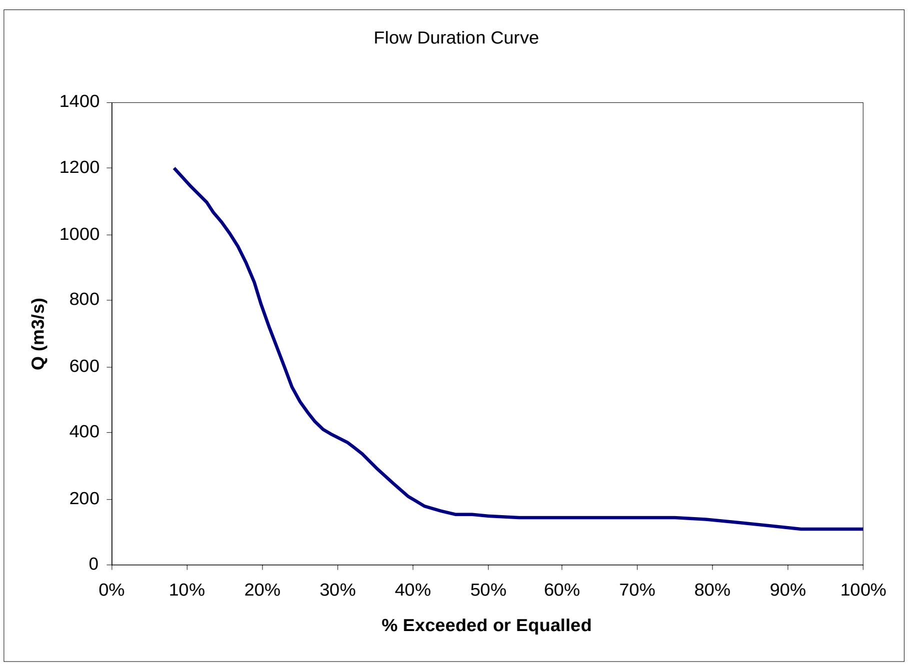


Figure 4.1: Flow Duration Curve

The shape of the flow-duration curve gives a good indication of a catchment's characteristics response to its average rainfall history. An initially steeply sloped curve results from a very variable discharge, usually from small catchments with little storage where the stream flow reflects directly the rainfall pattern. Flow duration curves that have very flat slope indicate little variation in flow regime, the resultant of the damping effects of large storages.

4.3 Flood Probability

4.3.1 Selection of Data

If probability analysis is to provide reliable answers, it must start with a data series that is relevant, adequate, and accurate. ***Relevance*** implies that the data must deal with the problem. Most flood studies are concerned with peak flows, and the data series will consist of selected observed peaks. However, if the problem is duration of flooding, e.g., for what periods of time a highway adjacent to a stream is likely to be flooded, the data series should represent duration of flows in excess of some critical value. If the problem is one of interior drainage of a leveed area, the data required may consist of those flood volumes occurring when the main river is too high to permit gravity drainage.

Adequacy refers primarily to length of record, but sparisty of data collecting stations is often a problem. The observed record is merely a sample of the total population of floods that have occurred and may occur again. If the sample is too small, the probabilities derived cannot be expected to be reliable. Available stream flow records are too short to provide an answer to the question: How long must a record be to define flood probabilities within acceptable tolerances?

Accuracy refers primarily to the problem of homogeneity. Most flow records are satisfactory in terms of intrinsic accuracy, and if they are not, there is little that can be done with them. If the reported flows are unreliable, they are not a satisfactory basis for frequency analysis. Even though reported flows are accurate, they may be unsuitable for probability analysis if changes in the catchment have caused a change in the hydrologic characteristics, i.e., if the record is not internally homogenous. Dams, levees, diversions, urbanization, and other land use changes may introduce inconsistencies. Such records should be adjusted before use to current conditions or to natural conditions.
There are two data series of floods:

(i) **The annual series, and**
(ii) **The partial duration series**.

The annual series constitutes the data series that the values of the single maximum daily/monthly/annually discharge in each year of record so that the number of data values equals the record length in years. For statistical purposes, it is necessary to ensure that the selected peak discharges are independent of one another. This data series is necessary if the analysis is concerned with probability less than 0.5. However as the interest are limited to relatively rare events, the analysis could have been carried out for a partial duration series to have more frequent events.

.

The partial duration series constitutes the data series with those values that exceed some arbitrary level. All the peaks above a selected level of discharge (a threshold) are included in the series and hence the series is often called **the Peaks Over Threshold (POT)** series. There are generally more data values for analysis in this series than in the annual series, but there is more chance of the peaks being related and the assumption of true independence is less valid.

4.3.2 Plotting Positions

Probability analysis seeks to define the flood flow with probability of *p* being equaled or exceed in any year. Return period T_r is often used in lieu of probability to describe a design flood. Return period and probability are reciprocals, i.e,

$$p = 1/T_r \tag{4.2}$$

To plot a series of peak flows as a cumulative frequency curves it is necessary to decide on a probability or return period to associate with each peak. There are various formulas for defining this value as shown in table 4.2.

The probability of occurrence of the event r times in n successive years can be obtained from:

$$P_{rn} = {}^{n}C_{r}\, P^{r} q^{n-r} = \frac{n!}{(n-r)!\,r!} P^{r} q^{n-r} \tag{4.3}$$

Where q = 1 - P.

Table 4.2 Plotting-position formulae

Method	P
California	m / N
Hazen	(m - 0.5) / N
Weibull	m / (N+1)
Chegodayev	(m - 0.3) / (N+0.4)
Gringorten	(m - 3/8) / (N + 1/4)

Consider, for example, a list of flood magnitudes of a river arranged in descending order as shown in Table 4.3. The length of record is 50 years.

Table 4.3: Calculation of Frequency T

Order No. m	Flood magnitude Q (m^3/s)	T in years = 51/m
1	160	51.00
2	135	25.50
3	128	17.00
4	116	12.75
.	.	.
.	.	.
.	.	.
49	65	1.04
50	63	1.02

The last column shows the return period T of various flood magnitude, Q. A plot of Q Vs T yields the probability distribution. For small return periods (i.e. for interpolation) or where limited extrapolation is required, a simple best-fitting curve through plotted points can be used as the probability distribution. A logarithmic scale for T is often advantageous. However, when larger extrapolations of T are involved, theoretical probability distributions (e.g. Gumbel extreme-value, Log-Pearson Type III, and log normal distributions) have to be used. In frequency analysis of floods the usual problem is to predict extreme flood events. Towards this, specific extreme-value distributions are assumed and the required statistical parameters calculated from the available data. Using these flood magnitude for a specific return period is estimated.

4.3.3 Theoretical Distributions of Floods

Statistical distributions are usually demonstrated by use of samples numbering in the thousands. No such samples are available for stream flow and it is not possible to state with certainty that a specific distribution applies to flood peaks. Numerous distributions have been suggested on the basis of their ability to "fit" the plotted data from streams.

Chow has shown that most frequency-distribution functions applicable in hydrologic studies can be expressed by the following equation known as the general equation of hydrologic frequency analysis:

$$x_T = \bar{x} + K\sigma \qquad (4.4)$$

Where x_T = value of the variate X of a random hydrologic series with a return period T, $\bar{x}$ = mean of the variate, σ = standard deviation of the variate, K = frequency factor which depends upon the return period, T and the assumed frequency distribution.

4.3.4 Extreme-Value Type I Distribution (Gumbel's Method)

This extreme value distribution was introduced by Gumbel (1941) and is commonly known as Gumbel's distribution. It is one of the most widely used probability-distribution functions for extreme values in hydrologic and meteorological studies for prediction of flood peaks, maximum rainfalls, and maximum wind speed, etc. Therefore, this extreme value theory of Gumbel is only applicable to annual extremes. In contrast to the previous example, in the Gumbel method the data are ranked in ascending order and it makes use of the probability of non-exceedence q=1-P (the probability that the annual maximum flow is less than a certain magnitude). The return period T is therefore given by T = 1 / P = 1 /(1-q).

Gumbel makes use of a reduced variate y as a function of q, which allows the plotting of the distribution as a linear function between y and X (the maximum flow in this case). Gumbel also defined a flood as the largest of the 365 daily flows and the annual series of flood flows constitute a series of largest values of flow. According to his theory of extreme events, the probability of occurrence of an event equal to or larger than a value x_0 is

$$P\left(X \geq x_0\right) = 1 - e^{-e^{-y}} \qquad (4.5)$$

In which y is a dimensionless variable given by

$$y = \alpha(X - a) \qquad (4.6)$$

$$a = \bar{x} - 0.45005\sigma_x$$

$$\alpha = \frac{1.2825}{\sigma_x}$$

Thus

$$y = \frac{1.2825\left(X - \bar{X}\right)}{\sigma_X} + 0.577 \qquad (4.7)$$

Where $\bar{x}$ = mean and σ_x = standard deviation of the variate X. In practice it is the value of X for a given P that is required and as such Eq.(4.7) is transposed as

$$y = -\ln(-\ln(q)) = -\ln(-\ln(1-p)) \qquad (4.8)$$

meaning that the probability of non-exceedence equals:

$$P(X \le x_0) = q = e^{-e^{-y}} \qquad (4.9)$$

Noting that the return period T = 1/P and designating; y_T = the value of y, commonly called the reduced variate, for a given T

$$y_T = -\left[\ln \ln \left(\frac{T}{T-1}\right)\right] \qquad (4.10)$$

Or

$$y_T = -\left[0.834 + 2.303 \log \log \left(\frac{T}{T-1}\right)\right] \qquad (4.10a)$$

Now rearranging Eq.(4.7), the value of the variate X with a return period T is

$$x_T = \bar{x} + K\sigma_x \qquad (4.11)$$

Where

$$K = \frac{(y_T - 0.577)}{1.2825} \qquad (4.12)$$

Note that Eq. (4.12) is of the same form as the general equation of hydrologic-frequency analysis, Eq. (4.4). Further eqs (4.11) and (4.12) constitute the basic Gumbel's equations and are applicable to an infinite sample size (i.e. N $\rightarrow \infty$).

Since practical annual data series of extreme events such as floods, maximum rainfall depths, etc., all have finite lengths of record; Eq. (4.12) is modified to account for finite N as given below for practical use.

4.3.5 Gumbel's Equation for Practical Use

Equation (4.11) giving the variate X with the return period T is used as

$$x_T = \bar{x} + K\sigma_{n-1} \qquad (4.13)$$

Where σ_{n-1} = standard deviation of the sample $= \sqrt{\dfrac{\sum (x-\bar{x})^2}{N-1}}$

K = frequency factor expressed as $K = \dfrac{y_T - \bar{y}_n}{S_n}$ (4.14)

In which y_T = reduced variate, a function of T and is given by

$$y_T = -\left[\ln \ln \left(\frac{T}{T-1}\right)\right] \qquad (4.15)$$

or

$$y_T = -\left[0.834 + 2.303 \log \log \left(\frac{T}{T-1}\right)\right]$$

$\bar{y}_n$ = reduced mean, a function of sample size N and is given in Table 4.4; for $N \rightarrow \infty$, $\bar{y}_n \rightarrow 0.577$.

S_n = reduced standard deviation, a function of sample size N and is given in Table 4.5; for $N \rightarrow \infty$, $S_n \rightarrow 1.2825$.

These equations are used under the following procedure to estimate the flood magnitude corresponding to a given return period based on annual flood series.

1. *Assemble the discharge data and note the sample size N. Here the annual flood value is the variate X. Find $\bar{x}$ and σ_{n-1} for the given data.*
2. *Using Tables 4.4 and 4.5 determine $\bar{y}_n$ and S_n appropriate to given N*
3. *Find y_T for a given T by Eq.(4.15).*
4. *Find K by Eq.(4.14).*
5. *Determine the required x_T by Eq.(4.13).*

To verify whether the given data follow the assumed Gumbel's distribution, the following procedure may be adopted. The value of x_T for some return periods T<N are calculated by using Gumbel's formula and plotted as x_T Vs T on a convenient paper such as a semi-log, log-log or Gumbel probability paper. The use of Gumbel probability paper results in a straight line for x_T Vs T plot. Gumbel's distribution has the property which gives T = 2.33 years for the average of the annual series when N is very large. Thus the value of a flood with T = 2.33 years is called the *mean annual flood.* In graphical plots this gives a mandatory point through which the line showing variation of x_T with T must pass. For the given data, values of return periods (plotting positions) for various recorded values, x of the variate are obtained by the relation T = (N+1)/m and plotted on the graph described above. A good fit of observed data with the theoretical variation line indicates the applicability of Gumbel's distribution to the given data series. By extrapolation of the straight-line x_T Vs T, values of x_T> N can be determined easily.

The Gumbel (or extreme-value) probability paper is a paper that consists of an abscissa specially marked for various convenient values of the return period T (or corresponding reduced variate y_T in arithmetic scale). The ordinate of a Gumbel paper represent x_T (flood discharge, maximum rainfall depth, etc.), which may have either arithmetic scale or logarithmic scale.

Table 4.4: Reduced mean $\bar{y}_n$ in Gumbel's extreme value distribution, N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.5070	0.5100	0.5128	0.5157	0.5181	0.5202	0.5220
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.5320	0.5332	0.5343	0.5353
30	0.5362	0.5371	0.5380	0.5388	0.5396	0.5402	0.5410	0.5418	0.5424	0.5430
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.5530	0.5533	0.5535	0.5538	0.5540	0.5543	0.5545
70	0.5548	0.5550	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.5570	0.5572	0.5574	0.5576	0.5578	0.5580	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.5600									

Table 4.5: Reduced standard deviation S_n in Gumbel's extreme value distribution, N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	1.1004	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2060
100	1.2065									

4.3.6 Confidence Limits for the fitted data

Since the value of the variate for a given return period, x_T determined by Gumbel's method can have errors due to the limited sample data used; an estimate of the confidence limits of the estimate is desirable. The confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only.

For a confidence probability c, the confidence interval of the variate x_T is bound by value x_1 and x_2 given by

$$x_{1/2} = x_T \pm f(c)\, S_e \qquad (4.16)$$

Where f(c) = function of the confidence probability c determined by using the table of normal variate as

C in per cent	50	68	80	90	95	99
f(c)	0.674	1.00	1.282	1.645	1.96	2.58

$$S_e = \text{probable error} = b\frac{\sigma_{n-1}}{\sqrt{N}} \qquad (4.16a)$$

$$b = \sqrt{1 + 1.3K + 1.1K^2}$$

K = frequency factor given by Eq.(4.14)

σ_{n-1} = standard deviation of the sample

N = sample size

It is seen that for a given sample and T, 80% confidence limits are twice as large as the 50% limits and 95% limits are thrice as large as 50% limits.

In addition to the analysis of maximum extreme events, there also is a need to analyze minimum extreme events; e.g. the occurrence of droughts. The probability distribution of Gumbel, similarly to the Gaussian probability distribution, does not have a lower limit; meaning that negative values of events may occur. As rainfall or river flows do have a lower limit of zero, neither the Gumbel nor Gaussian distribution is an appropriate tool to analyze minimum values. Because the logarithmic function has a lower limit of zero, it is often useful to first transform the series to its logarithmic value before applying the theory. Appropriate tools for analyzing minimum flows or rainfall amounts are the Log-Normal, Log-Gumbel, or Log-Pearson distributions.

4.3.7 Log-Pearson Type III Distribution

This distribution is widely used in USA. In this distribution the variate is first transformed into logarithmic form (base 10) and the transformed data is then analysed. If X is the variate of a random hydrologic series, then the series of Z variates where

$Z = \log x$ (4.18)

are first obtained. For this z series, for any recurrence interval T, equation (4.4) gives

$$z_T = \bar{z} + K_z \sigma_z \quad (4.19)$$

Where K_z = a frequency factor which is a function of recurrence interval T and the coefficient of skew C_s,

σ_z = standard deviation of the Z variate sample = $\sqrt{\dfrac{\sum (z-\bar{z})^2}{N-1}}$ (4.19a)

and C_s = coefficient of skew of variate Z = $\dfrac{N\sum (z-\bar{z})^3}{(N-1)(N-2)(\sigma_z)^3}$ (4.19b)

$\bar{z}$ = mean of the z values

N = sample size = number of years of record

The variations of $K_z = f(C_s, T)$ is given in Table 4.6. After finding z_T by Eq.(4.19), the corresponding value of x_T is obtained by Eq.(4.18) as $x_T = \text{antilog}(z_T)$ (4.20)

Sometimes, the coefficient of skew C_s, is adjusted to account for the size of the sample by using the following relation proposed by Hazen (1930)

$$\hat{C}_s = C_s\left(1+\frac{8.5}{N}\right) \quad (4.21)$$

Where $\hat{C}_s$ = adjusted coefficient of skew. However the standard procedure for use of Log-Pearson Type III distribution adopted by U.S. Water Resources Council does not include this adjustment for skew.

When the skew is zero, i.e. $C_s = 0$, the Log-Pearson Type III distribution reduces to *Log-normal distribution*. The Log-normal distribution plots as a straight line on logarithmic probability paper.

Table 4.6: $K_z = F(C_s, T)$ for use in Log-Pearson Type III Distribution

Coef. of skew, C_s	**Return Period T in years**						
	2	**10**	**25**	**50**	**100**	**200**	**1000**
3.0	-0.396	1.180	2.278	3.152	4.051	4.970	7.250
2.5	-0.360	1.250	2.262	3.048	3.845	4.652	6.600
2.2	-0.330	1.284	2.240	2.970	3.705	4.444	6.200
2.0	-0.307	1.302	2.219	2.912	3.605	4.298	5.910
1.8	-0.282	1.318	2.193	2.848	3.499	4.147	5.660
1.6	-0.254	1.329	2.163	2.780	3.388	3.990	5.390
1.4	-0.225	1.337	2.128	2.706	3.271	3.828	5.110
1.2	-0.195	1.340	2.087	2.626	3.149	3.661	4.820

Table 4.6: (Cont'd)

Coef. of skew, C_s	**Return Period T in years**						
	2	**10**	**25**	**50**	**100**	**200**	**1000**
1.0	-0.164	1.340	2.043	2.542	3.022	3.489	4.540
0.9	-0.148	1.339	2.018	2.498	2.957	3.401	4.395
0.8	-0.132	1.336	1.998	2.453	2.891	3.312	4.250
0.7	-0.116	1.333	1.967	2.407	2.824	3.223	4.105
0.6	-0.099	1.328	1.939	2.359	2.755	3.132	3.960
0.5	-0.083	1.323	1.910	2.311	2.686	3.041	3.815
0.4	-0.066	1.317	1.880	2.261	2.615	2.949	3.670
0.3	-0.050	1.309	1.849	2.211	2.544	2.856	3.525
0.2	-0.033	1.301	1.818	2.159	2.472	2.763	3.380
0.1	-0.017	1.292	1.785	2.107	2.400	2.670	3.235
0.0	**0.000**	**1.282**	**1.751**	**2.054**	**2.326**	**2.576**	**3.090**
-0.1	0.017	1.270	1.716	2.000	2.252	2.482	2.950
-0.2	0.033	1.258	1.680	1.945	2.178	2.388	2.810
-0.3	0.050	1.245	1.643	1.890	2.104	2.294	2.675
-0.4	0.066	1.231	1.606	1.834	2.029	2.201	2.540
-0.5	0.083	1.216	1.567	1.777	1.955	2.108	2.400
-0.6	0.099	1.200	1.528	1.720	1.880	2.016	2.275
-0.7	0.116	1.183	1.488	1.663	1.806	1.926	2.150
-0.8	0.132	1.166	1.448	1.606	1.733	1.837	2.035
-0.9	0.148	1.147	1.407	1.549	1.660	1.749	1.910
-1.0	0.164	1.128	1.366	1.492	1.588	1.664	1.880
-1.4	0.225	1.041	1.198	1.270	1.318	1.351	1.465
-1.8	0.282	0.945	1.035	1.069	1.087	1.097	1.130
-2.2	0.330	0.844	0.888	0.900	0.905	0.907	0.910
-3.0	0.396	0.660	0.666	0.666	0.667	0.667	0.668

The flood-frequency analysis described above is a direct means of estimating the desired flood based upon the available flood-flow data of the catchment. The results of the frequency analysis depend upon the length of data. The minimum number of years of record required to obtain satisfactory estimates depends upon the variability of data and hence on the physical and climatological characteristics of the basin. Generally a minimum of 30 years of data is considered as essential. Smaller lengths of records are also used when it is unavoidable. However, frequency analysis should not be adopted if the length of records is less than 10 years.

Flood-frequency studies are most reliable in climates that are uniform from year to year. In such cases a relatively short record gives a reliable picture of the frequency distribution. With increasing lengths of flood records, it affords a viable alternative method of flood-flow estimation in most cases.

A final remark of caution should be made regarding to frequency analysis. None of the frequency distribution functions have a real physical background. The only information having physical meaning are the measurements themselves. Extrapolation beyond the period of observation is dangerous. It requires a good engineer to judge the value of extrapolated events of high return periods. A good impression of the relativity of frequency analysis can be acquired through the comparison of result obtained from different statistical methods. Generally they differ considerably.

Example 4.1

Annual maximum recorded floods in a certain river, for the period 1951 to 1977 is given below. Verify whether the Gumbel extreme-value distribution fit the recorded values. Estimate the flood discharge with return period of (i) 100 years and (ii) 150 years by graphical extrapolation.

Solutions: The flood discharge values are arranged in descending order and the plotting position return period T_P for each discharge is obtained as $T_P = \frac{N+1}{m} = \frac{28}{m}$. Where m = order number. The discharge magnitude Q can be plotted against the corresponding T_P on a Gumbel extreme probability paper.

Year	Max. flood (m^3/s)	Year	Max. flood (m^3/s)	Year	Max. flood (m^3/s)
1951	2947	1960	4798	1969	6599
1952	3521	1961	4290	1970	3700
1953	2399	1962	4652	1971	4175
1954	4124	1963	5050	1972	2988
1955	3496	1964	6900	1973	2709
1956	2947	1965	4366	1974	3873
1957	5060	1966	3380	1975	4593
1958	4903	1967	7826	1976	6761
1959	3757	1968	3320	1977	1971

The statistics $\bar{x}$ *and* σ_{n-1} for the series are next calculated and are shown in table below.

Order number M	Flood discharge x (m^3/s)	T_P (years)	Order number M	Flood discharge x (m^3/s)	T_P (years)
1	7826	28.00	15	3873	1.87
2	6900	14.00	16	3757	1.75
3	6761	9.33	17	3700	1.65
4	6599	7.00	18	3521	1.56
5	5060	5.60	19	3496	1.47
6	5050	4.67	20	3380	1.40
7	4903	4.00	21	3320	1.33
8	4798	3.50	22	2988	1.27
9	4652	3.11	23	2947	-
10	4593	2.80	24	2947	1.17
11	4366	2.55	25	2709	1.12
12	4290	2.33	26	2399	1.08
13	4175	2.15	27	1971	1.04
14	4124	2.00			
N = 27 years, $\bar{x}$ = 4263 m^3/s, σ_{n-1} = 1432.6 m^3/s					

Using these the discharge x_T for some chosen return interval is calculated by using Gumbel's formulae [Eqs.(4.15), (4.14) and (4.13)]. From Tables 4.4 and 4.5, for N = 27, y_n = 0.5332 and S_n = 1.1004.

Choosing T = 10 years, by Eq.(4.15), y_T = -[ln(ln(10/9))] = 2.25037 and
K = (2.25307-0.5332)/1.1004 = 1.56 and x_T = 4263 + (1.56*1432.6) = 6499m^3/s.
Similarly, values of x_T are calculated for two more T values as shown below.

T (Years)	X_T[obtained by Eq.(3.13)] (m^3/s)
5.0	5522
10.0	6499
20.0	7436

When these values are plotted on Gumbel probability paper, it is seen that these points lie on a straight line according to the property of the Gumbel's extreme probability paper. Then by extrapolation of the theoretical x_T Vs T relationship, from this plot, *at T = 100 years, x_T = 9600m^3/s and at T = 150 years, x_T = 10700m^3/s.* [By using Eq. (4.13) to (4.15), x_{100} = 9558m^3/s and x_{150} = 10088m^3/s.]

Example 4.2:
Data covering a period of 92 years for a certain river yielded the mean and standard deviation of the annual flood series as 6437 and 2951 m^3/s respectively. Using Gumbel's method, estimate the flood discharge with a return period of 500 years. What are the (a) 95% and (b) 80% confidence limits for this estimate?

Solution: From Table 4.4 and 4.5 for N = 92 years, $\bar{y}_n$ = 0.5589, and S_n = 1.2020. Then
y_{500} = -[ln((ln(500/499))] = 6.21361
K_{500} = (6.21361 - 0.5589)/1.2020 = 4.7044,
Hence, x_{500} = 6437 + 4.7044*2951 = 20320m^3/s.
From Eq.(6.16a), $b = \sqrt{1 + 1.3(4.7044) + 1.1(4.7044)^2}$ = 5.61

$$S_e = \text{probable error} = 5.61 * \frac{2951}{\sqrt{92}} = \underline{1726}$$

(a) For the 95% confidence probability *f(c)* = 1.96 and by Eq.(4.16) $x_{1/2}$ = 20320x_T± (1.96*1726), which results in x_1 = 23703m^3/s and x_2 = 16937m^3/s. Thus the estimated discharge of 20320m^3/s has a 95% probability of lying between 23700 and 16940m^3/s.

(b) For 80% confidence probability, f(c) = 1.282 and by Eq.(4.16) $x_{1/2}$ = 20320x_T± (1.282*1726), which results in x_1 = 22533m^3/s and x_2 = 18107m^3/s. Thus the estimated discharge of 20320 m^3/s has an 80% probability of lying between 22533 and 18107m^3/s.

For the data of Example 4.2, the values of x_T for different values of T are calculated and can be shown plotted on a Gumbel probability paper.

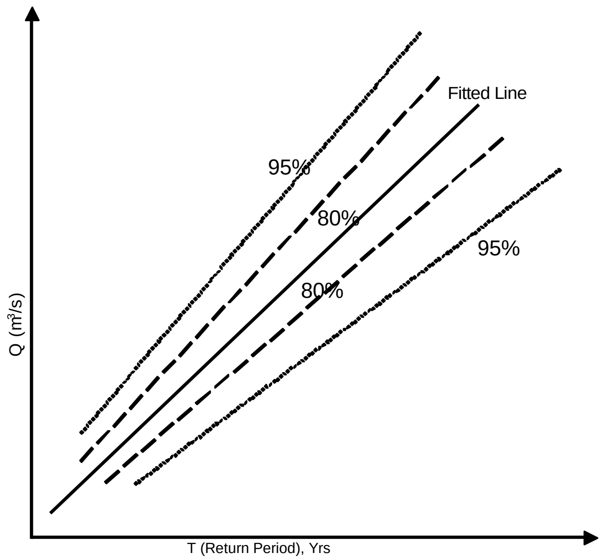


Figure 4.2: Confidence bands for Gumbel's distribution: Example 4.2.

Example 4.3: For the annual flood series data given in Example 3.1, estimate the flood discharge for a return period of (a) 100 years (b) 200 years and (c) 1000 years by using Log-Pearson Type III distribution.
Solution: The variate z = log x is first calculated for all the discharges in table below. Then the statistics $\bar{z}$, σ_z and C_s are calculated from table 4.6 to obtain

$\sigma_z = 0.1427$ $$C_s = \frac{27 * 0.0030}{(26)(25)(0.1427)^3}$$

$\bar{z} = 3.6071$ $C_s = 0.043$

The flood discharge for a given T is calculated as below. Here, values of K_z for given T and $C_s = 0.043$ are read from table 4.6.

$\bar{z}$ =3.6071, σ_z = 0.1427, C_s = 0.043				
T (years)	K_z (from table 3.5) (for C_s = 0.043)	$K_z\sigma_z$	$Z_T = \bar{z} + K_z\sigma_z$	X_T = antilog z_T (m^3/s)
100	2.358	0.3365	3.9436	8782
200	2.616	0.3733	3.9804	9559
1000	3.152	0.4498	4.0569	11400

Year	Flood x(/s)	z = log x	Year	Flood x(m^3/s)	z = log x
1951	2947	3.4694	1965	4366	3.6401
1952	3521	3.5467	1966	3380	3.5289
1953	2399	3.3800	1967	7826	3.8935
1954	4124	3.6153	1968	3320	3.5211
1955	3496	3.5436	1969	6599	3.8195
1956	2947	3.4694	1970	3700	3.5682
1957	5060	3.7042	1971	4175	3.6207

Year	Flood x(/s)	z = log x	Year	Flood x(m^3/s)	z = log x
1958	4903	3.6905	1972	2988	3.4754
1959	3751	3.5748	1973	2709	3.4328
1960	4798	3.6811	1974	3873	3.5880
1961	4290	3.6325	1975	4593	3.6621
1962	4652	3.6676	1976	6761	3.8300
1963	5050	3.7033	1977	1971	3.2947
1964	6900	3.8388			

4.4 Regional Frequency Analysis

A regional frequency analysis usually involves a regression analysis of gauged watersheds within the general region. Through this powerful technique, sufficiently reliable equations can be derived for peak flow of varying frequency given quantifiable physical basin characteristics and rainfall intensity for a specific duration. Once these equations are developed, they can then be applied to ungauged basins within the same region.

A regional analysis usually consists of the following steps:

1. Select components of interest, such as mean and peak flows.
2. Select definable basin characteristics of gauged watershed: drainage area, length, slope, etc.
3. Derive prediction equations with single-or multiple-linear regression analysis.
4. Map and explain the residuals (differences between computed and observed values) that constitute "unexplained variances" in statistical analysis on a regional basis.

The equation can then be used in ungauged areas within the same region and for data of similar magnitude to that used in the development process.

4.5 Low Flow Analysis

Characterization of the magnitude, frequency, and duration of low stream flows and droughts is vital for assessing the reliability of flows for all in-stream and withdrawal uses and for defining resource shortages and drought.

4.5.1 Definitions and Basic Concepts

Low Stream flows

The objective of low-flow analysis is to estimate the frequency or probability with which stream flow in a given reach will be less than various levels. Thus the flow-duration curve; is an important tool of low-flow analysis; from it one can readily determine the flow associated with any exceedence or non-exceedence probability. Most of the time, the flow exceeded 95% of the time, q_{95}, is a useful index of water availability that is often used for design purposes.

For purposes of statistical analysis, low flows are defined as annual minimum flows averaged over consecutive-day periods of varying length. The most commonly used averaging period is d = 7days, but analyses are often carried out for d = 1,3, 15, 30, 60, 90 and 180 days as well. Low-flow quantile values are cited as "dQp," where p is now the annual non-exceedence probability (in percent) for the flow averaged over d-days. The 7-day average flow that has an annual non-exceedence probability of 0.10 (a recurrence interval of 10 yr), called "7QI0," is commonly used as a low-flow design value. the "7Q10" value is interpreted as follows:

In any year there is a 10% probability that the lowest 7-consecutive-day average flow will be less than the 7QIO value.

Droughts

Droughts are extended severe dry periods. To qualify as a drought, a dry period must have duration of at least a few months and be a significant departure from normal. Drought must be expected as part of the natural climate, even in the absence of any long term climate change. However, "permanent" droughts due to natural climate shifts do occur, and appear to have been responsible for large scale migrations and declines of civilizations through human history. The possibility of regional droughts associated with climatic shifts due to warming cannot be excluded.

As shown in Figure 3.3, droughts begin with a deficit in precipitation that is unusually extreme and prolonged relative to the usual climatic conditions **(meteorological drought)**. This is often, but not always, accompanied by unusually high temperatures, high winds, low humidity, and high solar radiation that result in increased evapotranspiration.

These conditions commonly produce extended periods of unusually low soil moisture, which affect agriculture and natural plant growth and the moisture of forest floor **(Agricultural drought)**. As the precipitation deficit continues, stream discharge, lake, wetland, and reservoir levels, and water-table decline to unusually low levels **(Hydrological drought).** When precipitation returns to more normal values, drought recovery follows the same sequence: meteorological, agricultural, and hydrological.

Meteorological drought is usually characterized as a precipitation deficit.

4.5.2 Low flow frequency analysis

As noted earlier, the objective of low flow frequency analysis is to estimate quantiles of annual d-day-average minimum flows. As with floods, such estimates are usually required for reaches without long-term stream flow records. These estimates are first developed by analyzing low flows at gauging stations.

Low flow analysis at gauging stations:

For gauged reach, low flow analysis involves development of a time series of annual d-day low flows, where d is the averaging period. As shown in the table below, the analysis begins with a time series of average daily flows for each year. Then the overlapping d-day averages are computed for the d values of interest. For each value of d, this creates 365-(d-1) values of consecutive d-day averages for each year. The smallest of these values is then selected to produce an annual time series of minimum d-day flows. It is this time series that is then subjected to frequency analysis to estimate the quantiles of the annual d-day flows.

Example 4.4: Computation of d-consecutive Day averages for low flow analysis. Values in **bold** are minimum for the period shown

Day	1-Day Average (m^3/s)	3-Day Average (m^3/s)	7-Day Average (m^3/s)	15-Day Average (m^3/s)
1	5.75			
2	5.55	5.59		
3	5.47	5.45		
4	5.32	5.34	5.35	
5	5.24	5.24	5.24	
6	5.15	5.13	5.29	
7	4.98	5.04	5.52	
8	4.98	5.29	5.70	5.60
9	5.89	5.97	5.78	5.58
10	7.05	6.52	5.82	5.63
11	6.63	6.48	5.87	5.73
12	5.78	5.95	5.93	5.77
13	5.44	5.51	5.87	5.78
14	5.32	5.39	5.76	5.77
15	5.41	5.41	5.81	5.76
16	5.49	5.71	5.85	5.75
17	6.23	6.25	5.83	5.67
18	7.02	6.42	5.78	5.54
19	6.00	6.12	5.70	**5.50**
20	5.32	5.43	5.61	5.52
21	4.96	5.04	5.39	5.53
22	4.84	4.89	**5.12**	6.08
23	4.87	**4.81**	5.13	6.79
24	**4.73**	4.91	5.22	
25	5.13	5.30	5.31	
26	6.03	5.71	6.55	
27	5.97	5.86	8.15	
28	5.58	8.37		
29	13.56	11.73		
30	16.06			

Low flow analysis at ungauged stations:

As with floods, estimates of low flow quantiles are usually required for stream reaches where there are no long-term gauging station records. There are two basic approaches to developing such estimates:

1. Relate dQp values to drainage-basins characteristics via regression analysis.
2. During the low flow season, make a number of spot measurements of discharge at the ungauged stream reach where the dQp estimate is needed. Then relate those flows to concurrent flows at a nearby gauging station using

   $q_u = a + bq_g$ (4.22)

where:

q_u is the flow at the ungauged site, q_g is the concurrent flow at the gauged site, and a and b are estimated via regression anlysis. Then estimate the dQp at the ungauged site, dQpu, as:

$dQp_u = a + b.dQ_g$ (4.23)

where:

dQq_g is the dQp value established by frequency analysis at the gauged site. In order to minimize errors when using this procedure, each pair of flows used to establish equation (4.22) should be from a separate hydrograph recession, the r^2 value for the relation of equation (4.22) should be at least 0.70 and the two basins should be similar in size, geology, topography, and climate.

4.5.3 Drought analysis

The objective of drought analysis is to characterize the magnitude, duration, and severity of meteorological, agricultural, or hydrological drought in a region of interest. The analysis process can be structured in terms of five questions:

1. What type of drought of interest?
2. What averaging period will be used?
3. How will "drought" be quantitatively defined?
4. What are the magnitude-frequency relations of drought characteristics?
5. How are regional aspects of drought addressed?

Drought type:

As noted, one may be interested in one or more of the basic types of drought, each reflected in time series of particular types of data: meteorological (precipitation); agricultural (soil moisture); or hydrological (stream flow, reservoir levels, or ground water levels)

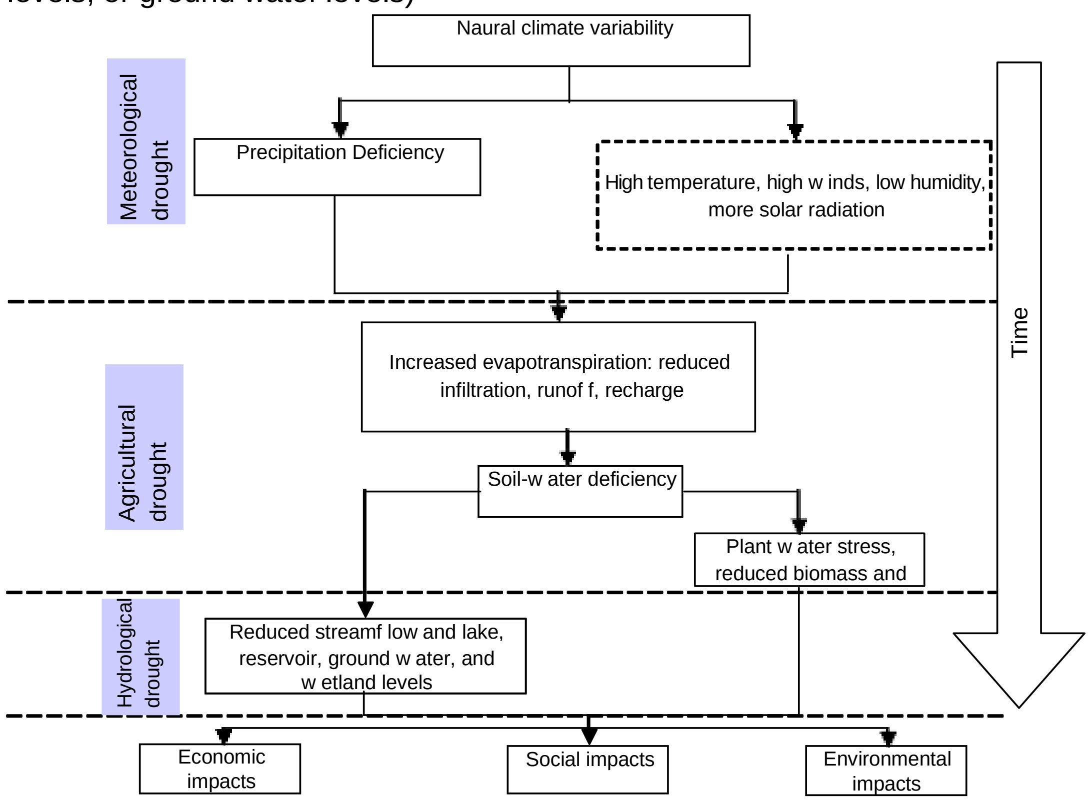


Figure 4.3: Sequence of drought impacts

Averaging Period:

As with time-series analysis generally, drought analysis requires selection of an averaging period (dt). Since drought by definition have significant duration, one would usually select dt = 1 month, 3 months, or 1 yr, with the choice depending on the available data and the purposes of the analysis. For a given record length the selection of dt involves a trade-off in uncertainty of the analysis:

Smaller sample size → greater uncertanity

Longer dt

Lower autocorrelation → smaller uncertainity

Drought definition:

Figure 4.4 shows a time series of a selected quantity, X (e.g., precipitation, stream flow, ground water level), averaged over an appropriate dt. The quantitative definition of drought is determined by the truncation level, X_0, selected by the analyst: Values of X < X_0 are defined as droughts. Typical values for X_0 might be:

$$X_0 = \bar{X}\,(\,mean\ value\ of\ X\,)$$

$$X_0 = X_{50}\,(\,median\ of\ X\,)\ \text{or}$$

$$X_0 = \bar{X} - \sigma_{n-1}\,(mean\ \text{min}\ us\ one\ s\ \text{tan}\ dard\ deviation)$$

Dracup et al. (1980) suggested choosing $X_0 = \bar{X}$ because it standardizes the analysis and gives more significance to extreme events, which are usually of most interest.

Once X_0 is determined, each period for which X < X_0 constitutes a "drought" and each "drought" is characterized by the following measures:

Duration, *D* = length of period for which X < X_0;

Severity, *S* = cumulative deviation from X_0;

Intensity (or magnitude), *I* = *S/D*.

Note that if X is stream flow **[L^3T^{-1}],** then the dimensions of ***S*** are **[L^3T^{-1}]x[T]** =**[L^3]** and the dimensions of ***I*** are **[L^3T^{-1}]**

Magnitude- Frequency Relations:

Once the severities, durations, and intensities of "drought" have been determined for a given time series, the magnitude-frequency characteristics of each of those quantities can be analyzed.

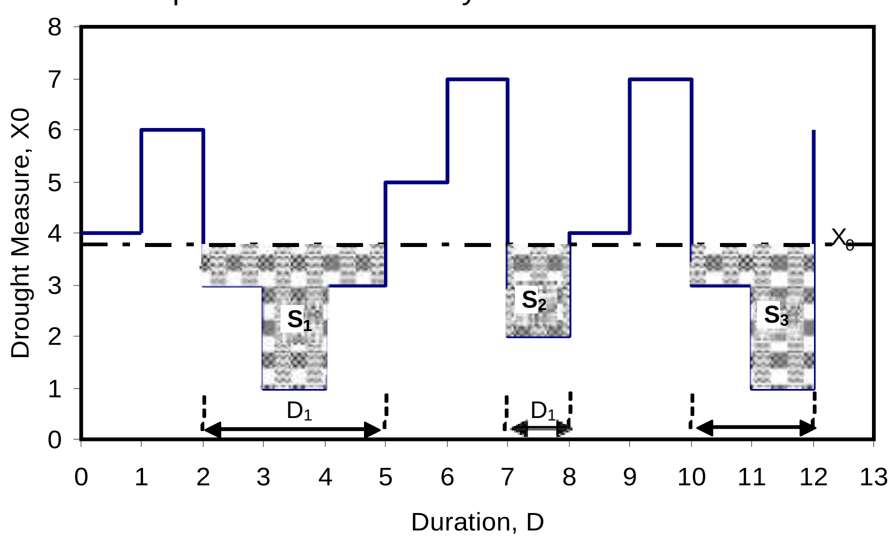


Figure 4.4: Quantitative definition of droughts. X is a drought measure, X_0 is the truncation level. D_1, D_2, D_3 are durations of droughts 1, 2 and 3. The areas S_1, S_2, S_3 are severities of droughts 1, 2 and 3.

4.6 Precipitation Probability

The preceding discussions on flood probability apply generally to precipitation. Annual maximum hourly or daily amounts ordinarily conform to Gumbel Type I, Log-Pearson or Log-Normal distribution. In humid areas, where the mean is high, monthly, seasonal, or annual totals will approximate a normal distribution. In drier areas a skew distribution such as the Log-Pearson, Log-Normal may give a better fit.

4.7 Reading Assignment

- Depth-Area-Duration Relationships
- Depth-Area Relation
- Maximum Depth-Area-Duration Curve (DAD Curve)
- Intensity-Duration-Frequency Relationships

4.8 Risk, Reliability and Safety factor

Risk and Reliability: The designer of a hydraulic structure always faces a nagging doubt about the risk of failure of his structure. This is because the estimation of the hydrologic design values (such as the design flood discharge and the river stage during the design flood) involve a natural or inbuilt uncertainty and as such a hydrological risk of failure. As an example, consider a weir with an expected life of 50 years and designed for a flood magnitude of return period T=100 years. This weir may fail if a flood magnitude greater than the design flood occurs within the life period (50 years) of the weir.

The probability of occurrence of an event ($x \geq x_T$) at least once over a period of n successive years is called the risk, $\bar{R}$. Thus the risk is given by $\bar{R}$ = 1 - (probability of non-occurrence of the event $x \geq x_T$ in n years)

$$\bar{R} = 1 - (1-P)^n = 1 - \left(1 - \frac{1}{T}\right)^n \qquad (4.24)$$

Where P = probability P($x \geq x_T$) = $\frac{1}{T}$ T = return period

The reliability R_e, is defined as

$$R_e = 1 - \bar{R} = \left(1 - \frac{1}{T}\right)^n \qquad (4.25)$$

It can be seen that the return period for which a structure should be designed depends upon the acceptable level of risk. In practice, the acceptable risk is governed by economic and policy considerations.

Safety Factor: In addition to the hydrologic uncertainty, as mentioned above, a water resource development project will have many other uncertainties. These may arise out of structural, constructional, operational and environmental causes as well as from non-technological considerations such as economic, sociological and political causes. As such, any water resource development project will have a safety factor for a given hydrological parameter M as defined below.

Safety factor (for the parameter M) = $(SF)_m$=

$$= \frac{\text{Actual value of the parameter M adopted in the design of the project}}{\text{Value of the parameter M obtained from hydrological considerations only}}$$

$= C_{am} / C_{hm}$ (4.26)

The parameter M includes such items as flood discharge magnitude, maximum river stage, reservoir capacity and free board. The difference (C_{am} - C_{hm}) is known as *Safety Margin.*

Example 4.5: A bridge has an expected life of 25 years and is designed for a flood magnitude of return period 100 years. (a) What is the risk of this hydrological design? (b) If 10% risk is acceptable, what return period will have to be adopted?

Solution:

(a) The risk $\bar{R}$ for n = 25 years and T = 100 years is: $\bar{R} = 1-\left(1-\frac{1}{100}\right)^{25} = 0.222$. Hence the inbuilt risk in this design is 22.2%.

(b) If $\bar{R}$ =10% = 0.10, $0.10 = 1-\left(1-\frac{1}{T}\right)^{25} \Rightarrow \left(1-\frac{1}{T}\right)^{25} = 0.90$ and T = 238 years

(c) say 240 years. Hence to get 10% acceptable risk, the bridge will have to be designed for a flood of return period T = 240 years.

Exercise: Annual flood data of a certain river covering the period 1948 to 1979 yielded for the annual flood discharges a mean of 29,600m^3/s and a standard deviation of 14,860m^3/s. for a proposed bridge on this river near the gauging site it is decided to have an acceptable risk of 10% in its expected life of 50 years. (a) Estimate the flood discharge by Gumbel's method for use in the design of this structure (b) If the actual flood value adopted in the design is 125,000m^3/s what are the safety factor and safety margin relating to maximum flood discharge? *(Answers (a) 105,000m^3/s and (b) $(SF)_{flood}$ = 1.19, Safety Margin for flood magnitude = 20,000m^3/s)*

5 STOCHASTIC HYDROLOGY

5.1 INTRODUCTION

Stochastic hydrology describes the physical processes involved in the movement of water onto, over, and through the soil surface. Quite often the hydrologic problems we face do not require a detailed discussion of the physical process, but only a time series representation of these processes. Stochastic models may be used to represent, in simplified form, these hydrologic time series. Some background in probability and statistics is necessary to fully understand this concept.

5.2 TIME SERIES

The measurements or numerical values of any variable that changes with time constitute a time series. In many instances, the pattern of changes can be ascribed to an obvious cause and is readily understood and explained, but if there are several causes for variation in the time series values, it becomes difficult to identify the several individual effects. In Fig. 5.1, the top graph shows a series of observations changing with time along the abscissa; the ordinate axis represents the changing values of y with time, t. From visual inspection of the series, there are three discernible features in the pattern of the observations. Firstly, there is a regular gradual overall increase in the size of values; this trend, plotted as a separate component $y_1(t)$, indicates a linear increase in the average size of *y* with time. The second obvious regular pattern in the composite series is a cyclical variation, represented separately by $y_2(t)$, the periodic component. The third notable feature of the series may be considered the most outstanding, the single high peak half way along the series. This typically results from a rare catastrophic event which does not from part of a recognizable pattern. The definition of the function $y_3(t)$ needs very careful consideration and may not be possible. The remaining hidden feature of the series is the random stochastic component, $y_4(t)$, which represents an irregular but continuing variation within the measured values and may have some persistence. It may be due to instrumental of observational sampling errors or it may come from random unexplainable fluctuations in a natural physical process. A time series is said to be a random or stochastic process if it contains a stochastic component. Therefore, most hydrologic time series may be thought of as stochastic processes since they contain both deterministic and stochastic components. If a time series contains only random/stochastic component is said to be a purely random or stochastic process.

The complete observed series, y(t), can therefore be expressed by:

$$y(t) = y_1(t) + y_2(t) + y_3(t) + y_4(t) \tag{5.1}$$

The first two terms are deterministic in form and can be identified and quantified fairly easily; the last two are stochastic with major random elements, and some minor persistence effects, less easily identified and quantified.

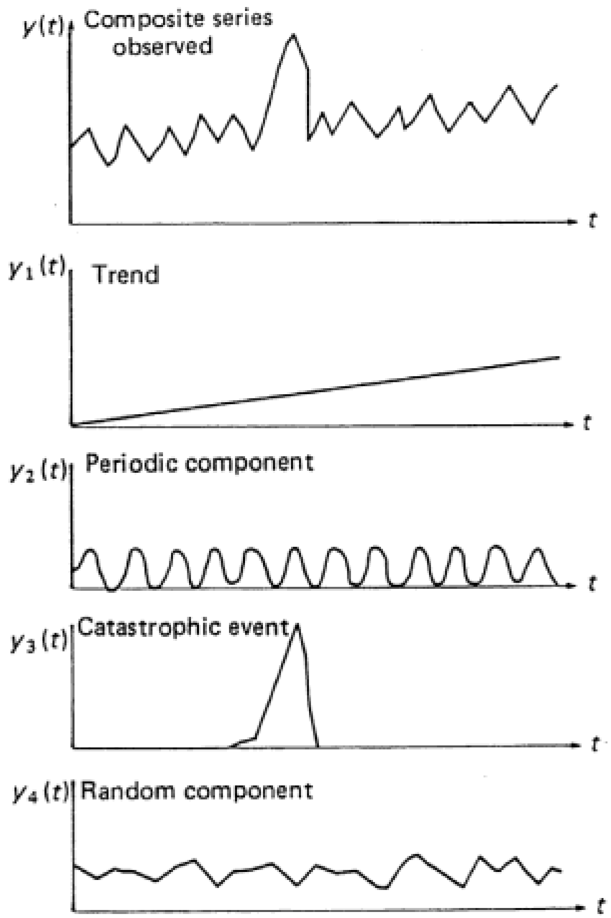


Figure 5.1: The time series components

5.3 PROPERTIES OF TIME SERIES

The purpose of a stochastic model is to represent important statistical properties of one or more time series. Indeed, different types of stochastic models are often studied in terms of the statistical properties of time series they generate. Examples of these properties include: trend, serial correlation, covariance, cross-correlation, etc. Therefore, before reviewing the different types of stochastic models used in hydrology, some distribution properties of stochastic processes need to be discussed. The following basic statistics are usually used for expressing the properties/characteristics of a time series.

$$Mean\ ,\ \mu = E(X_1) = \bar{X} = \frac{1}{n}\sum_{t=1} X_t \tag{5.2}$$

$$Variance,\ \sigma^2 = S^2 = \frac{1}{n-1}\sum_{t=1}^{n} (X_t - \bar{X})^2 \tag{5.3}$$

$$Co\,variance,\ \lambda = Cov(X_t, X_{t+L}) = \frac{1}{n-L}\sum_{t=1}^{n-L}(X_t - \bar{X})(X_{t+L} - \bar{X}) \tag{5.4}$$

Where L is the time lag.

Stationary time series:

If the statistics of the sample (mean, variance, covariance, etc.) as calculated by equations (5.2)-(5.4) are not functions of the *timing* or the *length* of the sample, then the time series is said to be stationary to the second order moment, weekly stationary, or stationary in the broad sense. Mathematically one can write as:

$$E(X_1) = \mu$$
$$Var(X_t) = \sigma^2$$
$$Cov(X_t, X_{t+L}) = \lambda_L$$

In hydrology, moments of the third and higher orders are rarely considered because of the unreliability of their estimates. Second order stationarity, also called covariance stationarity, is usually sufficient in hydrology. A process is strictly stationary when the distribution of X_t does not depend on time and when all simultaneous distributions of the random variables of the process are only dependent on their mutual time-lag. In another words, a process is said to be strictly stationary if its n-th (n for any integers) order moments do not depend on time and are dependent only on their time lag.

Non-stationary time series:

If the values of the statistics of the sample (mean, variance, covariance, etc.) as calculated by equations (5.2)-(5.4) are dependent on the *timing* or the *length* of the sample, i.e. if a definite trend is discernible in the series, then it is a non-stationary series. Similarly, periodicity in a series means that it is non-stationary. Mathematically one can write as:

$$E(X_1) = \mu_t$$
$$Var(X_t) = \sigma_t^2$$
$$Cov(X_t, X_{t+L}) = \lambda_{L,t}$$

White noise time series:

For a stationary ties series, if the process is purely random and stochastically independent, the time series is called a white noise series. Mathematically one can write as:

$$E(X_1) = \mu$$
$$Var(X_t) = \sigma^2$$
$Cov(X_t, X_{t+L}) = 0$ for all $L \neq 0$

Gaussian time series:

A Gaussian random process is a process (not necessarily stationary) of which all random variables are normally distributed, and of which all simultaneous distributions of random variables of the process are normal. When a Gaussian random process is weekly stationary, it is also strictly stationary, since the normal distribution is completely characterized by its first and second order moments.

5.4 ANALYSIS OF HYDROLOGIC TIME SERIES

Records of rainfall and river flow form suitable data sequences that can be studied by the methods of *time series analysis*. The tools of this specialized topic in mathematical statistics provide valuable assistance to engineers in solving problems involving the frequency of occurrences of major hydrological events. In particular, when only a relatively short data record is available, the formulation of a time series model of those data can enable long sequences of comparable data to be generated to provide the basis for better estimates of hydrological behaviour. In addition, the time series analysis of rainfall, evaporation, runoff and other sequential records of hydrological variables can assist in the evaluation of any irregularities in those records. Cross-correlation of different hydrological time series may help in the understanding of hydrological processes.

Tasks of time series analysis include:

(1) Identification of the several components of a time series.

(2) Mathematical description (modelling) different components identified.

If a hydrological time series is represented by X_1, X_2, $X_{3,}$ ..., X_t, ..., then symbolically, one can represent the structure of the X_t by:

$X \Leftrightarrow [T_t, P_t, E_t]$

Where T_t is the trend component, P_t is the periodic component and E_t is the stochastic component. The first two components are specific deterministic features and contain no element of randomness. The third, stochastic, component contains both random fluctuations and the self-correlated persistence within the data series. These three components form a basic model for time series analysis.

The aims of time series analysis include but not limited to:

(1) description and understanding of the mechanism,

(2) Monte-Carlo simulation,

(3) forecasting future evolution,

Basic to stochastic analysis is the assumption that the process is stationary. The modelling of a time series is much easier if it is stationary, so identification, quantification and removal of any non-stationary components in a data series is under-taken, leaving a stationary series to be modelled.

5.4.1 Trend component

This may be caused by long-term climatic change or, in river flow, by gradual changes in a catchment's response to rainfall owing to land use changes. Sometimes, the presence of a trend cannot be readily identified.

Methods of trend identification:

Different statistical methods, both *nonparametric tests and parametric tests,* for identifying trend in time-series are available in the literature. Two commonly used methods for identifying the trend are discussed briefly in this section.

(1) Mann-Kendall test

The test uses the raw (un-smoothed) hydrologic data to detect possible trends. The Kendall statistic was originally devised by Mann (1945) as a non-parametric test for trend. Later the exact distribution of the test statistic was derived by Kendall (1975).

The Mann-Kendall test is based on the test statistic S defined as follows:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}(x_j - X_i) \qquad (5.5)$$

Where the X_j are the sequential data values, *n* is the length of the data set, and

$$\operatorname{sgn}(\theta)=\begin{cases}1 & if\ \theta>0\\ 0 & if\ \theta=0\\ -1 & if\ \theta<0\end{cases} \qquad (5.6)$$

Mann (1945) and Kendall (1975) have documented that when, the statistic S is approximately normally distributed with the mean and the variance as follows:

E(S) =0 (5.7)

$$V(S)=\frac{n(n-1)(2n+5)-\sum_{p=1}^{q}t_p(t_p-1)(2t_p+5)}{18} \qquad (5.8)$$

Where n = number of data
t_p = the number of ties for the p^{th} value (number of data in the p^{th} group)
q = the number of tied values (number of groups with equal values/ties)
The standardized Mann-Kendall test statistic Z_{MK} is computed by

$$\operatorname{sgn}(\theta)=\begin{cases}\dfrac{S-1}{\sqrt{Var(s)}} & S>0\\ 0 & S=0\\ \dfrac{S+1}{\sqrt{Var(s)}} & S<0\end{cases} \qquad (5.9)$$

The standardized MK statistic Z follows the standard normal distribution with mean of zero and variance of one.
The hypothesis that there has not trend will be rejected if

$$\left|Z_{mk}\right|>Z_{1-\alpha/2} \qquad (5.10)$$

Where $Z_{1-\alpha/2}$ is the value read from a standard normal distribution table with α being the significance level of the test.

(2) Linear regression method

Linear regression method can be used to identify if there exists a linear trend in a hydrologic time series. The procedure consists of two steps, fitting a linear regression equation with the time T as independent variable and the hydrologic data, Y as dependent variable, i.e.

$$Y = \alpha + \beta.T \qquad (5.11)$$

and testing the statistical significance of the regression coefficient β.
Test of hypothesis concerning β can be made by noting that $(\beta-\beta_0)/S_\beta$ has t distribution with n-2 degrees of freedom. Thus the hypothesis H_0: $\beta=\beta_0$ versus H_0: $\beta\neq\beta_0$ is tested by computing

$$t=\frac{\beta-\beta_0}{S_\beta} \qquad (5.12)$$

Where S_β is the standard deviation of the coefficient β with

$$S_\beta=\frac{S}{\sqrt{\sum_{i=1}^{n}(T_i-\bar{T})^2}} \qquad (5.13)$$

and

$$S = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2} \qquad (5.14)$$

Where S is the standard error of the regression, Y_i *and* $\hat{Y}$ are observed and estimated hydrologic variable from the regression equation, respectively.
The hypothesis, i.e. no trend, is rejected if $|t| > t_{1-\alpha/2,\, n-2}$

Models for trend:

The shape of the trend depends on the background of the phenomenon studied. Any smooth trend that is discernible may be quantified and then subtracted from the sample series. Common models for trend may take the following forms:

$T_t = a + bt$ (a linear trend, as in Fig. 5.1) (5.15)

or

$T_t = a + bt + ct^2 + dt^3 + ...$ (a non-linear trend) (5.16)

The coefficients a, b, c, d ... are usually evaluated by least-squares fitting. The number of terms required in a polynomial trend being primarily imposed by the interpretation of the studied phenomenon. The number of terms is usually based on statistical analysis, which determines the terms contributing significantly to the description and the interpretation of the time series. Restriction is made to the significant terms because of the principle of parsimony concerning the number of unknown parameters (constants) used in the model. One wishes to use as small a number of parameters as possible, because in most cases the addition of a complementary parameter decreases the accuracy of the other parameters. Also prediction- and control procedures are negatively influenced by an exaggerated number of parameters. This principle of parsimony is not only important with respect to the selection of the trend function but also with respect to other parts of the model.

5.4.2 Periodic component

In most annual series of data, there is no cyclical variation in the annual observations, but in the sequences of monthly data distinct periodic seasonal effects are at once apparent. The existence of periodic components may be investigated quantitatively by (1) Fourier analysis, (2) spectral analysis, and (3) autocorrelation analysis. Of which, the autocorrelation analysis method is widely used by hydrologists and will be discussed briefly in this section.

Identification of periodic component by autocorrelation analysis:

The procedure consists of two steps, calculating the autocorrelation coefficients and testing their statistical significance. For a series of data, X_t, the autocorrelation coefficient r_L between X_t and X_{t+L} are calculated and plotted against values of L (known as the lag), for all pairs of data L time units apart in the series:

$$r_L = \frac{1}{n-L}\sum_{t=1}^{n-L}(X_t - \bar{X})(X_{t+L} - \bar{X}) \Big/ \frac{1}{n}\sum_{t=1}^{n}(X_t - \bar{X})^2 \qquad (5.17)$$

Where $\bar{X}$ is the mean of the sample of n values of X_t and L is usually taken for values from zero up to n/4. A plot of r_L versus L forms the correlogram. The characteristics of a time series can be seen from the correlogram. Examples of correlograms are given in Fig 54.2. Calculation of equation (5.17) for different L gives the following cases:

- If L = 0, r_L = 1. That is, the correlation of an observation with itself is one.

- If $r_L \approx 0$ for all $L \neq 0$, the process is said to be a purely random process. This indicates that the observations are linearly independent of each other. The correlogram for such a complete random time series is shown in Fig 5.2(a).
- If $r_L \neq 0$ for some $L \neq 0$, but after $L > \tau$, then , the time series is still referred to as simply a random one (not purely random) since it has a 'memory' up to $L = \tau$. When r_τ , the process is said to have no memory for what occurred prior to time $t-\tau$. The correlogram for such a non-independent stochastic process is shown in Fig 5.2(b). This is representative of an auto regressive process. Typically, such a correlogram could be produced from a series described by the Autoregressive model:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3} + \ldots + \varepsilon_t \qquad (4.18)$$

where a_i are related to the autocorrelation coefficients r_i and ε_t is a random independent element.

- In the case of data containing a cyclic (deterministic) component, then $r_L \neq 0$ for all $L \neq 0$, the correlogram would appear as in Fig. 5.2(c). Where T is the period of the cycle.

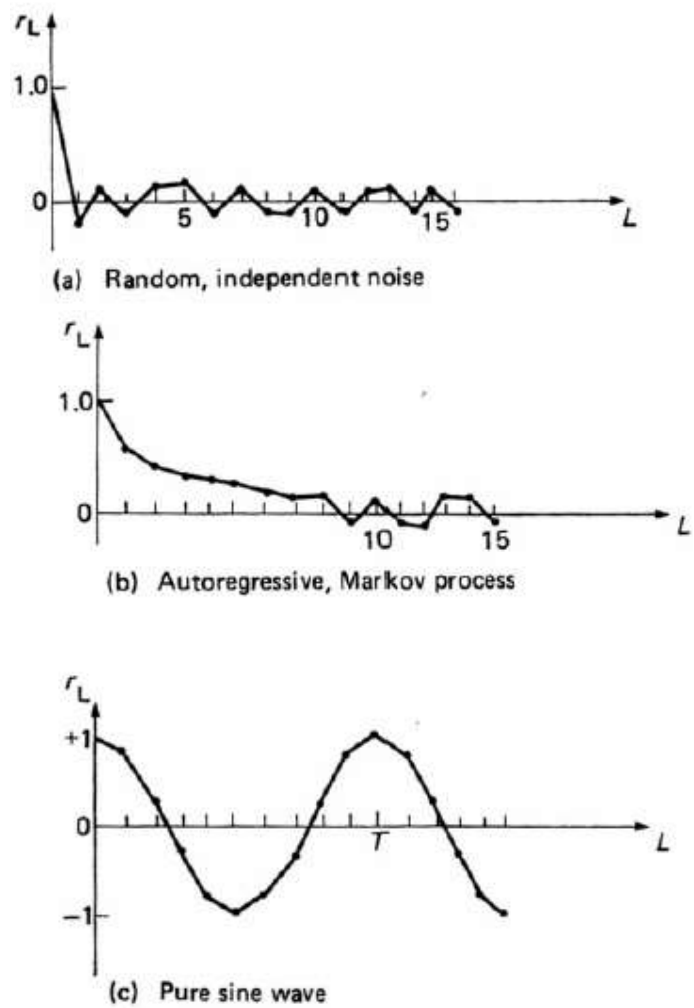


Figure 5.2 Examples of Correlograms

Modelling of periodic component:

A periodic function P_t is a function such that

$P_{t+T} = P_t$ for all t

The smallest value of T is called the period. The dimension of T is time, T thus being a number of time-units (years, months, days or hours, etc.) and we also have

$P_{t+nT} = P_t$ for all t and for all integer n.

The frequency is defined as the number of periods per time-unit:

$$frequency = \frac{1}{period}$$

Trigonometric functions are simple periodic functions. For example,

$\alpha \sin(\omega t + \beta)$

has a period of $2\pi/\omega$, because

$\alpha \sin[\omega(t+2\pi/\omega)+\beta] = \alpha \sin(\omega t+2\pi+\beta) = \alpha \sin(\omega t + \beta)$

The pulsation or angular frequency is defined as

$$\omega = \frac{2\pi 1}{period} = 2\pi \, . \, frequency$$

the constant α is termed the amplitude and β the phase (with respect to the origin) of the sine-function.

A simple model for the periodic component may be defined as (for more discussions refer to the literature of Time Series Analysis):

$Pt = m + C\sin(2\quad t/T)$ (5.19)

Where C is the amplitude of the sine wave about a level m and of wavelength T. The serial (auto) correlation coefficients for such a P_t are given by:

$r_L = \cos(2\quad L/T)$ (5.20)

The cosine curve repeats every T time units throughout the correlogram with $r_L = 1$ for L = 0, T, 2T, 3T,… Thus periodicities in a time series are exposed by regular cycles in the corresponding correlograms.

Once the significant periodicities, P_t, have been identified and quantified by μ_t (the means) and σ_t (the standard deviations) they can be removed from the original times series along with any trend, T_t, so that a new series of data, E_t, is formed:

$$E_t = \frac{X_t - T_t - m_t}{S_t} \qquad (5.21)$$

Simple models for periodic component in hydrology can be seen in the literature. For example, in many regions, typical monthly potential evapotranspiration variation during the year can be modelled more or less by a sinusoidal function, with a couple of parameters to tune the annual mean and the amplitude (Xu and Vandewiele, 1995).

This behavior leads to the idea to model by a truncated Fourier series:

ep_t = {a + bsin[(2 /12)(t-c)]}+

where again *t* is time in month. The plus sign at the end is necessary for avoiding negative values of *ep* which otherwise may occur in rare cases. Again parameters a, b and c are characteristics of the basin.

5.4.3 Stochastic component

E_t represents the remaining stochastic component of the time series free from non-stationary trend and periodicity and usually taken to be sufficiently stationary for the next stage in simple time series analysis. This E_t component is analysed to explain and quantify any persistence (serial (auto) correlation) in the data and any residual independent randomness. It is first standardized by:

$$Z_t = \frac{E_t - \bar{E}}{S_E} \tag{5.22}$$

Where $\bar{E}$ and s_E are the mean and standard deviation of the E_t series. The series, Z_t, then has zero mean and unit standard deviation. The autocorrelation coefficients of Z_t are calculated and the resultant correlogram is examined for evidence and recognition of a correlation and/or random structure.

For example, in Fig. 5.3a for a monthly flow, the correlogram of the Z_t stationary series (with the periodicities removed) has distinctive features that can be recognized. Comparing it with Fig. 5.2, the Z_t correlogram resembles that of an auto regressive (Markov) process. For a first order Markov model:

$$Z_t = r_1 Z_{t-1} + e_t \tag{5.23}$$

Where r_1 is the autocorrelation coefficient of lag 1 of the Z_t series and e_t is a random independent residual. A series of the residuals e_t may then be formed from the Z_t series and its known lag 1 autocorrelation coefficient, r_1:

$$e_t = Z_t - r_1 Z_{t-1} \tag{5.24}$$

The correlogram of residuals is finally computed and drawn (Fig.5.3b). For this data this resembles the correlogram of 'white noise', i.e. independently distributed random values. If there are still signs of autoregression in the e_t correlogram, a second-order Markov model is tried, and the order is increased until a random e_t correlogram is obtained. The frequency distribution diagram of the first order e_t values (Fig. 5.3c) demonstrates an approximate approach to the normal (Gussian) distribution.

At this stage, the final definition of the recognizable components of the time series has been accomplished including the distribution of the random residuals. As part of the analysis, the fitted models should be tested by the accepted statistical methods applied to times series. Once the models have been formulated and quantified to satisfactory confidence limits, the total mathematical representation of the time series can be used for solving hydrological problems by synthesizing non-historic data series having the same statistical properties as the original data series.

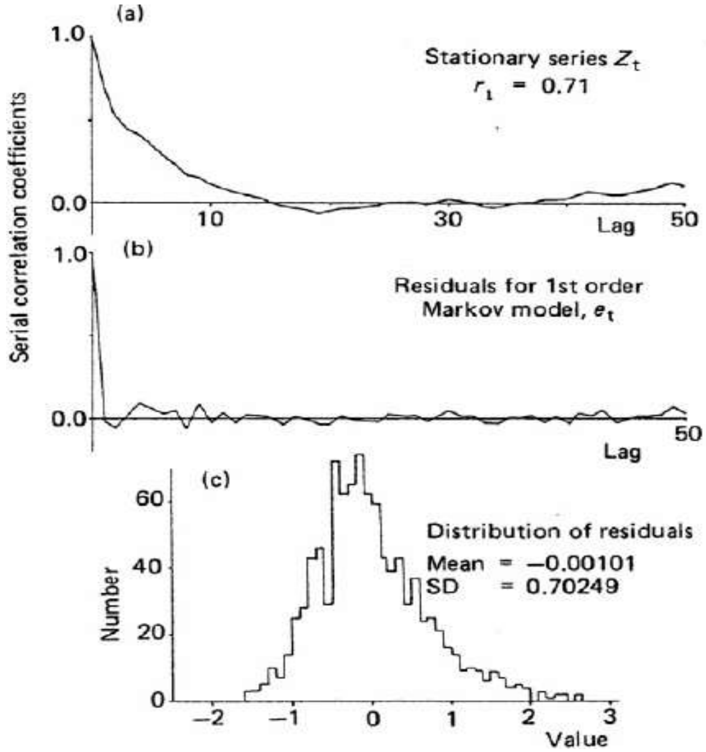


Figure 5.3 River Thames at Teddington Weirs (82 years of monthly flows, from Shaw, 1988)

5.5 TIME SERIES SYNTHESIS

The production of a synthetic data series simply reverses the procedure of the time series analysis. First, for as many data items as are required, a comparable sequence of random numbers, drawn from the e_t distribution, is generated using a standard computer package. Second, the corresponding synthetic Z_t values are recursively calculated using equation 5.23 (starting the series with the last value of the historic Z_t series as the Z_{t-1} value). Third, the E_t series then derives from equation 5.22 in reverse:

$$E_t = Z_t S_E + \bar{E} \qquad (5.25)$$

The periodic component P_t represented by m_t and s_t for time period *t* is then added to the E_t values to give:

$$X_t = T_t + E_t S_t + m_t \qquad \text{(from equation 5.21)} \qquad (5.26)$$

The incorporation of the trend component T_t then produces a synthetic series of X_t having similar statistical properties to the historic data series.

5.6 SOME STOCHASTIC MODELS

Ultimately design decisions must be based on a stochastic model or a combination of stochastic and deterministic models. This is because any system must be designed to operate in the future. Deterministic models are not available for generating future watershed inputs in the form of precipitation, solar radiation, etc., nor is it likely that deterministic models for these inputs will be available in the near future. Stochastic models must be used for these inputs.

5.6.1 Purely random stochastic models

Possibly the simplest stochastic process to model is where the events can be assumed to occur at discrete times with the time between events constants, the events at any time are independent of the events at any other time, and the probability distribution of the event is known. Stochastic generation from a model of this type merely amounts to generating a sample of random observations from a univariate probability distribution. For example, random observations for any normal distribution can be generated from the relationship,

$$y = \sigma R_N + \mu \qquad (5.27)$$

Where R_N is a standard random normal deviate (i.e. a random observation from a standard normal distribution) and μ and σ are the parameters of the desired normal distribution of y. Computer routines are available for generating standard random normal distribution.

5.6.2 Autoregressive models

Where persistence is present, synthetic sequences cannot be constructed by taking a succession of sample values from a probability distribution, since this will not take account of the relation between each number of sequences and those preceding it. Consider a second order stationary time series, such as an annual time series, made up of a deterministic part and a random part. The deterministic part is selected so as to reflect the persistence effect, while it is assumed that the random part has a zero mean and a constant variance. One of the models to simulate such a series is the Autoregressive model. The general form of an autoregressive model is:

$$(y_t - \mu) = \beta_1 (y_{t-1} - \mu) + \beta_2 (y_{t-2} - \mu) + ... + \beta_k (y_{t-k} - \mu) + \varepsilon_t \qquad (5.28)$$

Where μ is mean value of the series, β is the regression coefficient, the $\{y_1, y_2, \ldots, y_t, \ldots\}$ is the observed sequence and the random variables ε_t are usually assumed to be normally and independently distributed with zero mean and variance . In order to determining the order k of autoregression required to describe the persistence adequately, it is necessary to estimate k+2 parameters: $\beta_1, \beta_2, \ldots \beta_k$, μ and the variance of residuals . Efficient methods for estimating these parameters have been described by Kendall and Stuart (1968), Jenkins and Watts (1968).

The first order autoregression:

$$y_t - \mu = \beta_1 (y_{t-1} - \mu) + \varepsilon_t \qquad (5.29)$$

has found particular application in hydrology. When equation (5.29) is used to model annual discharge series, the model states that the value of y in one time period is dependent only on the value of y in the preceding time period plus a random component. It is also assumed that ε_t is independent of y_t.

Equation (5.29) is the well-known <u>first order Markov Model</u> in the literature. It has three parameters to be estimated: μ, β_1 , and σ_E^2.

For the moment method of parameter estimation, parameter μ can be computed from the time series as the arithmetic mean of the observed data.
As for β_1, the Yule-Walker equation (Delleur, 1991) shows that:

$$\rho_k = \sum_{j=1}^{P} \beta_j \rho_{k-j} \qquad k>0 \tag{5.30}$$

the above equation, written for k = 1, 2, …, yields a set of equations. Where ρ_k is the autocorrelation coefficient for time lag k. As the autocorrelation coefficients ρ_1, ρ_2, …, can be estimated from the data using equation (4.17), these equations can be solved for the autoregressive parameters β_1, β_2, …, β_p. This is the estimation of parameters by the method of moments. For example, for the first order autoregressive model, AR(1), the Yule-Walker equations yield

$$\rho_1 = \beta_1 . \rho_0 = \beta_1 \quad \sin ce\ \rho_0 = 1 \tag{5.31}$$

Similarly we can derive the equations for computing β_1 and β_2 for the AR(2) model as

$$\beta_1 = \frac{\rho_1 (1-\rho_2)}{1-\rho_1^2} \tag{5.32}$$

It can be shown that σ_E^2. is related to (the variance of the y_t series) by:

$$o_\varepsilon^2 = \sigma_y^2 (1-\beta_1^2) \tag{5.33}$$

If the distribution of y is $N(\mu_y, \sigma_y^2)$ then distribution of ε is N(0, σ_E^2.). Random values y_t can now be generated by selecting ε_t randomly from a N(0, o_E^2.) distribution. If z is N(0,1) then $Z\sigma_\phi or\ Z\sigma_y \sqrt{1-\beta_1^2}\ is\ N(0,\sigma_\varepsilon^2$. Thus, a model for generating Y's that are $N(\mu_y, \sigma_y^2)$ and follow the first order Markov model is

$$y_t = \mu_y + \beta_1 (y_{t-1} - \mu_y) + Z_t \sigma_y \sqrt{1-\beta_1^2} \tag{5.34}$$

The procedure for generating a value for y_t is:

(1) estimate μ_y, σ_y, and β_1 by $\bar{y}$, s_x and r_1(eq.5.17) respectively,
(2) select a z_t at random from a N(0, 1) distribution, and
(3) calculate y_t by eq. (5.34) based on $\bar{y}$, s_x and β_1, and y_{t-1}.

The first value of y_t, i.e. y_1, might be selected at random from a N(μ_y, σ_y^2).

To eliminate the effect of y_1 on the generated sequence, the first 50 or 100 generated values might be discarded.
Equation (5.34) has been widely used for generating annual runoff from watersheds

5.6.3 First order Markov process with periodicity: Thomas - Fiering model

The first order Markov model of the previous section assumes that the process is stationary in its first three moments. It is possible to generalise the model so that the periodicity in hydrologic data is accounted for to some extent. The main application of this generalisation has been in generating monthly streamflow where pronounced seasonality in the monthly flows exists. In its simplest form, the method consists of the use of twelve linear regression equations. If, say, twelve years of record are available, the twelve January flows and the twelve December flows are abstracted and January flow is regressed upon December flow; similarly, February flow is regressed upon January flow, and so on for each month of the year.

$$q_{jan} = \bar{q}_{jan} + b_{jan}(q_{dect} - \bar{q}_{dec}) + \varepsilon_{jant}$$

$$q_{feb} = \bar{q}_{feb} + b_{feb}(q_{jan} - \bar{q}_{jan}) + \varepsilon_{feb}$$

……….

Fig.4.4 shows a regression analysis of q_{j+1} on q_j, pairs of successive monthly flows for the months (j+1) and j over the years of record where j = 1, 2, 3, ..., 12 (Jan, Feb, ... Dec) and when j = 12, j+1 = 1 = Jan (there would be 12 such regressions). If the regression coefficient of month j+1 on j is bj, then the regression line values of a monthly flow, $\hat{y}_{j+1}$, can be determined from the previous months flow q_j, by the equation:

$$\hat{q}_{j+1} = \bar{q}_{j+1} + b_j(q_j - \bar{q}_j) \qquad (5.35)$$

To account for the variability in the plotted points about the regression line reflecting the variance of the measured data about the regression line, a further component is added:

$$Z.S_{j+1}\sqrt{(1-r_j^2)}$$

where is the standard deviation of the flows in month j+1, r_j is the correlation coefficient between flows in months j+1 and j throughout the record, and Z = N(0, 1), a normally distributed random deviate with zero mean and unit standard deviation. The general form may written as

$$\hat{q}_{j+1} = \bar{q}_{j+1} + b_j(q_j - \bar{q}_j) + Z_{j+1,i}.S_{j+1}\sqrt{(1-r_j^2)} \qquad (5.36)$$

Where $b_j = r_j * S_{j+1}/S_j$, there are 36 parameters for the monthly model (*q,* for each month). The subscript *j* refers to month. For monthly synthesis *j* varies from 1 to 12 throughout the year. The subscript *i* is a serial designation from year 1 to year *n*. Other symbols are the same as mentioned earlier.

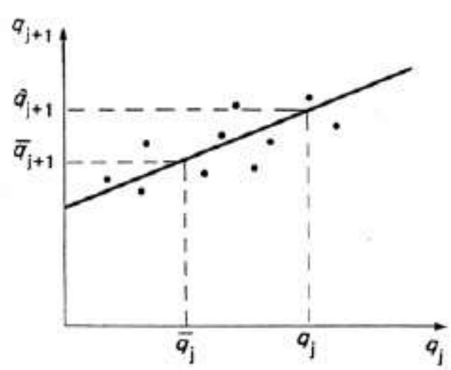


Fig.5.4: Thomas-Fiering model

The procedure for using the model is as follows:

(1) For each month, j = 1, 2, … 12, calculate

(a) The mean flow, $\bar{q}_{\beta} = \frac{1}{n}\sum q_{j,i;}$ $(i = j, 12j, 24 + j, ...)$

(b) The standard deviation, $S_j = \sqrt{\frac{\sum(q_{j,i} - \bar{q}_j)^2}{n-1}}$

(c) The correlation coefficient with flow in the preceding month,

$$r_j = \frac{\sum(q_{j,i} - \bar{q}_j)(q_{j+1,i} - \bar{q}_{j+1})}{\sqrt{\sum_i (q_{j,i} - \bar{q}_j)^2 \sum_i (q_{j+1,i} - \bar{q}_{j+1})^2}}$$

(d) The slope of the regression equation relating the month's flow to flow in the preceding month:

$$b_j = r_j \frac{S_{j+1}}{S_j}$$

(2) The model is then set of twelve regression equations

$$\hat{q}_{j+1,i} = \bar{q}_{j+1} + b_j(q_{j,i-1} - \bar{q}_j) + Z_{j+1,i} \cdot s_{j+1}\sqrt{(1 - r_j^2)}$$

where Z is a random Normal deviate N(0, 1).

(3) To generate a synthetic flow sequence, calculate (generate) a random number sequence $\{Z_1, Z_2, \ldots\}$, and substitute in the model.

5.6.4 Moving average models

The model form:

The moving average has frequently been used to smooth various types of hydrologic time series such as daily or weekly air temperature, evaporation rates, wind speed, etc. The moving average process used in the stochastic generation hydrologic data is somewhat different. In this use, the moving average process describes the deviations of a sequence of events from their mean value.

A process $\{x_1\}$ defined as

$$x_t = e_t + \Phi_1 e_{t-1} + \Phi_2 e_{t-2} + \ldots + \Phi_q e_{t-q} \quad (5.37)$$

Where $\{x_t\}$ is an uncorrelated stationary process, is called a moving average process of order q, denoted MA(q)-process.

It can also be written as

$$x_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \ldots - \theta_q e_{t-q} \quad (5.38)$$

with $\Phi 1 = -\theta 1, \Phi 2 = -\theta 2, \ldots, \Phi q = -\theta q$.

The properties of the moving average process:

The autocovariance of the process is obtained by forming the product and taking the expectation:

$$\gamma_k = E\left[(e_t - \theta_1 e_{t-1} - \ldots - \theta_q e_{t-q})(e_{t-k} - \theta_1 e_{t-k-1} - \ldots - \theta_q e_{t-k-q})\right] \quad (5.39)$$

For k = 0 we obtain the variance of the process

$$\sigma^2 = \gamma_o = \sigma_e^2(1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2) = \sigma_e^2 \sum_{j=0}^{q} \theta_j^2 \quad (5.40)$$

With the convention $\theta o = -1$

$$\gamma_k = \sigma_e^2(-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \ldots + \theta_{q-k}\theta_q) = \sigma_e^2 \sum_{j=0}^{q-k} \theta_j \theta_{j+k}$$ for k ≤ q (5.41)

$$\gamma_k = 0$$ for k > q (5.42)

The autocorrelation function is then

$$\rho_k = \frac{\gamma_k}{\gamma_o} = \frac{\sum_{j=0}^{q-k} \theta_j \theta_{j+k}}{\sum_{j=0}^{q} \theta_j^2}, \qquad k \le q$$

$$= 0 \qquad k > q$$ (5.43)

Equations (5.40) and (5.41) can be used for the estimation of the parameters by method of moments. For this purpose they are rewritten as follows:

$$\sigma_e^2 = \frac{\gamma_o}{1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2}$$ (5.44)

$$\theta_j = -(\frac{\gamma_j}{\sigma_e} - \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \ldots + \theta_{q-j}\theta_q)$$ (5.45)

Equ. (5.44) and (5.45) are used recursively. For example for the MA(1) model

$x_t = e_t - \theta_1 e_{t-1}$ (5.46)

we have

$$\hat{\sigma}_e^2 = \frac{\hat{\gamma}_o}{1 + \hat{\theta}_1^2} \qquad \hat{\theta}_1 = \frac{\hat{\gamma}_1}{\hat{\sigma}_e^2}$$ (5.47)

Where $\hat{\gamma}_o$ and $\hat{\gamma}_1$ are estimates of the auto-covariance and computed from the data.

5.6.5 ARMA models

<u>*Model form:*</u>

In stochastic hydrology ARMA models are known as Auto-Regressive Moving Average (ARMA) models. They combine any direct autocorrelation properties of a data series with the smoothing effects of an updated running mean through the series. The two components of the model for a data series x_t, e.g. annual river flows, are described by:

Auto-regression (AR(p))

$$x_t = \beta_1 x_{t-1} + \beta_2 x_{t-2} + \cdots + \beta_p x_{t-p} + e_t$$ (5.48)

Moving average (MA(q))

$x_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \ldots - \theta_q e_{t-q}$ (5.49)

Where e_t are random numbers with zero mean and variance σ^2_e.

The Auto-regressive moving average (ARMA(p, q)) model is defined as:

$$x_t = \beta_1 x_{t-1} + \beta_2 x_{t-2} + \cdots + \beta_p x_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \ldots - \theta_q e_{t-q}$$ (5.50)

One of the merits of the ARMA process is that, in general, it is possible to fit a model with a small number of parameters, i.e. p+q. This number is generally smaller than the number of parameters that would be necessary using either an AR model or a MA model. This principle is called the parsimony of parameters.
The first order model ARMA(1, 1) is:

$$x_t = \beta_1 x_{t-1} + e_t + \theta_1 e_{t-1} \qquad (-1 < \beta_1 < 1) \text{ and } (-1 < \theta_1 < 1) \tag{5.51}$$

Properties of ARMA model:

Consider in the ARMA(1, 1) model which has been used extensively in hydrology:

$$x_t = \beta_1 x_{t-1} + e_t + \theta_1 e_{t-1} \tag{5.52}$$

Multiplying both sides of (5.52) by X_{t-k}

$$x_{t-k}x_t = \beta_1 x_{t-k}x_{t-1} + x_{t-k}e_t + \theta_1 x_{t-k}e_{t-1}$$

and taking the expectation of both sides we obtain the autocovariance

$$\gamma_k = \beta_1\gamma_{k-1} + E(x_{t-k}\, e_t) - \theta_1 E(x_{t-k}\, e_{t-1}) \tag{5.53}$$

For k = 0, equ (5.53) becomes

$$\gamma_o = \beta_1\gamma_1 + E(x_t\, e_t) - \theta_1 E(x_t\, e_{t-1})$$

but

$$E(x_t e_t) = E\left|\beta_1 x_{t-1}e_t + e_t^2 + \theta_1 e_{t-1}e_t\right| = \sigma_e^2$$

and

$$\begin{aligned} E(x_t e_{t-1}) &= E\left|\beta_1 x_{t-1}e_{t-1} + e_t e_{t-1} + \theta_1 e_{t-1}^2\right| \\ &= E\beta_1[x_{t-1}e_{t-1}] - \theta_1\sigma_e^2 \\ &= (\beta_1 - \theta_1)\sigma_e^2 \end{aligned} \tag{5.54}$$

Thus

$$\gamma_o = \beta_1\gamma_1 + \sigma_e^2 - \theta_1(\beta_1 - \theta_1)\sigma_e^2 \tag{5.55}$$

For k = 1 equ (5.53) becomes

$$\gamma_1 = \beta_1\gamma_0 + 0 - \theta_1\sigma_e^2$$

Combining with the previous equation

$$\gamma_o = \beta_1^2\gamma_o - \beta_1\theta_1\sigma_e^2 + \sigma_e^2 - \theta_1(\beta_1 - \theta_1)\sigma_e^2$$

or

$$\gamma_o = \frac{1 + \theta_1^2 - 2\beta_1\theta_1}{1 - \beta_1^2} \tag{5.56}$$

and

$$\gamma_1 = \frac{(\beta_1 - \theta_1)(1 - \beta_1\theta_1)}{1 - \beta_1^2}\sigma_e^2 \tag{5.57}$$

For k ≥ 2

$$\gamma_k = \beta_1\gamma_{k-1} \qquad k \ge 2 \tag{5.58}$$

the autocorrelation function (ACF) is obtained by dividing (5.56), (5.57) and (5.58) by γ to obtain

$$\rho_k = 1 \qquad k = 0 \tag{5.59a}$$

$$= \frac{(\beta_1 - \theta_1)(1 - \beta_1\theta_1)}{1 + \theta_1 - 2\beta_1\theta_1} \qquad k = 1 \tag{5.59b}$$

$$= \beta_1 \gamma_{k-1} \qquad k \geq 2 \tag{5.59c}$$

Observe that the MA parameter θ_1 enters only in the expression for ρ_1. For ρ_2 and beyond the behaviour of the autocorrelation is identical to that of the AR(1) model.

Estimates of the parameters θ_1 and β_2 and can be obtained from equations (5.59b) and (5.59c), since the serial (auto) correlation coefficients ρ_1 and ρ_2 can be computed from data.

In general for an ARMA(p, q) model the autocovariance is

$$\gamma_k = \beta_1\gamma_{k-1} + \ldots + \beta_p\gamma_{k-p} + E[x_{t-k}e_t] - \theta_1 E[x_{t-k}e_{t-1}] - \ldots - \theta_q E[x_{t-k}e_{t-q}] \qquad k < q+1 \tag{5.60a}$$

$$\gamma_k = \beta_1\gamma_{k-1} + \ldots + \beta_p\gamma_{k-p} \qquad k \geq q+1 \tag{5.60b}$$

and the ACF is

$$\rho_k = \beta_1\rho_{k-1} + \ldots + \beta_p\rho_{k-p} \qquad k \geq q+1 \tag{5.61}$$

after lag q+1 the ACF tails off as for an AR(p) process. For the first q lags, the ACF depends on AR and MA parameters.

Hydrologic justification of ARMA models

A physical justification of ARMA models for annual streamflow simulation is as follows. Consider a watershed with annual precipitation X_t, infiltration aX_t and evapotranspiration bX_t. The surface runoff is $(1\text{-}a\text{-}b)X_t = dX_t$. (See Fig 5.5).

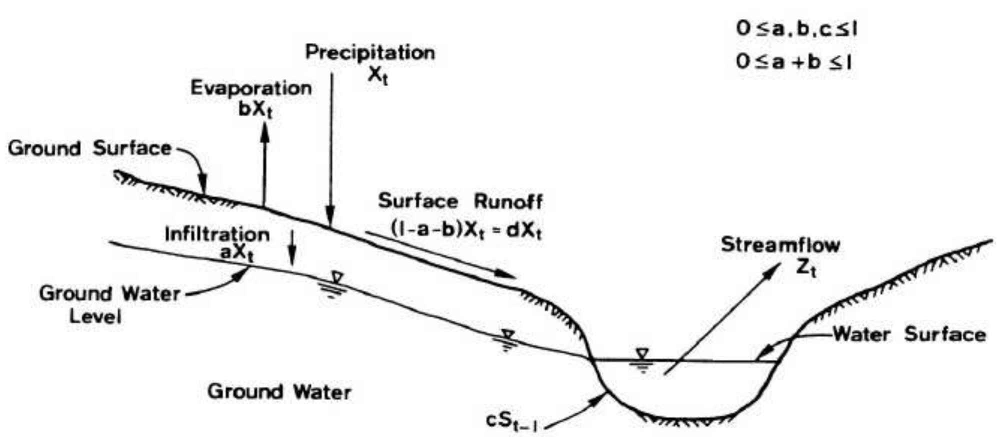


Fig.5.5 Conceptual representation of the precipitation-streamflow process after Salas and Smith (1980)

Let the groundwater contribution to the stream be cS_{t-1}.

Thus,

$$Z_t = cS_{t-1} + dX_t \tag{5.62}$$

The conservation of mass for the groundwater storage is

$$S_t = S_{t-1} + aX_t - cS_{t-1} \tag{5.63}$$

or

$$S_t = (1-c)S_{t-1} + aX_t \tag{5.64}$$

Rewriting (5.62)

$$Z_{t-1} = cS_{t-2} + dX_{t-1}$$

or

$$S_{t-2} = \frac{1}{c}Z_{t-1} + \frac{d}{c}X_{t-1} \tag{5.65}$$

and rewriting (5.64) as

$$S_{t-1} = (1-c)S_{t-2} + aX_{t-1} \tag{5.66}$$

Combining (5.62), (5.66) and (5.65) we obtain

$$Z_t = c(1-c)S_{t-2} + acX_{t-1} + dX_t$$

$$Z_t = (1-c)Z_{t-1} - d(1-c)X_{t-1} + acX_{t-1} + dX_t$$

$$Z_t = (1-c)Z_{t-1} + dX_t - \left[d(1-c) - ac\right]X_{t-1} \tag{5.67}$$

which has the form of an ARMA (1, 1), i.e. equation (5.52) model when the precipitation, X_t is an independent series and when (1-c) = β_1, d = 1, and [d(1-c)-ac)] = θ_1..

5.7 THE USES OF STOCHASTIC MODELS

(1) To make predictions of frequencies of extreme events

Stochastic models have been used to make predictions about the frequency of occurrence of certain extreme events of interest to the hydrologist. Models such as that given by equation (5.29) are selected, and the residual is taken to be random variable with probability distribution whose parameters are specified. The parameters are estimated from data; so-called "synthetic" sequence $\{y_t\}$ can then be constructed, and the frequency with which the extreme event occurs in them can be taken as an estimate of the "true" frequency with which it would occur in the long run.

(2) For the investigation of system operating rules

A further use for synthetic sequences generated by stochastic models is in reservoir operation, such as the investigation of the suitability of proposed operating rules for the release of water from complex systems of interconnected reservoirs. By using the generated sequence as inputs to the reservoir system operated according to the proposed rules, the frequency with which demands fail to be met can be estimated. This may lead to revision of the proposed release rules; the modified rules may be tested by a similar procedure.

(3) To provide short-term forecasts

Stochastic models have been used to make forecasts. Given the values x_t, x_{t-1},

x_{t-2}, ...; y_t, y_{t-1}, y_{t-2}, ... assumed by the input and output variables up to time t, stochastic models have been constructed from this data for forecasting the output from the system at future times, t+1, t+2, ..., t+k, ... In statistical terminology, k is the lead-time of the forecast. Many stochastic models have a particular advantage for forecasting purposes in that they provide, as a by-product of the procedure for estimating model parameters, confidence limits for forecasts (i.e. a pair of values, one less than the forecast and one greater, such that there is a given probability P that these values will bracket the observed value of the variable at time t+k). Confidence limits therefore express the uncertainty in forecasts; the wider apart the confidence limits, the less reliable the forecast. Furthermore, the greater the lead-time k for which forecasts is required, the greater will be the width of the confidence interval, since the distant future is more uncertain than the immediate.

(4) To "extend" records of short duration, by correlation

Stochastic models have been used to "extend" records of basin discharge where this record is short. For example, suppose that it is required to estimate the instantaneous peak discharge with a return period of T years (i.e. such that it would recur with frequency once in T years, in the long run). One approach to this problem is to examine the discharge record at the site for which the estimate is required, to abstract the maximum instantaneous discharge for each year of record, and to represent the distribution of annual maximum instantaneous discharge by a suitable probability density function. The abscissa, Yo, say, that is exceeded by a proportion 1/T of the distribution then estimates the T-year flood.

It, however, frequently happens that the length of discharge record available is short, say ten years or fewer. On the other hand, a much longer record of discharge may be available for another gauging site, such that the peak discharges at the two sites are correlated. In certain circumstances, it is then permissible to represent the relation between the annual maximum discharges at the two sites by a regression equation and to use this fitted equation to estimate the annual maximum instantaneous discharges for the site with short record.

(5) To provide synthetic sequences of basin input

Suppose that the model has been developed for a system consisting of a basin with rainfall as input variable, streamflow as output variable. If a stochastic model were developed from which a synthetic sequence of rainfall could be generated having statistical properties resembling those of the historic rainfall sequence, the synthetic rainfall sequence could be used as input to the main model for transformation to the synthetic discharge sequence. The discharge so derived could then be examined for the frequency of extreme events.

This approach to the study of the frequency of extreme discharge events is essentially an alternative to that described in paragraph (1) above. In the latter, a synthetic sequence is derived from a stochastic model of the discharge alone; in the former, a synthetic discharge sequence is derived by using a model to convert a synthetic sequence of rainfall into discharge.

5 Reservoir Capacity Determination

The reservoir capacity is a term used to represent the reservoir storage capacity. Its determination is performed using historical inflow records in the stream at the proposed dam site. There are several methods to determine a reservoir storage capacity. The most common ones are presented below.

5.1 Mass curve (ripple's) method:

A mass curve (or mass inflow curve) is a plot of accumulated flow in a stream against time. As indicated below a mass curve can be prepared from the flow hydrograph of a stream for a large number of consecutive previous years. Figure 5.1 (a) shows a typical flow hydrograph of a stream for six consecutive years. The area under the hydrograph from the starting year (i.e., 1953) up to any time t_l (shown by hatching) represents the total quantity of water that has flown through the stream from 1953 up to time t_1 and hence it is equal to the ordinate of the mass curve at time t_1. The ordinates of mass curve corresponding to different times are thus determined and plotted at the respective times to obtain the mass curve as shown in fig. 5.1(b). A mass curve continuously rises as it shows accumulated flows. The slope of the curve at any point indicates the rate of flow at that particular time. If there is no flow during certain period the curve will be horizontal during that period.

Fig. 6.1: a) Mass curve b) flow hydrograph

A demand curve (or mass curve of demand) on the other hand is a plot between accumulated demand and time (Fig. 6.2). If the demand is at a constant rate then the demand curve is a straight line having its slope equal to the demand rate. However, if the demand is not constant then the demand will be curved indicating a variable rate of demand.

Figure 6.2: Demand curve

The demand and supply curves discussed above are thus the basis of the mass curve (or ripple's) method of reservoir capacity determination.

The reservoir capacity required for a specified yield or demand may be determined by using mass curve and demand curve using the following steps.

1) A mass curve is prepared from the flow hydrograph for a number of consecutive years selected from the available stream flow record such that it includes the most critical or the driest period. Figure 6.3 shows a mass curve for a typical steam for a 6 years period;
2) Corresponding to the given rate of demand, a demand curve is prepared. If the rate of demand is constant then the corresponding demand curve is a straight line as shown in figure 6.3
3) Lines such as GH, FJ, etc are drawn parallel to the demand curve and tangential to the high points G, F etc, of the mass curve (or the points at the beginning of the dry periods);
4) The maximum vertical intercepts X_1Y_1, X_2Y_2, etc between the tangential lines drawn in step 3 and the mass curves are measured. The vertical intercepts indicate the volume by which the total flow in the stream falls short of the demand and hence required to be provided from the reservoir storage. For example assuming the reservoir to be full at G, for a period corresponding to points G and Z_1 there is a total flow in the stream represented by $Y_1Z_1$and there is a total demand represented by X_1Z_1, leaving a gap of volume represented by X_1Y_1which must be met with from the reservoir storage;
5) The largest of the maximum vertical intercepts X_1Y_1, X_2Y_2 etc, determined in step 4 represents the reservoir capacity required to satisfy the given demand. However, the requirement of storage so obtained would be the net storage that must be available for utilization and it must be increased by the amount of water lost by evaporation and seepage.

Figure 6.3: Use of a mass curve to determine the reservoir capacity required to produce a specified yield.

If the tangential lines drown (GH, FJ, etc) do not intersect the mass curve, the reservoir will not be filled again. Moreover, if the reservoir is very large the time interval between the points G and H, F and J, etc, may be several years. This graphical solution of the mass method can also be done in tabular calculation easily using computer spreadsheet programs.

Example 6.1: The following table gives the mean monthly flows in a river during certain year. Calculate the minimum storage required for maintaining a demand rate of 40m^3/s: (a) using graphical solution (b) using tabular solution.

Month	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept	Oct	Nov	Dec
Q(m^3/s)	60	45	35	25	15	22	50	80	105	90	80	70

Solution:-

a) Graphical solution:

Month	Mean flow (m^3/s)	Monthly flow volume (m^3/s.day)	Accumulated flow volume (m^3/s.day)
Jan(31)	60	1860	1860
Feb(28)	45	1260	3120
Mar(31)	35	1085	4205
Apr(30)	25	750	4955
May(31)	15	465	5420
Jun(30)	22	660	6080
Jul(31)	50	1550	7630
Aug(31)	80	2480	10110
Sept(30)	105	3150	13260
Oct(31)	90	2790	16050
Nov(30)	80	2400	18450
Dec(31)	70	2170	20620

b) Tabular calculation (sequent-peak algorithm)

Month	Volume in flow (cumec-day)	Volume demand (Cumec-day)	Departure (2)-(3) (cumec-day)	Cumulative Departure (cumec-day)	Storage (Cumec-day)
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Jan	1860	1240	620	620	
Feb	1260	1120	140	760	
Mar	1085	1240	-155	605	
Apr	750	1200	-450	155	
May	465	1240	-775	-620	
Jun	660	1240	-540	-1160	
Jul	1550	1240	310	-850	
Aug	2480	1240	1240	390	
Sept	3150	1200	1950	2340	
Oct	2790	1240	1550	3890	
Nov	2400	1200	1200	5090	
Dec	2170	1240	930	6020	

760-(-1160)
=1920
cumec-day

As shown fig 6.4 if the end points of the mass curve are joined by a straight line AB, then its slope represents the average discharge of the stream over the total period for which the mass curve has been plotted. If a reservoir is to be constructed to permit continuous release of water at this average value of discharge for the period, then the capacity required for the reservoir is represented by the vertical intercept between the two straight lines A^1B^1 and $A^{11}B^{11}$ drawn parallel to AB and tangent to the mass curve at the lowest tangent point C and the highest tangent point D, respectively. If the reservoir having this capacity is assumed to contain a volume of water equal to AA^1 at the beginning of the period, then the reservoir would be full at D and it would be empty at C. However, if the reservoir was empty in the very beginning, then it would be empty again at point E and also during the period from F to K. On the other hand if the reservoir was full in the very beginning it would be full again at points F and K, and between points A and E' there will be spill of water from the reservoir.

In the earlier discussions the rate of demand has been assumed to be constant. However, the rate of demand may not be always constant, in which case the demand curve will be curve with its slope varying from point to point in accordance with the variable rate of demand at different times. In this case also the required capacity of the reservoir can be determined in the same way by super imposing the demand curve on the mass curve from the high points (or beginning of the dry period) till the two meet again. The largest vertical intercept between the two curves gives the reservoir capacity. It is however essential that the demand curve for the variable demand coincide chronologically with the mass curve of stream flow, i.e. June demand must coincide with June inflow and so on.

Figure 6.4: Use of a mass curve to determine the reservoir capacity required to produce yield equal to the average discharge of the stream.

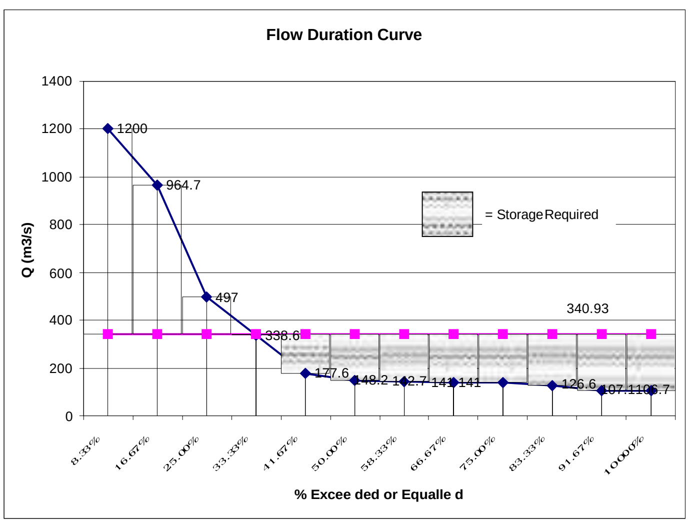
Example 6.2: Reservoir Capacity determination by the use of flow duration curve

Determine the reservoir capacity required if a hydropower plant is designed to operate at an average flow.

Solution: The average flow is 340.93 m^3/s.

i) First option: Storage is same as the hatched area under flow duration curve.

Discharge (m3/s)	Descending Order	Rank	%Exceeded or Equaled
106.70	1200	1	8.33%
107.10	964.7	2	16.67%
148.20	497	3	25.00%
497.00	338.6	4	33.33%
1200.00	177.6	5	41.67%
964.70	148.2	6	50.00%
338.60	142.7	7	58.33%
177.60	141	8	66.67%
141.00	141	9	75.00%
141.00	126.6	10	83.33%
142.70	107.1	11	91.67%
126.60	106.7	12	100.00%



(1)	(2)	(3)	(1) -(3)	
Ordinate of Flow Duration Curve (m3/s)	%Exceded or Equalled	Average Flow		Area
1200	8.33%	340.93	0.00	0.00
964.7	16.67%	340.93	0.00	0.00
497	25.00%	340.93	0.00	0.00
338.6	33.33%	340.93	2.33	6.90
177.6	41.67%	340.93	163.33	14.84
148.2	50.00%	340.93	192.73	16.29
142.7	58.33%	340.93	198.23	16.59
141	66.67%	340.93	199.93	16.66
141	75.00%	340.93	199.93	17.26
126.6	83.33%	340.93	214.33	18.67
107.1	91.67%	340.93	233.83	19.50
106.7	100.00%	340.93	234.23	
				1013.73
		Storage (Mm3)		**2662.61**

ii) 2nd option (Mass Curve/Ripple's Diagram/Sequent Peak Algorithm.)

Month	Monthly Flow	Monthly Volume	Average Flow	Average Volume	Departure	Cumulative Departure
Jan	106.70	280.25	340.93	895.47	-615.22	-615.22
Feb	107.10	281.30	340.93	895.47	-614.17	-1229.39
Mar	148.20	389.26	340.93	895.47	-506.22	-1735.60
Apr	497.00	1305.40	340.93	895.47	409.93	-1325.68
May	1200.00	3151.87	340.93	895.47	2256.40	930.72
Jun	964.70	2533.84	340.93	895.47	1638.37	**2569.09**
Jul	338.60	889.35	340.93	895.47	-6.12	2562.97
Aug	177.60	466.48	340.93	895.47	-429.00	2133.97
Sep	141.00	370.34	340.93	895.47	-525.13	1608.85
Oct	141.00	370.34	340.93	895.47	-525.13	1083.72
Nov	142.70	374.81	340.93	895.47	-520.66	563.06
Dec	126.60	332.52	340.93	895.47	-562.95	0.11
					ANS (Mm3)	**2569.00**

5.2 Reservoirs and sediments

A river entering a water reservoir will loose its capacity to transport sediments. The water velocity decreases, together with the shear stress on the bed. The sediments will therefore deposit in the reservoir and decrease its volume.

In the design of dam, it is important to assess the magnitude of sediment deposition in the reservoir. The problem can be divided I two parts:

1. How much sediments enter the reservoir
2. What is the trap efficiency of the reservoir

In a detailed study, the sediment size distributions also have to be determined for question 1. Question 2 may also involve determining the location of the deposits and the concentration and grain size distribution of the sediments entering the water intakes.

In general, there are two approaches to the sedimentation problem:

1. The reservoir is constructed so large that it will take a very long time to fill. The economical value of the project will thereby be maintained.
2. The reservoir is designed relatively small and the dam gates are constructed relatively large, so that it is possible to remove the sediments regularly by flushing. The gates are opened, lowering the water level in the reservoir, which increases the water velocity. The sediment transport capacity is increased, causing erosion of the deposits.

A medium sized reservoir will be the least beneficial. Then it will take relatively short time to fill the reservoir, and the size is so large that only a small part of the sediments are removed by flushing.

The flushing has to be done while the water discharge in to the reservoir is relatively high. The water will erode the deposits to a cross-stream magnitude similar to the normal width of the river. A long and narrow reservoir will therefore be more effectively flushed than a short and wide geometry. For the later, the sediment deposits may remain on the sides.

The flushing of a reservoir may be investigated by physical model studies.

Another question is the location of sediment deposits. Figure 5.5 shows a longitudinal profile of the reservoir. There is a dead storage below the lowest level the water can be withdrawn. This storage may be filled with sediments without affecting the operation of the reservoir.

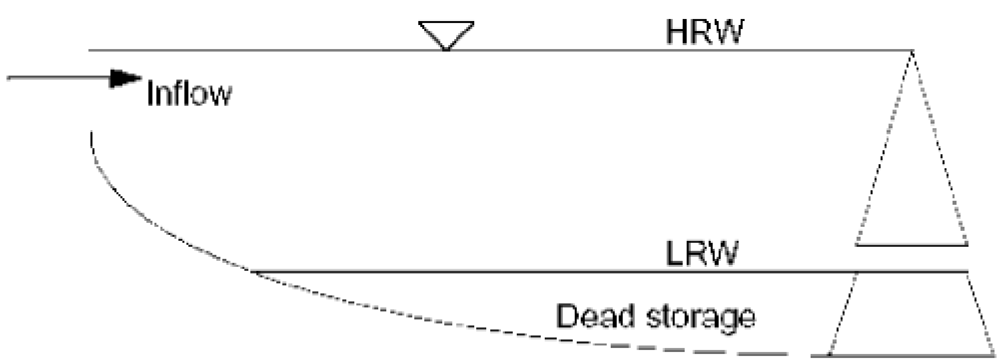


Figure 6.5: Longitudinal profile of a reservoir. HRW is the highest regulated water level. The reservoir volume below LRW is called the dead storage, as this can be used.

5.3 Sediment Load Prediction

Rough estimates of sediment load may be taken from regional data. Often the sediment yield in the area is known from neighboring catchments. It is then possible to assess the seriousness of the erosion in the present catchment and estimate rough figures of sediment yield. The land use, slope and size of the catchment are important factors.

For a more detailed assessment, measurements of the sediment concentration in the river have to be used. Sediment concentrations are measured using standard sampling techniques, and water discharges are recorded simultaneously. The measurements are taken at varying water discharges. The values of water discharge and sediment concentrations are plotted on a graph, and a rating curve is made. This is often on the form:

$$Q_s = a\, Q_w^b \tag{6.1}$$

Q_s is the sediment load, Q_w is the water discharge and a and b are constants, obtained by curve fitting

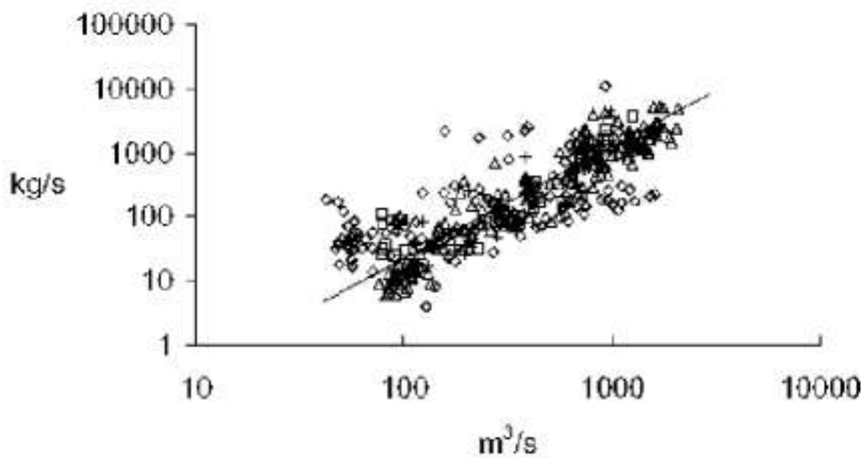


Figure 6.6: Example of sediment rating curve.

The annual average sediment transport is obtained by using a time series of the water discharge over the year together with equation 6.1.

7 URBAN HYDROLOGY

Storm magnitudes and their frequency of occurrence are of greater importance than annual rainfall totals in urban hydrology.

7.1 Catchment Response Modifications

The changes made to a rural area by the construction of a concentration of building have a direct effect on its surface hydrology. The covering of the land surface by a large proportion of impervious materials means that a much larger proportion of any rainfall forms immediate runoff. In addition to extensive ground coverage by building in a city, the paved streets and car parks contribute large areas to the impervious surface. Any slope of the land also greatly enhances the runoff response of a paved area. In a defined catchment area, the effect on the stream discharge is dependent on the extent on the remaining pervious surfaces, where normal infiltration in to the soil and percolation in to the underlying strata can take place. Thus, after major urban development in a catchment, the following differences in the river flow from that of an equivalent rural catchment can be identified.

a. there is a higher proportion of rainfall appearing as surface runoff, and so the total volume of discharge is increased
b. for a specific rainfall event, the response of the catchment is accelerated, with a steeper rising limb of the flow hydrograph; the lag time and time to peak is reduced
c. flood peak magnitudes are increased, but for the very, but for the very extreme events (when the rural runoff coefficient > 50%) these increase in urban areas are diminished
d. in times of low flows, discharges are decreased since there is reduced contribution from the groundwater storage that has received less replenishment; and
e. water quality in streams and rivers draining urban area is degraded by effluent discharges, increased water temperature and danger from other forms of pollution

Many of these modifications are promoted by structural changes made to drainage channels. It is essential to remove rain water quickly from developed areas, and surface water drainage systems are included in modern town extensions.

The interaction of the artificial nature of urban catchments and the need to accommodate the changed hydrological characteristics is complex. The solving of one drainage problem may easily exacerbate another feature of the catchment runoff, e.g. rain events on the planned surface drainage of a new housing estate could produce higher peaks downstream than formerly, and these might cause flooding at previously safe points along the channel.

Due to urbanization effect, the runoff volume and time distribution of the runoff hydrograph is modified. The various hydrograph parameters such as peak discharge, Q_p, time to peak, t_p and lag time are usually related to catchment characteristics including area of impervious surfaces or proportion of area urbanized, in order to obtain quantitative rainfall-runoff relationships.

7.2 Urban development planning

In the development of new urban centers, hydrological knowledge of the areas is required at two stages. The first is planning stage when the general layout of the new town is being decided. The second stage of hydrological involvement occurs at the detailing stage, the designing of storm water drainage channels and pipes to carry the surface water in to the rivers.

The principal objective at the planning stage is the determination of the size of flood, with its related return period, that the developing authority is prepared to accommodate. The design of the drainage system is dependent on a satisfactory assessment of the flood magnitude-return period relationship and the subsequent choice of a design flood.

7.3 Drainage design

Once the broad outline of the hydrological consequences of an urban development of an area have been determined at the planning stage and major remedial works considered, then the detailed design of the drainage systems is required. The engineering hydrologist is fully concerned with evaluating the runoff from sub areas to be drained in order to design the necessary storm water sewers. The peak runoff from the selected design storm determines the size sewer pipe which is dependent on the extent of each sub area to be drained. At the head of the catchment sub area, the required pipe size may e quite small, but downstream, as the sewer receives water from a growing are through a series of junctions, the pipe size gradually needs to be increased.

The problem of estimating the runoff from the storm rainfall is very much dependent on the character of the catchment surface. The degree of urbanization (extent of impervious area) greatly affects the volume of runoff obtained from a given rainfall. Retention of rainfall by initial wetting of surfaces and absorption by vegetation and pervious areas reduces the amount of storm runoff. These surface conditions also affect the time distribution of the runoff. Thus the computational method used to obtain the runoff from the rainfall should allow for the characteristics of the surface are to be drained.

7.3.1 Impervious areas

These comprises the roof areas and large expanses of paved surfaces of city centers and industrial sites, in which there is very little or even no part of the ground surface into which rainfall could infiltrate. The calculation of the runoff from these relatively small catchments is the most straight forward, since the area can be easily defined and measured. Over such limited areas, the storm rainfall can be assumed to be uniformly distributed with 100% runoff occurring. The response of the impervious surface is rapid, resulting in a short time of concentration of the flow in the drainage system. The rational formula can thus provide the peak drainage.

$$Q(l/s) = \frac{A(m^2)*i(mm/h)}{3600} \qquad (7.1)$$

Looking the simple pipe design in the figure below, the computation of the required pipe size can be done as shown in the table.

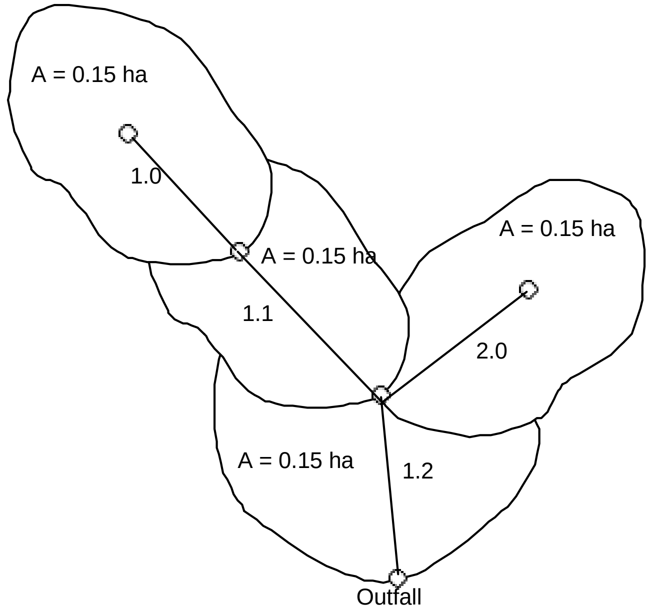


Figure 7.1: Simple Pipe Design

At the outset of the design procedure, the selected return period for a design storm will have been decided. Storm water sewers are usually designed for 1 in 1, 1 in 2 or 1 in 5 year storm return periods. The type of pipe will also have been chosen; the internal roughness governs the flow characteristics, and roughness coefficient. Velocities and discharges for standard sized pipes can be found from published tables, assuming full bore conditions, a hydraulic gradient equal to the pipe gradient and appropriate roughness coefficient. Design charts for the velocities and discharges are also available and provide for easier interpolation. Flows larger than those derived from the tables or charts would require hydraulic gradients greater than the pipe gradient, and these could only occur by ponding (or surcharging) of water in the manholes at the pipe junctions. ***The design objective is to avoid such surcharging***.

Referring to figure 7.1, the design procedure begins with the choice of a trial pipe size for pipe 1.0, say 150 mm is chosen (the smallest used in practice) (Refer Table 7.1). From published tables and for k_s = 0.6 for a normal concrete pipe, the velocity and discharge for a gradient of 1 in 65 are noted, 1.26 m/s and 23.0 l/s, respectively. A flow greater than 23.0 l/s would result in surcharging.

The time of flow along the pipe is next calculated from the velocity and length of pipe and comes to 0.86 min. the time of concentration at the end of the first pipe is then 0.86 min plus an assumed allowance of 2 min, for the time of entry, which is assumed to cover the lag time between the beginning of the storm rainfall and the entry of the overland flow in to the leading manhole. With the time of concentration of the drainage to the end of the first pipe known, the design return period rainfall intensity (i) over this duration to give the peak flow an be obtained from intensity-duration-frequency data. The storm peak discharge is then calculated using equation 6.1 for comparison with the unsurcharged full bore pipe flow. The first trial pipe of 150 mm diameter would clearly be surcharged, so the calculations are repeated with the next size pipe, diameter 225 mm. the calculated storm discharge, 28.8 l/s would be easily contained by larger pipe.

The calculations proceed for each pipe in turn, with the previous time of concentration being added to the new time of flow to give the combined times of concentration at the end of sequential pipes. The drainage areas are also accumulated. It will be noted that the 2.0 min time of entry is also added to the flow time of pipe 2.0 since it is at the start of a branch of pipeline. The time of concentration for the last pipe, is then the sum of the time of concentration of pipe 1.1 and the flow time of pipe 1.2. the extra contribution from the greatly increased area drained by the tributary pipe results in a much larger discharge requiring the next size larger pipe, 300 mm diameter.

Table 7.1: Rational method Drainage Design

Pipe No	Level Diff. (m)	Pipe Leng. (m)	Grad. (1 in)	Trial Pipe Dia (mm)	Pipe v (m/s)	Pipe Q (l/s)	Time of Flow (min)	Time of Conc. (min)	Rate of Rain (mm/h)	Imp. A Cum. (ha)	Storm, Q (l/s)	Comm.
1.0	1.00	65.00	65.00	150.00	1.26	23.00	0.86	2.86	67.50	0.15	28.13	S
				225.00	1.64	67.50	0.66	2.66	69.20		28.83	PF
1.1	0.90	70.00	78.00	225.00	1.50	61.70	0.78	3.44	63.20	0.25	43.89	PF
2.0	1.50	60.00	40.00	150.00	1.61	29.40	0.62	2.62	69.50	0.20	38.61	S
				225.00	2.10	86.00	0.48	2.48	70.70		39.28	PF
1.2	0.90	50.00	56.00	225.00	1.77	72.80	0.47	3.91	60.20	0.53	88.63	S
				300.00	2.13	156.00	0.39	3.83	60.70		89.36	PF

7.4 The Transport and Road Research Laboratory (TRRL) Hydrograph Method (Watkins, 1962)

In the time-area method, the total catchment are is deemed to be contributing to the flow after the time of concentration, T_c, the time it takes for the rain on the furthest part of the catchment to reach the outfall. Thus in figure 7.2, for two drains receiving uniform rainfall from areas A_1 and A_2 with drain 2 joining the main channel, drain 1, a relationship of contributing area, A, versus time, T, is constructed. From the beginning of the flow in drain 1 at T=0 there is steady increase in area contributing until $T = T_1$ which is the value of T_c for area A_1. Drain 2 begins to contribute to the outfall flow T_c at time $T = (T_2+T_3)$. Between times T_3 and T_1 both drains have been flowing and the joint contributing area (at C) at $T = T_1$ is given by:

$$A_1 + \frac{(T_1 - T_3)}{T_2} . A_2 \qquad (7.2)$$

From $T = (T_2+T_3)$, both areas are contributing fully. The time-area curve for the combined drains is composite line OBCD.

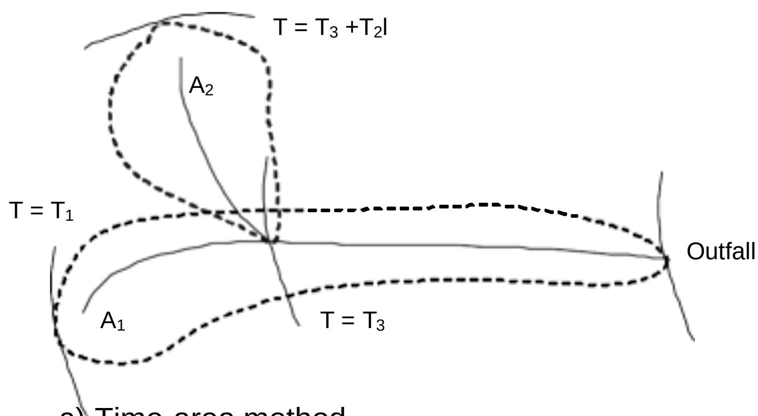
The principle of TRRL as outlined as in figure 6.2, a catchment are, divided into four sub areas, is drained by a single channel to the outfall where the hydrograph is required. Subarea 1 begins contributing to the flow first, to be followed sequentially by the other three subareas. The composite time-area curve for the whole catchment is drawn by summing the subarea contributions at regular time intervals. The incremental contributing areas after each time interval are then read from the composite curve, a_1, a_2, a_3, etc. from the diagram, the time of concentration for the whole area is determined.

The effective rainfall intensity is computed for each of the chosen time unit intervals from gauge measurements. The discharge rates after each time unit interval are given by:

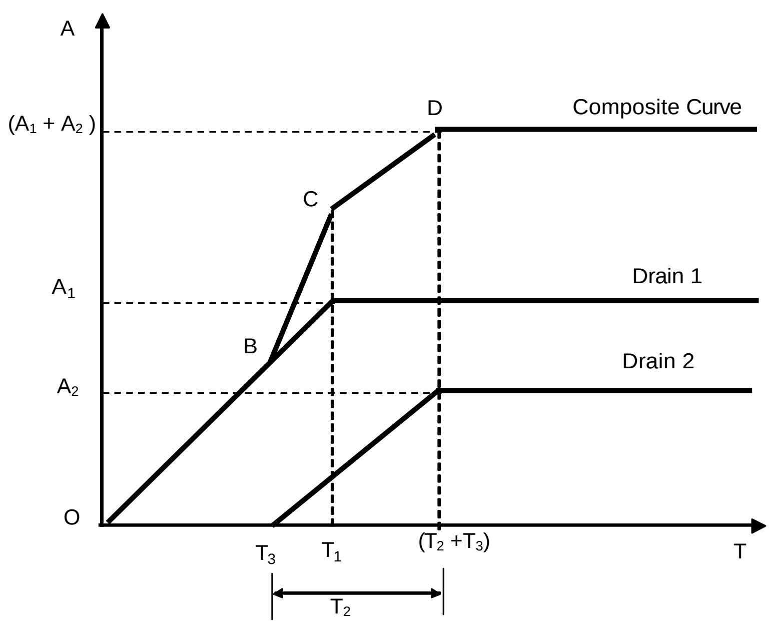
$$q_1 = i_1 a_1$$
$$q_2 = i_2 a_1 + i_1 a_2$$
$$q_3 = i_3 a_1 + i_2 a_2 + i_1 a_3 \qquad (7.3)$$

etc.

The peak flow is then considered as design flow.



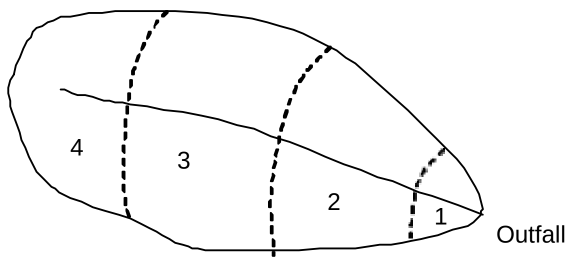
a) Time-area method



b) Time-area diagram

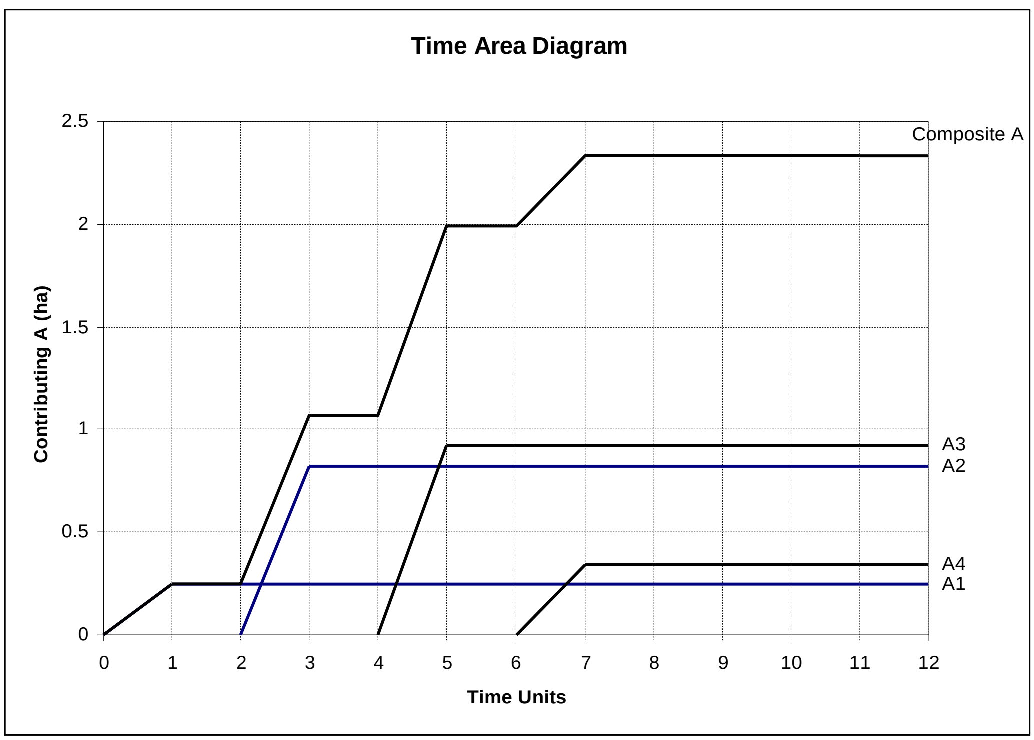
Figure 7.2: Time-area method for sewer design

Example 7.1: The TRRL Hydrograph method



a) Catchment boundary

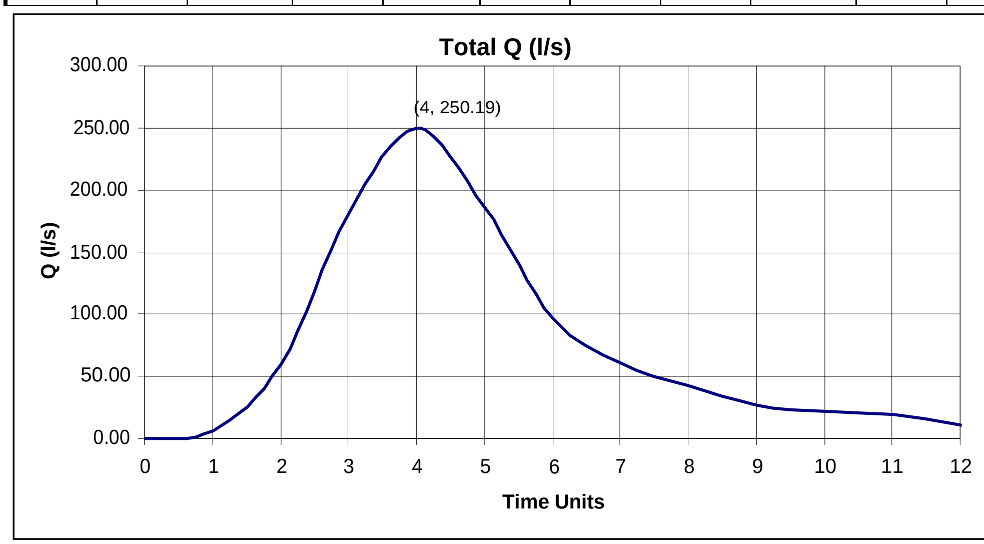
Time Units	Contributing Area (ha)				
	a1	a2	a3	a4	Composite A
0.00	0				0
1.00	0.25				0.25
2.00	0.25	0			0.25
3.00	0.25	0.82			1.07
4.00	0.25	0.82	0.00		1.07
5.00	0.25	0.82	0.92		1.99
6.00	0.25	0.82	0.92	0.00	1.99
7.00	0.25	0.82	0.92	0.34	2.33
8.00	0.25	0.82	0.92	0.34	2.33
9.00	0.25	0.82	0.92	0.34	2.33
10.00	0.25	0.82	0.92	0.34	2.33
11.00	0.25	0.82	0.92	0.34	2.33
12.00	0.25	0.82	0.92	0.34	2.33



b) Time-area diagram

Time Unit	Eff. RF(mm)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)	A (ha)
1.0	8.8	0.25	0.25									
2.0	57.6	0.82	0.82	0.25								
3.0	38.0	0.92	0.92	0.82	0.25							
4.0	11.7	0.34	0.34	0.92	0.82	0.25						
5.0	10.8			0.34	0.92	0.82	0.25					
6.0	8.8				0.34	0.92	0.82	0.25				
7.0	3.4					0.34	0.92	0.82	0.25			
8.0	3.3						0.34	0.92	0.82	0.25		
9.0	3.9							0.34	0.92	0.82	0.25	
10.0	2.9								0.34	0.92	0.82	0.25
11.0										0.34	0.92	0.82
12.0											0.34	0.92
13.0												0.34

Time Units	*Aili*	*Aili*	*Aili*	*Aili*	*Aili*	*Aili*	*Aili*	*Aili*	*Aili*	*Aili*	Total Q (l/s)
0	*0.00*										0.00
1	6.11										6.11
2	20.04	40.00									60.04
3	22.49	131.20	26.39								180.08
4	8.31	147.20	86.56	8.13							250.19
5		54.40	97.11	26.65	7.50						185.66
6			35.89	29.90	24.60	6.11					96.50
7				11.05	27.60	20.04	2.36				61.06
8					10.20	22.49	7.74	2.29			42.73
9						8.31	8.69	7.52	2.71		27.23
10							3.21	8.43	8.88	2.01	22.54
11								3.12	9.97	6.61	19.69
12									3.68	7.41	11.09
13										2.74	2.74



c) Discharge hydrograph

The procedure of the TRRL method applies to one drainage unit, i.e. one pipe, in a system. The calculations have to be carried out for each pipe in a sewerage network.