APPLIED HYDROLOGY

(HE 612)

Adane Abebe (Dr.-Ing)
Hydrology and Water Resources

Course Outline

1. Hydrologic principles
   - Introduction
   - Hydrologic cycle
   - Catchment morphometry
   - Hydrometric measurement
   - Quality control of data
2. Linear System Response function
   - Rainfall-runoff relationships
   - Hydrograph analysis
   - Unit hydrograph
   - Synthetic unit hydrograph
   - Instantaneous unit hydrograph
   - Conceptual models

1. Introduction

- It is the science that deals with the occurrence, circulation, and distribution of water of the earth and earth's atmosphere.

Applications of Hydrology in Engineering

- The capacity of storage structures such as reservoirs
- The magnitude of flood flows to enable safe disposal of the excess flow
- The minimum flow and quantity of flow available at various seasons
- The interaction of the flood wave and hydraulic structures, such as dams, levees, weirs, bridges, and culverts.

1. Hydrologic Cycle

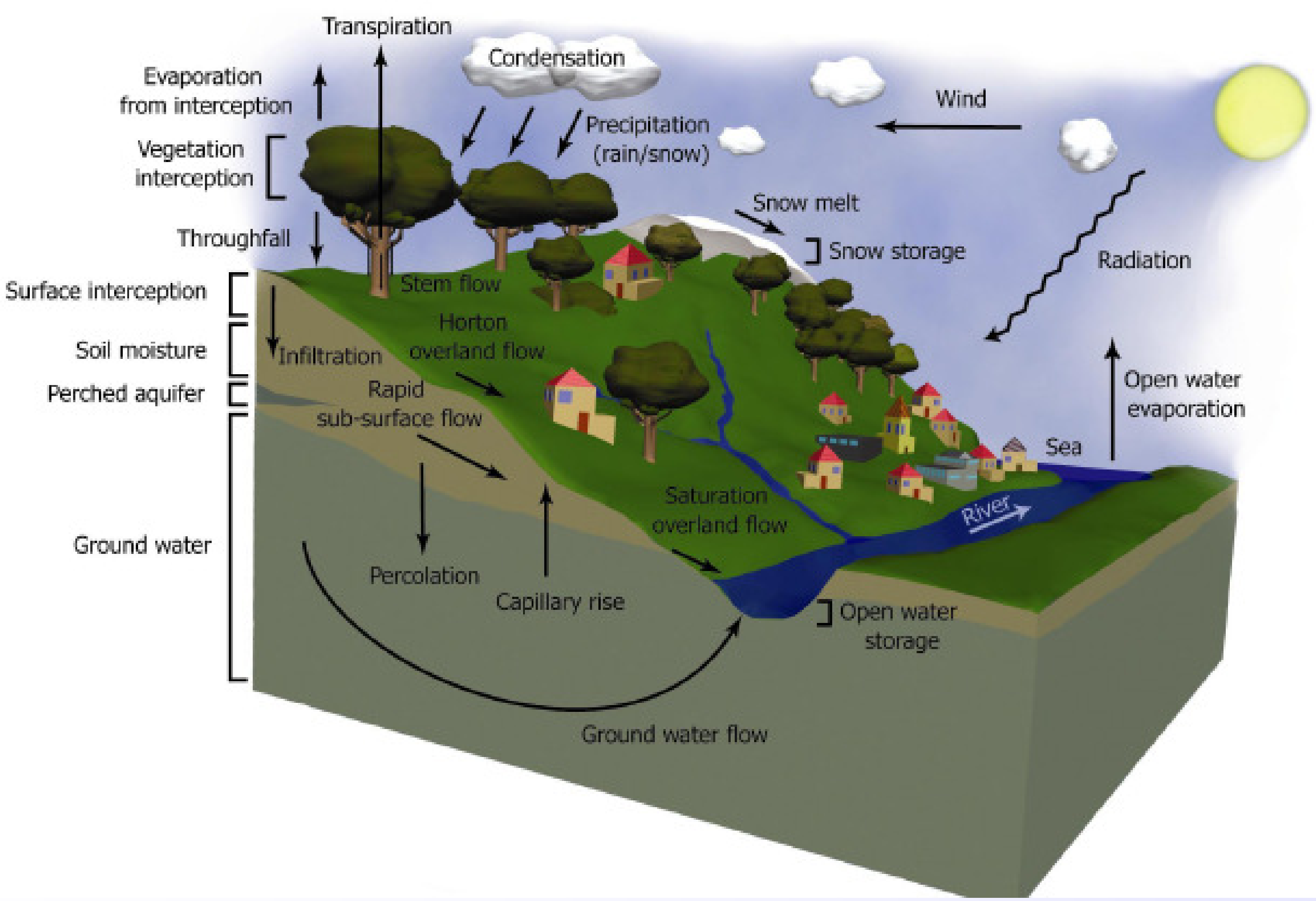


Fig. 1.2 Schematic of the hydrologic cycle.

1. Catchment morphometry

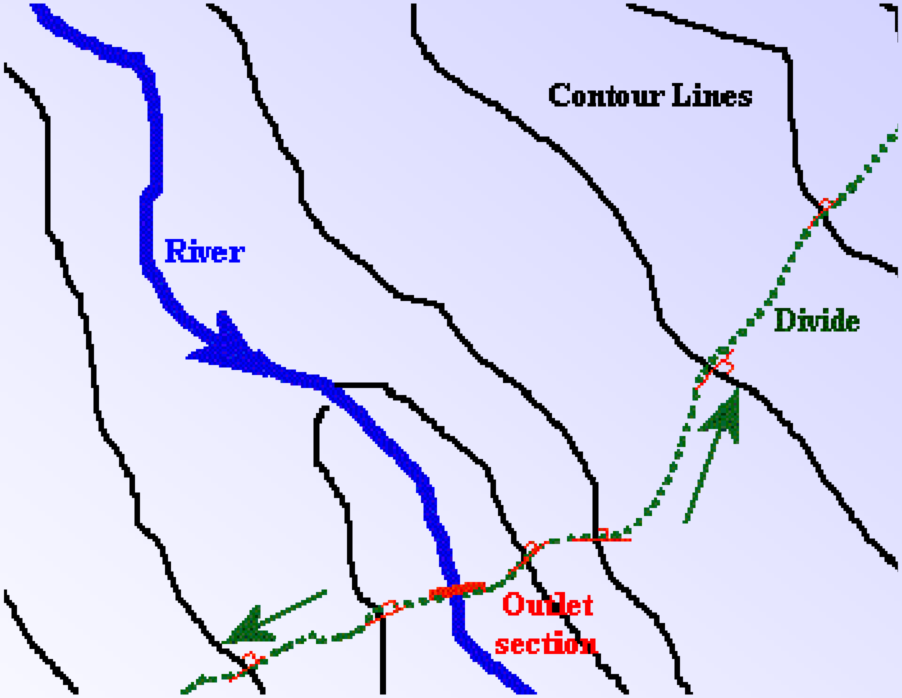


Figure 1.2 Watershed delimitation method - detail.

1. Catchment morphometry

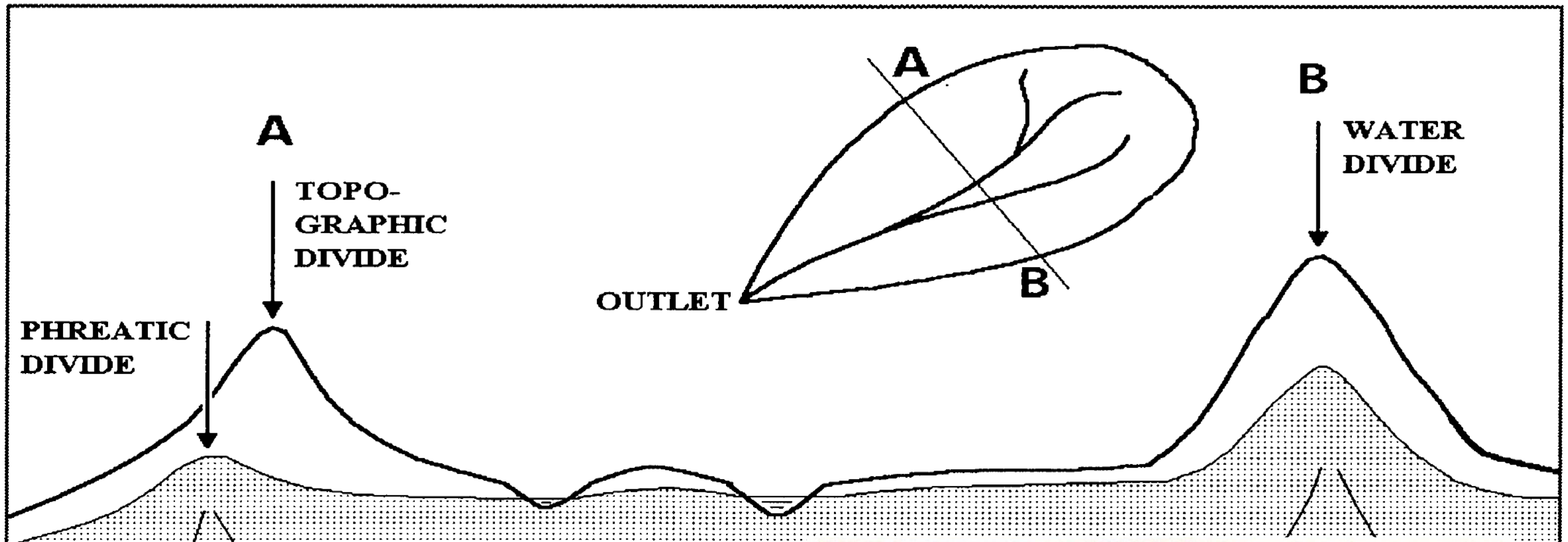


Figure 1.2a Watershed and cro
showing a phreatic and topogra

1. Catchment morphometry (cont'd)

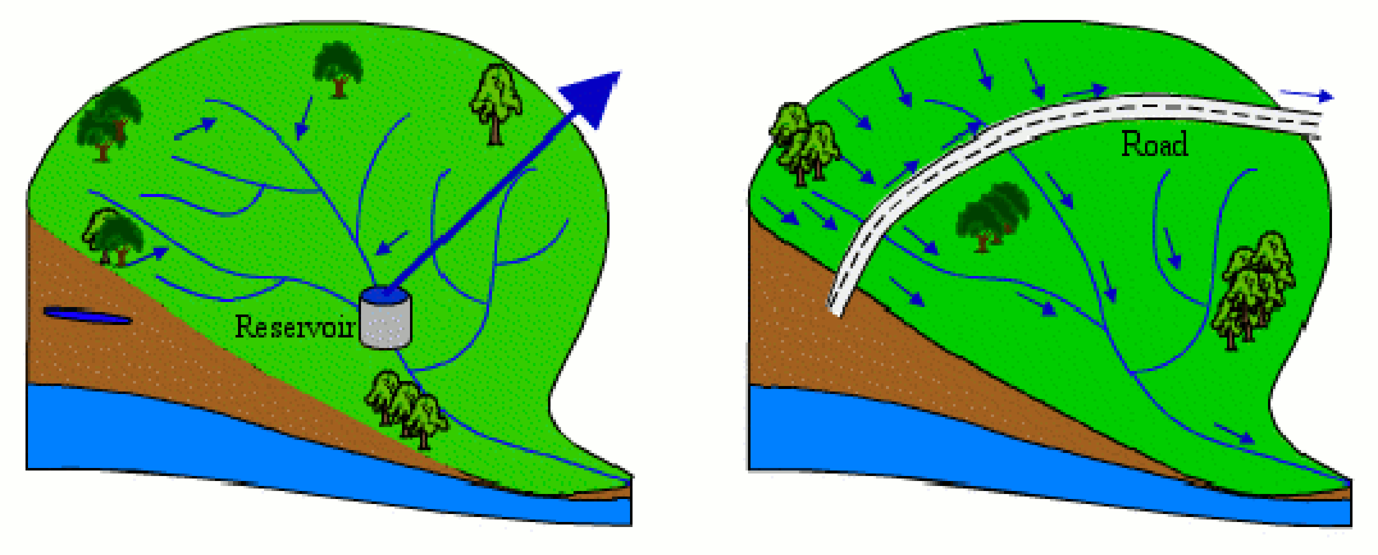


Figure 1.3 Artificial changes that occur in a watershed [Musy, 2001] .

1. Catchment morphometry (cont‘d)

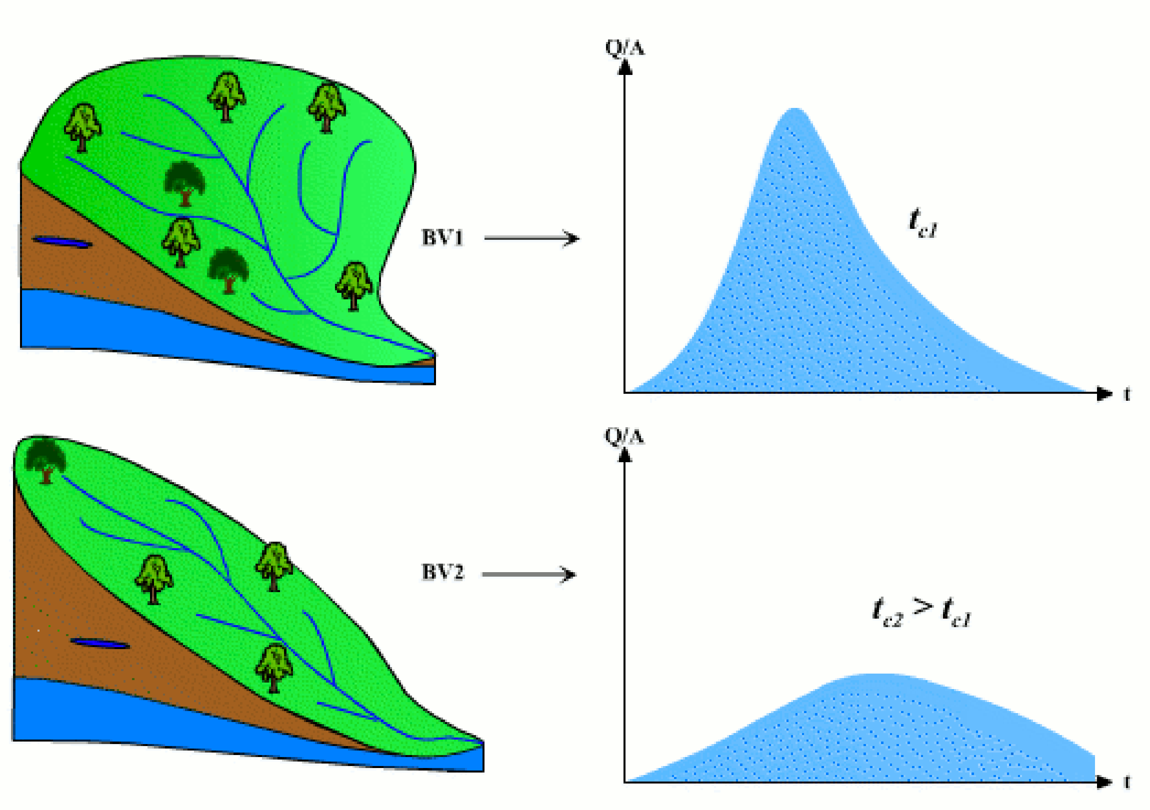


Figure 1.4 The influence of watershed shape on the hydrograph

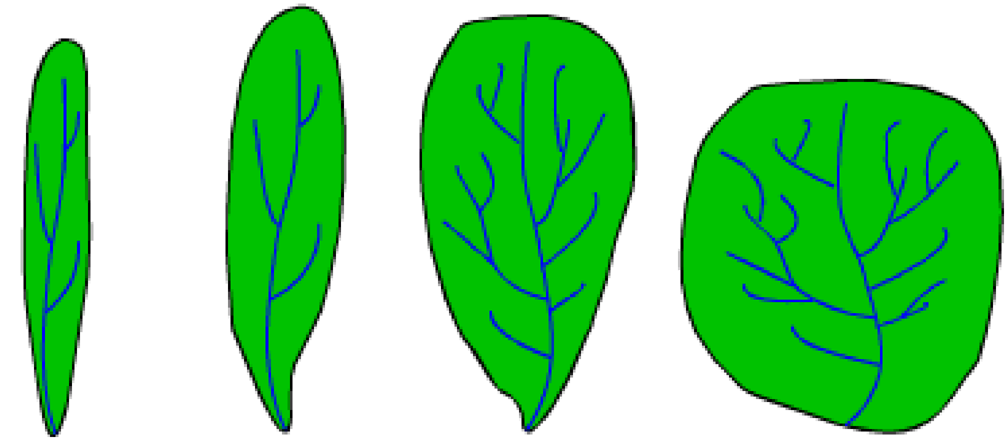
1. Catchment morphometry (cont'd)

Gravelius's index, *KG*, which is defined as the relation between the perimeter of the watershed and that of a circle having a surface equal to that of a watershed.

$$K_G = \frac{P}{2 \cdot \sqrt{\pi \cdot A}} \approx 0.28 \frac{P}{\sqrt{A}}$$

where:

K_G	Gravelius's shape index
A	watershed area [km^2]
P	watershed perimeter [km]



K_G= 1,6 K_G= 1,3 K_G= 1,2 K_G= 1,1

Figure 1.5. Some KG values for different watershed shapes

1. Catchment morphometry (cont‘d)

Average altitude Ha

$$H_a = \sum \frac{A_{i,i+1} \cdot (h_i + h_{i+1})}{2 \cdot A}$$

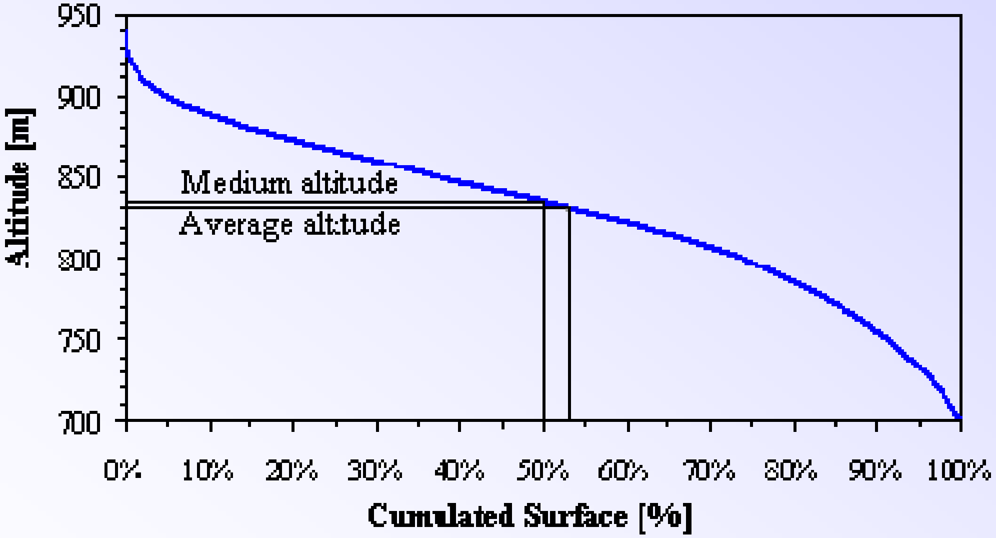


Figure 1.6 Hypsographical curve of a watershed

1. Catchment morphometry (cont'd)

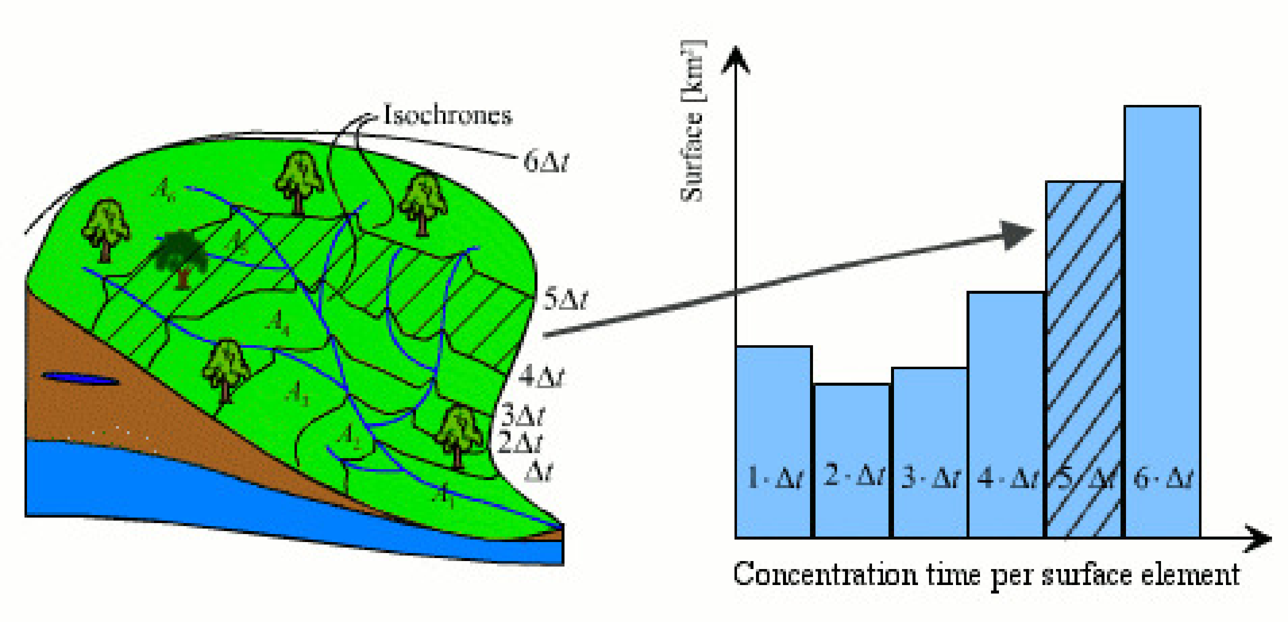


Figure 1.7 Representation of isochrones from a watershed

1. Catchment morphometry (cont‘d)

- The quantitative study of stream networks was originated by Horton (1945) and modified by Strahler (1964).

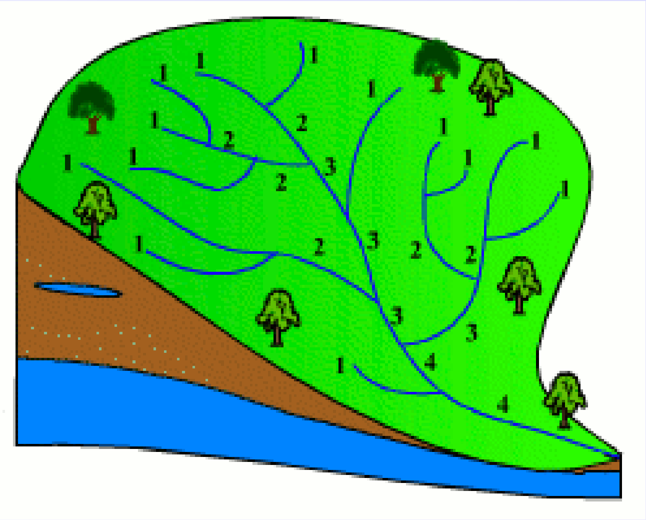


Figure 1.8 Strahler's system of hydrographic network classification

1. Catchment morphometry (cont‘d)

- The smallest recognizable channels are designated as Order 1.
- In general when two channels of order i join, a channel of order i+1 results.
- When a channel of lower order joins a channel of higher order , the channel downstream retains the higher of the two orders.
- The order of the drainage basin is designated as the order of the stream draining its outlet, the highest stream order in the basin.

1. Catchment morphometry (cont‘d)

- Horton’s law of Stream orders: Bifurcation ratio R_B or ratio of the number N_i of channels of order I to the number N_{i+1} of channels of order i+1 is relatively constant from one order to another.

$$\frac{N_i}{N_{i+1}} = R_B \qquad i = 1,2,\ldots,i-1$$

- Law of stream lengths: the average lengths of stream of successive orders are related by a length ratio R_L

$$\frac{L_{i+1}}{L_i} = R_L \qquad i = 1,2,\ldots,i-1$$

- Law of stream areas: Schumm(1956) related the average areas A_i drained by streams of successive orders

$$\frac{A_{i+1}}{A_i} = R_A \qquad i = 1,2,\ldots,i-1$$

1. Catchment morphometry (cont‘d)

- The drainage density, D is the ratio of the total length of stream channels in a watershed to its area

$$D = \frac{\sum_{i=1}^{I}\sum_{j=1}^{N_i} L_{ij}}{A_I}$$

Where L_{ij} is the length of the j stream of order i

- The average length of overland flow, L_o is given approximately by

$$L_o = \frac{1}{2D}$$

1. Exercise

Question:

- Consider a rainstorm with a constant intensity of 14 mm/hr falls on a certain area for a duration of 3 hours. For a particular conditions the infiltration capacity of a wet soil in the area is described with empirical Horton equation as:

  $$fp=3+17e^{-0.5t}$$

  where fp, fc and fo are in mm./hr, t in hours and k in hr^{-1}.
- Estimate the total amount of rain that will infiltrate into the soil at the end of the storm.

1. Hydrometric measurement

Discharge in streams can be measured directly by:

- Area - velocity method
- Moving - Boat method
- Dilution or chemical method
- Electromagnetic method
- Ultrasonic method

1. Hydrometric measurement (cont'd)

- Area velocity method

$$Q = \sum_{i=1}^{N-1} \Delta Q_i$$

$$\Delta Q_i = Y_i * (W_i/2 + W_{i+1}/2) * V_i$$

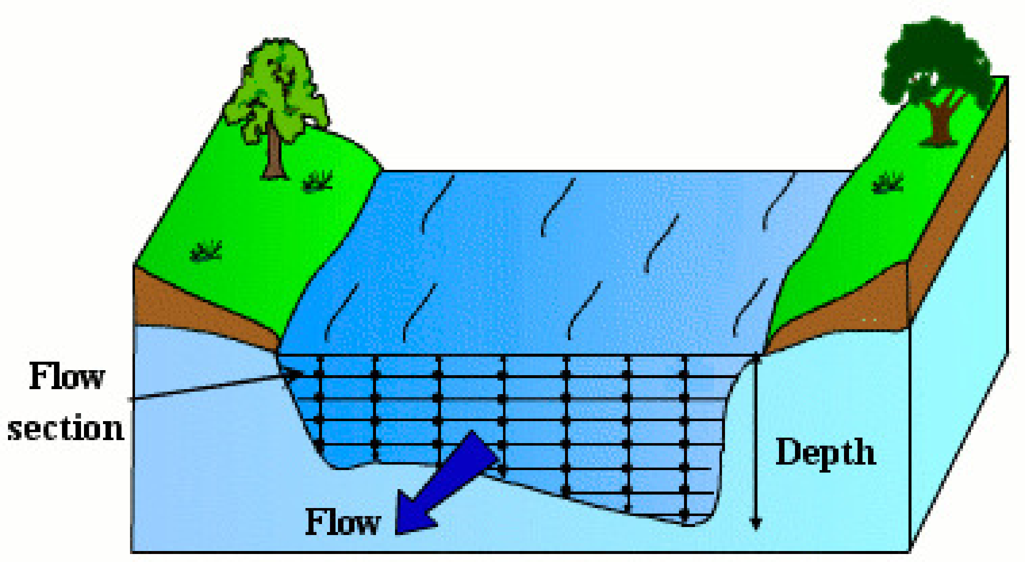
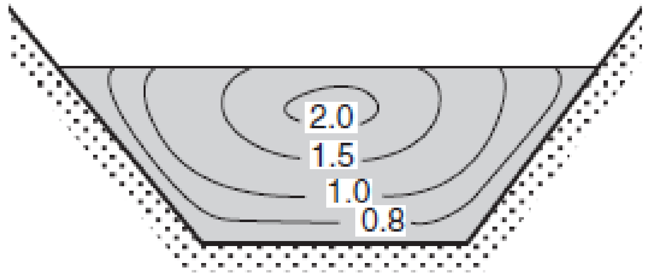


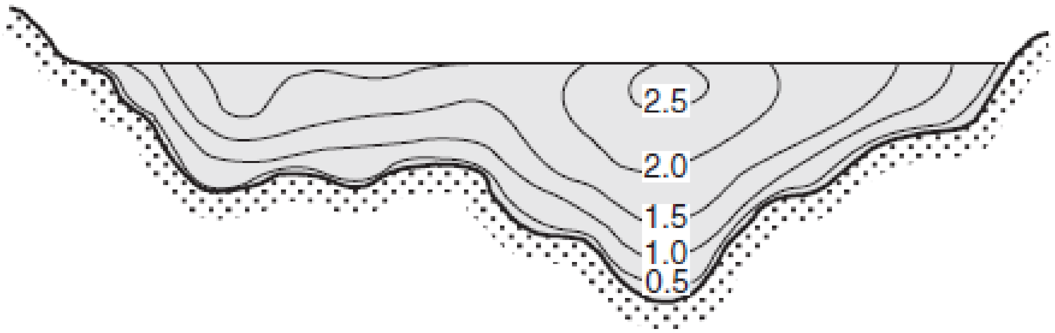
Figure 1.10 Flow and field of velocities through a cross section

1. Hydrometric measurement

- The velocity distribution in a cross-section of a stream



Trapezoidal channel



Natural irregular channel

1. Hydrometric measurement

(1) Three-points measuring method ・・・・

$$Vm = 0.25\times(V_{0.2} + 2V_{0.6} + V_{0.8})$$

(2) Two-points measuring method ・・・・

$$Vm = 0.50\times(V_{0.2} + V_{0.8})$$

(3) One-point measuring method ・・・・・ $Vm = V_{0.6}$

(4) Surface measuring method ・・・・・・ $Vm = 0.8\times Vs$

where, Vm: Mean velocity Vs: Surface velocity

$V_{0.2}$: Velocity at the depth of 20% below the water surface

$V_{0.6}$: Velocity at the depth of 60% below the water surface

$V_{0.8}$: Velocity at the depth of 80% below the water surface

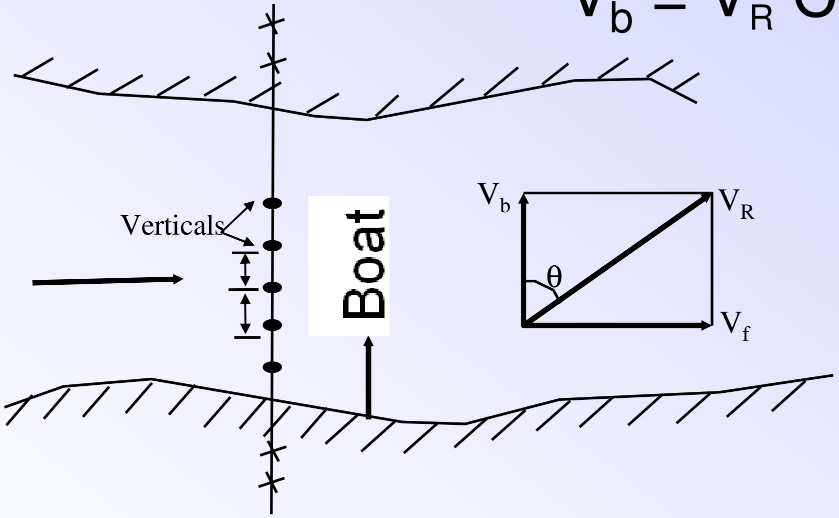
1. Hydrometric measurement (cont’d)

- Moving boat method

1. Hydrometric measurement (cont'd)

- Moving boat method

$$V_b = V_R \cos\theta \text{ and } V_f = V_R \sin\theta$$



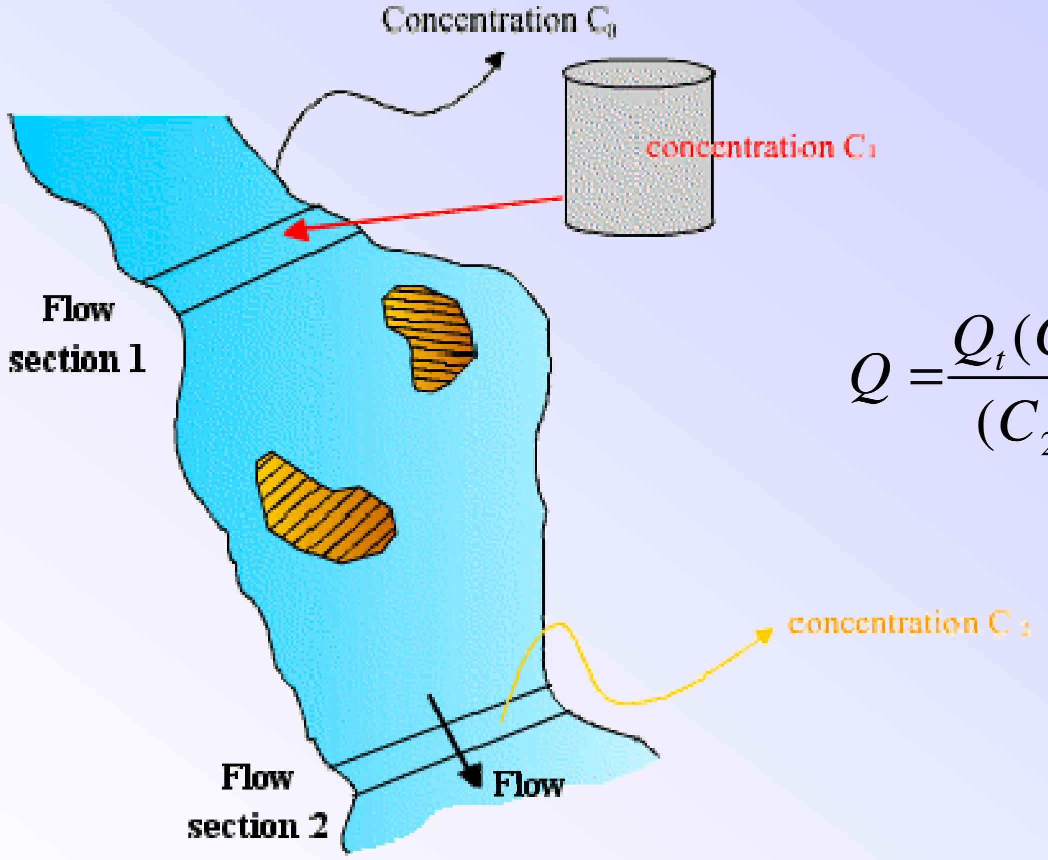
$$W = V_b \Delta t$$

$$\Delta Qi = \left(\frac{y_i + y_{i+1}}{2}\right) W_{i+1} V_f$$

$$\Delta Qi = \left(\frac{y_i + y_{i+1}}{2}\right) V_R^2 \, Sin\,\theta . Cos\,\theta . \Delta t$$

1. Hydrometric measurement (cont'd)

- Dilution gauging



$$Q = \frac{Q_t(C_1 - C_2)}{(C_2 - Co)}$$

Figure 1.12 Principle of dilution measuring [Musy, 2001]

1. Hydrometric measurement (cont'd)

The following criteria are adopted:

- The stream should have a well-defined cross-section that does not change in various seasons.
- It should be easily accessible all through the year.
- The site should be in a straight, stable reach.
- The gauging site should be free from backwater effects in the channel.

1. Hydrometric measurement (cont'd)

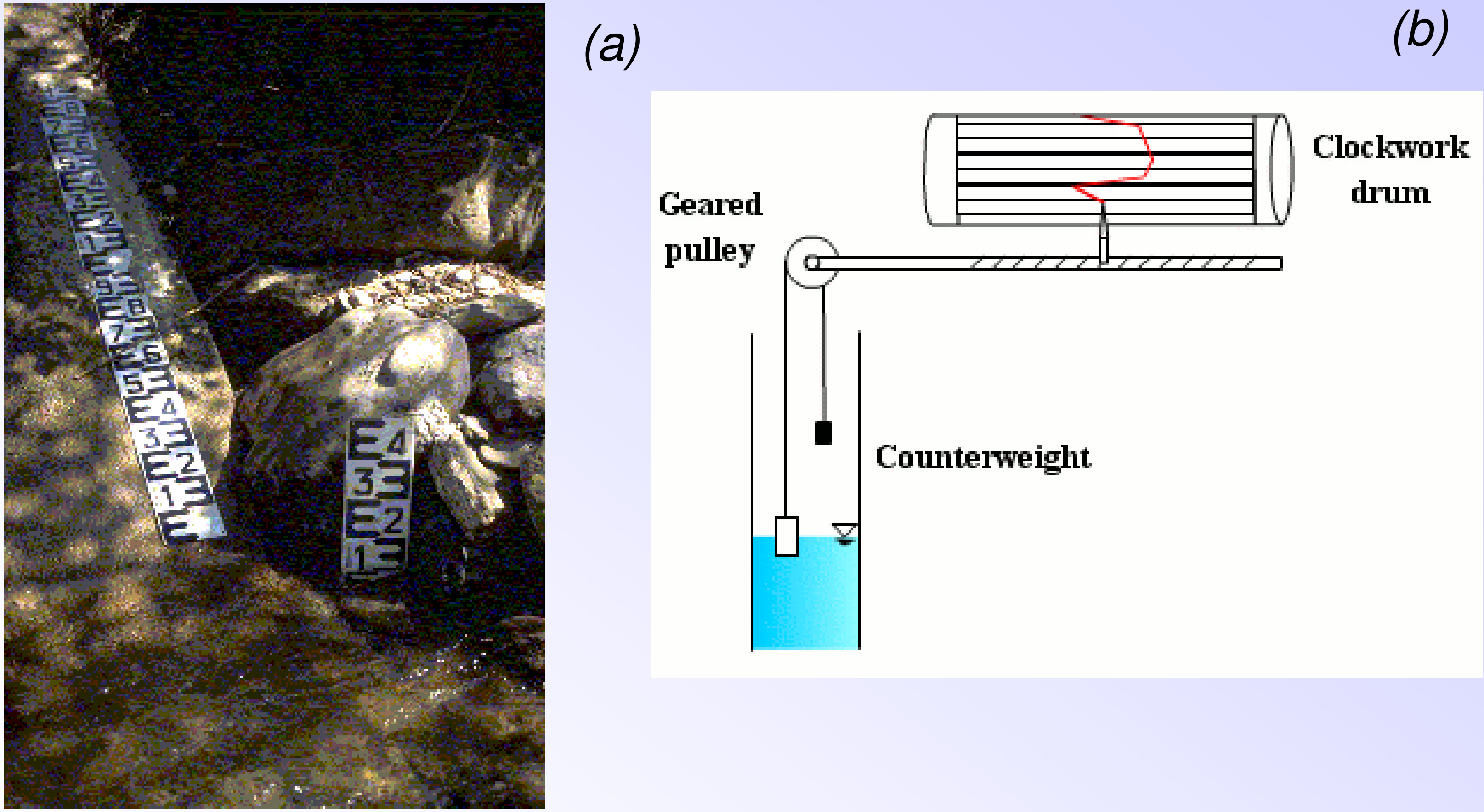


Figure 1.13 (a) Staff gauges (b) Float recorder

1. Hydrometric measurement (cont'd)

- Stage-discharge curve

$$Q = a\,(H - H_o)^b$$

$$\Rightarrow \log Q = b \log (H-H_o) + \log a$$

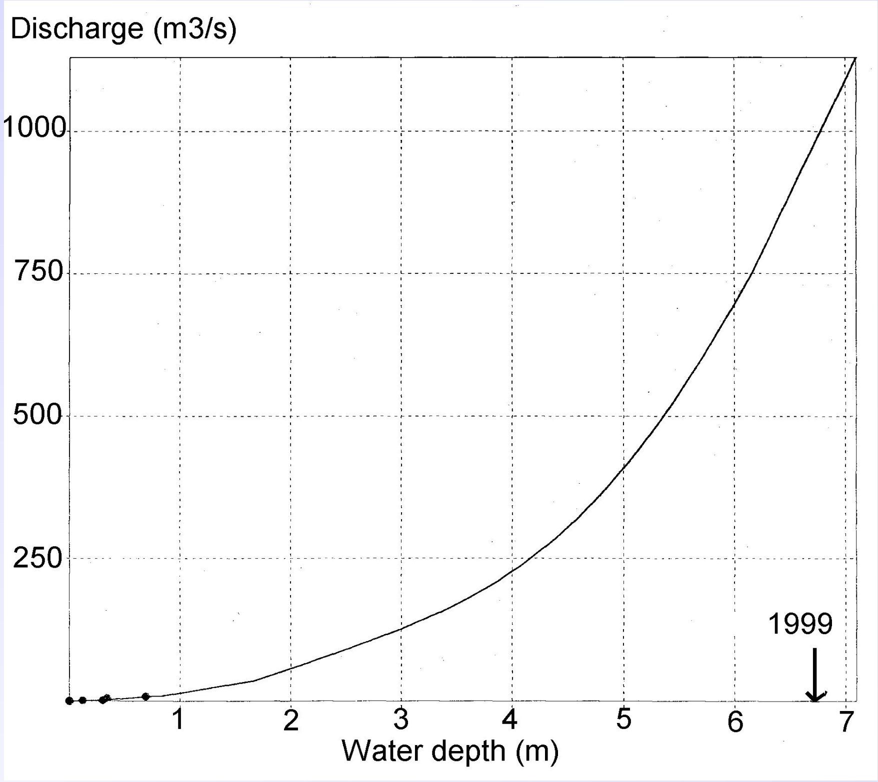
$$\text{or} \quad Y = bx + C$$

$Y = \log Q$ and $x = \log (H - H_o)$,
$C = \log a$

$$b = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2}$$

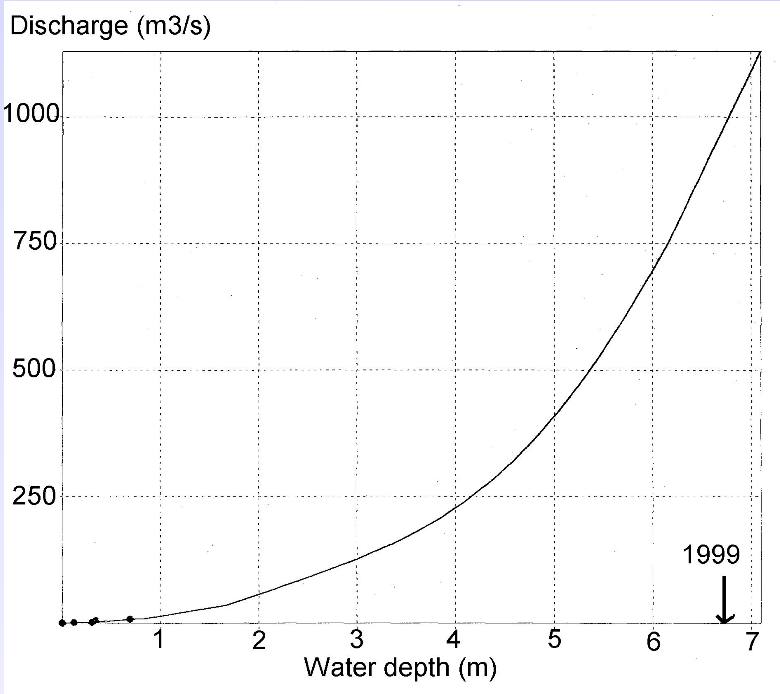
$$c = \frac{\sum Y - b(\sum X)}{N}$$

$$r = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[N(\sum X^2) - (\sum X)^2][N(\sum Y^2) - (\sum y)^2]}}$$



1. Hydrometric measurement (cont'd)

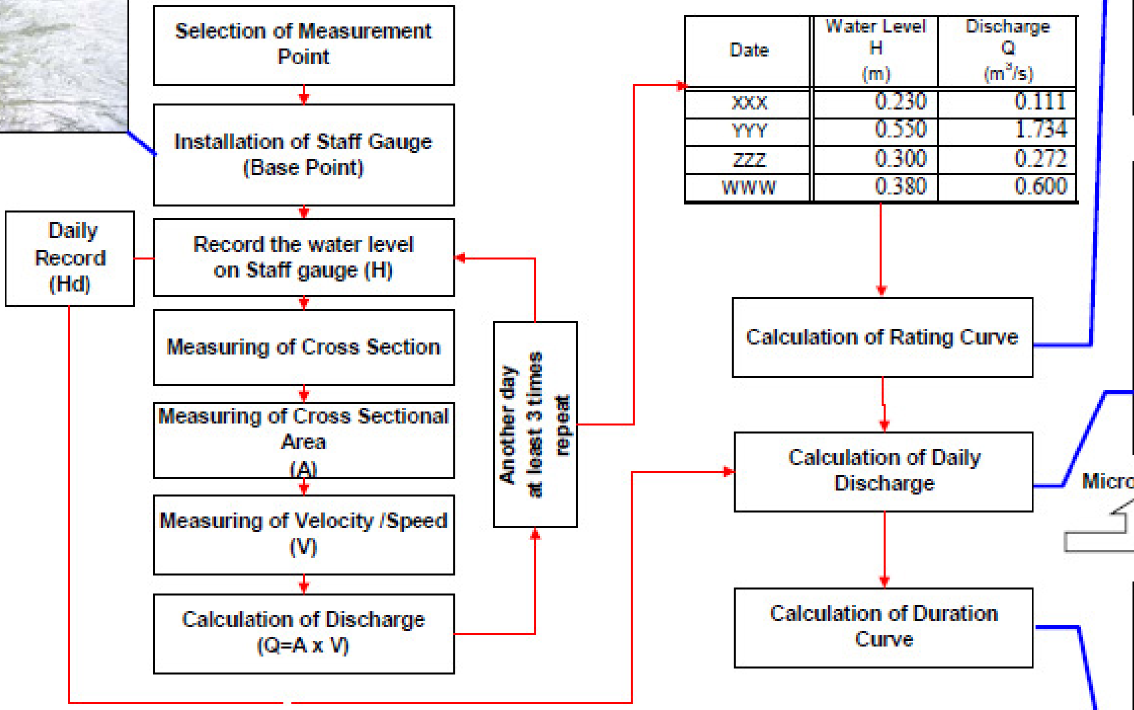
Stage for zero Discharge, Ho:



$$\frac{Q_A}{Q_B} = \frac{Q_B}{Q_C}$$

$$H_0 = \frac{H_1 H_3 - H_2^{\,2}}{(H_1 + H_3) - 2H_2}$$

1. Hydrometric measurement (cont'd)



Selection of Measurement Point

Installation of Staff Gauge (Base Point)

Record the water level on Staff gauge (H)

Daily Record (Hd)

Measuring of Cross Section

Measuring of Cross Sectional Area (A)

Measuring of Velocity /Speed (V)

Calculation of Discharge (Q=A x V)

Another day at least 3 times repeat

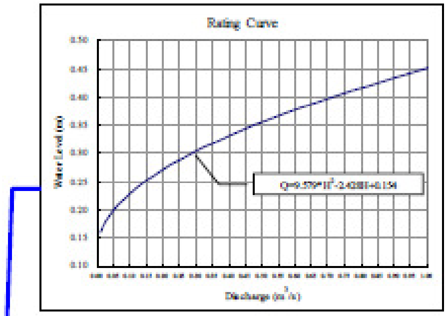
Date	Water Level H (m)	Discharge Q (m^3/s)
XXX	0.230	0.111
YYY	0.550	1.734
ZZZ	0.300	0.272
WWW	0.380	0.600

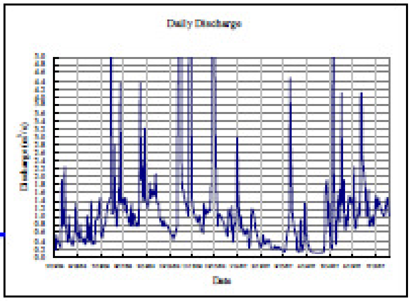
Calculation of Rating Curve

Calculation of Daily Discharge

Calculation of Duration Curve

Micro-Hydro





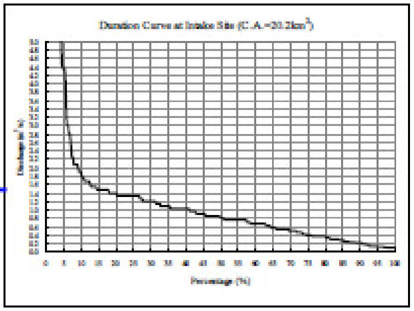
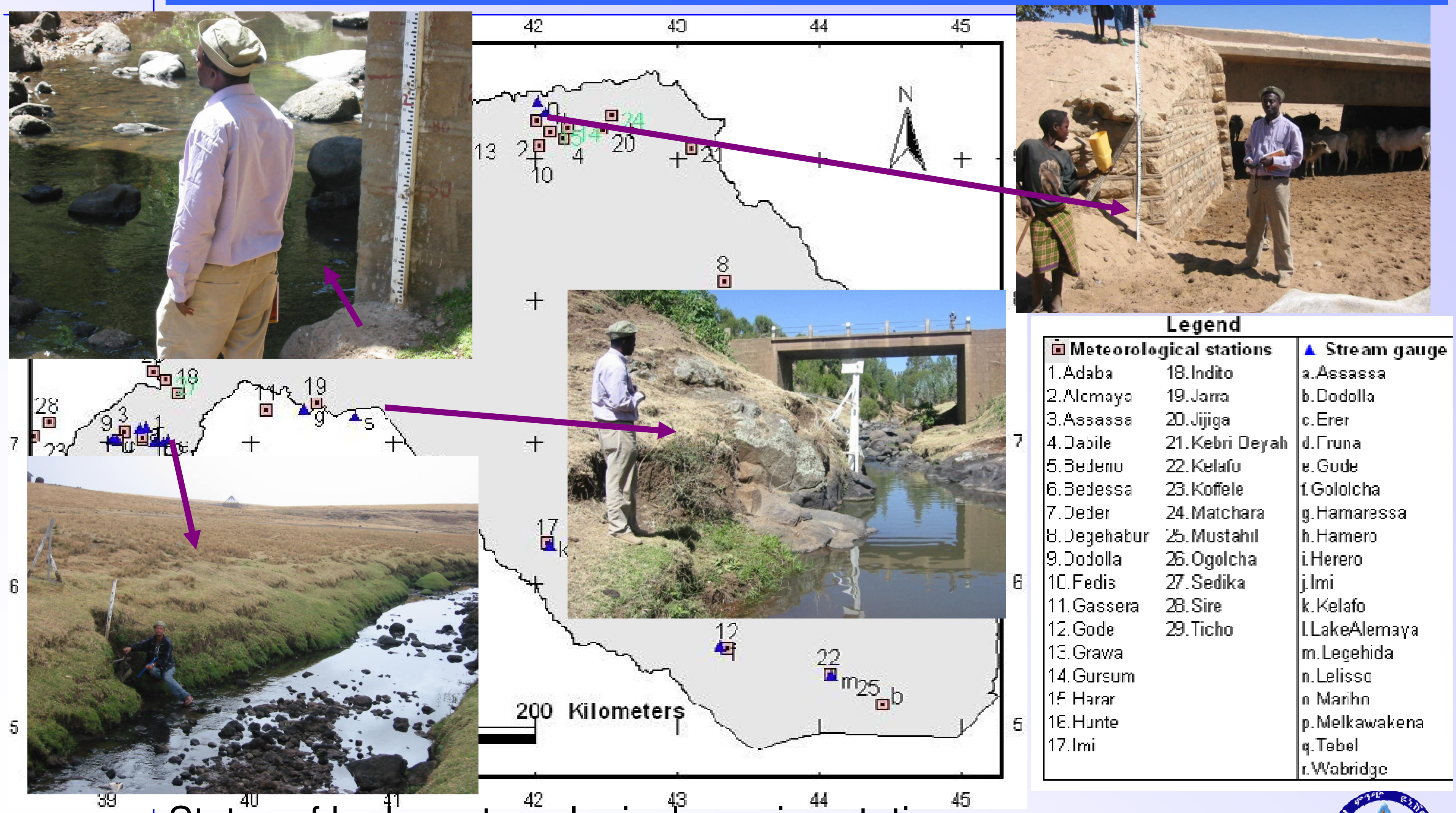


Fig.3.4.2 Flowchart to check Minimum Flow/ Duration Curve

1. Quality control of data

Hydrologic principles
Hydrologic analysis
Frequency analysis
Flood routing
Hydrologic design



Legend

Meteorological stations		Stream gauge
1.Adaba	18.Indito	a.Assassa
2.Alemaya	19.Jarra	b.Dodolla
3.Asassa	20.Jijiga	c.Erer
4.Dabile	21.Kebri Deyah	d.Fruna
5.Bedenu	22.Kelafu	e.Gude
6.Bedessa	23.Koffele	f.Gololcha
7.Deder	24.Matchara	g.Hamaressa
8.Degehabur	25.Mustahil	h.Hamero
9.Dodolla	26.Ogolcha	i.Herero
10.Fedis	27.Sedika	j.Imi
11.Gassera	28.Sire	k.Kelafo
12.Gode	29.Ticho	l.LakeAlemaya
13.Grawa		m.Legehida
14.Gursum		n.Lelisso
15.Harar		o.Marho
16.Hunte		p.Melkawakena
17.Imi		q.Tebel
		r.Wabridge

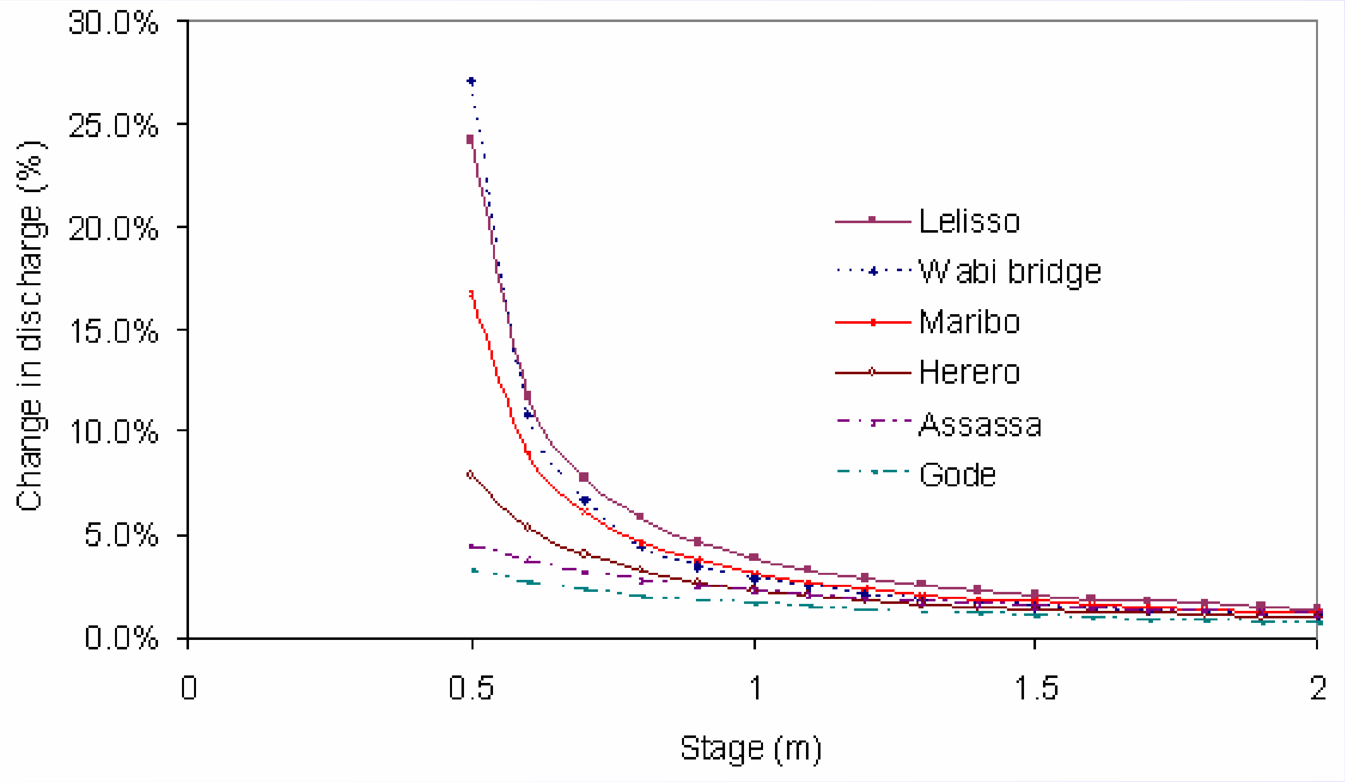
Status of hydrometeorological gauging stations

1. Quality control of data (cont'd)

- Sources of uncertainity in streamflow gauging
  - Random uncertainities in measurements of width, depth and velocity
  - With 20 verticals and price type current meter
    - In shallow water (<76cm) where v is measured at 0.6d have CV of random errors about 4.3 %
    - In deeper water (>76cm) where v is measured at 0.2 and 0.8d have CV of random errors about 3%.
- Systematic uncertainity-bias in velocity area gaugings
  - Errors in calibrated tapes, cables for depth and width measurement & incorrect current meter calibration

1. Quality control of data (cont‘d)

- Percentage change in discharge for a systematic error of 10 mm at different stations



1. Quality control of data (cont‘d)

- Test for independence and stationarity
- Test for homogeneity
- Test for outliers

1. Quality control of data (cont‘d)

- **Test for independence and stationarity**
  - Wald-Wolfowitz (W-W) test

$$R = \sum_{i=1}^{N-1} x_i x_{i+1} + x_1 x_N$$

$$\bar{R} = \frac{(s_1^2 - s_2)}{N-1}$$

$$\operatorname{var}(R) = \frac{s_2^2 - s_4}{N-1} - \bar{R}^2 + \frac{(s_1^4 - 4 s_1^2 s_2 + 4 s_1 s_3 + s_2^2 - 2 s_4)}{(N-1)(N-2)}$$

$$s_r = N m_r' \qquad m_r' = \frac{1}{n} \sum_{i=1}^{n} x_i^r$$

m_r' is the r th moment of the sample about the origin

$$u = \frac{(R - \bar{R})}{(\operatorname{var}(R))^{1/2}}$$ normally distribute d with mean zero and variance 1.

1. Quality control of data (cont‘d)

- **Test for homogeneity and stationarity**
  - Mann-Whitney (M-W) test (Two samples of size p and q, $p \le q$)
  - N=p+q is ranked in increasing order, R=sum of the ranks of the elements of the first sample (size p) in the combined series (size N)

$$V = R - \frac{(p(p+1))}{2}$$

$$W = pq - V \qquad U = \min(V, W)$$

$$\bar{U} = \frac{pq}{2}$$

$$\operatorname{var}(U) = \left[\frac{pq}{N(N-1)}\right]\left[\frac{N^3 - N}{12} - \sum T\right]$$

$$T = \frac{(J^3 - J)}{12}$$

J is the number of observations tied at a given rank.

T is summed over all groups of tied observations in both samples of size p and q

$$u = \frac{(U - \bar{U})}{(\operatorname{var}(U))^{1/2}}$$

compare u with the standard normal variate at a significance level of α

1. Quality control of data (cont‘d)

- **Tests for trend**
- Mann-Kendall (non-parametric)
- Spearman’s Rho (non-parametric)
- Linear Regression (parametric)

- **Tests for step change in mean/median**
- Distribution Free CUSUM (non-parametric)
- Cumulative Deviation (parametric)
- Worsley Likelihood Ratio (parametric)

1. Quality control of data (cont‘d)

- **Tests for randomness**
- Median Crossing (non-parametric)
- Turning Points (non-parametric)
- Rank Difference (non-parametric)
- Autocorrelation (parametric)

1. Quality control of data (cont‘d)

- **Mann-Kendall (non-parametric**)
- The n time series values (X1, X2, X3, ….., Xn) are replaced by their relative ranks (R1, R2, R3,….., Rn) (starting at 1 for the lowest up to n).
- The test statistic S is:

$$S = \sum_{i=1}^{n-1} [\sum_{j=i+1}^{n} \text{sgn}(R_j - R_i)]$$

- where

$$\text{sgn}(x) = 1 \quad \text{for} \quad x > 0$$
$$\text{sgn}(x) = 0 \quad \text{for} \quad x = 0$$
$$\text{sgn}(x) = -1 \quad \text{for} \quad x < 0$$

If the null hypothesis Ho is true, then S is approximately normally distributed with:

$$\mu = 0$$
$$\sigma = n(n-1)(2n+5)/18$$

$$z = |S| / \sigma^{0.5}$$

1. Quality control of data (cont‘d)

- **Spearman’s Rho (non-parametric)**
- This is a rank-based test that determines whether the correlation between two variables is significant.
- where

$$\rho_s = S_{xy} / (S_x S_y)^{0.5}$$

$$S_x = \sum_{i=1}^{n} (x_i - \overline{x})^2$$

$$S_y = \sum_{i=1}^{n} (y_i - \overline{y})^2$$

$$S_{xy} = \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})$$

$\rho_s \sqrt{n-1}$ approximately normally distributed with mean of 0 and variance of 1

1. Quality control of data (cont‘d)

- **Linear Regression (parametric)**
- This is a parametric test that assumes that the data are normally distributed.
- The regression gradient is estimated by

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

and the intercept is estimated as

$$a = \bar{y} - b\,\bar{x}$$

The test statistic S is:

$$S = b/\sigma$$

$$\text{where } \sigma = \sqrt{\frac{12\sum_{i=1}^{n}(y_i - a - bx_i)}{n(n-2)(n^2-1)}}$$

The test statistic S follows a Student-t distribution with n-2 degrees of freedom under the null hypothesis

2. Linear system response

- Rainfall-runoff relationships
- Hydrograph analysis
- Unit hydrograph
- Synthetic unit hydrograph
- Instantaneous unit hydrograph
- Conceptual models

2. Rainfall-runoff relationships

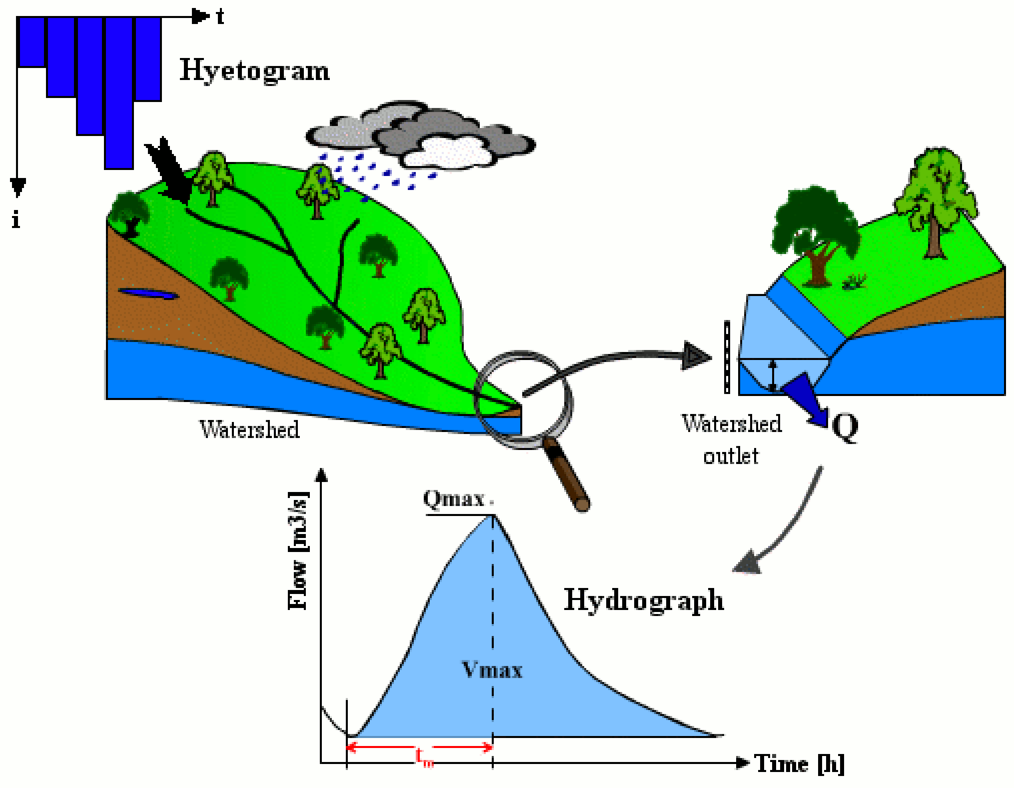


Figure 2.1 Hydrological response of a catchment

2. Rainfall-runoff relationships

- Hydrological response on a catchment is influenced by many factors that are related to:
  - climatic conditions of the environment
  - rain (spatial and temporal distribution, intensity and rain duration)
  - catchment morphology (shape, dimension, slopes' orientation)
  - physical properties of the catchment (soil nature, vegetal coverage)
  - structure of the hydrographic network (dimensions, hydraulic properties)
  - previous soil humidity state.

2. Rainfall-runoff relationships

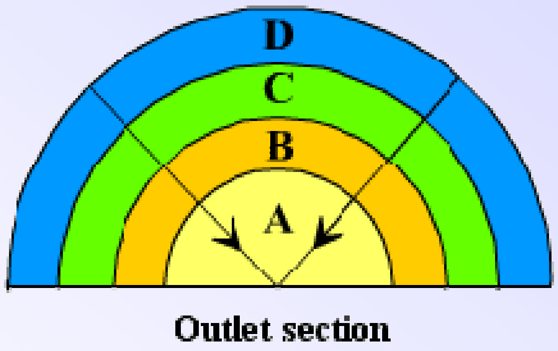


Figure 2.2 Catchment representation

$$Q_T = \sum_{k=1}^{T} i_{(T-K)} \, \Delta A_{(k)}$$

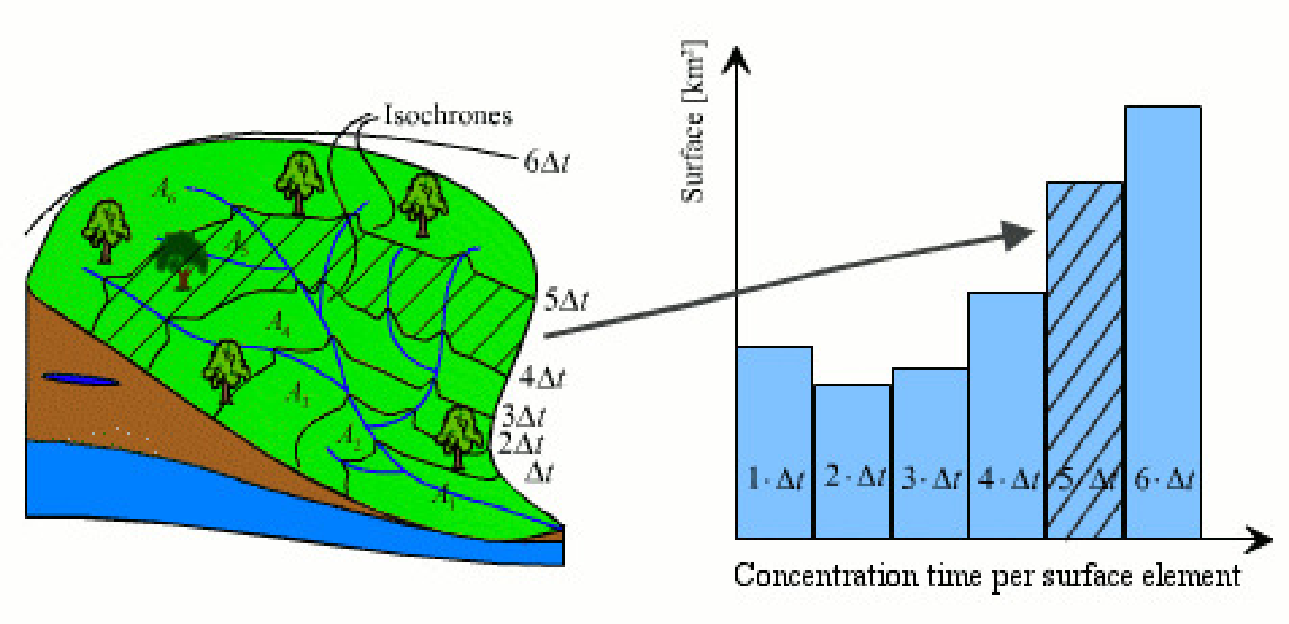
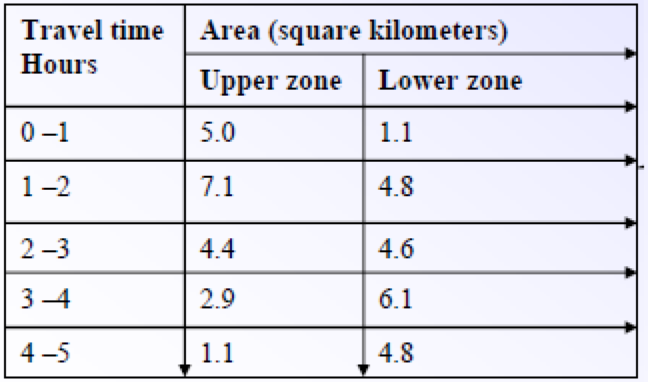


Figure 2.3 Representation of isochrones from a watershed

2. Rainfall-runoff relationships

Example 1. On a catchment where the travel time concept can be applied, a rain of two hours precipitated. An analysis of the depths shows that due to some orographic effect the area can roughly be divided into two rainfall zones. The upper zone in the figure receives 8 cm/hr as average net intensity; the lower one gets 6 cm/hr. the isochrones are also indicated in the figure. Assume runoff coefficient for upper zone and lower zone are 0.5 and 0.7 respectively. The areas in between isochrones are:



Travel time Hours	Area (square kilometers)	
	Upper zone	Lower zone
0 –1	5.0	1.1
1 –2	7.1	4.8
2 –3	4.4	4.6
3 –4	2.9	6.1
4 –5	1.1	4.8

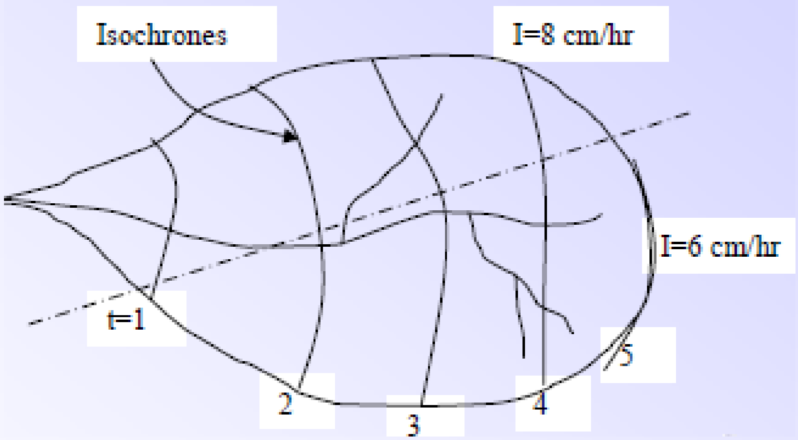


Figure 2.3 Compute the first five ordinates of the direct runoff generated by the given storm of 2 hours.

2. Rainfall-runoff relationships

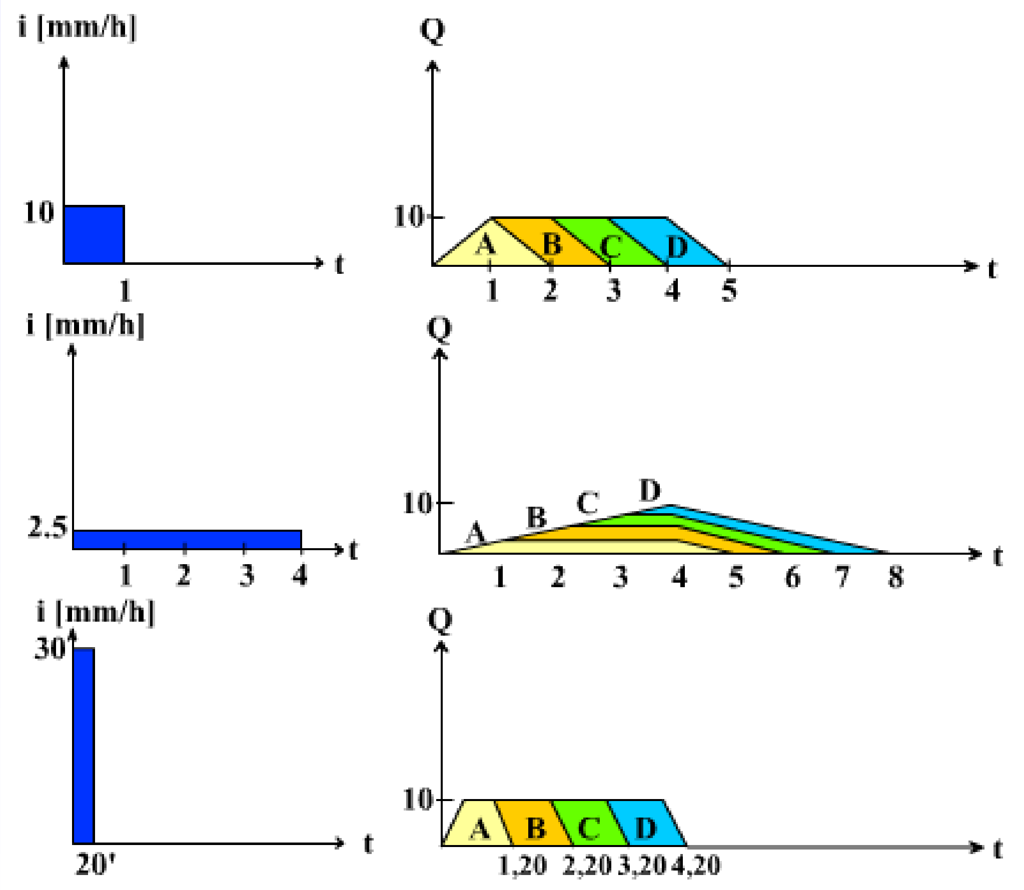


Figure 2.3 The rainfall influence in time, on the hydrological response of a catchment (spatially uniform)

2. Rainfall-runoff relationships

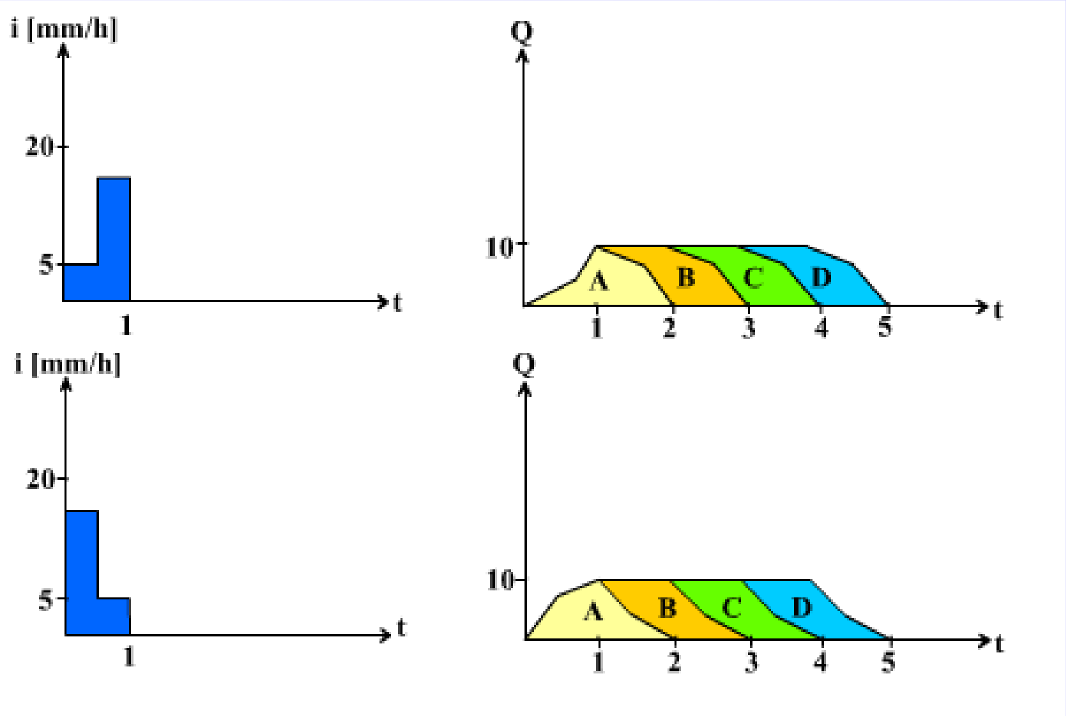


Figure 2.4 The influence of rainfall intensity variations on the hydrological response on a catchment

2. Rainfall-runoff relationships

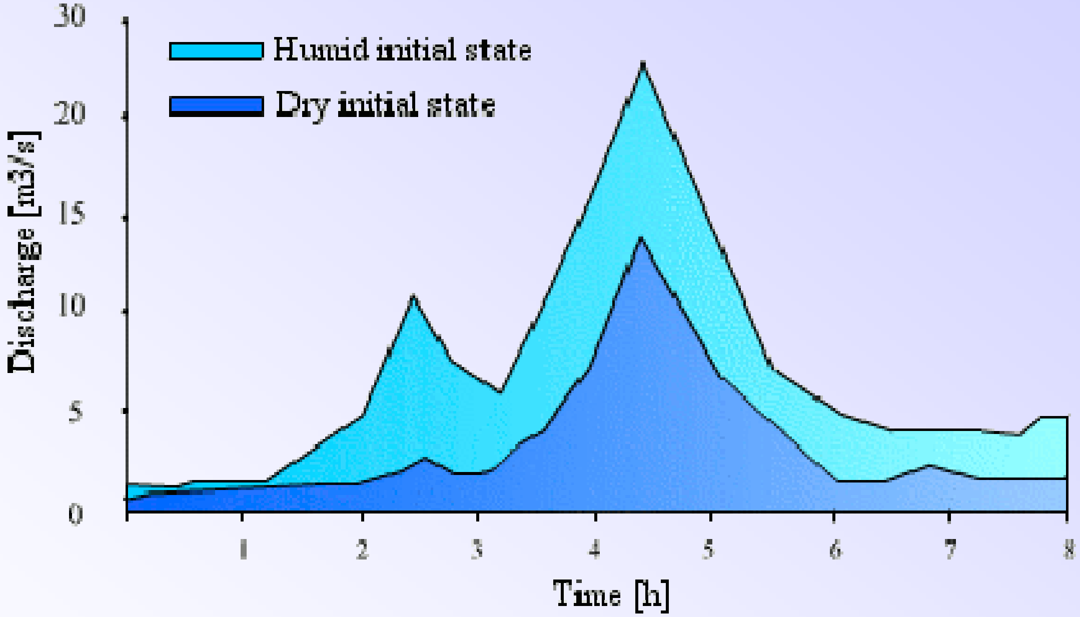


Figure 2.5 Influence of initial humidity conditions on the hydrograph's response of a catchment.

2. Hydrograph analysis

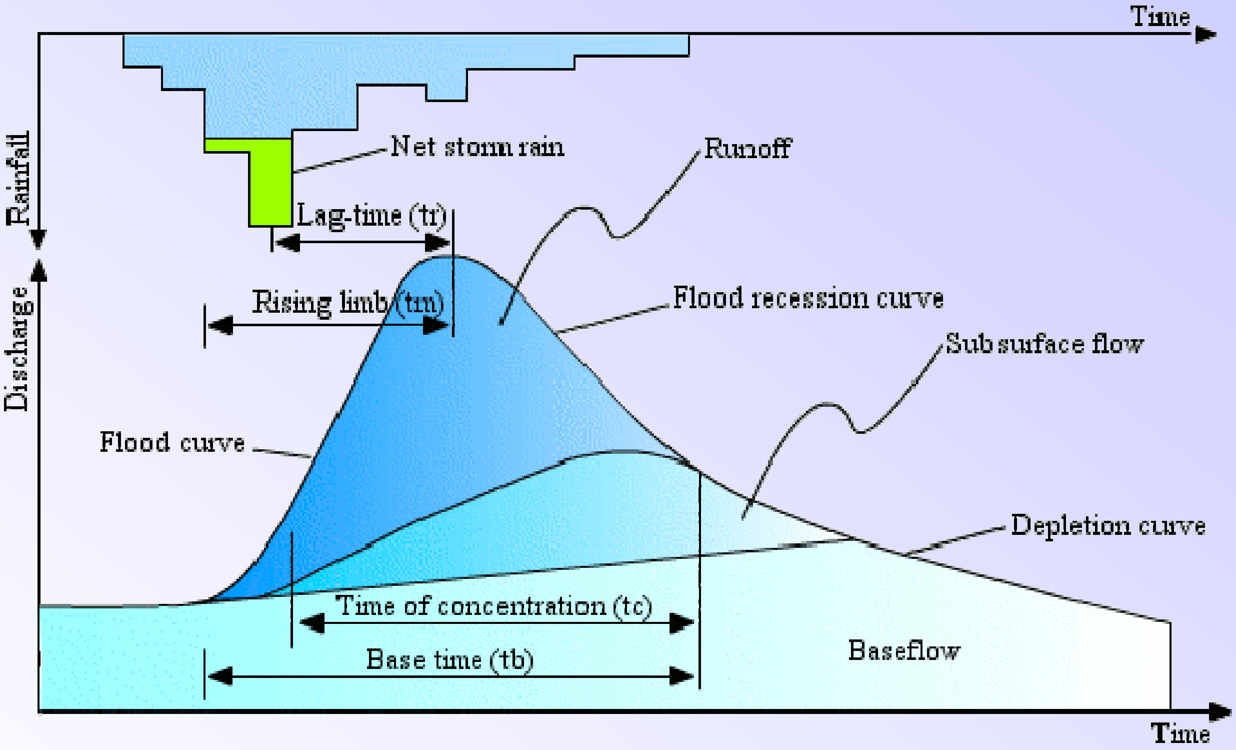


Figure 2.6 Hyetogram and hydrograph resulting from a storm event (rainfall – runoff)

2. Hydrograph analysis

Groundwater acts as reservoir (retards runoff from wet season and smoothes the shape of the hydrograph)

The way the outflow from the GW behaves is as linear reservoir where outflow is proportional to the storage.

- $S=KQ$
- For aquifer without recharge $Q= - dS/dt$
- $Q= - kdQ/dt$

$$- dt/k=dQ/Q$$

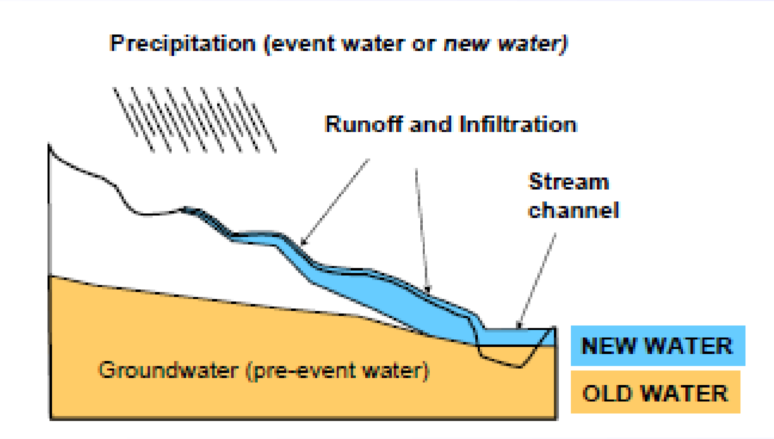
- $Q=Q_0e^{-(t-to)/k}$

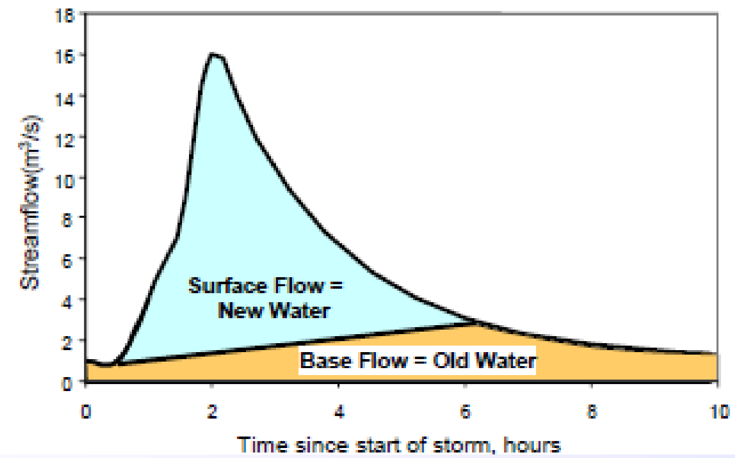
2. Hydrograph analysis

Base flow separation

- Method I: Straight line method
- Method II: Fixed base method
  - $N=0.83A^{0.2}$ [A]=square kilometer, [N]=days
- Method III: Variable slope method
  - Inflection point

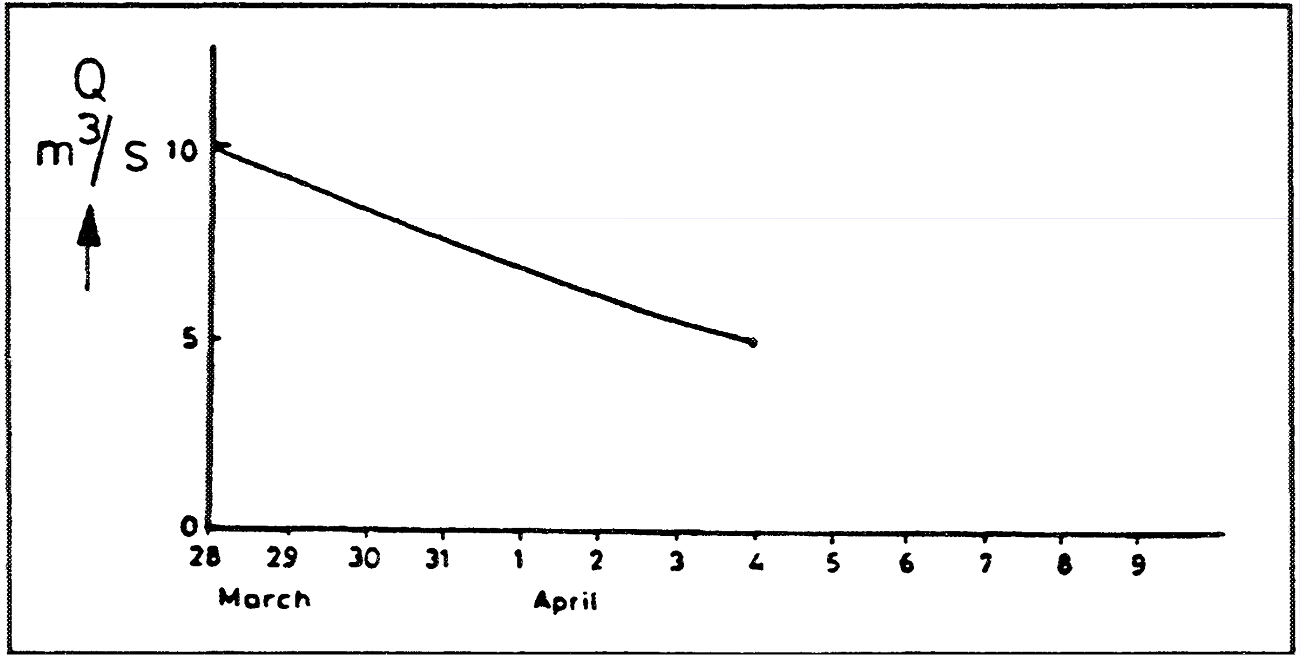
Application of Isotope Hydrology





2. Hydrograph analysis

Example: A wastewater treatment plant is allowed to discharge its effluent into a river, provided the flow in the river is more than 1 m^3/s. After a long dry period the flow in the river on April 4 has decreased to only 5 m^3/s. The depletion curve during the preceding week is given in the figure below. The curve may be described with the equation $Q_t=Q_0e^{-t/k}$. Compute the date on which the treatment plant has to terminate the discharge of the effluent into the river, assuming that the drought continues.

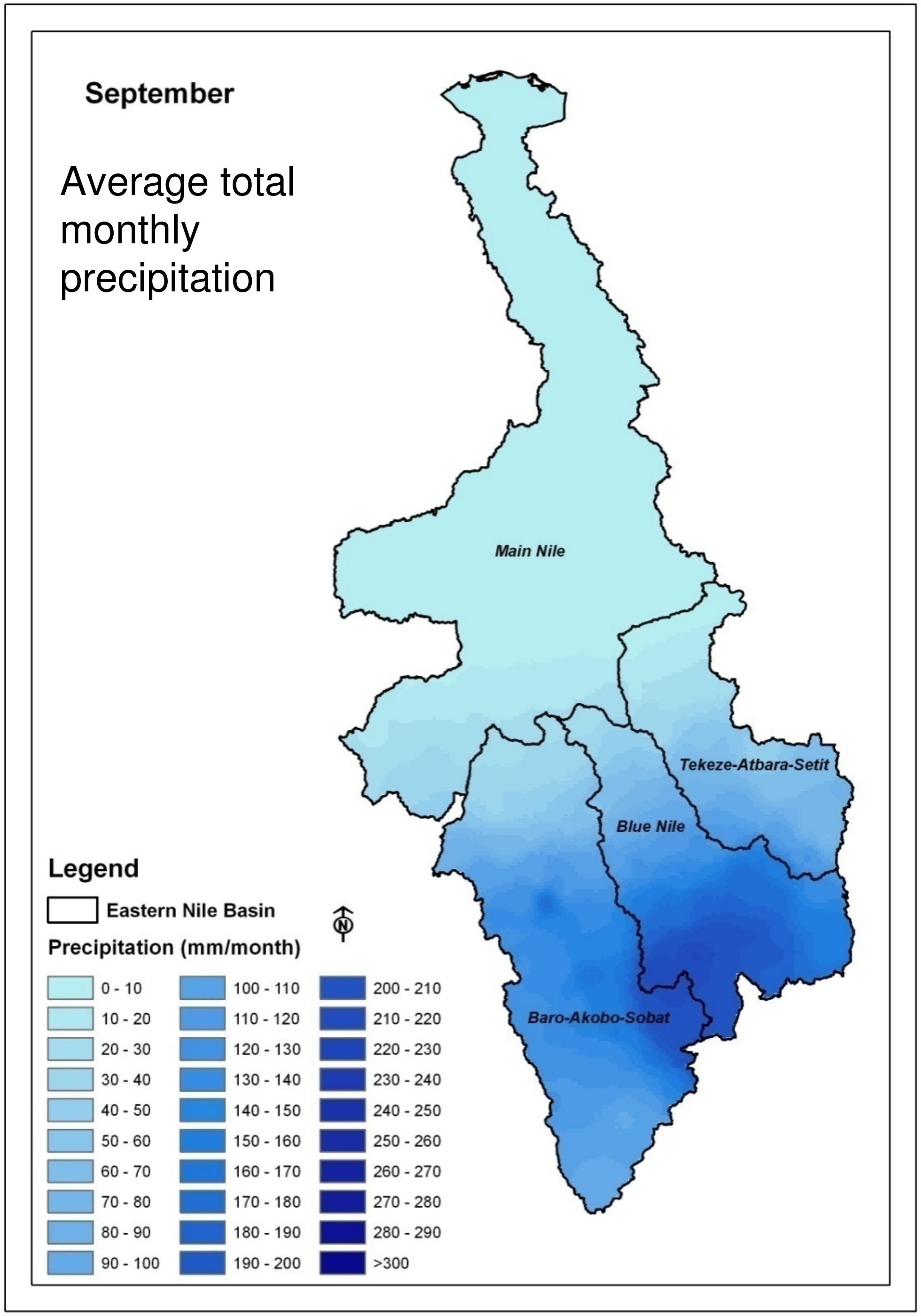


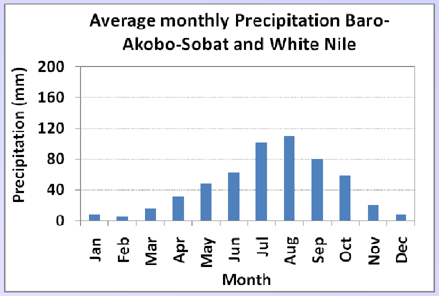
2. Hydrograph analysis

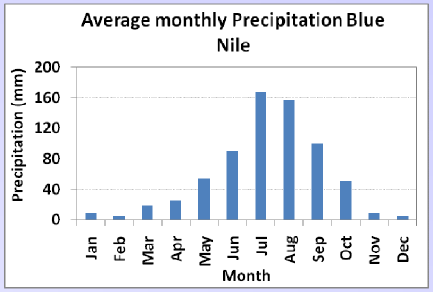
Due regard needs to be given to both:

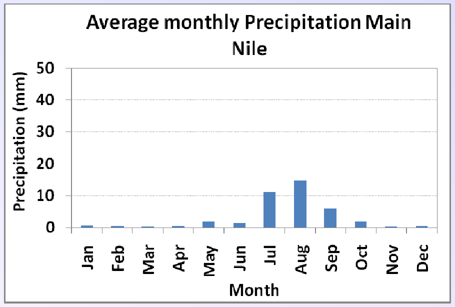
- Temporal variability
- Spatial variability

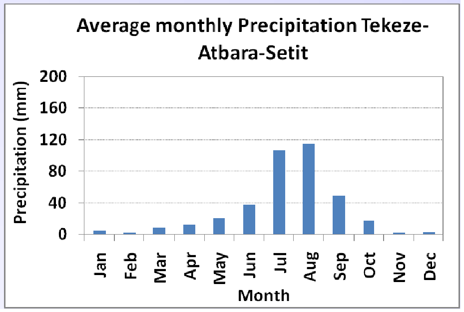
Precipitation of Eastern Nile



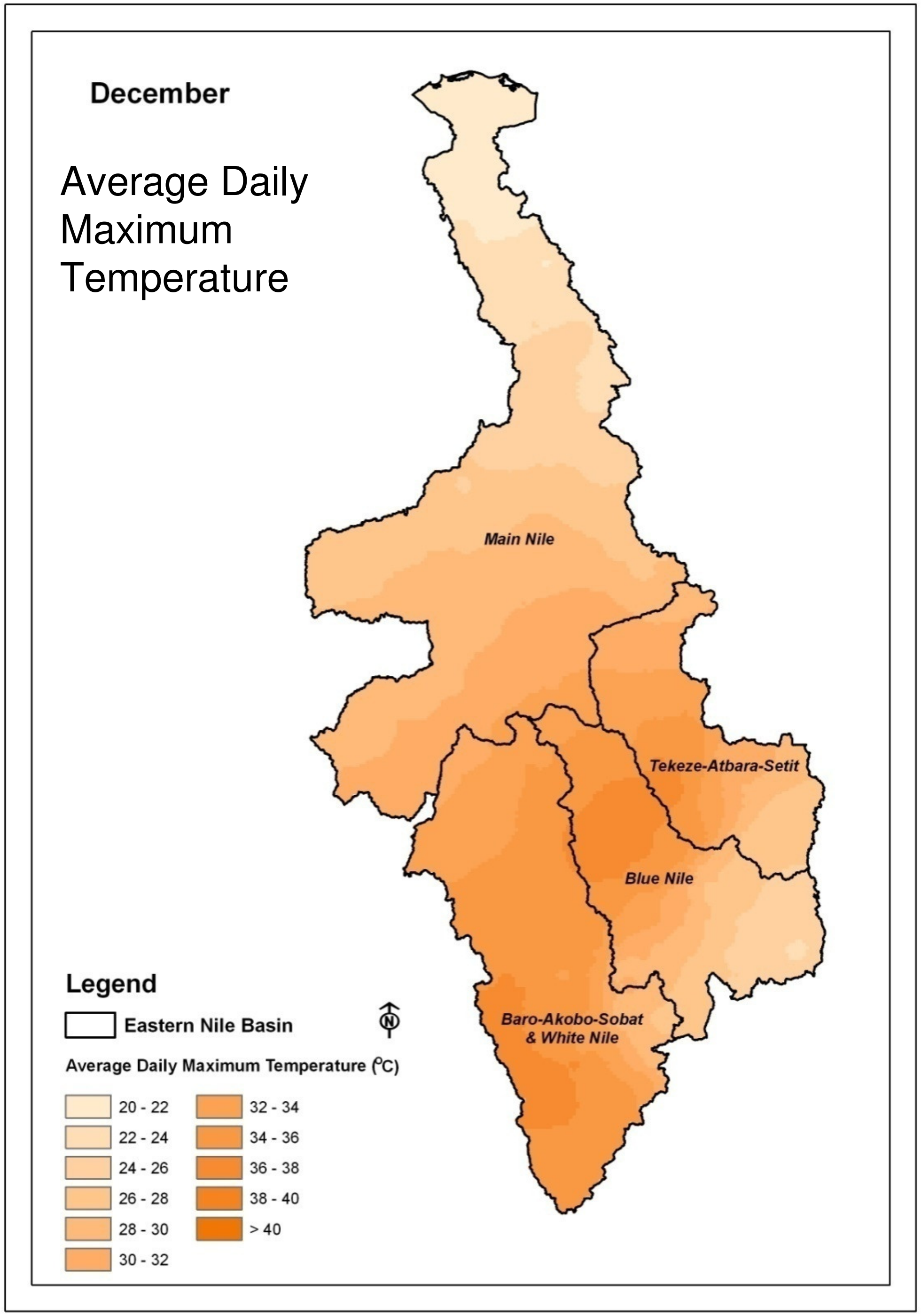

Average monthly Precipitation Baro-Akobo-Sobat and White Nile

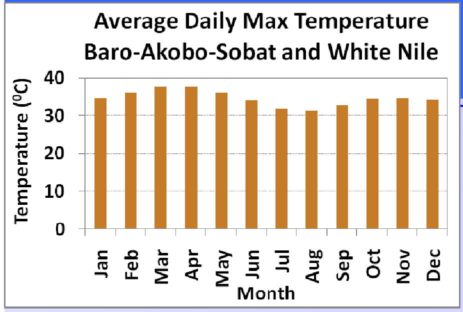

Average monthly Precipitation Blue Nile

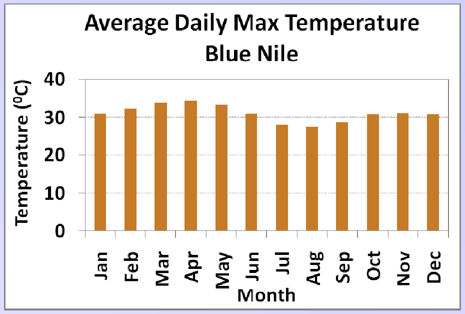

Average monthly Precipitation Main Nile

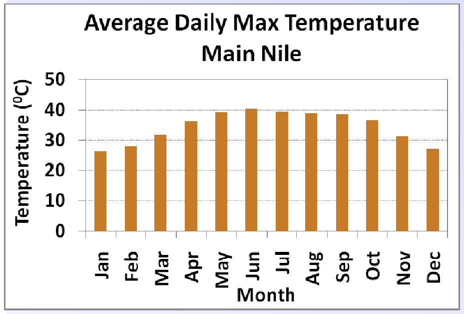

Average monthly Precipitation Tekeze-Atbara-Setit

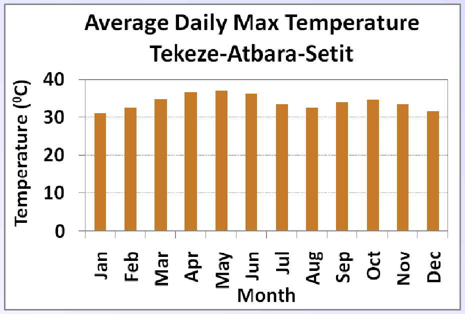
Temperature of EN




Average Daily Max Temperature Baro-Akobo-Sobat and White Nile

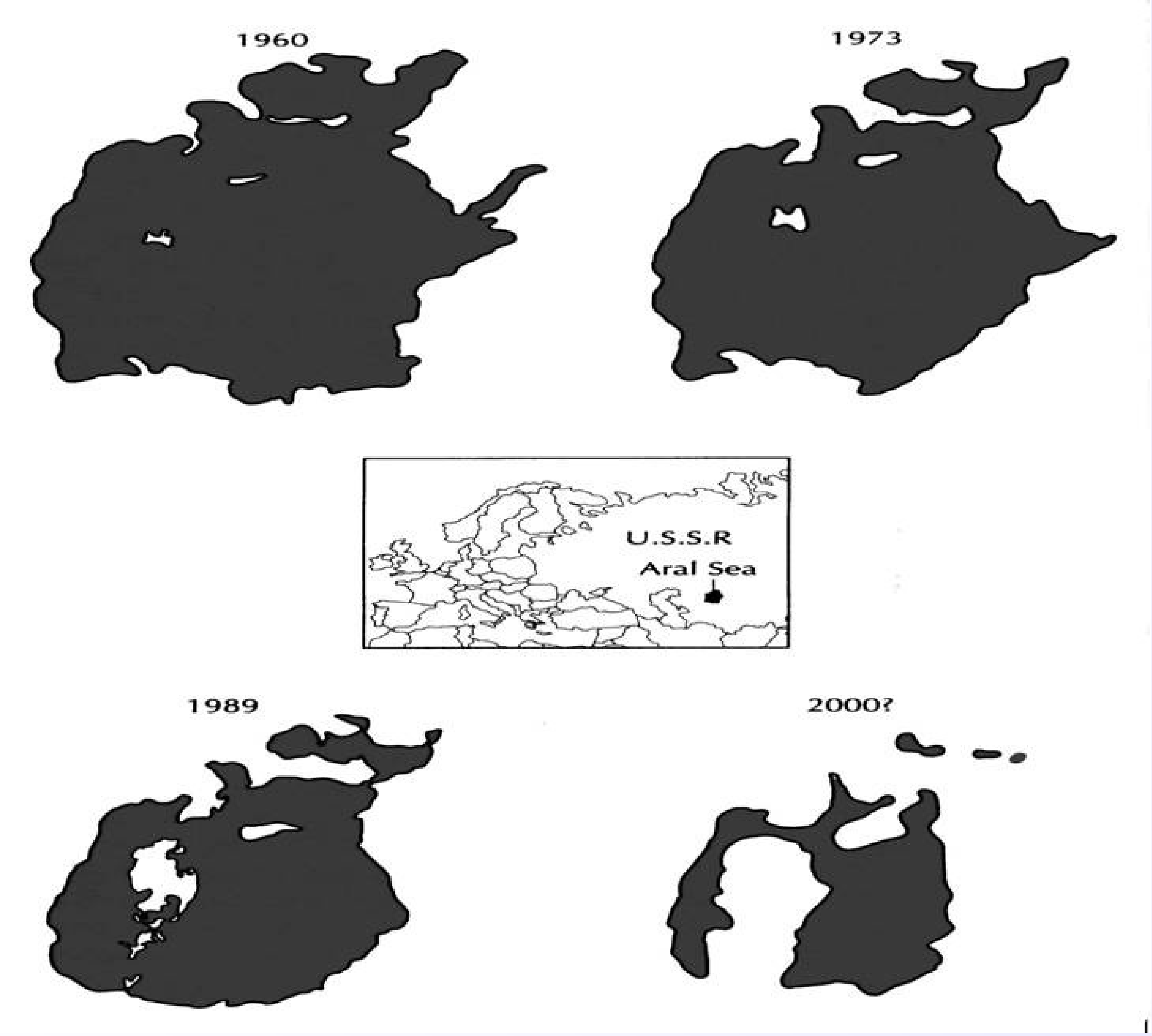

Average Daily Max Temperature Blue Nile


Average Daily Max Temperature Main Nile


Average Daily Max Temperature Tekeze-Atbara-Setit

2. Hydrograph analysis

Aral sea:



2. Hydrograph analysis

Example: Aral sea

2. Linear Systems response

Linear systems and basin response

- Continuity equation

$$\frac{dS}{dt} = I - Q$$

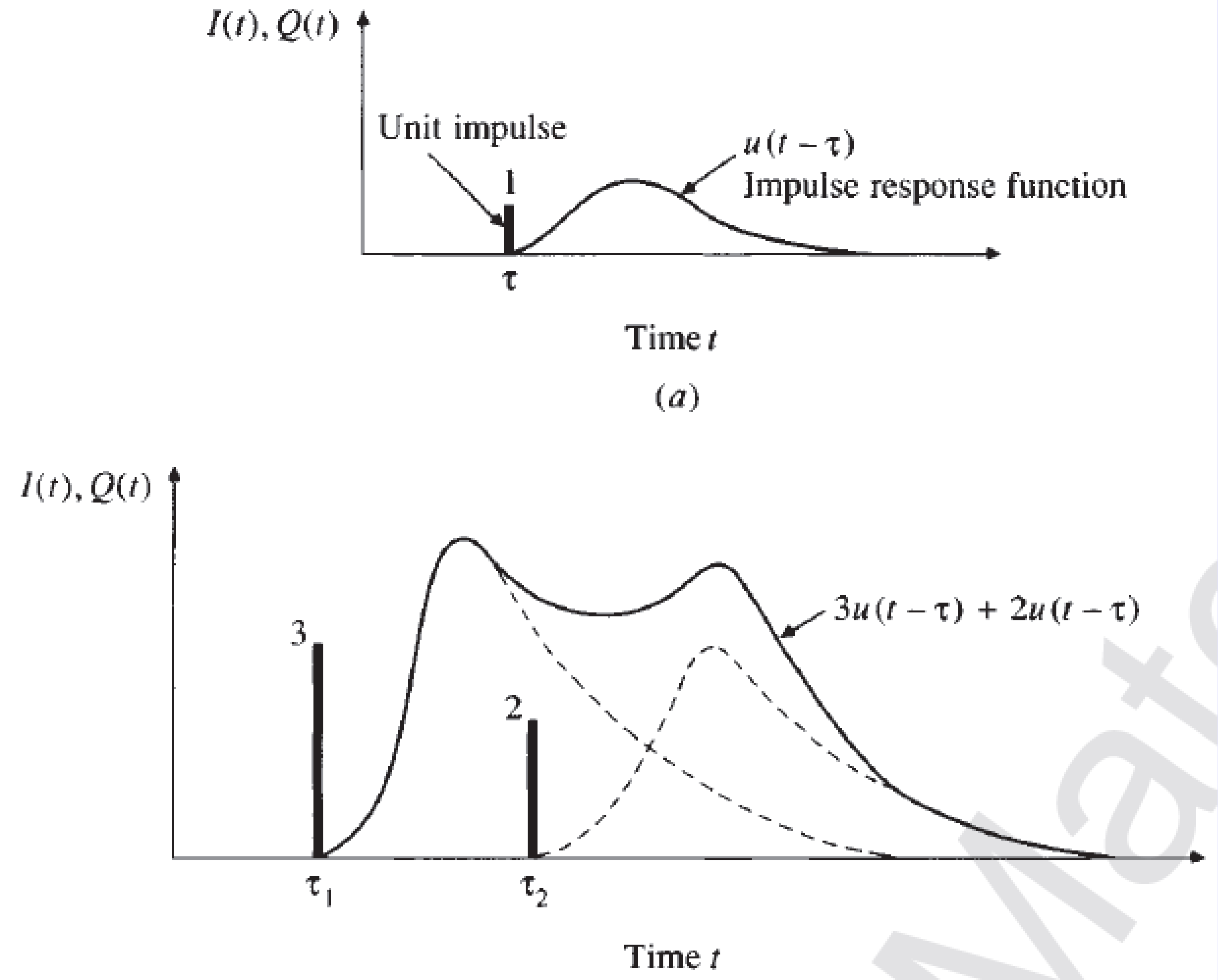
- General storage equation

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, \ldots, Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, \ldots\right)$$

- Convolution equation

$$Q(t) = \int_0^t I(\tau)u(t-\tau)\,d\tau$$

2. Linear systems response



2. Linear systems response

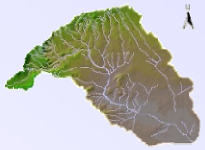
Linear systems and basin response

- Impulse response function
- Unit step response function

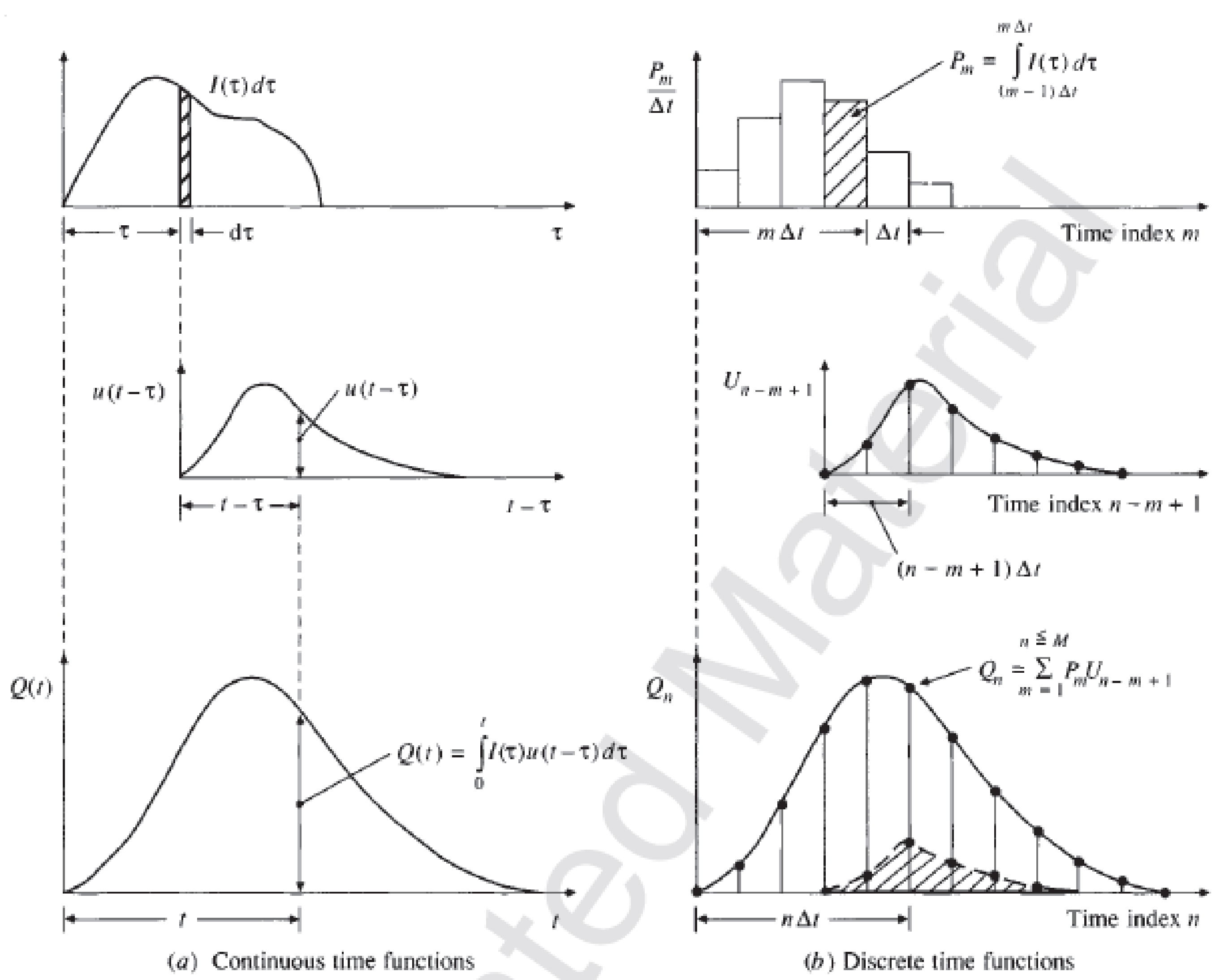
$$g(t) = \int_0^t u(l)\,dl$$

- Unit pulse response function

$$\begin{aligned} h(t) &= \frac{1}{\Delta t}[g(t) - g(t-\Delta t)] \\ &= \frac{1}{\Delta t}\left[\int_0^t u(l)\,dl - \int_0^{t-\Delta t} u(l)\,dl\right] \\ &= \frac{1}{\Delta t}\int_{t-\Delta t}^{t} u(l)\,dl \end{aligned}$$



2. Linear systems response



(a) Continuous time functions

(b) Discrete time functions

Fig. Continuous and discrete convolutions

2. Linear systems response

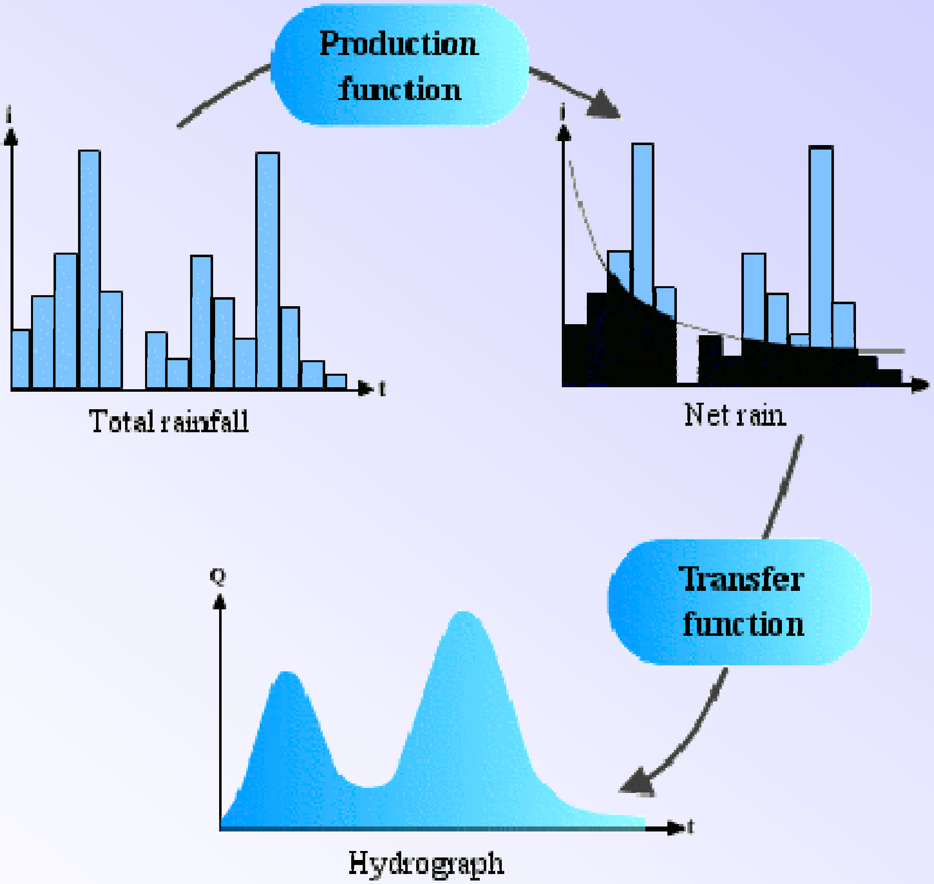


Figure 2.7 Transformation of total rain in hydrograph.

2. Unit Hydrograph

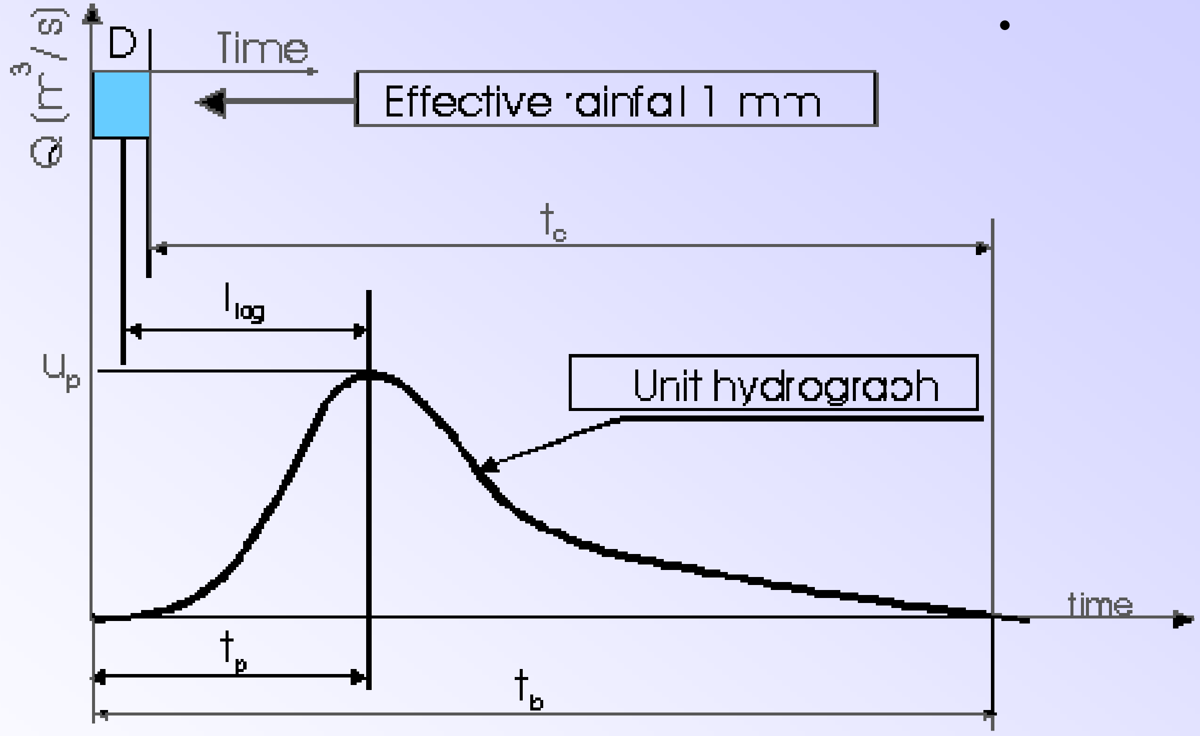


Figure 2.8 Characteristics of the unit hydrograph

2. Unit Hydrograph

Basic proposition of the unit hydrograph theory

(i) Principle of Time invariance
(ii) Principle of Linearity of response, and
(iii) Principle of Superposition

2. Unit Hydrograph

The following hypotheses are considered while the linear unit hydrograph is used as a transfer function:

- The unit hydrograph reflects the ensemble of the physical characteristics of the river basin.
- The effective rainfall is uniformly distributed over the catchment.
- The shape characteristics of the unit hydrograph are independent of time. Therefore duration of the unit hydrograph is constant regardless the effective rainfall intensity.

2. Unit Hydrograph

- The response of the catchment to effective rainfall is linear. For a certain reference duration of an effective rainfall having the amount h, the ordinate of the catchment hydrograph at the time t is h.u(t), where u(t) is the ordinate of the unit hydrograph at time t. This property is called proportionality.

2. Unit Hydrograph (cont’d)

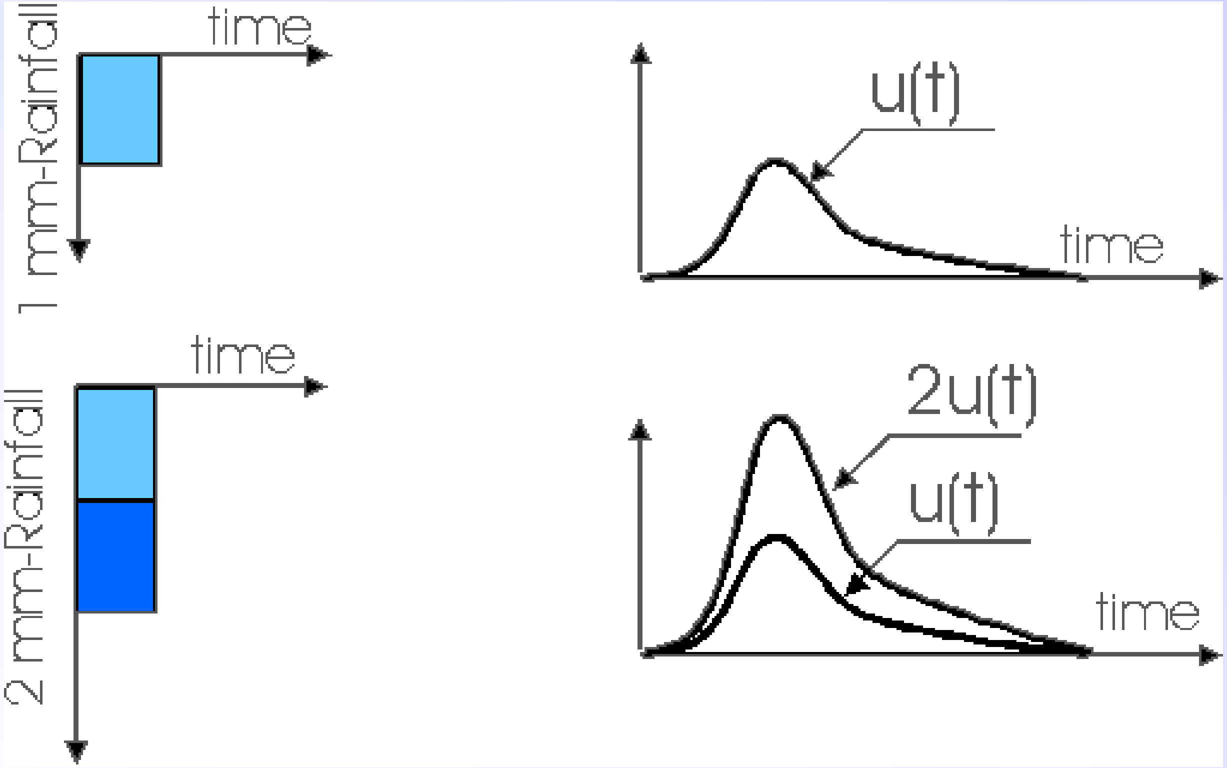


Figure 2.9 Linearity hypothesis of the unit hydrograph

2. Unit Hydrograph (cont'd)

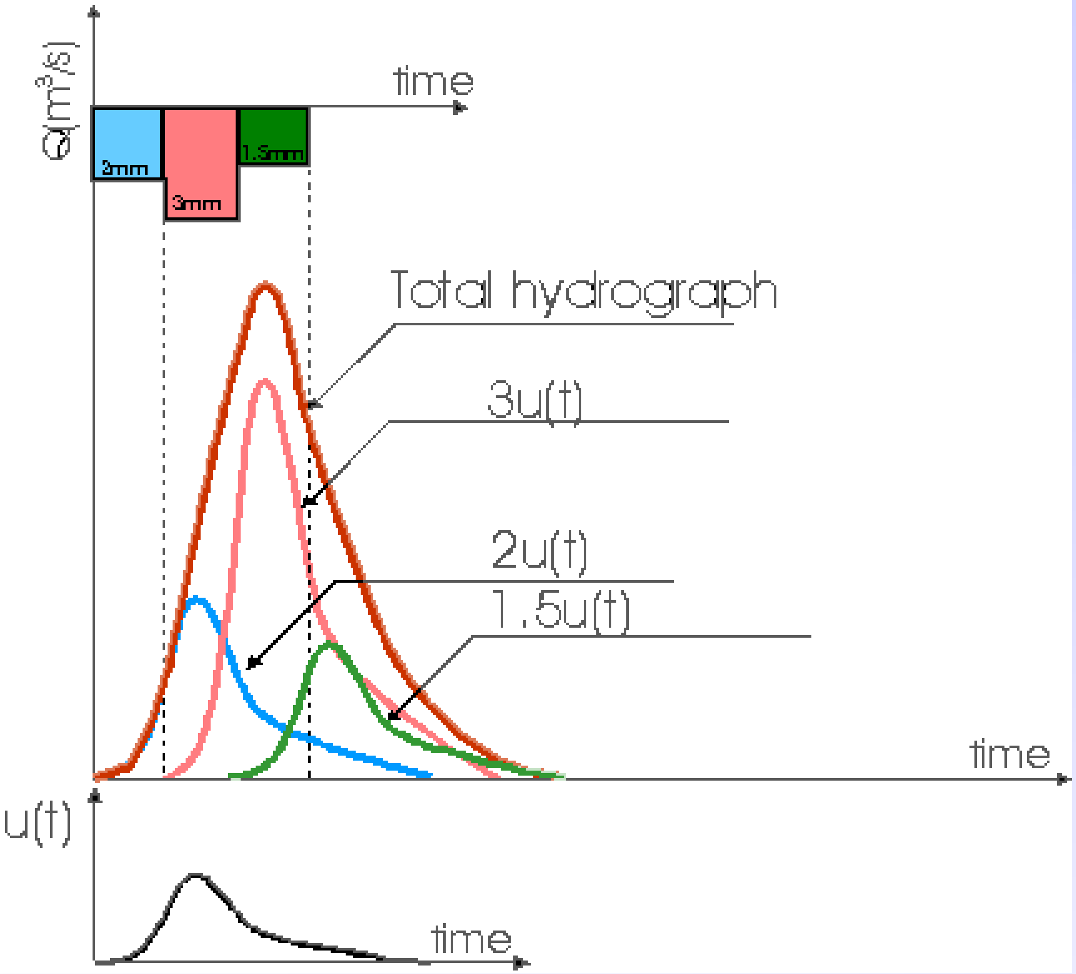


Figure 2.10 Principle of superposition

2. Unit Hydrograph (cont'd)

Limitations of UH

- Precipitation must be from rain only
- Catchment should not have unusually large storage
- If precipitation is non-uniform
- Non-proportional behavior of rivers- ER depends on the state of the catchment before storm
- Response vary per season
- Assumption that ER is produced uniformly in time and space

2. Unit Hydrograph (cont’d)

Derivation of a Unit Hydrograph from an isolated Storm hydrograph

1. Separation of the measured hydrograph into direct runoff hydrograph and baseflow.
2. Calculation of direct runoff volume by integrating the direct runoff hydrograph.
3. Calculation of direct runoff depth by dividing the direct runoff volume by the catchment area.
4. Calculation of unit hydrograph ordinates by dividing the ordinates of the direct runoff hydrograph by the direct runoff depth.
5. Estimation of the unit hydrograph duration.

2. Unit Hydrograph

Table 2.1 Application of the convolution method

Time	1τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	10τ
h1.u(t)	h1.u1	h1.u2	h1.u3	h1.u4	h1.u5	h1.u6	h1.u7	h1.u8		
h2.u(t)		h2.u1	h2.u2	h2.u3	h2.u4	h2.u5	h2.u6	h2.u7	h2.u8	
h3.u(t)			h3.u1	h3.u2	h3.u3	h3.u4	h3.u5	h3.u6	h3.u7	h3.u8
Total hydrograph ordinates	h1.u1	h1.u2 + h2.u1	h1.u3 + h2.u2 + h3.u1	h1.u4 + h2.u3 + h3.u2	h1.u5 + h2.u4 + h3.u3	h1.u6 + h2.u5 + h3.u4	h1.u7 + h2.u6 + h3.u5	h1.u8 + h2.u7 + h3.u6	h2.u8 + h3.u7	h3.u8

2. Unit Hydrograph

$$Q_i = \sum_{j=1}^{i} h_j \cdot u_{i-j+1}$$

$$Q_1 = h_1 \cdot u_1$$

$$Q_2 = h_1 \cdot u_2 + h_2 \cdot u_1$$

$$Q_3 = h_1 \cdot u_3 + h_2 \cdot u_2 + h_3 \cdot u_1$$

$$Q_4 = h_1 \cdot u_4 + h_2 \cdot u_3 + h_3 \cdot u_2 \quad (h_4 \cdot u_1 = 0$$

$$Q_5 = h_1 \cdot u_5 + h_2 \cdot u_4 + h_3 \cdot u_3$$

$$Q_6 = h_1 \cdot u_6 + h_2 \cdot u_5 + h_3 \cdot u_4$$

$$Q_7 = h_1 \cdot u_7 + h_2 \cdot u_6 + h_3 \cdot u_5$$

$$Q_8 = h_1 \cdot u_8 + h_2 \cdot u_7 + h_3 \cdot u_6$$

$$Q_9 = h_2 \cdot u_8 + h_3 \cdot u_7 \quad (h_1 \cdot u_9 = 0 \qquad u_9 = 0)$$

$$Q_{10} = h_3 \cdot u_8$$

The total no. of ordinates of the unit hydrograph is N-M+1, hence 10-3+1=8

2. Unit Hydrograph

Method of substitution forward:

$$u_1 = \frac{Q_1}{h_1}$$

$$u_2 = \frac{Q_2 - h_2 \cdot u_1}{h_1}$$

$$u_3 = \frac{Q_3 - h_3 \cdot u_1 - h_2 \cdot u_2}{h_1}$$

$$u_4 = \frac{Q_4 - h_2 \cdot u_3 - h_3 \cdot u_2}{h_1}$$

$$u_5 = \frac{Q_5 - h_2 \cdot u_4 - h_3 \cdot u_3}{h_1}$$

$$u_6 = \frac{Q_6 - h_2 \cdot u_5 - h_3 \cdot u_4}{h_1}$$

$$u_7 = \frac{Q_7 - h_2 \cdot u_6 - h_3 \cdot u_5}{h_1}$$

$$u_8 = \frac{Q_8 - h_2 \cdot u_7 - h_3 \cdot u_6}{h_1}$$

Method of substitution backward:

$$u_8 = \frac{Q_{10}}{h_3}$$

$$u_7 = \frac{Q_9 - h_2 \cdot u_8}{h_3}$$

$$u_6 = \frac{Q_8 - h_2 \cdot u_7 - h_1 \cdot u_8}{h_3}$$

$$u_5 = \frac{Q_7 - h_2 \cdot u_6 - h_1 \cdot u_7}{h_3}$$

$$u_4 = \frac{Q_6 - h_2 \cdot u_5 - h_1 \cdot u_6}{h_3}$$

$$u_3 = \frac{Q_5 - h_2 \cdot u_4 - h_1 \cdot u_5}{h_3}$$

$$u_2 = \frac{Q_4 - h_2 \cdot u_3 - h_1 \cdot u_4}{h_3}$$

$$u_1 = \frac{Q_3 - h_2 \cdot u_2 - h_1 \cdot u_3}{h_3}$$

2. Unit Hydrograph

$$\begin{bmatrix} h_1 & 0 & 0 & \ldots 0 & 0\ldots & 0 & 0 \\ h_2 & h_1 & 0 & \ldots 0 & 0\ldots & 0 & 0 \\ \ldots & & & & & & \\ h_M & h_{M-1} & h_{M-2} & \ldots h_1 & 0\ldots & 0 & 0 \\ 0 & h_M & h_{M-1} & \ldots h_2 & h_1\ldots & 0 & 0 \\ \ldots & & & & & & \\ 0 & 0 & 0 & \ldots 0 & 0\ldots & h_M & h_{M-1} \\ 0 & 0 & 0 & \ldots 0 & 0\ldots & 0 & h_M \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \ldots \\ \ldots \\ u_{N-M+1} \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ \ldots \\ Q_M \\ Q_{M+1} \\ \ldots \\ Q_{N-1} \\ Q_N \end{bmatrix}$$

$$[h] \times [u] = [Q]$$

$$[h]^T.[Q] = [h]^T.[h].[u]$$

$$[u] = ([h]^T.[h])^{-1}.[h]^T.[Q]$$

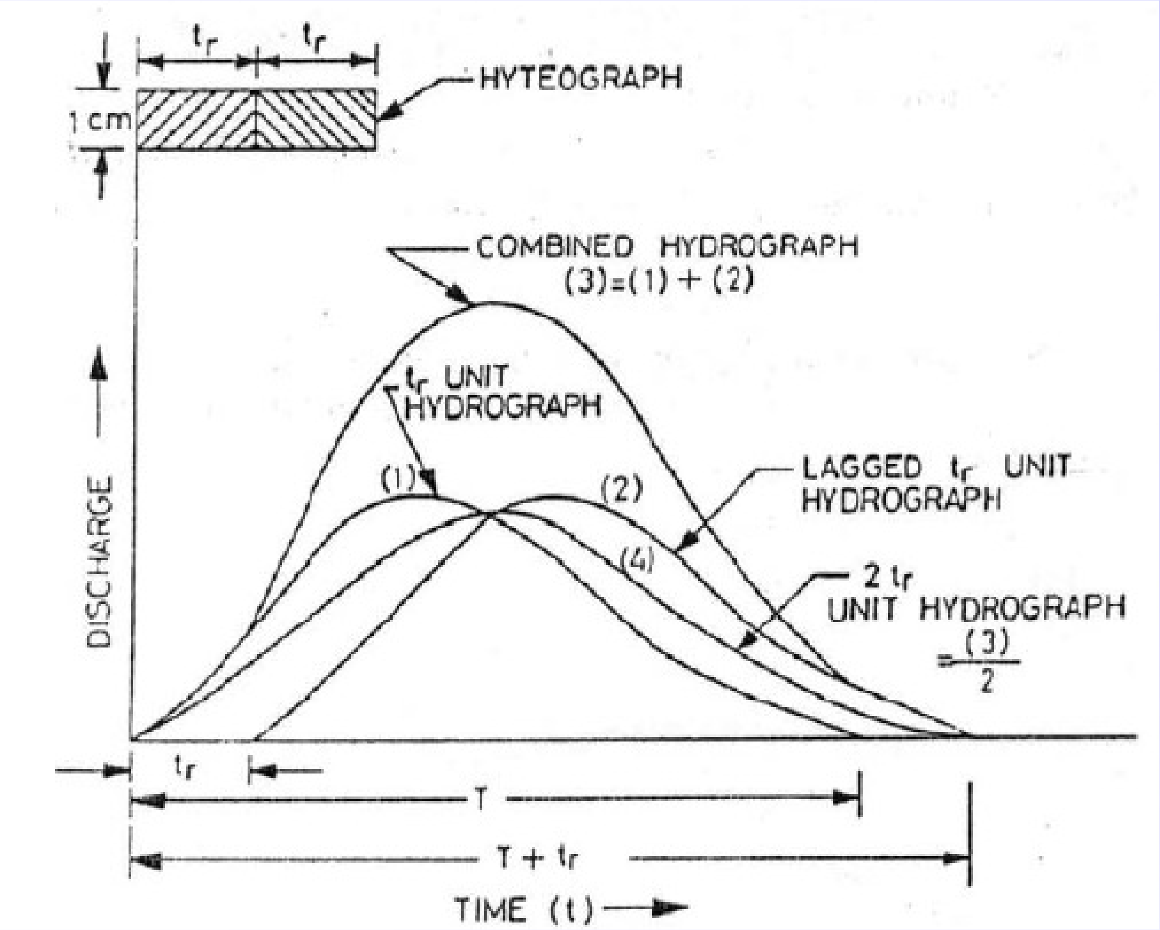
2. Unit Hydrograph

Change in Unit Hydrograph Duration

(1) the superposition method and

(2) the S-hydrograph method

2. Unit Hydrograph

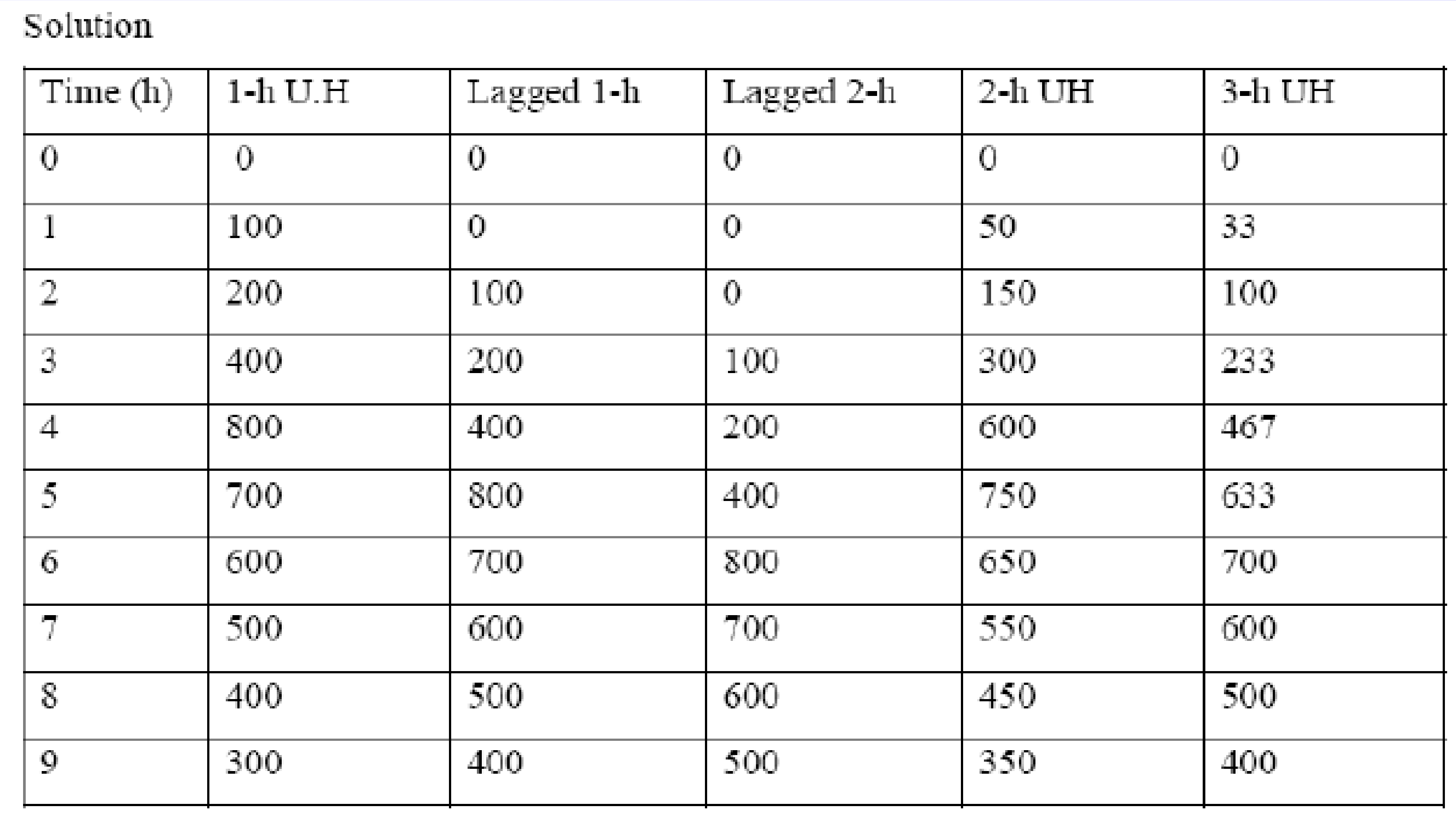


2. Unit Hydrograph

Example Use the superposition method to calculate the 2-h and 3-h unit hydrograph of a catchment, based on the following1-h unit hydrograph.

Time (h)	0	1	2	3	4	5	6	7	8	9	10	11	12
Flow (m3/s)	0	100	200	400	800	700	600	500	400	300	200	100	0

2. Unit Hydrograph



Solution

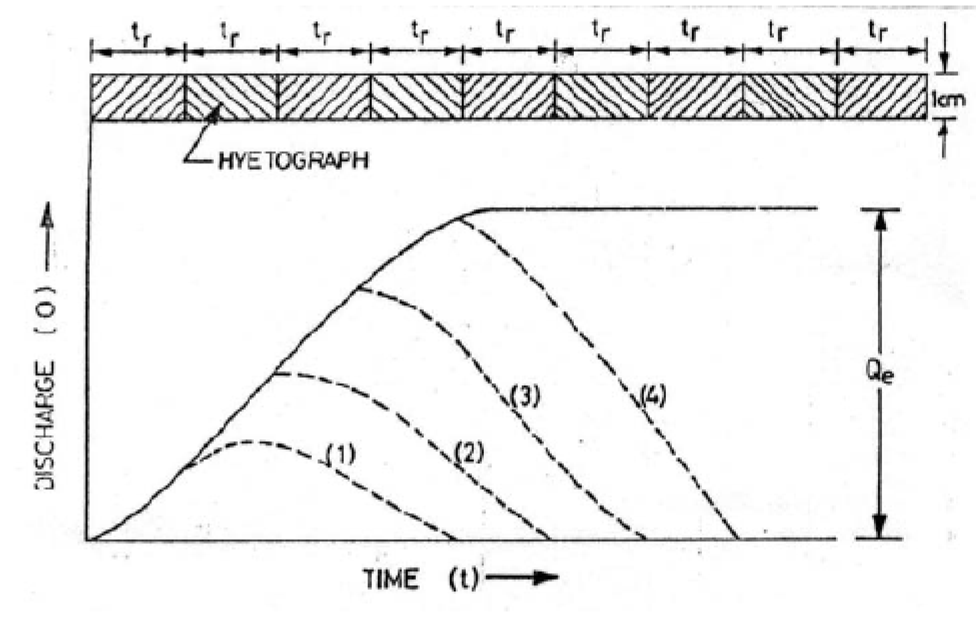
Time (h)	1-h U.H	Lagged 1-h	Lagged 2-h	2-h UH	3-h UH
0	0	0	0	0	0
1	100	0	0	50	33
2	200	100	0	150	100
3	400	200	100	300	233
4	800	400	200	600	467
5	700	800	400	750	633
6	600	700	800	650	700
7	500	600	700	550	600
8	400	500	600	450	500
9	300	400	500	350	400

2. Unit Hydrograph

S-hydrograph:

S-curve is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of X-hr U.H's spaced X-hr apart.

2. Unit Hydrograph



$$Q_e = \frac{2.78A}{t_r} \quad cumecs$$

2. Unit Hydrograph

S-hydrograph:

1 Determine the X-hour S-hydrograph. The X-hour S-hydrograph is derived by accumulating the unit hydrograph ordinates at intervals equal to X.
2 Lag the X-hour S hydrograph by a time interval equal to Y-hours.
3 Subtract ordinates of the two pervious S- hydrographs.
4 Multiply the resulting hydrograph ordinates by X/Y to obtain the Y-hour unit hydrograph.

2. Unit Hydrograph

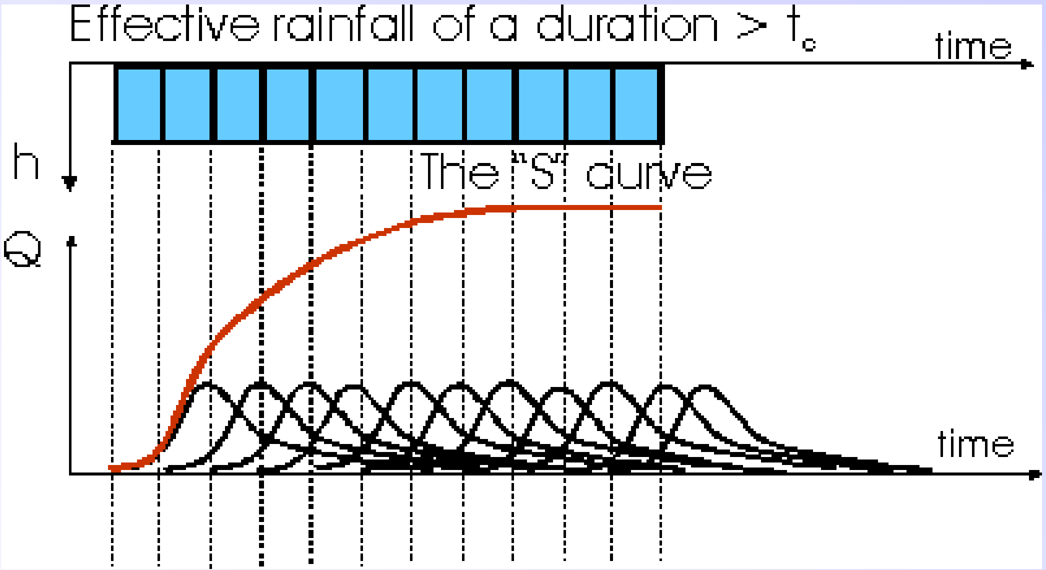


Figure 2.11. The construction of the "S" curve

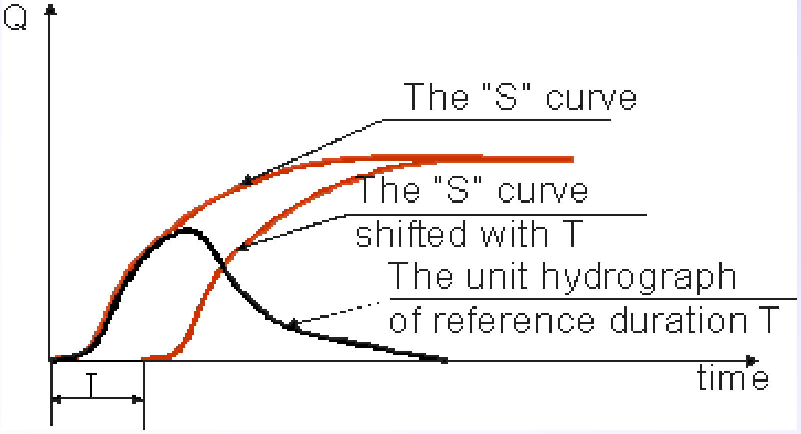
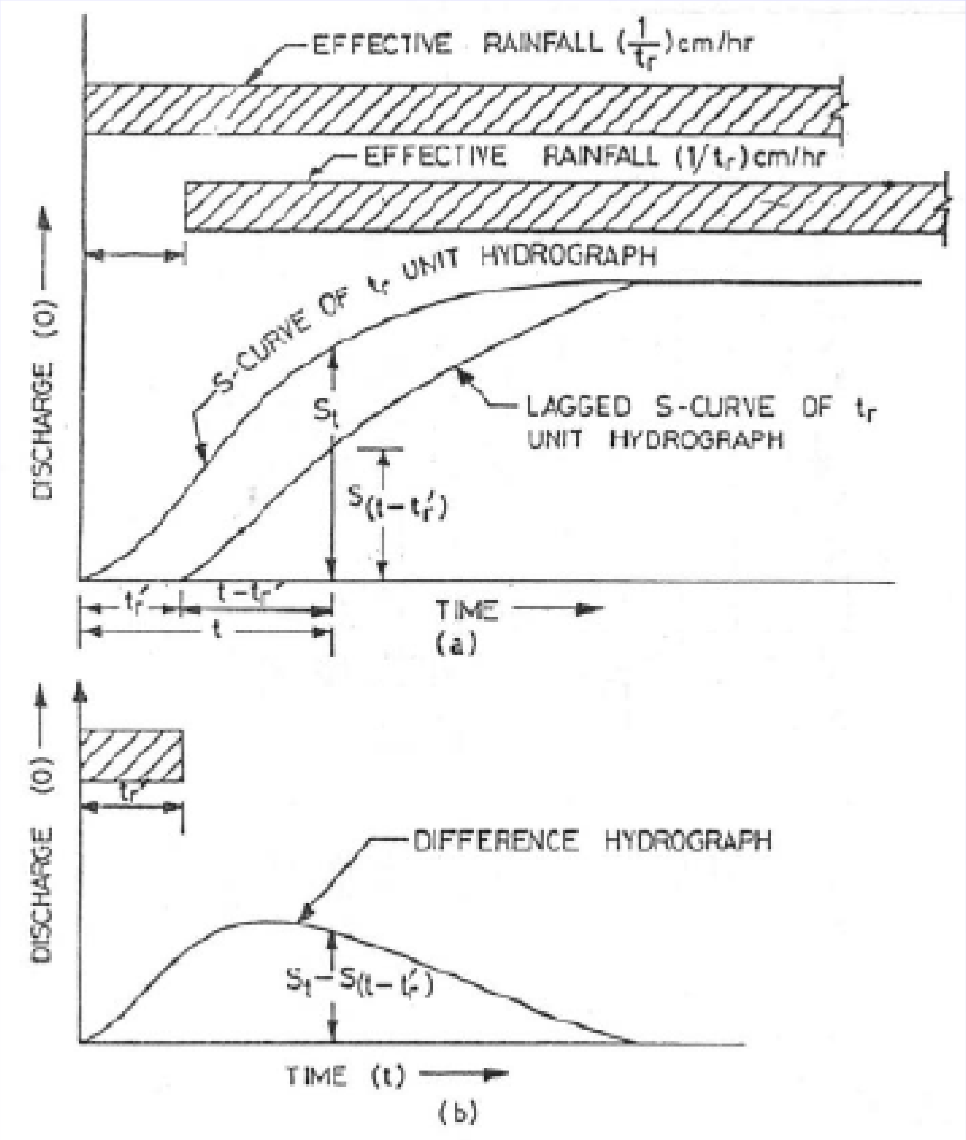


Figure 2.12 Unit hydrograph of reference duration

2. Unit Hydrograph

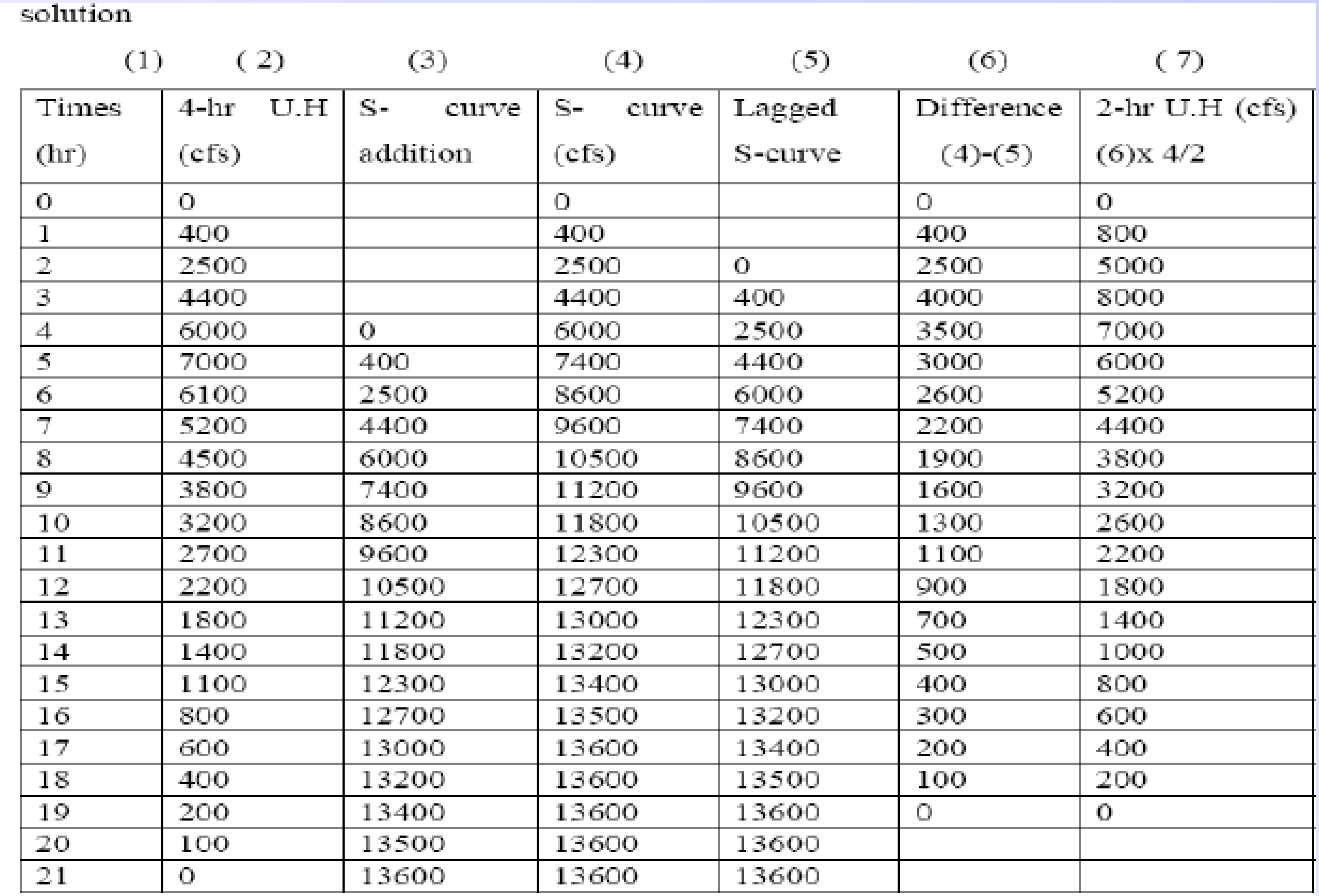


2. Unit Hydrograph

Example Given below is the 4-hr U.H for a basin of 84 sq.km. Derive S-curve and find the 2-hr unit hydrograph.

Time (hr)	Flow, cfs	Time (hr)	Flow ,cfs	Time (hr)	Flow, cfs
0	0	8	4500	15	1100
1	400	9	3800	16	800
2	2500	10	3200	17	600
3	4400	11	2700	18	400
4	6000	12	2200	19	200
5	7000	13	1800	20	100
6	6100	14	1400	21	0
7	5200	15	1100		

2. Unit Hydrograph



solution

(1) Times (hr)	(2) 4-hr U.H (cfs)	(3) S-curve addition	(4) S-curve (cfs)	(5) Lagged S-curve	(6) Difference (4)-(5)	(7) 2-hr U.H (cfs) (6)x 4/2
0	0		0		0	0
1	400		400		400	800
2	2500		2500	0	2500	5000
3	4400		4400	400	4000	8000
4	6000	0	6000	2500	3500	7000
5	7000	400	7400	4400	3000	6000
6	6100	2500	8600	6000	2600	5200
7	5200	4400	9600	7400	2200	4400
8	4500	6000	10500	8600	1900	3800
9	3800	7400	11200	9600	1600	3200
10	3200	8600	11800	10500	1300	2600
11	2700	9600	12300	11200	1100	2200
12	2200	10500	12700	11800	900	1800
13	1800	11200	13000	12300	700	1400
14	1400	11800	13200	12700	500	1000
15	1100	12300	13400	13000	400	800
16	800	12700	13500	13200	300	600
17	600	13000	13600	13400	200	400
18	400	13200	13600	13500	100	200
19	200	13400	13600	13600	0	0
20	100	13500	13600	13600		
21	0	13600	13600	13600		

2. Synthetic unit hydrograph

Snyder's Synthetic unit hydrograph

Snyder standardised the unit hydrograph as one those rainfall duration *tr* is related to the catchment lag time *tlag* by

$$t_r = \frac{t_p}{5.5} hrs$$

For a standard unit hydrograph the following relations have been derived:

$$t_p = c_t (L.L_c)^{0.3} \quad hrs$$

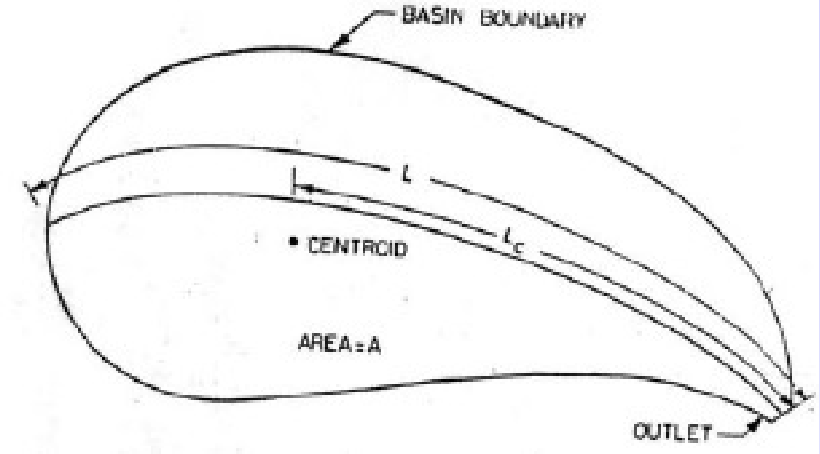
Where

L = distance from gaging station (outlet) to catchment boundary (divide) along the main stream (km)

Lc = distance from gaging station to centroid of catchment area, measured along the main stream to the nearest point(km)

Ct = a coefficient depending on units and drainage characteristics.

2. Synthetic unit hydrograph



The peak discharge Q_p (cumec) for the unit hydrograph is

$$Q_p = \frac{2.78 c_p A}{t_p}$$

2. Synthetic unit hydrograph

$$W_{50} = \frac{5.9}{(q_p)^{1.08}}$$

$$W_{75} = \frac{W_{50}}{1.75} = \frac{3.4}{(q_p)^{1.08}}$$

where q_p' is the peak discharge per unit area (cumecs/km2) and W_{50} and W_{75} are in hours.

The duration of surface runoff, or the time base of unit hydrograph is

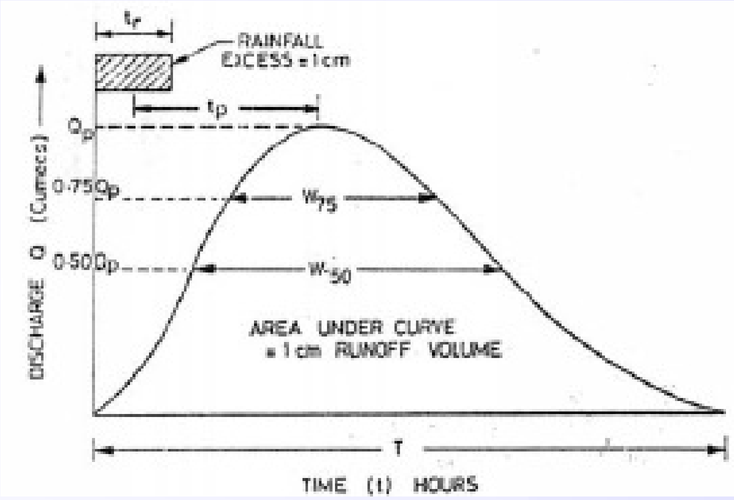
$$T = 3 + 3\left(\frac{t_p}{24}\right) \ days$$

For any other duration tr′, a modified basin lag time tp′ is

$$t_p' = t_p + \frac{t_r' - t_r}{4} \ hrs$$

Where tp′ = basin lag for a storm duration tr′.

2. Synthetic unit hydrograph



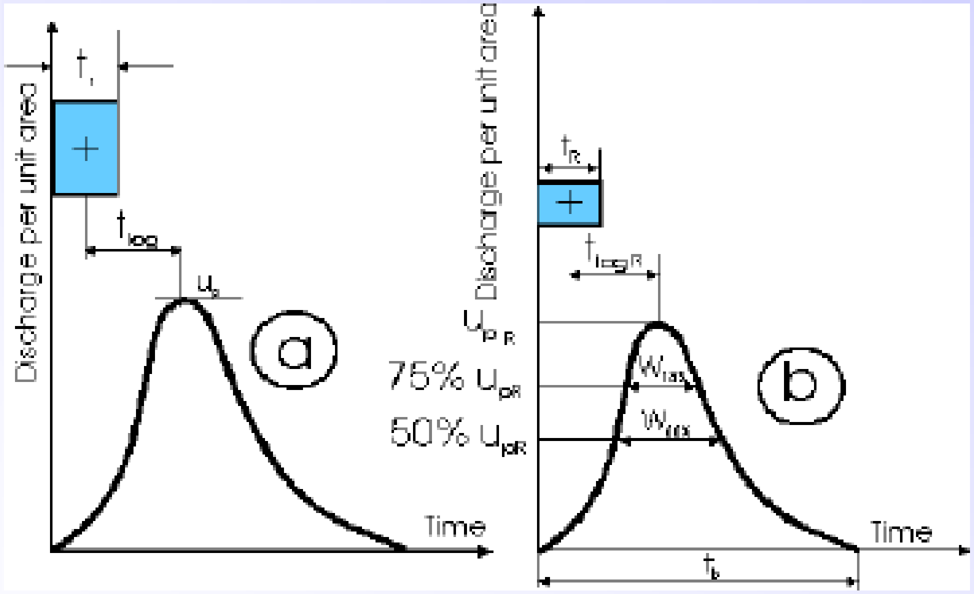


Figure . Snyder's unit hydrograph; (a) Standard unit hydrograph (tlag= 5.5 tr)
(b) Required unit hydrograph (tlag ≠ 5.5 tr).
(Chow et al., 1988)

2. Synthetic unit hydrograph

Example: Derive a 3-hr unit hydrograph for an ungaged basin from the following data.
Length L =32 km; length Lc =25 km; Area of catchment =325 km^2 Assume Ct =0.9 and Cp =1.8

Solution

$Tp = Ct\,(LLc)^{0.3}$

$= 0.9\,(32 \times 25)^{0.3} = 6.7$ hrs

$tr = tp/5.5 = 1.2$ hrs

As tr is not equal to the desired unit duration tr′, we have to calculate the value of tp′.

2. Synthetic unit hydrograph

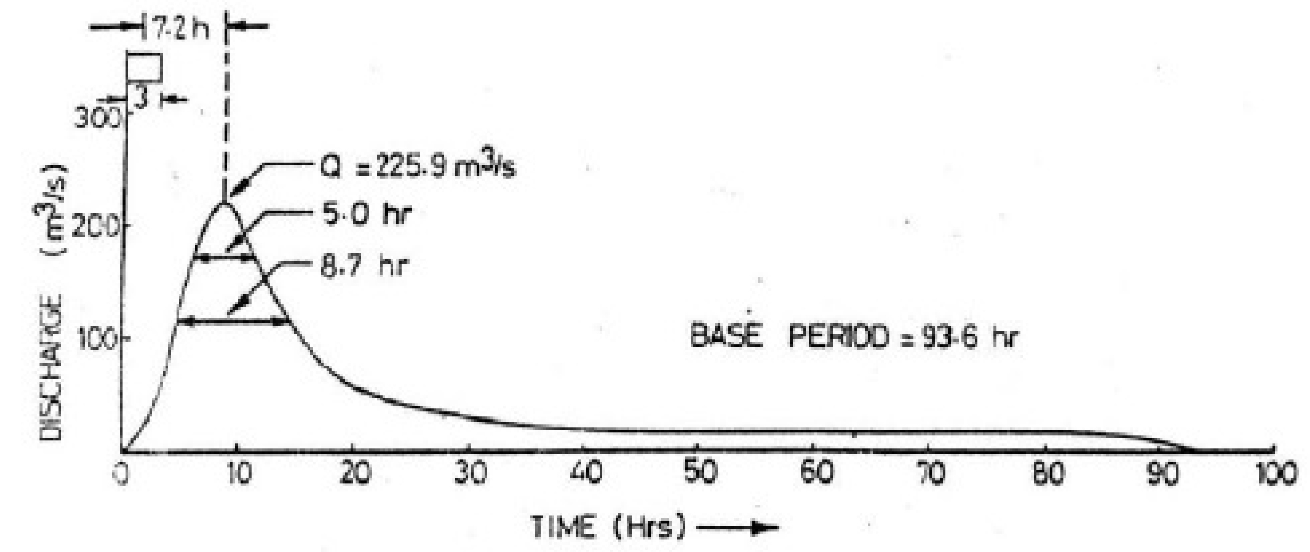
$$t'_p = t_p + \left(\frac{t'_r - t_r}{4}\right) = 6.7 + \left(\frac{3-1.2}{4}\right) = 7.2\ hrs$$

$$Q'_p = \frac{2.78 C_p A}{t'_p} = \frac{278x1.8x325}{7.2} = 225.9\ cumec$$

$$q'_p = \frac{225.9}{325} = 0.695\ cumec/km^2$$

$$T' = 3 + 3\left(\frac{t'_p}{24}\right) = 3 + 3\left(\frac{7.2}{24}\right) = 3.9\ days = 93.6\ hrs$$

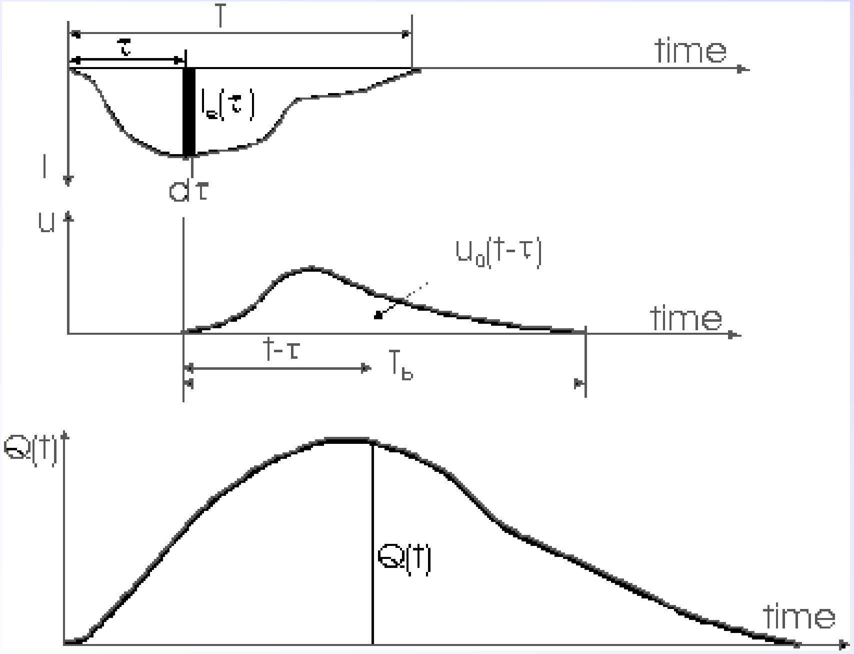
$$W_{50} = \frac{5.9}{(q'_p)^{1.08}} = 8.7\ hrs\ ; W_{75} = \frac{3.4}{(q'_p)^{1.08}} = 5.0\ hrs$$



2. Instantaneous unit hydrogra

The instantaneous unit hydrograph (IUH)

The instantaneous unit hydrograph is defined as a unit hydrograph having an infinitesimal reference duration (in other words the duration tends towards zero).



$$Q(t) = \int_0^t I_e(\tau) \cdot u(t-\tau) \cdot d\tau$$

Figure 2.13 Convolution of the intensity function $I_e(t)$ with the instantaneous unit hydrograph $u_0(t)$ giving the discharge hydrograph Q(t)

2. Instantaneous unit hydrogra

The ordinates of the instantaneous hydrograph u_0 *corresponds* to a duration $\tau \to 0$

$$\tau \to 0 \Rightarrow \frac{S(t) - S(t-\tau)}{\tau} \to \frac{dS(t)}{dt}$$

$$u_0(t) = \frac{dS(t)}{dt}$$

The boundaries of integration are:

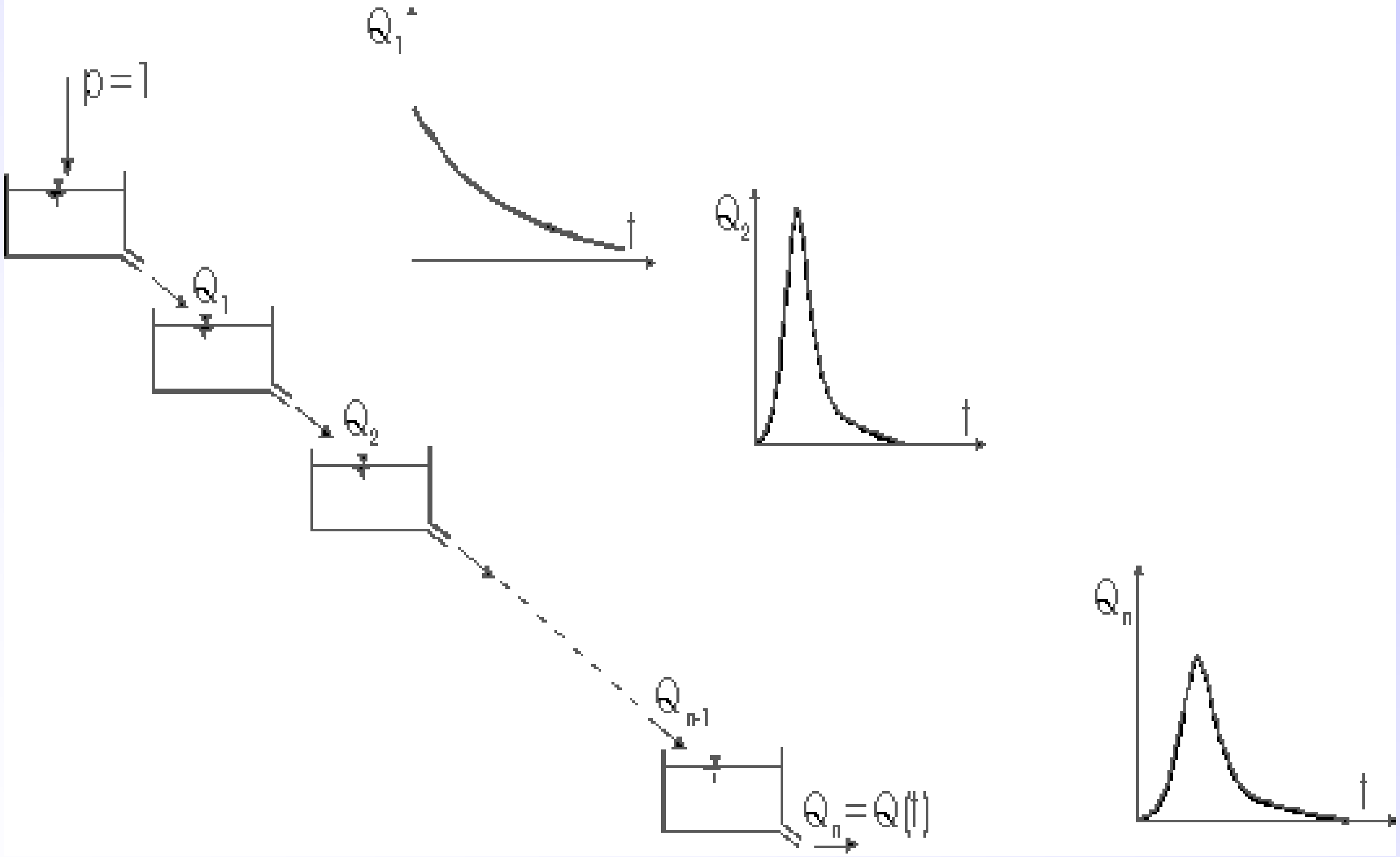
τ should not exceed the total duration of effective rainfall, that is $0 \le \tau \le T$

t - τ should not exceed the base time of the unit hydrograph T_b , that is $0 \le t - \tau \le T_b$

hence the following condition will be satisfied: $0 \le t \le T_b - T$

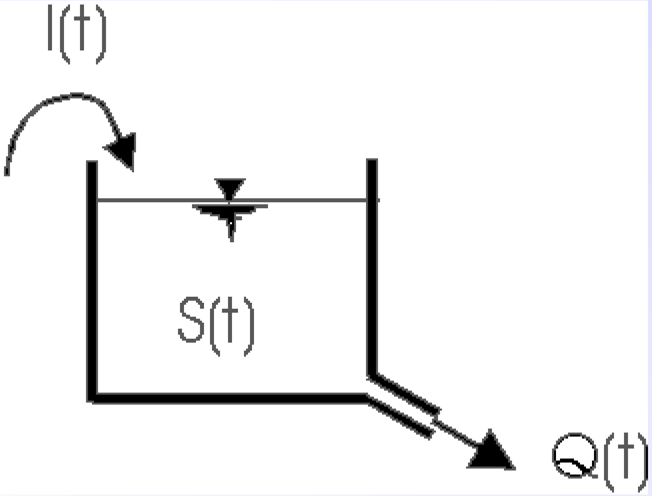
2. Conceptual models

The instantaneous unit hydrograph of Nash



2. Conceptual models

Instantaneous unit hydrograph resulting from conceptual models



Continuity law: $$\frac{dS}{dt} = I(t) - Q(t)$$

Storage law: $$S(t) = K \cdot Q(t)$$

$$K \cdot \frac{dQ}{dt} + Q = i$$

2. Conceptual models

The solution of the above equation

$$Q = \frac{1}{k} e^{-t/k} \int e^{t/k} . I dt$$

For the first reservoir

$$Q_1 = \frac{1}{k} e^{-t/k} \qquad for\ I = 0, t \neq 0$$

Thus the output from the first reservoir becomes the input in the second one and so on.

2. Conceptual models

Again the convolution equation allows for determining the output from the second reservoir:

$$Q_2 = \frac{1}{k} e^{-t/k} \int e^{t/k} . \frac{1}{k} e^{-t/k} dt$$

$$Q_2 = \frac{1}{k^2} e^{-t/k} .t$$

$$Q_3 = \frac{1}{2k} e^{-t/k} . \left(\frac{t}{k} \right)^2$$

$$Q_n = \frac{1}{k} .e^{-t/k} . \left(\frac{t}{k} \right)^{n-1} . \frac{1}{(n-1)!}$$

2. Conceptual models

Reiterating the procedure for *n reservoirs the general* expression of the ordinates of the Nash instantaneous unit hydrograph is obtained as:

$$Q(t) = \frac{1}{K.\Gamma(n)} . \left(\frac{t}{K}\right)^{(n-1)} .e^{\left(\frac{-t}{K}\right)}$$

where *Γ(n) is the gamma function defined as:*

$$\Gamma(n) = \int_0^\infty x^{n-1}.e^{-x}.dx \qquad \forall n \in \Re$$

with the following iterative relation:

$$\Gamma(n+1) = n.\Gamma(n)$$

2. Conceptual models

The increase time t_p *and the peak discharge* Q_p *are* derived by solving the equation *dQ/dt = 0, as follows:*

$$t_p = K(n-1)$$

and:

$$Q_p = \frac{1}{K.\Gamma(n)}.(n-1)^{(n-1)}.e^{(1-n)}$$

Finally, the ordinates of the instantaneous unit hydrograph might be expressed as a function of the increase time and the peak discharge, as given below:

$$Q(t) = Q_p.\left(\frac{t}{t_p}\right).e^{(1-n)\frac{t}{t_p}}.e^{n-1}$$

2. Conceptual models

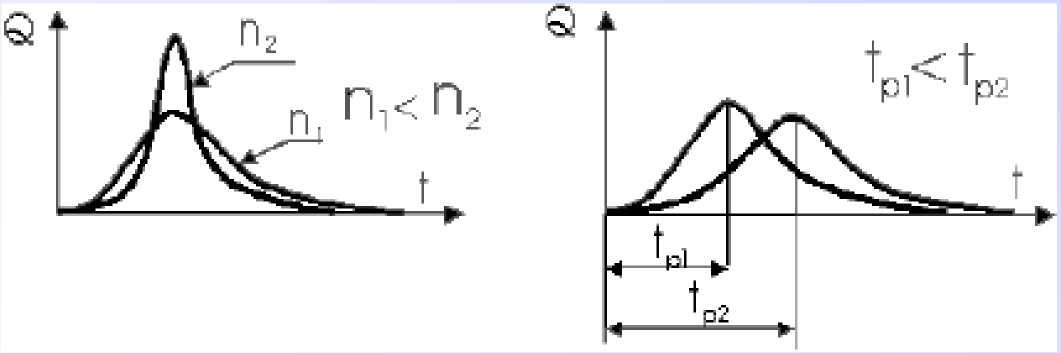


Figure . Sensitivity of the Nash model to the parameters n and K(tp)

$$nk = M_{1Q} - M_{1h}$$

$$n(n+1)k^2 + 2nk.M_{1h} = M_{2Q} - M_{2h}$$

2. Conceptual models

$$M_m = \frac{\int_A f(x)dx.x^m}{\int f(x)dx}$$

$$M_m = \frac{\int_0^\infty u(t)dt.t^m}{\int_0^\infty u(t)dt}$$

$$M_m = \frac{1}{k\Gamma(n)}\int_0^\infty e^{-t/k}.(\frac{t}{k})^{n-1}t^m dt$$

$$M_m = \frac{k^m}{\Gamma(n)}\int e^{-t/k}.(\frac{t}{k})^{n-1}(\frac{t}{k})^{m.}d(\frac{t}{k})$$

$$M_m = \frac{k^m\Gamma(m+n)}{\Gamma(n)}$$

$$M_1 = nk$$

$$M_2 = n(n+1)k^2$$

2. Conceptual models

Exercise: Determine the peak of the instantaneous unit hydrograph (IUH) using Nash model for a watershed having a drainage area of 36 square kilometers, assuming abstractions of 0.5 cm/hr and a constant base flow of 5m^3/s. Use the following data:

6 hour period	1^1	2	3	4	5	6	7	8	9	10
Rainfall in cm/hr	1.5	3.5	2.5	1.5						
Stream flow, m^3/s	15	75	170	185	147	84	43	18	8	

[*Hint: Peak of IUH is at a time of t when* $\frac{du(0,t)}{dt} = 0.$]

3. Frequency Analysis in Hydrology

- Hydrological processes-chance and time
- Data types
  - Peak annual discharge
  - Annual flood series
  - Partial duration series
    - POT
    - M highest peaks (if M=N, annual exceedences)

3. Frequency Analysis in Hydrology

- Assumptions
  - Independence
  - Stationary with time
  - Population parameters from sample

Data requirement

- Relevant
- Adequate
- Accurate

3. Frequency Analysis in Hydrology

- Descriptive statistics

$$F(x) = P(X \le x) = \sum_{i} P(x_i)$$

$$F(x_1) = P(-\infty \le x \le x_1) = \int_{-\infty}^{x^1} f(x)dx$$

$$P(x_1 \le x \le x_2) = F(x_2) - F(x_1)$$

Conditions for pdf to meet

$$f(x) \ge 0$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

3. Frequency Analysis in Hydrology

- **Moments of a Distribution**

n$^{\text{th}}$moment

$$\mu_N' = \sum_{-\infty}^{\infty} x_i^N P(x_i)$$

$$\mu_N' = \int_{-\infty}^{\infty} x^N f(x)dx$$

First Moment about the Origin

$$E(x) = \mu = \sum x_i P(x_i)$$

Discrete

$$E(x) = \mu = \int_{-\infty}^{\infty} x f(x) dx$$

Continuous

3. Frequency Analysis in Hydrology

- **Var(x) = Variance**
  Second moment about mean

$$Var(x) = \sigma^2 = \sum_{-\infty}^{\infty} (x_i - \mu)^2 P(x_i)$$

$$Var(x) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$cv = \frac{\sigma}{\mu} \quad = \text{ Coeff. of Variation}$$

3. Frequency Analysis in Hydrology

Exercise 1: The probability density function of a random variable is given by

$$f(x) = kx \qquad \text{for} \qquad 0 \leq x \leq 9$$

Evaluate k and find the mean, standard deviation and skewness coefficient.

Exercise 2: Consider the following pdf

$$f(x) = \frac{1}{5} e^{-x/5} \qquad x > 0$$

a. Derive the cdf
b. What is the probability that x lies between 3 and 5
c. Determine ‘x’ such that P[X < x] = 0.5
d. Determine ‘x’ such that P[X > x] = 0.75

3. Frequency Analysis in Hydrology

The probability of at least one success in n years,where the probability of success in any year is 1/T,is called the RISK.

- Prob success = $p = 1/T$ *and* Prob failure = $1-p$
- $RISK = 1 - P(0)$

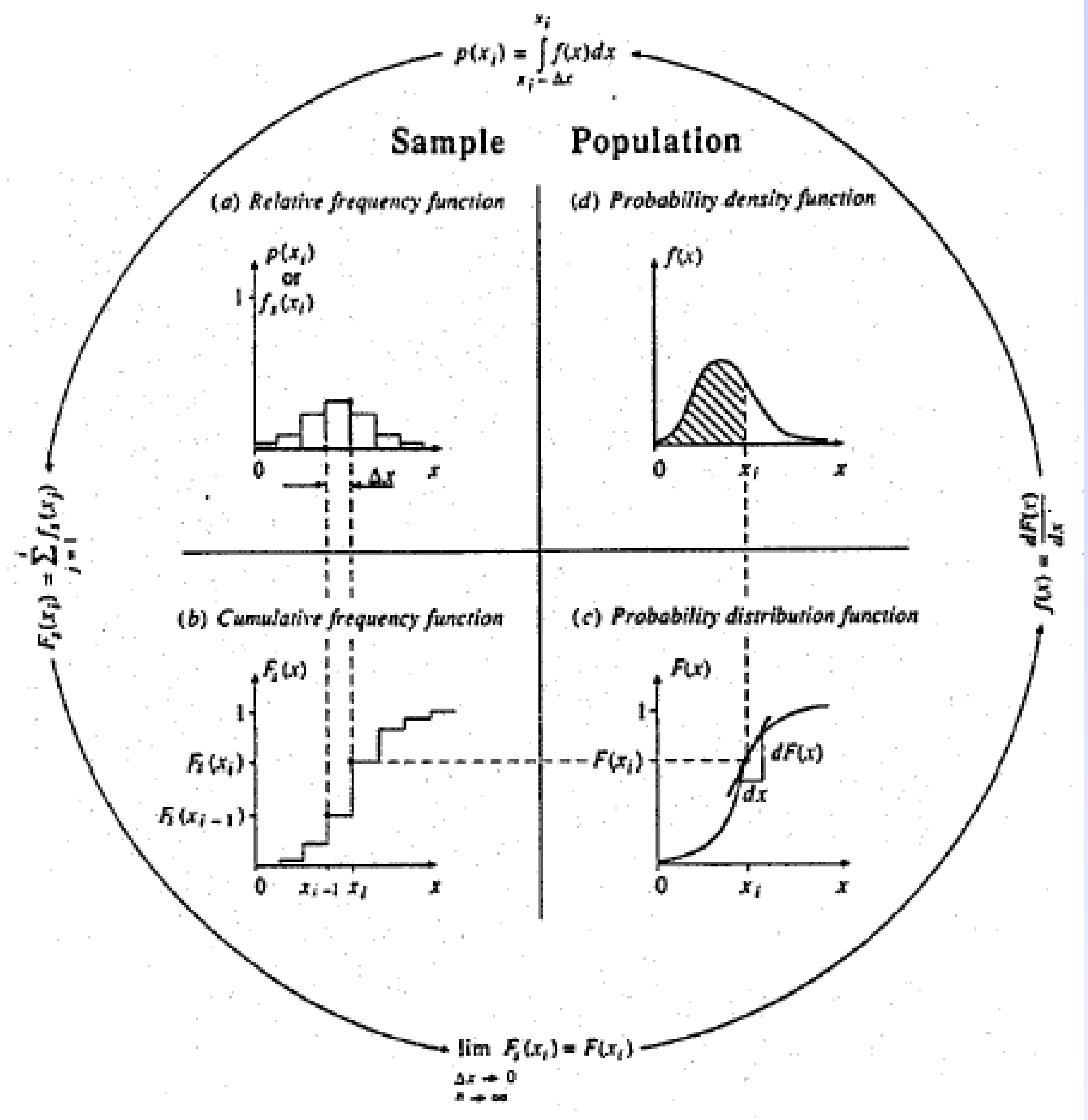
$$= 1 - Prob(\text{no success in } n \text{ years})$$

$$= 1 - (1-p)^n$$

$$= 1 - (1 - 1/T)^n$$

- $Reliability = (1 - 1/T)^n$

3. Frequency Analysis in Hydrology



3. Frequency Analysis in Hydrology

- Procedures
  - Select distribution & estimate parameters
  - Choose a distribution
    - X^2 test
    - Kolmogrov-Simirnov test
    - Coefficient of Skewness and Kurtosis
    - Moment ratio tests
  - Use selected distribution to estimate T year event

Distribution	Parameter est.
Normal	Least squares
Gamma	MOM
Pearson type III	ML
Exponential	Maximum entropy
Pareto	PWM
Logistic	
EV I, II, III, GEV	
Wakeby	
Kappa	

3. Frequency Analysis in Hydrology

- Normal distribution
- Probability density function

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

- Symmetric about the mean
- Varies over a continuous range
- Applicable - such as annual precipitation
- Limited use since most hydrologic variables are skewed.

3. Frequency Analysis in Hydrology

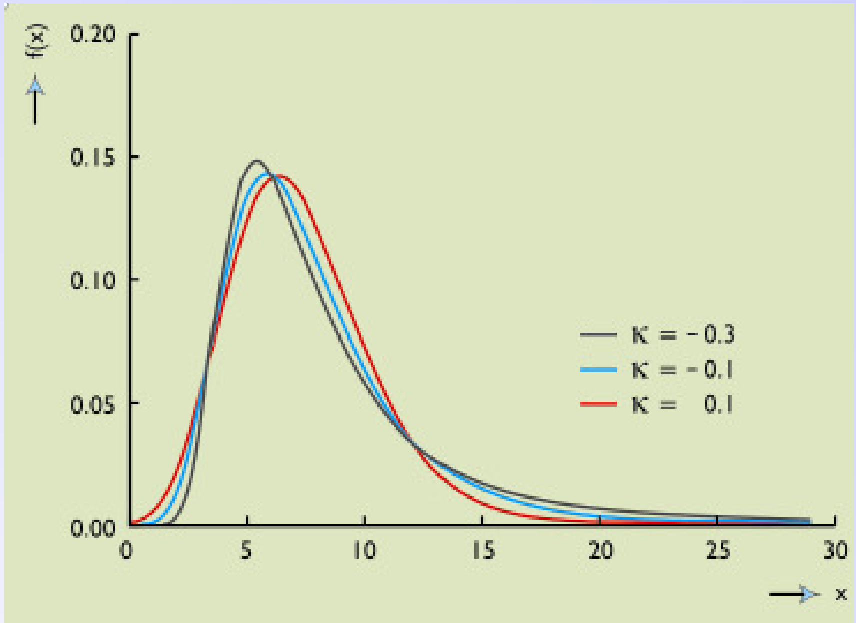
- Extreme values: largest and smallest
- Limiting distributions: EV-I, EV-II and EV-III

$$F(x) = \exp\left[-\left(1 - k\frac{x-u}{\alpha}\right)^{1/k}\right]$$

  - K=0, EV-I (Gumbel)

$$F(x) = \exp\left[-\exp\left(-\frac{x-u}{\alpha}\right)\right]$$

  - K<0, EV-II (Frechet)
  - K>0, EV-III (Weibull)



3. Frequency Analysis in Hydrology

- EV-I (Gumbel) distributions
- Maximum type events

$$f(x) = \frac{1}{\alpha}\exp\left[-\frac{x-\mu}{\alpha} - \exp\left(-\frac{x-\mu}{\alpha}\right)\right]$$

$$F(x) = \exp\left[-\exp\left(-\frac{x-u}{\alpha}\right)\right] \qquad \alpha = \frac{\sqrt{6}s}{\pi} \qquad u = \bar{x} - 0.5772\alpha$$

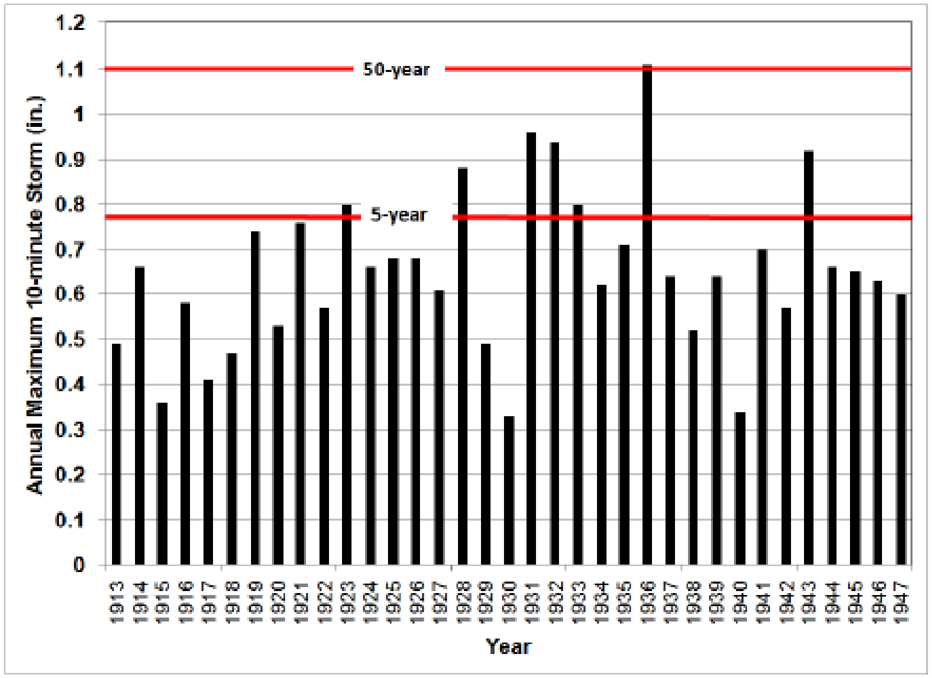
- Reduced variate

$$y = \frac{x-u}{\alpha} \qquad \Longrightarrow \qquad F(x) = \exp[-\exp(-y)]$$

$$y = -\ln\left[\ln\left(\frac{1}{F(x)}\right)\right] \Longrightarrow \begin{array}{l} p = \Pr(X \geq x_T) = \dfrac{1}{T} \\ \dfrac{1}{T} = 1 - F(x_T) \end{array}$$

3. Frequency Analysis in Hydrology

- Example
- Given annual maxima for 10-minute storms, find 5 and 50 year return period 10 minute storms.



$$\bar{x} = 0.649\,in \qquad s = 0.177\,in$$

$$y_5 = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right]$$

$$= -\ln\left[\ln\left(\frac{5}{5-1}\right)\right] = 1.5$$

$$\alpha = \frac{\sqrt{6}s}{\pi} = \frac{\sqrt{6}*0.177}{\pi} = 0.138$$

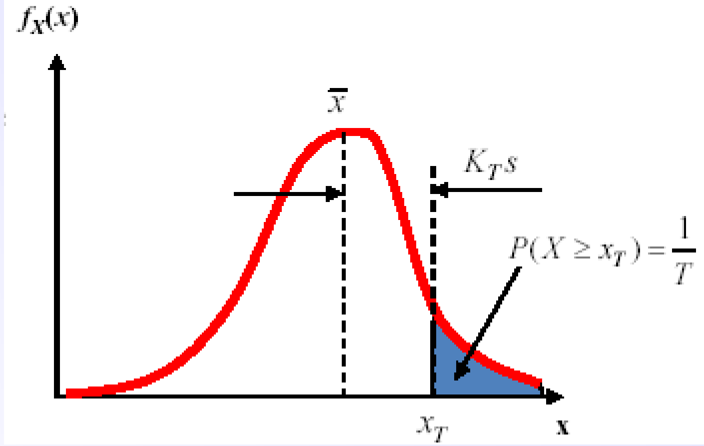
$$u = \bar{x} - 0.5772\alpha = 0.649 - 0.5772*0.138 = 0.569$$

$$x_5 = u + \alpha y_5 = 0.569 + 0.138*1.5 = 0.78\,in \qquad x_{50} = 1.11\,in$$

3. Frequency Analysis in Hydrology

- Frequency factors
- Once a distribution has been selected and parameters estimated, then how do we use it?
- Chow proposed using $\boxed{x_T = \bar{x} + K_T s}$
- Where

x_T = Estimated event magnitude
K_T = Frequency factor
T = Return period
$\bar{x}$ = Sample mean
s = Sample standard deviation



3. Frequency Analysis in Hydrology

- Normal distribution
- Probability density function

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$K_T = \frac{x_T - \overline{x}}{s} = z_T$$

- The frequency factor is the standard normal variate

$$x_T = \overline{x} + K_T s = \overline{x} + z_T s$$

- Given the mean and standard deviation, e.g. the 50 year return period annual precipitation at a place can be computed

$$T = 50;\ p = \frac{1}{50} = 0.02;\ K_{50} = z_{50} = 2.054$$

3. Frequency Analysis in Hydrology

- EV-I (Gumbel) distributions

$$x_T = u + \alpha y_T$$

$$= \bar{x} - 0.5772\frac{\sqrt{6}}{\pi}s + \frac{\sqrt{6}}{\pi}s\left\{-\ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

$$= \bar{x} - \frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}s$$

$$x_T = \bar{x} + K_T s$$

$$\alpha = \frac{\sqrt{6}}{\pi}s$$

$$u = \bar{x} - 0.5772\alpha$$

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right]$$

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

3. Frequency Analysis in Hydrology

- Gamma distribution
- Gamma distribution is distribution of sum of n IID exponential variables

$$f_X(x) = \frac{\lambda^{\beta} x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)}$$

- Used to model many natural phenomenon including streamflow.
- Has lower bound of 0
- 2 special cases:
  - Exponential $\beta = 1$
  - Chi-squared $\lambda = \frac{1}{2};\ 2\beta$

3. Frequency Analysis in Hydrology

- Generalizations of Gamma distribution

- Consider a random variable x, subtract a constant e from x.
- If (x-e) has a Gamma distribution, then x has a Pearson type 3 distribution (3 parameter Gamma distribution)
- If Ln(x-e) has a Gamma distribution, then x has a Log-Pearson type 3 distribution

3. Frequency Analysis in Hydrology

- Log Pearson Type III distribution

- The pdf is:

$$f(x)=\frac{\lambda^{B}(y-\epsilon)^{B-1}e^{-\lambda(y-\epsilon)}}{x\Gamma_{B}}$$

- Similar to Gamma distribution, introduces lower bound ϵ
- Common for annual maximum floods
- Event magnitudes are calculated as

$$y_T=\bar{y}+K_T s_y$$

$\bar{y}=$ mean of logs of x (ln *or* $\log_{10}$)

$s_y=$ standard deviation of logs of x

- K_T=frequency factor (quantiles with p=1/T) of an LP III distribution with skewness coefficient Cs.

3. Frequency Analysis in Hydrology

- Exponential distribution
- Probability density function and distribution function

$$f(x)=\frac{1}{\alpha}e^{-\left(\frac{x-\varepsilon}{\alpha}\right)}$$

$$f(x)=\lambda e^{-\lambda x}$$

$$F(x)=1-e^{-\left(\frac{x-\varepsilon}{\alpha}\right)}$$

- Using MOM

$$\alpha=\sigma$$

$$\varepsilon=\mu-\alpha$$

- Describes events that occur instantaneously and independently along a time horizon
- λ is the mean rate of occurrence of events
- time intervals between major precipitation events, etc

3. Frequency Analysis in Hydrology

- Parameter estimation techniques

- Method of Moments
- Method of Maximum likelihood
- Probability weighted Moment

3. Frequency Analysis in Hydrology

- Method of Moments

- common
- parameters of a probability distribution function are obtained by equating the moments of the sample with the moments of the probability distribution function.
- higher order moment estimates are biased (Wallis, et. al. 1974).

3. Frequency Analysis in Hydrology

- Method of Maximum likelihood

- involves the choice of parameter estimates that produce a maximum probability of occurrence of the observations.
- The most efficient parameter estimates
- The parameter estimates that maximize the likelihood function are computed.

$$L = \prod_{i=1}^{n} f(x_i)$$

Log-likelihood function

$$\ln L = \sum_{i=1}^{n} \ln\left[f(x_i)\right]$$

3. Frequency Analysis in Hydrology

Distribution	Maximum likelihood	
	Severity	Duration
Exponential	-3.9487	-288.757
Gamma	-3.7586	-272.183
Weibull	-3.5463	-274.465
Lognormal	-12.3910	-271.105
Log-logistic	-37.7698	-278.621

Table 2. MLE of severity & duration

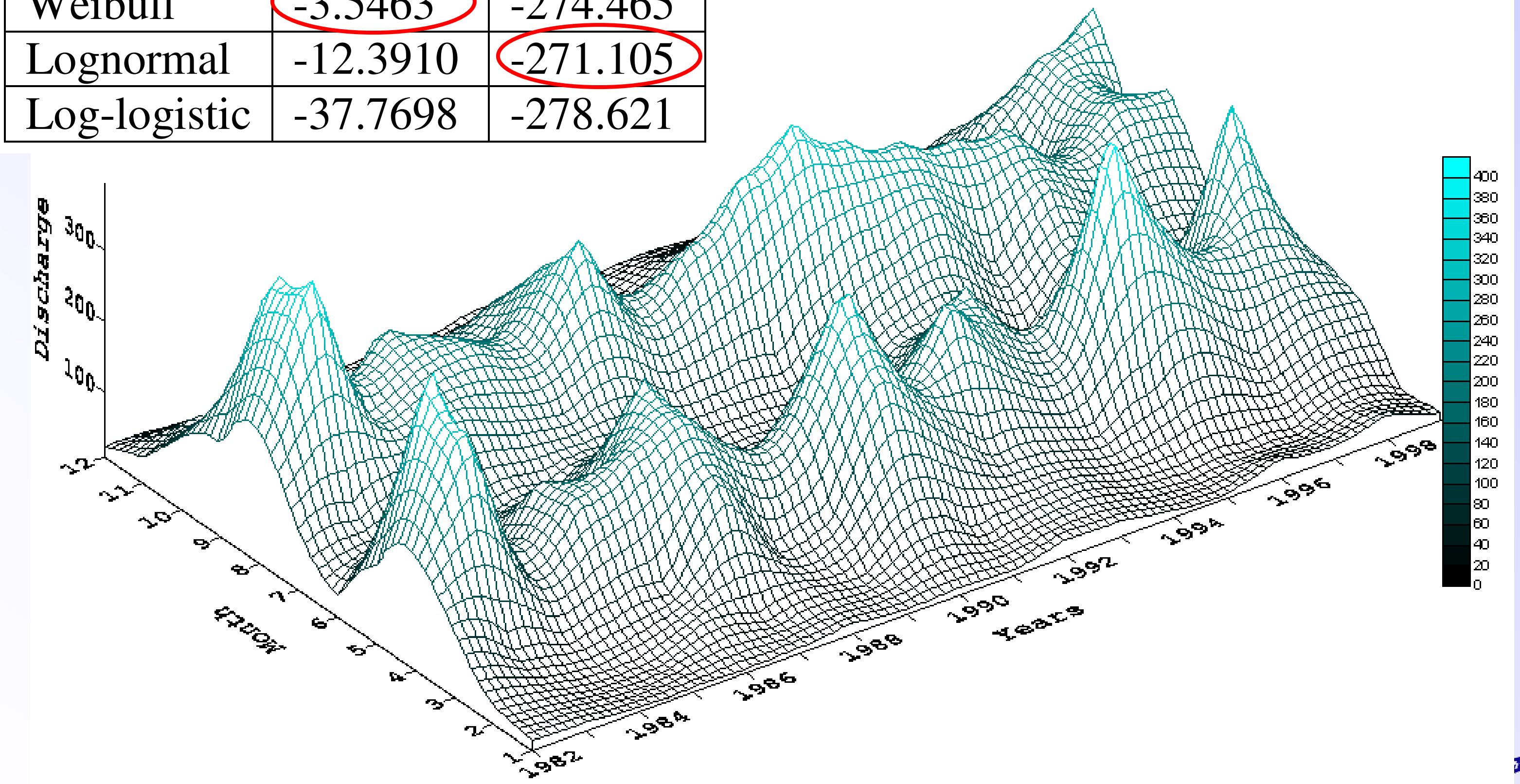


Fig 16. Discharge (m^3/s) at Imi gauging station

3. Frequency Analysis in Hydrology

- Probability weighted Moment

- As in the case of MOM, by equating moments of the distributions with the corresponding sample moments of observed data.
- Useful for distributions whose fn has an inverse form
- in small samples PWM may be as efficient as ML.
- PWM method can be easily used in regional estimation schemes.

3. Frequency Analysis in Hydrology

Probability weighted moments (PWMs) are defined by Greenwood et al. (1979) as

$$M_{p,r,s} = E[x^p F^r (1-F)^s] = \int_0^1 [x(F)]^p F^r (1-F)^s \, dF$$

In particular, the following two moments M1,0,s and M1,r,0 are often considered:

$$M_{1,0,s} = \alpha_s = \int_0^1 x(F)(1-F)^s \, \mathrm{dF}$$

$$M_{1,r,0} = \beta_r = \int_0^1 x(F)F^r \, \mathrm{dF}$$

When p= 1 and either r or s is equal to zero, then $M_{1,r,0}$=βr and $M_{1,0,s}$=αs are linear in x and of sufficient generality for parameter estimation.

3. Frequency Analysis in Hydrology

For an ordered sample x1≤ … ≤N,N>r, N>s unbiased sample PWMs are given by

$$a_s = \hat{\alpha}_s = \hat{M}_{1,0,s} = \frac{1}{N}\sum_{i=1}^{N}\binom{N-i}{s}x_i\Big/\binom{N-1}{s}$$

$$b_r = \hat{\beta}_r = \hat{M}_{1,r,0} = \frac{1}{N}\sum_{i=1}^{N}\binom{i-1}{r}x_i\Big/\binom{N-1}{r}$$

Alternatively, consistent but biased estimators of PWMs may be obtained by using the plotting position $F_i=(i–0.35)/N$.

$$a_s = \hat{\alpha}_s = \hat{M}_{1,0,s} = \frac{1}{N}\sum_{i=1}^{N}(1-F_i)^s x_i$$

$$b_r = \hat{\beta}_r = \hat{M}_{1,r,0} = \frac{1}{N}\sum_{i=1}^{N}F_i^r x_i$$

3. Frequency Analysis in Hydrology

The PWMs α_s and β_r are related as

$$\alpha_s = \sum_{i=0}^{s} \binom{s}{i} (-1)^i \beta_i, \quad \beta_r = \sum_{i=0}^{r} \binom{r}{i} (-1)^i \alpha_i$$

In particular:

$$\begin{array}{lll}
\alpha_0 = \beta_0 & . & \beta_0 = a_0 \\
\alpha_1 = \beta_0 - \beta_1 & . & \beta_1 = \alpha_0 - \alpha_1 \\
\alpha_2 = \beta_0 - 2\beta_1 + \beta_2 & . & \beta_2 = \alpha_0 - 2\alpha_1 + \alpha_2 \\
\alpha_3 = \beta_0 - 3\beta_1 + 3\beta_2 - \beta_3 & . & \beta_3 = \alpha_0 - 3\alpha_1 + 3\alpha_2 - \alpha_3
\end{array}$$

3. Frequency Analysis in Hydrology

L-moments are defined by Hosking in terms of the PWMs α and β as

$$\lambda_{r+1} = (-1)^r \sum_{k=0}^{r} p^*_{r,k}\,\alpha_k = \sum_{k=0}^{r} p^*_{r,k}\,\beta_k$$

where:

$$p^*_{r,k} = (-1)^{r-k}\binom{r}{k}\binom{r+k}{k}$$

In particular:

$$\begin{aligned}
\lambda_1 &= \alpha_0 && = \beta_0 \\
\lambda_2 &= \alpha_0 - 2\alpha_1 && = 2\beta_1 - \beta_0 \\
\lambda_3 &= \alpha_0 - 6\alpha_1 + 6\alpha_2 && = 6\beta_2 - 6\beta_1 + \beta_0 \\
\lambda_4 &= \alpha_0 - 12\alpha_1 + 30\alpha_2 - 20\alpha_3 && = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0
\end{aligned}$$

3. Frequency Analysis in Hydrology

L-moment ratios, which are analogous to conventional moment ratios are defined by Hosking (1990) as

$$\tau = \lambda_2 / \lambda_1$$

$$\tau_r = \lambda_r / \lambda_2, \; r \geq 3$$

where λ1 is a measure of location, τ is a measure of scale and dispersion (LCv), τ3 is a measure of skewness (LCs), and τ4 is a measure of kurtosis (LCk).are defined by Hosking in terms of the PWMs α and β as

3. Frequency Analysis in Hydrology

Extreme Value (EV type I)

$$f(x) = \frac{1}{\alpha}\exp\left[-\left(\frac{x-\mu}{\alpha}\right) - e^{-\left(\frac{x-\mu}{\alpha}\right)}\right]$$

$$F(x) = \exp\left[-e^{-\left(\frac{x-\mu}{\alpha}\right)}\right]$$

$$x = \mu - \alpha \ln\left(-\ln F\right)$$

$$\beta_r = \frac{\mu}{1+r} + \frac{\alpha\{\ln(1+r)+\varepsilon\}}{1+r}$$

$$\alpha = \frac{2b_1 - b_0}{\ln 2} = \frac{l_2}{\ln 2}$$

$$\mu = b_0 - 0.5772157\alpha$$

3. Frequency Analysis in Hydrology

Generalized Extreme Value (GEV)

$$f(x) = \frac{1}{\alpha}\left[1 - k\left(\frac{x-u}{\alpha}\right)\right]^{1/k-1} e^{-\left[1-k\left(\frac{x-u}{\alpha}\right)\right]^{1/k}}$$

$$F(x) = \exp\left\{-\left[1-k\left(\frac{x-u}{\alpha}\right)\right]^{1/k}\right\}$$

$$x = u + \frac{\alpha}{k}\,[1-(-\log F)^{k}]$$

$$\beta_r = (r+1)^{-1}\left[u+\frac{\alpha}{k}\,\{1-(r+1)^{-k}\,\Gamma(1+k)\}\right]$$

3. Frequency Analysis in Hydrology

Generalized Extreme Value (GEV)

$$\frac{3+t_3}{2} = (1-3^{-k})/(1-2^{-k})$$

which is approximated by

$$\hat{k} = 7.8590C + 2.9554\ C^2$$

$$\text{where } C = \frac{2b_1-b_0}{3b_2-b_0} - \frac{\log 2}{\log 3} = \frac{2}{3+t_3} - \frac{\log 2}{\log 3}$$

$$\hat{\alpha} = \frac{(2b_1-b_0)\hat{k}}{\Gamma(1+\hat{k})\ (1-2^{-\hat{k}})} = \frac{l_2\hat{k}}{\Gamma(1+\hat{k})\ (1-2^{-k})}$$

$$\hat{u} = b_0 + \frac{\hat{\alpha}}{\hat{k}}\left[\Gamma(1+\hat{k})-1\right] = l_1 + \frac{\hat{\alpha}}{\hat{k}}\left[\Gamma(1+\hat{k})-1\right]$$

3. Frequency Analysis in Hydrology

- Selection of parent distribution- Conventional moments- approximations can be used for constructing C_s-C_k

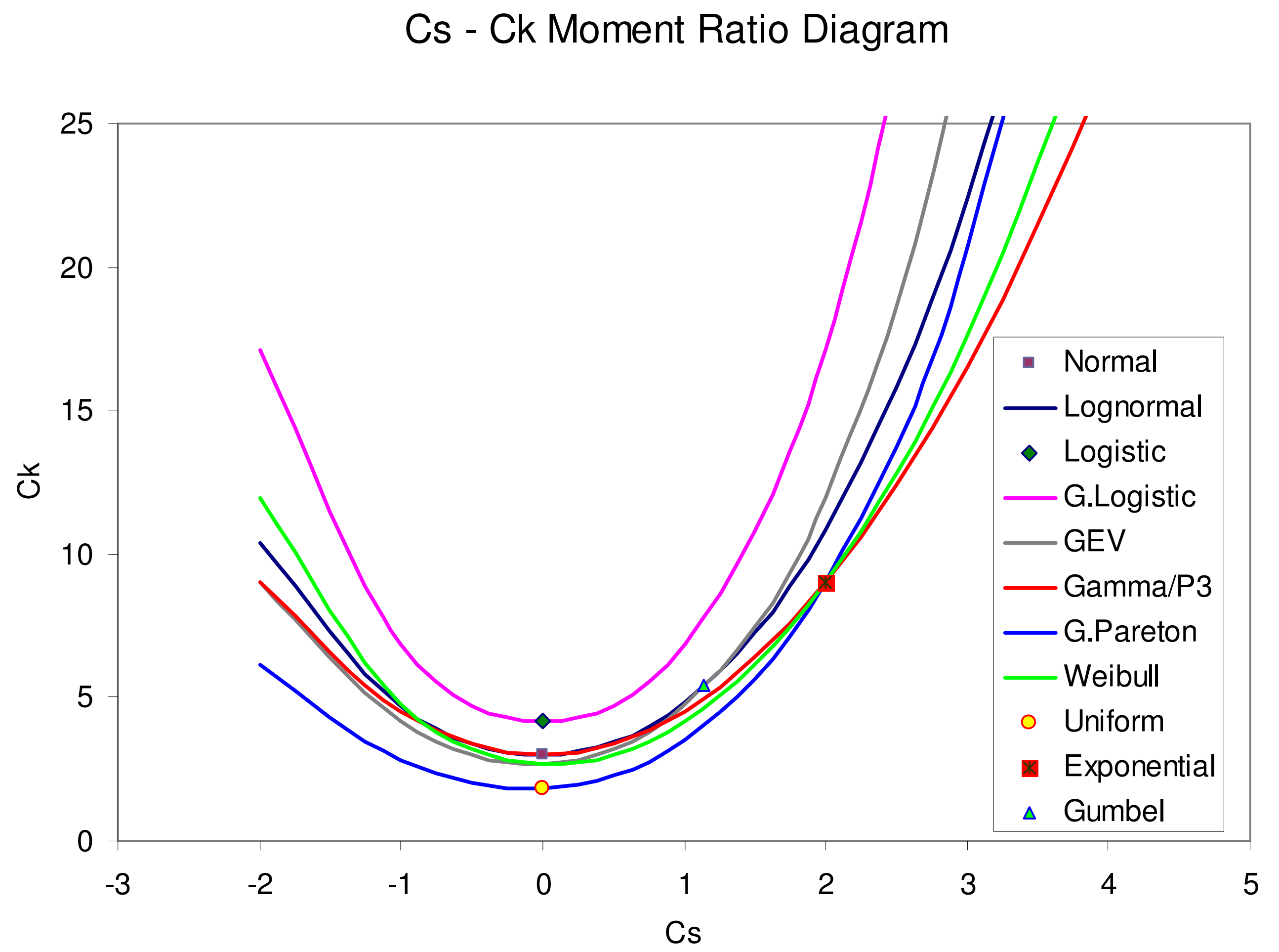
1. Normal: $C_s=0$, $C_k=3.0$
2. Logistic: $C_s=0$, $C_k=4.2$
3. Gumbel: $C_s=1.1396$, $C_k=5.4002$
4. Exponential: $C_s=2.0$, $C_k=9.0$
5. Gamma and Pearson III: $C_k=3+1.5C_s^2$
6. Lognormal:

   $C_k=3+0.025653C_s+1.720551C_s^2+0.041755C_s^3+0.046052C_s^4-0.00478C_s^5+0.000196C_s^6$
7. GEV:

   $C_k=2.695079+0.185768C_s+1.753401C_s^2+0.110735C_s^3+0.037691C_s^4+0.0036C_s^5+0.00219C_s^6+0.000663C_s^7+0.000056C_s^8$
8. Weibull: same as GEV but with C_s replaced by $-C_s$

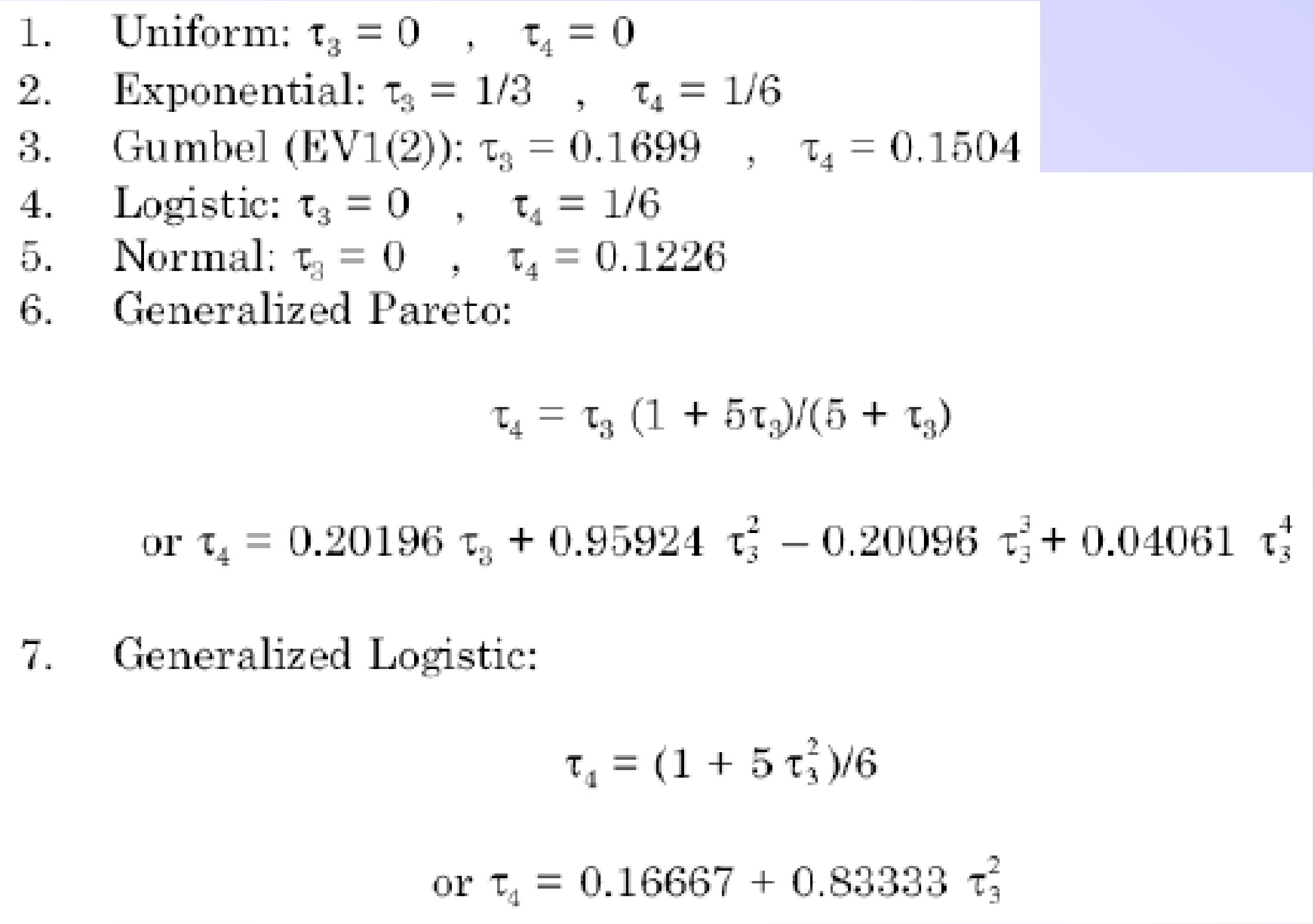
3. Frequency Analysis in Hydrology

- Selection of parent distribution- Moment ratio diagrams



3. Frequency Analysis in Hydrology

L-moment ratio diagrams (LMRD)



1. Uniform: $\tau_3 = 0$, $\tau_4 = 0$
2. Exponential: $\tau_3 = 1/3$, $\tau_4 = 1/6$
3. Gumbel (EV1(2)): $\tau_3 = 0.1699$, $\tau_4 = 0.1504$
4. Logistic: $\tau_3 = 0$, $\tau_4 = 1/6$
5. Normal: $\tau_3 = 0$, $\tau_4 = 0.1226$
6. Generalized Pareto:

$$\tau_4 = \tau_3\,(1 + 5\tau_3)/(5 + \tau_3)$$

$$\text{or } \tau_4 = 0.20196\ \tau_3 + 0.95924\ \tau_3^2 - 0.20096\ \tau_3^3 + 0.04061\ \tau_3^4$$

7. Generalized Logistic:

$$\tau_4 = (1 + 5\ \tau_3^2)/6$$

$$\text{or } \tau_4 = 0.16667 + 0.83333\ \tau_3^2$$

3. Frequency Analysis in Hydrology

L-moment ratio diagrams (LMRD)

8. Generalized Extreme Value:

$$\tau_4 = 0.10701 + 0.11090\ \tau_3 + 0.84838\ \tau_3^2 - 0.06669\ \tau_3^3$$

$$+ 0.00567\ \tau_3^4 - 0.04208\ \tau_3^5 + 0.03763\ \tau_3^6$$

9. Gamma and Pearson III:

$$\tau_4 = 0.1224 + 0.30115\ \tau_3^2 + 0.95812\ \tau_3^4 - 0.57488\ \tau_3^6 + 0.19383\ \tau_3^8$$

10. Lognormal (two and three parameters):

$$t_4 = 0.12282 + 0.77518\ \tau_3^2 + 0.12279\ \tau_3^4 - 0.13638\ \tau_3^6 + 0.11368\ \tau_3^8$$

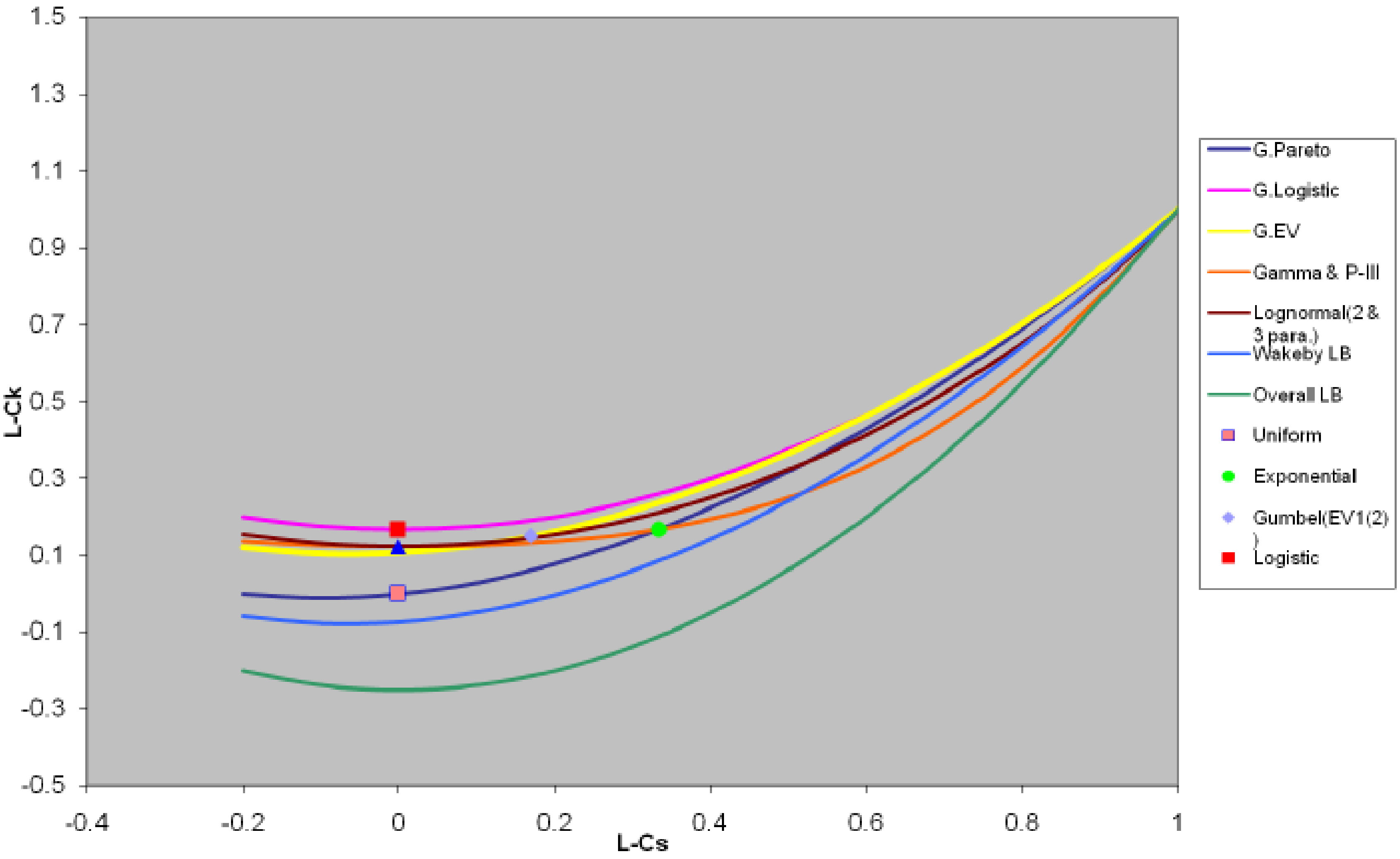
11. Wakeby lower bound:

$$\tau_4 = -0.07347 + 0.14443\ \tau_3 + 1.03879\ \tau_3^2 - 0.14602\ \tau_3^3 + 0.03357\ \tau_3^4$$

12. Overall lower bound: $\tau_4 = -0.25 + 1.25\ \tau_3^2$

3. Frequency Analysis in Hydrology

L-moment ratio diagrams (LMRD)



3. Frequency Analysis in Hydrology

- Selection of parent distribution

- Chi-square test: data are first divided into k class intervals.

$$x^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j}$$

$$x^2 = \frac{k}{n} \sum_{i=1}^{k} O_j^2 - n$$

O_j	observed number of events
E_j	expected number of events
K	number of classes

If class intervals are chosen : each interval corresponds to an equal probability

3. Frequency Analysis in Hydrology

- Selection of parent distribution

- Kolmogorov-Smirnov test: deviation of sample distribution $F_n(x)$ from completely specified continuous hypothetical distribution function $F_o(x)$.

- D_N=max /$F_n(x)$-$F_o(x)$/
- $F_n(x)$ estimated as n_j/n
- $F_o(x)$ is 1/k,2/k…

3. Frequency Analysis in Hydrology

- Assuming an exponential distribution for the following data check goodness of fit test using chi-square and kolmogorov-smirnov test.

- Soln:

Class	F_o	Class limit	O_i	$n_j=\sum O_i$	$F_n=n_j/n$	$/F_n-F_o/$

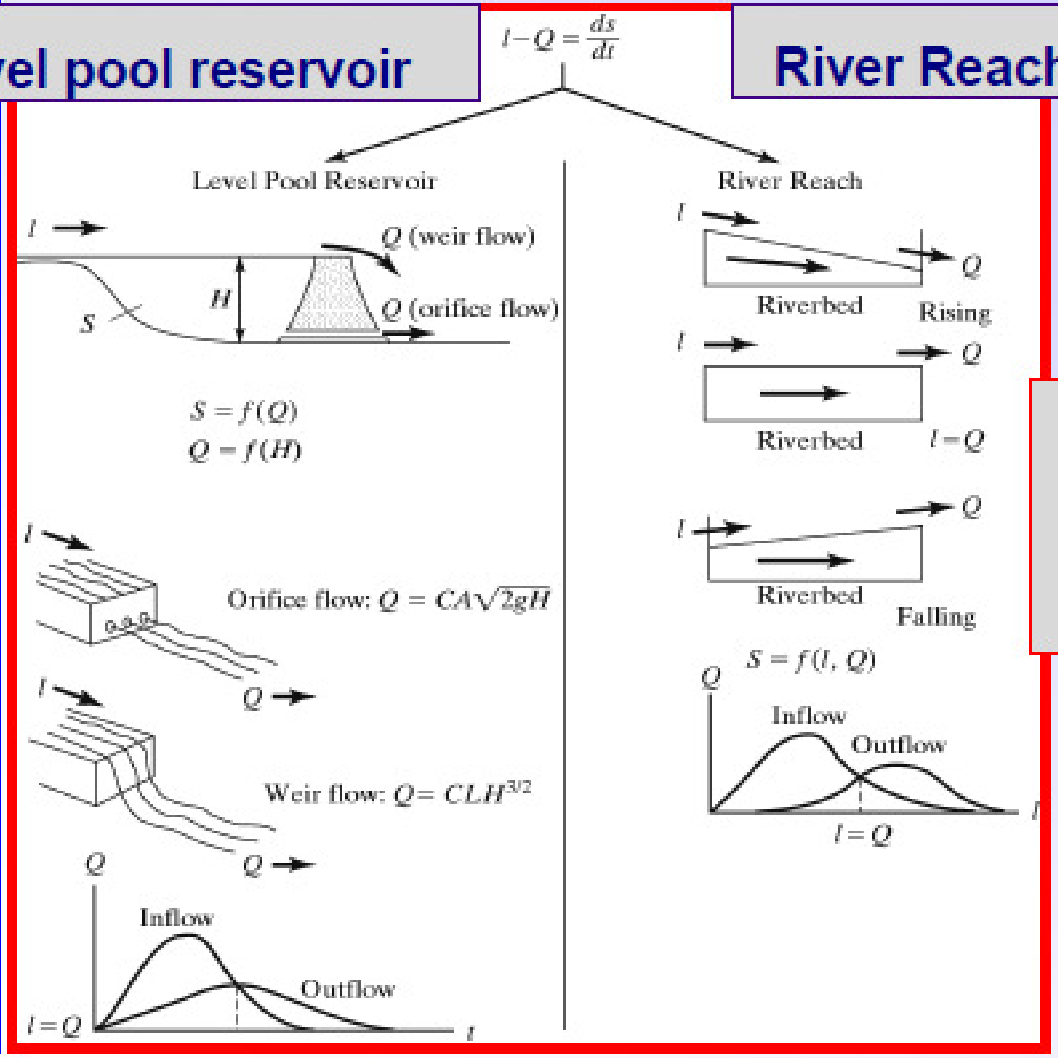
4. Flood Routing

- Routing is a procedure that predicts the flow at a point from known upstream information.
- Hydrologic routing employs the continuity equation and a relationship between storage and discharge to accomplish this.

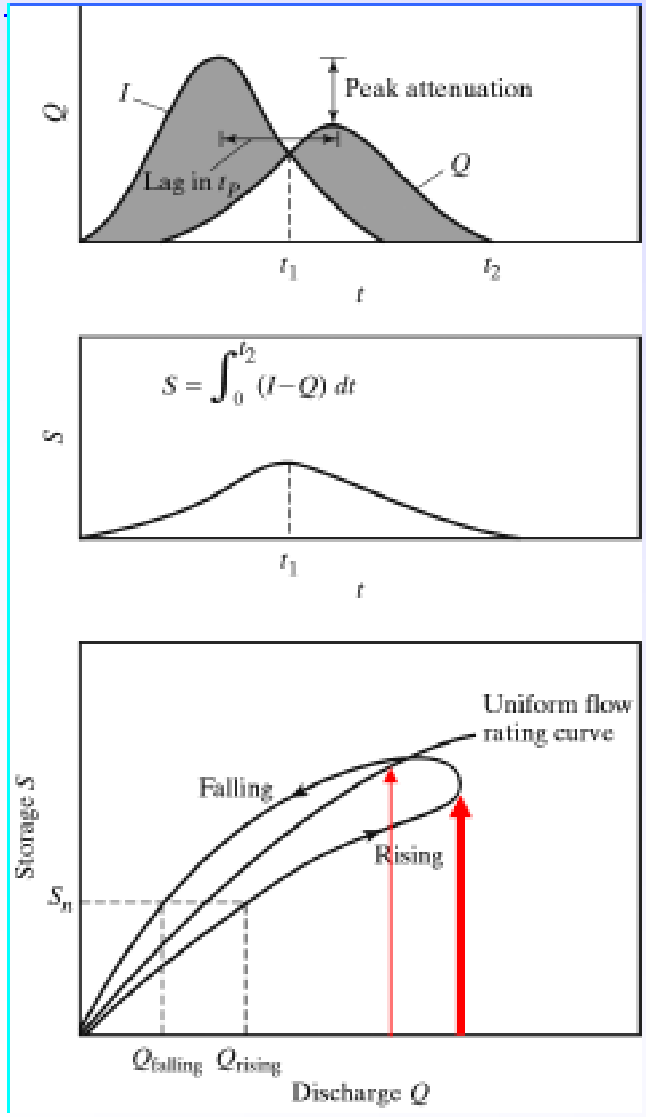
4. Flood Routing

Level pool reservoir

River Reach



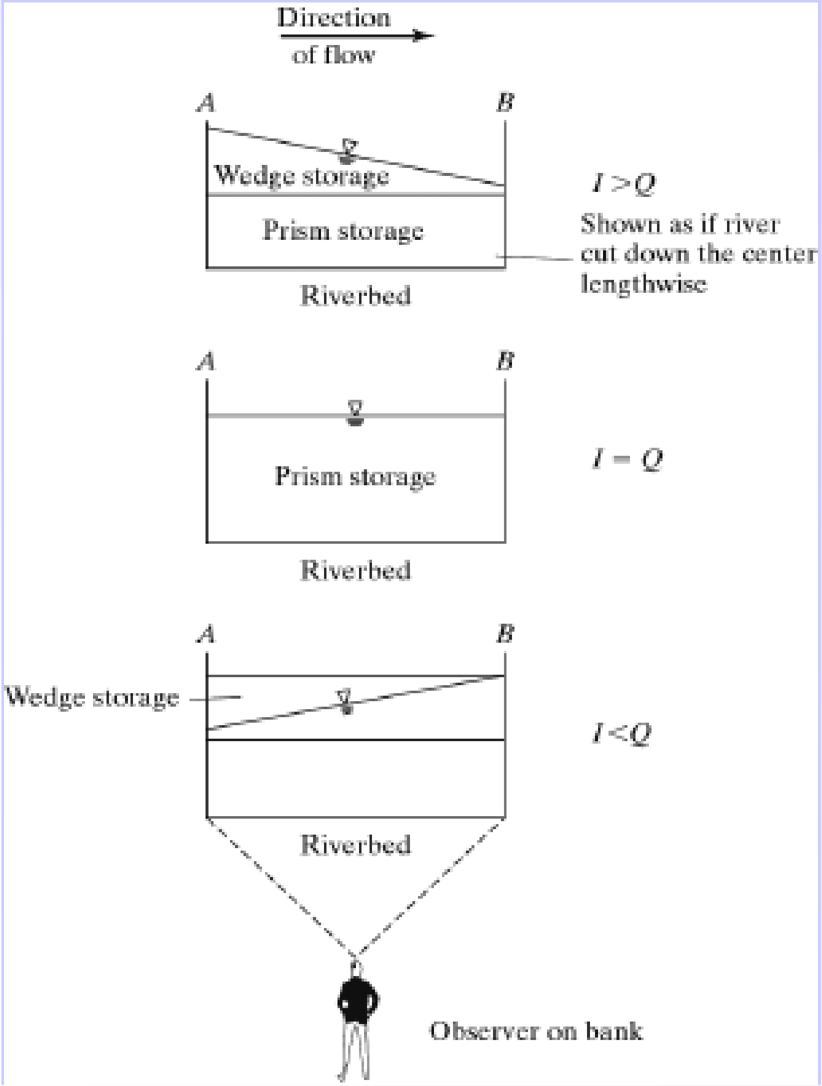
4. Flood Routing



River Rating Curves

- Inflow and outflow are complex
- Wedge and prism storage occurs
- Peak flow Qp greater on rise limb than on the falling limb
- Peak storage occurs later than Qp

4. Flood Routing



Wedge and Prism Storage

- Positive wedge I > Q
- Maximum S when I = Q
- Negative wedge I < Q

4. Flood Routing

Muskingum Method - 1938

- Continuity Equation $I - Q = dS / dt$

- Storage Eqn $S = K \{x I + (1-x)Q\}$

- Parameters are:

x = weighting Coeff
K = travel time or time between peaks
x = ranges from 0.2 to about 0.5 (pure trans)
and assume that initial outflow = initial inflow

4. Flood Routing

Muskingum Method – 1938

- Continuity Equation $I - Q = dS / dt$
- Storage Eqn $S = K \{x I + (1-x)Q\}$
- Combine 2 eqns using finite differences for I, Q, S

$S2 - S1 = K [x(I2 - I1) + (1 - x)(Q2 - Q1)]$

Solve for Q2 as fcn of all other parameters

4. Flood Routing

Muskingum Equations

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

Where

$C0 = (-Kx + 0.5\Delta t) / D$

$C1 = (Kx + 0.5\Delta t) / D$

$C2 = (K - Kx - 0.5\Delta t) / D$

and

$D = (K - Kx + 0.5\Delta t)$

Repeat for Q3, Q4, Q5 and so on.

4. Flood Routing

x and k can be determined if upstream and downstream hydrographs are available:

1.Compute the change in storage for each time interval

$$\left[\left(\frac{I_1+I_2}{2}\right)-\left(\frac{O_1+O_2}{2}\right)\right]dt$$

2. Compute cumulative storage over time
3. Plot storage vs (xI + (1-x)O) for a range of values of x
4. Select value of x which produces the narrowest "loop“ and calculate K as the best fit slope

4. Flood Routing

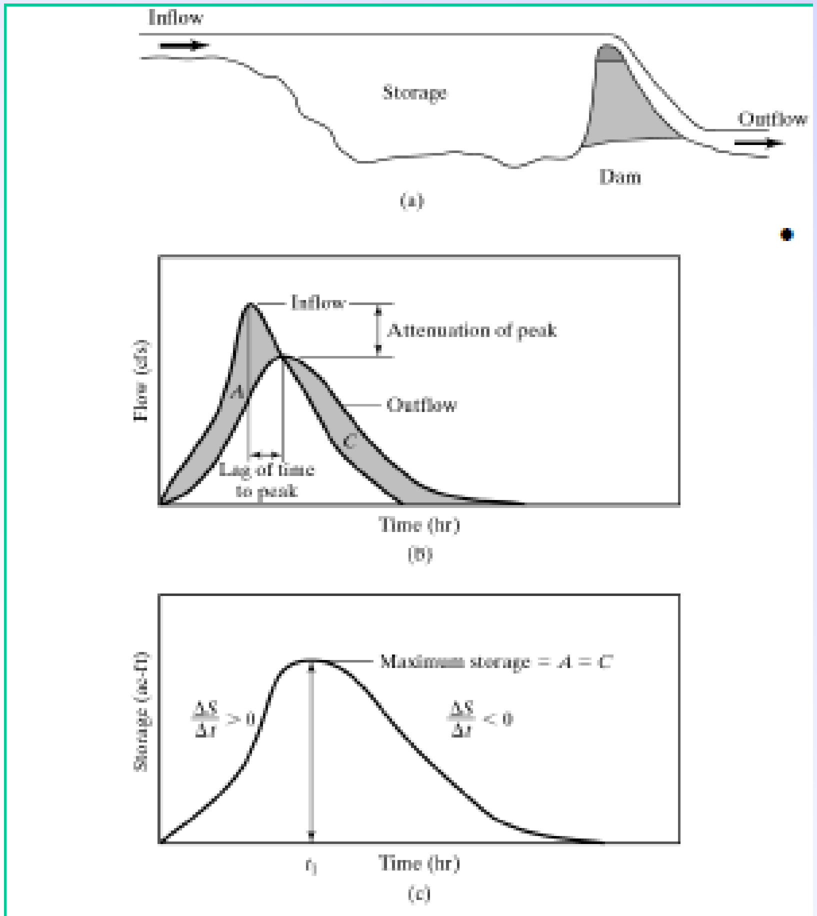


Figure 4.1

Reservoir concepts. (a) Reservoir storage. (b) Inflow to and outflow from the reservoir. (c) Storage in the reservoir.

Reservoir Routing

Reservoir acts to store water and release through control structure later.

- Inflow hydrograph
- Outflow hydrograph
- S - Q Relationship
- Outflow peaks are reduced
- Outflow timing is delayed

4. Flood Routing

Inflow and Outflow

$$I - Q = \frac{dS}{dt}$$

$$\frac{I1 + I2}{2} - \frac{Q1 + Q2}{2} = \frac{S2 - S1}{\Delta t}$$

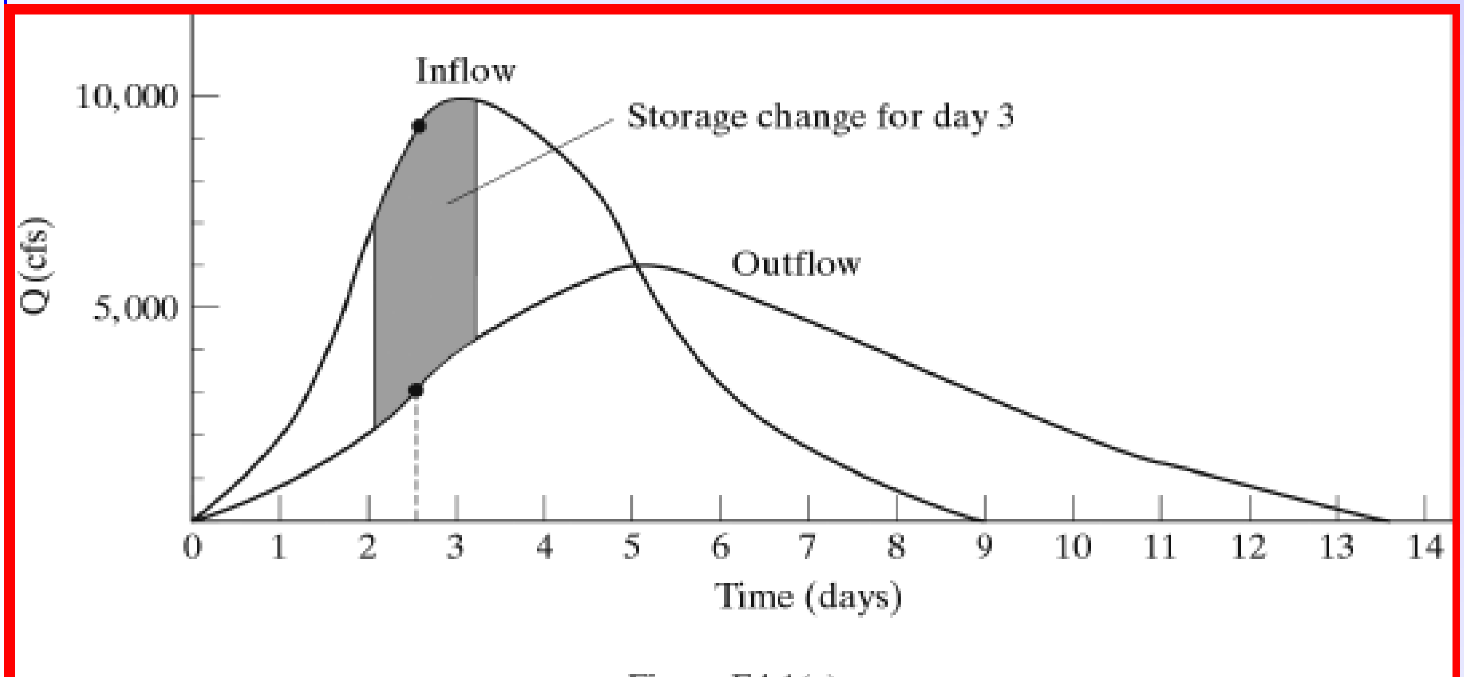
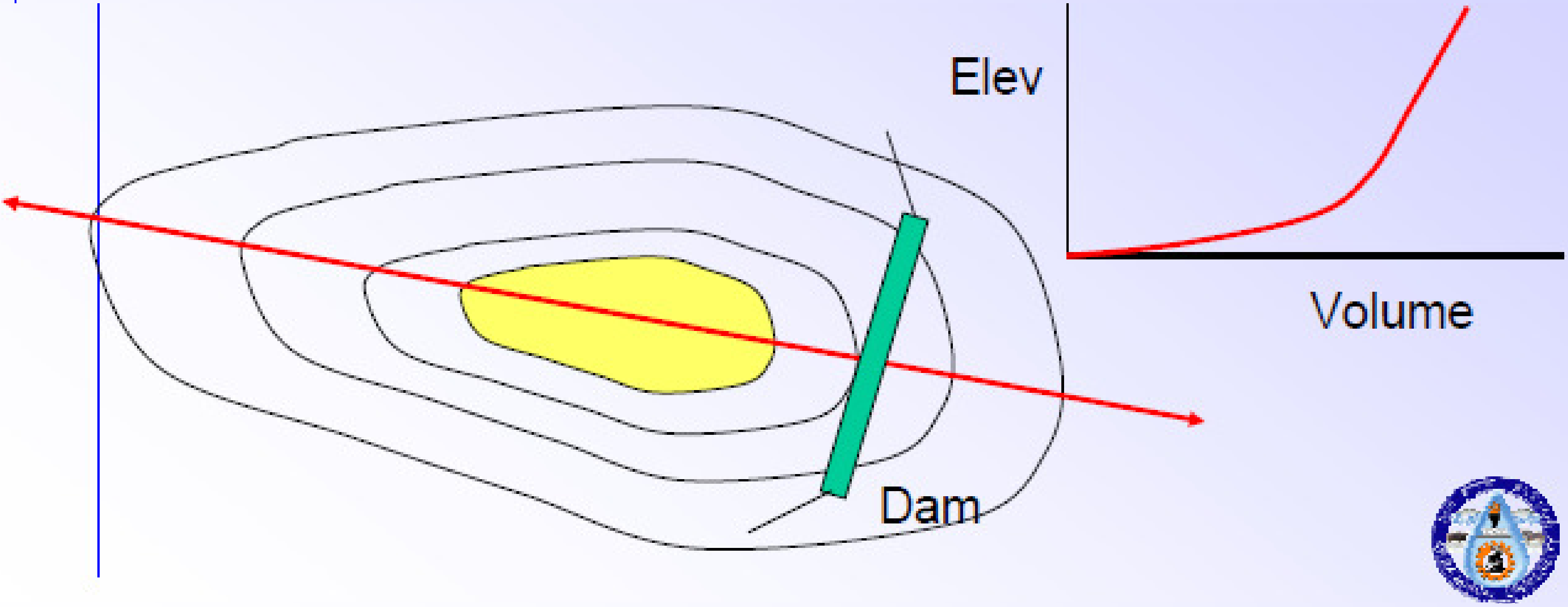


Figure E4.1(a)

4. Flood Routing

Determining Storage

- Evaluate surface area at several different depths
- Use available topographic maps or GIS based DEM sources (digital elevation map)
- Storage and area vary directly with depth of pond



4. Flood Routing

Determining Outflow

- Evaluate area & storage at several different depths
- Outflow Q can be computed as function of depth for

Pipes - Manning’s Eqn
Orifices - Orifice Eqn
Weirs or combination outflow structures - Weir Eqn

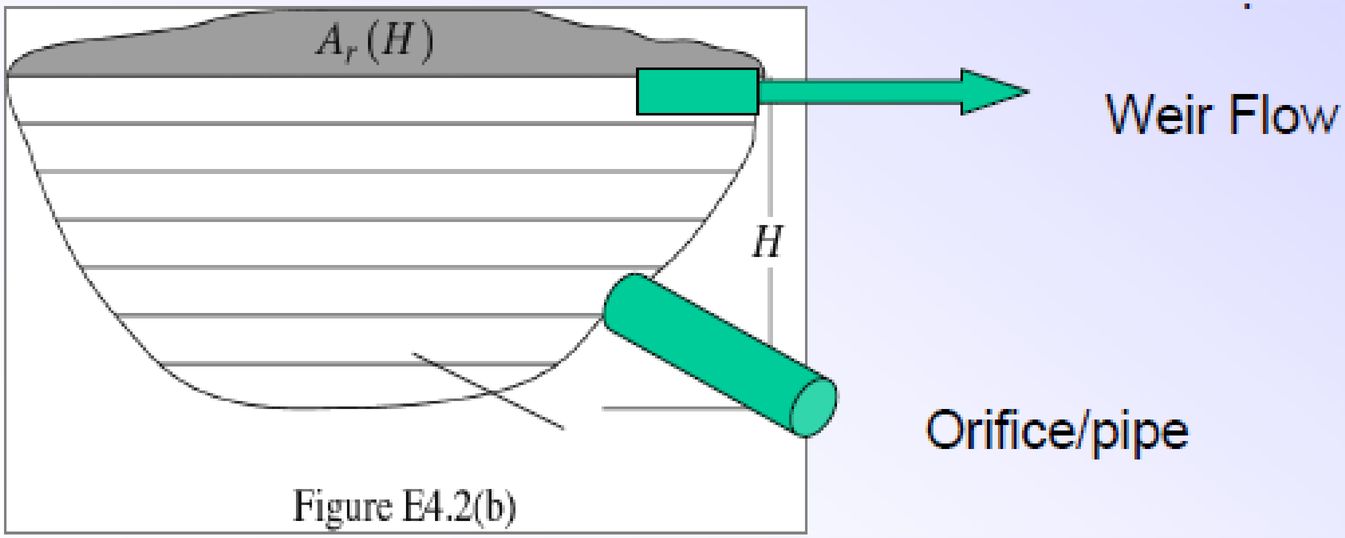


Figure E4.2(b)

4. Flood Routing

Determining Outflow

$$Q = CA\sqrt{2gH} \text{ for orifice flow}$$

$$Q = CLH^{3/2} \text{ for weir flow}$$

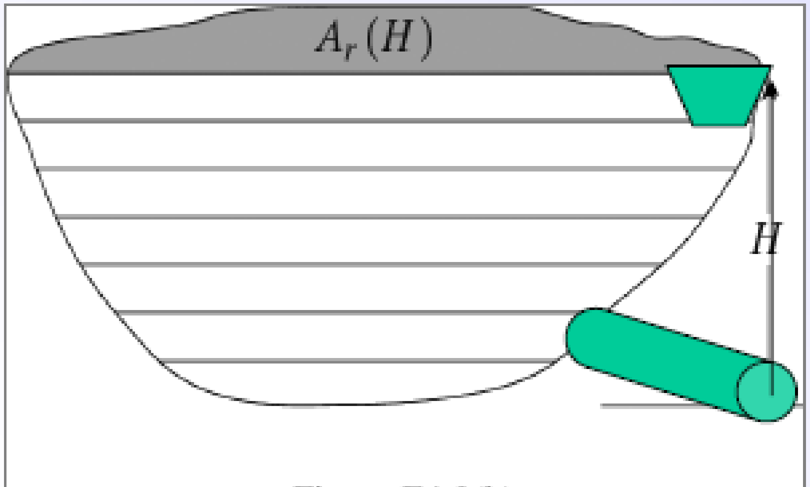
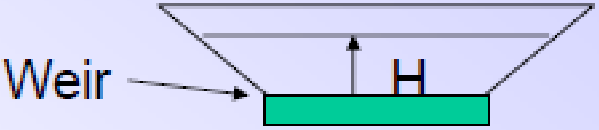


Figure E4.2(b)



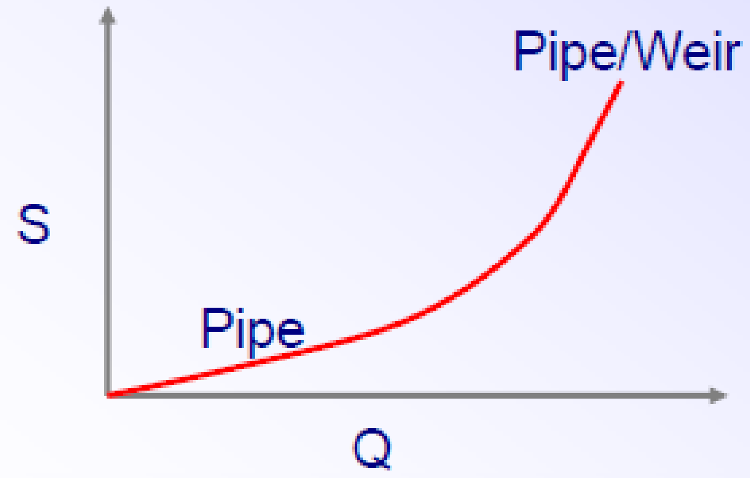
Orifice H measured above

Center of the orifice/pipe

4. Flood Routing

Typical Storage -Outflow

- Plot of Storage in cumec-days vs. Outflow in cumecs
- Storage is largely a function of topography
- Outflows can be computed as function of elevation for either pipes or weirs



4. Flood Routing

Reservoir Routing

$$I_1 + I_2 + \left(\frac{2S_1}{dt} - Q_1\right) = \left(\frac{2S_2}{dt} + Q_2\right)$$

1. LHS of Eqn is known
2. Know S as fcn of Q
3. Solve Eqn for RHS
4. Solve for Q2 from S2

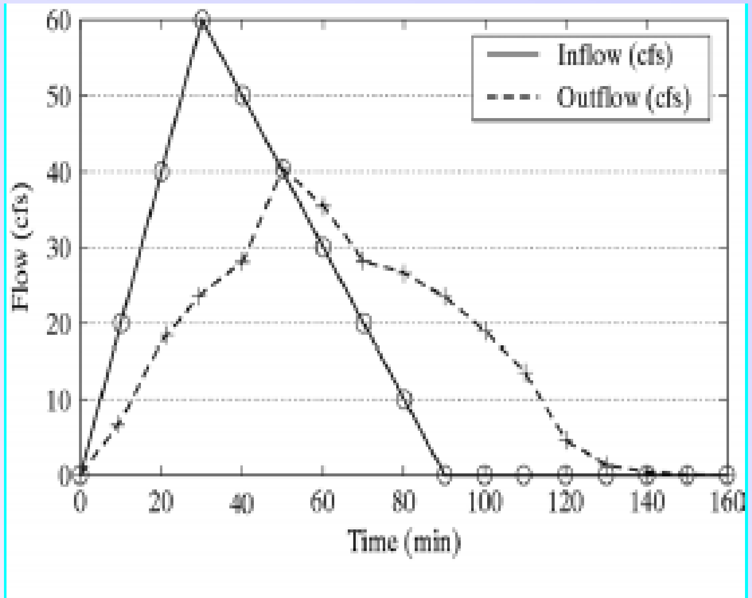


Figure E4.5(a)

Hydrographs.

Repeat each time step

4. Flood Routing

Examples

The topographic surveys at a proposed site yielded the following data:

Table 1

Contour elevation m	470	472	474	476	478	480	482	484	486
Contour area ha	219	227	240	257	278	303	330	362	396

There are 2 circular sluices with diameter of 2.5 m and with their centers at an elevation of 470 m. A spillway with an effective crest length of 20 m is also provided with its sill at 480 m. The Cd for sluices may be taken as 0.8 and for spillway C=2.25. Prepare the storage-discharge curve for the reservoir and route the following hydrograph through the reservoir. Determine the attenuation and the reservoir lag.

Table 2

Time h	0	6	12	18	24	30	36	42	48	54	60	66	72
Flow m3/s	50	180	270	360	410	370	300	230	155	90	60	35	20

Also find out what is the maximum elevation reached by the water surface in the reservoir. Assume that the outflow from the reservoir just before the flood arrived was 200 m3/s.

5. Hydrology in Design

- Hydrologic design standards and criteria
- Synthesizing design storms
  - IDF curves of design storms
- Urban hydrology and design
  - Effects of urbanization on runoff
  - Urban Hydrograph methods and models
    - Detention pond for urban areas
  - Urban Storm water drainage design

5. Hydrology in Design

- Hydrologic design standards and criteria

  Assess the impact of hydrologic events on designs

  – Design Scale – Range of design variables

  – Design Level – Magnitude of hydrologic event considered for the design

  - Return periods for various structures
    - 1 – 100 years (Minor structures) – Highway culverts & Bridges, small irrigation structures, urban drainage air fields small dams (w/o LOL)
    - 100 – 1000 years (Intermediate structures) – Major levees, intermediate dams
    - 500 – 100,000 years (Major structures) – Large dams, intermediate & small dams (w LOL)
  - Probable Maximum Precipitation (PMP)
  - Probable Maximum Flood (PMF)

5. Hydrology in Design

- Intensity-duration frequency curves

In many hydrologic design it´s essential to know the rainfall intensity of d/t duration and d/t return periods.

$$i = \frac{C}{T_d^e + f}$$

USA

$$i = \frac{CT^m}{T_d{}^e + f}$$

$$I = \frac{a}{(t+b)^c}$$

Commonly used

5. Hydrology in Design

- Intensity-duration frequency curves

The values of a, b and c are obtained through regression analysis

$$I = \frac{a}{(t+b)^c}$$

Commonly used

$$\log I = \log a - c\log(t+b)$$

The best value of a, b and c are those for which the sum of the squared deviations is minimum.

$$S = \sum\left[\log I - \{\log a - c\log(t+b)\}\right]^2$$

5. Hydrology in Design

- Intensity-duration frequency curves

Partial differentiation of S with respect to a and c yields

$$\sum \log I = n \log a - c \sum \log(t+b)$$

$$\sum \left[\log I \log(t+b)\right] = \log a \sum \log(t+b) - c \sum \left[\log(t+b)\right]^2$$

$$c = \frac{\sum \left[\log I \log(t+b)\right] - \dfrac{\sum \log I \sum \log(t+b)}{n}}{\dfrac{\left(\sum \log(t+b)\right)^2}{n} - \sum \left[\log(t+b)\right]^2}$$

5. Hydrology in Design

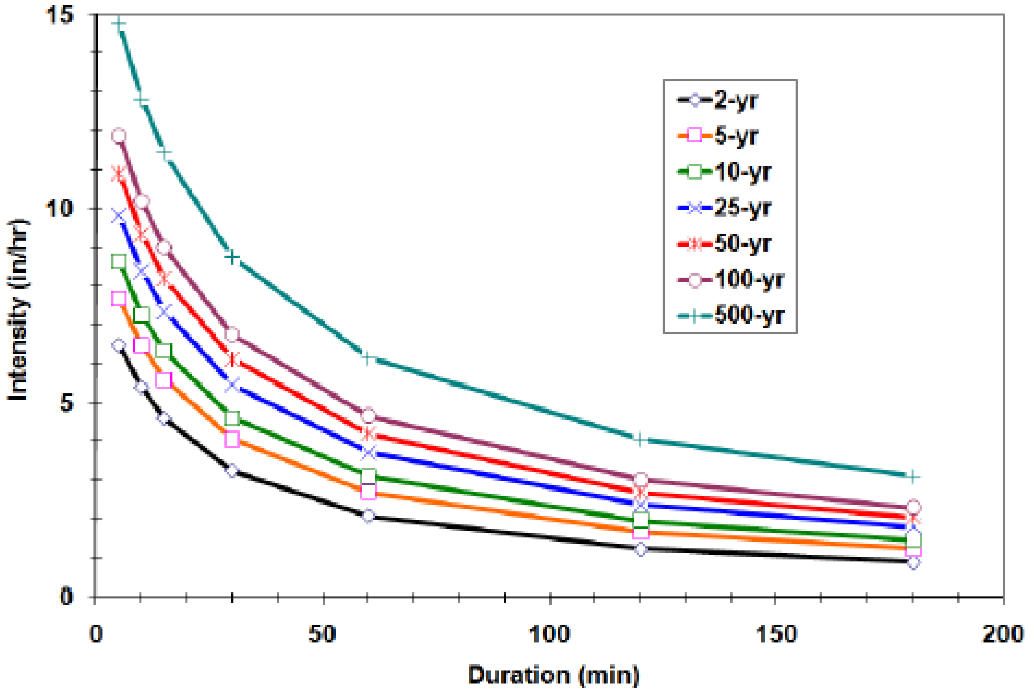
- Intensity-duration frequency curves

Duration t min	Intensity I mm/hr	log I	log (t+a)	log I.log(t+a)	$\{\log(t+a)\}^2$	Est I	ΔI	ΔI^2
	Σ	Σ	Σ	Σ	Σ			Σ

5. Hydrology in Design

- Intensity-duration frequency curves

Storm Frequency	a	b	c
2-year	106.29	16.81	0.9076
5-year	99.75	16.74	0.8327
10-year	96.84	15.88	0.7952
25-year	111.07	17.23	0.7815
50-year	119.51	17.32	0.7705
100-year	129.03	17.83	0.7625
500-year	160.57	19.64	0.7449



5. Hydrology in Design

- Intensity-duration frequency curves

Example: The cumulative rainfall depth with time during a storm as obtained from a recording rain gauge is given below.

Time (hr)	Rainfall (mm)
10:00:00	0
10:30:00	6
11:00:00	11
11:30:00	16
12:00:00	24
12:30:00	29
13:00:00	38
13:30:00	51
14:00:00	57
14:30:00	61
15:00:00	66
15:30:00	67
16:00:00	67

5. Hydrology in Design

- Intensity-duration frequency curves

1. construct the hyetograph of this storm using uniform time interval of 30 minutes and also 2 hours.
2. compute the maximum average intensities of rainfall for durations of 30 minutes, 1 hour, 2 hours, 3 hours and 5 hours in this storm and plot the resulting intensity duration curve.
3. fit an appropriate regression equation for the intensity duration curve obtained above. Assume the best value of b as 16.

5. Hydrology in Design

- Developed areas tend to have the following characteristics:
- High percentage of impervious surfaces
- Presence of —hydraulically improvedll drainage (sewers)
- Increased levels of pollutants available for runoff

5. Hydrology in Design

–Effects of urbanization on runoff

Affects the elements of the hydrological cycle

- Climate modifications
- Catchment response

- Climate
  - increase in temperature (saving of fuel in winter in temperate areas
  - decrease in wind speed
  - Precipitation total increase up to 10% (greater concentrates of condensation nuclei)

5. Hydrology in Design

–Effects of urbanization on runoff

- Catchment
  - Large proportion of impervious material
  - Higher volume of runoff
  - Steeper rising limb
  - Peaks are increased in magnitude
  - Lowflow discharge is decreased

5. Hydrology in Design

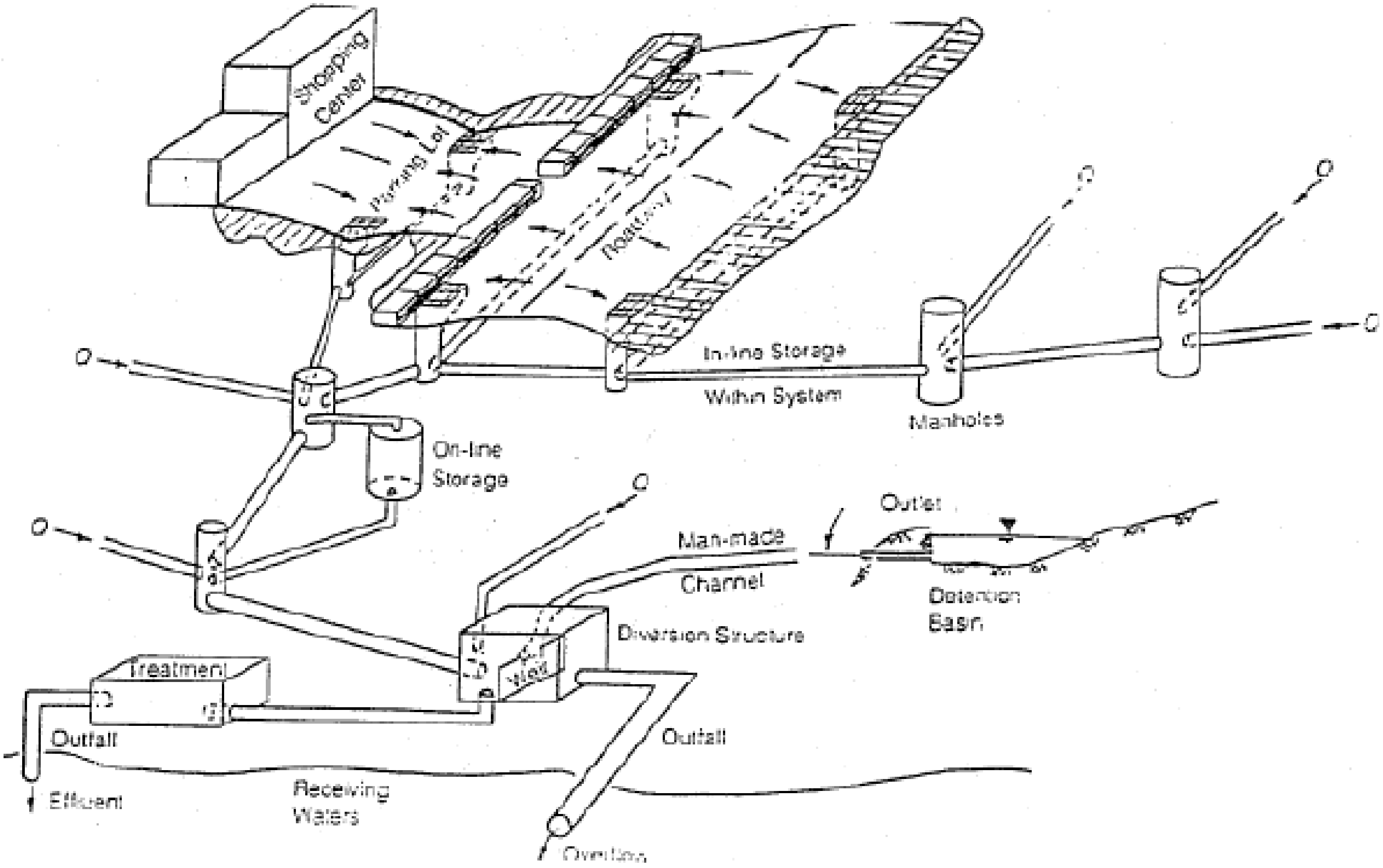


Figure 5.2: A Typical Stormwater Management System

5. Hydrology in Design

- Detention ponds design

A storm water management facility installed on, or adjacent to tributaries of rivers, streams, lakes or bays

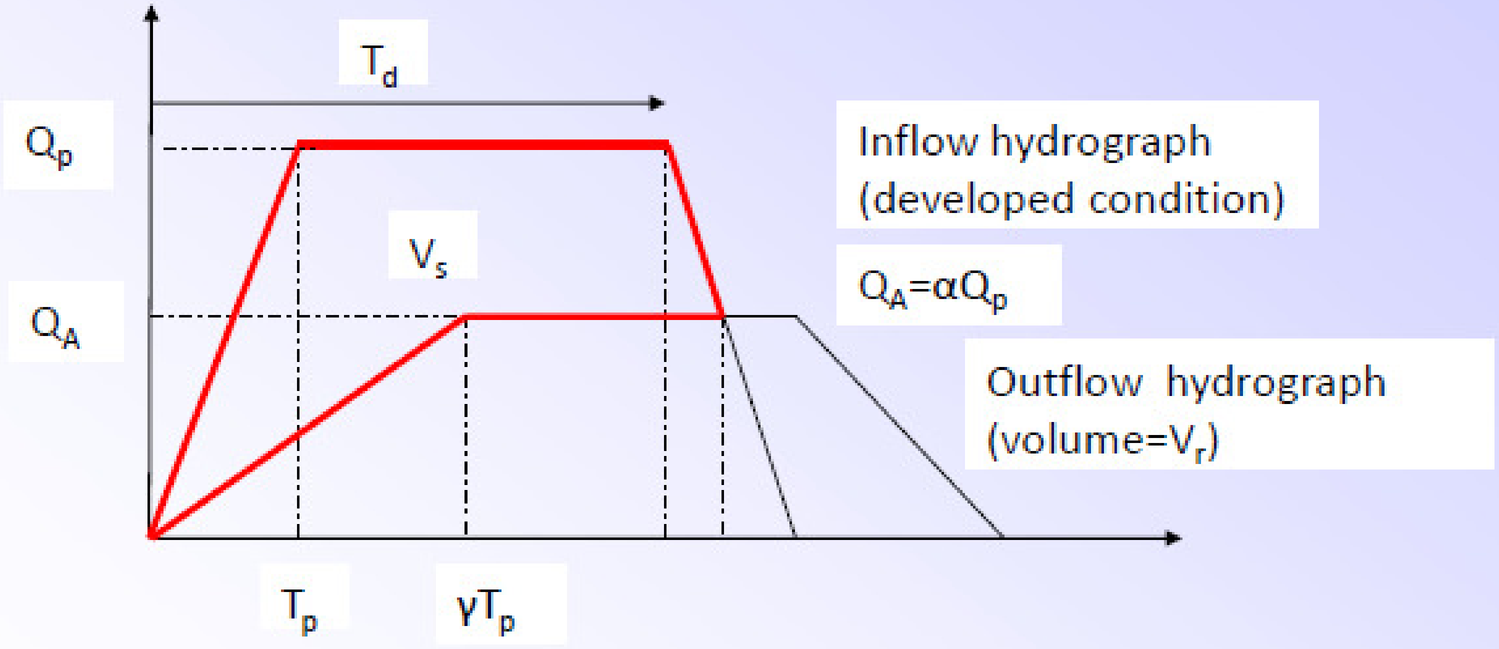
do not significantly reduce the volume of surface runoff but simply reduce the peak flow rates

Considerations in design of storm water detention facilities:

- The selection of a design rainfall event
- the volume of storage needed
- the maximum permitted release rate
- pollution control requirements and opportunities
- the outlet works for releasing the detained water

5. Hydrology in Design

- Detention ponds design



γ is the ratio of the times to peak in the two hydrographs
V_r is the volume of runoff after development
V_s is the volume of storage needed in the basin
Td is the duration of the precipitation

5. Hydrology in Design

- Detention ponds design

Using geometry of the trapezoidal hydrographs, the ratio of the volume of the storage to the volume of the runoff:

$$\frac{V_s}{V_r} = 1 - \alpha\left[1 + \frac{T_p}{T_d}\left(1 - \frac{\gamma + \alpha}{2}\right)\right]$$

Consider a rainfall intensity-duration relationship of the form

$$i = \frac{a}{T_d + b}$$

The volume of runoff after development is equal to the volume under the inflow hydrograph.

$V_r = Q_p T_d$

5. Hydrology in Design

- Detention ponds design

$$V_s = Q_p T_d \left\{1-\alpha\left[1+\frac{T_p}{T_d}\left(1-\frac{\gamma+\alpha}{2}\right)\right]\right\}$$

$$= Q_p T_d - Q_A T_d - Q_A T_p + \frac{\gamma Q_A T_p}{2} + \frac{Q_A^2 T_p}{2}\frac{1}{Q_p}$$

Where $\alpha = Q_A / Q_p$

The duration that results in the maximum detention is determined by substituting

$$Q_p = CiA = CAa/(T_d + b)$$

then differentiating the above equation with respect to T_d and setting the derivative equal to zero:

5. Hydrology in Design

- Detention ponds design

$$\frac{dV_s}{dT_d} = 0 = T_d\frac{dQ_p}{dT_d} + Q_p - Q_A + \frac{Q_A^2 T_p}{2}\left[\frac{d\left(1/Q_p\right)}{dT_d}\right]$$

$$= \frac{-T_d CAa}{\left(T_d + b\right)^2} + \frac{CAa}{T_d + b} - Q_A + \frac{Q_A^2 T_p}{2CAa}$$

$$= \frac{bCAa}{\left(T_d + b\right)^2} - Q_A + \frac{Q_A^2 T_p}{2CAa}$$

5. Hydrology in Design

- Detention ponds design

Where it is assumed that Q_A, T_p and γ are constants.
Solving for Td:

$$T_d = \left(\frac{bCAa}{Q_A - \dfrac{Q_A^2 T_p}{2CAa}} \right)^{1/2} - b$$

The time to peak Tp is set equal to the time of concentration.

5. Hydrology in Design

- Detention ponds design

Example: Determine the maximum detention storage for a 25 acre watershed with a developed runoff coefficient 0.825. The allowable discharge is the predevelopment discharge of 18 cfs. The time of concentration for the developed conditions is 20 min and for the undeveloped conditions is 40 min. The applicable rainfall intensity duration relationship is

$$i = \frac{96.6}{T_d + 13.9}$$

Soln:

$$T_d = \left(\frac{(13.9)(0.825)(25.0)(96.6)}{18 - \dfrac{(18)^2 (20)}{2(0.825)(25)(96.6)}} \right)^{1/2} - 13.9$$

$$= 27.23\,\text{min}$$

$$\gamma = 40/20 = 2$$

5. Hydrology in Design

- Detention ponds design

The peak discharge for the duration of 27.23 min is

$$Q_p = CA\left(\frac{a}{T_d + b}\right)$$

$$= (0.825)(25)\left(\frac{96.6}{27.23 + 13.9}\right) = 48.44 cfs$$

$$then$$

$$V_s = (27.23)(48.44) - (18)(27.23) - (18)(20) + (18)(20)\left(\frac{2}{2}\right) + \frac{(18)^2(20)}{2}\frac{1}{48.44}$$

$$= 895.77 cfs.\min \times 60 s/\min$$

$$= 53{,}746\, ft^3$$

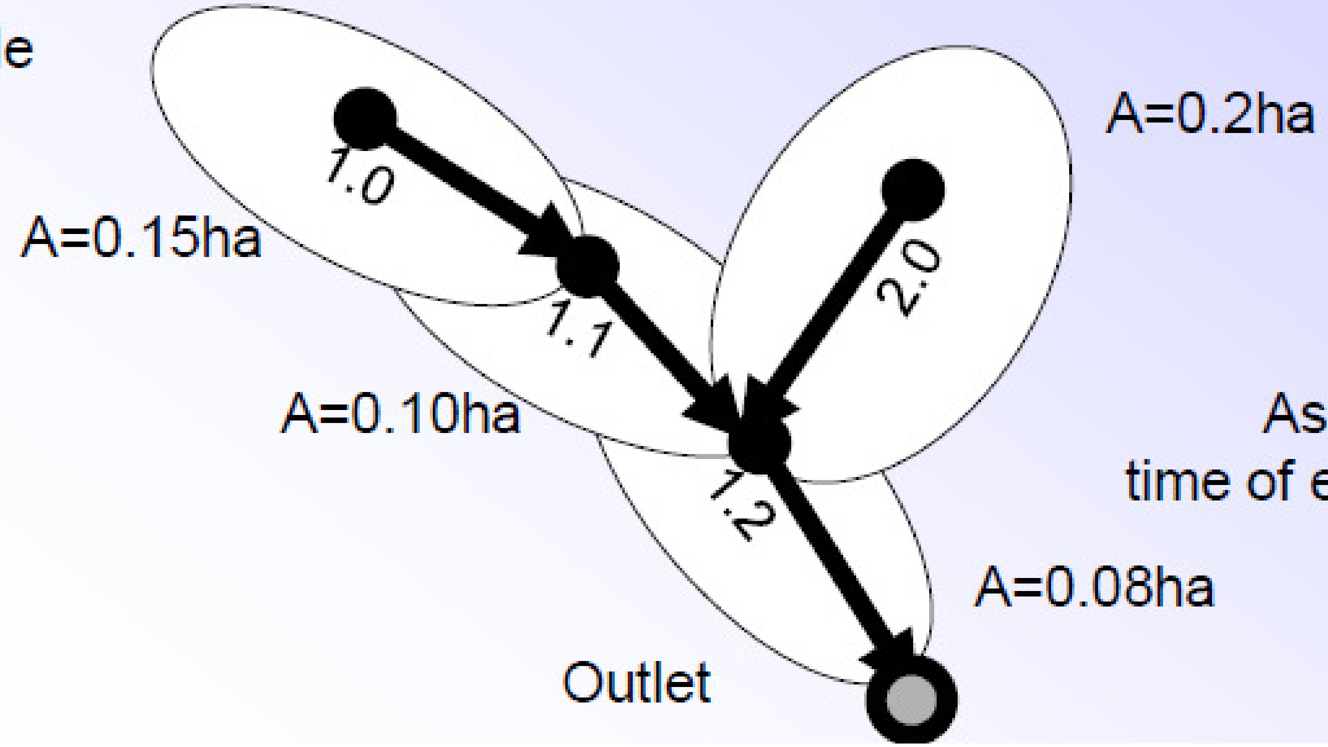
Since $V_r=Q_pT_d$=1319cfs.min=79,140ft^3, V_s/V_r=0.68. Hence the detention pond will store 68% of its inflow hydrograph in this example.

5. Hydrology in Design

- Storm drainage systems

  The recommended method for piped storm sewer drainage system. Storm water sewers are usually designed for 1 in 1, 1 in 2 or 1 in 5 years events

Example



Assume
time of entry=2 min

5. Hydrology in Design

- Storm water drainage design

Pipe no.	Level Diff m	Length L m	Trial Dia mm	Pipe		Time of Flow min	Tc min	I mm/hr	Imper Area ha	Storm Q l/s	Comment
				V m/s	Q l/s						
1	1	65	150	1.26	23.0	0.86	2.86	67.5	0.15	28.1	Surcharg
			250	1.64	67.5	0.66	2.66	69.2		28.8	Partial
1.1	0.9	70	225	1.50	61.7	0.78	3.44	63.2	0.25	43.9	Partial
2	1.5	60	150	1.61	29.4	0.62	2.62	69.5	0.20	38.6	Surcharg
			225	2.10	86.0	0.48	2.48	70.7		39.3	Partial
1.2	0.90	50	225	1.77	72.8	0.47	3.91	60.2	0.53	88.6	Surcharg
			300	2.13	156	0.39	3.83	60.7		89.4	Partial

Missing …for the last n days!
(where n>>0)

I, THANK YOU!

Supplementary

ANNEX

3. Regionalization in Hydrology

- Large variations associated with small sample sizes cause the estimates to be unrealistic.

- Regional analysis is based on the concept of regional homogeneity which assumes that annual maximum flow populations at several sites in a region are similar in statistical characteristics and are not dependent on catchment size (Cunnane, 1989).

  Regionalization serves two purposes.

- For sites where data are not available and sites with short record

3. Regionalization in Hydrology

- One of the simplest procedures which has been used for a long time is the index flood method. The key assumption in the index flood method is that the distribution of floods at different sites in a region is the same except for a scale or index flood parameter, which reflects rainfall and runoff characteristics of each region.

- Regional quantile estimates at a given site for a given return period T can be obtained as

$$\hat{Q}_T = \mu_i q_T$$

where q_T is the quantile estimate from the regional distribution for the given return period, and μ_i is the mean flow at the site.

3. Regionalization in Hydrology

- The regional distribution parameters are obtained by using the regional weighted average of dimensionless moments obtained by using the dimensionless rescaled data.
- Another method of obtaining the regional distribution parameters is the station year approach where all the data are pooled, after dividing them by the mean at each site, and are treated as a single sample.

  Catchments of similar hydrologic characteristics were delineated.

  Cluster analysis methods were used to define “homogeneous” hydrologic regions.
    - Hierarchical clustering
    - Kmean clustering
    - Fuzzy clustering

3. Regionalization in Hydrology

- The Regional frequency analysis methods are based on the assumption that the standardized variable $q_t = Q_T/\mu_i$ at each station (i) has the same distribution at every site in the region under consideration.
- A method of assigning homogeneous regions is geographical similarity in soil types, climate and topography. However, geographically similar regions may prove regions may not be similar from the flood frequency point of view (Cunnane, 1989).

3. Regionalization in Hydrology

Regional Homogeneity Tests

Discordancy measure, intended to identify those sites that are grossly discordant with the group as a whole. The discordancy measure D estimates how far a given site is from the center of the group.

If $u_i = \left[t^{(i)}, t_3^{(i)}, t_4^{(i)}\right]^T$ is the vector containing the t, t_3 and t_4 values for site (i), then the group average for NS sites is given as:

$$\bar{u} = \frac{1}{NS}\sum_{i=1}^{NS} u_i$$

The sample covariance matrix is given

$$S = (NS-1)^{-1}\sum_{i=1}^{NS}(u_i - \bar{u})(u_i - \bar{u})^T$$

3. Regionalization in Hydrology

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3. Regionalization in Hydrology

The discordancy measure is defined by:

$$D_i = \frac{1}{3}\left(u_i - \overline{u}\right)^T S^{-1}\left(u_i - \overline{u}\right)$$

A site (i) is declared to be unusual if D_i is large. A suitable criterion to classify a station as discordant is that D_i should be greater than or equal to 3.

3. Regionalization in Hydrology

Heterogeneity measure

It is intended to estimate the degree of heterogeneity in a group of sites and to assess whether they might reasonably be treated as homogeneous. Specifically, the heterogeneity measure compares the between-site variations in sample L-moments for the group of sites with that expected for a homogeneous region. Three measures of variability V_1, V_2, and V_3 are available.

3. Regionalization in Hydrology

Based on LCv(t), the weighted standard deviation of (t) is

$$V_1 = \left(\sum_{i=1}^{NS} N_i \left(t^{(i)} - \bar{t}\right)^2 / \sum_{i=1}^{NS} N_i \right)^{1/2}$$

where, NS in eq. above is the number of sites, Ni is the record length at each site and $\bar{t}$ is the average value of t(i) given by

$$\bar{t} = \left(\sum_{i=1}^{NS} N_i t^{(i)} \right) / \sum_{i=1}^{NS} N_i$$

Based on LCv and LCs, the weighted average distance from the site to the group weighted mean on a t vs. t3 graph is computed.

$$V_2 = \sum_{i=1}^{NS} N_i \left\{ \left(t^{(i)} - \bar{t}\right)^2 + \left(t_3^{(i)} - \bar{t}_3\right)^2 \right\}^{1/2} / \sum_{i=1}^{NS} N_i$$

3. Regionalization in Hydrology

Based on L-skewness (t_3) and L-kurtosis (t_4), the weighted average distance from the site to the group weighted mean on a t_3 vs. t_4 graph is computed as

$$V_3 = \sum_{i=1}^{NS} N_i \left\{ \left(t_3^{(i)} - \bar{t}_3\right)^2 + \left(t_4^{(i)} - \bar{t}_4\right)^2 \right\}^{1/2} / \sum_{i=1}^{NS} N_i$$

To evaluate the heterogeneity measures, a Kappa distribution (Hosking,1988) is fitted to the group average L-moments 1, m_1.m_2 Simulations of a large number of regions, N_{sim}, from this Kappa distribution are performed.

3. Regionalization in Hydrology

For each simulated region, Vi (where Vi is any of the three measures V1, V2, or V3defined above) is calculated. From the simulated data the mean μ_v and standard deviation σ_v of the Nsim values of Vi are determined.

The heterogeneity measure is defined

$$H_i = (V_i - \mu_v) / \sigma_v$$

3. Regionalization in Hydrology

A region is declared to be heterogeneous if H_i is sufficiently large. Hosking and Wallis (1991b) suggest that a region be regarded as:

- acceptably homogeneous if H_i is less than 1,
- possibly heterogeneous if it is between 1 and 2, and
- definitely heterogeneous if H_i is greater than 2.

Hosking and Wallis (1991) observed that statistics H_2 and H_3 based on the measures V_2 and V_3 lack the power to discriminate between homogeneous and heterogeneous regions and that H_i based on V_1 had much better discriminating power.

3. Regionalization in Hydrology

Therefore the H_1statistic based on V_1 is recommended as a principal indicator of heterogeneity. If a Kappa distribution cannot be fitted is too large relative to , the generalized logistic distribution, a special case of the Kappa distribution, is used for simulation.

Regional growth curves : procedures

i. Find the pwms for each of the sites
ii. Standardize the pwms by their corresponding M100. Thus obtaining 1, M1, M2…
iii. Find regionally weighted pwms and regional parameters
iv. Determine the regional estimates of the quantiles
v. Evaluate the at-site quantile estimates