Student ID: ....

ADDIS ABABA INSTITUTE OF TECHNOLOGY
አዲስ አበባ ቴክኖሎጂ ኢንስቲትዩት
ADDIS ABABA UNIVERSITY
አዲስ አበባ ዩኒቨርሲቲ

Academic contact during examination: Tewodros Gemechu

Phone: +251911394427

SCEE, Geotechnical Engineering Chair

Solution Set for CEng3143 Fundamentals of Geotechnical Engineering - III

Examination date: 13rd January 2020

Examination time (from-to): 13:30 - 16:30

Permitted exam support material: No printed or handwritten material,

Number of pages enclosed: **10 including cover page**

Instruction:
- Write your full ID number on each page of this examination.
- Provide concise answers for theoretical questions.
- Show your steps clearly for problems involving calculations.

Examination paper set checked by: **Asrat Worku Setegn (Dr.-Ing.)**

Date Signature

Question #	Weight [%]	Score [%]
1	10	
2	10	
3	20	
4	30	
5	30	

Question 1: Soil Compressibility & Settlement Analysis [10%]

1.1 Explain the difference between soil deformation modulus and the modulus of subgrade reaction. Comment on which one is a true soil property and why. (6%)

The modulus of deformation (kN/m²) is defined by the equations of elasticity and as the slope of the line of a stress-strain curve of a material in the case of an unconfined compression test.

[2 marks]

The modulus of subgrade reaction (kN/m³) is the ratio of pressure p applied to the boundary through a loading area divided by the displacements experienced by the loaded area.

[2 marks]

Only the modulus of deformation is a true soil property, because modulus of subgrade reaction depends on the size of the loaded area.
The results of stiffness and modulus of subgrade reaction in one test will be different from the results of other tests with different areas. The modulus of deformation for the same material is not affected by the size of the loaded area.

[2 marks]

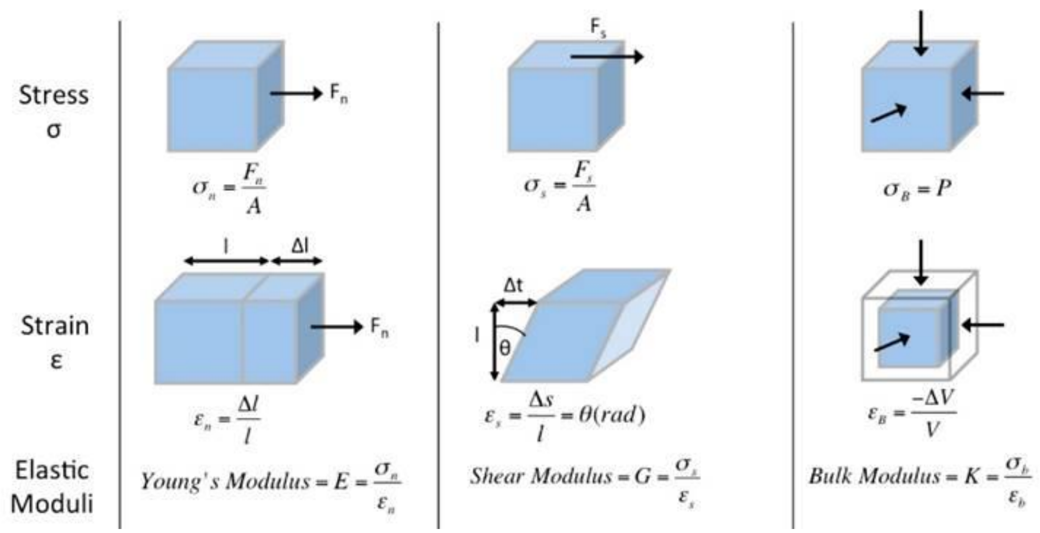
1.2 Differentiate between bulk modulus and shear modulus of soil. Use diagrams to illustrate your answer. (4%)

The shear modulus, G, relates to strain response of a body to shear or torsional stress. It involves change of shape without change of volume.

[1 mark]

On the other hand, the bulk modulus, K, describes the strain response of a body to hydrostatic stress involving change in volume without change of shape.

[1 mark]

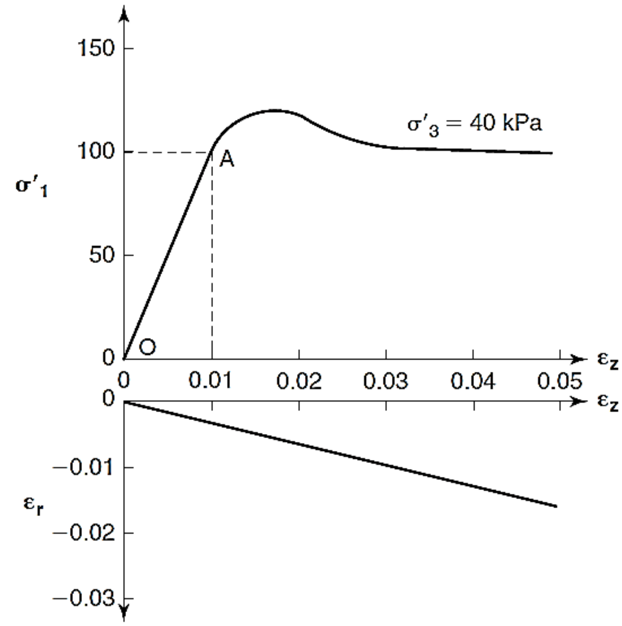


[2 marks]

Question 2: Shear Strength of Soils [10%]

2.1 Consider the stress-strain curve from a triaxial test shown in figure below:

a. Why is $\varepsilon_r < 0$ when $\varepsilon_z > 0$? (2%)



In a triaxial test, when we apply an incremental vertical load to the cylindrical specimen, it will compress in longitudinal direction and the radius will increase.
A negative sign is usually used for expansion and a positive sign for compression.

[2 marks]

b. Calculate Poisson's ratio. (2%)

$$\upsilon = \frac{-\varepsilon_r\sigma_1 + \varepsilon_z\sigma_3}{\varepsilon_z\sigma_1 + \varepsilon_z\sigma_3 - 2\varepsilon_r\sigma_3} = \frac{-(-0.004)\times 100 + 0.01\times 40}{0.01\times 100 + 0.01\times 40 - 2\times(-0.004)\times 40} = 0.47$$

[2 marks]

2.2 It is generally recommended not to use direct shear test for determination of shear strength parameters of fine-grained soils. Articulate the reason behind this recommendation. If you absolutely need to use direct shear test for fine-grained soil (for whatever reason), how would you go about doing that to achieve unbiased outputs? (4 %)

The major reason for geotechnical engineers opt to not generally use direct shear test for determination of shear strength parameters of fine grained soils is because the excess pore pressure developed in the specimen will offset the results significantly. If one had no choice but use the test, she would have to synchronize the rate of loading with that of the rate of expulsion of water from the specimen so as to avoid (minimize) the excess pore pressure.

[2 marks]

2.3 Imagine you are driving on a rural road constructed of predominantly fine-grained soil. You have come across a section logged with water. Should you drive fast or slowly to avoid getting stuck in the mud? Explain your answer from the perspective of loading conditions associated with soil shear strength. (2%)

Driving faster would be the logical choice in this context so as to activate undrained loading condition and utilize the undrained shear strength of the soil.

[2 marks]

Question 3: Lateral Earth Pressure [20%]

3.1 Explain the conditions at which Rankine's earth pressure theory and Coulomb's earth pressure theory give equal results. (5%)

Upon fulfillment of the following conditions / assumptions

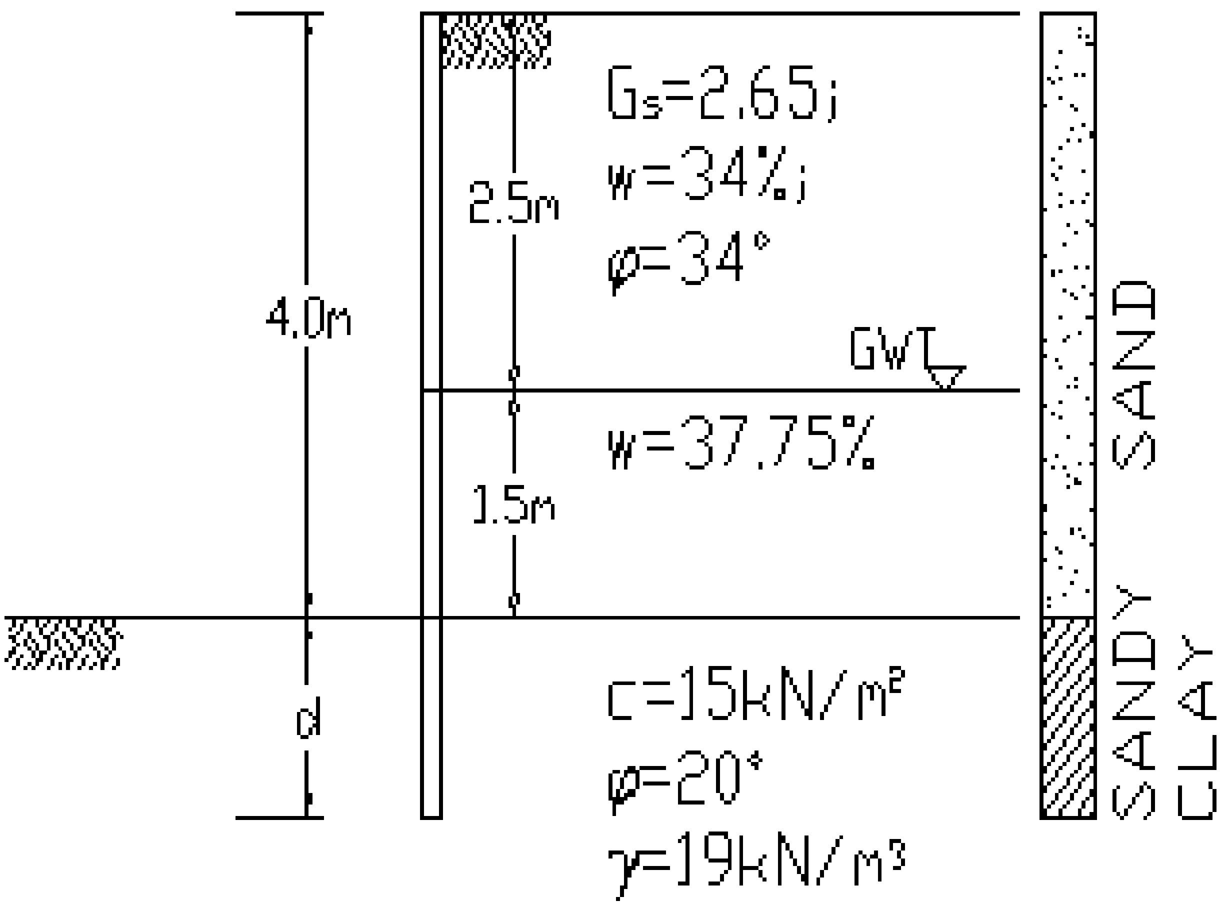
- ***Soil mass is semi-infinite, homogeneous, dry, cohesionless.***

[3 marks]

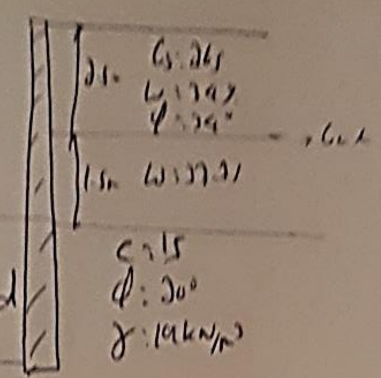
- ***The face of the wall in contact with the backfill is vertical and smooth.***

[2 marks]

3.2 Find the minimum depth of embedment, d_{min}, of the rigid wall shown in the figure below so that it will not fail by sliding. Use Rankine's theory for lateral earth pressure distribution and neglect the sliding resistance at the base of the wall. (15 %)



Given



Solution

assuming fully saturated (for the layer below the GWT.)

$e = \frac{wG_s}{S} = 0.3773 \times 2.65$

$e \approx 1$

$\gamma_{dry} = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 10}{2} = 13.25\ kN/m^3$ (1)

$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+1) \times 10}{2} = 18.25\ kN/m^3$ (1)

layer 1

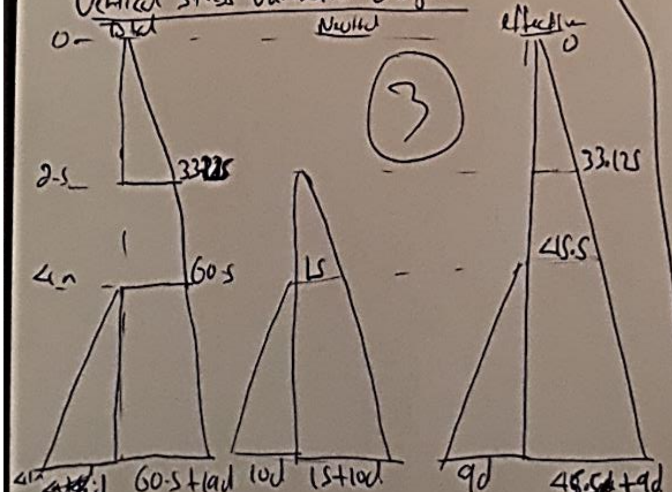
$K_{a_1} = 1-\sin\phi / 1+\sin\phi = \frac{1-\sin 34}{1+\sin 34} = 0.283$ — (1)

layer 2

$K_{a_2} = 1-\sin\phi / 1+\sin\phi = \frac{1-\sin 20}{1+\sin 20} = 0.49$ — (1)

$K_{p_1} = 1+\sin\phi / 1-\sin\phi = 1+\sin 20 / 1-\sin 20 = 2.04$ — (1)

Vertical Stress Variation diagram

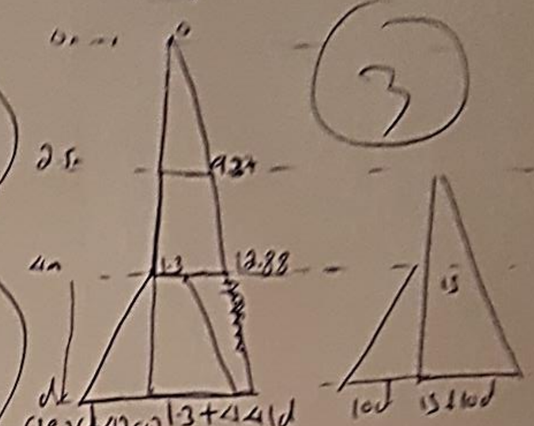


Lateral earth pressure

$P_a = \sigma_h - 2c\sqrt{k_a}$

$P_p = \sigma_h + 2c\sqrt{k_p}$

(1) drawing the lateral earth pressure diagram



(2) $P_a = P_p$

for active

$P_a = \frac{1}{2} \times 2.5 \times 9.37 + 3.51 \times 1.5 \times \frac{1}{2} + 9.37 \times 1.5 + 1.3 \times d + \frac{1}{2} \times 4.41 \times d$ (1)

for the neutral

$= \frac{1}{2} \times 1.5 \times 15$

$P_a = 39.65 + 1.3d + 2.205d^2$ — eq 1

for passive

$P_p = \frac{1}{2} \times d \times (18.36d + 42.84)$ (1)

$P_p = \frac{1}{2} d \times (18.36d + 42.84)$

$P_p = 9.18d^2 + 21.42d$ — eq 2

equation 1 & 2

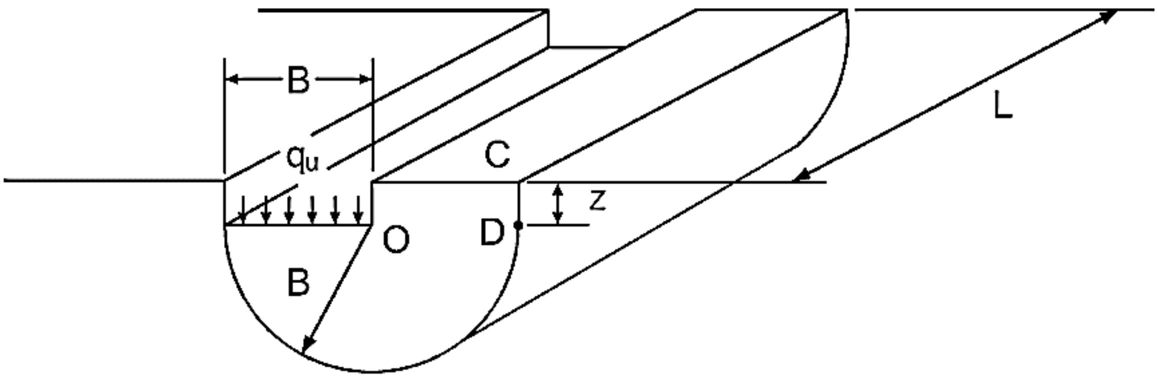
$9.18d^2 + 21.42d = 2.205d^2 + 1.3d + 39.65$

$6.975d^2 + 20.12d - 39.65 = 0$

$d = 1.34m$ (2)

Question 4: Bearing Capacity of Soils [30%]

4.1 Consider a foundation failing by rotation about one edge and founded at a depth z below the surface of a saturated clay of unit weight γ and undrained strength c_u as depicted in the figure below.



Derive the following bearing capacity formula for a strip footing by considering disturbing and resisting moments about Point O.

$$q_u = 6.28c_u\left(1 + 0.32\frac{z}{B} + 0.16\frac{\gamma z}{c_u}\right)$$

(10%)

Disturbing moment about O: $q_u \times LB \times \frac{B}{2} = \frac{q_u LB^2}{2}$

[2 marks]

Resisting moments about O:
Cohesion along cylindrical sliding surface $= c_u \pi LB$
Moment $= \pi c_u LB^2$

[2 marks]

Cohesion along CD $= c_u ZL$
Moment $= c_u ZLB$

[2 marks]

Weight of the soil above the foundation $= \gamma ZLB$
Moment $= \frac{\gamma ZLB^2}{2}$

[2 marks]

For equilibrium

$$\frac{q_u LB^2}{2} = \pi c_u LB^2 + c_u ZLB + \frac{\gamma ZLB^2}{2}$$

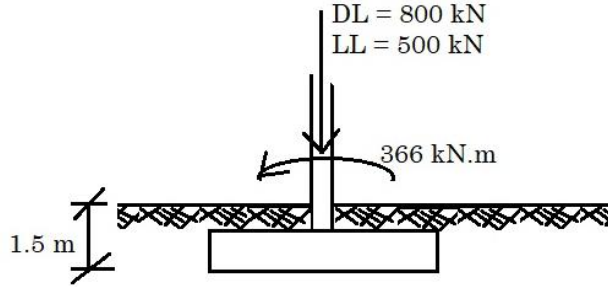
$$q_u = 2\pi c_u + \frac{2c_u z}{B} + \gamma Z$$

$$= 2\pi c_u\left(1 + \frac{1}{\pi}\frac{z}{B} + \frac{1}{2\pi}\frac{\gamma Z}{c_u}\right)$$

$$= 6.28c_u\left(1 + 0.32\frac{z}{B} + 0.16\frac{\gamma Z}{c_u}\right)$$

[2 marks]

4.2 Check whether or not overall stability (ULS) requirement is satisfied for the drained bearing resistance for the isolated foundation shown in the figure. A column exerts an unfactored 800 kN of deadload, 500 kN of unfactored live loads, and 366 kN.m of uniaxial moment.



- Use DA 2 (A1+M1+R2) for load combination
- Ignore self-weight of the foundation
- Foundation Area = 3m x 3m
- Effective unit weight of the soil (silty sand)– $\gamma' = 16\ kN/m^3$
- Other parameters – c' = 10 kPa, $\phi' = 20^0$

Partial factors on actions (γ_F) or the effects of actions (γ_E)

Action		Symbol	Set	
			A1	**A2**
Permanent	Unfavourable	γ_G	1.35	1.0
	Favourable		1.0	1.0
Variable	Unfavourable	γ_Q	1.5	1.3
	Favourable		0	0

$$s_q = 1 + \sin\phi'$$
$$s_c = \frac{s_q N_q - 1}{N_q - 1}$$
$$s_\gamma = 1 - 0.3\frac{B'}{L'}$$

Partial factors for soil parameters (γ_M)

Soil parameter	Symbol	Value	
		M1	**M2**
Shearing resistance	γ_φ[1]	1.0	1.25
Effective cohesion	γ_c	1.0	1.25
Undrained strength	γ_{cu}	1.0	1.4
Unconfined strength	γ_{qu}	1.0	1.4
Weight density	γ_γ	1.0	1.0

[1] This factor is applied to tan φ'

Partial resistance factors for spread foundations (γ_R)

Resistance	Symbol	Set		
		R1	**R2**	**R3**
Bearing	γ_{Rv}	1.0	1.4	1.0
Sliding	γ_{Rh}	1.0	1.1	1.0

Drained Bearing Capacity Equation

$$q = c'N_c b_c s_c i_c + q'N_q b_q s_q i_q + 1/2\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma$$

$$N_q = e^{\pi \times \tan\varphi'} \tan^2(45° + \varphi'/2)$$
$$N_c = (N_q - 1)\cot\varphi'$$
$$N_\gamma = 2(N_q - 1)\tan\varphi'$$

$$i_q = (1 - 0.70 \times H / (V + A' \times c' \times \cot\phi'))^m$$
$$m = m_B = [2 + (B'/L')]/[1 + (B'/L')]$$
$$m = m_L = [2 + (L'/B')]/[1 + (L'/B')]$$
$$m = m_\theta = m_L \cos^2\theta + m_B \sin^2\theta$$

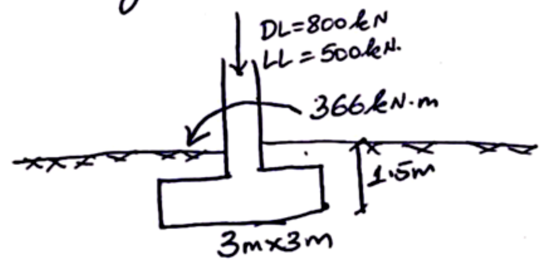
$$i_c = (i_q \times N_q - 1) / (N_q - 1)$$
$$i_\gamma = (1 - H / (V + A' \times c' \times \cot\phi'))^3$$

$$b_c = b_q - (1 - b_q) / (N_c \tan\phi')$$
$$b_q = b_\gamma = (1 - \alpha \tan\phi')^2$$

(20 %)

Solution Keys - Final Exam

20/20



$\gamma' = 16\ kN/m^3$
$\phi' = 20°$
$c' = 10kPa$

DA2 (A1 + M1 + R2) ▶ ULS

Factored Load → A1 → $\gamma_G = 1.35$ $\gamma_Q = 1.5$ (1.5)

$P_f \Rightarrow [1.35 \times 800] + [1.5 \times 500] = 1830kN$ (2)

$e = \frac{M}{P_f} = \frac{366kN.m}{1830kN} = 0.2m$ (1)

Contact Pressure

$q_{max/min} = \frac{P}{A}\left(1 \pm 6 \cdot \frac{e_b}{B} \pm 6\frac{e_L}{L}\right)$ ($e_L \to 0$)

$= \frac{1830kN}{9m^2}\left(1 \pm 6 \cdot \left(\frac{0.2m}{3m}\right)\right) = 284.67kPa / 122kPa$ (2)

Drained Bearing Resistance ... γ_ϕ & $\gamma_c = 1.0$ [M1] (1.5)

$q = c'N_c s_c i_c + q' N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma$

- No load inclination → $i = 1_{(c,q,\gamma)}$
- No base inclination → $b = 1_{(c,q,\gamma)}$

$N_q = e^{\pi \cdot \tan 20} \cdot \tan^2(45 + 20/2) = 6.399 \sim 6.4$ (1)

$N_c = (N_q - 1) \cdot \cot \phi = (6.4 - 1) \cdot \cot 20 = 14.836$ (1)

$N_\gamma = 2(N_q - 1)\tan\phi = 2(6.4 - 1) \tan 20 = 3.93$ (1)

Shape Factors

$S_q = 1 + \sin\phi' = 1 + \sin 20° = 1.342$ (1)

$S_c = \dfrac{S_q \cdot N_q - 1}{N_q - 1} = \dfrac{(1.342 \times 6.4) - 1}{6.4 - 1} = 1.405$ (1)

$S_\gamma = 1 - 0.3\left(\dfrac{B'}{L'}\right)$; there is a uniaxial moment, i.e eccentricity is along one direction only (B);

$B' = B - 2 \cdot e_b = 3m - 2(0.2) = 2.6m$
$L' = L = 3m$.

$S_\gamma = 1 - 0.3\left(\dfrac{2.6}{3.0}\right) = 0.74$ (1)

$16 kN/m^3 \times 1.5m = 24 kPa$

$\dfrac{R}{A'} = c'N_c S_c i_c + q'N_q b_q S_q i_q + \frac{1}{2}\gamma' B' N_\gamma S_\gamma i_\gamma$

$= (10 kPa \cdot 14.836 \cdot 1.405) + (24 kPa) \cdot (6.4 \cdot 1.342) + \frac{1}{2}(16)(2.6)(3.93)(0.74)$

$= 208.446$ (1) $+ 206.1312$ (1) $+ 60.49$ (1)

$= 475.0672 \, kPa$

Resistance Factor for bearing, $\gamma_R = 1.4$ (R2) (1.5)

$\gamma_R \overset{?}{\leq} \dfrac{q_{all(fact)}}{P_{fact}} \rightarrow 1.4 \overset{?}{\leq} \dfrac{475 kPa}{284.67 kPa}$

$1.4 \leq 1.67$ .... SAFE! (1.5)

(!!) This answer is applicable is the moment given is assumed to be factored. If not,

$M_{fact} = 494.1 \, kN \cdot m$
$e = 0.27m$
$q_{max} = 313.3$,

$S_\gamma = 0.754$
$q_u = 476.2 \, kPa$
$\gamma_R = 1.4 \leq \dfrac{476.2}{313.3} = 1.52$ ... SAFE!

-2-

Question 5: Soil Slope Stability [30%]

5.1 Investigation of the stability of finite slopes generally involves three steps according to the commonly adopted procedure. List these steps. (3 %)

a) Assuming a possible slip surface,

[1 mark]

b) Studying the equilibrium of forces acting on this surface (activating & resisting)

[1 mark]

c) Repeating the process until the worst slip surface, that is, the one with minimum margin of safety is found.

[1 mark]

5.2 An infinite slope is made of sand with a friction angle of 32◦ and a unit weight of 17 kN/m^3 as part of the currently under-construction Modjo-Hawassa express route. The slope angle is 2.5 horizontal to 1 vertical.

Calculate the factor of safety

a. in the “spring/በጋ/birraa” when the slope has no water,

b. in the “summer/ክረምት/bona” when the slope is filled with water,

c. for the same slope on the moon. (The acceleration due to gravity on the surface of the Moon is about 1.625 m/s^2 i.e. 16.6% of that on Earth’s surface. Assume that there is no water on the moon:) (7%)

For the case of sand with no water during the “spring/በጋ/birraa” when the slope has no water:

$$FS = \frac{\tan\phi'}{\tan\beta} = \frac{\tan 32}{1/2.5} = 1.56$$

[2 marks]

For the case of the sand filled with water during the “summer/ክረምት/bona” when the slope is filled with water, no cohesion, and assuming a saturated unit weight of 17 kN/m3:

$$FS = \frac{(\gamma_{sat} - \gamma_w)}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta} = \frac{(17 - 9.81)}{17} \frac{\tan 32}{1/2.5} = 0.66$$

[2 marks]

NB. The presence of water significantly reduces the factor of safety of the slope.

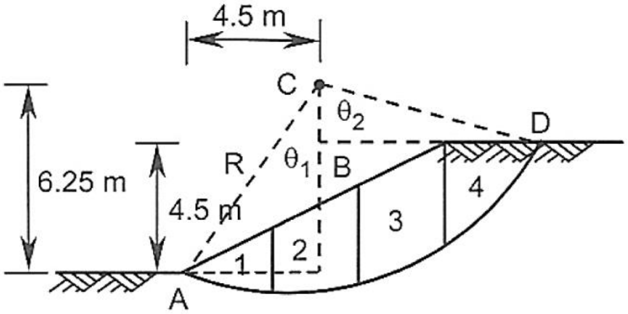
The factor of safety for dry sand is $FS = \frac{\tan\phi'}{\tan\beta}$

It is independent of gravity acceleration (g), so it would be the same on the moon.

[3 marks]

5.3 Using Fellenius' method, determine the factor of safety for a slope of 1 vertical to 2 horizontal and height H=4.5 m using a trial toe circle for which $x_c = 4.5$ m and $y_c = 6.25$ m. The soil mass is divided into 4 slices all having identical width of b = 3 m, whose average height and angle α are tabulated below. (Show a sample calculation for one of the slices)
The soil properties are as follows: c'= 6.75 kPa, ϕ'= 17° and ρ = 1.96 Mg/m³

(20 %)



$$F = \frac{\sum_{i=1}^{i=4}\left[c'l + (w\cos\alpha\tan\phi')\right]_i}{\sum_{i=1}^{i=4}(w\sin\alpha)_i}$$

Slice	h (m)	α (deg.)	w (kN) [4 marks]	wcosα (kN) [4 marks]	wsinα (kN) [4 marks]
1	1.6	-23	*92.29*	*84.95*	*-36.06*
2	3.7	0	*213.43*	*213.43*	*0.00*
3	4.6	23	*265.34*	*244.25*	*103.68*
4	3.0	51	*173.05*	*108.90*	*134.48*
Total				*651.53*	*202.10*

Sample calculation for slice 3:
w = b * h * 1.0 * ρ * 9.81 = 3.0 * 4.6 * 1.96 * 9.81 = 265.34 kN
wcosα = 265.34 * cos 23.0° = 244.25 kN
wsinα = 265.34 * sin 23.0° = 103.68 kN

$\boldsymbol{R = \sqrt{4.5^2 + 6.25^2} = 7.7\ m}$

[1 mark]

$\boldsymbol{\tan\theta_1 = 4.5/6.25 = 0.72 \rightarrow \theta_1 = 35.75° \ \&\ \cos\theta_2 = (6.25 - 4.5)/7.7 = 0.2273 \rightarrow \theta_2 = 76.86°}$

$\boldsymbol{\theta_1 + \theta_2 = 35.75° + 76.86° = 112.61°}$

[2 marks]

$$F = \frac{\sum_{i=1}^{i=4}\left[c'l + (w\cos\alpha\tan\phi')\right]_i}{\sum_{i=1}^{i=4}(w\sin\alpha)_i} = \frac{c'L_a + \tan\phi'\sum_{i=1}^{i=4}(w\cos\alpha)_i}{\sum_{i=1}^{i=4}(w\sin\alpha)_i},$$

$$F = \frac{6.75 \times 7.7 \times \pi(112.61°/180.0°) + 651.53 \times \tan 17.0°}{202.1} = 1.49$$

[5 marks]