Addis Ababa institute of Technology

Reinforced Concrete Structures 2

(CEng-3122)

Chapter Four

Column

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School of Civil and Environmental Engineering
Concrete Material and Structures Chair

Presentation Outline

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Introduction

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Introduction

A column is a vertical structural member supporting axial compressive loads, with or without moments.

The cross-sectional dimensions of a column are generally considerably less than its height.

Columns support vertical loads from the floors and roof and transmit these loads to the foundations.

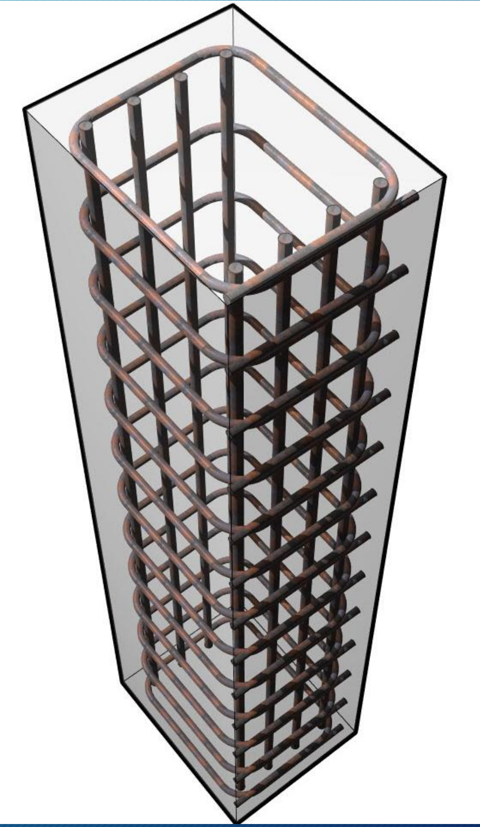
Columns may be classified based on the following criteria:

a) On the basis of geometry; rectangular, square, circular, L-shaped, T-shaped, etc. depending on the structural or architectural requirements
b) On the basis of composition; composite columns, in-filled columns, etc.
c) On the basis of lateral reinforcement; tied columns, spiral columns.
d) On the basis of manner by which lateral stability is provided to the structure as a whole; braced columns, un-braced columns.
e) On the basis of sensitivity to second order effect due to lateral displacements; sway columns, non-sway columns.
f) On the basis of degree of slenderness; short column, slender column.
g) On the basis of loading: axially loaded column, columns under uni-axial bending, columns under biaxial bending.

Introduction

Introduction

The more general terms compression members and members subjected to combined axial loads & bending are used to refer to columns, walls, and members in concrete trusses and frames.



These may be vertical, inclined, or horizontal.

A column is a special case of a compression member that is vertical.

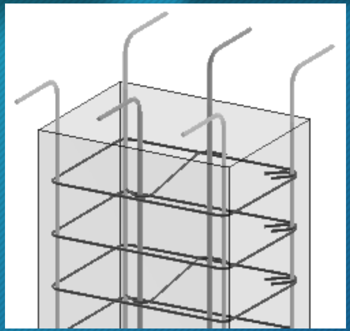
Although the theory developed in this chapter applies to columns in seismic regions, such columns require special detailing to resist the shear forces and repeated cyclic loading from the EQ.

In seismic regions the ties are heavier & more closely spaced.

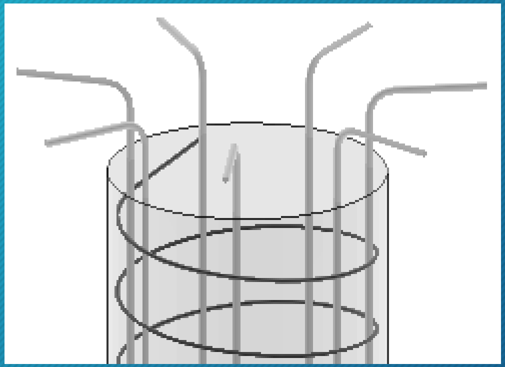
Tied and Spiral Columns

Tied and Spiral Columns

Most of the columns in buildings in nonseismic regions are tied columns.



Occasionally, when high strength and/or ductility are required, the bars are placed in a circle, and the ties are replaced by a bar bent into a helix or a spiral, with a pitch from 35 to 85 mm.



Such a column, is called a spiral column.

The spiral acts to restrain the lateral expansion of the column core under axial loads causing crushing and, in doing so, delays the failure of the core, making the column more ductile.

Behavior of Tied and Spiral Columns

The figure on the right shows a portion of the core of a spiral column enclosed by one and a half turns of a spiral.

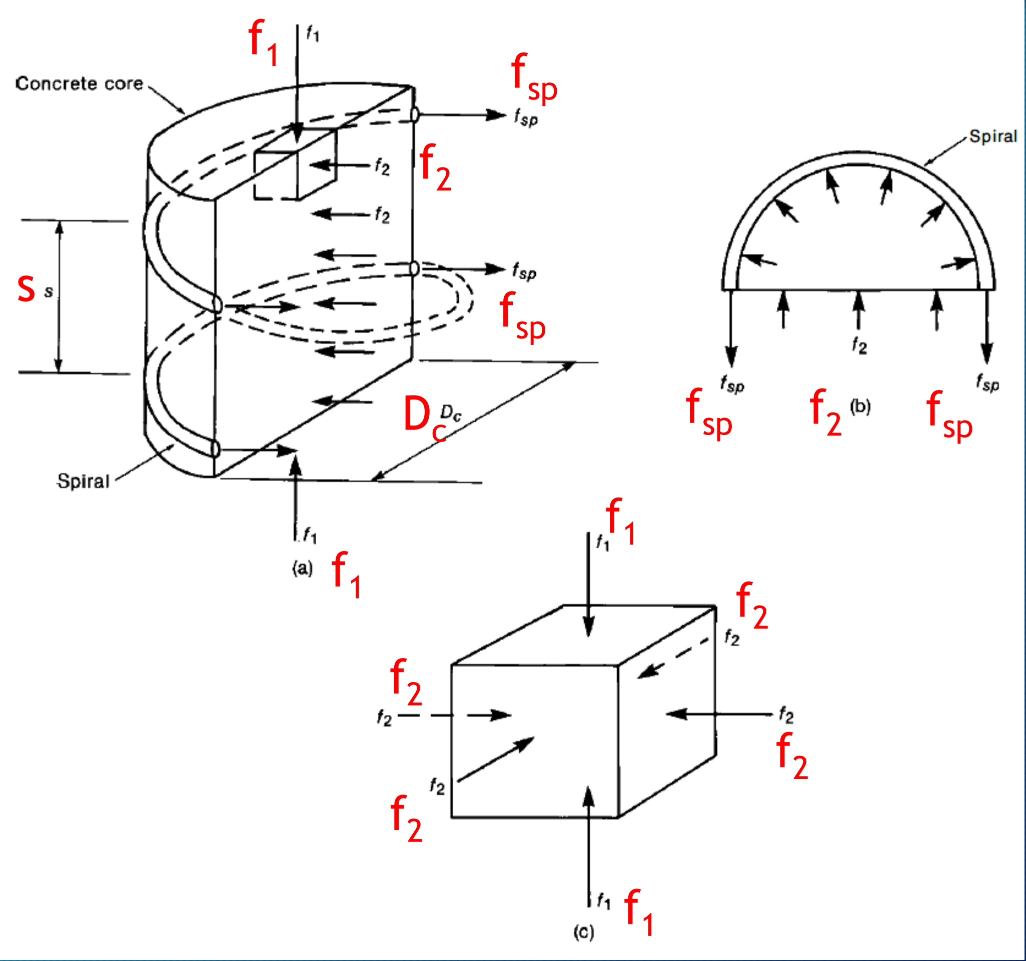
Under a compressive load, the concrete in this column shortens longitudinally under the stress f_1 and so, to satisfy the Poisson's ratio, it expands laterally.

This lateral expansion is especially pronounced at stresses in excess of 70% of the cylinder strength.

In spiral column, the lateral expansion of the concrete inside the spiral (the core) is restrained by the spiral.

These stresses in the spiral are in tension

For equilibrium the concrete is subjected to lateral compressive stresses, f_2.



Behavior of Tied and Spiral Columns

An element taken out of the core (see fig) is subjected to triaxial compression which increases the strength of concrete: $f_1 = f_{ck}+4.1f_2$.

In a tied column in a non-seismic region, the ties are spaced roughly the width of the column apart and thus provide relatively little lateral restraint to the core.

Hence, normal ties have little effect on the strength of the core in a tied column.

They do, however, act to reduce the unsupported length of the longitudinal bars, thus reducing the danger of buckling of those bars as the bar stresses approach yield.

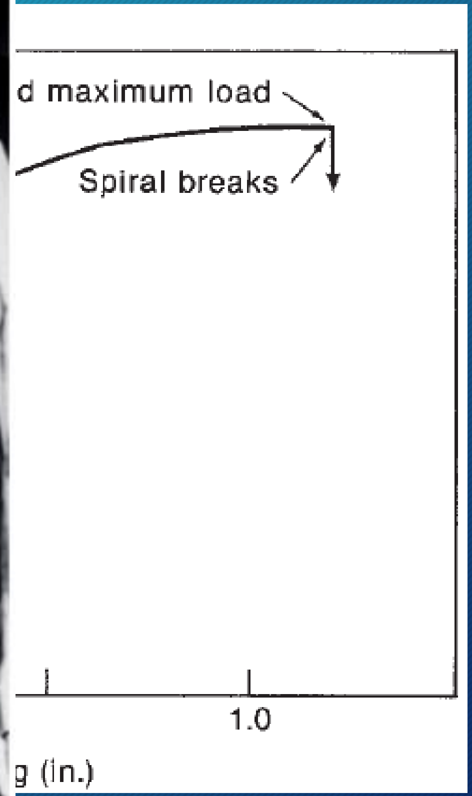
Behavior of T ns

The figure below presents loa spiral column subjected to axial loads.

The initial parts of these di similar.

As the maximum load is reach cracks and crushing devel concrete shell outside the tie and this concrete spalls off.

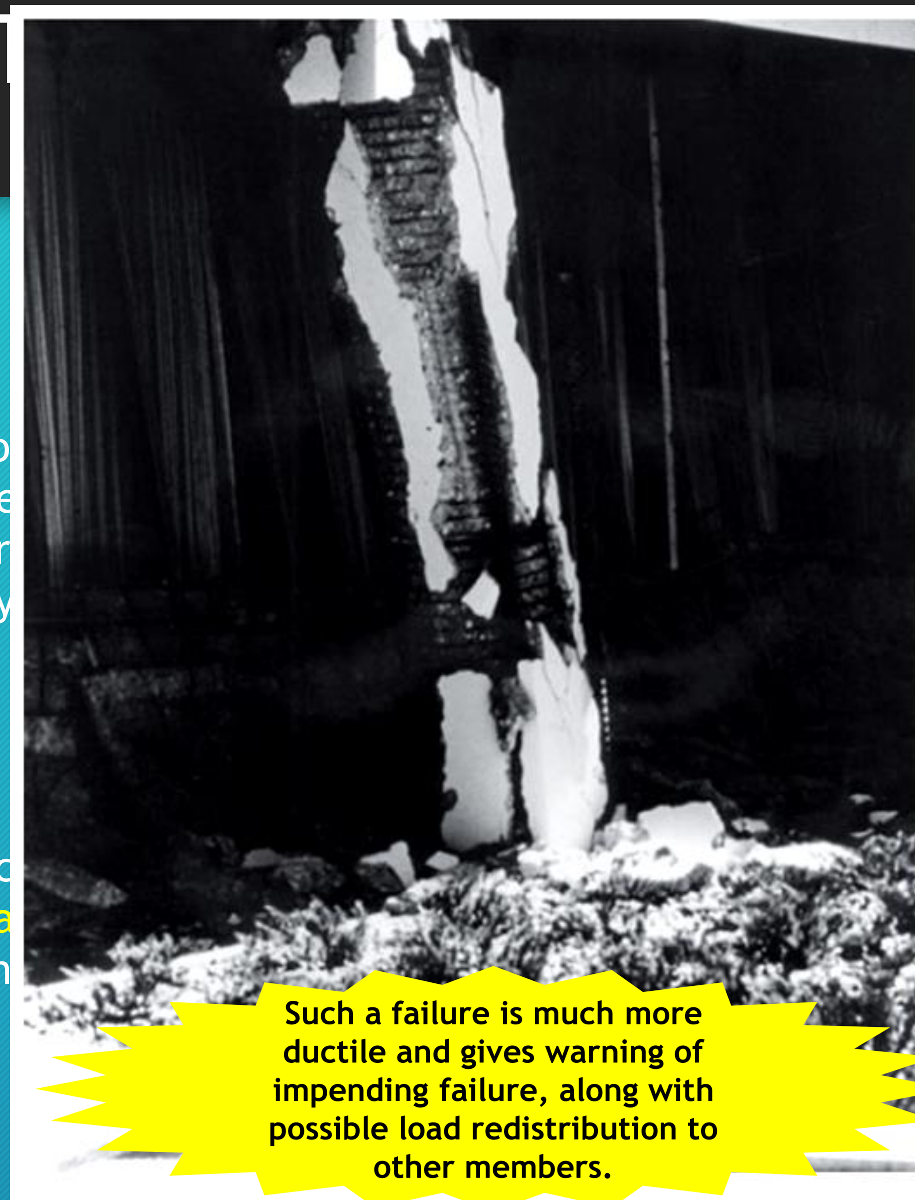
When this occurs in a tied co capacity of the core that rema than the load on the colu concrete core is crushed, reinforcements buckle outv ties.



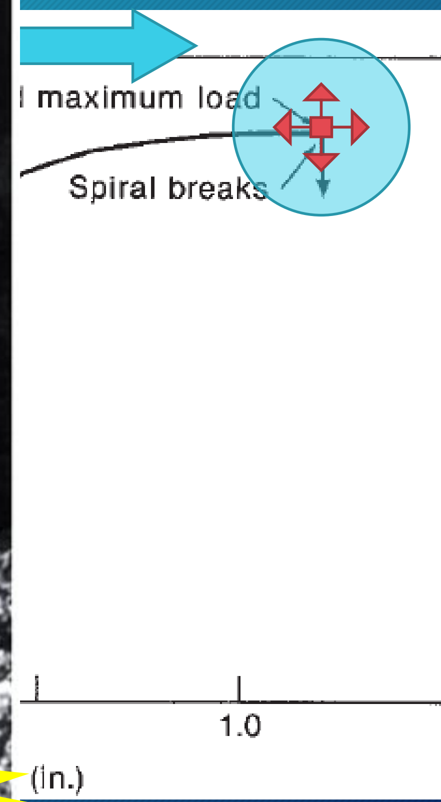
Behavior of T ns

When the shell spalls o
column, the column doe
immediately because the str
cores has been enhanced by
stresses.

As a result, the column c
large deformations, eventua
a 2nd maximum load, when
yield and the column finally



Such a failure is much more ductile and gives warning of impending failure, along with possible load redistribution to other members.



Sway Frames vs. Non-sway Frames

Sway Frames vs. Non-sway Frames

For design purpose, a given story in a frame can be considered "non-sway" if horizontal displacements do not significantly reduce the vertical load carrying capacity of the structure.

In other words, a frame can be "non-sway" if the P-Δ moments due to lateral deflections are small compared with the first order moments due to lateral loads.

In sway frames, it is not possible to consider columns independently as all columns in that frame deflect laterally by the same amount.

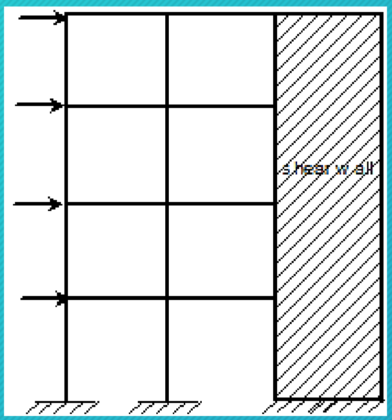


Fig. Non-sway Frame / Braced columns

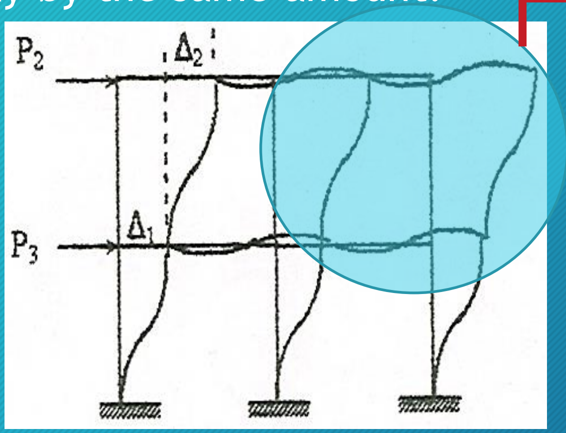
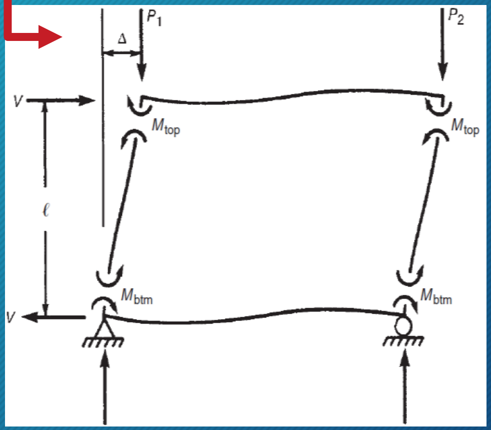
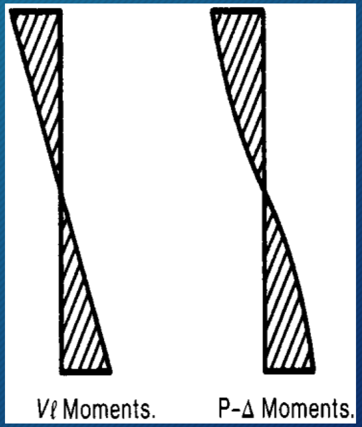


Fig. Sway Frame/ Un-braced columns





Slender Columns vs. Short Columns

Slender Columns vs. Short Columns

Columns are broadly categorized in to two as short and slender columns.

Short columns are columns for which the strength is governed by the strength of the materials and the geometry of the cross section.

In short columns, Second-order effects are negligible.

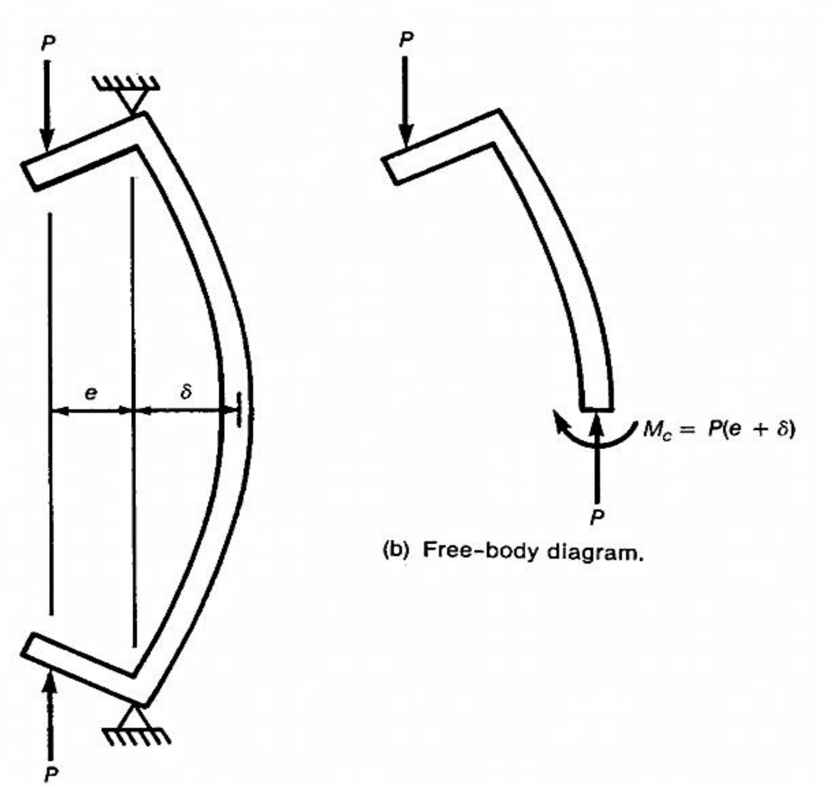
In these cases, it is not necessary to consider slenderness effects and compression members can be designed based on forces determined from first-order analyses.

When the unsupported length of the column is long, lateral deflections shall be so high that the moments shall increase and weaken the column.

Such a column, whose axial load carrying capacity is significantly reduced by moments resulting from lateral deflections of the column, is referred to as a slender column or sometimes as a long column.

Slender Columns vs. Short Columns

When slenderness effects cannot be neglected, the design of compression members, restraining beams and other supporting members shall be based on the factored forces and moments from a second-order analysis.



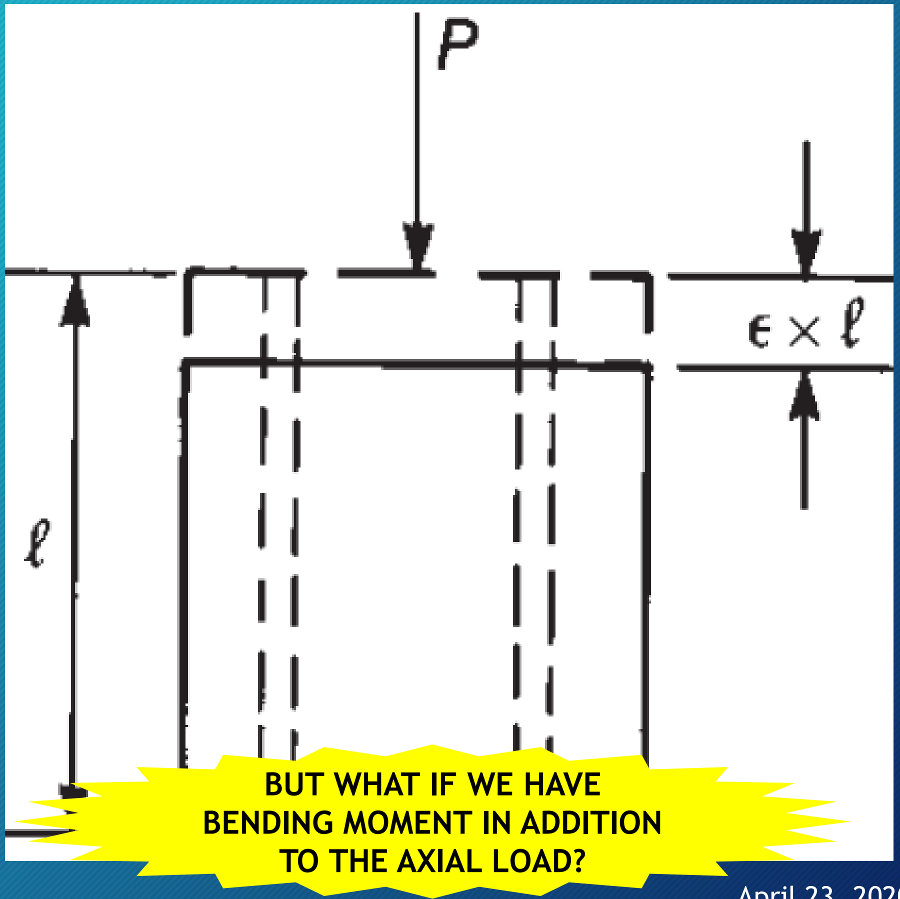
(a) Column.

(b) Free-body diagram.

Strength of Axially Loaded Columns

Strength of Axially Loaded Columns

When a symmetrical column is subjected to a concentric axial load, P, longitudinal strains ε, develop uniformly across the section as shown.



Because the steel & concrete are bonded together, the strains in the concrete & steel are equal.

For any given strain, it is possible to compute the stresses in the concrete & steel using the stress-strain curves for the two materials.

Failure occurs when P_o reaches a maximum:

$P_o = f_{cd}(A_g - A_{s,tot}) + f_{yd}A_{s,tot}$ (in compression)

$P_o = -f_{yd}A_{s,tot}$ (in tension)

Interaction Diagrams/ M-N Relationship

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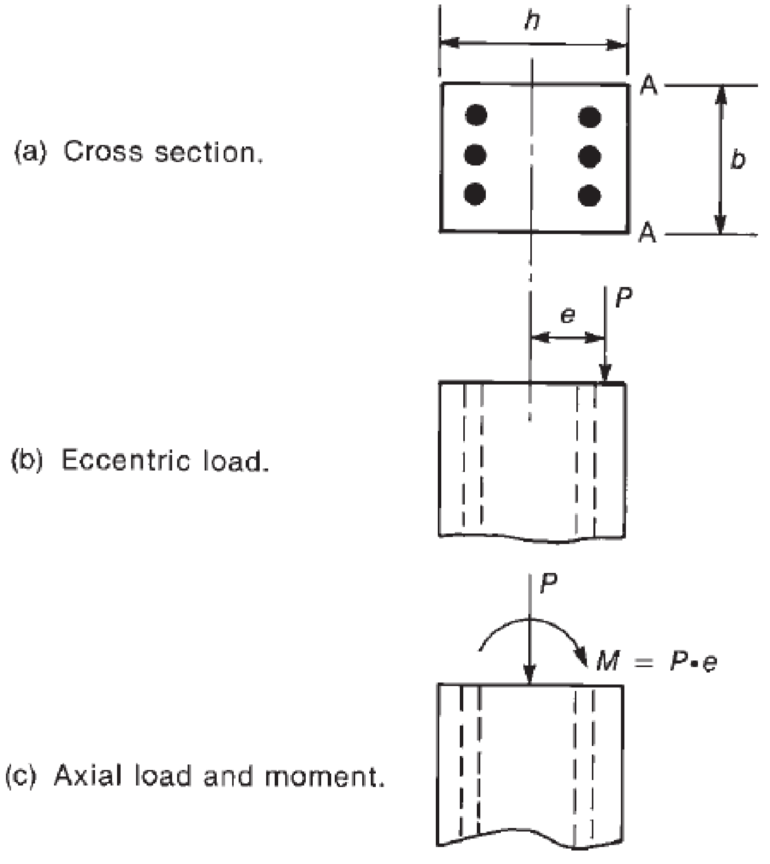
Interaction Diagrams/ M-N Relationship

Almost all compression members in concrete structures are subjected to moments in addition to axial loads.

These may be due to misalignment of the load on the column, or may result from the column resisting a portion of the unbalanced moments at the ends of the beams supported by the columns .

The distance e is referred to as the eccentricity of load.

These two cases are the same, because the eccentric load can be replaced by an axial load P plus a moment M=Pe about the centroid.



Interaction Diagrams/ M-N Relationship

The load P and moment M are calculated w.r.t. the geometric centroidal axis because the moments and forces obtained from structural analysis are referred to this axis.

For an idealized homogeneous and elastic column with a compressive strength, f_{cu}, equal to its tensile strength, f_{tu}, failure would occur in compression when the maximum stresses reached f_{cu}, as given by:

$$\frac{P}{A} + \frac{My}{I} = f_{cu}$$

Dividing both sides by f_{cu} gives:

$$\frac{P}{f_{cu}A} + \frac{My}{f_{cu}I} = 1$$

The maximum axial load the column can support occurs when M = 0 and P is $P_{max} = f_{cu}A$.

Similarly, the maximum moment that can be supported occurs when P = 0 and M is $M_{max} = f_{cu}I/y$.

Substituting P_{max} and M_{max} gives:

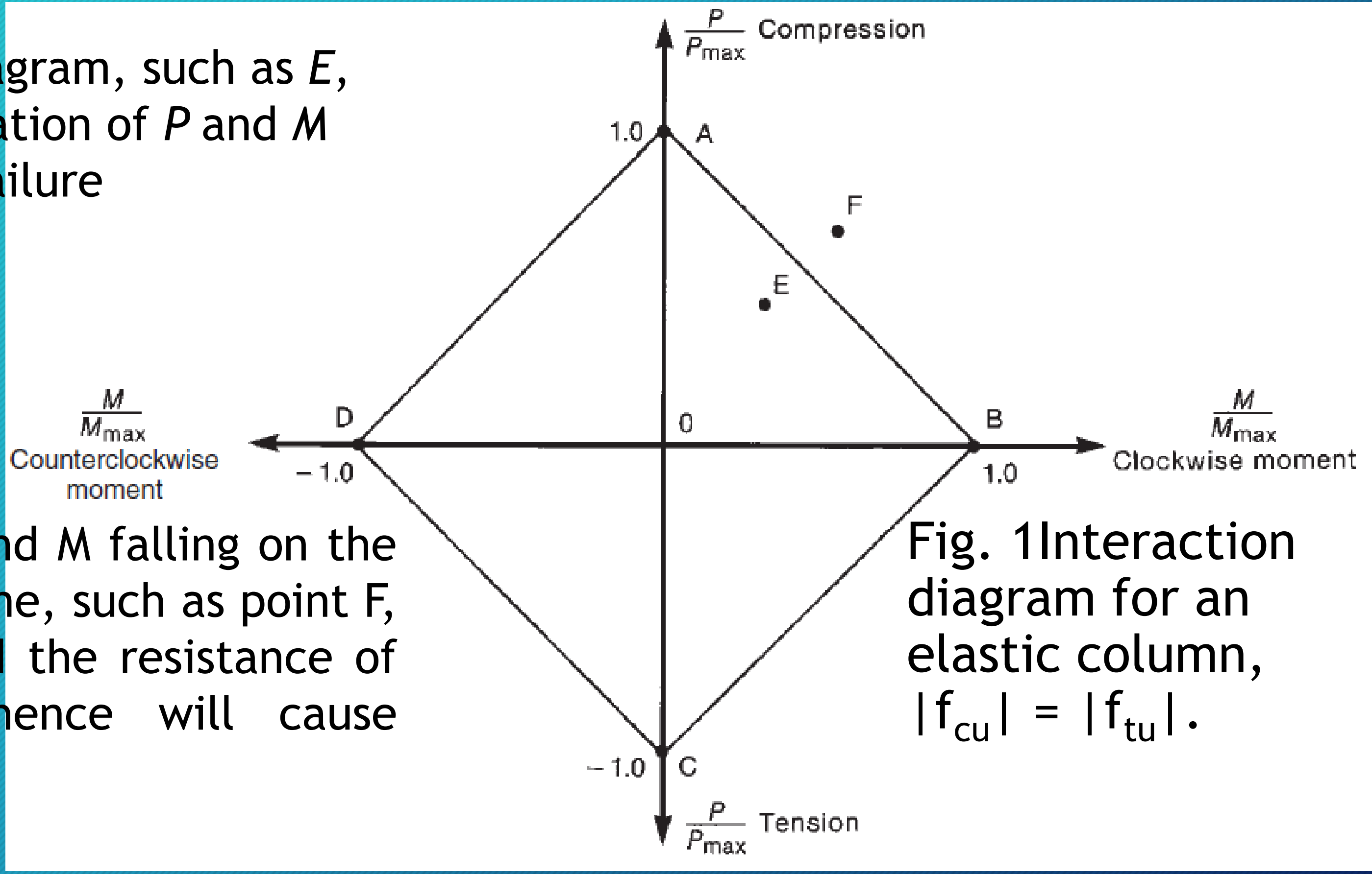
$$\frac{P}{P_{max}} + \frac{M}{M_{max}} = 1$$

This equation is known as an interaction equation, because it shows the interaction of, or relationship between, P and M at failure.

Interaction Diagrams/ M-N Relationship

Points on the lines plotted in this figure represent combinations of P and M corresponding to the resistance of the section.

A point inside the diagram, such as *E*, represents a combination of *P* and *M* that will not cause failure



Combinations of P and M falling on the line or outside the line, such as point F, will equal or exceed the resistance of the section and hence will cause failure

Fig. 1Interaction diagram for an elastic column, $|f_{cu}| = |f_{tu}|$.

Interaction Diagrams/ M-N Relationship
for Reinforced Concrete Columns

Since reinforced concrete is not elastic & has a tensile strength that is lower than its compressive strength, the general shape of the diagram resembles the figure slide #21 :

So, how can we address the M-N Relationship of RC Sections?

Strain Compatibility Solution

Interaction diagrams for columns are generally computed by assuming a series of strain distributions at the ULS, each corresponding to a particular point on the interaction diagram, and computing the corresponding values of P and M.

Once enough such points have been computed, the results are summarized in an interaction diagram.

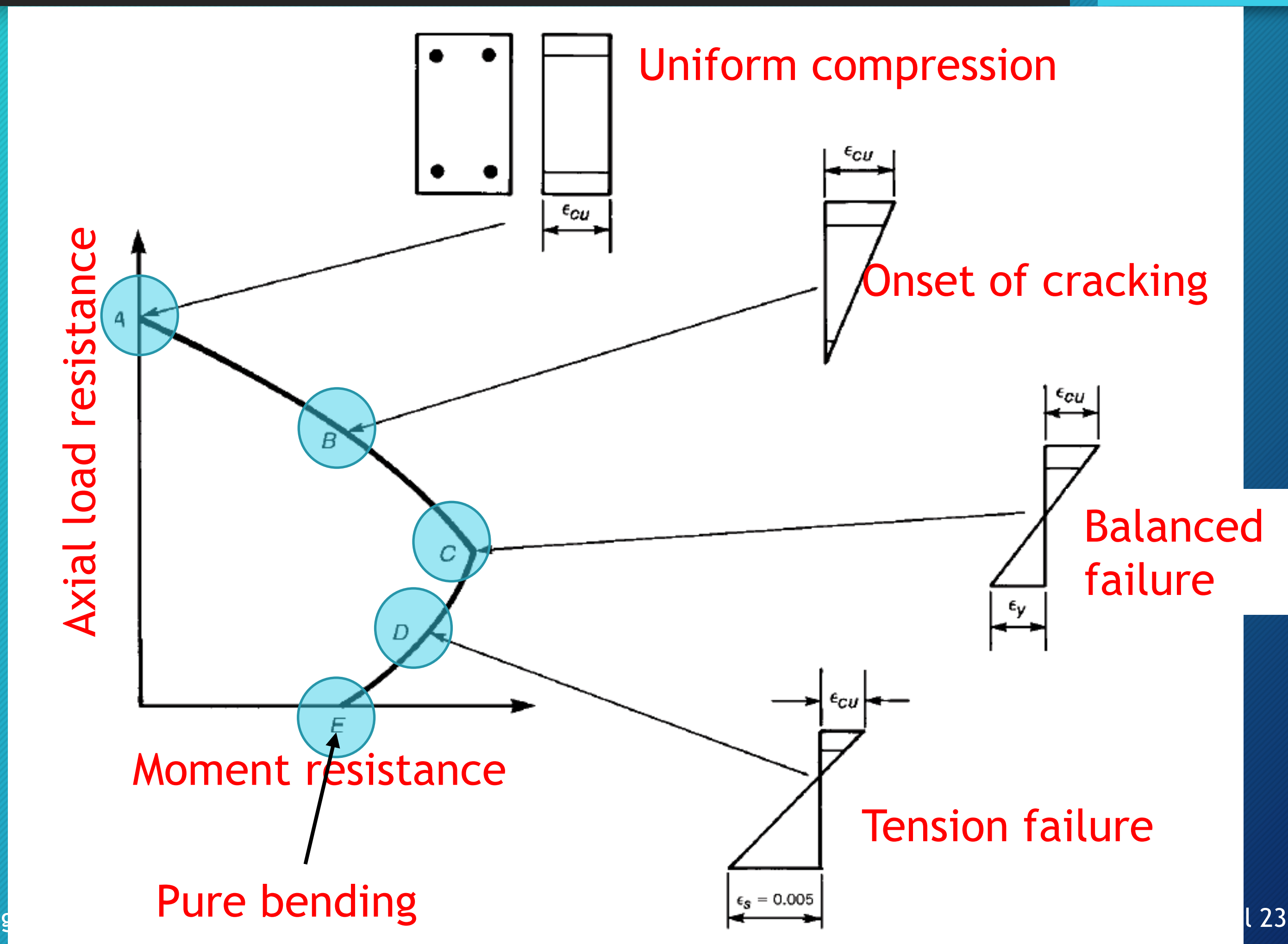
Interaction Diagrams/ M-N Relationship
for Reinforced Concrete Columns

Strain Compatibility Solution - Procedures to follow

1. Assume strain distribution and select the location of the neutral axis.
2. Compute the strain in each level of reinforcement from the strain distribution.
3. Using this information, compute the size of the compression stress block and the stress in each layer of reinforcement.
4. Compute the forces in the concrete and the steel layers, by multiplying the stresses by the areas on which they act.
5. Finally, compute the axial force P_n by summing the individual forces in the concrete and steel, and the moment M_n by summing the moments of these forces about the geometric centroid of the cross section.
6. These values of P_n and M_n represent one point on the interaction diagram.

Other points on the interaction diagram can be generated by selecting other values for the depth, *c*, to the neutral axis from the extreme compression fiber.

Interaction Diagrams/ M-N Relationship
for Reinforced Concrete Columns



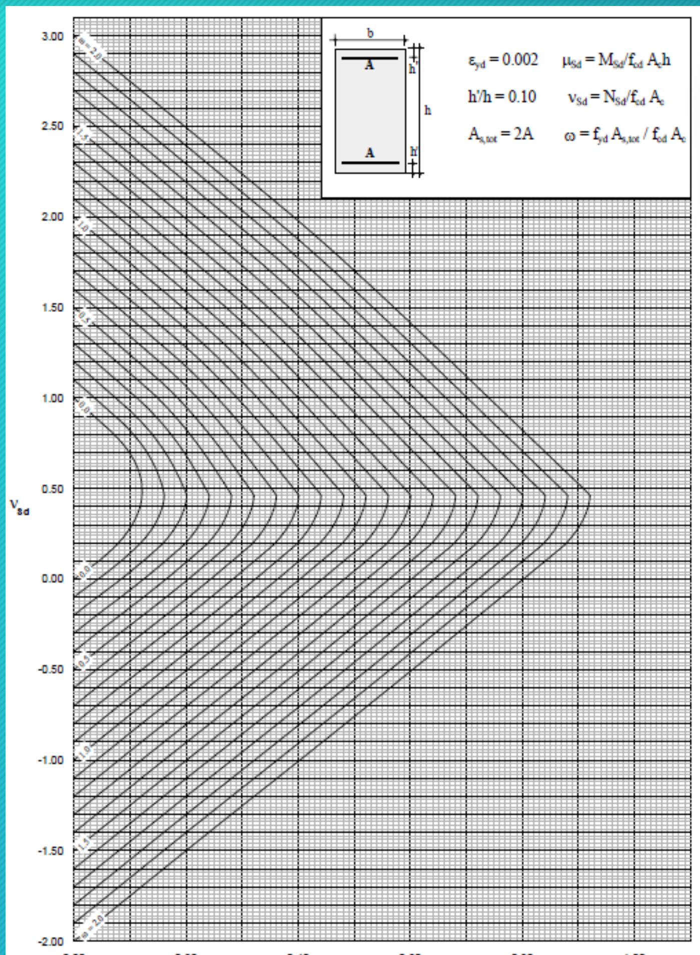
Interaction Diagrams/ M-N Relationship
for Reinforced Concrete Columns

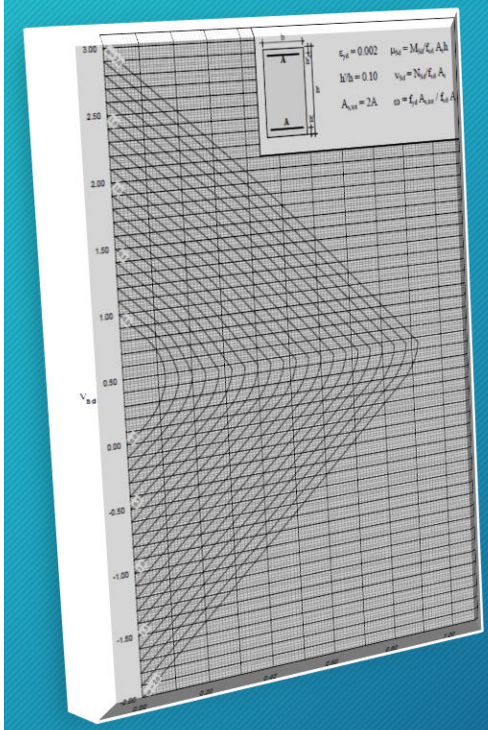
Interaction chart in Design

In the actual design, interaction charts prepared for uniaxial bending can be used. The procedure involves:

1. Assume a cross section, d’ and evaluate d’/h to choose appropriate chart.
2. Compute:
   Normal force ratio: $\nu = N_u / f_{cd}\, bh$
   Moment ratios: $\mu = M_u / f_{cd}\, bh^2$
3. Enter the chart and pick ω (the mechanical steel ratio), if the coordinate (ν, μ) lies within the families of curves. If the coordinate (ν, μ) lies outside the chart, the cross section is small and a new trail need to be made.
4. Compute $A_{s,tot} = \omega A_c f_{cd} / f_{yd}$
5. Check A_{tot} satisfies the maximum and minimum provisions
6. Determine the distribution of bars in accordance with the charts requirement

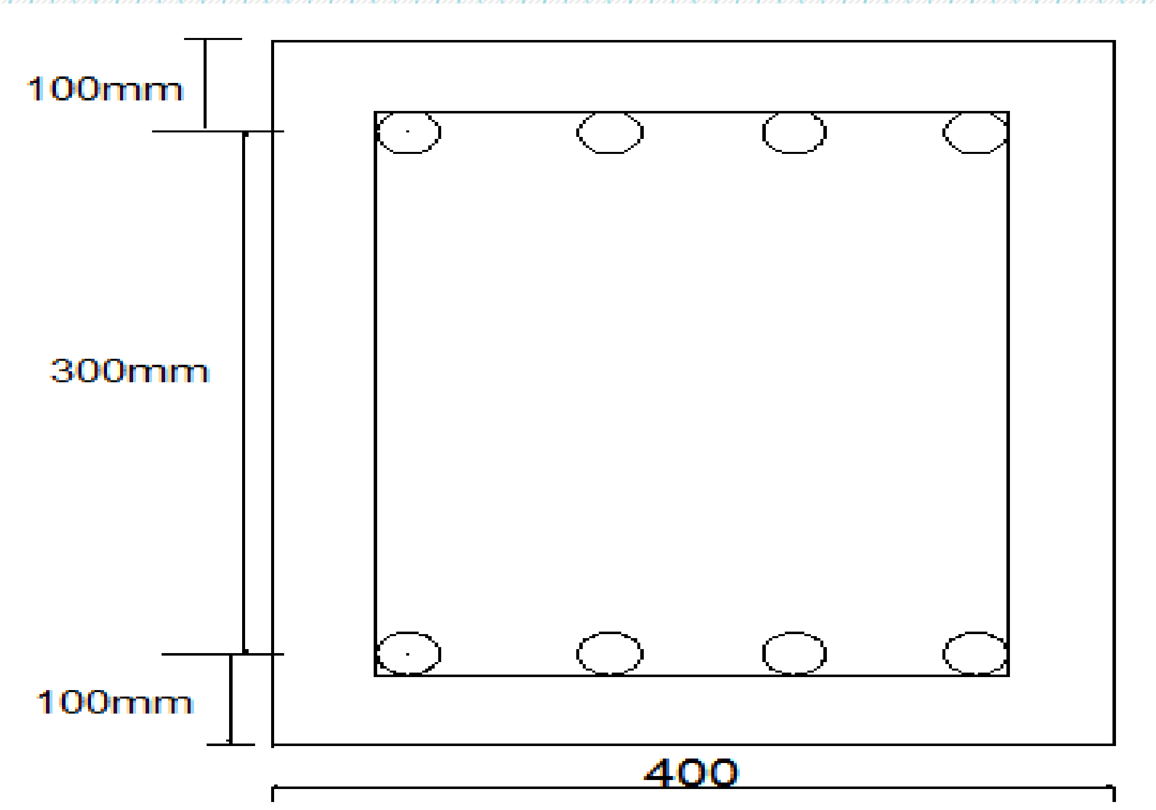
Interaction Diagrams/ M-N Relationship
for Reinforced Concrete Columns





A set on interaction charts are prepared by by Dr-Ing Girma Z., for both uniaxial and biaxial bending.

Example 4.1. Draw the interaction diagram for column cross section. Use: C 25/30, S – 460,



$$A_{s,total} = 6800\ mm^2$$

$$\omega = \frac{A_s f_{yd}}{f_{cd} bh} = \frac{6800*400}{14.1666*400*400} = 0.96$$

Show at least a minimum of 6 points in the interaction diagram.

i. Pure axial compression
ii. Balanced failure
iii. Zero tension (Onset of cracking)
iv. Pure flexure
v. A point b/n balanced failure and pure flexure
vi. A point b/n pure axial compression and zero tension

Thank you for the kind attention!

Questions?

Biaxially Loaded Columns

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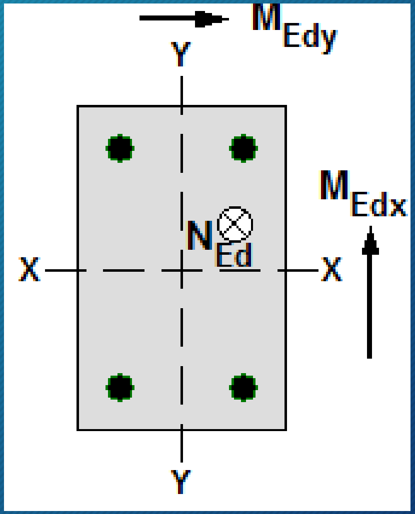
Biaxially Loaded Columns

Up to this point in the chapter we have dealt with columns subjected to axial loads accompanied by bending about one axis.

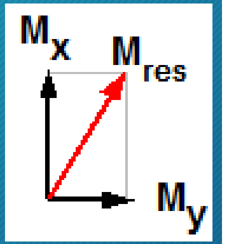
It is not unusual for columns to support axial forces and bending about two perpendicular axes.

One common example is a corner column in a frame.

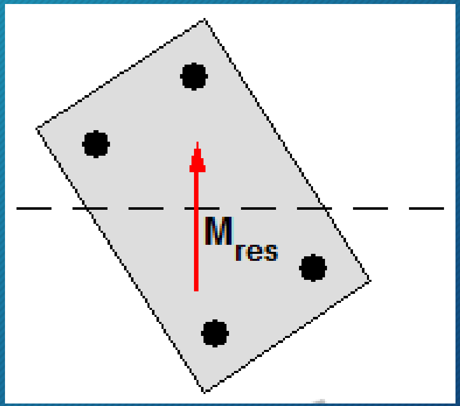


So, how can we construct the interaction chart or design such a section?

One approach is, we can convert biaxial bending into uniaxial bending by finding the resultant moment vector thus:



If we now rotate the section anticlockwise so that M_{res} points vertically, we have uniaxial bending of a non-rectangular section.



This is a mathematically complicated analysis!

Biaxially Loaded Columns

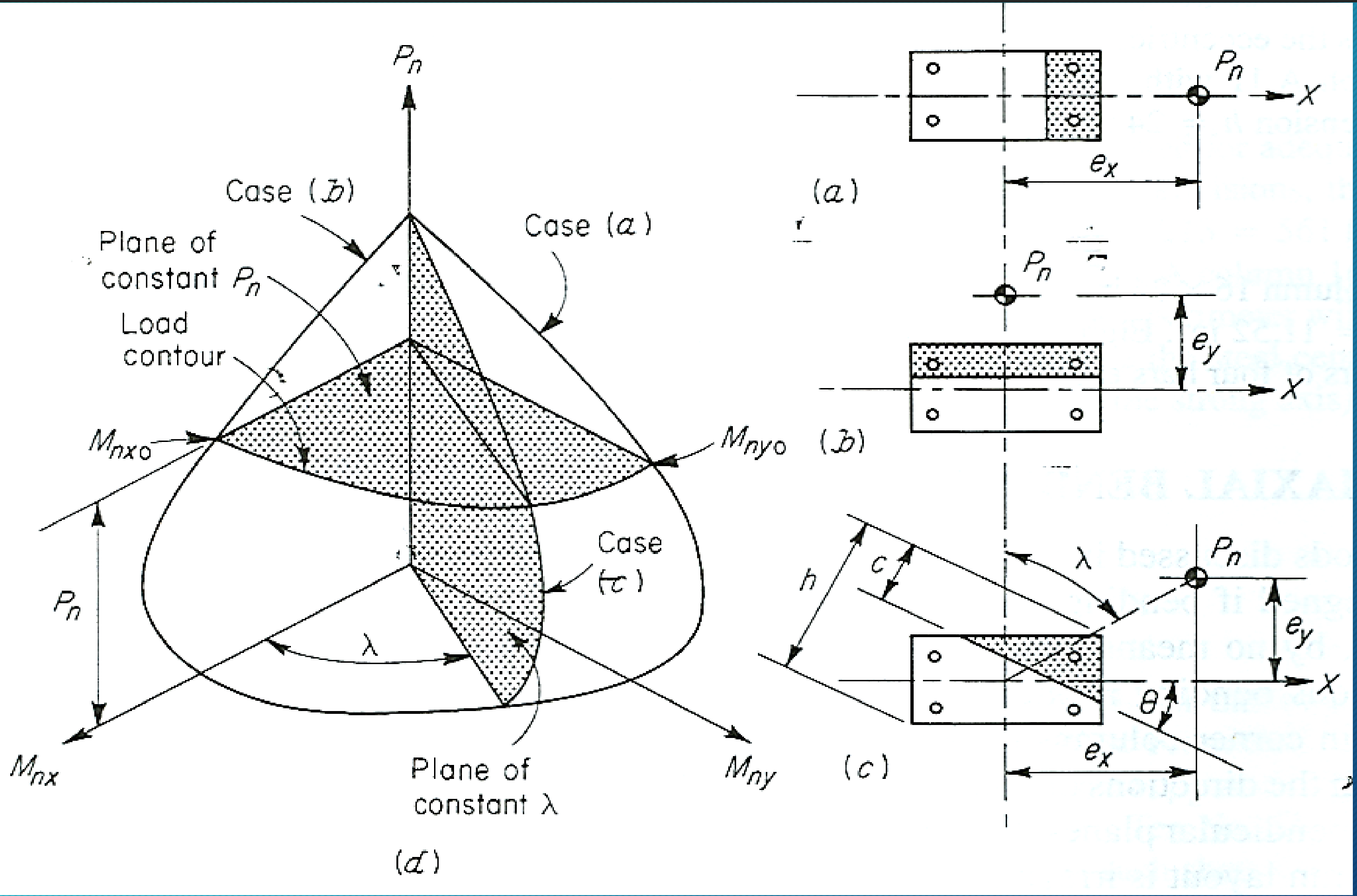
Another approach is constructing or making use of the interaction diagram (M-M-N Relationship) of biaxilly loaded column sections.

For a given cross section and reinforcing pattern, one can draw an interaction diagram for axial load and bending about either axis.

These interaction diagrams form the two edges of an interaction surface for axial load and bending about 2 axes.

These interaction diagrams form the two edges of an interaction surface for axial load and bending about 2 axes.

Biaxially Loaded Columns



Biaxially Loaded Columns

As shown in the figure above, the interaction diagram involves a three-dimensional interaction surface for axial load and bending about the two axes.

The calculation of each point on such a surface involves a double iteration:

- The strain gradient across the section is varied, and
- The angle of the neutral axis is varied.

There are different methods for the design of Biaxially loaded columns:

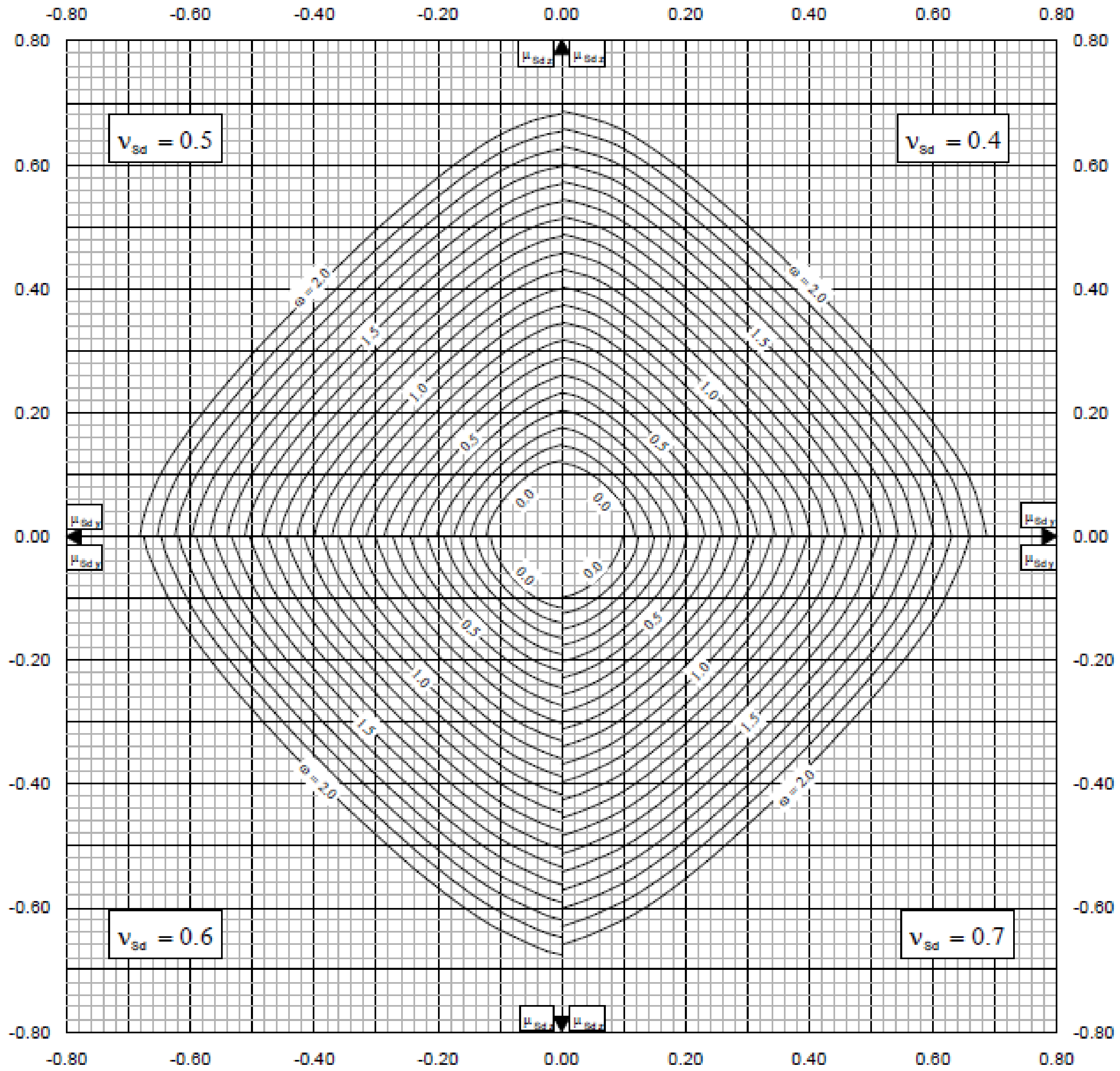
- Strain compatibility method
- The equivalent eccentricity method
- Load contour method
- Bresler reciprocal load method

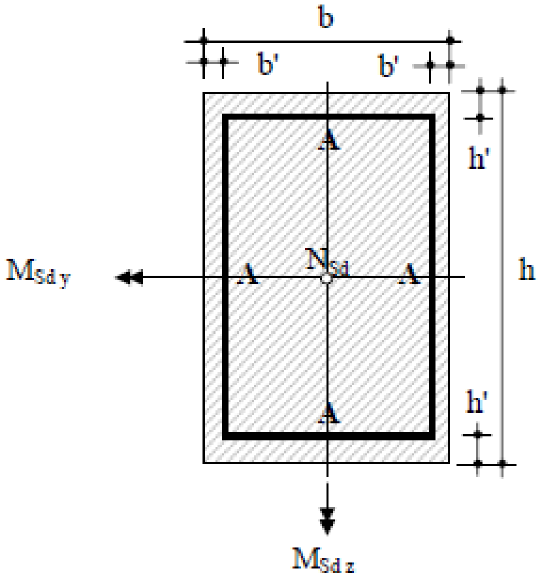
Can you name one more method?

Biaxially Loaded Columns

Biaxial interaction diagrams calculated and prepared as load contours or P-M diagrams drawn on planes of constant angles relating the magnitudes of the biaxial moments are more suitable for design (but difficult to derive).

Interaction diagrams are prepared as load contours for biaxially loaded columns with different reinforcement arrangement (4-corner reinforcement, 8-rebar arrangement, uniformly distributed reinforcement on 2-edges, uniformly distributed reinforcement on 4-edges and so on.)





$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$\mu_{Sd\,y} = \mu_{Sd\,y} / A_c\, h\, f_{cd}$

$\mu_{Sd\,z} = \mu_{Sd\,z} / A_c\, b\, f_{cd}$

$\nu_{Sd} = N_{Sd} / A_c\, f_{cd}$

$\omega = A_{s,tot}\ f_{yd} / A_c\, f_{cd}$

Fig. Sample Interaction Diagram

Biaxially Loaded Columns

Interaction chart in Design

In the actual design, interaction charts prepared for biaxial bending can be used. The procedure involves:

1. Select suitable chart which satisfy and h'/h and b'/b ratio.
2. Compute:
   Normal force ratio: $\nu = N_u / f_{cd}\, bh$
   Moment ratios: $\mu_h = M_{uh} / f_{cd}\, A_c h$
   $\mu_b = M_{ub} / f_{cd}\, A_c b$
3. Enter the chart and pick ω (the mechanical steel ratio),
4. Compute $A_{s,tot} = \omega A_c f_{cd} / f_{yd}$
5. Check A_{tot} satisfies the maximum and minimum provisions
6. Determine the distribution of bars in accordance with the charts requirement

Thank you for the kind attention!

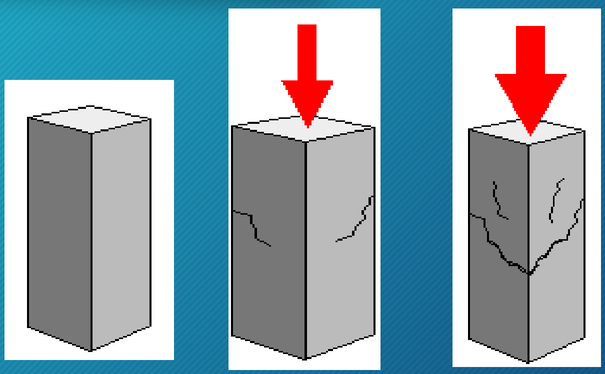
Questions?

Please read on the concept of buckling of axially loaded columns.

Buckling in Columns

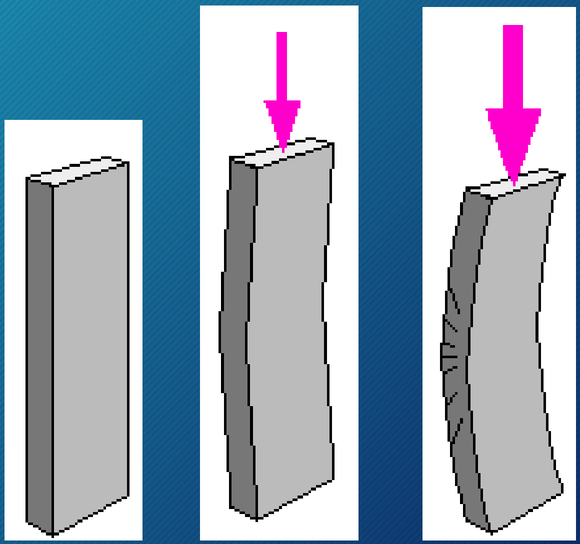
Buckling in Columns:
What is buckling?

Consider a column subjected to an increasing axial load:
Eventually the compressive strength is excceded and a compressive failure *occurs.*



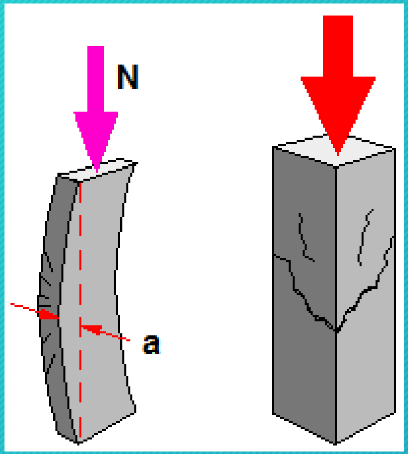
Now consider a column with a thinner section subjected to an increasing axial load
when the load reaches a certain value the column begins to bend about the weaker axis and deflect sideways. The column is said to have buckled.

Increasing the load the column to deflect further until eventually a bending failure *occurs*



Buckling in Columns:
What is buckling?

The effect of the sideways deflection is to increase the moment at any section by an amount equal to the axial load times the deflection at that section, thus:



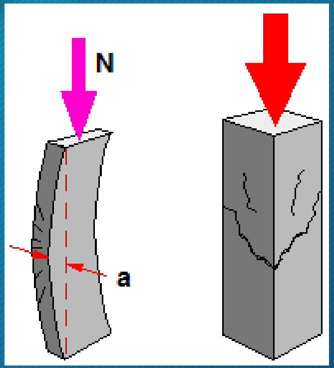
$$M_{add} = N\,a$$

In many, If not most, practical situations the effect of deflections is so small that it can be ignored. Where it is significant the member is described as being slender.

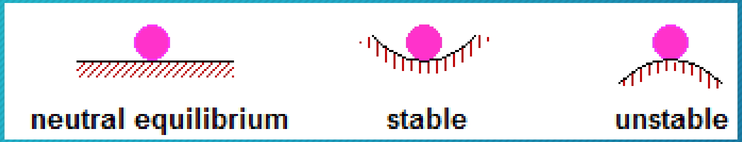
However, if the load is applied through the centroid of the column section and aligned with the longitudinal axis, and if the column is perfectly straight and fully elastic, in other words an ideal column, then buckling will not occur.

There needs to be some lateral disturbance to produce a rotation of the column, no matter how slight.

Buckling in Columns:
What is buckling?



Of course an ideal column does not exist in the real world – columns will not be absolutely vertical , loads are always slightly eccentric, and reinforced concrete is not a fully elastic material – but their behavior gives an insight into the behavior of actual columns.



Leonhard Euler (1707-1783) was a great Swiss mathematician who first investigated buckling. He discovered that for an ideal column there is a critical load, N_{cr} (now called the Euler load) when the column is in a state of *neutral equilibrium*. When the load is less the column is stable, and when the load is more the column is unstable.

He found the critical load for *single curvature* buckling of a pin-ended ideal column is:

$$N_{cr} = \frac{\pi^2 E I}{L^2}$$

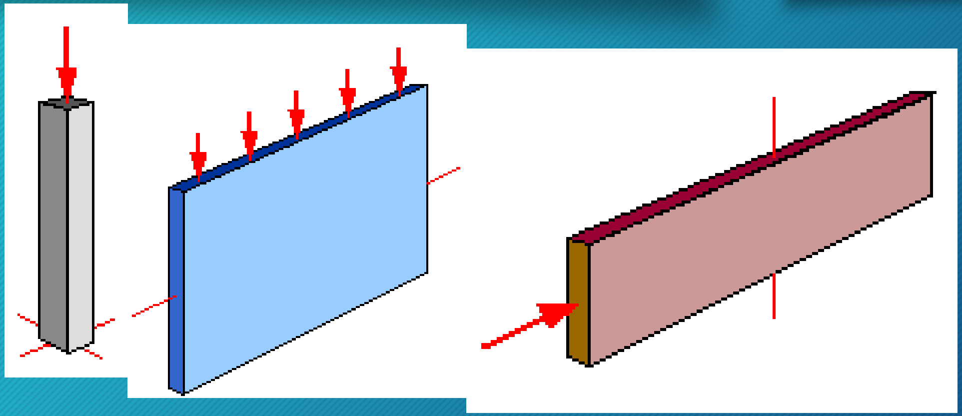
Thus, a column is more likely to buckle (N_{cr} is reduced) when either the length is increased or the flexural rigidity (EI) is reduced the π^2 term is result of the sinusoidal deflected shape.

Slenderness Effects in Structures

Slenderness Effects in Structures

In structures, slenderness effects occur mainly in …

- *Columns, about one or both axes,*
- *Walls, about the minor axis, and*
- *Sometimes in beams which are narrow compared to either the span or the depth.*



Methods for assessing slenderness effects in structures generally involves three basic steps :

- determine how the structure will deflect,
  the Code does this by classifying structures.
- establish whether slenderness effects are significant,,
  the Code defines 'significance' as increasing critical moment by at least 10%, but as this would involve carrying out a full analysis to determine the significance, the Code provides simplified rules based on the effective length and the slenderness ratio.
- take account of significant effects in the design.,
  This is achieved, essentially, by designing for additional bending moments.

Classification of Structures

Classification of Structures

The sensitivity of a structure and its parts to deflection depends to the large extent on the general layout and detailing of the structure as a whole, and of its component parts. It is not possible to define all cases which are likely to occur in practice, so the Code provides a limited classification of structures, distinguishing only between :- braced and unbraced

A braced structure is one which contains bracing elements. These are vertical elements, usually walls, which are so stiff relative to other vertical elements that may be assumed to attract all horizontal forces. A braced structure may be defined as -

> **"one where the bracing element(s) attract and transmit to the foundations, at least 90% of all horizontal forces applied to the structure"**

An unbraced structure relies on the stiffness of the frame to transmit the horizontal forces to the foundation.

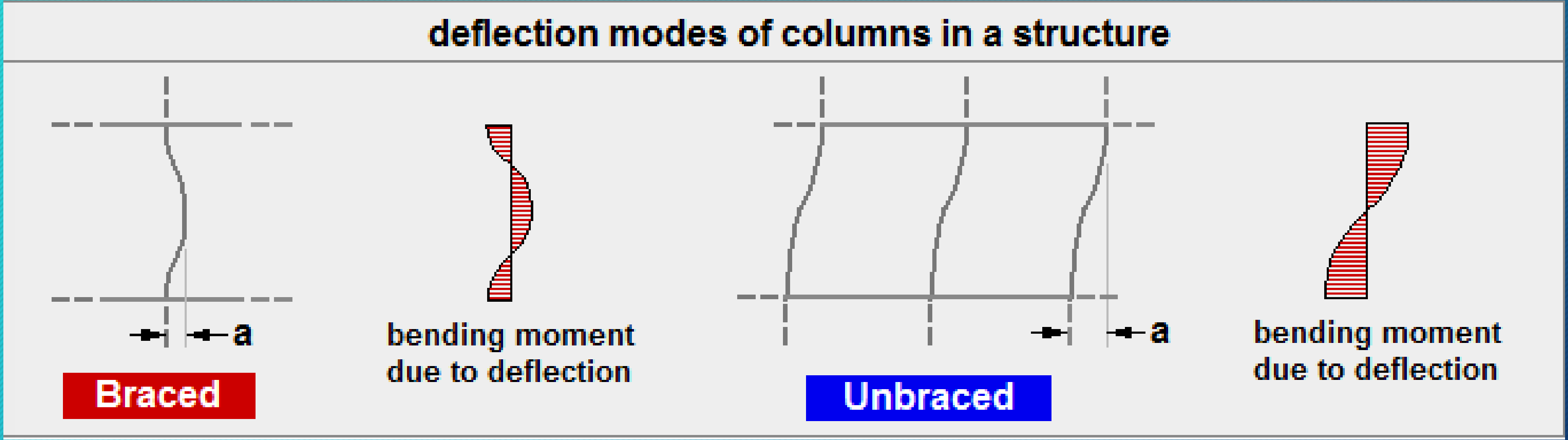
A braced structure is one where sidesway of the whole structure is unlikely to be significant, while in unbraced structures sidesway is likely to be significant in the context of the Code significance is defined as :-

> **"lateral displacement of the ends of the columns increasing the critical bending moments by more than 10% above that calculated by ignoring the displacements"**

Classification of Structures

The classification of a structure as braced or unbraced is very important in the design of columns because it establishes the mode of deflection.

The two types of structure give very different forms bending moment in the columns.



In an **unbraced** structure, all columns within a storey height are subjected to the **same deflection**.

Effective Length

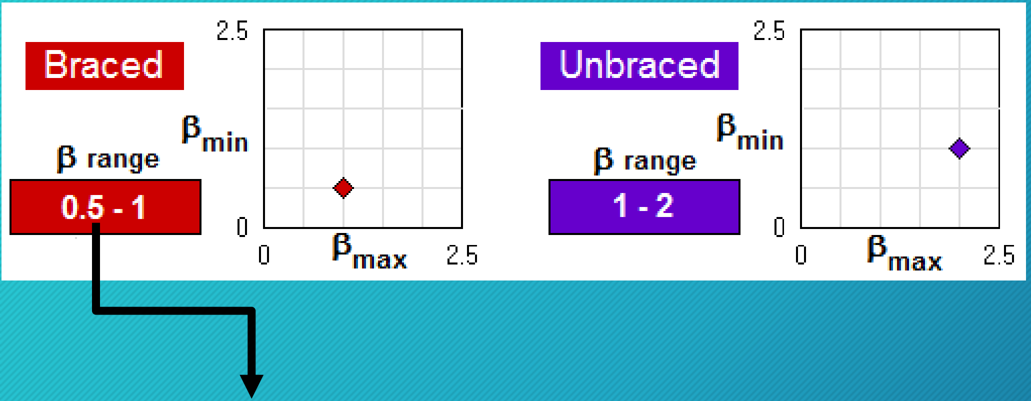
Effective Length

The effective length, l_o of a member is defined as the length of a pin-ended strut with constant normal force having the same cross-section and buckling load.

This can be expressed as :

$$l_o = \beta l$$

(where, l is the clear height between end restraints).

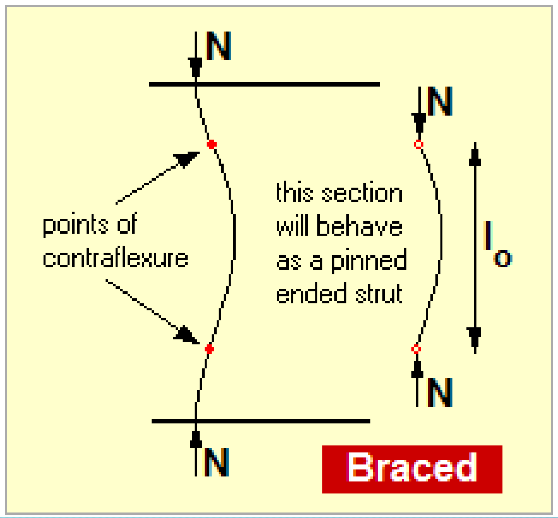


In practice the lower limit is more likely to be about 0.7

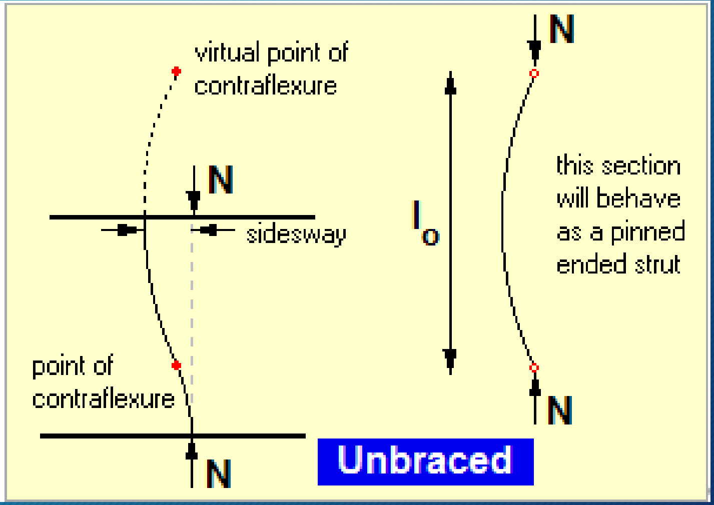
Effective Length

The effective length is dependent on the deflected shape and the end restraints, and is the length between points of contraflexure.

For a braced structure the deflected shape gives two points of contraflexure within the length of the member. If both ends are fully restrained ß=0.5, and if both ends are pinned ß =1.0



For an unbraced structure only one point of contraflexure is within the length of the member. If both ends are fully restrained ß=1.0. If one end is fully restrained and the other is free ß=2.0.



Effective Length

For braced members -

$$\beta = 0.5 \left[\left(1 + \frac{k_1}{0.45 + k_1}\right)\left(1 + \frac{k_2}{0.45 + k_2}\right) \right]^{0.5}$$

For unbraced members -

Greater of :

$$\beta = \left(1 + \frac{k_1}{1 + k_1}\right)\left(1 + \frac{k_2}{1 + k_2}\right)$$

and

$$\beta = \left(1 + 10\, \frac{k_1 k_2}{k_1 + k_2}\right)^{0.5}$$

Where,

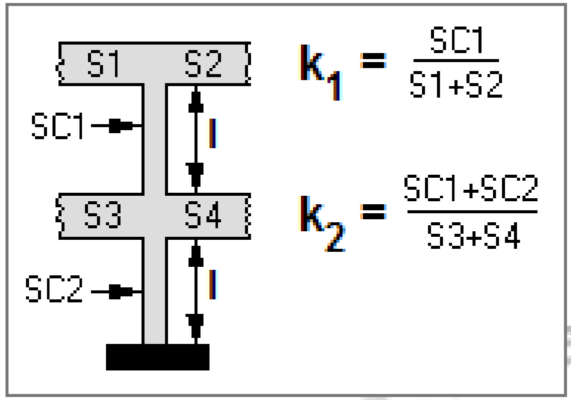
k_1, k_2 are the relative flexibilities of rotational restraints at ends 1 and 2, $= (\theta/M) \times (EI/L)$ (where $\theta = 0$ then $k = 0$, but a rigid rotational restraint is unlikely in practice so take $k = 0.1$)

Θ is the rotation of the restraining members for bending moment M,

EI is the sum of the bending stiffness of the columns which contribute to the restraints.

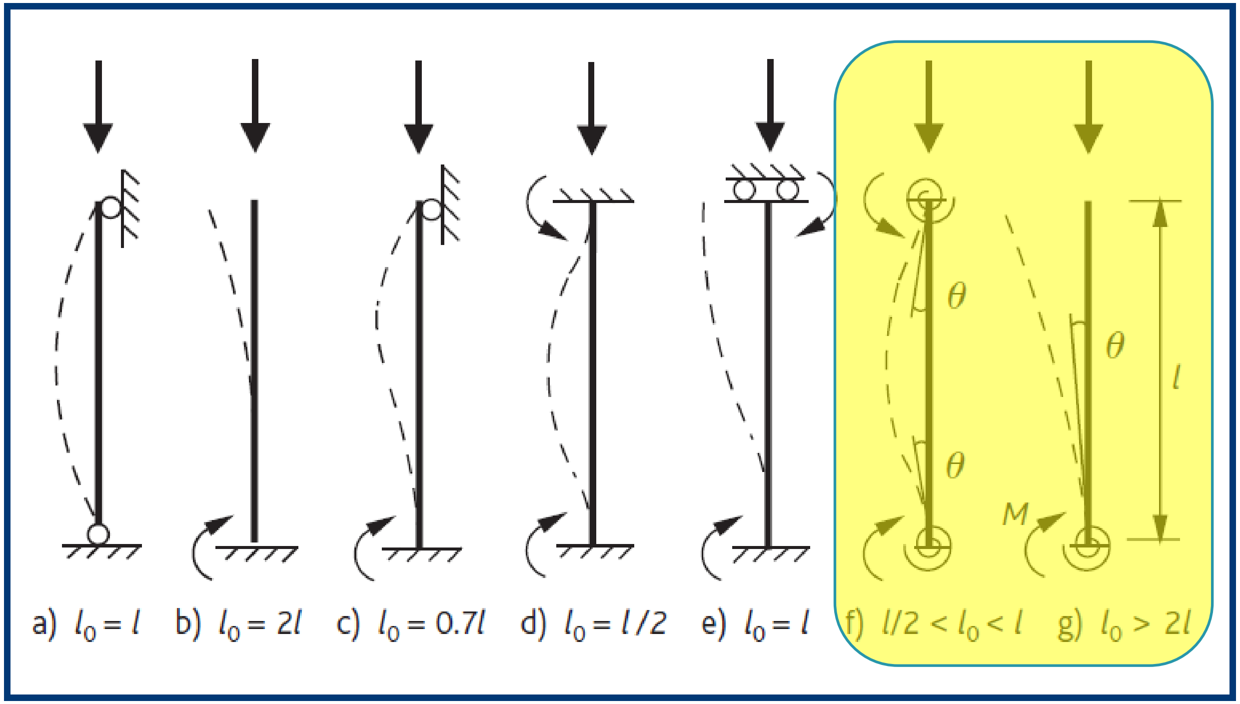
Where θ is not known k_1, k_2 can be calculated from the ratio of the column bending stiffness to beam/slab bending stiffness, but taking only 50% of beam stifnesses to allow for cracking (see opposite).

Note: in this calculation of k_1 and k_2 only members properly framed into the end of the column in the appropriate plane of bending should be considered.



Effective Length

Effective lengths for isolated members



The figure above gives guidance on the effective length of the column. However, for most real structures figures (f) and (g) only are applicable.

Slenderness Ratio

Slenderness Ratio

The slenderness ratio, λ is defined in the Code as the ratio of the effective length to the radius of gyration, thus

$$\lambda = l_0 / i$$

Where,

$$i = (I / A)^{0.5}$$

Some code use the section depth rather than the radius of gyration. This gives lower values. There is no intrinsic reason to say is better than the other, they are both related to the flexural rigidity (although as mentioned before the problem is not one of classical buckling). Using the radius of gyration, however, does have the advantage of providing a suitable way of dealing with non-rectangular sections.

<u>Example 4.4.</u> If the effective length of a 350mm square column is 2.63m, what is its slenderness ratio?

Solution

<u>Step1</u> Compute the cross sectional parameters of the square section.

$A = b \times h = h^2 = 0.1225m^2$; where $h=b=0.35m$
$I = bh^3/12 = h^4/12 = 0.001251m^4$
$i = (I/A)^{0.5} = 0.2887h = 0.1013m$

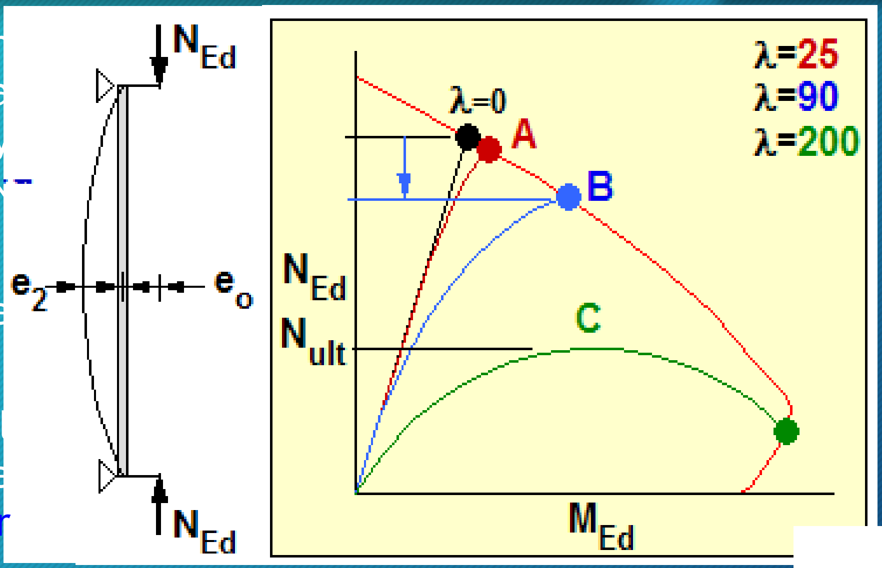
<u>Step2</u> Compute the compute the slenderness ratio

$\Lambda = l_o/i = 2.63m / 0.1013m = 26.03\ (26)$

Slenderness Ratio

Consider a stiff column (say $\lambda=0$) subjected to an eccentric axial force N_{Ed}. This produces a moment $N_{Ed}e_o$ in the column. Ignoring any deflection effects we can plot this on the M-N relationship (described in previous lessons), thus

Now if we include the buckling deflection of the column for a range of slenderness ratios.



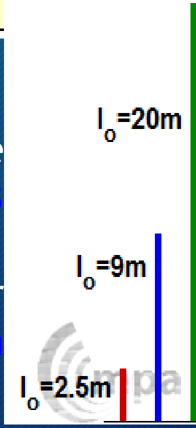
The 'buckling deflection' produces an additional moment $N_{Ed}e_2$ in the column. This reduces the load carrying capacity of the column – the greater the slenderness the greater the reduction.

When the reduction is significant, it must be allowed for in the design.

When $\lambda=25$ the reduction is relatively small and can be ignored, but when $\lambda=90$ the reduction is considerable and must be taken into account. For these slenderness ratios the column is stable and will suffer a material failure at points A and B respectively.

When $\lambda=200$ the precise behavior is uncertain. The column may fail due to buckling prior to reaching the ultimate load N_{ult} at point C. If this does not occur then material failure will occur almost immediately due to instability.

These are the 3 effective length s for the 350mm square column.



Design Considerations

Design Considerations

The Code states that second order (slenderness) effects may be ignored if they are less than 0% of the corresponding first order effects, or as an alternative for isolated members if he slenderness ratio, λ is less than λ_{lim} given by:-

$$\lambda_{lim} = 20\,A\,B\,C\,\eta^{0.5}$$

where, $A = 1 / (1 + 0.2\phi_{ef})$ [if ϕ_{ef} is not known take **A=0.7**]

$B = (1 + 2\omega)^{0.5}$ [if ω is not known take **B=1.1**]

$C = 1.7 - r_m$ [if r_m is not known take **C=0.7**]

$\eta = N_{Ed} / (A_c f_{cd})$

ϕ_{ef} is the *effective* creep ratio $= \phi(\infty,t_o)\, M_{0Eqp} / M_{0Ed}$

M_{0Eqp} is the 1st order moment due to quasi-permanent load combination (SLS)

M_{0Ed} is the 1st order moment due to design load combination (ULS)

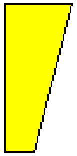
ω is the mechanical reinforcement ratio $= A_s f_{yd} / (A_c f_{cd})$

A_s is the total area of longitudinal reinforcement

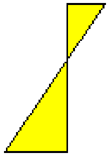
r_m is the *moment ratio* $= M_{01} / M_{02}$

M_{01}, M_{02} are the 1st order end moments, $|M_{02}| \geq |M_{01}|$

If the end moments give tension on the same side then r_m is taken positive, otherwise negative.



$C \leq 1.7$



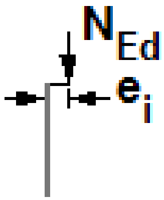
$C > 1.7$

In the following cases r_m should be taken as **1** -

- for **braced** members where the 1st order moments are predominantly due to geometric imperfections or transverse loading,
- for **unbraced** members in general.

Design Considerations

When designing for 'slenderness' the calculation of deformations should take into account the effects of cracking, creep, non-linear material properties and geometric imperfections, which normally means considering the structure being constructed 'out of plumb' (not vertical), which in isolated members is allowed for by introducing an additional eccentricity, e_i of the axial load.



$$e_i = 0.5\,\theta_i\, l_o$$

The Code provides three methods of analysis :-

- A rigorous, non-linear analysis of the structure.
  This complex and beyond the scope of this package.

- A simplified method based on the estimation of curvatures.
  This will be discussed in the upcoming sections.

- A simplified second order analysis based on nominal siffnesses.
  To avoid confusion this is not described in this course.

Thank you for the kind attention!

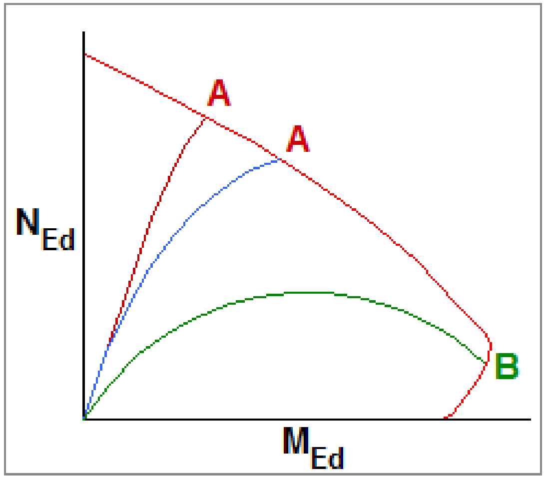
Questions?

Please read on the concept simplified design method based on the estimation of curvatures.

Design of Columns for second order effects

Design of Slender Columns:
Simplified Design Method based on Nominal Curvature

The aim of a rigorous non-linear analysis is to calculate the maximum load capacity of a slender column by ascertaining the load-deflection relationship. A simplified method must aim to find a close estimate of this ultimate load in a single calculation.



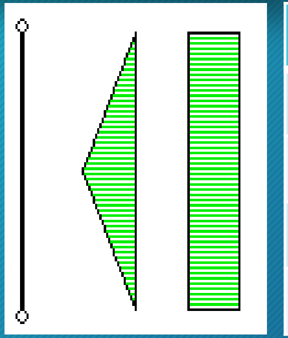
The nominal curvature method proposed in the Code aims to predict the deflection at which failure of the concrete commences, that is at the maximum compressive strain.

This point either corresponds to the actual ultimate load(A), or to a lower value (B). The method will therefore, in principle, give a lower bound estimate of the strength.

The deflection of a pin-ended strut is calculated from its curvature, but to calculate the curvature at each section along the length of the strut requires a rigorous analysis. Therefore, the shape of the curvature distribution must be assumed. Various shapes produce a central deflection of $\beta l_o^2(1/r)$.

Design of Slender Columns:
Simplified Design Method based on Nominal Curvature

$\beta l_o^2(1/r)$



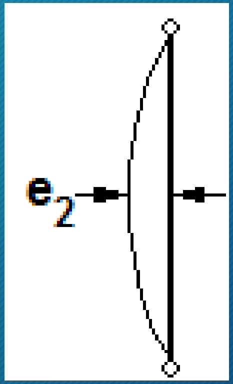
β	Distribution
1/12	Triangular
1/9.6	Parabolic
1/9.87	Sinusoidal
1/8	Rectangular

The triangular and rectangular distributions give the extremes,

The parabolic and the sinusoidal ($9.87=\pi^2$) more closely represents the actual distribution.,

For simplicity, the Code uses a value of 1/10, so that.

$$\boxed{e_2 = 0.1\, l_o^2\, (1/r)}$$



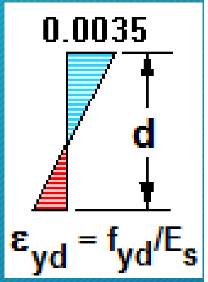
Design of Slender Columns:

Simplified Design Method based on Nominal Curvature

The next step is to calculate the curvature (1/r). This can be done by considering the M-N interaction curve.

At the balance point the compressive strain in the concrete is at its maximum and the tensile strain in the steel is at its yield point, thus

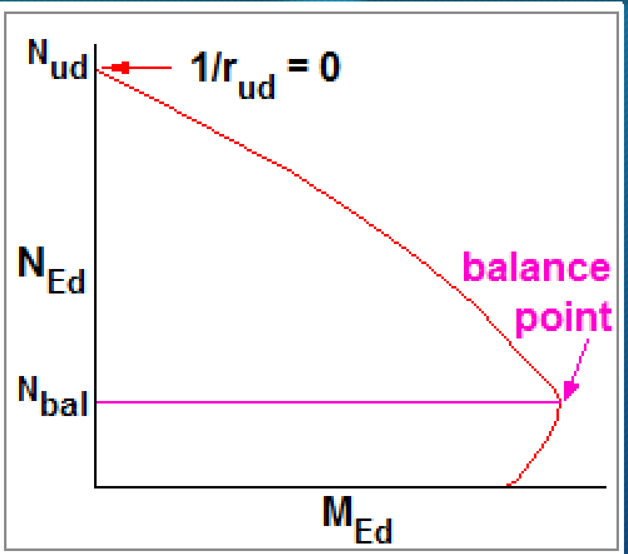
Now the curvature of the section is equal to the change in strain over the depth of the section, …



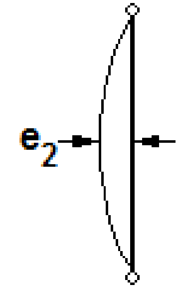
$$1/r_{bal} = (0.0035 + f_{yd}/E_s)/d$$

$$\text{In the Code ... } 1/r_{bal} = 1/r_o = \varepsilon_{yd}/(0.45d)$$

The curvature at the ultimate axial load N_{ud} of the section is zero



$$e_2 = 0.1\, l_0^2\, (1/r)$$



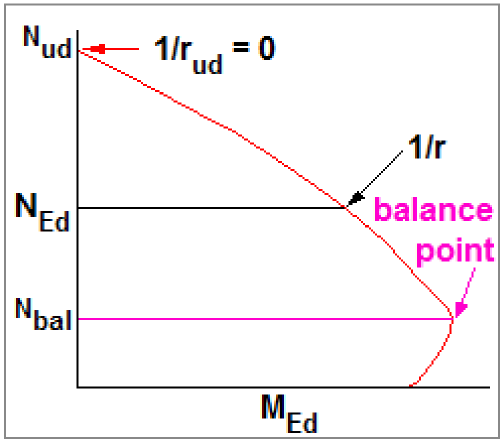
Design of Slender Columns:
Simplified Design Method based on Nominal Curvature

Assuming the curvature to vary linearly between $1/r_{ud}$ and $1/r_{bal}$, then for any load N_{Ed} the curvature can be expressed as …

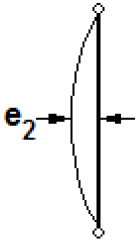
The Code uses a simplified expression for $1/r_{bal}$ by assuming that for a balanced section the neutral axis depth is 0.55d, thus

The coefficient K_ϕ allows for the effects of creep, but if the following three conditions are satisfied then creep can be ignored, i.e. $K_\phi = 1$.

- $\phi(\infty, t_o) \leq 2$
- $\lambda \leq 75$
- $M_{0Ed} / N_{Ed} \geq h$



$$e_2 = 0.1\, l_0^2\, (1/r)$$



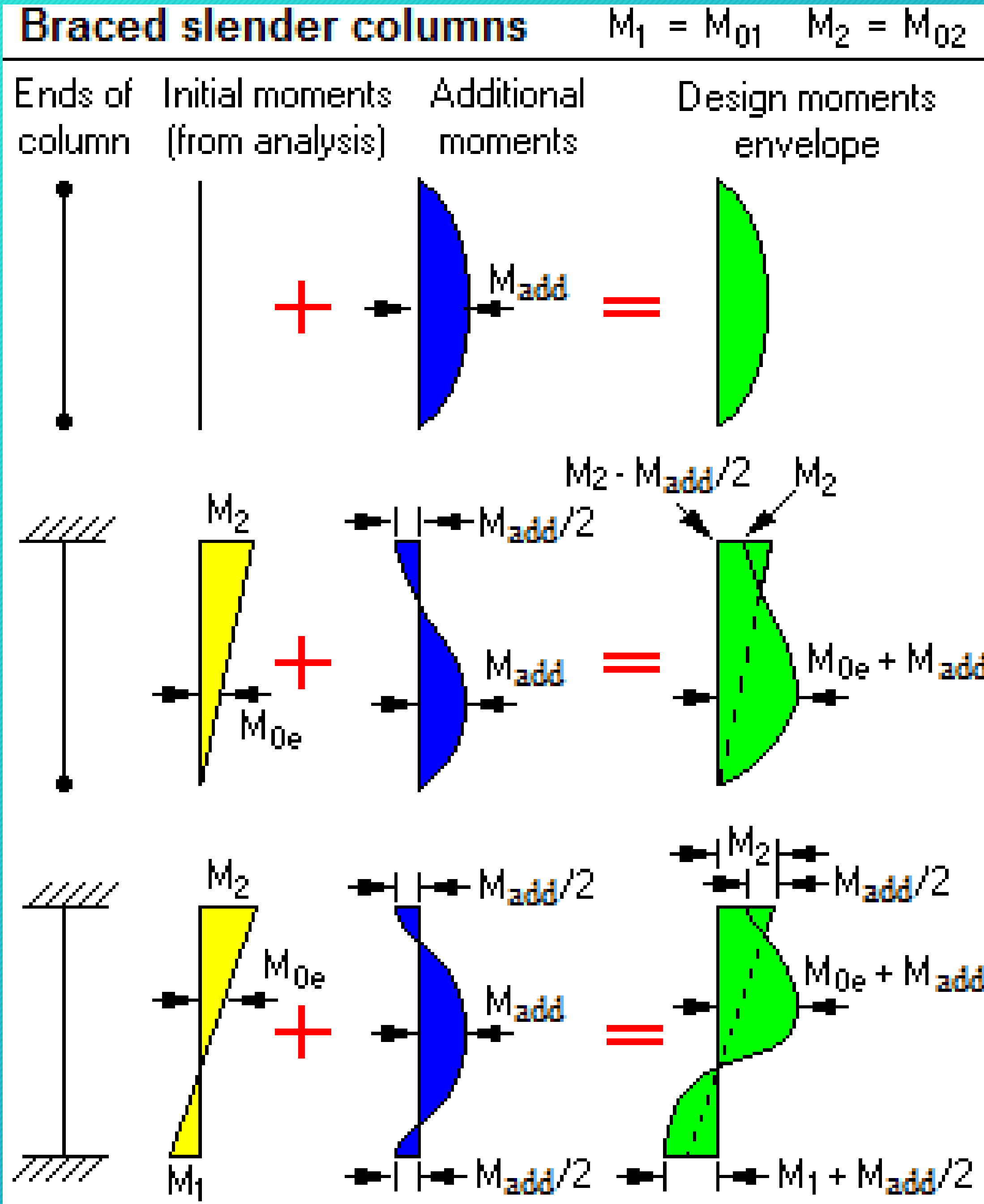
$$1/r = K_r K_\phi (1/r_{bal})$$

$$K_r = \frac{N_{ud} - N_{Ed}}{N_{ud} - N_{bal}} \leq 1$$

$$1/r_{bal} = \varepsilon_{yd} / 0.45d$$

$$K_\phi = 1 + \beta\phi_{ef} \geq 1$$

$$\beta = 0.35 + f_{ck}/200 - \lambda/150$$



Finally, we need to find the maximum design moment, M_{Ed}.

1. For braced columns the maximum additional moment due to deflection, $M_{add}=(N_{Ed}e_2)$ occurs close to the mid-height of the actual column (as shown opposite).

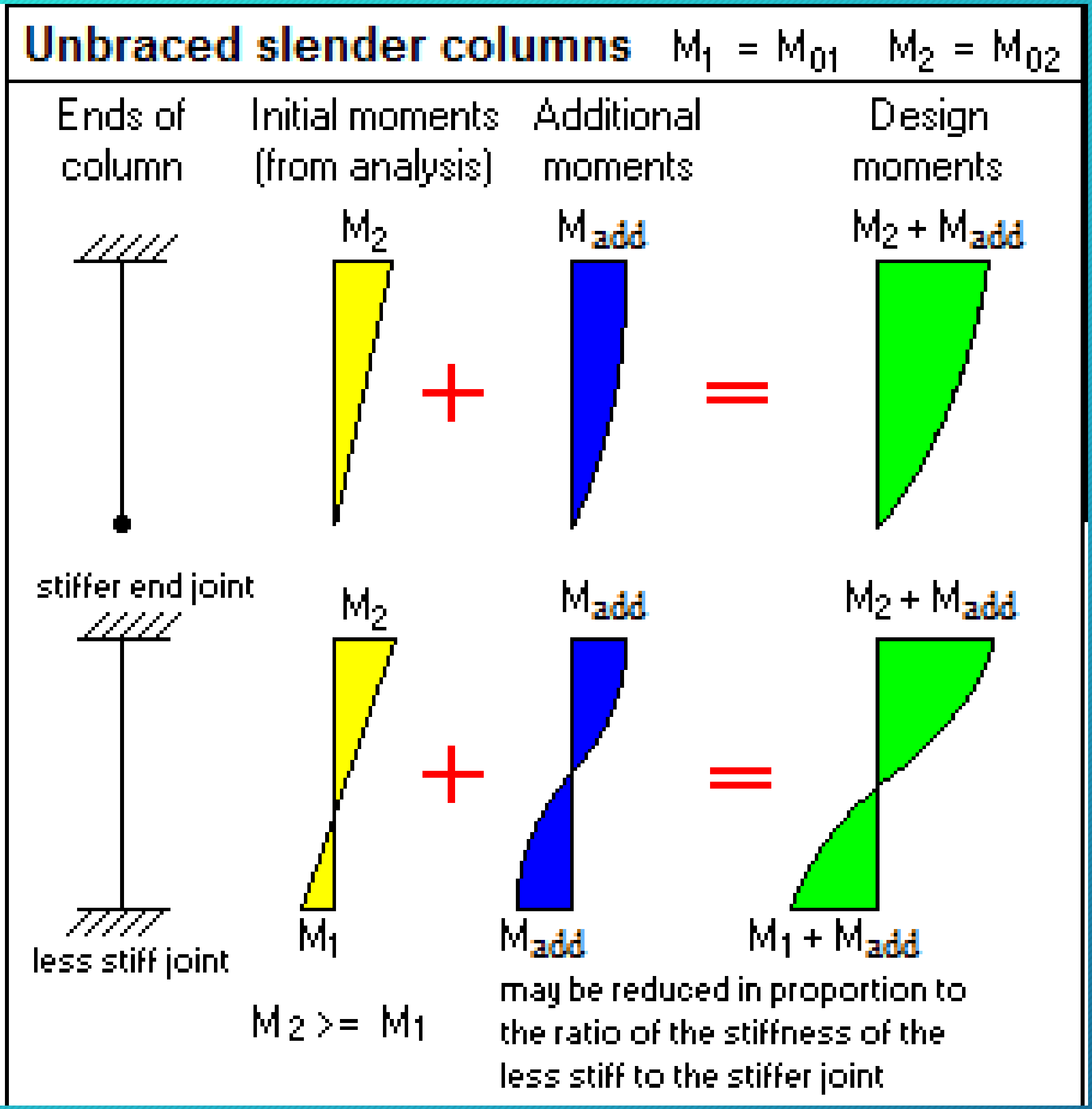
A reasonable estimate of the first order moment at this point for end moments M_{01}, M_{02} is given by …

$$M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02} \quad \text{with} \quad |M_{02}| \geq |M_{01}|$$

Including the additional moment for imperfection, $N_{Ed}e_i$, $(e_i=0.5\theta_i l_o)$ this gives a maximum design moment of …

$$M_{Ed} = MAX\,(M_{0e} + N_{Ed}(e_2 + e_i)\,,\, M_{02} + N_{Ed}\,e_i)$$

Finally, we need to find the maximum design moment, M_{Ed}.



2. For unbraced columns the maximum moment due to deflection, $N_{Ed}e_2$ occurs at the column end as does the additional moment for imperfections, $N_{Ed}e_i$ giving a total design moment of …

$$M_{Ed} = M_{02} + N_{Ed}(e_2 + e_i)$$

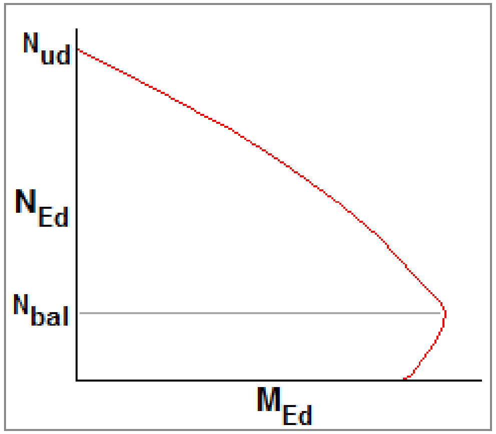
Design of Slender Columns: *Simplified Design Method based on Nominal Curvature*

To determine the maximum moment is not straightforward because M_{Ed} depends on e_2 which in turn depends on K_r.

This cannot be calculated until the area of steel is known because $N_{ud} = A_c f_{cd} + A_s f_{yd}$ (for a symmetrically reinforced rectangular section N_{bal} can be taken as $0.4A_c f_{cd}$).

Therefore an iterative procedure must be adopted, thus

1) Assume K_r (usually = 1).
2) Calculate M_{Ed}.
3) Find area of steel required in column section for axial load N_{Ed} and moment M_{Ed}.
4) Re-calculate K_r.
5) If this value of K_r differs significantly from the previous value return to 2).



$$e_2 = 0.1\, l_0^2\, (1/r)$$

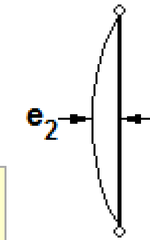
$$1/r = K_r K_\phi (1/r_{bal})$$

$$K_r = \frac{N_{ud} - N_{Ed}}{N_{ud} - N_{bal}} \le 1$$

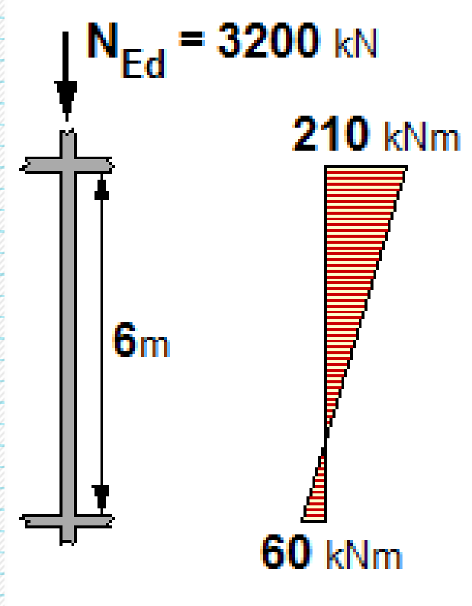
$$1/r_{bal} = \varepsilon_{yd} / 0.45d$$

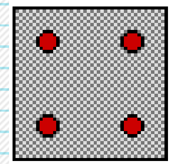
$$K_\phi = 1 + \beta\phi_{ef} \ge 1$$

$$\beta = 0.35 + f_{ck}/200 - \lambda/150$$



Example 4.6: Calculate the area of steel required in the column below by addressing the questions I to V sequentially.





400 mm square
d_2 = 40 mm

USE

- C40/50 and S-500 bars for all reinforcement.
- $\theta_i = 1/200$
- Ignore creep $\phi_{ef} = 0$
- All beam stiffnesses and column stiffnesses are equal

I. What is the effective height of the column, l_o(m)?
II. What is the slenderness ratio, λ of the column and its slenderness limit, λ_{lim} ?
III. What is the value of the equivalent first order moment, M_{oe} (kNm)?
IV. What is the value of the coefficient, K_r?
V. What area of reinforcement is required, A_s (mm²)?

Solution:

Step1: Summarize the given parameters

Material C40/50

f_{ck}=40MPa; f_{cd}=22.66MPa;

$f_{ctk,0.05}$=2.5MPa; f_{ctd}=1.4MPa

E_{cm}=35,000MPa

S-500

f_{yk}=500MPa;

f_{yd}=434.78MPa;

E_s=200,000MPa; ε_y=2.17‰

Action N_{Ed}=3200 kN

M_{01}=60 kNm

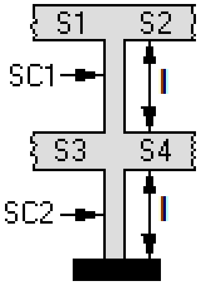
M_{02}=210 kNm

Step2: Compute the effective height of the column.

$l_o = \beta\, l$ where l is the clear height

$$\beta = 0.5\left[\left(1 + \frac{k_1}{0.45 + k_1}\right)\left(1 + \frac{k_2}{0.45 + k_2}\right)\right]^{0.5}$$

Beam stiffnesses and column stiffnesses are euqal therefore -



$$k_1 = k_2 = \frac{SC1+SC2}{S3+S4} = 1$$

$$\beta = 0.5 \times [1.69 \times 1.69]^{0.5} = 0.845$$

$l_o = 0.845 \times 6 = \mathbf{5.07\ m}$

Step3: Compute the slenderness ratio, λ of the column

$\lambda = l_o / i$ For a rectangular section $\mathbf{i} = 0.2887\mathbf{h}$

Hence, $\lambda = 5070 / (0.2887 \times 400) = \mathbf{43.9}$

Step4: Compute the slenderness ratio limit, λ_{lim} of the column

$$\lambda_{lim} = 20\, A\, B\, C\, \eta^{0.5}$$

$A = 1 / (1 + 0.2\phi_{ef})$ $\phi_{ef} = 0$ therefore $\mathbf{A=1.0}$

$B = (1 + 2\omega)^{0.5}$ $A_s(\omega)$ is unknown so $\mathbf{B=1.1}$

$C = 1.7 - r_m$ $r_m = M_{01} / M_{02} = -60/210 = -0.286$

$C = 1.986$

$$\eta = N_{Ed} / (A_c\, f_{cd}) = \frac{3200 \times 1000}{400 \times 400 \times 0.85 \times 40 / 1.5}$$

$\eta = 0.88$

$$\lambda_{lim} = 20 \times 1 \times 1.1 \times 1.986 \times 0.88^{0.5} = \mathbf{41.0}$$

:- this is <43.9 therefore the column is slender.

Step5: Compute the equivalent first order moment.

$$M_{0e} = 0.6M_{02} + 0.4M_{01} \leq 0.4M_{02}$$

$$= 0.6 \times 210 + 0.4 \times -60 = 126 - 24 = \mathbf{102\ kNm}$$

$$(> 0.4 \times 210)$$

The eccentricity due to imperfections is :-

$$e_i = 0.5\, \theta_i\, l_0$$

$$= 0.5 \times (1/200) \times 5070$$

$$= \mathbf{12.675\ mm}$$

giving an additional moment of :-

$$N_{Ed}\, e_i = 3200 \times 12.675 / 1000 = \mathbf{40.56}\ \text{kNm}$$

Step6: Compute the value of the coefficient, K_r

$$e_2 = K_r K_\phi 0.1 l_o^2 e_{yd} / 0.45d \quad (\text{with } K_\phi = 1)$$

$$= 0.1 \times 5070^2 \times (500/1.15/200000) / (0.45 \times 360)\, K_r$$

$$= 34.49\, K_r$$

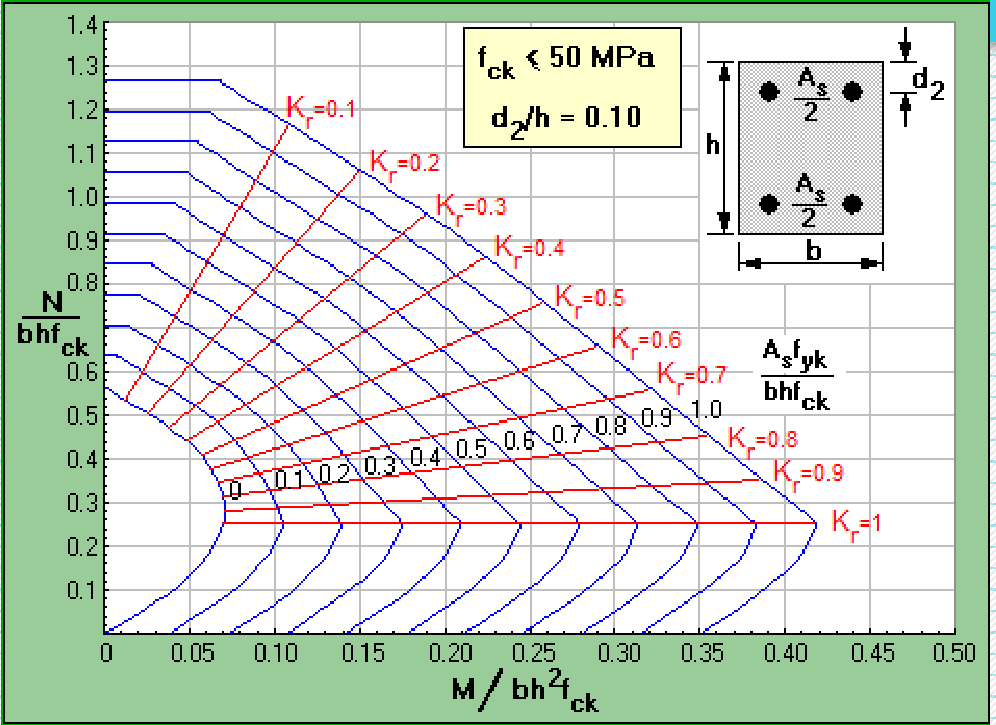
$$M_{Ed} = M_{0e} + N_{Ed}e_i + N_{Ed}e_2 = 102 + 40.56 + 3.2e_2$$

$$N/bhf_{ck} = 3200 \times 1000 / (400 \times 400 \times 40) = \mathbf{0.5}$$

If the effects of creep are ignored design charts cab be produced with K_r lines superimposed to show their influences, but they are of little practical value.

For h'/h =40/400=0.1; $v=N_{Ed}/bhf_{ck}$ = 0.5 and $\mu=M_{Ed}/bh^2f_{ck}$ = 0.174;

Using the chart we have the new K_r=0.47



:- By iterating the value of K_r

assumed K_r	M_{Ed}	M_{Ed}/bh^2f_{ck}	new K_r
1.0	252.9	0.099	0.47
0.47	187.8	0.076	0.41
0.41	188.1	0.073	**0.40**

Step7: Compute area of reinforcement required.

$e_2 = 34.49\,K_r = 34.49 \times 0.40 = 13.8$ mm

$M_{Ed} = M_{0e} + N_{Ed}e_i + N_{Ed}e_2$

$= 102 + 40.56 + 3200 \times 13.8/1000 = \mathbf{186.7}$ kNm

This is **less than** larger end moment plus the imperfection moment = 210 + 40.56 = 250.56 kNm

Therefore, $M_{Ed} = \mathbf{250.6}$ kNm

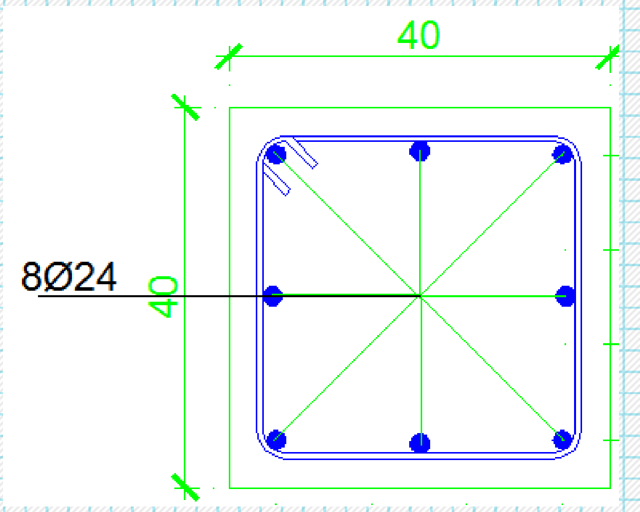
$N/bhf_{ck} = 3200 \times 1000 / (400 \times 400 \times 40) = \mathbf{0.5}$

$M/bh^2f_{ck} = 250.6 \times 1000^2 / (400^3 \times 40) = \mathbf{0.098}$

From **chart** for $d_2/h=0.1$

$w = A_s f_{yk}/bhf_{ck} = 0.23$ approximately

$A_s = 0.23 \times 400 \times 400 \times 40 / 500 = \mathbf{2944\ mm^2}$



Thank you for the kind attention!

Questions?

Please read on the concept simplified design method nominal stiffness!

Homework: redo example 4.6 (previous slides) based on nominal stiffness.

Please read on the concept of Torsion for next class.