THEORY OF STRUCTURE II

CHAPTER 2

INFLUENCE LINES FOR INDETERMINATE STRUCTURES

1. *INFLUENCE LINES FOR BEAMS AND TRUSSES*

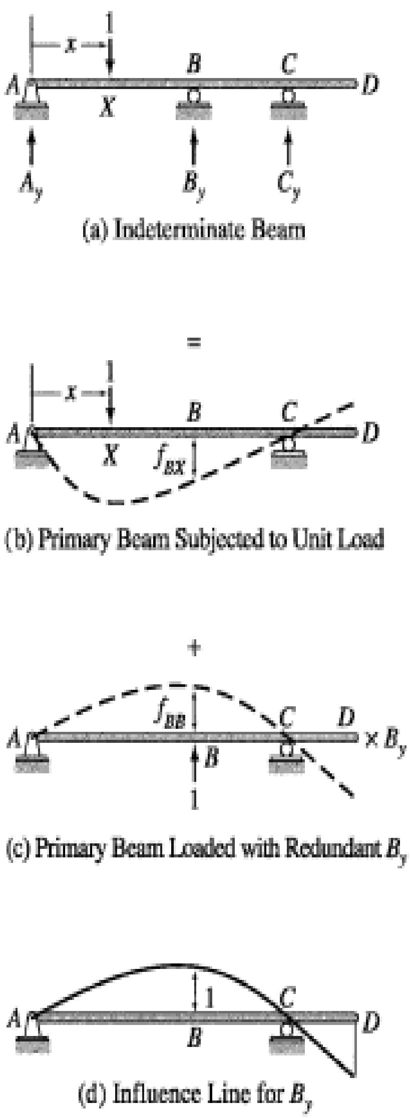


FIG. 15.1

Consider the continuous beam shown in Fig. 15.1(a). Suppose that we wish to draw the influence line for the vertical reaction at the interior support B of the beam. The beam is subjected to a downward-moving concentrated load of unit magnitude, the position of which is defined by the coordinate x measured from the left end A of the beam, as shown in the figure.

To develop the influence line for the reaction B_y, we need to determine the expression for B_y in terms of the variable position x of the unit load. Noting that the beam is statically indeterminate to the first degree, we select the reaction B_y to be the redundant. The roller support at B is then removed from the actual indeterminate beam to obtain the statically determinate primary beam shown in Fig. 15.1(b). Next, the primary beam is subjected, separately, to the unit load positioned at an arbitrary point X at a distance x from the left end, and the redundant B_y, as shown in Fig. 15.1(b) and (c), respectively. The expression for B_y can now be determined by using the compatibility condition that the deflection of the primary beam at B due to the combined effect of the external unit load and the unknown redundant B_y must be equal to zero. Thus

$$f_{BX} + f_{BB}B_y = 0$$

from which

$$B_y = -\frac{f_{BX}}{f_{BB}} \qquad (15.1)$$

in which the flexibility coefficient f_{BX} denotes the deflection of the primary beam at B due to the unit load at X (Fig. 15.1(b)), whereas the flexibility coefficient f_{BB} denotes the deflection at B due to the unit value of the redundant B_y (Fig. 15.1(c)).

We can use Eq. (15.1) for constructing the influence line for B_y by placing the unit load successively at a number of positions X along the beam, evaluating f_{BX} for each position of the unit load, and plotting the values of the ratio $-f_{BX}/f_{BB}$. However, a more efficient procedure can be devised by applying *Maxwell's law of reciprocal deflections* (Section 7.8), according to which the deflection at B due to a unit load at X must be equal to the deflection at X due to a unit load B; that is, $f_{BX} = f_{XB}$. Thus, Eq. (15.1) can be rewritten as

$$B_y = -\frac{f_{XB}}{f_{BB}} \tag{15.2}$$

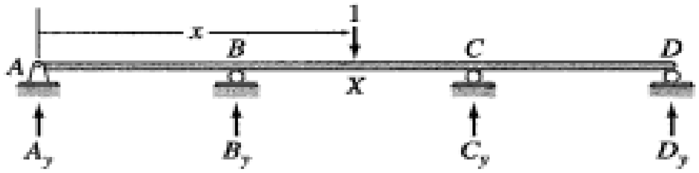
which represents the equation of the influence line for B_y. Note that the deflections f_{XB} and f_{BB} are considered to be positive when in the upward direction (i.e., in the positive direction of the redundant B_y) in accordance with the sign convention adopted for the method of consistent deformations in Chapter 13.

Equation (15.2) is more convenient to apply than Eq. (15.1) in constructing the influence line, because according to Eq. (15.2), the unit load needs to be placed on the primary beam only at B, and the deflections f_{XB} at a number of points X along the beam are to be computed. The influence line can then be constructed by plotting the values of the ratio $-f_{XB}/f_{BB}$ as ordinates against the distance x, which represents the position of point X, as abscissa.

The equation of an influence line, when expressed in the form of Eq. (15.2), shows the validity of *Müller-Breslau's principle* for statically indeterminate structures. It can be seen from Eq. (15.2) for the influence line for B_y that since f_{BB} is a constant, the ordinate of the influence line at any point X is proportional to the deflection f_{XB} of the primary beam at that point due to the unit load at B. Furthermore, this equation indicates that the influence line for B_y can be obtained by multiplying the deflected shape of the primary beam due to the unit load at B by the scaling factor $-1/f_{BB}$. Note that this scaling yields a deflected shape, with a unit displacement at B, as shown in Fig. 15.1(d). The foregoing observation shows the validity of Müller-Breslau's principle for indeterminate structures. Recall from Section 8.2 that, according to this principle, the influence line for B_y can be obtained by removing the support B from the original beam and by giving the released beam a unit displacement in the direction of B_y. Also, note from Fig. 15.1(d) that, unlike the case of statically determinate structures considered in Chapter 8, the removal of support B from the indeterminate beam does not render it statically unstable; therefore, the influence line for its reaction B_y is a curved line. Once the influence line for the redundant B_y has been determined, the influence lines for the remaining reactions and the shears and bending moments of the beam can be obtained through equilibrium considerations.

Influence Lines for Structures with Multiple Degrees of Indeterminacy

The procedure for constructing the influence lines for structures with multiple degrees of indeterminacy is similar to that for structures with a single degree of indeterminacy. Consider, for example, the three-span continuous beam shown in Fig. 15.2(a). Because the beam is statically



(a) Indeterminate Beam

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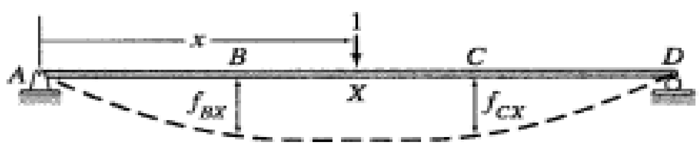

(b) Primary Beam Subjected to Unit Load

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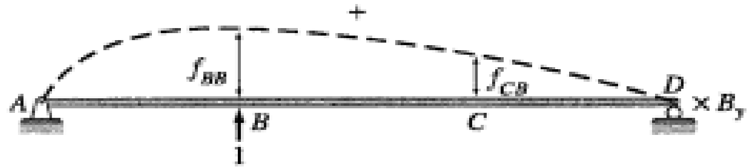

(c) Primary Beam Loaded with Redundant B_y

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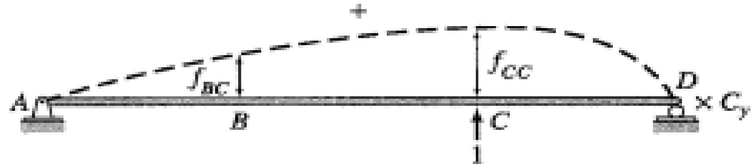


(d) Primary Beam Loaded with Redundant C_y

indeterminate to the second degree, we select the reactions B_y and C_y to be the redundants. To determine the influence lines for the redundants, we place a unit load successively at a number of positions X along the beam; and for each position of the unit load, the ordinates of the influence lines for B_y and C_y are evaluated by applying the compatibility equations (see Fig. 15.2(a) through (d))

$$f_{BX} + f_{BB}B_y + f_{BC}C_y = 0 \tag{15.3}$$

$$f_{CX} + f_{CB}B_y + f_{CC}C_y = 0 \tag{15.4}$$

Once the influence lines for the redundants have been obtained, the influence lines for the remaining reactions and the shears and bending moments of the beam can be determined by statics.

As discussed previously, the analysis can be considerably expedited by the application of *Maxwell's law of reciprocal deflections*, according to which $f_{BX} = f_{XB}$ and $f_{CX} = f_{XC}$. Thus, the unit load needs to be placed successively only at points B and C, and the deflections f_{XB} and

f_{XC} at a number of points X along the beam are computed instead of computing the deflections f_{BX} and f_{CX} at points B and C, respectively, for each of a number of positions of the unit load.

Procedure for Analysis

The procedure for constructing influence lines for statically indeterminate structures by the method of consistent deformations can be summarized as follows:

1. Determine the degree of indeterminacy of the structure and select redundants.
2. Select a number of points along the length of the structure at which the numerical values of the ordinates of the influence lines will be evaluated.
3. To construct the influence lines for the redundants, place a unit load successively at each of the points selected in step 2; and for each position of the unit load, apply the method of consistent deformations to compute the values of the redundants. Plot the values of the redundants thus obtained as ordinates against the position of the unit load as abscissa, to construct the influence lines for the redundants. (Evaluation of the deflections involved in the compatibility equations can be considerably expedited by the application of *Maxwell's law of reciprocal deflections*, as illustrated by Examples 15.1 through 15.3.)
4. Once the influence lines for the redundants have been determined, the influence lines for the other force and/or moment response functions of the structure can be obtained through equilibrium considerations.

Example 15.1

Draw the influence lines for the reaction at support B and the bending moment at point C of the beam shown in Fig. 15.3(a).

Solution

The beam has one degree of indeterminacy. We select the vertical reaction B_y at the roller support B to be the redundant. The ordinates of the influence lines will be computed at 3-m intervals at points A through E, as shown in Fig. 15.3(a).

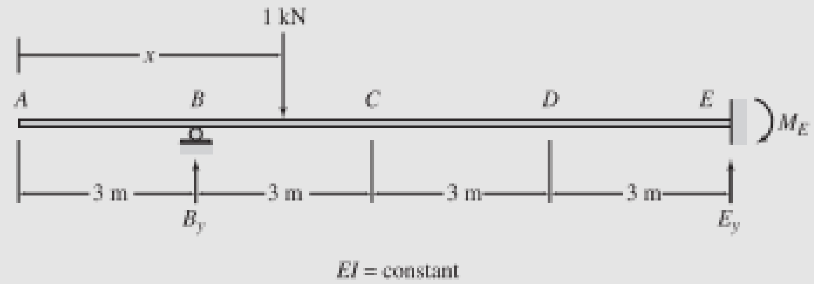
Influence Line for Redundant B_y The value of the redundant B_y for an arbitrary position X of the unit load can be determined by solving the compatibility equation (see Fig. 15.3(b) and (c))

$$f_{BX} + \bar{f}_{BB} B_y = 0$$

from which

$$B_y = -\frac{f_{BX}}{\bar{f}_{BB}} \tag{1}$$

continued



(a) Indeterminate Beam

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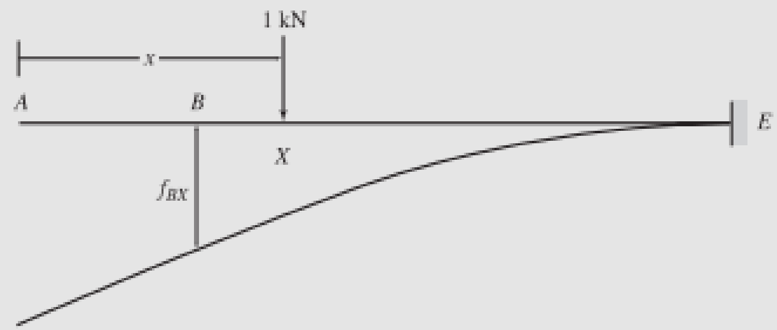

(b) Primary Beam Subjected to Unit Load

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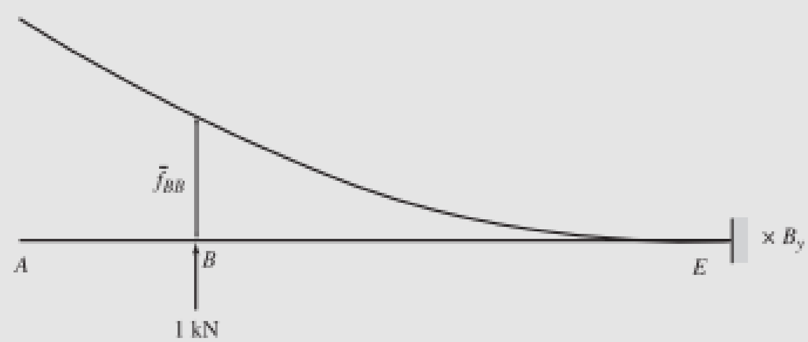

FIG. 15.3 (c) Primary Beam Loaded with Redundant B_y

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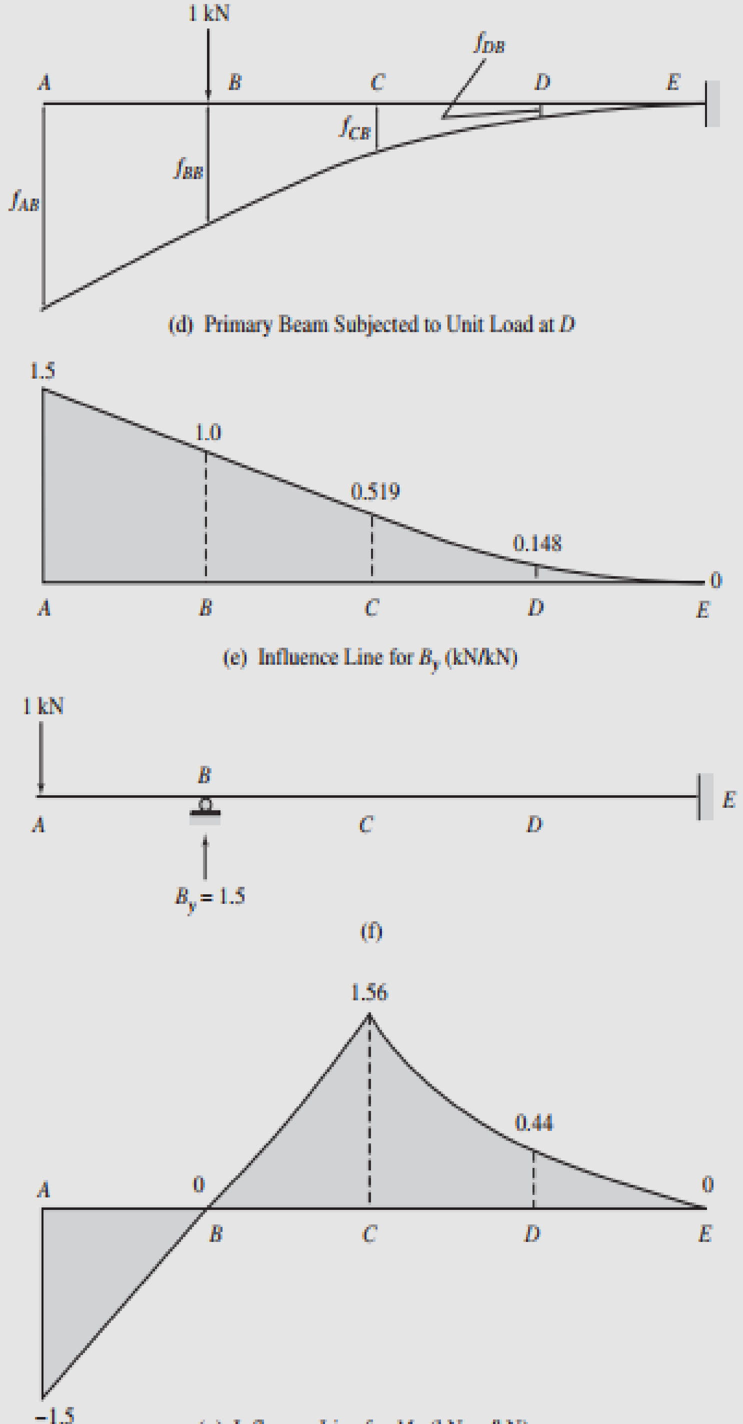

(d) Primary Beam Subjected to Unit Load at *D*

(e) Influence Line for B_y (kN/kN)

(f)

(g) Influence Line for M_C (kN–m/kN)

FIG. 15.3 (contd.)

continued

Since by Maxwell's law of reciprocal deflections, $f_{BX} = f_{XB}$, we place the unit load at B on the primary beam (Fig. 15.3(d)) and compute the deflections at points A through E by using the beam-deflection formulas given inside the front cover of the book. Thus,

$$f_{BA} = f_{AB} = -\frac{364.5 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{BB} = -\frac{243 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{BC} = f_{CB} = -\frac{126 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{BD} = f_{DB} = -\frac{36 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{BE} = f_{EB} = 0$$

in which the negative signs indicate that these deflections are in the downward direction. Note that the flexibility coefficient $\bar{f}_{BB}$ in Eq. (1) denotes the upward (positive) deflection of the primary beam at B due to the unit value of the redundant B_y (Fig. 15.3(c)), whereas the deflection f_{BB} represents the downward (negative) deflection at B due to the external unit load at B (Fig. 15.3(d)). Thus,

$$\bar{f}_{BB} = -f_{BB} = +\frac{243 \text{ kN–m}^3/\text{kN}}{EI}$$

The ordinates of the influence line for B_y can now be evaluated by applying Eq. (1) successively for each position of the unit load. For example, when the unit load is located at A, the value of B_y is obtained as

$$B_y = -\frac{f_{BA}}{\bar{f}_{BB}} = \frac{364.5}{243} = 1.5 \text{ kN/kN}$$

The remaining ordinates of the influence line for B_y are calculated in a similar manner. These ordinates are tabulated in Table 15.1, and the influence line for B_y is shown in Fig. 15.3(e). Ans.

TABLE 15.1

Unit Load at	Influence Line Ordinates: B_y (kN/kN)	Influence Line Ordinates: M_C (kN–m/kN)
A	1.5	−1.5
B	1.0	0
C	0.519	1.56
D	0.148	0.44
E	0	0

Influence Line for M_C With the influence line for B_y known, the ordinates of the influence line for the bending moment at C can now be evaluated by placing the unit load successively at points A through E on the indeterminate beam and by using the corresponding values of B_y computed previously. For example, as depicted in Fig. 15.3(f), when the unit load is located at point A, the value of the reaction at B is $B_y = 1.5$ kN/kN. By considering the equilibrium of the free body of the portion of the beam to the left of C, we obtain

$$M_C = -1(6) + 1.5(3) = -1.5 \text{ kN–m/kN}$$

The values of the remaining ordinates of the influence line are calculated in a similar manner. These ordinates are listed in Table 15.1, and the influence line for M_C is shown in Fig. 15.3(g). Ans.

Example 15.2

Draw the influence lines for the vertical reactions at the supports and the shear and bending moment at point C of the two-span continuous beam shown in Fig. 15.4(a).

Solution

The beam is indeterminate to the first degree. We select the vertical reaction D_y at the interior support D as the redundant. The influence line ordinates will be evaluated at 2 m intervals at points A through F shown in Fig. 15.4(a).

Influence Line for Redundant D_y The value of the redundant D_y for an arbitrary position X of the unit load can be determined by solving the compatibility equation (see Fig. 15.4(b) and (c))

$$f_{DX} + \bar{f}_{DD} D_y = 0$$

from which

$$D_y = -\frac{f_{DX}}{\bar{f}_{DD}} \tag{1}$$

Since $f_{DX} = f_{XD}$ in accordance with Maxwell's law, we place the unit load at D on the primary beam (Fig. 15.4(d)) and compute the deflections at points A through F by using the conjugate-beam method. The conjugate beam is shown in Fig. 15.4(e), from which we obtain the following:

$$f_{DA} = f_{AD} = 0$$

$$f_{DB} = f_{BD} = -\frac{1}{EI}\left[3.44(2) - \frac{1}{2}(2)(0.4)\left(\frac{2}{3}\right)\right] = -\frac{6.613 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DC} = f_{CD} = -\frac{1}{EI}\left[3.44(4) - \frac{1}{2}(4)(0.8)\left(\frac{4}{3}\right)\right] = -\frac{11.627 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DD} = -\frac{1}{EI}\left[3.44(6) - \frac{1}{2}(6)(1.2)\left(\frac{6}{3}\right)\right] = -\frac{13.44 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DE} = f_{ED} = -\frac{1}{EI}\left[4.96(2) - \frac{1}{2}(2)(1.2)\left(\frac{2}{3}\right)\right] = -\frac{9.12 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DF} = f_{FD} = 0$$

in which the negative signs indicate that these deflections occur in the downward direction. Note that the flexibility coefficient $\bar{f}_{DD}$ in Eq. (1) denotes the upward (positive) deflection of the primary beam at D due to the unit value of the redundant D_y (Fig. 15.4(c)), whereas the deflection f_{DD} represents the downward (negative) deflection at D due to the external unit load at D (Fig. 15.4(d)). Thus

$$\bar{f}_{DD} = -f_{DD} = +\frac{13.44 \text{ kN-m}^3/\text{kN}}{EI}$$

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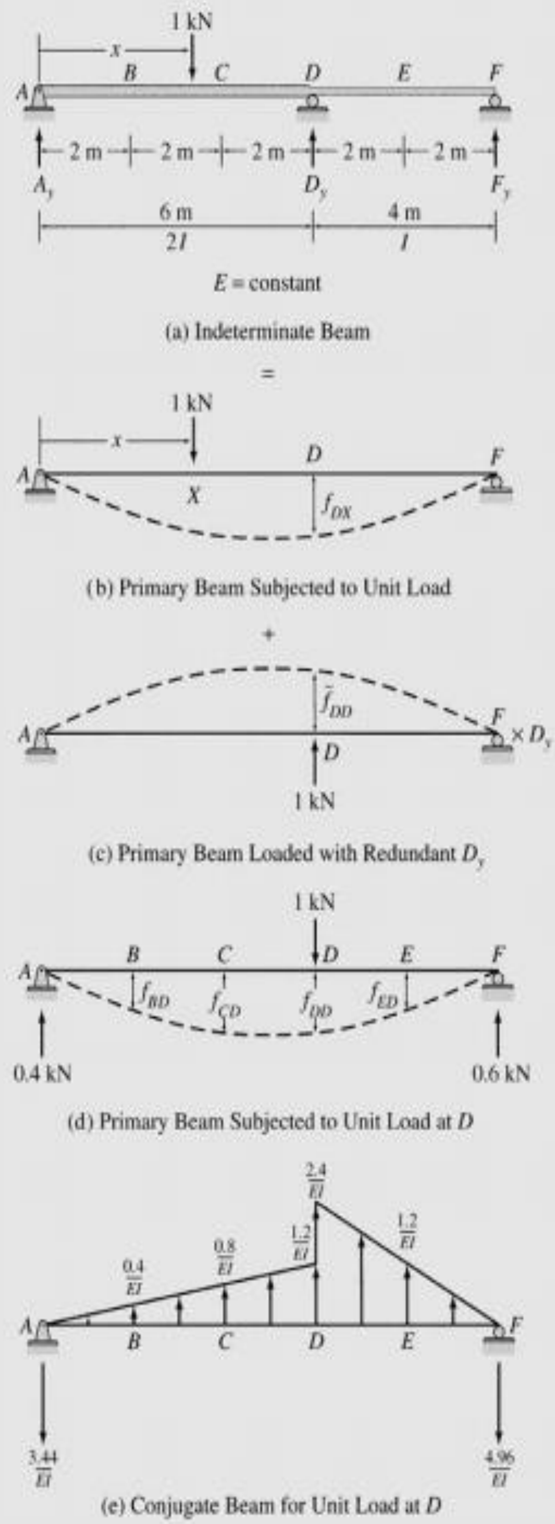

FIG. 15.4

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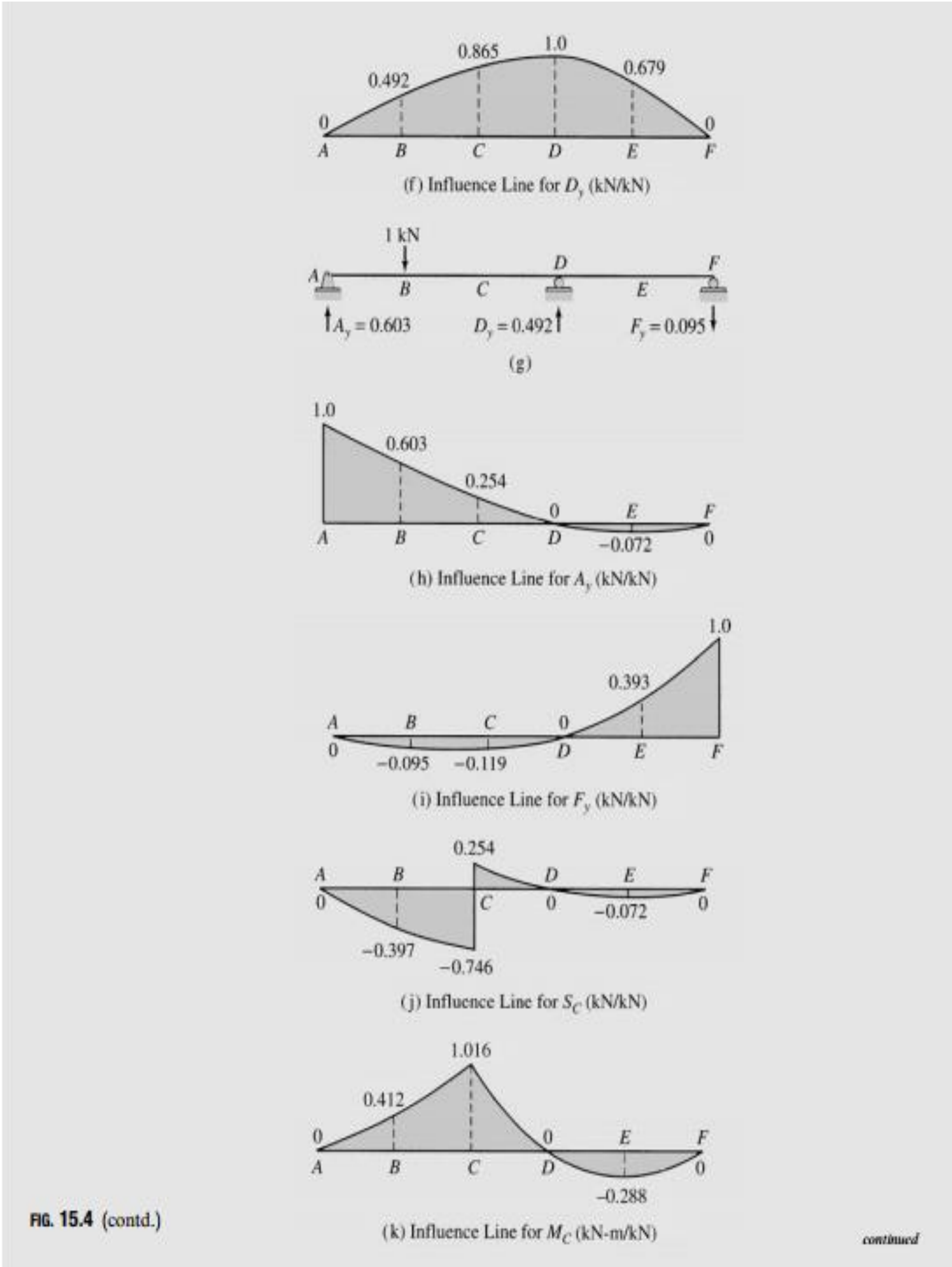

(f) Influence Line for D_y (kN/kN)

(g)

(h) Influence Line for A_y (kN/kN)

(i) Influence Line for F_y (kN/kN)

(j) Influence Line for S_C (kN/kN)

(k) Influence Line for M_C (kN-m/kN)

FIG. 15.4 (contd.)

TABLE 15.2

Unit Load at	Influence Line Ordinates				
	D_y (kN/kN)	A_y (kN/kN)	F_y (kN/kN)	S_C (kN/kN)	M_C (kN-m/kN)
A	0	1.0	0	0	0
B	0.492	0.603	−0.095	−0.397	0.412
C	0.865	0.254	−0.119	−0.746 (left) 0.254 (right)	1.016
D	1.0	0	0	0	0
E	0.679	−0.072	0.393	−0.072	−0.288
F	0	0	1.0	0	0

The ordinates of the influence line for D_y can now be computed by applying Eq. (1) successively for each position of the unit load. For example, when the unit load is located at *B*, the value of D_y is given by

$$D_y = -\frac{f_{DB}}{f_{DD}} = \frac{6.613}{13.44} = 0.492 \text{ kN/kN}$$

The remaining ordinates of the influence line for D_y are computed in a similar manner. These ordinates are tabulated in Table 15.2, and the influence line for D_y is shown in Fig. 15.4(f). Ans.

Influence Lines for A_y and F_y With the influence line for D_y known, the influence lines for the remaining reactions can now be determined by applying the equations of equilibrium. For example, for the position of the unit load at point *B* as shown in Fig. 15.4(g), the value of the reaction D_y has been found to be 0.492 kN/kN. By applying the equilibrium equations, we determine the values of the reactions A_y and F_y to be

$$+\circlearrowleft \sum M_F = 0 \qquad -A_y(10) + 1(8) - 0.492(4) = 0$$

$$A_y = 0.603 \text{ kN/kN} \uparrow$$

$$+\uparrow \sum F_y = 0 \qquad 0.603 - 1 + 0.492 + F_y = 0$$

$$F_y = -0.095 \text{ kN/kN} = 0.095 \text{ kN/kN} \downarrow$$

The values of the remaining influence line ordinates are computed in a similar manner. These ordinates are listed in Table 15.2, and the influence lines for A_y and F_y are shown in Fig. 15.4(h) and (i), respectively. Ans.

Influence Lines for S_C and M_C The ordinates of the influence lines for the shear and bending moment at *C* can now be evaluated by placing the unit load successively at points *A* through *F* on the indeterminate beam and by using the corresponding values of the reactions computed previously. For example, as shown in Fig. 15.4(g), when the unit load is located at point *B*, the values of the reactions are $A_y = 0.603$ kN/kN; $D_y = 0.492$ kN/kN; and $F_y = -0.095$ kN/kN. By considering the equilibrium of the free body of the portion of the beam to the left of *C*, we obtain

$$S_C = 0.603 - 1 = -0.397 \text{ kN/kN}$$

$$M_C = 0.603(4) - 1(2) = 0.412 \text{ kN-m/kN}$$

The values of the remaining ordinates of the influence lines are computed in a similar manner. These ordinates are listed in Table 15.2, and the influence lines for the shear and bending moment at *C* are shown in Fig. 15.4(j) and (k), respectively. Ans.

Example 15.3

Draw the influence lines for the reactions at supports for the beam shown in Fig. 15.5(a).

Solution

The beam is indeterminate to the second degree. We select the vertical reactions D_y and G_y at the roller supports D and G, respectively, to be the redundants. The influence line ordinates will be evaluated at 5-m intervals at points A through G shown in Fig. 15.5(a).

Influence Lines for Redundants D_y and G_y The values of the redundants D_y and G_y for an arbitrary position X of the unit load can be determined by solving the compatibility equations (see Fig. 15.5(b) through (d)):

$$f_{DX} + \bar{f}_{DD}D_y + \bar{f}_{DG}G_y = 0 \quad (1)$$

$$f_{GX} + \bar{f}_{GD}D_y + \bar{f}_{GG}G_y = 0 \quad (2)$$

Since by Maxwell's law, $f_{DX} = f_{XD}$, we place the unit load at D on the primary beam (Fig. 15.5(e)) and compute the deflections at points A through G by using the beam-deflection formulas given inside the front cover of the book. Thus,

$$f_{DA} = f_{AD} = 0$$

$$f_{DB} = f_{BD} = -\frac{166.667 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{DC} = f_{CD} = -\frac{583.333 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{DD} = -\frac{1{,}125 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{DE} = f_{ED} = -\frac{1{,}687.5 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{DF} = f_{FD} = -\frac{2{,}250 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{DG} = f_{GD} = -\frac{2{,}812.5 \text{ kN–m}^3/\text{kN}}{EI}$$

Similarly, the deflections $f_{GX} = f_{XG}$ are computed by placing the unit load at G (Fig. 15.5(f)):

$$f_{GA} = f_{AG} = 0$$

$$f_{GB} = f_{BG} = -\frac{354.167 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{GC} = f_{CG} = -\frac{1{,}333.333 \text{ kN–m}^3/\text{kN}}{EI}$$

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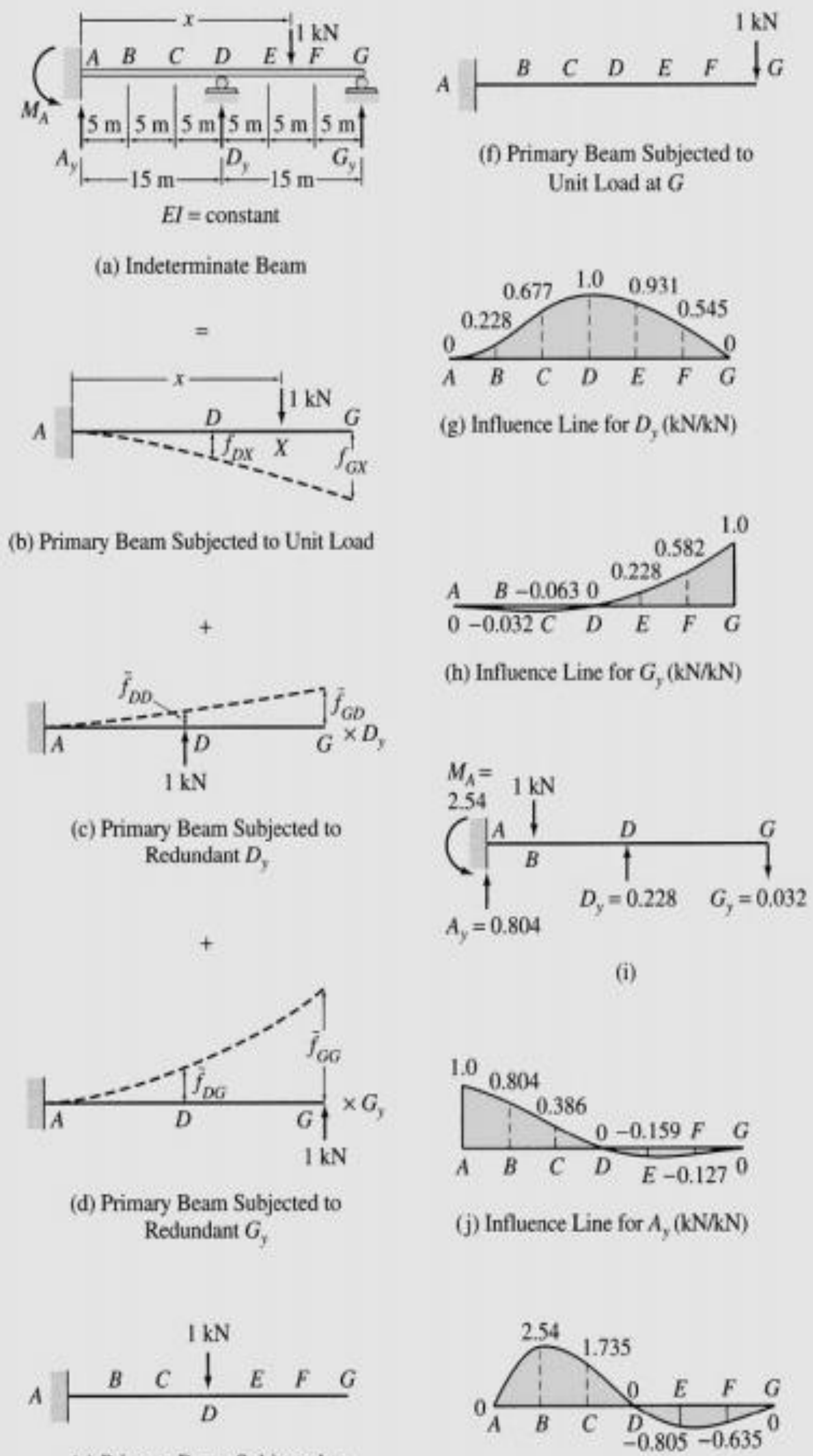


(a) Indeterminate Beam

(b) Primary Beam Subjected to Unit Load

(c) Primary Beam Subjected to Redundant D_y

(d) Primary Beam Subjected to Redundant G_y

(e) Primary Beam Subjected to Unit Load at D

(f) Primary Beam Subjected to Unit Load at G

(g) Influence Line for D_y (kN/kN)

(h) Influence Line for G_y (kN/kN)

(i)

(j) Influence Line for A_y (kN/kN)

(k) Influence Line for M_A (kN–m/kN)

FIG. 15.5

continued

$$f_{GE} = f_{EG} = -\frac{4{,}666.667 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{GF} = f_{FG} = -\frac{6{,}770.833 \text{ kN–m}^3/\text{kN}}{EI}$$

$$f_{GG} = -\frac{9{,}000 \text{ kN–m}^3/\text{kN}}{EI}$$

In these equations the negative signs indicate that these deflections are in the downward direction.

The upward deflections due to the unit values of the redundants (Fig. 15.5(c) and (d)) are given by

$$\bar{f}_{DD} = +\frac{1{,}125 \text{ kN–m}^3/\text{kN}}{EI}$$

$$\bar{f}_{DG} = \bar{f}_{GD} = +\frac{2{,}812.5 \text{ kN–m}^3/\text{kN}}{EI}$$

$$\bar{f}_{GG} = +\frac{9{,}000 \text{ kN–m}^3/\text{kN}}{EI}$$

By substituting the numerical values of these flexibility coefficients into the compatibility equations (Eqs. (1) and (2)) and solving for D_y and G_y, we obtain

$$D_y = \frac{EI}{1{,}968.75}(-8f_{DX} + 2.5f_{GX}) \tag{3}$$

$$G_y = \frac{EI}{1{,}968.75}(2.5f_{DX} - f_{GX}) \tag{4}$$

The values of the redundants D_y and G_y for each position of the unit load can now be determined by substituting the corresponding values of the deflections f_{DX} and f_{GX} into Eqs. (3) and (4). For example, the ordinates of the influence lines for D_y and G_y for the position of the unit load at B can be computed by substituting $f_{DX} = f_{DB} = -166.667/EI$ and $f_{GX} = f_{GB} = -354.167/EI$ into Eqs. (3) and (4):

$$D_y = \frac{EI}{1{,}968.75}\left[-8\left(-\frac{166.667}{EI}\right) + 2.5\left(-\frac{354.167}{EI}\right)\right] = 0.228 \text{ kN/kN} \uparrow$$

$$G_y = \frac{EI}{1{,}968.75}\left[2.5\left(-\frac{166.667}{EI}\right) + \frac{354.167}{EI}\right] = -0.032 \text{ kN/kN}$$

$$= 0.032 \text{ kN/kN} \downarrow$$

The remaining ordinates of the influence lines for the redundants are computed in a similar manner. These ordinates are tabulated in Table 15.3, and the influence lines for D_y and G_y are shown in Fig. 15.5(g) and (h), respectively. Ans.

Influence Lines for A_y and M_A The ordinates of the influence lines for the remaining reactions can now be determined by placing the unit load successively at points A through G on the indeterminate beam and by applying the equations of equilibrium. For example, for the position of the unit load at B (Fig. 15.5(i)), the values of the reactions D_y and G_y have been found to be 0.228 kN/kN and −0.032 kN/kN, respectively. By considering the equilibrium of the beam, we determine the values of the reactions A_y and M_A to be as follows:

$$+\uparrow \sum F_y = 0 \qquad A_y - 1 + 0.228 - 0.032 = 0$$

$$A_y = 0.804 \text{ kN/kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0 \qquad M_A - 1(5) + 0.228(15) - 0.032(30) = 0$$

$$M_A = 2.54 \text{ kN–m/kN} \circlearrowleft$$

continued

TABLE 15.3

	Influence Line Ordinates			
Unit Load at	D_y (kN/kN)	G_y (kN/kN)	A_y (kN/kN)	M_A (kN–m/kN)
A	0	0	1.0	0
B	0.228	−0.032	0.804	2.540
C	0.677	−0.063	0.386	1.735
D	1.0	0	0	0
E	0.931	0.228	−0.159	−0.805
F	0.545	0.582	−0.127	−0.635
G	0	1.0	0	0

The values of the remaining influence line ordinates are computed in a similar manner. These ordinates are listed in Table 15.3, and the influence lines for A_y and M_A are shown in Fig. 15.5(j) and (k), respectively. Ans.

Example 15.4

Draw the influence lines for the forces in members *BC*, *BE*, and *CE* of the truss shown in Fig. 15.6(a). Live loads are transmitted to the top chord of the truss.

Solution

The truss is internally indeterminate to the first degree. We select the axial force F_{CE} in the diagonal member *CE* to be the redundant.

Influence Line for Redundant F_{CE} To determine the influence line for F_{CE}, we place a unit load successively at joints *B* and *C* of the truss, and for each position of the unit load, we apply the method of consistent deformations to compute the value of F_{CE}. The primary truss, obtained by removing member *CE*, is subjected separately to the unit load at *B* and *C*, as shown in Fig. 15.6(b) and (c), respectively, and a unit tensile force in the redundant member *CE*, as shown in Fig. 15.6(d).

When the unit load is located at *B*, the compatibility equation can be expressed as

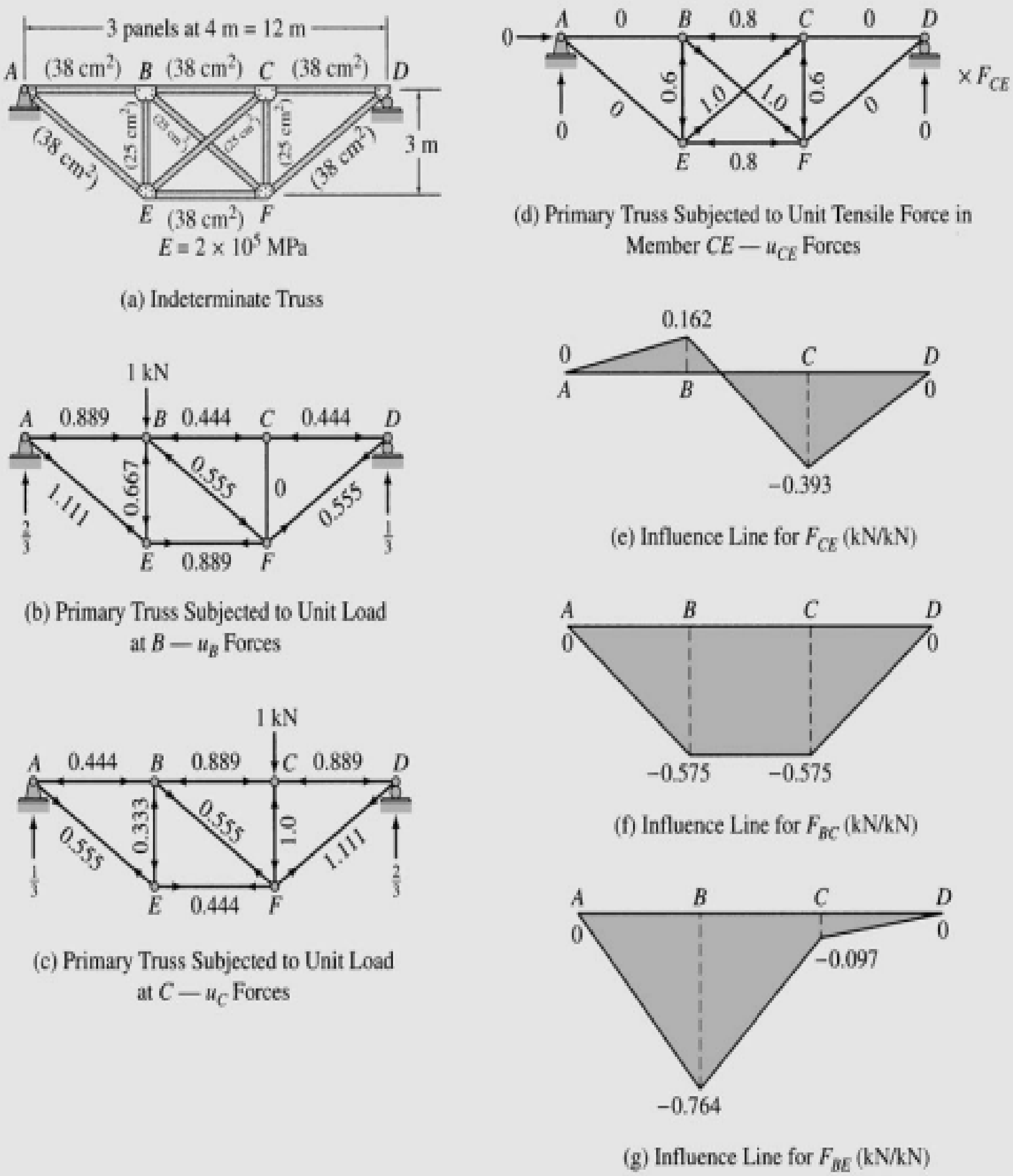
$$f_{CE,B} + f_{CE,CE}F_{CE} = 0$$

in which $f_{CE,B}$ denotes the relative displacement between joints *C* and *E* of the primary truss due to the unit load at *B* and $f_{CE,CE}$ denotes the relative displacements between the same joints due to a unit value of the redundant F_{CE}. Applying the virtual work method (see Fig. 15.6(b) and (d) and Table 15.4), we obtain

$$f_{CE,B} = \frac{1}{E}\sum \frac{u_B u_{CE} L}{A} = -\frac{1004.49}{E}$$

$$f_{CE,CE} = \frac{1}{E}\sum \frac{u_{CE}^2 L}{A} = \frac{6211.36}{E}$$

continued



(a) Indeterminate Truss

(b) Primary Truss Subjected to Unit Load at B — u_B Forces

(c) Primary Truss Subjected to Unit Load at C — u_C Forces

(d) Primary Truss Subjected to Unit Tensile Force in Member CE — u_{CE} Forces

(e) Influence Line for F_{CE} (kN/kN)

(f) Influence Line for F_{BC} (kN/kN)

(g) Influence Line for F_{BE} (kN/kN)

FIG. 15.6

By substituting these numerical values into the compatibility equation, we determine the ordinate of the influence line for F_{CE} at B to be

$$F_{CE} = 0.162 \text{ kN/kN (T)}$$

Similarly, when the unit load is located at C, the compatibility equation is given by

$$f_{CE,C} + f_{CE,CE}F_{CE} = 0$$

continued

TABLE 15.4

Member	L (m)	A (cm^2)	u_B (kN/kN)	u_C (kN/kN)	u_{CE} (kN/kN)	$\frac{u_B u_{CE} L}{A}$	$\frac{u_C u_{CE} L}{A}$	$\frac{u_{CE}^2 L}{A}$
AB	4	38	−0.889	−0.444	0	0	0	0
BC	4	38	−0.444	−0.889	−0.8	373.9	748.63	673.68
CD	4	38	−0.444	−0.889	0	0	0	0
EF	4	38	0.889	0.444	−0.8	−748.63	−373.9	673.68
BE	3	25	−0.667	−0.333	−0.6	480.24	239.8	432.0
CF	3	25	0	−1.0	−0.6	0	720.0	432.0
AE	5	38	1.111	0.555	0	0	0	0
BF	5	25	−0.555	0.555	1.0	−1110	1110	2000
CE	5	25	0	0	1.0	0	0	2000
DF	5	38	0.555	1.111	0	0	0	0
					$\sum$	−1004.49	2444.53	6211.36

(see Fig. 15.6(c) and (d) and Table 15.4) in which

$$f_{CE,C} = \frac{1}{E}\sum \frac{u_C u_{CE} L}{A} = \frac{2444.53}{E}$$

By substituting the numerical values of $f_{CE,C}$ and $f_{CE,CE}$ into the compatibility equation, we determine the ordinate of the influence line for F_{CE} at C to be

$$F_{CE} = -0.393 \text{ kN/kN} = 0.393 \text{ kN/kN (C)}$$

The influence line for F_{CE} is shown in Fig. 15.6(e). Ans.

Influence Lines for F_{BC} and F_{BE} The ordinate at B of the influence line for force in any member of the truss can be determined by the superposition relationship (see Fig. 15.6(b) and (d) and Table 15.4)

$$F = u_B + u_{CE} F_{CE}$$

in which F_{CE} denotes the ordinate at B of the influence line for the redundant F_{CE}. Thus the ordinates at B of the influence lines for F_{BC} and F_{BE} are

$$F_{BC} = -0.444 + (-0.8)(0.162) = -0.575 \text{ kN/kN} = 0.575 \text{ kN/kN (C)}$$

$$F_{BE} = -0.667 + (-0.6)(0.162) = -0.764 \text{ kN/kN} = 0.764 \text{ kN/kN (C)}$$

Similarly, the ordinates of the influence lines for F_{BC} and F_{BE} at C can be determined by using the superposition relationship (see Fig. 15.6(c) and (d) and Table 15.4)

$$F = u_C + u_{CE} F_{CE}$$

in which F_{CE} now denotes the ordinate at C of the influence line for the redundant F_{CE}. Thus

$$F_{BC} = -0.889 + (-0.8)(-0.393) = -0.575 \text{ kN/kN} = 0.575 \text{ kN/kN (C)}$$

$$F_{BE} = -0.333 + (-0.6)(-0.393) = -0.097 \text{ kN/kN} = 0.097 \text{ kN/kN (C)}$$

The influence lines for F_{BC} and F_{BE} are shown in Fig. 15.6(f) and (g), respectively. Ans.