Chapter 1: Introduction to Comm. Eng’g

Addis Ababa Institute of Technology
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Undergraduate Program

School of Electrical and Computer Engineering

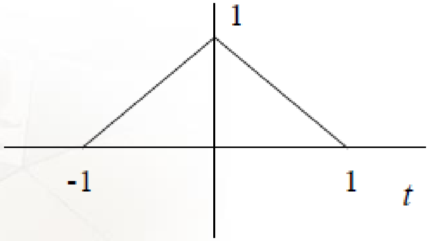
Overview

- **Introduction of Communication Engineering**
  - Elements of communication system
    - Channel characteristics
    - Signals and systems – ***Review***
    - Mathematical models of a channel
  - Fundamentals of Analog Transmission
  - The Hilbert Transform & Bandpass Signals

Common Signals

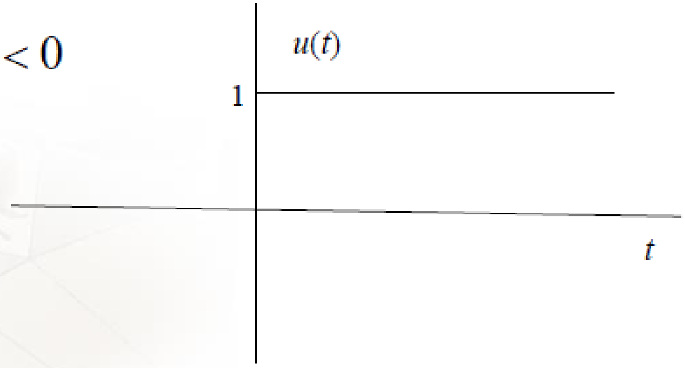
Triangle pulse

$$\Lambda(t) = \begin{cases} 1-|t| & |t|<1 \\ 0 & |t|>1 \end{cases}$$



Step function

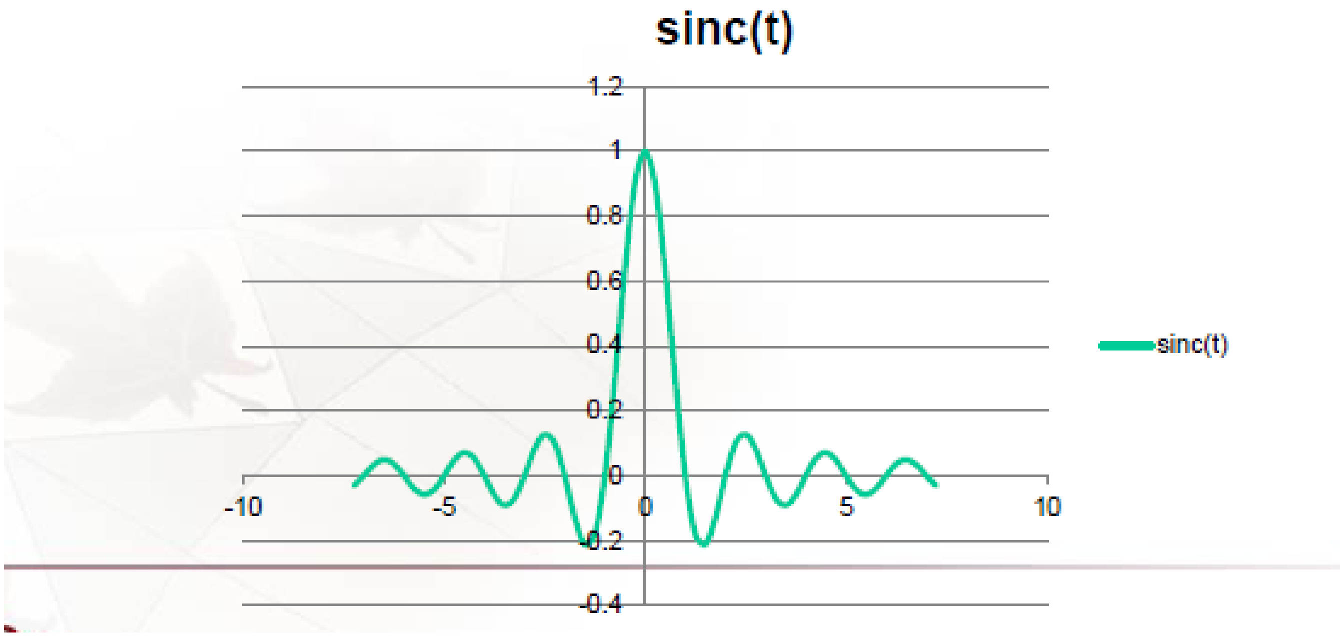
$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$



Common Signals

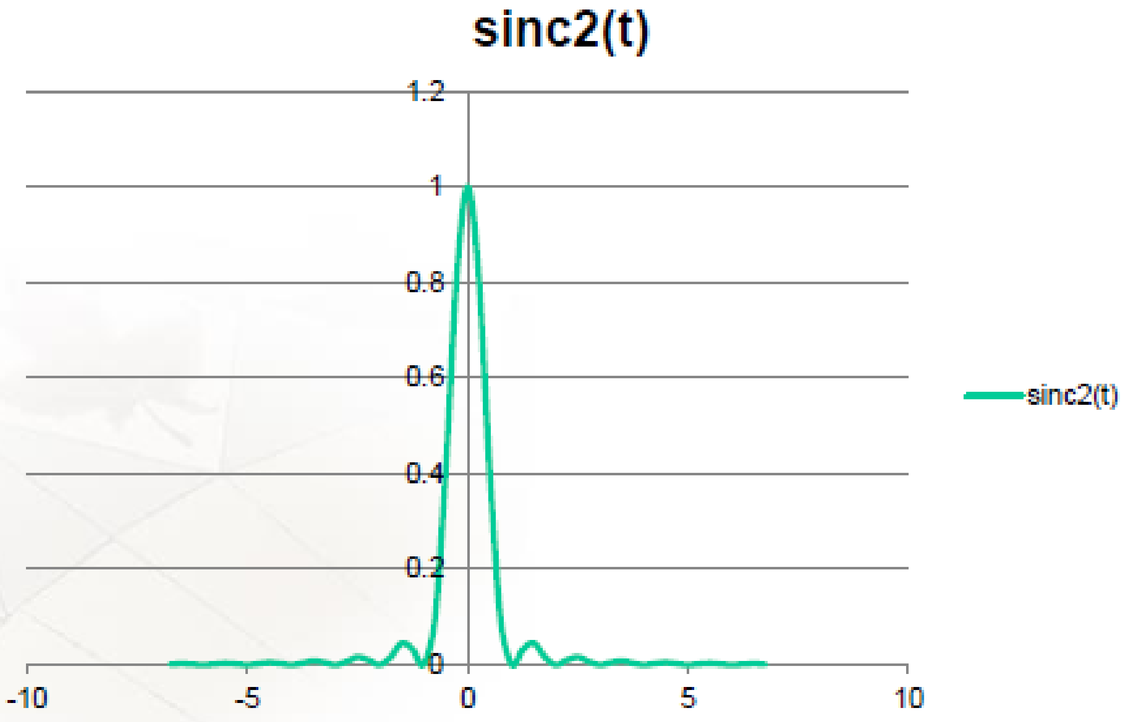
Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Common Signals

Sinc squared function



Frequency Domain Analysis of Signal and Systems

- Fourier series
- Fourier transform
- Power and energy
- Sampling Theorem

Periodic Signals

- Periodic signals are important class of signals (widely used), where smallest T is a period

$$x(t) = x(t+T), \text{ for all } t$$

- Examples: $\cos(\omega_0 t)$ & $e^{j\omega_0 t}$. Period $T = 2\pi / \omega_0$
- Introduce a set of harmonically-related complex exponentials

$$\phi_n(t) = e^{jn\omega_0 t} = e^{jn\frac{2\pi}{T}t}, \quad n = 0, \pm 1, \pm 2, \dots$$

DC (n = 0) 1st harmonic (±1)

- Construct a periodic signal

$$x'(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$$

Fourier Series of Periodic Signal

- Can x'(t) be made the same as x(t)?
- Yes, by adjusting c_n,

$$c_n = \frac{1}{T}\int_T x(t)\, e^{-j2\pi \frac{n}{T} t}\, dt \quad \Rightarrow \quad x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}, \omega_0 = \frac{2\pi}{T}$$

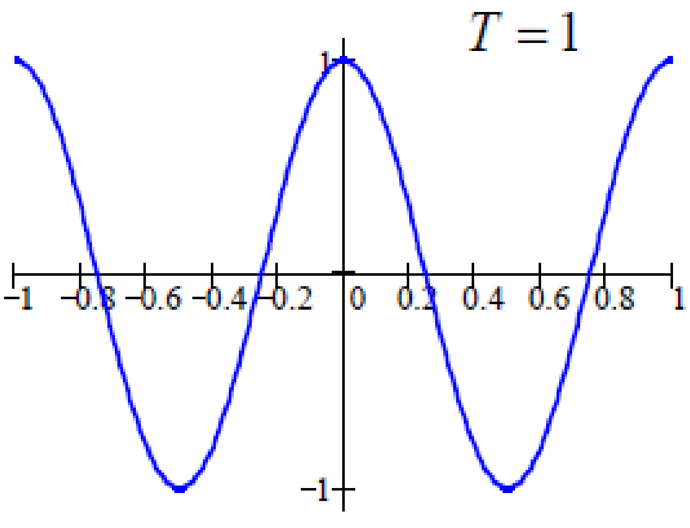
- $\{c_n\}$ – Fourier series coefficient (or spectral coefficients or discrete spectrum of the signal)
- c_0 – DC component or average value of x(t)

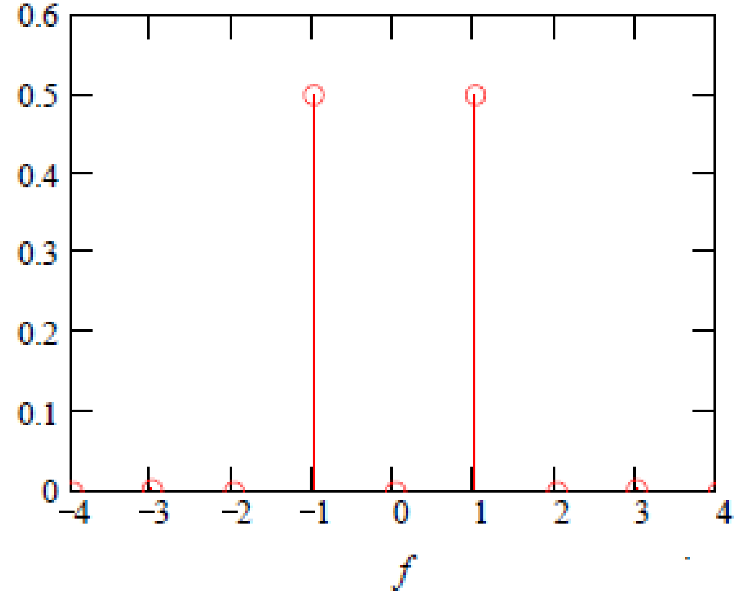
$$c_0 = \frac{1}{T}\int_T x(t)\, dt$$

Fourier Series – Example 1

$$x(t) = \cos(\omega_0 t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$c_1 = \frac{1}{2}, c_{-1} = \frac{1}{2},$$
$$c_k = 0, k \neq \pm 1$$



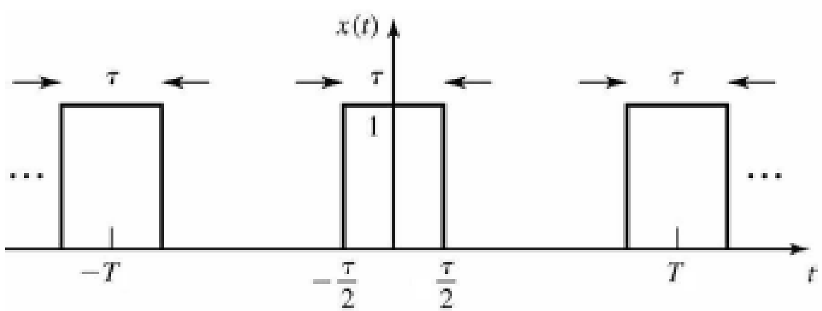


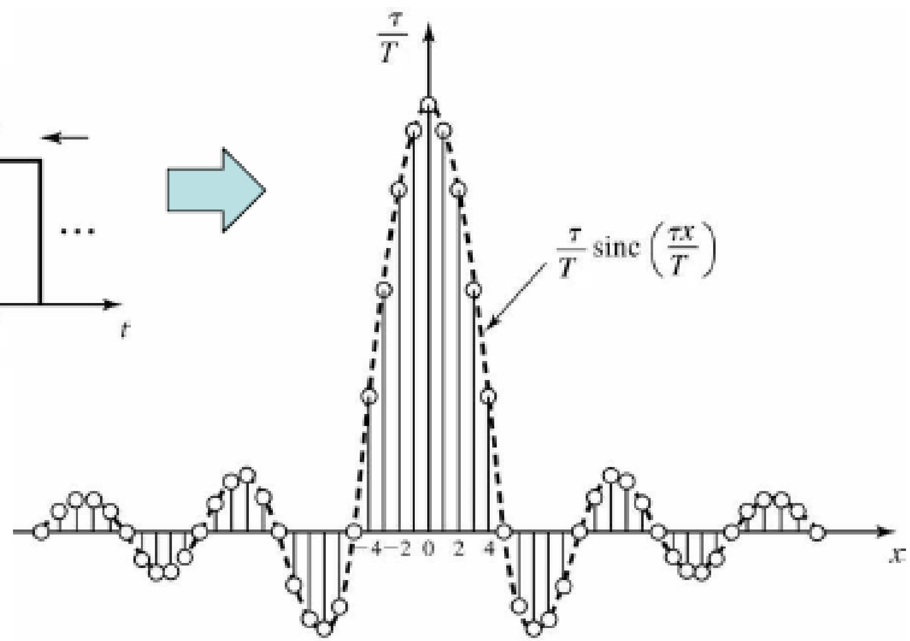
Fourier Series – Example 2

$$x(t)=\begin{cases}1, & |t|<\tau/2\\ 0, & \tau/2<|t|<T/2\end{cases}$$

$$c_n=\frac{\tau}{T}\operatorname{sinc}\left(\frac{n\tau}{T}\right),$$

$$\operatorname{sinc}(t)=\frac{\sin(\pi t)}{\pi t}$$





Q.: How does c_n scale with the pulse amplitude? Duration? Period?

Convergence of Fourier Series

- Dirichlet conditions:
  - x(t) must be absolutely integrable (finite power)

$$\int_T |x(t)|\,dt < \infty$$

  - x(t) must be of bounded variation; that is the number of maxima and minima during a period is finite
  - In any finite interval of time, there are only a finite number of discontinuities, which are finite.
- Dirichlet conditions are only sufficient, but are not necessary.
- All physically-reasonable (practical) signals meet these conditions.

Fourier Series of Real Signals

- For a real signal, Im{ x(t)} = 0 $\Rightarrow c_{-n} = c_n^*$
- Then the trigonometric Fourier series is given as

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right], \quad \omega_0 = \frac{2\pi}{T}$$

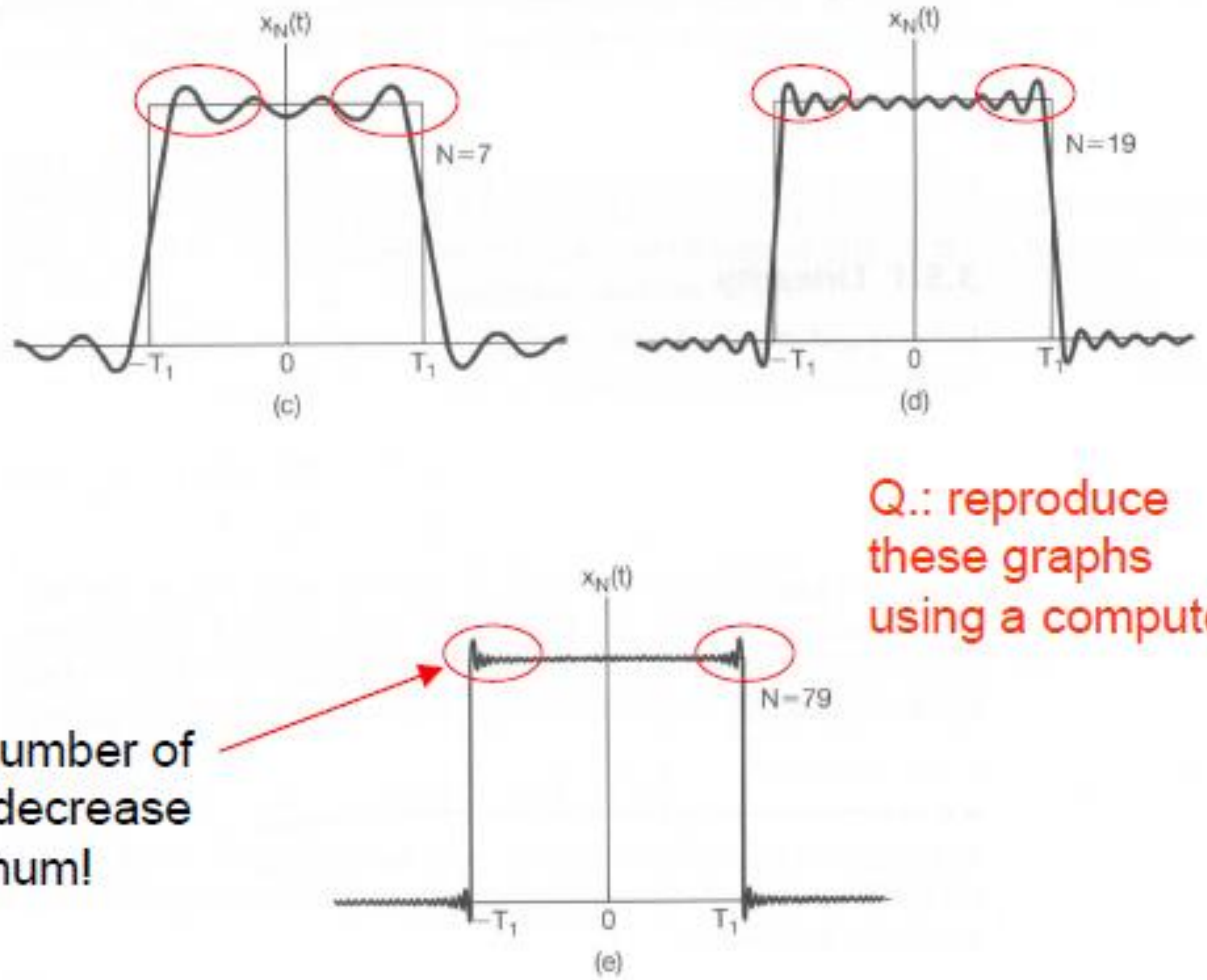
$$a_n = 2\,\mathrm{Re}\{c_n\} = \frac{2}{T}\int_T x(t)\cos(n\omega_0 t)dt, \quad b_n = -2\,\mathrm{Im}\{c_n\} = \frac{2}{T}\int_T x(t)\sin(n\omega_0 t)dt$$

- Another form of it is

$$x(t) = x_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \varphi_n)$$

$$A_n = |c_n| = \sqrt{a_n^2 + b_n^2}, \quad \varphi_n = -\arg(c_n) = -\tan^{-1}(b_n / a_n)$$

Gibbs Phenomenon



Q.: reproduce these graphs using a computer

increasing the number of terms does not decrease the ripple maximum!

Properties of Fourier Series

- Linearity: $$\mathrm{F}\left[\alpha x_1(t)+\beta x_2(t)\right]=\alpha\mathrm{F}\left[x_1(t)\right]+\beta\mathrm{F}\left[x_2(t)\right]$$
- Time shifting: $$x(t)\overset{F}{\longleftrightarrow}c_n \Leftrightarrow x(t-t_0)\overset{F}{\longleftrightarrow}e^{-jn\omega_0 t_0}c_n$$
- Time reversal: $$x(t)\overset{F}{\longleftrightarrow}c_n \Leftrightarrow x(-t)\overset{F}{\longleftrightarrow}c_{-n}$$
- Time scaling: $$x(\alpha t)=\sum_{n=-\infty}^{+\infty}c_n e^{jn(\alpha\omega_0)t}$$

Properties of Fourier Series

- Multiplication: $$x(t)y(t) \overset{F}{\longleftrightarrow} \sum_{k=-\infty}^{\infty} c'_k c''_{n-k}$$
- Convolution: $$\int_T x(\tau)y(t-\tau)d\tau \overset{F}{\longleftrightarrow} Tc'_n c''_n$$
- Differentiation: $$\frac{dx(t)}{dt} \overset{F}{\longleftrightarrow} jn\omega_0 c_n$$
- Integration: $$\int_{-\infty}^{t} x(\tau)d\tau \overset{F}{\longleftrightarrow} \frac{c_n}{jn\omega_0},$$ for $c_0 = 0$

Properties of Fourier Series

- Real x(t): $$c_{-n} = c_n^*$$
- Real & even x(t): $$c_{-n} = c_n, \mathrm{Im}\{c_n\} = 0$$
- Real & odd x(t): $$c_{-n} = -c_n, \mathrm{Re}\{c_n\} = 0$$
- Parseval's Theorem: $$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Frequency Domain Analysis of Signal and Systems

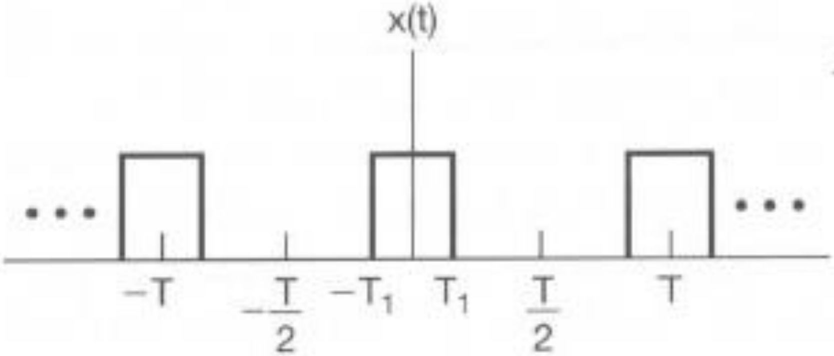
- Fourier series
- Fourier transform
- Power and energy
- Sampling Theorem

Fourier Transform - Review

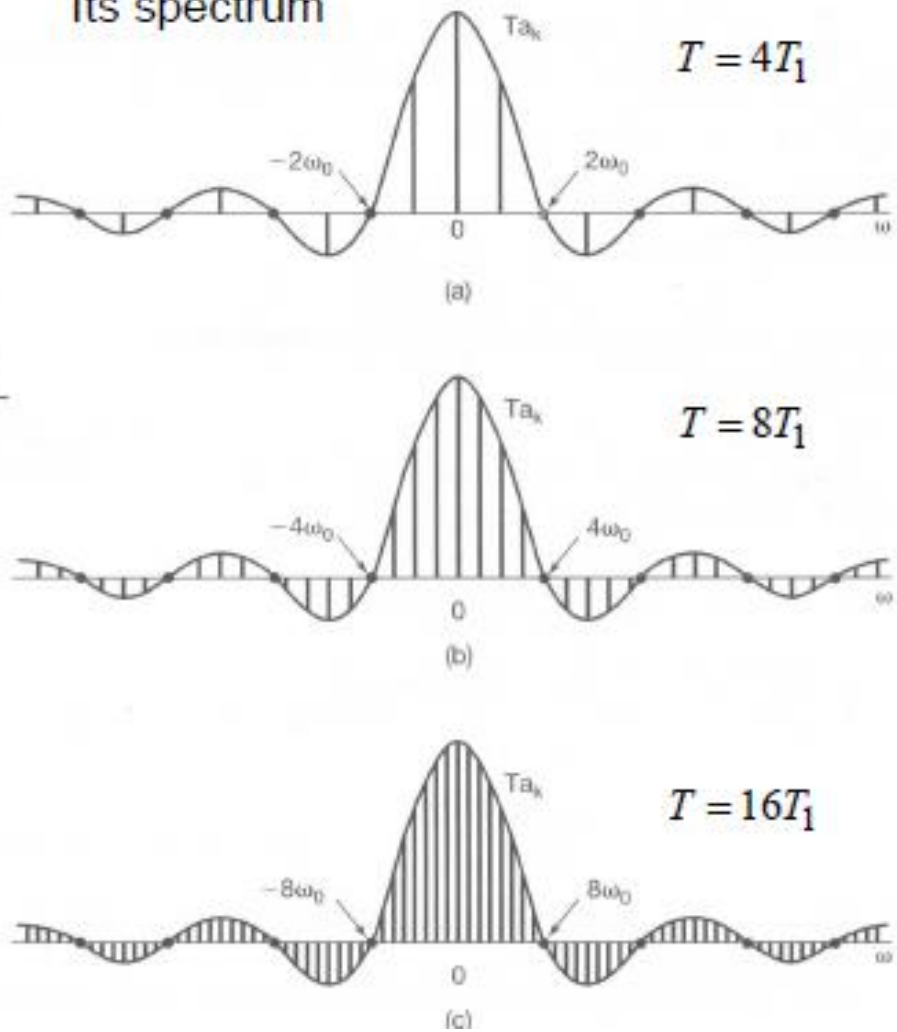
- Fourier series works for periodic signals only
- What about aperiodic signals?
  - This is very large and important class of signals
- Aperiodic signal can be considered as periodic for $T \to \infty$
- Fourier series changes to Fourier transform
- Complex exponents are infinitesimally close in frequency
- Discrete spectrum becomes a continuous one
  - Also known as spectral density

Fourier Series -> Fourier Transform

Periodic signal



Its spectrum



As T increases, spectral components are getting closer and closer, becoming the continuous spectrum at the limit

Fourier Transform

- Fourier transform (spectrum)

$$S_x(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

radial frequency

$$S_x(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

- Inverse Fourier transform

$$x(t) = \int_{-\infty}^{+\infty} S_x(f) e^{j2\pi ft} df$$

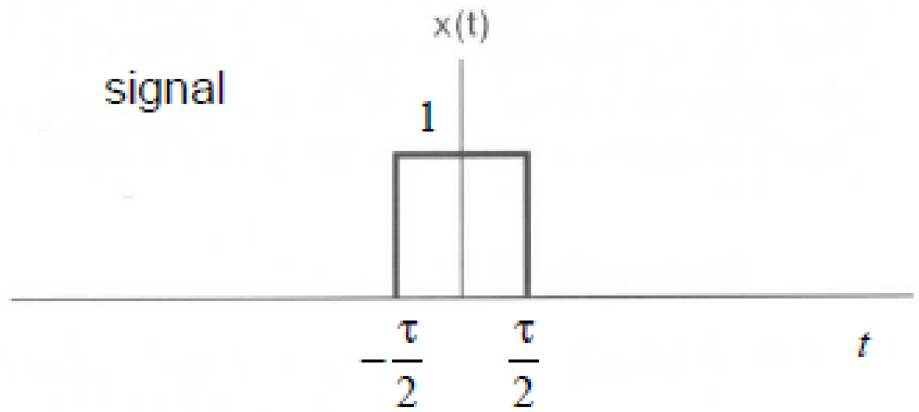
radial frequency

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_x(\omega) e^{j\omega t} d\omega$$

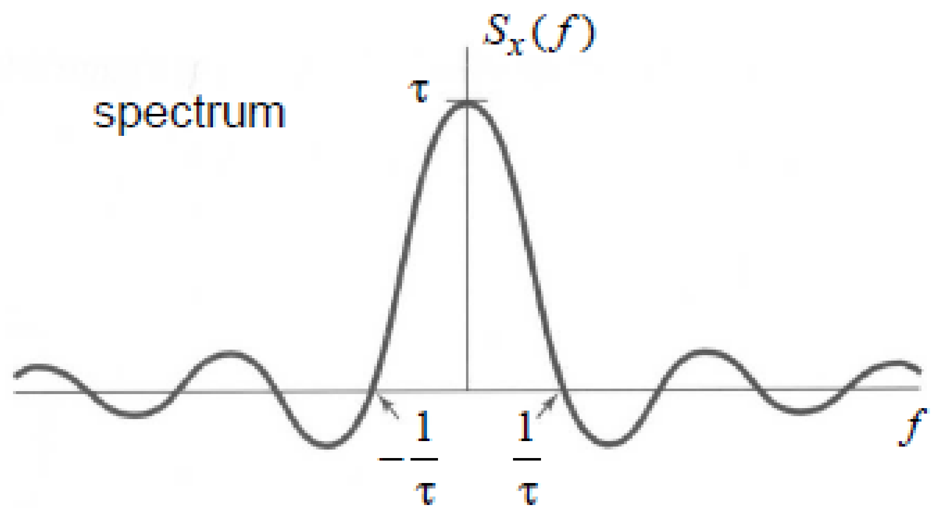
- Convergence of Fourier Transform - Dirichlet conditions
  - x(t) is absolutely integrable
  - X(t) has a fine number of maxima and minima within any finite interval
  - X(t) has a finite number of discontinuities within any finite interval
  - These discontinuities must be finite

Examples: Rectangular Pulse

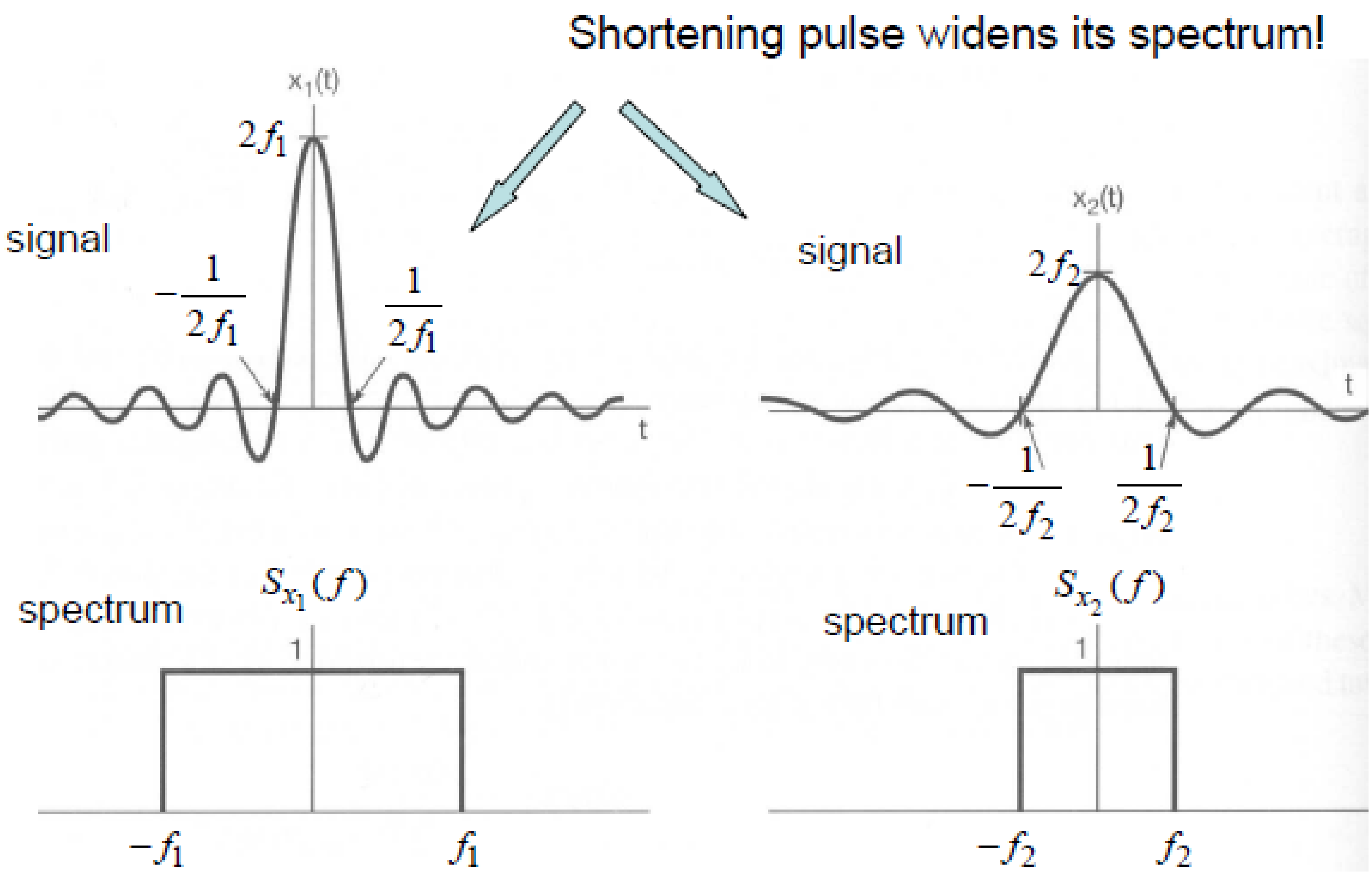
$$x(t) = \Pi(t) = \begin{cases} 1, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$



$$S_x(f) = \tau \frac{\sin \pi f \tau}{\pi f \tau} = \tau \operatorname{sinc}(f\tau)$$



Example: Sinc(t)



Fourier Transform of Periodic Signal

- FT of a complex exponent:

$$x(t) = e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0) = \delta(f - f_0)$$

- Important property:

$$\delta(f) = \int_{-\infty}^{+\infty} e^{\pm j2\pi ft}\, dt$$ ← prove this property

- FT of a periodic signal:

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t} \overset{FT}{\leftrightarrow} 2\pi \sum_{n=-\infty}^{+\infty} c_n \delta(\omega - n\omega_0) = \sum_{n=-\infty}^{+\infty} c_n \delta(f - nf_0)$$

- FT of $\cos(\omega_0 t)$?

Properties of Fourier Transform*

- *Very similar* to those of Fourier series!
- Linearity: $$\alpha x_1(t)+\beta x_2(t) \overset{F}{\leftrightarrow} \alpha S_{x_1}(f)+\beta S_{x_2}(f)$$
- Time shifting: $$x(t) \leftrightarrow S_x(\omega) \Rightarrow x(t-t_0) \leftrightarrow e^{-j\omega t_0} S_x(\omega)$$
- Time reversal: $$x(t) \leftrightarrow S_x(\omega) \Rightarrow x(-t) \leftrightarrow S_x(-\omega)$$
- Time scaling: $$x(at) \leftrightarrow \frac{1}{|a|} S_x\left(\frac{\omega}{a}\right)$$

Prove these properties.

*properties are useful for evaluating Fourier transform in a simple way

Properties of Fourier Transform

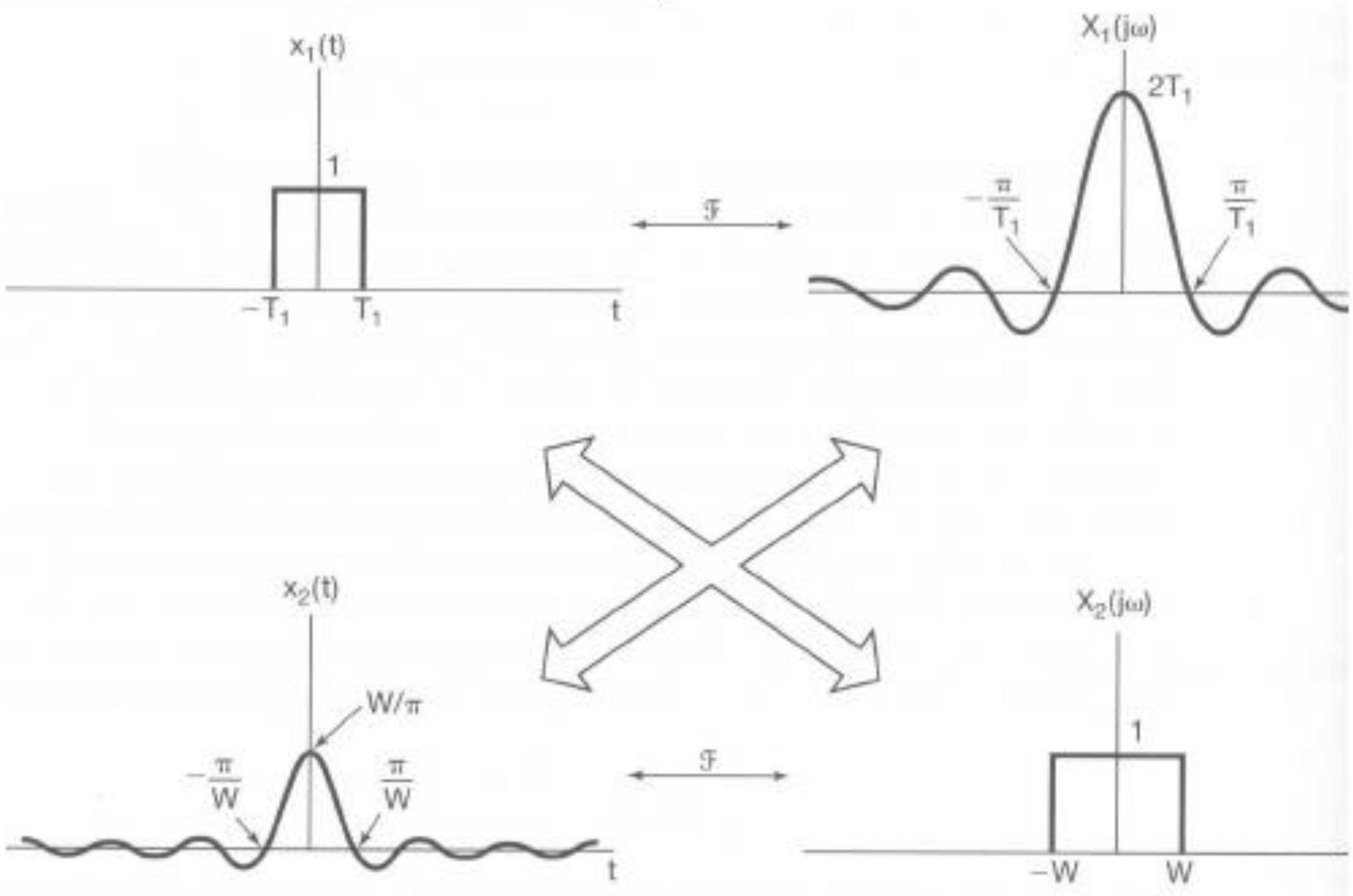
- Conjugation: $x(t) \leftrightarrow S_x(\omega) \Rightarrow x^*(t) \leftrightarrow S_x^*(-\omega)$
- Differentiation: $x(t) \leftrightarrow S_x(\omega) \Rightarrow \frac{dx(t)}{dt} \leftrightarrow j\omega S_x(\omega)$
- Integration: $\int_{-\infty}^{t} x(t)dt \leftrightarrow \frac{1}{j\omega} S_x(\omega) + \pi S_x(0)\delta(\omega)$
- Multiplication: $x(t)y(t) \leftrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega')S_y(\omega-\omega')d\omega' = S_x(\omega) * S_y(\omega)$
- Frequency shift (modulation): $x(t)e^{j\omega_0 t} \leftrightarrow S(\omega-\omega_0)$

Prove these properties

Duality of Fourier Transform

$$x(t) \leftrightarrow S_x(\omega) \Rightarrow S_x(t) \leftrightarrow 2\pi x(-\omega)$$

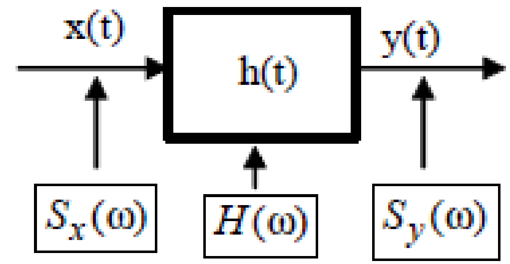
$$x(t) \leftrightarrow S_x(f) \Rightarrow S_x(t) \leftrightarrow x(-f)$$



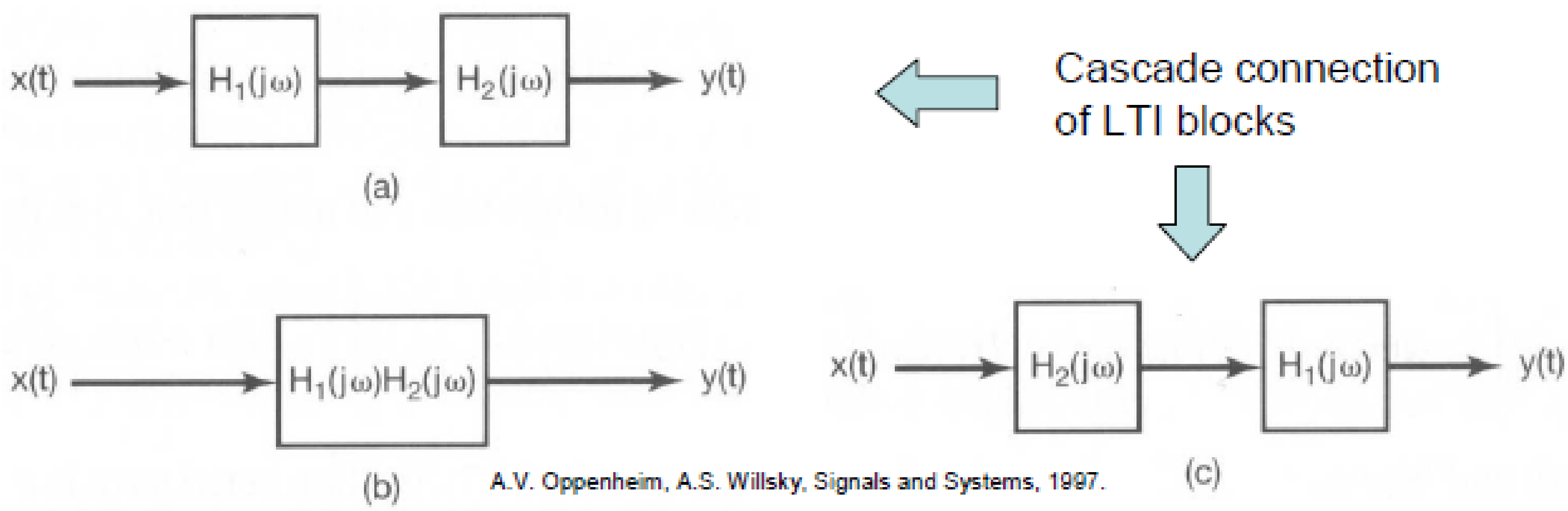
Convolution Property

- This property is of great importance

$$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \leftrightarrow S_x(\omega)H(\omega)=S_y(\omega)$$



Cascade connection of LTI blocks



A.V. Oppenheim, A.S. Willsky, Signals and Systems, 1997.

Parseval Theorem

- Total energy in time domain is the same as the total energy in frequency domain

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |S_x(f)|^2 df = \frac{1}{2\pi} \int_{-\infty}^{\infty} |S_x(\omega)|^2 d\omega$$

- $E(f) = |S_x(f)|^2$. energy spectral density (ESD) of x(t)
- It represents the amount of energy per Hz of bandwidth
- Counterpart of Parseval theorem for periodic signal is the
- Autocorrelation property

$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t)\, x^*(t-\tau)\, dt \leftrightarrow |S_x(\omega)|^2$$

$$R_x(0) = E$$

Fourier Transform of Real Signals

- If x(t) is real $\mathrm{Im}\{x(t)\} = 0 \Rightarrow S_x(-\omega) = S_x^*(\omega)$

- Fourier transform can be presented as

$$x(t) = 2\int_0^\infty |S_x(f)| \cos\left(2\pi f + \varphi(f)\right) df,$$

$$\varphi(f) = \tan^{-1}\left(\frac{\mathrm{Im}[S_x(f)]}{\mathrm{Re}[S_x(f)]}\right)$$

No negative frequencies!

Frequency Domain Analysis of Signal and Systems

- Fourier series
- Fourier transform
- Power and energy

Power and Energy

- Power P_x and energy E_x of a signal x(t) are

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \qquad E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- Energy-type signals: $E_x < \infty$
- Power-type signals $0 < P_x < \infty$
- Signal cannot be both energy and power type
- Signal energy: if x(t) is voltage or current, E_x is the energy dissipated in 1 Ohm resistor
- Signal power: if x(t) is voltage or current, P_x is the power dissipated in 1 Ohm resistor

Energy-type Signals (Summary)

- Signal energy in time and frequency domains:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |S_x(f)|^2 df = \frac{1}{2\pi} \int_{-\infty}^{\infty} |S_x(\omega)|^2 d\omega$$

- Energy spectral density (energy per Hz of bandwidth)

$$E_x(f) = |S_x(f)|^2$$

- ESD is FT of autocorrelation function

$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t) x^*(t-\tau) dt \leftrightarrow E_x(f)$$

$$R_x(0) = E_x$$

Power-type Signals: PSD

- Definition of the power spectral density (PSD) (power per Hz of bandwidth):

$$P_x(f) = \lim_{T\to\infty} \frac{\left|S_T(f)\right|^2}{T} \Rightarrow P_x = \int_{-\infty}^{\infty} P_x(f)\,df < \infty$$

- where $x_T(t)$ is the truncated signal (to [-T/2,T/2]),

$$x_T(t) = x(t)\Pi\left(\frac{t}{T}\right) = \begin{cases} x(t), & -T/2 \le t \le T/2 \\ 0, & \text{otherwise} \end{cases}$$

- and $S_T(f)$ is its spectrum (FT),

$$S_T(f) = FT\left\{x_T(t)\right\}$$

Signal Bandwidth and Negative Frequencies

- What negative frequency means?
- It means that there is $e^{-j2\pi ft}$ term in signal spectrum
- Convenient mathematical tool
  - Do not exist in practice (i.e., cannot be measured on spectrum analyzer)

- What is the signal bandwidth? There are many definitions
- Defined for positive frequency only
- Determines the frequency band over which a substantial part of the signal power/energy is concentrated
- For bandlimited signals

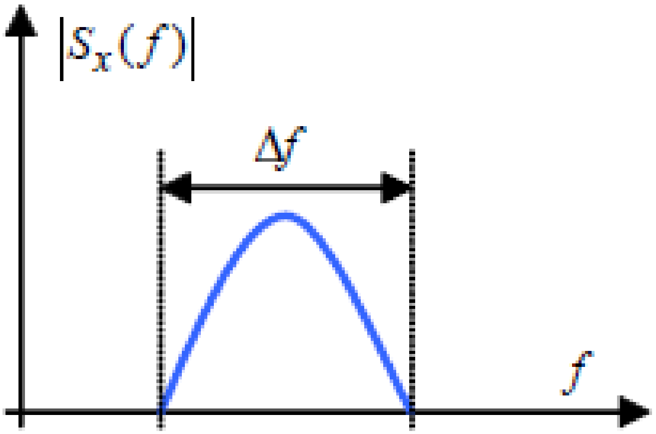
$$\Delta f = f_{\max} - f_{\min}, \quad f_{\max}, f_{\min} \geq 0$$

Signal Bandwidth

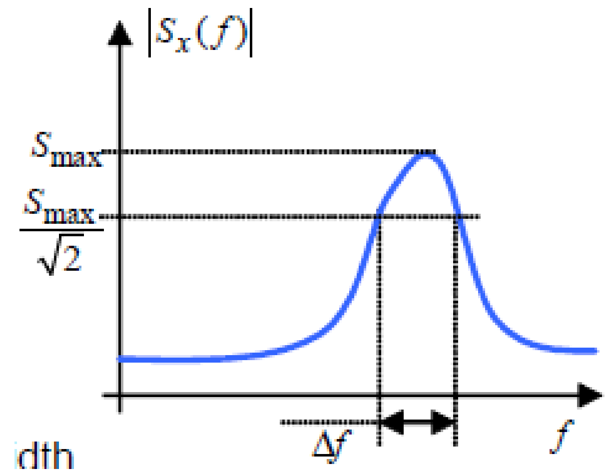
- Defined for positive frequencies only
- Informal: a frequency band over which a substantial (or all) signal power is concentrated
- Absolute bandwidth: for band-limited signals, frequency band where spectrum is not zero
  - For all other frequencies, the spectrum must be zero
- 3 dB (half-power) bandwidth: frequency band where PSD (or ESD) is not lower than -3dB with respect to the maximum
- Zero-crossing bandwidth: frequency band limited by 1st zero(s) in the spectrum

Signal Bandwidth

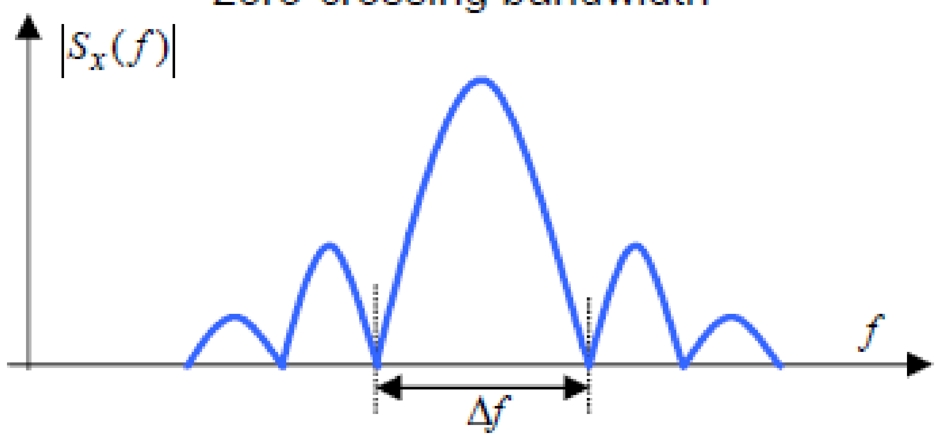
absolute bandwidth



3 dB bandwidth



zero-crossing bandwidth



Overview

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    - Mathematical models of a channel
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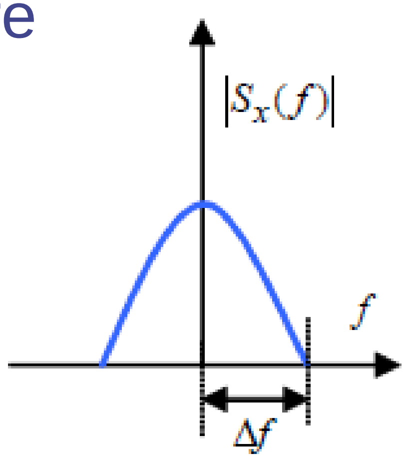
Baseband & Bandpass Signals

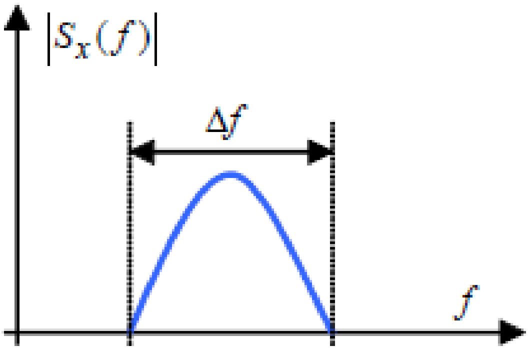
- Baseband (lowpass) signal: Spectrum is nonzero around the origin (f=0) and zero (negligible) elsewhere

$$S_X(f) \neq 0, \quad |f| \leq f_{\max}$$



- Bandpass (narrowband) signal: spectrum is nonzero around the carrier frequency f_c and zero (negligible) elsewhere

$$S_X(f) \neq 0, \quad |f - f_c| \leq B$$



Complex Envelope Representation

- Any narrowband (bandpass) signal can be presented as

$$x(t) = \mathrm{Re}\left\{C(t)e^{j\omega_c t}\right\} = A(t)\cos\left(\omega_c t + \varphi(t)\right)$$

- where
  - $c(t) = A(t)e^{j\varphi(t)}$ is complex envelope (phasor)
  - $A(t)=|c(t)|$ is amplitude
  - $\varphi(t) = \angle c(t)$ is phase
- Amplitude and phase vary in time, but much slower than the carrier
- Equivalent form (in-phase (I) and quadrature (Q))

$$x(t) = a_I(t)\cos\left(\omega_c t\right) - a_Q(t)\sin\left(\omega_c t\right)$$

- where

$$a_I(t) = \mathrm{Re}\left\{C(t)\right\} = A(t)\cos\left(\varphi(t)\right)$$
$$a_Q(t) = \mathrm{Im}\left\{C(t)\right\} = A(t)\sin\left(\varphi(t)\right)$$

Complex Envelope Representation …

- $C(t)$, $A(t)$, $\varphi(t)$, $a_I(t)$, $a_Q(t)$ – are baseband signals
- Some additional relations:

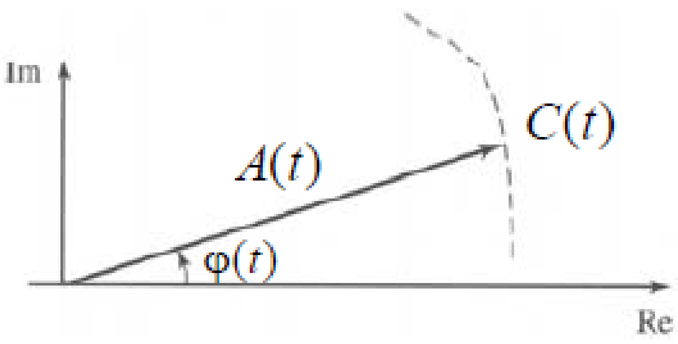
$$C(t) = a_I(t) + j a_Q(t)$$

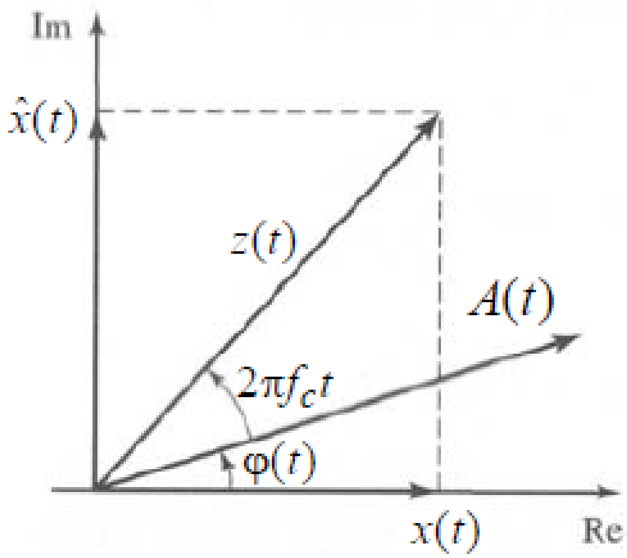
$$A(t) = \sqrt{a_I^2(t) + a_Q^2(t)}$$

$$\varphi(t) = \tan^{-1}\left(\frac{a_Q(t)}{a_I(t)}\right)$$

- Very useful for analysis and simulation of modulated signals

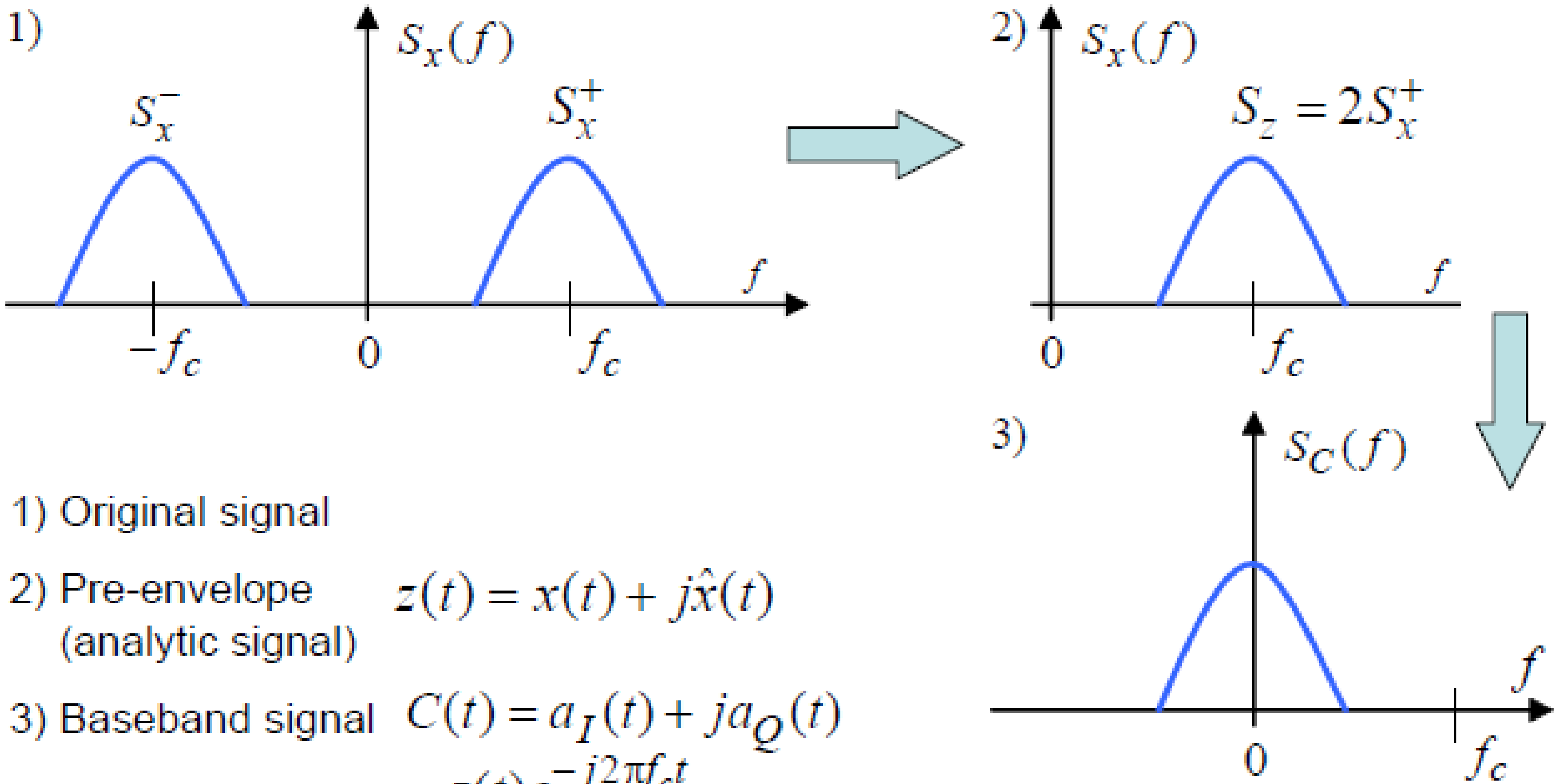
Geometric Viewpoint of Narrowband Signals





- $A(t)$ is rotating at $d\varphi(t)/dt$ (rad/s)
- $z(t)$ is rotating at $2\pi f_c$ (rad/s) w.r.t. *A(t)*

Frequency-Domain Viewpoint



1) Original signal

2) Pre-envelope (analytic signal) $z(t) = x(t) + j\hat{x}(t)$

3) Baseband signal $C(t) = a_I(t) + ja_Q(t)$
$= z(t)e^{-j2\pi f_c t}$

Hilbert transform: $\hat{x}(t) \rightarrow S_{\hat{x}}(f) = -j\,\mathrm{sgn}(f) \cdot S_x(f)$

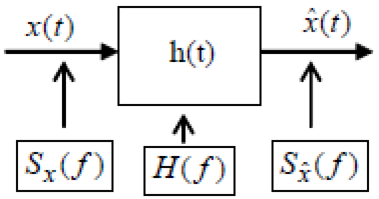
Hilbert Transform (Extra!)

- Frequency-domain representation

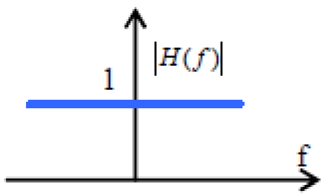
$$S_{\hat{x}}(f) = \begin{cases} -jS_x(f), f > 0 \\ jS_x(f), f < 0 \end{cases}$$

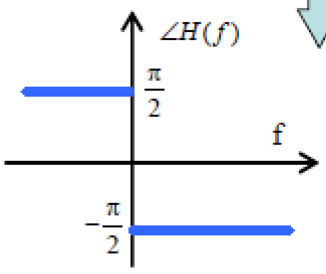
$$H(f) = \begin{cases} -j(-90^0), f > 0 \\ j(90^0), f < 0 \end{cases}$$

- Time-domain representation



$$\hat{x}(t) = \frac{1}{\pi t} * x(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$





- Example: $x(t) = A\cos(\omega t + \theta) \rightarrow \hat{x}(t) = ?$

Example

- Consider the signal

$$x(t) = cos(2\pi f_m t)cos(2\pi f_c t)$$

A. Obtain and sketch the spectrum of the analytical signal (pre-envelop) $xp(t) = x(t) + j\hat{x}(t)$

B. Obtain and sketch the spectrum of the complex envelope (or complex baseband representation) $\tilde{x}(t)$

Ex.

- Consider the signal

$$x(t) = 2W\, sinc(2W\, t)\, cos(2\pi f_0 t)$$

A. Is the signal narrowband or wideband? Justify your answer.

B. Obtain and sketch the spectrum of the analytical signal $xp(t) = x(t) + j\hat{x}(t)$

C. Obtain and sketch the spectrum of the complex envelope (or complex baseband representation) $\tilde{x}(t)$

Overview

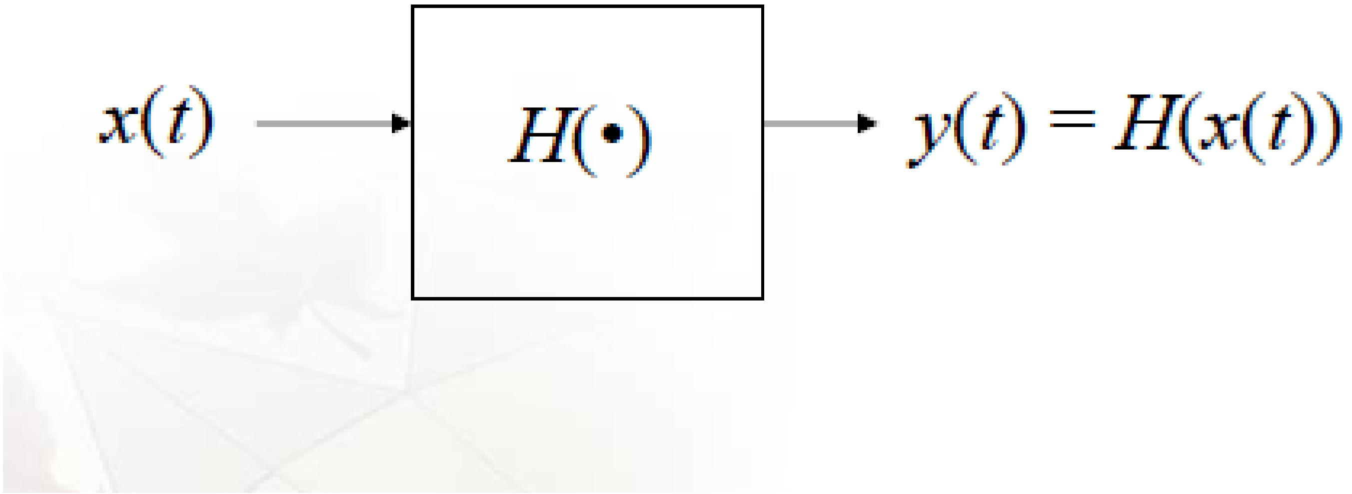
- Elements of communication system
- Channel characteristics
- Signals and systems – Review
- Mathematical models of a channel

Channel Impairments

- There are factors that limit the performance of the communication system.
  - *Attenuation*: radio signal strength decreases as it propagates through matter.
  - *Interference*
  - *Noise* : Undesired or unwanted signal
    - *Shot noise*: the electrons are discrete and are not moving in a continuous steady flow, so the current is randomly fluctuating.
    - *Thermal noise:* caused by the rapid and random motion of electrons within a conductor due to thermal agitation. $(Thermal\ Noise\ Power = KB.T.BW)$
  - *Phase delays*
    - due multipath propagation: radio signal reflects off objects ground, arriving at destination at slightly different times
- Can be modeled as realizable (LTI) system

Review of Linear Time Invariant Systems

- A system performs a transformation on an input x(t) to produce an output y(t)



Review of Linear Time Invariant Systems

- Linear Systems
  - A linear system is a system for which the **superposition** property applies

– Consider a system that produces output $y_1(t)$ for input $x_1(t)$ and output $y_2(t)$ for input $x_2(t)$ then we write

- $y_1(t) = H(x_1(t))$ and
- $y_2(t) = H(x_2(t))$

– Then the system H is linear if for $x_3(t) = ax_1(t)+bx_2(t)$, $y_3(t)=H(x_3(t)) = aH(x_1(t))+bH(x_2(t)) = ay_1(t)+by_2(t)$.

Time Invariant Systems

- A system is time invariant if a time shift to the input results in no changes other than the same time shift being applied to the output

- If $y_1(t)$ *is the output of the system when* $x_1(t)$ *is the input let* $x_2(t) = x_2(t-\tau)$ *be the input that produces output* $y_2(t)$

- The system is time invariant if $y_2(t) = y_1(t-\tau)$

Linear time Invariant Systems

- A system is LTI if it is both linear and time invariant
- An LTI system is described by its impulse response
- The system's impulse response is $h(t)$ *and it is the output of the system when the input is* $x(t) = \delta(t)$

Output of LTI system

- For any input $x(t)$, the output of an LTI system is $y(t) = x(t)*h(t)$, where * denotes convolution.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

Causality

- A system is causal if it's output depends only on past and present values of the input (it does not depend on future values of the input).
- For LTI system:

$$y(t) = \int_{-\infty}^{\infty} h(\lambda) x(t-\lambda) d\lambda$$

- When $\lambda < 0$, $y(t)$ depends on future values of the input. Therefore an LTI system is causal if $h(\lambda)=0$ for all $\lambda < 0$.

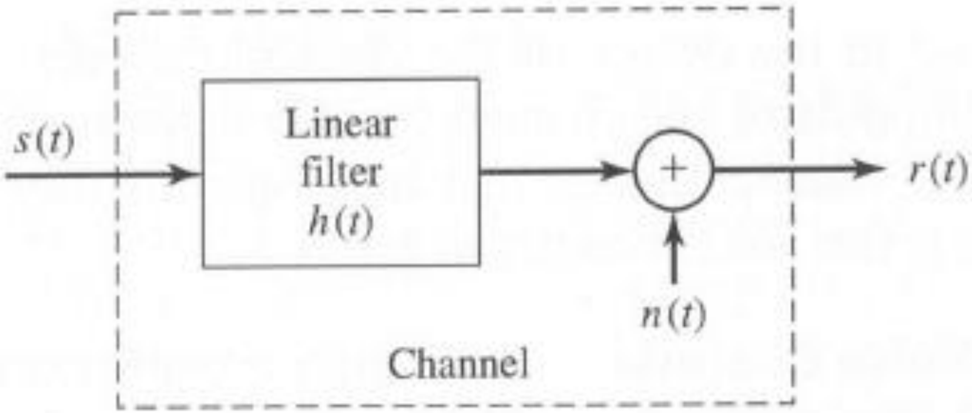
Stability

- A system is stable if for any bounded input, it's output is also bounded.
- For LTI system, this implies that

$$\int_{-\infty}^{\infty} |\, h(\lambda) \,|\, d\lambda \leq \infty$$

Mathematical Models of Channels

- System-level model: linear time-invariant system



$$r(t) = s(t) * h(t) + n(t) =$$
$$= \int_{-\infty}^{+\infty} h(\tau) s(t-\tau) d\tau + n(t)$$

- Detailed model: based on Electromagnetics (i.e., radio wave propagation)

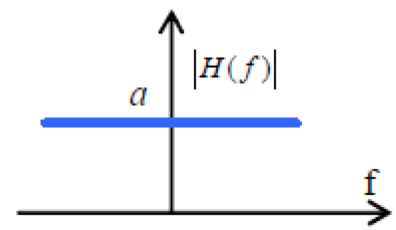
Distortionless Transmission

- When a signal is not distorted by a filter?
- Output is a shifted and scaled copy of the input

$$y(t) = \mathbf{L}\left[x(t)\right] = a \cdot x(t - t_0)$$

- In the frequency domain:

$$S_y(f) = a \cdot e^{-j2\pi f t_0} S_x(f)$$

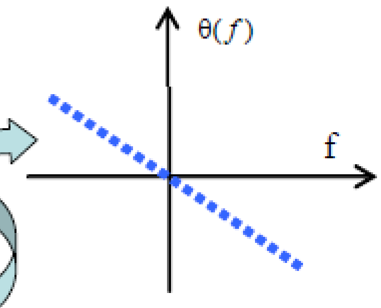


- Filter frequency response

$$H(f) = a \cdot e^{-j2\pi f t_0}$$

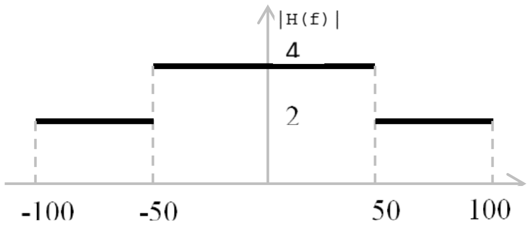
$$\begin{cases} |H(f)| = const \\ \theta(f) = -2\pi f t_0 \end{cases}$$

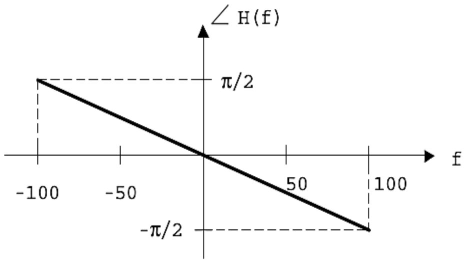
$$t_0 = -\theta(f)/(2\pi f)$$



Example

- A given communication channel has amplitude and phase responses as shown in the figure below:





- For which cases is the transmission distortion-less?
- With a plot of amplitude and phase spectrum of the output indicate what type of distortion in imposed.

  a) $cos(48\pi t) + 5\,cos(126\pi t)$

  b) $\cos(10\pi t) + 4\cos(50\pi t)$

Distortionless Transmission: Narrowband Signals

- Output is a shifted and scaled copy of the input + constant phase shift of the carrier is permitted

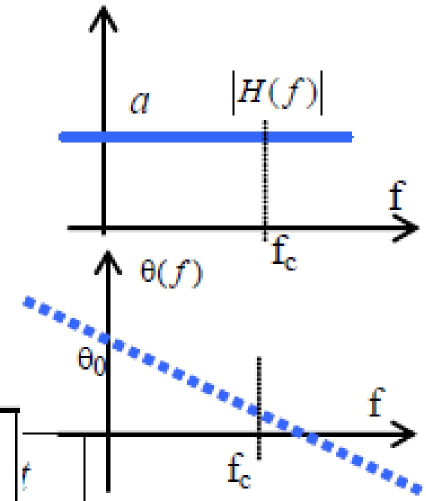
$$x(t) = A(t)\cos\left(\omega_c t + \varphi(t)\right) \Rightarrow$$
$$y(t) = a \cdot A(t-t_0)\cos\left(\omega_c (t-t_0) + \varphi(t-t_0) + \theta_0\right)$$

- In the frequency domain

$$S_y^+(f) = a \cdot e^{j(-2\pi f t_0 + \theta_0)} S_x^+(f)$$

- Filter frequency response (over the signal bandwidth)

$$\begin{cases} |H(f)| = const \\ \theta(f) = -2\pi f t_0 + \theta_0 \end{cases}$$



group time delay: $t_g = -\frac{1}{2\pi}\frac{d\theta(f)}{df}$

carrier time delay: $t_c = t_0 - \theta_0/(2\pi f_c)$

Summary

- Baseband (lowpass) and narrowband (bandpass) signals and systems
- Complex envelope representation
  - Time-varying amplitude and phase
- Hilbert transform
  - In-phase and quadrature signals
- Geometric representation of narrowband signals
- Transmission of narrowband signals through bandpass systems
- Distortionless transmission