Chapter 7

ECONOMICS OF INPUT AND PRODUCT SUBSTITUTION

The essential requisites of production are three—labor, capital, and natural agents; the term capital *including all external and physical requisites which are products of* labor, *the term* natural agents *all those which are not.*

John Stuart Mill (1806–1873)

Chapter Outline

The example of labor use by TOP-AG in Chapter 6 focused on varying use of only one input. This allowed us to introduce a number of important production concepts, their relationship to the cost of production, and the profit-maximizing level of output and input use in the short run. Let us now expand this discussion to include two variable inputs and input substitution. This requires shifting the bar appearing after the first input (labor) in Equation 6.2 so that it appears after the second input (capital).

In virtually every setting, a business can alter the combination of capital and labor used in production. For example, weeds can be pulled or hoed (a labor-intensive practice) as they were at the turn of the century, or they can be killed with herbicides (a capital-intensive practice).[1] The choice between capital-intensive and labor-intensive operations becomes an issue in the long run and is influenced by such things as the relative cost of capital and labor, and changes in technology.

As illustrated in Chapter 2, farming operations have become much more capital intensive during the post–World War II period. This trend not only has implications for farm input manufacturers and farm laborers but also has environmental consequences, which will be discussed in Chapter 13.

The purpose of this chapter is to explain the economics of input substitution in the short and long run. In the short run, we determine the least-cost combination of labor and variable capital inputs, given the business's existing fixed resources and technology. Because all inputs are variable in the long run, the business will also have an interest in the optimal expansion path of labor and all capital over time.

Concept and Measurement of Isoquants

If we attempted to graph a total physical product curve for two inputs, it would take three dimensions: two dimensions for the two inputs and one dimension for output. However, three-dimensional figures are difficult to draw and understand; therefore, in this chapter two-dimensional figures will be used. This can be done by focusing on the combination of two inputs that, when used together, result in a specific level of output.

An isoquant captures unique combinations of two inputs that result in the same level of output. The prefix "iso" is the Greek word meaning equal.

A curve that reflects the combinations of two inputs that result in a particular level of output is called an **isoquant** curve. The term *iso* here has the same meaning (i.e., equal) as it did in Chapter 3 when we were discussing iso-utility, indifference curves for two goods faced by consumers. An isoquant consists of a locus of points that correspond to an equal or identical level of output. Along any isoquant, an infinite number of combinations of labor and capital that result in the same level of output are depicted. As the quantity of labor (capital) increases, less capital (labor) is necessary to produce a given level of output.

To illustrate this point, think of quantities of capital as being divisible units of fuel and machinery (e.g., hours of tractor use). When the tractor and its complementary equipment and fuel are increased in quantity, fewer hours of

[1] Remember the term *capital* can include both variable inputs such as fuel, fertilizer, and rented land or machinery, and fixed inputs such as owned machinery, buildings, and land.

labor are required to produce a given level of wheat production, for example. Similarly, with less capital available, more hours of labor are required to produce the same amount of wheat. We can conclude from this discussion that capital and labor are technical substitutes.

Rate of Technical Substitution

To determine the rate of substitution between two inputs, which represents (the negative of) the slope of an isoquant, we must measure the **marginal rate of technical substitution**. This concept is illustrated in Figure 7.1. As we move from range *A* to range *B* on the isoquant corresponding to ten units of output, we see that less capital and more labor are required.

The slope of an isoquant will play a key role later in this chapter when we determine the optimal or least-cost combination of two inputs when producing a product.

Consider the three separate one-unit changes in labor illustrated: ranges *A*, *B*, and *C* each represent different reductions in the use of capital for three separate one-hour increases in labor use on the isoquant associated with 10 units of output. Figure 7.1 implies that the marginal rate of technical substitution of capital for labor falls from approximately 4 over range *A* to 1 over range *B*, and to 0.25 over range *C*. The rate of substituting capital for labor can be expressed mathematically as

$$\frac{\Delta \text{ capital}}{\Delta \text{ labor}} = \frac{\text{MPP}_{\text{labor}}}{\text{MPP}_{\text{capital}}} \tag{7.1}$$

in which $\text{MPP}_{\text{capital}}$ and $\text{MPP}_{\text{labor}}$ represent the marginal physical products for capital and labor, and Δ represents the change in a variable.

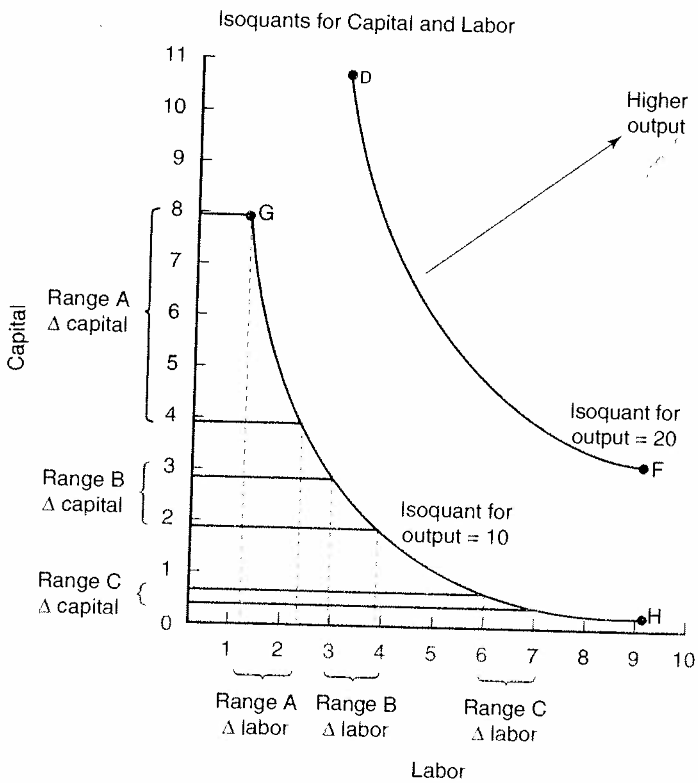

FIGURE 7.1 The slope of an isoquant for a particular level of output typically changes over the full range of the curve.

The expression in Equation 7.1 indicates that changes in labor must be compensated by changes in capital, if the level of output is to remain unchanged.[2] For example, if output is to remain unchanged and the marginal rate of technical substitution of capital for labor is equal to three, capital use must be reduced by three hours if labor is increased by one hour.[3]

These observations illustrate that when labor is substituted for capital along an isoquant (output remaining unchanged), the marginal rate of technical substitution of capital for labor falls. A declining marginal rate of technical substitution is a consequence of the law of diminishing returns (discussed in Chapter 6). When labor increases, its marginal physical product falls. Reductions in capital imply an increase in its marginal physical product.

How do the isoquants in Figure 7.1 relate to the stages of production discussed in Chapter 6? Focusing on the isoquant for 10 units of output, the marginal physical product of capital is negative above point *G* and to the right of point *H*. You will recall that the marginal physical product was negative in stage III. Because stage III is not of economic interest, the economic region of production is bounded by points *G* and *H* for the isoquant corresponding to an output of 10 units, and by points *D* and *F* for the isoquant associated with an output of 20 units. Thus, only certain regions of input-output relationships are of interest to businesses seeking to maximize their profit.

Higher isoquants represent higher levels of output. Does this mean that the firm can or desires to be on a higher isoquant? The answer so far is that we cannot tell with the information we have.

Increases in output are reflected in Figure 7.1 by isoquants that lie farther away from the origin. In this figure, the isoquant for an output of 20 units lies farther from the origin than the isoquant associated with an output of 10 units.

Finally, isoquants at the extreme can be either perfect substitutes or perfect complements. Each case is illustrated in Figure 7.2.

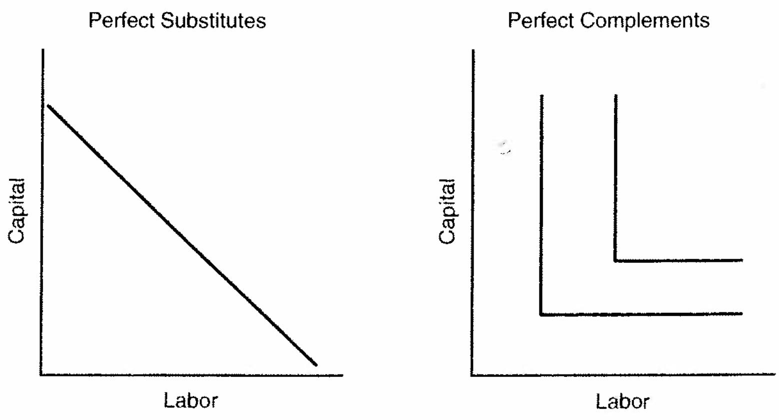

FIGURE 7.2 A graphical illustration of extreme perfect substitutes and complements.

[2] If output is to remain unchanged (i.e., remain on the same isoquant), the loss in output from the decrease in labor must equal the gain in output from the increase in capital $-\Delta$ labor $\times$ $MPP_{labor} = +\Delta$ capital $\times$ $MPP_{capital}$. Equation 7.1 simply represents a rearrangement of this statement.

[3] Because the marginal rate of technical substitution is negative in all rational areas of production (i.e., stage II), most economists do not bother to include the minus sign.

A set of isoquants for perfect substitutes is a straight line, which implies a constant marginal rate of technical substitution. This differs from the imperfect substitution implied by the isoquants in Figure 7.1, which have a decreasing marginal rate of technical substitution as one moves down the isoquants. A set of isoquants for perfect complements forms 90-degree angles, indicating that both capital and labor are required to produce a specific level of output. That is, it takes a certain proportion of labor and capital to produce a product.

THE ISO-COST LINE

Assume that a business uses two inputs (labor and capital) to produce a particular product. The total cost of production in this case would be equal to the wage rate times the hours of labor used plus the cost of capital times the amount of capital used. The concept of wage rates paid to labor is familiar, but the cost of capital will require further explanation.

All inputs to production (land, labor, capital, and management) have a cost. The cost of two inputs can be captured in something called an iso-cost line.

We have learned that capital includes both variable and fixed inputs. We cited fuel as an example of a variable input and land as an example of a fixed input. The cost of capital therefore equals the price of fuel and other variable inputs purchased times the amount purchased and the **rental rate of capital** for using fixed inputs such as tractors and other machinery, buildings, and land. In the short run, the business can rent an additional tractor or land. The annual rental payment for leased fixed inputs is a variable cost of production. Owned capital has its costs, too. The owner of a building, for example, has the option of leasing or selling the building to someone else and using those monies in their next best alternative. The revenue forgone from not selling or leasing the building is a cost. Economists call this an implicit or opportunity cost. Thus, our cost of capital is a composite variable that reflects in the short run the cost of both variable inputs, labor and rented capital. The prices for these two inputs, or the wage rate for labor and the rental rate for capital, are treated as fixed in the short run; they will not vary with the level of input use by a single firm.

Suppose Frank Farmer has $1,000 available daily to finance a business's production costs. The wage rate for labor is $10 per hour, and the rental rate for capital is $100 per day. The business's daily budget constraint therefore is

$$(\$10 \times \text{use of labor}) + (\$100 \times \text{use of capital}) = \$1{,}000 \quad (7.2)$$

Frank's choice of how much capital and labor to employ must be no more than $1,000. The combination of labor and capital Frank can afford for a given level of total cost is illustrated by line *AB* in Figure 7.3. This relationship is referred to as an **iso-cost line**.

The slope of the iso-cost line is equal to the negative ratio of the wage rate to the rental rate of capital,[4] or

$$\text{slope of iso-cost line} = -\frac{\text{wage rate}}{\text{rental rate}} \quad (7.3)$$

If these two input prices change by a constant proportion, the total cost will change, but the slope of the iso-cost line will remain constant.

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[4] Equation 7.3 can be rearranged algebraically to give the iso-cost line and its slope as follows:

$$\text{hours of capital} = \frac{\$1{,}000}{\text{rental rate}} - \frac{\text{wage rate}}{\text{rental rate}} \times \text{hours of labor}$$

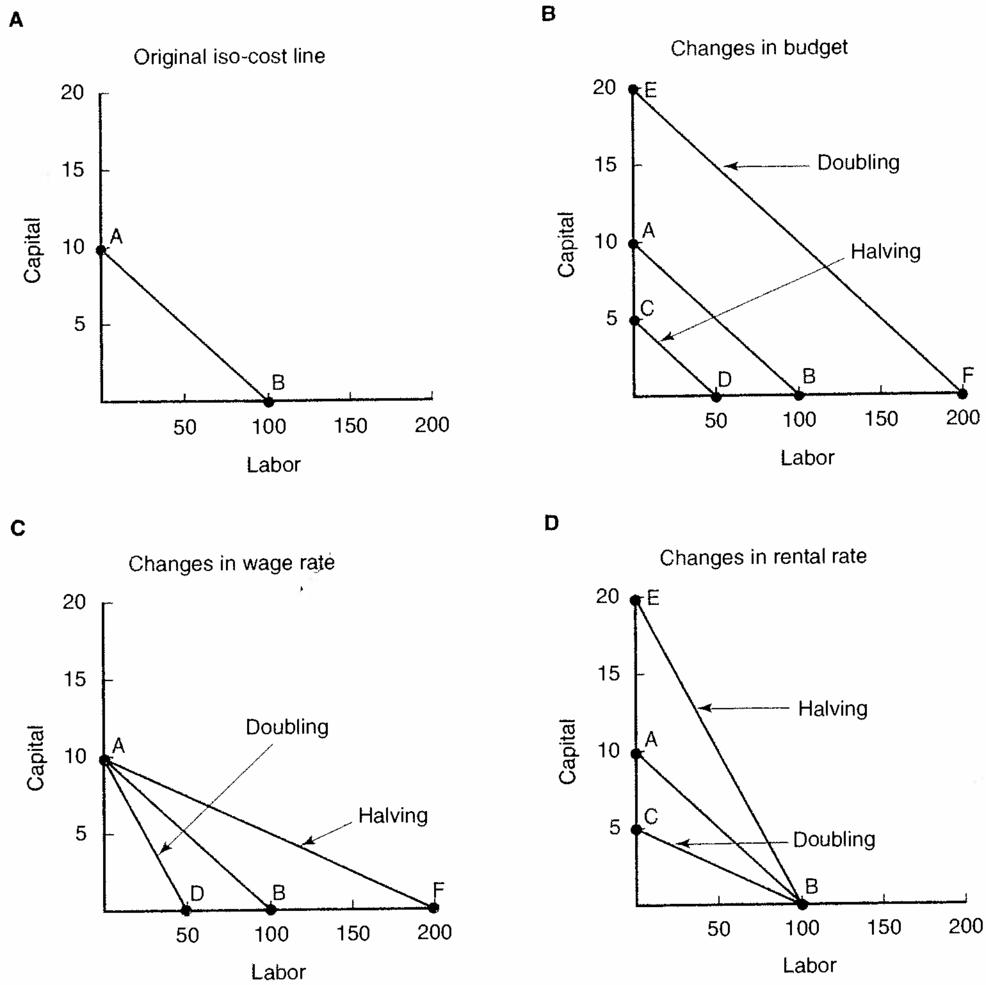


FIGURE 7.3 The iso-cost line plays a key role in determining the least-cost combination of input use. *A,* The slope of curve *AB* is given by the ratio of the wage rate for labor to the rental rate for capital. *B,* A doubling of the production budget changes both intercepts but not the slope of the iso-cost line. *C,* A doubling of the wage rate (holding the rental rate constant) would make the iso-cost line steeper as shown by line *AD. D,* A doubling of the rental rate (holding the wage rate constant) would make the iso-cost line flatter as shown by line *CB.* Declines in these input prices would have the opposite effect.

The slope of an iso-cost line is represented by the ratio of two inputs. This line allows us to use economics to determine the least-cost combination of two inputs.

To illustrate the nature of the iso-cost line, suppose the budget the firm allocated to these two inputs was doubled. Total costs may double, but the iso-cost line *EF* would still have the same slope as line *AB* (Figure 7.3, *B*). Only changes in the relative price of inputs (or input price ratio) will alter the slope of the iso-cost line.

For a given total cost, a rise (fall) in the price of capital relative to that of labor will cause the iso-cost line to become flatter (steeper) (Figure 7.3, *D*). If labor's wage rises (falls), the iso-cost line would become steeper (flatter) (Figure 7.3, *C*).

Suppose that the wage rate was $20 an hour instead of $10. The new iso-cost line *AD* would be steeper than line *AB* (Figure 7.3, *C*). The new iso-cost

The use of herbicides and other chemicals have replaced hand weeding and use of harrows to control weeds as well as insects and diseases in crops today. This illustrates the substitution relationship between a number of inputs to produce a raw agricultural product.

line would still intersect the capital axis at point *A*, because a maximum of 10 units of capital can be purchased, if the producer's total budget is limited to $1,000. If the capital price rose to $200 per unit, the iso-cost line *CB* would be flatter than the original iso-cost line *AB* (Figure 7.3, *D*).

Least-Cost Use of Inputs for a Given Output

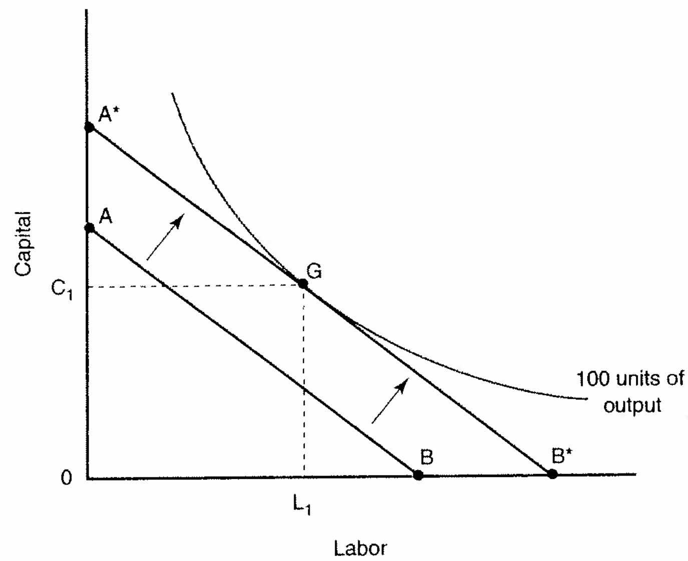
There are essentially two input decisions a business faces in the short run that pertain to input use. One concerns the least-cost combination of inputs to produce a given level of output. Here, the level of production is not constrained by the business's budget. The other is the least-cost combination of inputs and output constrained by a given budget. This section focuses on the first of these two concerns.

Short-Run Least-Cost Input Use

The first of these two perspectives on the least-cost use of inputs requires that we find the lowest possible cost of producing a given level of output with a business's existing plant and equipment. Technology and input prices are assumed to be known and constant. We know from Figure 7.1 that the alternative combinations of capital and labor produce a given level of output that forms an isoquant, and that the relative prices of inputs help shape the iso-cost line in Figure 7.3.

We need to find the least-cost combination of inputs that will allow the business to produce a given level of output in the current period. Any additional

FIGURE 7.4 The least-cost choice of input use is given by the point where the iso-cost curve is tangent to the isoquant for the desired level of output. If the iso-cost line is line *AB*, the least-cost combination would occur at point *G* (where 100 units of output are produced).



The point of tangency of an isoquant and an iso-cost line, and not where they might cross, represents the least-cost combination of two inputs to produce a particular level of output.

capital is rented (variable cost) through a short-term leasing arrangement rather than owned (fixed cost), or represents nonlabor variable inputs (e.g., fuel and chemicals). Graphically, the least-cost combination of inputs is found by shifting the iso-cost line in a parallel fashion until it is tangent to (i.e., just touches) the desired isoquant. This point of tangency represents the least-cost capital/labor combination of producing a given level of output and the total cost of production.

Figure 7.4 can be used to determine the least-cost combination of labor and capital to produce 100 units of output using the business's current productive capacity. Assume that iso-cost line *AB* reflects the existing input prices for labor and capital and current total costs of production. The least-cost combination of labor and capital to produce 100 units of output is found graphically by shifting line *AB* out in a parallel fashion to the point where it is just tangent to the desired isoquant.

Figure 7.4 shows that line A^*B^* is tangent to the isoquant associated with 100 units of output at point *G*. The new total cost at point *G* in Figure 7.4 can be determined by multiplying the quantity of labor (L_1) times the wage rate and adding that to the product of the quantity of capital (C_1) times the rental rate for capital.

A fundamental interpretation to the conditions underlying the least-cost combination of input use is illustrated in Figure 7.4. The slope of the isoquant is equal to the slope of the iso-cost line at point *G*. At this point, the marginal rate of technical substitution of capital (fertilizer, fuel, feed, rental payments, etc.) for labor, or the negative of the slope of the isoquant, is equal to the input price ratio, or the negative of the slope of the iso-cost line. Thus, the least-cost combination of inputs requires that the market rate of exchange of capital for labor (i.e., the ratio of input prices) equal their rate of exchange in production (i.e., their marginal rate of technical substitution).

We can express the foregoing conditions for the least-cost combination of labor and capital in mathematical terms as

$$\frac{MPP_{labor}}{MPP_{capital}} = \frac{\text{wage rate}}{\text{rental rate}} \tag{7.4}$$

We can rearrange Equation 7.4 as

$$\frac{MPP_{labor}}{\text{wage rate}} = \frac{MPP_{capital}}{\text{rental rate}} \tag{7.5}$$

Equation 7.5 suggests that the marginal physical product per dollar spent on labor must equal the marginal physical product per dollar spent on capital. This is analogous to the condition for consumer equilibrium described in Equation 4.2, and it represents a recurring theme in economics. In the present context, a firm should allocate its expenditures on inputs so the marginal benefits per dollar are spent on competing equally.[5]

The discussion presented above can be summarized as follows: input use depends on input prices, desired output, and technology. Such cost-minimizing input use is often referred to as conditional demand because it is conditioned by the desired level of output.

Effects of Input Price Changes

Now let us see what would happen to these input demands if we allow the price of an input to change. Because total production costs equal the sum of expenditures on each input, total cost will also change. A fundamental principle of economic behavior is that a firm will use less of an input as its per unit cost rises (Figure 7.5).

Figure 7.5 shows that as the relative price of labor (wage rate divided by price of capital) falls, the iso-cost line becomes flatter, as illustrated by the shift of iso-cost line *AB* to line *AB**. We know from the previous discussion that our next step must be to find the point of tangency with the desired isoquant. Moving line *AB** inward in a parallel fashion to the point where it (let's use a new label; line *DE*) is tangent to the isoquant, we see that the least-cost combination of inputs for 100 units of output shifts from point *G* to point *H*.

A change in one or both of the prices of two inputs will cause the iso-cost line to shift in one fashion or another. This will affect the desired use of inputs and even the level of production.

Therefore, when the price of labor falls (rises) relative to the price of capital, labor is substituted for capital, causing the capital/labor ratio to fall (rise). Because of diminishing marginal products, equilibrium is attained by reducing capital (from C_1 to C_2) use and using more labor (L_2 instead of L_1).

An increase in the budget available to purchase two inputs will mean that the firm can reach a higher isoquant, or produce more output. The reverse will occur if the firm's budget is cut.

[5] Another way to think of this equilibrium is that marginal benefit equals marginal cost. Suppose that the marginal value product of labor usage (marginal physical product times the price of output) is \$5 and the corresponding marginal benefit is \$7 for capital. The opportunity cost of expending a dollar on increased labor usage is the \$7 gain if this expenditure were instead used to purchase another unit of capital services. Therefore, the marginal benefit (\$5) is less than marginal cost (\$7), and labor usage should be reduced. If output is to remain constant when labor is reduced, capital must be expanded until marginal benefit equals marginal cost.

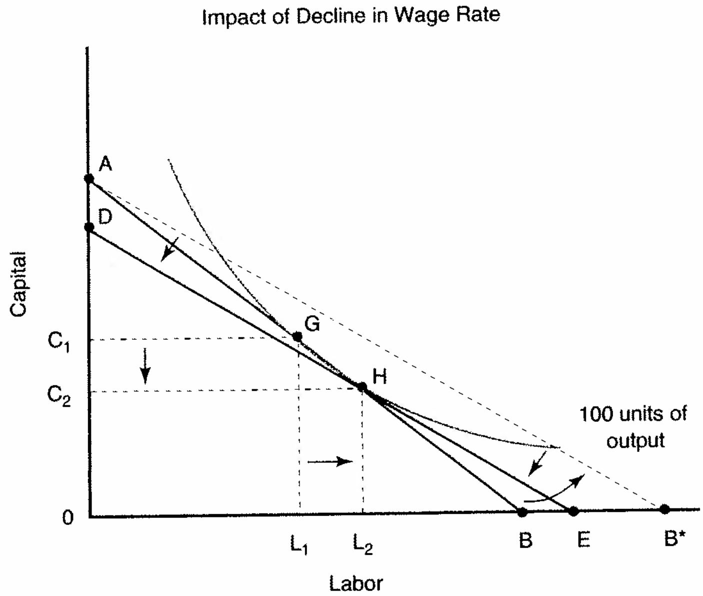


FIGURE 7.5 The shift in the iso-cost line from *AB* to *AB** caused by a lower wage rate suggests that labor should be increased to L_2 and capital use should be reduced to C_2. Line *DE* represents a parallel shift of line *AB** to a point of tangency with the desired isoquant.

■ LEAST-COST INPUT USE FOR A GIVEN BUDGET

The previous section illustrated how to determine the least-cost combination of inputs in the current period to produce a given level of output. A somewhat different twist to this analysis is to determine the least-cost combination of inputs and output in the current period for a given production budget. We will continue to use the concept of the iso-cost line and the isoquant for specific levels of output.

Assume that a firm has a specific amount of money to spend on current production activities and wants to know the least-cost combination of capital (currently owned plus rented) and labor to employ. Figure 7.6 contains four isoquants that present all the information we need to answer this firm's question. The isoquant for 50 units of output shows all the combinations of labor and capital that are needed to produce this level of output. Similar isoquants are shown for 75, 100, and 125 units of output.

Line *MN* in Figure 7.6 represents a total cost of production that completely exhausts the amount of money the firm desires to spend. The point of tangency between this iso-cost line and the highest possible isoquant will indicate the least-cost combination of inputs associated with the firm's current budget constraint. This occurs at point *P* in Figure 7.6, which suggests that the firm would utilize C_3 units of capital and L_3 units of labor. The economic conditions set forth in Equation 7.4 are satisfied by this combination of inputs (remember, the left-hand side of this equation represents the slope of the isoquant, and the right-hand side represents the slope of the iso-cost line).

Point *P* also suggests that the firm would produce 75 units of output. The firm simply could not afford to operate on a higher isoquant in the current period. The only way the firm could move out to the isoquant associated with 100 units of output is if it were able to attract additional funds or if both input prices declined to the point where the iso-cost line became tangent with this higher isoquant.

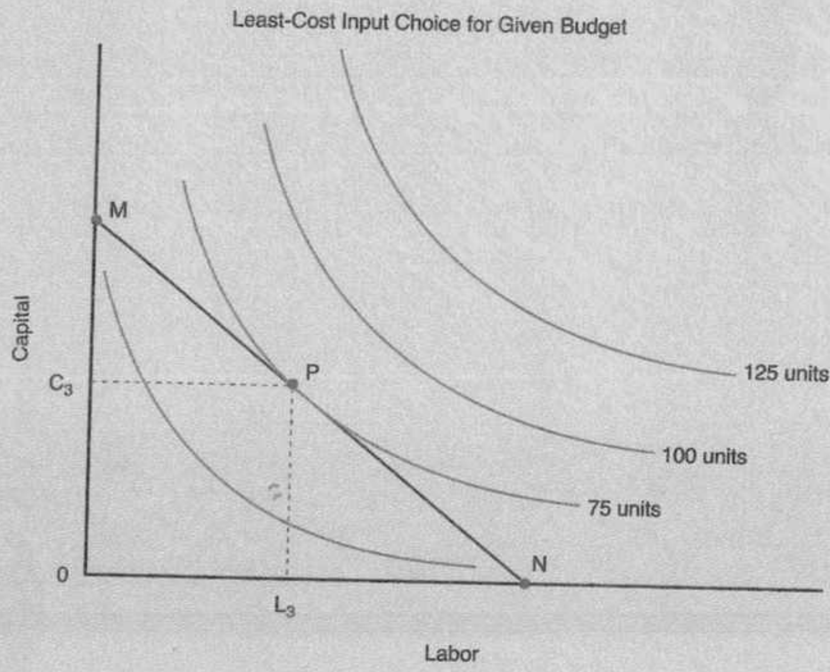


FIGURE 7.6 The least-cost choice of input use for a given budget is found by plotting the iso-cost line associated with this budget and observing the point of tangency with the highest possible isoquant.

■ Long-Run Expansion of Input Use

In the previous section, we learned that some costs are variable in the short run, and other costs are fixed. In the long run, however, a business has the time to expand the size of its operations, and all costs become variable. The purpose of this section is to discuss the long-run average cost curve and the factors that influence its shape.

Long-Run Average Costs

Figure 7.7 depicts three short-run average cost (*SAC*) curves. The presence of fixed inputs in the short run ensures that these short-run average cost curves are U-shaped. Each short-run average cost curve reflects the full average cost of the business for three separate sizes. Size *A* is the smallest, with costs represented by SAC_A. This curve might correspond to TOP-AG, using a specific amount of capital to produce 12 units of output in the example discussed previously.

Size *B* is larger and can operate at much lower costs. Curve SAC_B is much lower, except at its extreme left end. This curve might reflect the capital needed by TOP-AG to minimize its cost of producing more than 12 units of output. Size *C* is still larger, but the curve SAC_C represents a higher cost structure than size *B*.

A business desiring to minimize its production costs will wish to operate at size *A* in Figure 7.7 if an output equal to *OA* or less is desired. The average cost of production will be substantially lower than size *B* at outputs less than *OA*. If output *OB* or *OC* is desired, however, the business will prefer size *B* or size *C*, respectively.

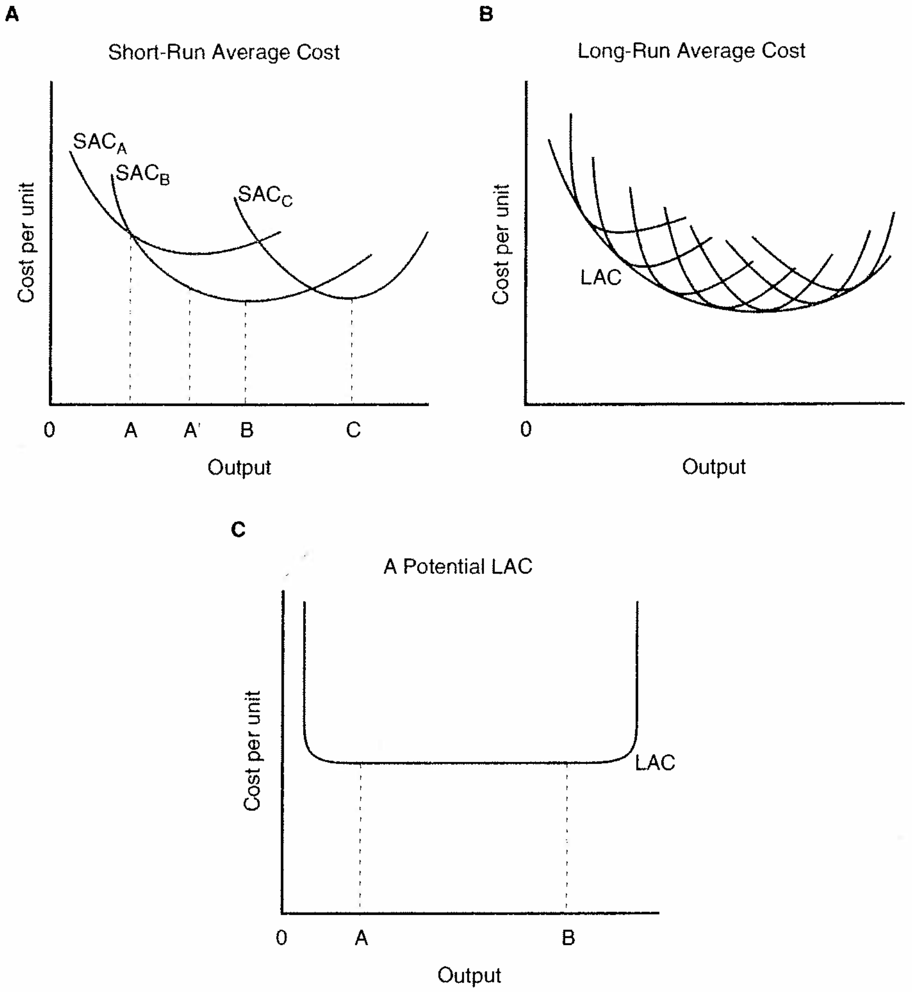


FIGURE 7.7 The long-run average cost (*LAC*) curve plays a key role in determining the minimum costs of operation in the long run. Often referred to as the *planning curve,* the *LAC* curve represents an envelope of a series of short-run average cost curves (*A* and *B*).

If the business desires to produce an output somewhat larger than *OA*, size *B* would be better than size *A* because its costs are lower. Output *OA′* has special significance for a business of size *A*, because it represents the minimum cost of operation. Obviously, it would be better, when producing more than *OA*, to operate a business of size *B* at less than capacity than it would be to continue operating a business of size *A*, because the costs of production would be lower.

The Long-Run Planning Curve

When deciding how big the business should be in the long run, management must consider a relevant range of minimum cost. Management may be aware of this range, either from its own experience or from economic feasibility studies conducted for other businesses of a similar nature. The short-run average cost curves associated with different sizes over this range enable the business to determine its long-run average cost (*LAC*) curve.

Often referred to as the long-run planning curve, the long-run average cost curve illustrates to the business how varying its size will affect the business's economic efficiency. It also indicates the minimum per unit cost at which any output can be produced after adjusting the business's size.

The long-run average cost curve is comprised of points on a series of short-run average cost curves. This curve helps economists determine the profitability of different sizes of operations.

What causes the long-run average cost curve to decline, become relatively flat, and then increase? The answer lies in what economists call returns to size. If an increase in output is exactly proportional to an increase in inputs, constant returns to size are said to exist. This means that a doubling or a tripling of inputs used by the firm will cause a doubling or tripling of its output.[6] If the increase in output is more (less) than proportional to the increase in input use, we say the returns to size are increasing (decreasing). Decreasing (increasing) returns to size will exist if the firm's long-run average costs are increasing (decreasing) when the firm is expanded.

Increasing Returns to Size. Some of the physical causes of increasing returns to size are purely dimensional in nature. If the diameter of a pipe is doubled, the flow through it is more than doubled. The carrying capacity of a truck also increases faster than its weight. After some point, such increases in dimensional efficiency stop. As the size of the pipe is increased, it has to be made out of thicker and stronger materials. The size of the truck will also be limited by the width of streets, the height of overpasses, and the capacity of bridges.

A closely related technical factor that helps to explain the existence of increasing returns to size is the indivisibility of inputs. In general, indivisibility means that equipment is available only in minimum sizes or in a specific range of sizes. As the size of the firm's operations increases, the firm's management can switch from using the minimum-sized piece of equipment to larger, more efficient equipment. Thus, the larger the size of the operation, the more the firm will be able to take advantage of large-size equipment that cannot be used profitably in smaller-size operations.

There are three measures of returns to size; increasing, constant, and decreasing. Each of these sizes has much to say about the "staying power" of the firm should prices fall.

Another technical factor contributing to increasing returns to size comes from the potential benefits from specialization of effort. For example, as the firm hires more labor, it can subdivide tasks and become more efficient.[7] When the firm expands the size of its operations, it can buy specialized pieces of equipment and assign special jobs to standardized types of machinery.

A frequently noted factor that helps us explain the existence of increasing returns to size is volume discounts on large purchases of production inputs. Lower input prices paid by larger farming operations could be a major reason why average costs decline as farm size is increased.

Constant Returns to Size. Increasing returns to size cannot go on indefinitely. Eventually, the firm will enter the phase of constant returns to size, where a doubling of all inputs doubles output. The phase of constant returns to size

[6] A word of caution: the phrase "the economies of mass production" carries several meanings, some of which are irrelevant here and therefore are potential sources of confusion. For example, the greater efficiency frequently observed for larger production units (in contrast to smaller ones) is often caused because larger units are newer and use better production techniques than the older and smaller units. However important this may be, improvements in technology are not part of the concept of returns to size, which assumes a given technology.

[7] The benefits gained from specialization are well known. Adam Smith, in his book *The Wealth of Nations*, published in 1776, addressed the gains from the division of labor.

can be brief before decreasing returns to size sets in. Empirical evidence suggests that the phase of constant or nearly constant returns to size can be fairly long and cover a large range of output levels.

Decreasing Returns to Size. Can a business keep on doubling its inputs indefinitely and expect its output to double? Most likely, the answer is no. Eventually, there must be a decreasing return to size. The farmer may be the reason for decreasing returns to size. While all other inputs can be increased, his or her ability to manage larger operations may not. The managerial skills needed to coordinate efforts and resources usually do not increase proportionately with the size of operations.

In Figure 7.7, *B*, the long-run average cost curve is the *envelope* of the set of short-run average cost curves; that is, the long-run average cost curve is tangent to the short-run average cost curve when it is declining. When the long-run average cost curve is rising, it touches the short-run average cost curves to the right of their minimum points. The minimum point on the long-run average cost curve is the only point that touches the minimum point on the short-run average cost curve. The declining portion of the long-run average cost curve suggests the existence of increasing returns to size. Beyond the minimum point on this curve, decreasing returns to size exist.

Economists are concerned with the shape of the long-run average and marginal cost curves. The minimum point on the long-run average cost curve represents the most efficient amount in the long run in the sense that the business's average costs of operation are minimized.

The long-run cost curve depicted in Figure 7.7, *B*, reflects the conventional shape illustrated in most textbooks. Although the long-run average cost curve no doubt decreases over some range of output before eventually turning up, its shape is not likely to be perfectly U-shaped.

Studies by agricultural economists suggest that there may be some range of output where the long-run average cost curve is relatively flat. In California, Hall and LaVeen (1978) found that the long-run average cost curve becomes relatively flat after initially declining rapidly. They reported that the costs of producing highly mechanized crops generally continued to decline slowly over the entire range of surveyed farm sizes. For vegetables and fruit crops, however, Hall and LaVeen found little or no decline after the initial benefits from expansion were achieved.

Long-run average costs curves are rarely if ever perfectly U-shaped. They are more likely to fall relatively sharply and then decline very slowly before turning upward when decreasing returns to size appear.

Figure 7.7, *C*, illustrates the general nature of these findings. Between outputs *OA* and *OB*, the long-run average cost curve is relatively flat. Over this range, all business sizes will have approximately the same costs. Thus, the long-run average cost curve in practice is more L-shaped than U-shaped.

■ Economics of Business Expansion

In the long run, businesses have time to expand (or contract) the size of their operations. Suppose that the short-run marginal cost and average cost curves for an existing business are represented in Figure 7.8 by SMC_1 and SAC_1, respectively. If the market price for the product is equal to *P* and the business produced at the point where $P = SMC_1$, the business would sustain a small economic loss on each unit of output produced. At this point, the business would

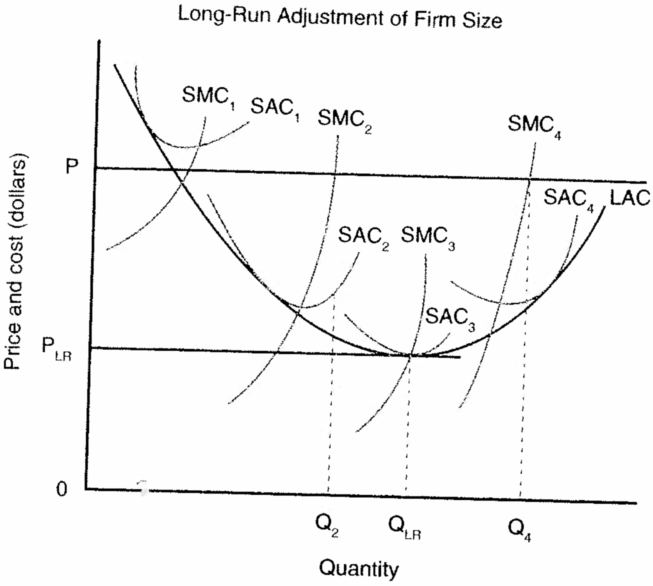


FIGURE 7.8 A profit-maximizing business may desire to expand the size of its operations to the level corresponding to the SAC_4 and SMC_4 cost curves, given the level of product price P. When others respond to the existence of economic profits, total market supply will expand and the market price will fall. In a free-market setting, businesses will cut back their output and size as best they can. Some may cease producing altogether. The market will be in long-run equilibrium at the point where $P = MC = AC$, which would result in the business producing Q_{LR} units of output at a market price of P_{LR}.

have two options: (1) it could go out of business, or (2) it could expand its existing size, if it could convince its banker of the benefits from this expansion.

If the business expanded to the size represented by SAC_2 and SMC_2, it could produce quantity Q_2 and it would earn an economic profit per unit equal to P minus short-run average cost at quantity Q_2. A profit-maximizing business may want to expand to the size represented by SAC_4 and SMC_4. By producing at Q_4, the business would be operating at the point where P is equal to SMC_4.

While this long-run adjustment for an existing business is taking place, the number of businesses may also increase because of attractive economic profits. Some of these entrants will be newly created businesses. Others may be firms that have shifted out of less profitable enterprises. As these new and modified operations begin to produce output, the market supply of the product will increase. This, in turn, will cause the price of the product to fall.[8] When each business responds to the new lower market price, the output of each will become smaller than before. Those businesses that were just preparing to expand their size in response to the earlier price will be able to adjust their size rapidly. Other businesses that have just completed expansion of their firm will obviously respond more slowly. Those businesses that cannot contract rapidly will lose more money than those that can. This process conceivably will continue until economic profits have been reduced to zero and the incentive for additional firms to enter the sector has been eliminated. Those existing businesses who are losing money will eventually cease producing this product.

A market achieves equilibrium at the minimum point on the industry's long-run average cost curve.

The market will be in long-run equilibrium at price P_{LR}. At this price, the firm will be operating at the point at which product price is just equal to the minimum point on the long-run average cost curve (LAC) in Figure 7.8. This figure shows that the optimal size of the business in the long run would see it

[8] Once we develop the market supply curve for all businesses producing a particular product in Chapter 8, this sequence of events will become more clear.

producing an output equal to Q_{LR}. Businesses expanding beyond this output run the risk of eventually scaling back their operations.

Capital Variable in the Long Run

Thus far, the firm has been limited to expanding its use of variable inputs (including rented capital like farmland). The ownership of capital, held constant in the short run, is allowed to vary in the long run. This input can be thought of as plant size (e.g., the number of manufacturing lines, the capacity of a feedlot, or the number of grain silos owned by an elevator). In the long run, the capital structure of the firm can be adjusted to its least-cost level that occurs when the marginal rate of technical substitution of capital for labor is equal to the input price ratio.

Figure 7.9 shows that at 1 unit of capital, labor is at its least-cost level of use only if output is equal to 10 units (point *A*). At this output level, the least-cost combination of inputs would be 5 units of labor and 1 unit of capital (see point *A*).

The firm may face the choice of increasing labor and/or capital when expanding the size of its operations. The choice will be influenced not only by the productivity of these inputs, but also their relative cost.

If an output of 20 units is desired in the long run, the firm has two options: (1) expand labor use to 15 units and operate at point *C* or (2) expand capital to 2 units and operate at point *B* at which you would employ 6 units of labor. The least-cost combination of inputs to produce 20 units would occur at point *B*. If capital is held constant at 1 unit, the only way this firm can produce 20 units of output would be to employ 15 units of labor (point *C*). The total cost of producing 20 units would be given by the iso-cost line *HI*. Because iso-cost line *FG* lies to the left of iso-cost line *HI*, the total cost of producing 20 units of output at point *B* would be less than producing 20 units of output at point *C*. Suppose the wage rate for labor was $10 per hour and the rental rate for capi-

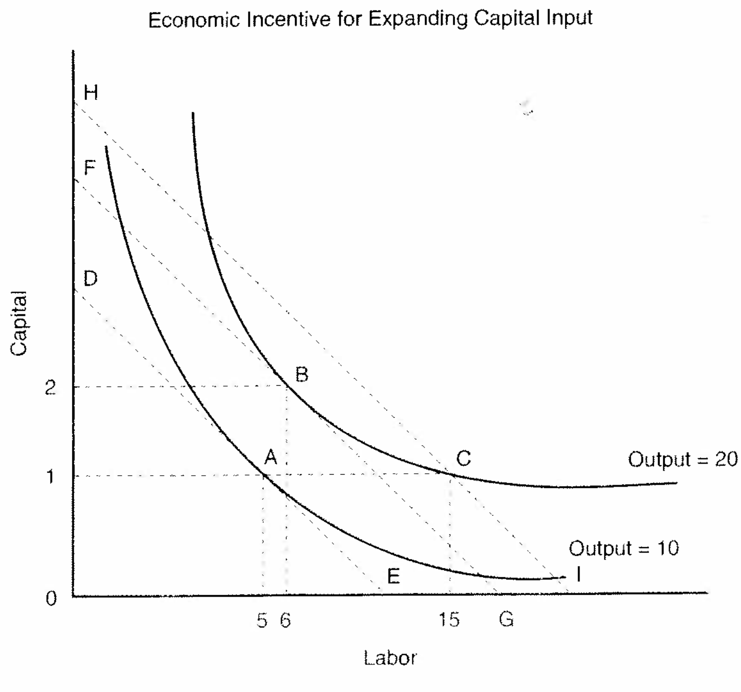


FIGURE 7.9 In the long run, the size of the firm's operations can be expanded if the economic incentive to do so is there. This requires increasing the capital input to its least-cost level (i.e., the point at which the marginal rate of technical substitution of capital for labor is equal to their price ratio). You will recall that this is the same stipulation made for the short-run case. Unlike the short run, however, the firm can now increase its use of capital beyond 1 unit. Producing 20 units of output instead of 10 units can be most efficiently done by operating at point *B*, not *C*.

Table 7.1 Total Production Costs

Iso-Cost Line	Calculation of Total Cost	Total Cost
DE	(1 × \$50) + (5 × \$10)	\$100
FG	(2 × \$50) + (6 × \$10)	\$160
HI	(1 × \$50) + (15 × \$10)	\$200

tal was \$50 per hour. The total production costs associated with the three iso-cost lines in Figure 7.9 appear in Table 7.1.

Thus, it would cost \$200 an hour to operate at point *C*, or to produce 20 units of output with 1 unit of capital and 15 units of labor. It would only cost \$160 dollars an hour to produce the same quantity of output with 2 units of capital and 6 units of labor. Therefore, there is economic incentive for the business desiring to produce 20 units of output to expand its capital to the level indicated by point *B*. Using 2 units of capital and 6 units of labor will minimize the cost of producing 20 units of output.

■ Concept and Measurement of the Production Possibilities Frontier

So far we have examined issues associated with the combination of inputs used by a business. Part of our interest focused on the degree to which one input could be substituted for another in producing a given level of output. It is also important to understand the substitution among the different products the business can produce. To do this we shall examine the production possibilities frontier (PPF) a business faces, both in the short run, when some inputs are fixed, and in the long run, when all inputs are variable.

Production Possibilities Frontier

Chapter 6 introduced the concept of technical efficiency by indicating the minimal number of hours required to produce given levels of output. In the multiproduct case, we can think of technical efficiency in terms of the maximum outputs possible from given levels of inputs.

Suppose SunSpot Canning Company has the option of canning either all fruit, all vegetables, or some combination of these two products as shown in Table 7.2. As fruit (vegetable) canning is increased, vegetable (fruit) canning must be decreased because of the plant's fixed current canning capacity. Thus, a substitution among products occurs in the same sense that inputs were substituted for one another in the preceding chapter.

The production possibilities curve illustrates the maximum output for different combinations of two products a firm can produce given its existing resources.

If SunSpot specialized in fruit canning, it could can 135,000 cases of canned fruit a week. If it specialized in vegetable canning, SunSpot could can 90,000 cases of canned vegetables a week.[9] Column 3 in Table 7.2 reflects the physical trade-off this canning plant faces for these two products,

[9] We will assume that the fruit pack or canning season and the vegetable canning season overlap and thus compete for use of SunSpot's existing resources.

Table 7.2 Production Possibilities for SunSpot Canning

	(1) Cases of Canned Fruit	(2) Cases of Canned Vegetables	(3) Marginal Rate of Product Transformation, Δ(1) ÷ Δ(2)
A	135,000	0	
B	128,000	10,000	−0.7
C	119,000	20,000	−0.9
D	108,000	30,000	−1.1
E	95,000	40,000	−1.3
F	80,000	50,000	−1.5
G	63,000	60,000	−1.7
H	44,000	70,000	−1.9
I	23,000	80,000	−2.1
J	0	90,000	−2.3

or the marginal rate of product transformation. It represents the slope of the **production possibilities frontier**.

Product Substitution

The **marginal rate of product transformation** represents the rate at which the canning of fruit must contract (expand) for a one-case increase (decrease) in vegetable canning. The marginal product rate of transformation in absolute terms is given by

$$\text{marginal rate of product transformation} = \frac{\Delta \text{ canned fruit}}{\Delta \text{ canned vegetables}} \quad (7.6)$$

In Table 7.2, the marginal rate of product transformation of vegetables for fruit is initially very small (i.e., Δ canned fruit relative to Δ canned vegetables is quite small). In column 3, however, the marginal rate of product transformation becomes much higher (i.e., Δ canned fruit relative to Δ canned vegetables becomes quite large). This increasing marginal rate of product transformation is a widely observed and measured phenomenon and has the same general lawlike acceptance as the declining marginal rate of technical substitution discussed for two inputs.

The substitution relationship between two products can be illustrated further by plotting the combinations of fruit and vegetables shown in Table 7.2. Points *A* through *J* in Figure 7.10 represent production levels of fruit and vegetables for a canning plant with a given technically efficient use of capital and labor. Point *A*, for example, represents specialization in the canning of fruit. Point *J*, on the other hand, represents specialization in vegetable canning. Point *C* would result in the canning of some of both commodities with the *same* inputs. Point *K* can be ruled to be technically inefficient because a smaller amount of output is being produced with the same quantity of inputs, leaving only points *A* through *J* as the efficient production points.

A curve drawn through these points is called a production possibilities frontier, which gives the product combinations that can be *efficiently* produced using the business's existing resources. Finally, point *L* is impossible to attain

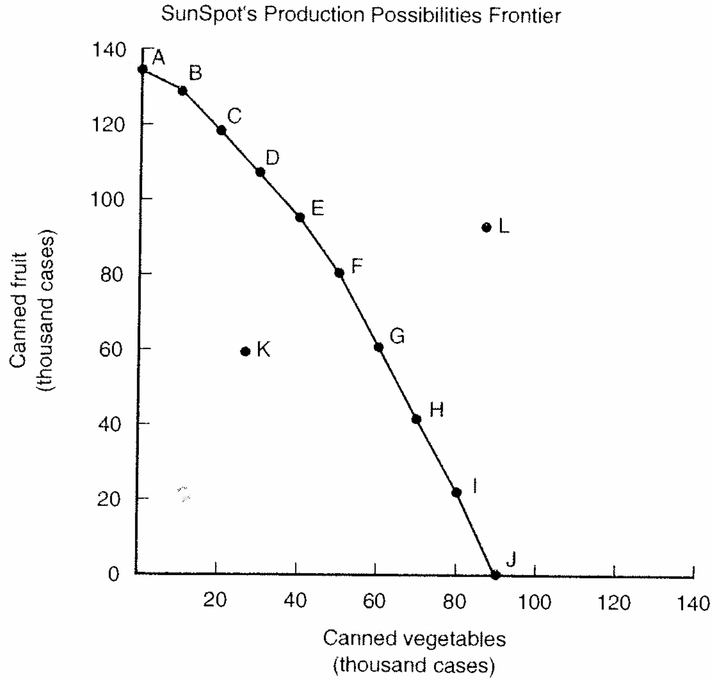


FIGURE 7.10 The downward-sloping production possibilities frontier illustrates the physical trade-offs this business faces in choosing between canning fruit or vegetables as documented in Table 7.2. The concave shape of this curve reflects the less-than-perfect substitutability of input use in switching from canning fruit to canning vegetables.

with SunSpot's existing resources because it lies outside the production possibilities frontier.

Figure 7.10 suggests that vegetable and fruit canning operations at SunSpot are close—but not perfect—substitutes in competing for the firm's scarce resources in production (i.e., the PPF is neither linear nor has a constant slope of − 1.0).

■ Concept and Measurement of the Iso-Revenue Line

We need to account for the price received by the canning firm for these two products before we may determine what combination maximizes SunSpot's profits. The **iso-revenue line** represents the rate at which the market is willing to exchange one product for another. We may begin to define an iso-revenue line for SunSpot by defining its total revenue, which, for fruit and vegetables, is given by

$$\text{total revenue} = (\text{price of canned fruit} \times \text{cases of canned fruit}) + (\text{price of canned vegetables} \times \text{cases of canned vegetables}) \tag{7.7}$$

If no canned vegetables are produced by this canning plant, then the number of cases of canned fruit produced for a specific level of revenue is given by the level of revenue divided by the price of canned fruit. Similarly, if no canned fruit is produced, then the number of cases of canned vegetables produced for a specific level of revenue is given by the level of revenue divided by the price of canned vegetables.

We cannot determine the optimal combination of two products to produce without knowing the price of these two products.

The slope of the iso-revenue line is the ratio of the price of the two products, or the price of canned vegetables this business receives by selling a case of canned vegetables in the market, divided by the price of canned fruit. Stated mathematically, the slope is given by

$$\text{slope of iso-revenue line} = -\frac{\text{price of vegetables}}{\text{price of fruit}} \tag{7.8}$$

For example, if the wholesale price of a case of canned fruit that SunSpot receives is \$33.33, and the price of a case of canned vegetables it receives is \$25.00, the slope of the iso-revenue line would be −0.75 (i.e., the negative of \$25.00 divided by \$33.33). One case of canned vegetables is worth three-fourths of a case of canned fruit.

Figure 7.11 illustrates the general nature of the iso-revenue line for these two products. You may wonder why the slope of the line plotted in Figure 7.11

The Iso-Revenue Line

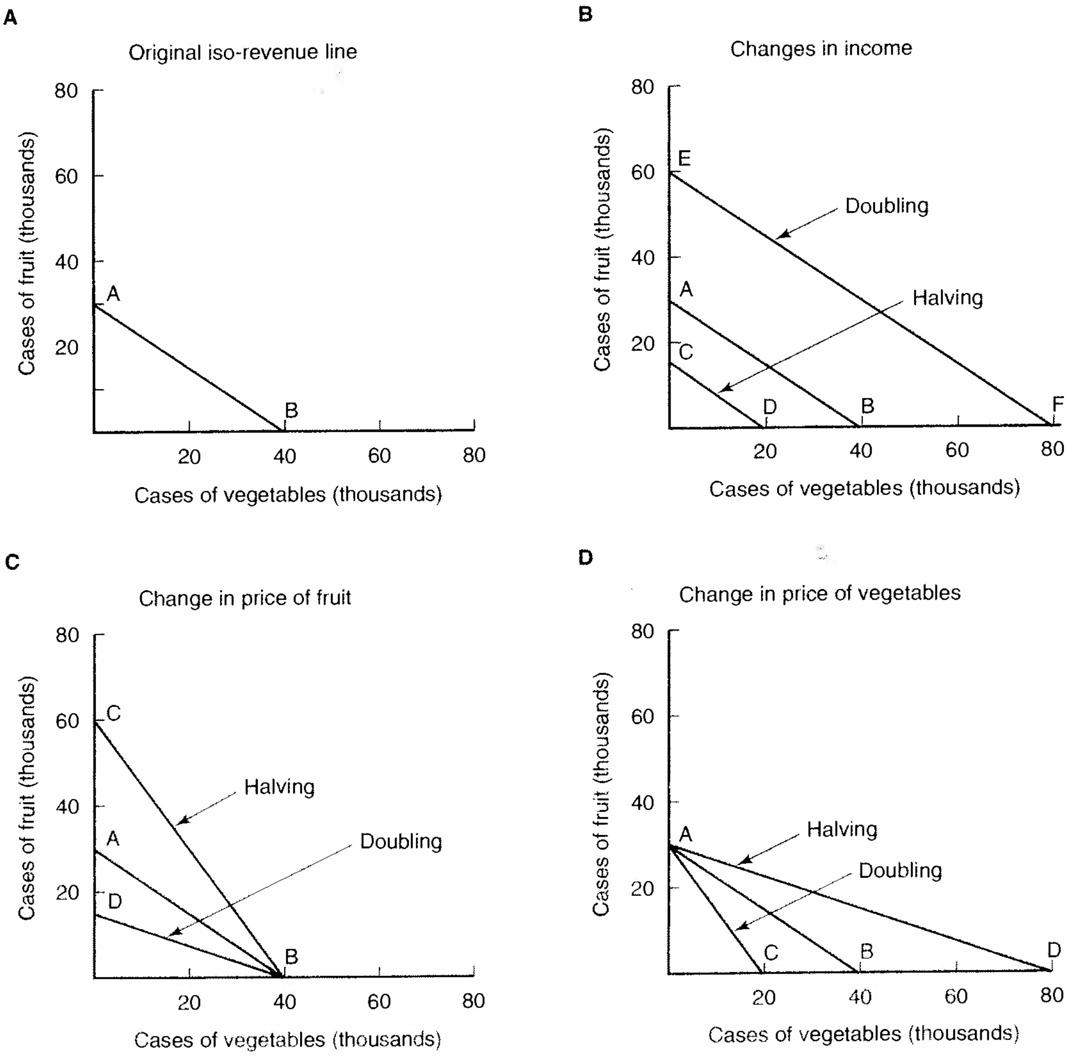


FIGURE 7.11 The iso-revenue line plays an important role in the determination of the profit-maximizing combination of two products. The slope of this line is equal to the negative of the price ratio for the two products.

is the negative of the ratio of the price of canned vegetables to the price of canned fruit, when the vertical axis is labeled "cases of fruit" and the horizontal axis is labeled "cases of vegetables."[10] This is entirely consistent with the discussion of the budget constraint in Chapter 3, in which we determined the slope of the budget line.

The iso-revenue curve will shift or change slope as the price of the two products change.

The original iso-revenue line associated with a revenue of $1 million, the price of a case of canned fruit of $33.33, and the price of a case of canned vegetables of $25.00 are the basis for iso-revenue line *AB* plotted in Figure 7.11, *A*. The maximum number of cases of canned fruit associated with this level of revenue is 30,000 cases (i.e., $1 million ÷ $33.33), and the maximum number of cases of canned vegetables is 40,000 cases (i.e., $1 million ÷ $25.00). Thus, SunSpot would achieve a revenue of $1 million if it could process 30,000 cases of fruit, 40,000 cases of vegetables, or the specific combinations of these two products that appear on the iso-revenue line.

Figure 7.11, *B*, shows that if consumer expenditures for SunSpot's products fell by half, the iso-revenue line would shift in a parallel fashion from line *AB* to line *CD*. Only 15,000 cases of fruit, 20,000 cases of vegetables, or specific combinations of the two products could be sold. A doubling of consumer expenditures for these products would shift the iso-revenue line from line *AB* to line *EF*. Under these conditions, SunSpot would sell 60,000 cases of canned fruit or 80,000 cases of canned vegetables, or specific combinations of these two products. Figure 7.11, *C*, shows what would happen to the iso-revenue line if the price of fruit were either doubled (line *BD*) or cut in half (line *BC*). Figure 7.11, *D*, shows what would happen if the price of vegetables doubled (line *AC*) or were cut in half (line *AD*). In Figures 7.11, *C* and *D*, the slope of the iso-revenue line became either flatter or steeper as the price of one of the commodities changed.

■ PROFIT-MAXIMIZING COMBINATION OF PRODUCTS

We can determine the profit-maximizing combination of products under conditions of perfect competition by considering both the physical and economic trade-offs from the alternatives currently available. This requires uniting the concepts of the production possibilities frontier and the iso-revenue line.

Choice of Products in the Short Run

The technical rate of exchange between canned fruit and canned vegetables for SunSpot in the current period is captured by the production possibilities frontier in Figure 7.10. We know from Equation 7.6 that the slope of this curve, called the marginal rate of product transformation, is equal to the ratio of the change in the production of these two products. The slope of this curve is negative, indicating an increasing opportunity cost of product substitution.

The profit-maximizing business seeks to maximize the revenue for the least-cost combination of inputs. In the present context, the business will want

[10] Equation 7.8 can be rearranged algebraically to give the iso-revenue line and its slope:

$$\text{cases of canned fruit} = \frac{\text{total revenue}}{\text{price of fruit}} - \frac{\text{price of vegetables}}{\text{price of fruit}} \times \text{cases of canned vegetables}$$

Table 7.3 Profit-Maximizing Combination of Products for SunSpot

(1) Cases of Canned Fruit	(2) Cases of Canned Vegetables	(3) Revenue, \$33.33 × (1) + \$25.00 × (2)	(4) Marginal Rate of Product Transformation, Δ(1) ÷ Δ(2)	(5) Ratio of Price of Vegetables to the Price of Fruit, \$25.00 ÷ \$33.33
135,000	0	\$4,499,550		
128,000	10,000	4,516,240	−0.70	0.75
119,000	20,000	4,466,270	−0.90	0.75
108,000	30,000	4,349,640	−1.10	0.75
95,000	40,000	4,166,350	−1.30	0.75
80,000	50,000	3,916,400	−1.50	0.75
63,000	60,000	3,599,790	−1.70	0.75
44,000	70,000	3,216,520	−1.90	0.75
23,000	80,000	2,766,590	−2.10	0.75
0	90,000	2,250,000	−2.30	0.75

The profit-maximizing firm will operate where the slope of the iso-revenue curve is tangent to the production possibilities curve.

to determine the point at which the marginal rate of product transformation is equal to the relative prices of the products being produced. Table 7.3 suggests that the profit-maximizing combination of canned fruit and vegetables for SunSpot in the current period given existing input prices would be between 119,000 and 128,000 cases of canned fruit, and between 10,000 and 20,000 cases of canned vegetables.

The absolute value of the marginal rate of product transformation in column 4 of Table 7.3 will equal the absolute value of the price ratio in column 5 of 0.75 in this range. At this point, the marginal rate of product transformation (slope of the production possibilities curve) for fruits and vegetables equals the ratio of the price of vegetables to the price of fruit (slope of the iso-revenue line). We can, therefore, state the conditions for the profit-maximizing combination of these two products in mathematical terms as

$$\frac{\Delta \text{ canned fruit}}{\Delta \text{ canned vegetables}} = \frac{\text{price of vegetables}}{\text{price of fruit}} \tag{7.9}$$

in which both sides of the equation will have a negative value. (Recall the negative values for the marginal rate of product transformation in Table 7.2 and Table 7.3.)

Figure 7.12 suggests the profit-maximizing combination of canned fruit and vegetables for SunSpot would be approximately 126,000 cases of fruit and 13,000 cases of vegetables, which lie in between the ranges discussed in the context of Table 7.3. Total revenue would reach approximately \$4,524,580. This combination also represents maximum profits because the business is on the production possibilities curve, which assures maximum technical efficiency.

Effects of Change in Product Prices

Let's assume that we are at point M in Figure 7.12, and the wholesale price of fruit suddenly falls to \$25. This will alter the slope of the iso-revenue line in Figure 7.13. The new iso-revenue line CB is extended out in a parallel fashion until it is tangent to the production possibilities curve at point N.

In Figure 7.13, SunSpot's profit-maximizing objective is to *decrease* its fruit canning operations from O_F (Figure 7.12) to O^*_F and *increase* its vegetable can-

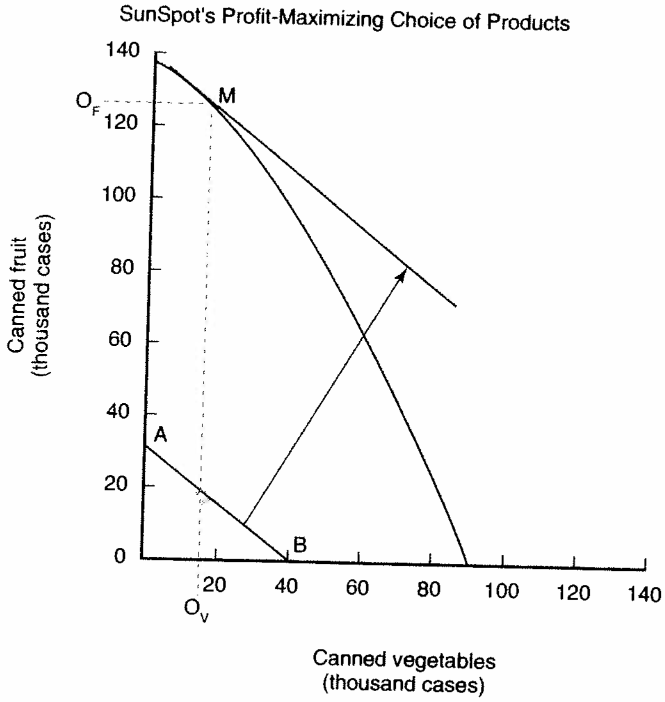


FIGURE 7.12 The profit-maximizing choice of how much fruit and vegetables to can is given by the point at which the iso-revenue line is tangent to this business's current production possibilities frontier (point *M*).

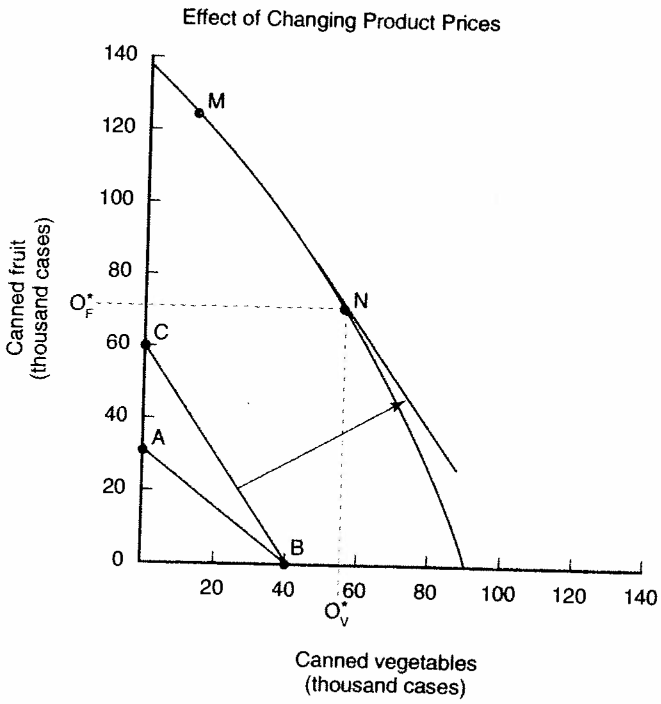


FIGURE 7.13 A decrease in the price of canned fruit would cause SunSpot to alter the combination of the products it cans.

ning operations from O_V (Figure 7.12) to O^*_V. This suggests that a business, with a given amount of resources, will alter the allocation of resources between the production of alternative products as their price ratio changes. The quantity of vegetables and fruit SunSpot chooses to can, therefore, will depend on the prices of all its inputs, the stock of its existing fixed inputs, the technology embodied in its labor and capital, and the relative price of fruit and the price of vegetables.

Summary

The purpose of this chapter was two fold: (1) consider several input demand issues facing the business, including the size of the business in the long run and the factors that can influence this size, and (2) illustrate the physical and economic relationships associated with the issue of product choice by a business under conditions of perfect competition. The major points made in this chapter may be summarized as follows:

1. The least-cost combination of inputs and level of output possible with a given budget is found graphically by looking for the point of tangency between a specific iso-cost line and the highest possible isoquant curve.
2. The cost of production associated with the least-cost combination to produce a given level of output is found graphically by looking for the point of tangency between a specific isoquant and an iso-cost line.
3. The least-cost combination of two inputs can be found numerically by searching for the equality between the ratio of the two input prices (slope of the iso-cost line) and marginal rate of technical substitution or ratio of the two input MPPs (slope of an isoquant).
4. Firms in the long run can expand the size of their operations by using more of all inputs, including forms of capital that were fixed in the short run.
5. The long-run average cost curve, often referred to as the planning curve, illustrates how varying the size of the firm will affect its efficiency.
6. The long-run equilibrium of the firm under conditions of perfect competition will occur at that output level where the product price is equal to both the firm's marginal and average total costs.
7. When businesses expand the size of their operations, they will incur returns to size.
8. If the increase in input is exactly proportional to the increase in input use, the returns to size are constant. If this increase was more (less) than proportional to the increase in input use, returns to size are increasing (decreasing). The business will normally pass through a phase of increasing returns to size or economies of size before encountering constant and then decreasing returns to size, or diseconomies of size.
9. The production possibilities frontier in the current period represents the different combinations of two products a business can produce given efficient use of its existing resources. When the business expands its operations in the long run, it will be on higher production possibilities frontiers that reflect the changing nature of its resources.
10. The slope of the production possibilities curve is called the marginal rate of product transformation. This slope reflects the rate at which the business can substitute between the production of two products in the current period. If two products are perfect substitutes, the marginal rate of product transformation will be constant at all points along the production possibilities frontier.
11. The iso-revenue line reflects the rate at which the market is willing to substitute between two products as their prices change. The slope of this line is therefore equal to the ratio of the prices of the two products. The intercept of this line on both axes reflects the maximum quantity of these two products that could be purchased if bought alone, and reflects a given amount of revenue and the prices of the products. Changes in the prices of these products will alter the slope of the iso-revenue line.

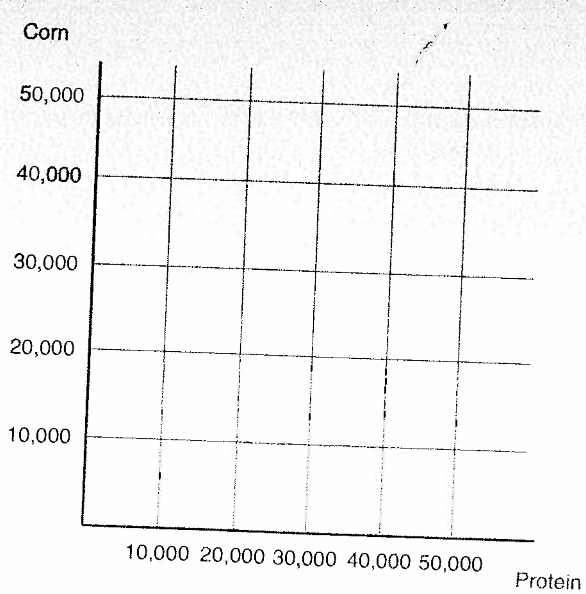
12. The profit-maximizing combination of two products to produce is determined by the point of tangency with the business's current production possibilities frontier and the iso-revenue line. At this point of tangency, the marginal rate of product transformation, or slope of the production possibilities frontier, will be equal to the ratio of the two product prices, or the slope of the iso-revenue line.

Key Terms

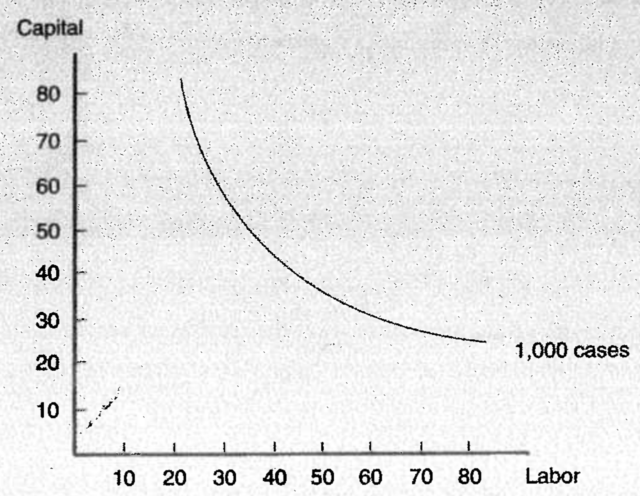
Iso-cost line
Iso-revenue line
Isoquant
Marginal rate of product transformation
Marginal rate of technical substitution
Production possibilities frontier
Rental rate of capital

Testing Your Economic Quotient

1. A firm uses corn and protein supplement to mix a particular type of hog feed. Corn costs $.08 per pound and protein supplement costs $.12 per pound. Let's assume the firm has $3,000 on these two inputs. Plot the iso-cost line suggested by this information in the graph below. What is the value of this curve's slope?

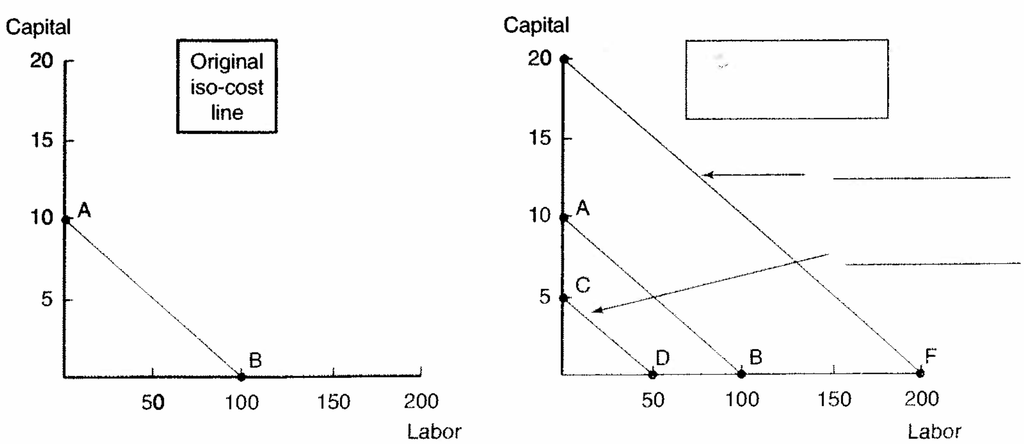


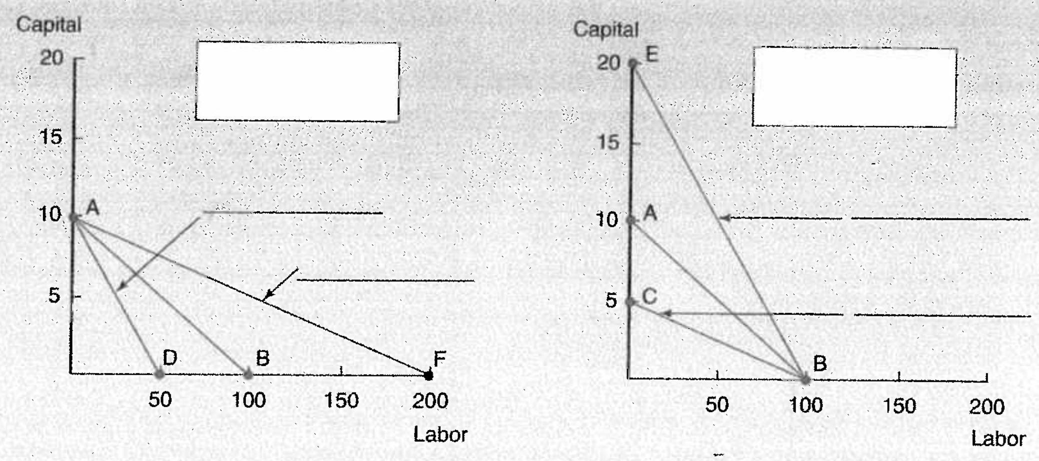
2. Suppose the wage rate for labor is $20 an hour and the rental rate for capital is $50 per hour. Based on this information, please answer the questions appearing below the graph.



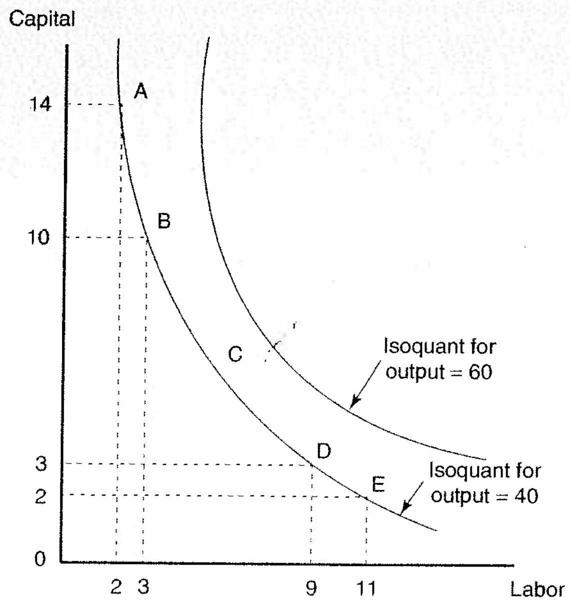
a. Draw the iso-cost line associated with an hourly budget of $1,000.
b. What is the least-cost level of capital and labor this business should utilize when packaging 1,000 cases of fruit juice? How did you arrive at this answer?
c. How much does it cost this business to package 1,000 cases of fruit juice? If the firm can sell the juice for $50 per case, what is its accounting profit?

3. In *each* of the following four graphs, please describe what caused the iso-cost line to change *in each box* and the nature of the change *on each line.*





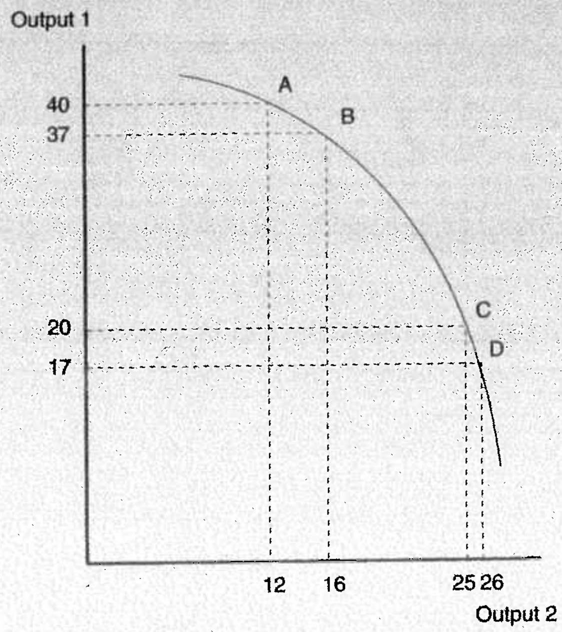
4. Given the following graph, briefly respond to the questions appearing directly below this graph.



a. For an output level of 40 units, calculate the marginal rate of technical substitution between points *A* and *B*.

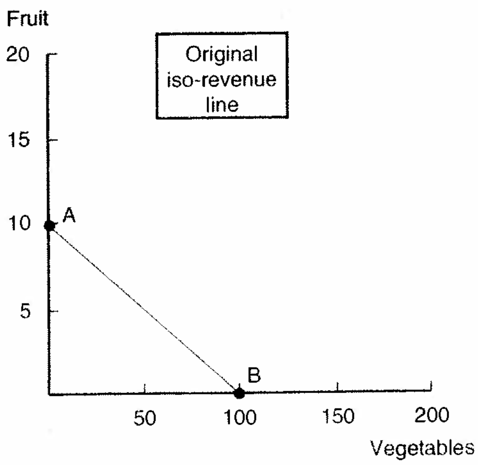
b. Also for an output level of 40 units, calculate the marginal rate of technical substitution between points *D* and *E*.

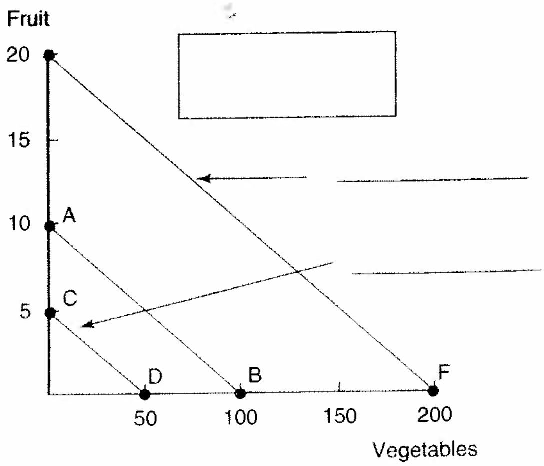
5. Given the following graph for two products, please respond to the questions below.

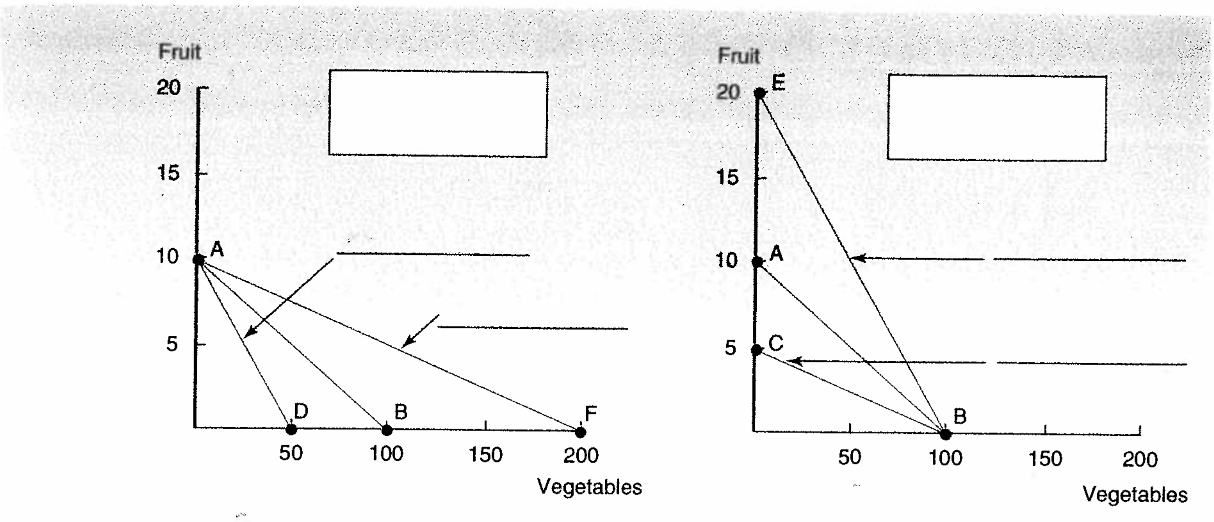


a. Calculate the marginal rate of product transformation between points *A* and *B*.
b. Calculate the marginal rate of product transformation between points *C* and *D*.

6. In *each* of the following four graphs, please describe what caused the iso-revenue line to change *in each box* and the nature of the change *on each line*.







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