Addis Ababa University
Addis Ababa Institute of Technology
School of Electrical and Computer Engineering
Electromagnetic Fields ECEG – 2122 Assignment – 2

Solve the following problems. *Show all relevant steps clearly and neatly. Justify your assumptions. Your results should include units. Unrelated and irrelevant text may result in deduction of marks.*

1. (a) Verify that

$$\nabla \times (V\mathbf{A}) = V(\nabla \times \mathbf{A}) + \nabla V \times \mathbf{A}$$

where V and A are scalar and vector fields, respectively.

(b) Evaluate $\nabla \times (V\mathbf{A})$ when $V = \frac{1}{r^2}$ and $\mathbf{A} = r\cos\theta\,\mathbf{a}_r + r\sin\theta\,\mathbf{a}_\theta + \sin\theta\cos\phi\,\mathbf{a}_\phi$.

2. Let $\mathbf{D} = 2\rho z^2\mathbf{a}_\rho + \rho\cos^2\phi\mathbf{a}_z$. Evaluate

(a) $\oint_S \mathbf{D}\cdot d\mathbf{S}$
(b) $\int_v \nabla\cdot\mathbf{D}dv$

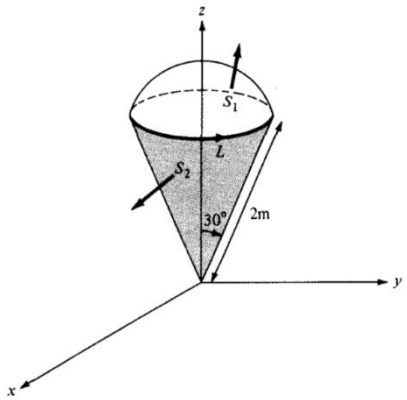
over the region defined by $0 \le \rho \le 5, -1 \le z \le 1, 0 < \phi < 2\pi$.

3. A vector field is given by

$$\mathbf{Q} = \frac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}}[(x-y)\mathbf{a}_x + (x+y)\mathbf{a}_y$$

Evaluate the following integrals:

(a) $\int_L \mathbf{Q}\cdot d\mathbf{l}$ where L is the circular edge of the volume in the form of an ice-cream cone shown in Figure



(b) $\int_{S_1} (\nabla\times\mathbf{Q})\cdot d\mathbf{S}$ where S_1 is the top surface of the volume
(c) $\int_{S_2} (\nabla\times\mathbf{Q})\cdot d\mathbf{S}$ where S_2 is the slanting surface of the volume

(d) $\int_{S_1} \mathbf{Q}\cdot d\mathbf{S}$
(e) $\int_{S_2} \mathbf{Q}\cdot d\mathbf{S}$
(f) $\int_v \nabla\cdot\mathbf{Q}\,dv$

How do your results in parts (a) to (f) compare?

4. (a) Prove that when a particle of constant mass and charge is accelerated from rest in an electric field, its final velocity is proportional to the square root of the potential difference through which it is accelerated.
   (b) Find the magnitude of the proportionality constant if the particle is an electron.
   (c) Through what voltage must an electron be accelerated, assuming no change in its mass, to require a velocity one-tenth that of light? (At such velocities, the mass of a body becomes appreciably larger than its "rest mass" and cannot be considered constant.)

5. In an electric field $\mathbf{E} = 20r \sin\theta\, \mathbf{a}_r + 10r \cos\theta\, \mathbf{a}_\theta$ V/m, calculate the energy expended in transferring a 10-nC charge
   (a) From $A(5, 30°, 0°)$ to $B(5, 90°, 0°)$
   (b) From A to $C(10, 30°, 0°)$
   (c) From A to $D(5, 30°, 60°)$
   (d) From A to $E(10, 90°, 60°)$

6. In free space, $V = x^2y(z + 3)$ V. Find
   (a) $\mathbf{E}$ at $(3, 4, -6)$
   (b) the charge within the cube $0 < x,y,z < 1$.