Research Methods and Experimental Design

Lecture 1: Research methods: The basics

INTRODUCTION

- ✓ Research Methods are the tools and techniques for doing research.
- ✓ any kind of investigation that is intended to uncover interesting or new facts.
- ✓ Research methods are a range of tools that are used for different types of enquiry,
- ✓ just as a variety of tools are used for doing different practical jobs

RESEARCH THEORY AND PRACTICE:**RESEARCH BASICS**

- ❖Research is a very general term for an activity that involves finding out, in a more or less systematic way, things you did not know.
- ❖A more academic interpretation is that research involves finding out about things that no-one else knew either.
- ❖It is about advancing the frontiers of knowledge.

Cont…

- Research methods are the techniques you use to do research.
- They represent the tools of the trade, and provide you with ways to collect, sort and analyse information so that you can come to some conclusions.
- If you use the right sort of methods for your particular type of research, then you should be able to convince other people that your conclusions have some **validity**, and that the new knowledge you have created is soundly based.

Cont…

- Being a researcher is as much about doing a practical job as being academically competent.
- Identifying a subject to research, finding and collecting information and analysing it, presents you with a range of practical problems that need to be solved.

WHAT YOU CAN DO WITH RESEARCH

- *So what can we use research to do in order to gain this new knowledge?*
  Some of the ways it can be used one to:

- ✓ **Categorise.** This involves forming a typology of objects, events or concepts, i.e. a set of names or 'boxes' into which these can be sorted.
- ✓ This can be useful in explaining which 'things' belong together and how.

Cont…

- ✓**Describe.** Descriptive research relies on observation as a means of collecting data.
- ✓It attempts to examine situations in order to establish what is the norm, i.e. what can be predicted to happen again under the same circumstances.

Cont…

- ✓ **Explain.** This is a descriptive type of research specifically designed to deal with complex issues. It aims to move beyond 'just getting the facts' in order to make sense of the myriad other elements.
- ✓ **Evaluate.** This involves making judgements about the quality of objects or events. Quality can be measured either in an absolute sense or on a comparative basis. To be useful, the methods of evaluation must be relevant to the context and intentions of the research.

Cont…

- ✓ **Compare.** Two or more contrasting cases can be examined to highlight differences and similarities between them, leading to a better understanding of phenomena.

- ✓ **Correlate.** The relationships between two phenomena are investigated to see whether and how they influence each other. The relationship might be just a loose link at one extreme or a direct link when one phenomenon causes another. These are measured as levels of association.

Cont…

- ✓ **Predict.** This can sometimes be done in research areas where correlations are already known. **Predictions** of possible future behaviour or events are made on the basis that if there has been a strong relationship between two or more characteristics or events in the past, then these should exist in similar circumstances in the future, leading to predictable outcomes.

Cont…

- ✓ **Control.** Once you understand an event or situation, you may be able to find ways to **control** it. For this you need to know what the cause and effect relationships are and that you are capable of exerting control over the vital ingredients. All of technology relies on this ability to control.

You can combine two or more of these objectives in a research project

RESEARCH DESIGNS

- There are numerous types of research design that are appropriate for the different types of research projects.
- The choice of which design to apply depends on the nature of the problems posed by the research aims.
- Each type of research design has a range of research methods that are commonly used to collect and analyse the type of **data** that is generated by the investigations.

Here is a list of some of the more common research designs, with a short explanation of the characteristics of each.

HISTORICAL

- This aims at a systematic and objective evaluation and synthesis of evidence in order to establish facts and draw conclusions about past events.

DESCRIPTIVE

- This design relies on observation as a means of collecting data. It attempts to examine situations in order to establish what is the norm, i.e. what can be predicted to happen again under the same circumstances. ‘Observation’ can take many forms. Depending on the type of information sought, people can be interviewed, questionnaires distributed, visual records made, even sounds and smells recorded.
- The scale of the research is influenced by two major factors: the level of complexity of the survey and the scope or extent of the survey.

CORRELATION

- This design is used to examine a relationship between two concepts. There are two broad classifications of relational statements:
- an association between two concepts – where there is some kind of influence of one on the other; and a causal relationship – where one causes changes to occur in the other. **Causal statements** describe what is sometimes called a 'cause and effect' relationship. The cause is referred to as the '**independent variable**', the variable that is affected is referred to as the '**dependent variable**'.

Cont…

- The correlation between two concepts can either be none (no correlation); positive (where an increase in one results in the increase in the other, or decrease results in a decrease); or negative (where the increase in one results in the decrease in the other or vice versa). The degree of association is often measurable.

COMPARATIVE

- This design is used to compare past and present or different parallel situations, particularly when the researcher has no control over events.
- **Analogy** is used to identify similarities in order to predict results – *assuming that if two events are similar in certain characteristics, they could well be similar in others too.* In this way comparative design is used to explore and test what conditions were necessary to cause certain events, so that it is possible, for example, to understand the likely effects of making certain decisions.

EXPERIMENTAL

- Experimental research attempts to isolate and control every relevant condition which determines the events investigated and then observes the effects when the conditions are manipulated.
- At its simplest, changes are made to an independent variable and the effects are observed on a dependent variable – i.e. cause and effect.

Cont…

- Although experiments can be done to explore a particular event, they usually require a hypothesis (prediction) to be formulated first in order to determine what variables are to be tested and how they can be controlled and measured.

SIMULATION

- **Simulation** involves devising a representation in a small and simplified form (**model**) of a system, which can be manipulated to gauge effects.
- It is similar to experimental design in the respect of this manipulation, but it provides a more artificial environment in that it does work with original materials at the same scale.

Cont…

- Models can be mathematical (number crunching in a computer) or physical, working with two- or three-dimensional materials. The performance of the model must be checked and calibrated against the real system to check that the results are reliable.
- Simulation enables theoretical situations to be tested – what if?

ACTION

- Essentially, this is an ‘on the spot’ procedure, principally designed to deal with a specific problem found in a particular situation. There is no attempt made to separate the problem from its context in order to study it in isolation. What are thought to be useful changes are made and then constant monitoring and evaluation are carried out to see the effects of the changes. The conclusions from the findings are applied immediately, and further monitored to gauge their effectiveness.

DECIDING ON YOUR TYPE OF RESEARCH

- It is your research interest that decides the nature of your research problem, and this will indicate the appropriate type of research to follow.
- Once the objectives of a research project have been established, the issue of how these objectives can be met leads to a consideration of which research design should be chosen.

Cont…

- The research design provides a framework for the collection and analysis of data and subsequently indicates which research methods are appropriate.

STRUCTURING THE RESEARCH PROJECT

- Research projects are set up in order to explain a phenomenon or to test a theory.
- Research methods are the practical techniques used to carry out research.
- They are the ‘tools of the trade’ that make it possible to collect information and to analyse it.

Cont…

- What information you collect and how you analyse it depends on the nature of the **research problem**, the central generating point of a research project.

THE RESEARCH PROCESS

- It is necessary to first define some kind of research problem in order to provide a reason for doing the research.
- The problem will generate the subject of the research, its aims and objectives, and will indicate what sort of data need to be collected in order to investigate the issues raised and what kind of analysis is suitable to enable you to come to conclusions that provide answers to the questions raised in the problem.

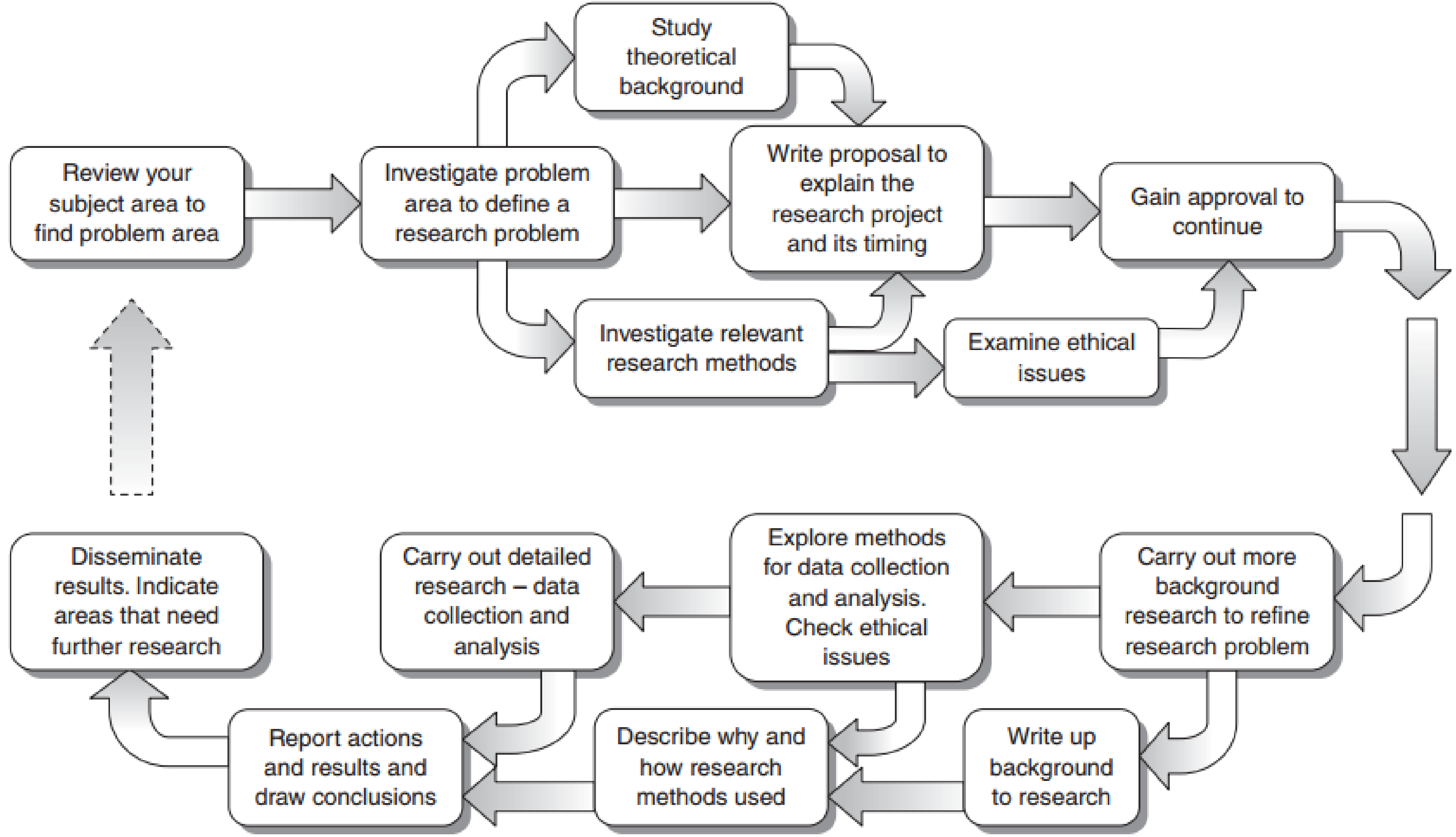
Cont…

- Some projects are aimed at testing and refining existing knowledge, others at creating new knowledge.
- The answers to four important questions underpin the framework of any research project:
  - What are you going to do? The subject of your research.
  - Why are you going to do it? The reason for this research being necessary or interesting.
  - How are you going to do it? The research methods that you will use to carry out the project.
  - When are you going to do it? The programme of the work.

Cont…

- The answers to these questions will provide a framework for the actual doing of the research. The answers to these questions are not simple.

THE STRUCTURE OF A TYPICAL RESEARCH PROJECT



THE RESEARCH PROBLEM

- There is no shortage of problems throughout the world, but for a problem to be researchable, it needs to have several crucial features.
  It must be:
  - stated clearly and concisely;
  - significant i.e. not trivial or a repeat of previous work;
  - delineated, in order to limit its scope to practical investigation
  - possible to obtain the information required to explore the problem;
  - possible to draw conclusions related to the problem, as the point of research is to find some answers.

Cont…

- A research problem can be based on a question, an unresolved controversy, a gap in knowledge or an unrequited need within the chosen subject.
- An awareness of current issues in the subject and an inquisitive and questioning mind and an ability to express yourself clearly is required in order to find and formulate a problem that is suitable for a research project.

Research Methods and Experimental Design

Lecture 2: Research methods: How to Write a scientific paper

How to write a scientific paper

- A scientific experiment is not complete until the results have been published and understood.

- A scientific paper is a written and published report describing *original research results.*

What is Scientific Writing

- The purpose of scientific writing is to communicate new scientific findings
- Thus it has to be clear, simple and well ordered communication to transmit new scientific findings
- Scientific writing must use proper English which gives the sense in the fewest short words

Origins of Scientific Writing

- Knowledge is lost without written records
- Cave paintings and inscriptions were the first attempts to leave records
- About 2000 BC, Papyrus paper was used as a medium of communication
- In 190 BC, parchment made from animal skin came into use
- In 105 AD, the Chinese invented paper

Origins of Scientific Writing

- Knowledge could not be widely circulated with no effective duplication
- In 1100 AD, the Chinese invented movable type
- In 1455 AD, Gutenberg printed his 42-line Bible from movable type on a printing press
- By the year 1500 thousands of copies of hundreds of books (called "incunabula") were printed
- In 1665, the first scientific journals were published

IMRAD Story

(Introduction, Methods, Results and Discussion)

- Early journals published **descriptive** papers (still used in case reports, geological surveys etc..)
- By the second half of the 19th century, reproducibility of experiments became a fundamental principle of the philosophy of science.
- The methods section became all important since Louis Pasteur confirmed the germ theory of disease
- IMRAD organization of a scientific paper started to develop
- IMRAD format slowly progressed in the latter half of the 19th century

IMRAD Format

- I = Introduction, what question (problem) was studied
- M = Methods, how was the problem studied
- R = Results, what are the findings
- A = and
- D = Discussion, what do these findings mean

Organization of a scientific paper

- The most common is the IMRAD
- If a number of methods were used to achieve directly related results:

  M + R = Experimental section
- The results are so complex that they need to be immediately discussed:

  R + D = Results and Discussion section

What is a scientific paper

- A scientific paper is a written and published report describing original research results.
- It must be the first publication of original research results,
- In a form whereby peers of the author can repeat the experiments and test the conclusions, and
- In a journal or other source document readily available within the scientific community

Definition of Scientific paper

- An accepted original scientific publication containing scientific information to enable peers:
- To assess observations
- To repeat experiments
- To evaluate intellectual processes
- Must have an impact
- Available to scientific community without restriction
- Available for regular screening by one or more of the major recognized secondary services (Biological abstracts, Index Medicus, Pub Med etc…)

Some important Language points:

- Poor experimentation cannot be masked by brilliant writing; however, poor writing can mask brilliant experimentation
- Avoid complex sentence structure
- Use simple and clear English
- Always keep in mind that the paragraph is the essential unit of thought

Before Starting to Write the Paper

- Record your readings (results)
- Make tables
- Draw graphs
- Keep file to record summaries of results and any observation however insignificant
- Date the files
- Revise your readings, you may need to repeat an experiment while you still have the materials.
- Write ideas when ever they come to you

Essential Parts of a Scientific paper

- Title: Describe concisely the core contents of the paper
- Abstract: Summarize the major elements of the paper
- Introduction: provide context and rationale for the study
- Materials: Describe the experimental design so it is reproducible
- Methods: Describe the experimental procedures
- Results: Summarize the findings without interpretation
- Discussion: Interpret the findings of the study
- Summary: Summarize the findings
- Acknowledgement: Give credit to those who helped you
- References: List all scientific papers, books and websites that you cited

The Title

- A good title is defined as the fewest possible words that adequately describe the contents of the paper.
- The title is extremely important and must be chosen with great care as it will be read by thousands, whereas few will read the entire paper
- Indexing and abstracting of the paper depends on the accuracy of the title. An improperly titled paper will get lost and will never be read.

The Title

- Titles should neither be too short nor too long as to be meaningless
- Waste words (studies on, investigations on, a, an, the etc) should not be used.
- Syntax (word order) must be very carefully considered
- It should contain the keywords that reflect the contents of the paper.
- It should be meaningful and not general
- It should be concise, specific and informative
- It should capture the fundamental nature of the experiments and findings

Examples

- Effect of semiconductor on bacteria
  - Effect: should be defined
  - Semiconductor: should be listed
  - Bacteria: should be listed
- Photocatalytic inactivation of TiO^2 on gram negative bacteria

How to Prepare the Title

- Make a list of the most important keywords
- Think of a title that contains these words
- The title could state the conclusion of the paper
- The title **NEVER** contains abbreviations, chemical formulas, proprietary names or jargon
- Think, rethink of the title before submitting the paper
- Be very careful of the grammatical errors due to faulty word order
- Avoid the use of the word "using"

The Abstract

- An abstract can be defined as a summary of the information in a document
- It is of fundamental importance that the abstract be written clearly and simply, as it is the first and sometimes the only part of the manuscript read.
- It should provide a brief summary of each of the main sections (IMRAD) of the paper:

1. State the principal objective and scope of the investigation
2. Describe the methods used

Cont…

1. Summarize the results, and
2. State the principal conclusions

- It is easier to write the abstract after completion of the paper

Criteria of the Abstract

- It should not exceed 250 words
- It should be written in one paragraph.
- It should be written in the past tense as it refers to work done.
- Long words should be followed by its abbreviation which would be used through out the abstract and paper.
- It should not cite any references (except in rare cases)
- It should never give any information or conclusion that is not stated in the paper
- Must be accurate with respect to figures quoted in the main text.

The Introduction

❖ The introduction should answer the following questions:

1. What was I studying?
2. Why was this an important question?
3. What did I know about this topic before I did this study?
4. What model was I testing? and
5. What approach did I take in this study?

Suggested rules for a good introduction:

- It should present the nature and scope of the problem investigated
- Review the pertinent literature
- State the method of investigation
- State the principal results of the investigation
- State the principal conclusion(s) suggested by the results

General rules

- Use the present tense when referring to work that has already been published, but past tense when referring to your own study.
- Use the active voice as much as possible
- Avoid lengthy or unfocused reviews of previous research.
- Cite peer-reviewed scientific literature or scholarly reviews. Avoid general reference works such as textbooks.
- Define any specialized terms or abbreviations

How to write the Materials and Methods section

- Provide full details so that the experiments are reproducible
- If the peer reviewer has doubts that the experiments could be repeated, the manuscript will be rejected.
- Organize the methods under subheadings, with related methods described together (e.g. subjects, experimental design, Measurement of…, Characterization of….,etc…).
- Describe the experimental design in detail
- Do *not* mix some of the Results in this section
- Write in the past tense

Materials

- ❖Must identify accurately experimental chemicals, reagents, equipment….etc
- ❖The source of subjects studied, number of expriments
- ❖For chemicals used, include exact technical specifications and source or method of preparation.
- ❖Avoid the use of trade names of chemicals, generic or chemical names are preferred.

Methods

- This part of the manuscript must be clear, precise and concise so that it can be reproducible
- If the method is new, ***all*** details must be provided
- If the method has been previously published in a scientific journal, only the reference should be given with some identification:

e.g. "The preparation of N-doped TiO2 was carried out using ammonia solution (28%) as nitrogen source based on optimized synthesis parameters from our previous work [reference] "

Cont…

- Questions such as "how" or "how much" must be answered and not left to be puzzled over
- Methods used for statistical analyses must be mentioned; ordinary ones without comments, but advanced or unusual ones require literature citation

How to write the Results

- Results section is written in the past tense
- It is the core or heart of the paper
- It needs to be clearly and simply stated since it constitutes the new knowledge contributed to the world
- The purpose of this section is to summarize and illustrate the findings in an orderly and logical sequence, **without interpretation**
- The text should guide the reader through the findings, stressing the major points
- Do not describe methods that have already been described in the M&M section or that have been inadvertently omitted

Methods of presenting the data

1. Directly in the text
2. In a table
3. In a figure

- All figures and tables ***must*** be accompanied by a textual presentation of the key findings
- Never have a table or figure that is not mentioned in the text

Tables and figures

- Tables are appropriate for large or complicated data sets that would be difficult to explain clearly in text.
- Figures are appropriate for data sets that exhibit trends, patterns, or relationships that are best conveyed visually.
- Any table or figure must be sufficiently described by its title and caption or legend, to be understandable without reading the main text of the results section.
- Do not include both a table and a figure showing the same information

How to write the Discussion

- It is the **hardest section to write**.
- Its primary purpose is to show the relationships among observed facts
- It should end with a short summary or conclusion regarding the significance of the work.

Components of the discussion

- Try to present the principles, relationships, and generalizations shown by the Results
- Point out any exceptions or any lack of correlation and define unsettled points.
- Show how your results and interpretations agree or contrast with previously published work
- Discuss the theoretical implications of your work, and any possible practical applications.
- State your conclusions as clearly as possible
- Summarize your evidence for each conclusion

How to State the Acknowledgments

- You should acknowledge:
  1. Any significant technical help that you have received from any individual in your lab or elsewhere
  2. The source of special equipment, or any other material
  3. Any outside financial assistance, such as grants, contracts or fellowships
- Do not use the word “wish”, simply write “I thank …..” and not “I wish to thank…”
- Show the proposed wording of the Acknowledgement to the person whose help you are acknowledging

References

What is referencing?

- Referencing is a standardized way of acknowledging the sources of information and ideas that you have used in your document.

- A list of ALL the references used in the text must be written.

- Reference format varies widely:
  - Harvard format (the name and year system) is the most widely used
  - Alphabet-Number system is a modification of name and year system
  - Citation order system

In-text citations

In name and year system:

- Citation in the text is followed by the author's last name and year of publication between parentheses.
  - If they were two authors then both last names are written.
  - If more than two then the only first author's name is written followed by the abbreviation *et al*
- If a single statement requires more than one citation then the references are arranged chronologically from oldest to more recent, separated by semicolons.
  - If more than one reference share the same year then they are arranged alphabetically within the year.

Cont…

In alphabet-number system:

- ❖Citation by number from an alphabetically arranged numbered reference list.

In Citation order system:

- ❖The references are numbered in the order they are mentioned in the text

Reference List

- Any papers not cited in the text should not be included.
- Reference lists allow readers to investigate the subject in greater depth.
- A reference list contains only the books, articles, and web pages etc that are cited in the text of the document. A bibliography includes all sources consulted for background or further reading.

Cont….

In name and year system:

- The reference list is arranged alphabetically by author. If an item has no author, it is cited by title, and included in the alphabetical list using the first significant word of the title.
- If more than one item has the same author, list the items chronologically, starting with the earliest publication.
- Each reference appears on a new line.
- There is no indentation of the references
- There is no numbering of the references

Cont…

In alphabet-number system:

It the same as above in addition each reference is given a number

In Citation order system:

The reference list is arranged by the number given to the citation by the order that it were mentioned in the text

Example

- **Book**
  - 1. Okuda M, Okuda D. *Star Trek Chronology: The History of the Future*. New York: Pocket Books; 1993.
- **Journal or Magazine Article** (with volume numbers)
  - 2. Wilcox RV. Shifting roles and synthetic women in Star trek: the next generation. *Stud Pop Culture*. 1991;13:53-65.
- **Newspaper, Magazine or Journal Article** (without volume numbers)
  - 3. Di Rado A. Trekking through college: classes explore modern society using the world of Star trek. *Los Angeles Times*. March 15, 1995:A3.
- **Encyclopedia Article**
  - 4. Sturgeon T. Science fiction. In: Lorimer LT, editorial director; Cummings C, ed-in-chief; Leish KW, managing ed. *The Encyclopedia Americana*. Vol 24. International ed. Danbury, Conn: Grolier Incorporated; 1995:390-392.

Example

- **Book Article or Chapter**
  - 5. James NE. Two sides of paradise: the Eden myth according to Kirk and Spock. In: Palumbo D, ed. *Spectrum of the Fantastic*. Westport, Conn: Greenwood; 1988:219-223.
- **ERIC Document**
  - 6. Fuss-Reineck M. *Sibling Communication in Star Trek: The Next Generation: Conflicts Between Brothers*. Miami, Fla: Annual Meeting of the Speech Communication Association; 1993. ERIC Document Reproduction Service ED364932.
- **Website**
  - 7. Lynch T. DSN trials and tribble-ations review. Psi Phi: Bradley's Science Fiction Club Web site. 1996. Available at: http://www.bradley.edu/campusorg/psiphi/DS9/ep /503r.htm. Accessed October 8, 1997.
- **Journal Article on the Internet**
  - 8. McCoy LH. Respiratory changes in Vulcans during pon farr. *J Extr Med* [serial online]. 1999;47:237-247. Available at: http://infotrac.galegroup.com/itweb/nysl_li_liu. Accessed April 7, 1999.

Research Methods and Experimental Design

Lecture 3: Research methods: How to Write a Thesis

How to Write a Thesis

- A PhD thesis in the science is supposed to present the candidate's original research i.e. it is a scientific paper
- Unlike the scientific paper, the thesis may describe more than one topic, and it may present more than one approach to some topics.
- The thesis may present all or most of the data obtained in the student's thesis related research.
- Thus it is more involved and longer than a scientific paper.
- Think of a thesis as a good thriller, and write in a logical way so that a reader will find it interesting and will not be bored.

Ethics, Rights and Permissions

- Beware of originality and copyrights of others.
- Do not copy anything without giving the credit to the owner by referencing it.
- In some cases permissions are needed
- Repetitive publication of the same data is considered plagiarism

Finally…Avoiding Plagiarism

- What is it?
  - All knowledge in your head has either been copied from some place or originally discovered by you.
  - Most knowledge was copied.
  - This is true in most settings. General knowledge is copied. Most teachers' lectures are copied knowledge.
  - Humans are naturally copiers, but this is not what we would typically call "plagiarism."

Cont…

– Among other things, plagiarism refers to taking others' work and representing it as if it were your own.

– In academics this is bad because with plagiarism:
  - One cannot assess students' development accurately
  - The person who makes his or her livelihood by scholarly pursuit is being robbed of credit
  - It masks the lineage of ideas and facts.

Cont…

Lineage of Ideas:

- Original sources of research are all the proof we have for some facts. Without the "paper trail" of academic thought:
  - People could pass incorrect ideas off as facts
  - We would have to keep "re-proving" things.
  - The contexts that generated facts and ideas get lost.
  - Research becomes highly inefficient as it becomes incredibly difficult to find "full information" on a topic.

To avoid plagiarism:

1. Document every source for information that is not "general knowledge"—this includes facts and ideas.
2. Cite every time a fact or idea is used unless it is clear that one citation is referring to a group of facts or ideas.
3. If you quote material, put quotation marks around the quoted stuff and include a page number within the citation.
4. It is alright to paraphrase material, but you still have to cite from where the paraphrased material came.
5. When in doubt, cite the source.

Plagiarism v. Paraphrasing Samples

- **Direct quote from research:**
- "Japan's beautiful Mount Fuji last erupted in 1707 and is now classified as dormant. Dormant volcanoes show no signs of activity, but they may erupt in the future."

- **Non-plagiarized paraphrase:**
- Mount Fuji, the highest mountain in Japan, is actually a dormant volcano. Dormant means that it is not active. The last time Mount Fuji erupted was in 1707, and there is always the possibility of a future eruption.

Cont…

- **Direct quote from research:**
- "Three weeks after Katrina, warnings of the arrival of Hurricane Rita sent residents of cities such as Houston, Texas, rushing to evacuate, fearing for their lives. Fortunately, Hurricane Rita turned out to be much less severe than Katrina. However, mass evacuations like this bring hazards of their own, as panicking drivers may cause accidents on the jammed roads."

- **Non-plagiarized paraphrase:**
- Shortly after Hurricane Katrina devastated the city of Houston, Texas, a warning for a new hurricane named Rita was broadcast, which caused many people to panic and flee the city. However, the mass departure of people leaving Houston at the same time could have caused many car accidents, even though the hurricane turned out to be not as dangerous as Katrina.

Some Technical Points of Style

- Page margins and line spacing - these can vary
- Mathematical and statistical notation – publications vary in what is acceptable
- Citations and reference notes – footnotes and referencing
- Tables, charts, and graphs – should be clear and “stand alone”
- Verb tense – don’t vary within a section
- Personal pronoun form – generally third person for formal papers and first of second person for less formal writing

Proofread, Proofread, & Proofread!!!

1. Are all words spelled correctly? (Use a paper or online dictionary is unsure!)
2. Did I capitalize the beginning of each sentence and all proper nouns?
3. Did I punctuate correctly?
4. Do I use grammar correctly?
5. Did I answer all of the topic questions, and fulfill all of the requirements on my rubric.
6. Did I include an introduction and conclusion?

Proofread, Proofread, & Proofread!!!

1. Did I type the paper using the correct font type, size, line spacing and margin requirements?
2. Did I paraphrase all content?
3. Did I use parenthetical notations for quotes?
4. Do my sentences make sense when read aloud?
5. Have I had my paper peer edited?
6. Does my paper flow well?
7. Did I include a bibliography page?

Important Software

- Mendeley Desktop
- Endnote
- OriginPro
- Sigma plot

Research Methods and Experimental Design

Lecture 4: Research methods: How to Write a research proposal

Research proposal

- A research plan is the key to successful research. The approach to the research needs to be carefully constructed and designed.

"the heart of the research plan is the research proposal"

- The intent is not to limit creativity … the most insightful discovery usually occur within structured inquiry.

Research proposal

- Statement of intent
  - Academically prepared to complete the research
  - Audience: peers, supervisors, examiners
- A research proposal is your PLAN
  - It describes in detail your study
  - Decisions about your study are based on the quality of the proposal
    - Approvals to proceed by the Institutional Review Board

Research proposal

- ❖It is like a blue print of a building plan before the construction starts
- ❖Writing a research proposal is both science and art
- ❖A good research proposal is based on scientific facts and on the art of clear communication
- ❖Writing a formal research proposal should be started by the time one has decided on the topic for the study

Research proposal

- Proposals are generally required by all entities that support or encourage research
- They can seek financial support or simply serve as a guide for the research. The most stringent and complex are Ph.D. dissertation proposals.
- Some funding sources (eg. industry groups), may prefer short, concise plans without "academic" aspects.
- Rarely a proposal may be delivered orally (but these are usually backed by a written proposal).

Research proposal

- Effective communication skills are essential. Thoughts must be clear and well developed.
- Proposals serve dual purposes:
  - Provides an operational plan for the researcher. This forces clear understanding of the intent of the research and anticipation of potential problems.
  - For evaluators (including graduate committees) a proposal clarifies the intent of the research and allows decisions on approval or disapproval.

Importance of a research proposal?

Contract between you and your committee

1. Serves to protect the student
   - Demanding additional requirements

2. Protects the committee from the student
   - From delivering a degree of poor quality

Elements of the Research Proposal

- Although varying in complexity and form, there are common elements to all proposals. The figure below shows components and linkages

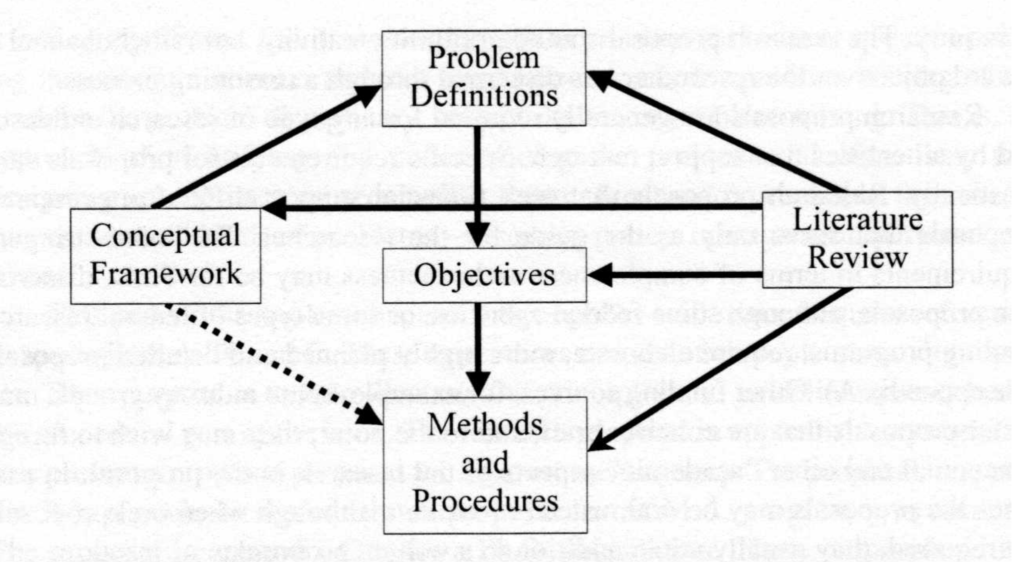
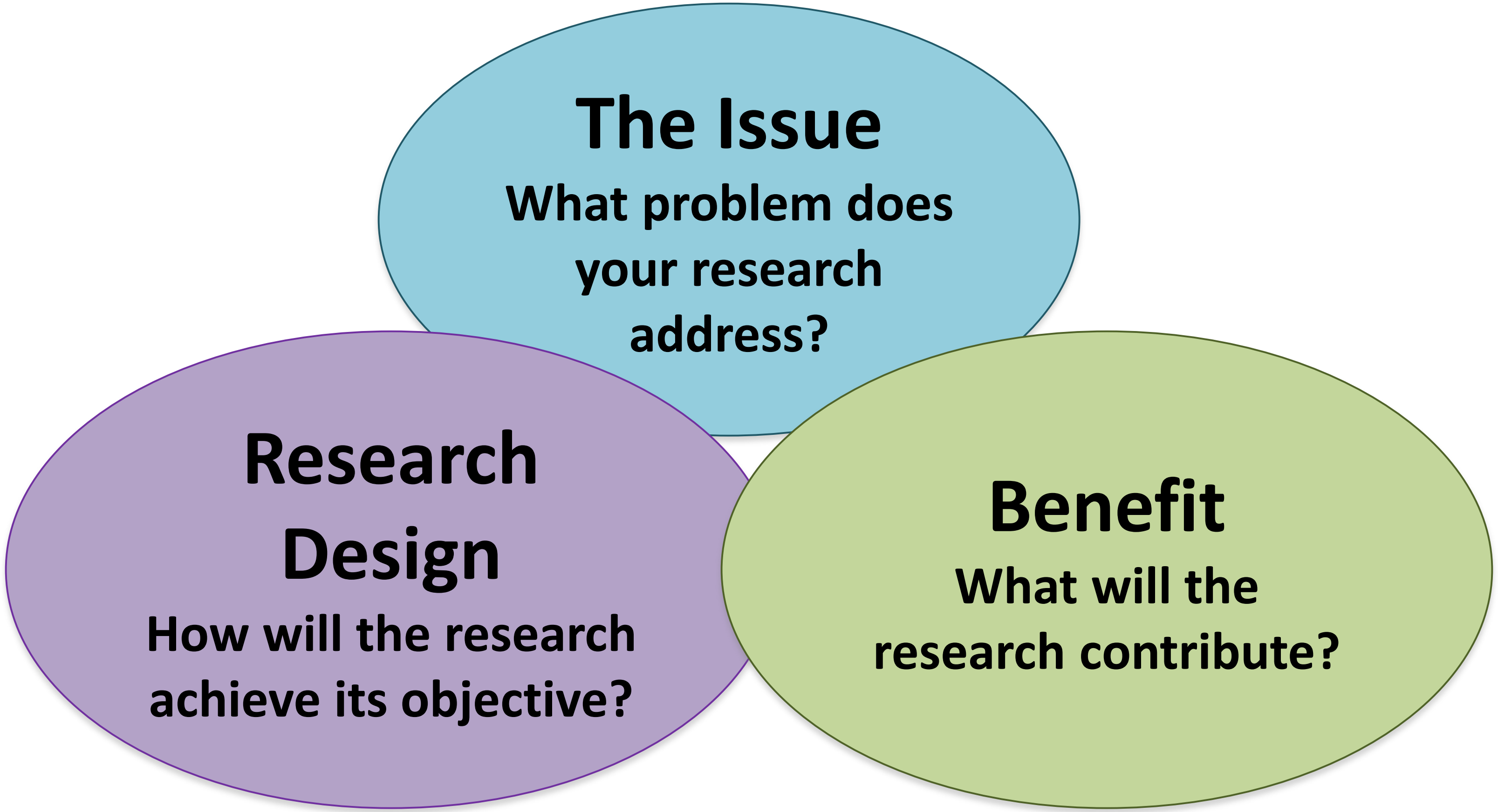


Figure 5.1. Linkages among components of a research project.

What are the essential ingredients?



Research proposals make you:

OUTLINE steps in your proposed research

Provide yourself with intellectual CONTEXT

JUSTIFY your research

Be CREATIVE

THINK through your experiments

Anticipate potential PROBLEMS

Anticipate a realistic TIMETABLE

Formal Structure of Scientific Proposal

- Title
- Introduction
- Background /Review of literature
- Justification/Problem statement
- Objective
- Methodology
- Time frame and work schedule/Gantt chart
- Personnel needed / available
- Facilities needed / available
- Budget
- Outcomes

Title

- ❖Project title should be descriptive of the main focus, but no longer than necessary
- ❖NOT a detailed description – but still provide an accurate impression of the central focus

(For further information refer slide # 45)

Identifying information

- Describes the people and organizations involved in the research, and other summary information
- Names, titles, addresses, phone numbers
- People may be grouped: Project Leader, Cooperators, students (committee members) or primary investigators and co- investigators, their qualifications, research experience etc
- Total budget amount, dates …

Introduction

- The problem proposed to be studied is introduced in this section
- It should help the reader to acquaint with the topic
- Introduction should be short about one or two pages
- The problem should be stated in such a way that it's importance and relevance is realized by any one who reads it

Background
(Review of Literature)

- Provides the base of knowledge on what is known about the problem to be studied
- This section reflects extensive review of literature done by the investigator
- In this section what is already known about the topic is written including the lacunae
- Just quoting the literature verbatim will not serve the purpose
- It is important to make it coherent, relevant and easily readable knowledge

Background
(Review of Literature)

- It helps the investigator to gain good knowledge in that field of inquiry
- It also helps the investigator to have insight on different methodologies that could be applied

Background
(Review of Literature)

- This is NOT just a summary of literature

Show how your project:

- Literature SUPPORTS your hypothesis
  - EXTENDS previous work
  - AVOIDS previous mistakes
- IS UNIQUE to previously followed paths

Background
(Review of Literature)

❖ Selecting Sources
- √ Select literature that is relevant or closely related to the problem and purpose
- √ Emphasize the primary sources
- √ Use secondary sources selectively
- √ Concentrate on scholarly research articles
- √ Discuss your criteria for inclusion of articles

Background
(Review of Literature)

- Organize the review by topics or ideas, not by author
- Organize the review logically (least to most relevant – evolution of topic –by key variables)
- Discuss major studies/theories individually and minor studies with similar results or limitation as a group

Background
(Review of Literature)

- Adequately criticize the design and methodology of important studies so readers can draw their own conclusions
- Compare and contrast studies.
- Note for conflicting and inconclusive results
- Explicitly show the relevance of each to the problem statement

Example

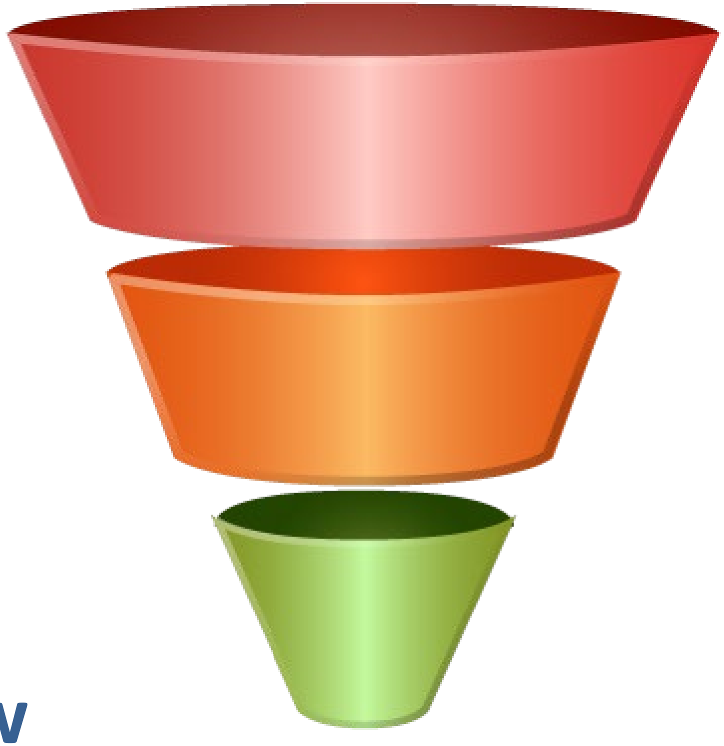
Authors	Study parameter	Experimental response	Optimization method
(Lin et al. 2013)	N atom and calcination temperature	XRD, BET, TG-DTA, TEM, optical property, and ethylene photooxidation	One parameter at a time approach
(Dunnill et al. 2011)	Sintering temperatures and dopant concentrations.	UV-Visible-NIR, XRD, Raman, FT-IR and Rate of stearic acid destruction	None
(Yu et al. 2007)	Acidity of the solution of precursor, calcination temperature,	photodegradation of methylene blue (MB), XRD, and TEM	One parameter at a time approach
(Nosaka et al. 2005)	Nitrogen source compound, calcination temperature	XRD, XPS, DRS, and decomposition of propanol	None
(Wang et al. 2011)	Source of precursor	photocatalytic oxida tion of propylene, XRD, XPS, DRS, and ESR	None

Example cont…

(Qin et al. 2008)	Ti/N molar ratio	TG-DSC, XRD, BET, TEM, DSC and photodegradation of Methyl orange (MO) and 2-mercaptobenzothiazole (MBT)	None
(Huang et al. 2005)	Calcination temperature	XRD, DRS, DT-TGA,DSC, and photocatalytic oxidation of methylene orange	None
(Ananpattarachai et al. 2009)	Types of nitrogen dopants	XRD, TEM, UV–vis spectra,XPS, and degradation of 2-chlorophenol (2-CP)	One parameter at a time approach
(Kuo et al. 2011)	Nitrogen sources, nitrogen source concentration, stirring time, and calcined temperature	photodecolorization of methyl blue (MB)	Taguchi method with an L9 orthogonal array
This report	N/Ti molar ratio, calcination temperature, calcination time	XRD, BET, photo decolorization of methyl blue (MB)	Response surface methodology, Box-Behnken design

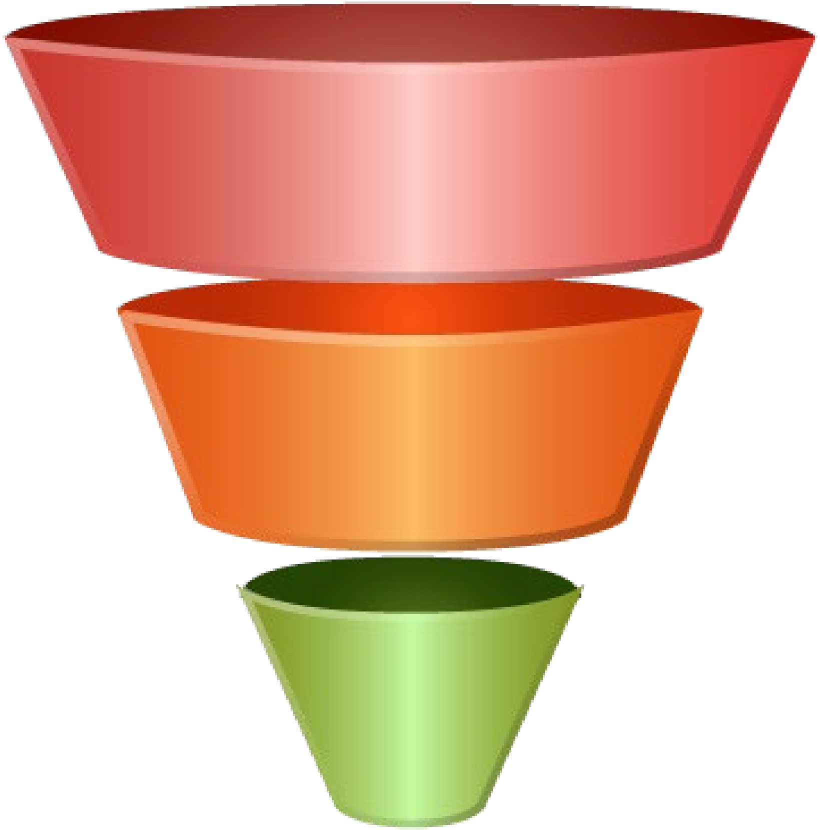
The narrative of a good literature review

Reader knows



Reader doesn't know

The narrative of a good literature review



Introduce the field: broad focus

Focus on certain aspects in field of interest

End with gap analysis

Problem statement

- ❖ From the literature review, gap analysis can be conducted in order to see how the propose research would fill in the gap in the area of research.
- ❖ How does the proposed research relate to the existing knowledge in the area.
- ❖ Explicitly state the significance of your purpose or the rationale for your study. A significant research is one that:
  - √ Develops knowledge of an existing practise
  - √ Develops theory
  - √ Expands the current knowledge or theory base
  - √ Advances current research methodology
  - √ Related to a current technological issue
  - √ Exploratory research on an unexamined issue

Problem statement

❖ Short SO WHAT statement

❖ Purpose
- Blueprint for your literature review
- Focus your committee at the beginning
- Keep them on track throughout your proposal

Problem statement

- Often, a two step procedure:
  1. Develop a general perspective of the broad problem area
  2. Focus on the part of the problem area to be studied, within resource constraints of the project
- This is the reason (justification) for the research.

Example

- "…. In most previous studies, it has been tried to investigate the effects of sol–gel synthesis parameters on the preparation of N-doped TiO_2 using the conventional "one-parameter-at-a-time approach". Although this approach is widely acceptable, it has a limitation in estimating the interaction effects between the factors and lacks a predictive capability.
  …. In this paper, optimization of some of the significant sol–gel synthesis parameters by using BBD is reported. "

Objectives

The purpose of this research is to……

❖Aims

- short but general statement of intent

❖Objectives

- very specific statements that define the practical steps you will take to achieve your aim(s)

Objectives

- This is a very important and pivotal section and everything else in the study is centered around it
- The objective of the proposed study should be stated very clearly
- The objective stated should be specific, achievable and measurable
- Too many objectives to be avoided
- Even just one clearly stated relevant objective for a study would be good enough
- If there is more than one objective the objectives can be presented in the appropriate order of importance

Research methodology

- Research methodology is a way to systematically solve the research problem. It may be understood as a science of studying how research is done scientifically
- It is necessary for the researcher to know not only the research methods/techniques but also the methodology.
- It is essential to discuss procedures clearly and completely with considerable amount of details

Research methodology

- ❖ Study design
- ❖ Study population / Sampling specifications
- ❖ Sample size needed
- ❖ Instrumentation
- ❖ Specific procedures
- ❖ Etc…

Study design

Definition: A study design is a specific plan or protocol for conducting the study, which allows the investigator to translate the conceptual hypothesis into an operational one.

- The study design should be clearly stated
- The study design to be used should be appropriate for achieving the objective of the study

Study population / Sample specifications

- It is important to describe which would be the study population
- How study subjects would be selected, randomization process and other details should be given

Sample size

- It is important to mention in the protocol what would be the minimum sample required and how it is arrived
- Determination of sample size is a bargain between precision and the price (Resources & expenses involved)

Description of process

- Proposal should include the details of all process to be adopted in the study
- How exposures, outcome variables and other variables are going to be measured should be described in detail
- A brief description of how the data will be processed and use of statistical package if any should be given
- What statistical tests of significance would be used?

Time Frame & Work Schedule

- The proposal should include the sequence of tasks to be performed, the anticipated length of time required for its completion and the personnel required
- It can be presented in tabular or graphic form (Gantt chart)
- Flow charts and other diagrams are often useful for highlighting the sequencing and interrelationship of different activities in the study

Facilities

- The proposal should also include the important facilities required / available for the study namely computers, laboratories, special equipment etc

Budget

- ❖Give you an appreciation of research costs
- ❖Prevents you from overspending!
- ❖Provide specific explanations for:
  - – Need for specific technologies
  - – Need for other financial requests (e.g. conference, instrumentation, staff, bursaries etc).
  - – Do you really need this kit?

Budget

- ❖ The budget translates project activities into monetary terms
- ❖ It is a statement of how much money will be required to accomplish the various tasks

Major items

- Payment for external laboratory analysis
- Travel (material collection)
- Labor cost
- Purchase of equipment
- Printing ,data storage
- Consultancy charges
- Institutional overheads

Outcomes

- What do you expect the results to be?
- Measurable
  - E.g. you will get a degree
  - New patent / paper
- Qualitative
  - Contribute understanding to subject / new technology / application

Research Methods and Experimental Design

Lecture 5: Experimental Design : Experimental Design and Analysis

Introduction

- It is important to obtain **maximum realistic** information with the **minimum number** of **well designed experiments**.
- An experimental program recognizes the major "**factors**" that affect the **outcome** of the experiment.
- The **factors** may be **identified** by **looking at all the quantities** that may **affect** the outcome of the experiment.
- The most important among these may be identified using:
  - A **few exploratory experiments** or
  - From **past experience** or
  - based on some **underlying theory** or **hypothesis**.

Engineering Experiments

❖Some of the **objectives**

- ❖ **Reduce time** to design/develop new products & processes
- ❖ **Improve performance** of existing processes
- ❖ **Improve reliability** and performance of products
- ❖ Achieve product & process **robustness**
- ❖ **Evaluation** of materials, design alternatives, **setting** component & system tolerances, etc.

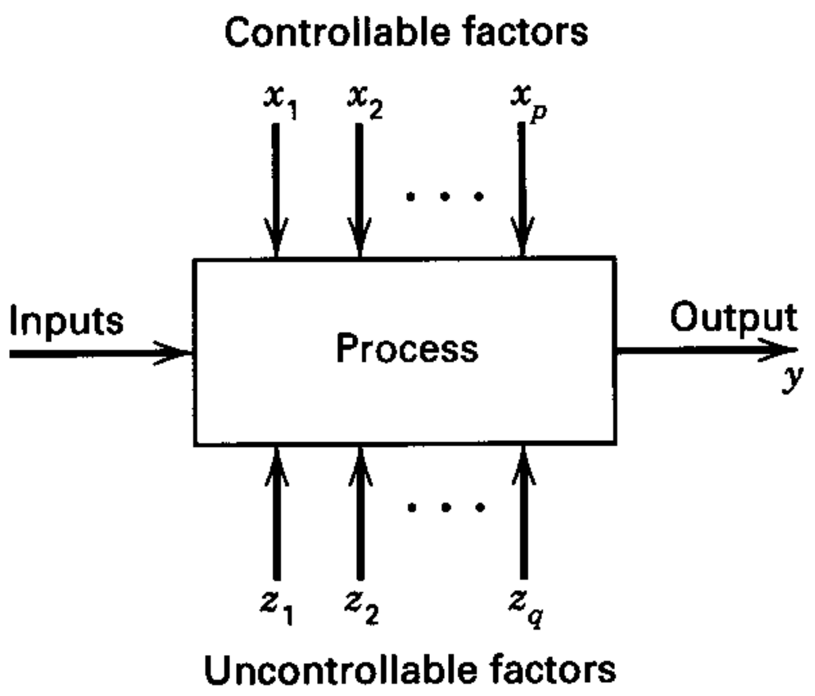


Figure 1-1 General model of a process or system.

Design of Experiments (DOE)

- A **statistics-based approach** to design experiments
- A **methodology** to achieve a **predictive knowledge** of a **complex**, **multi-variable process** with the **fewest acceptable trials**.
- An **optimization** of the experimental process itself

Design of Experiments (DOE)

- An experiment is a test or a series of tests
- Experiments are used widely in the engineering world
  - Process characterization & optimization
  - Evaluation of material properties
  - Product design & development
  - Component & system tolerance determination
- "All experiments are designed experiments, some are **poorly designed**, some are **well-designed**"

Some major players in DOE

- ❖ Sir Ronald A. Fisher - pioneer
  - – invented ANOVA and used of statistics in experimental design while working at Rothamsted Agricultural Experiment Station, London, England.
- ❖ George E. P. Box - married Fisher's daughter
  - – still active (86 years old)
  - – developed response surface methodology (1951)
  - – plus many other contributions to statistics
- ❖ Others
  - – Raymond Myers, J. S. Hunter, W. G. Hunter, Yates, Montgomery, Finney, etc..

Four eras of DOE

- The **agricultural** origins, 1918 – 1940s
  - R. A. Fisher & his co-workers
  - Profound impact on agricultural science
  - Factorial designs, ANOVA
- The **first industrial** era, 1951 – late 1970s
  - Box & Wilson, response surfaces
  - Applications in the chemical & process industries
- The **second industrial** era, late 1970s – 1990
  - Quality improvement initiatives in many companies
  - Taguchi and robust parameter design, process robustness
- The **modern** era, beginning circa 1990
  - Wide use of computer technology in DOE
  - Expanded use of DOE in Six-Sigma and in business
  - Use of DOE in computer experiments

Strategy of Experimentation

- Strategy of experimentation
  - Best guess approach (trial and error)
    - can continue indefinitely
    - cannot guarantee best solution has been found
  - One-factor-at-a-time (OFAT) approach
    - inefficient (requires many test runs)
    - fails to consider any possible interaction between factors
  - Factorial approach (invented in the 1920's)
    - Factors varied together
    - Correct, modern, and most efficient approach
    - Can determine how factors interact
    - Used extensively in industrial R and D, and for process improvement.

Statistical Design of Experiments

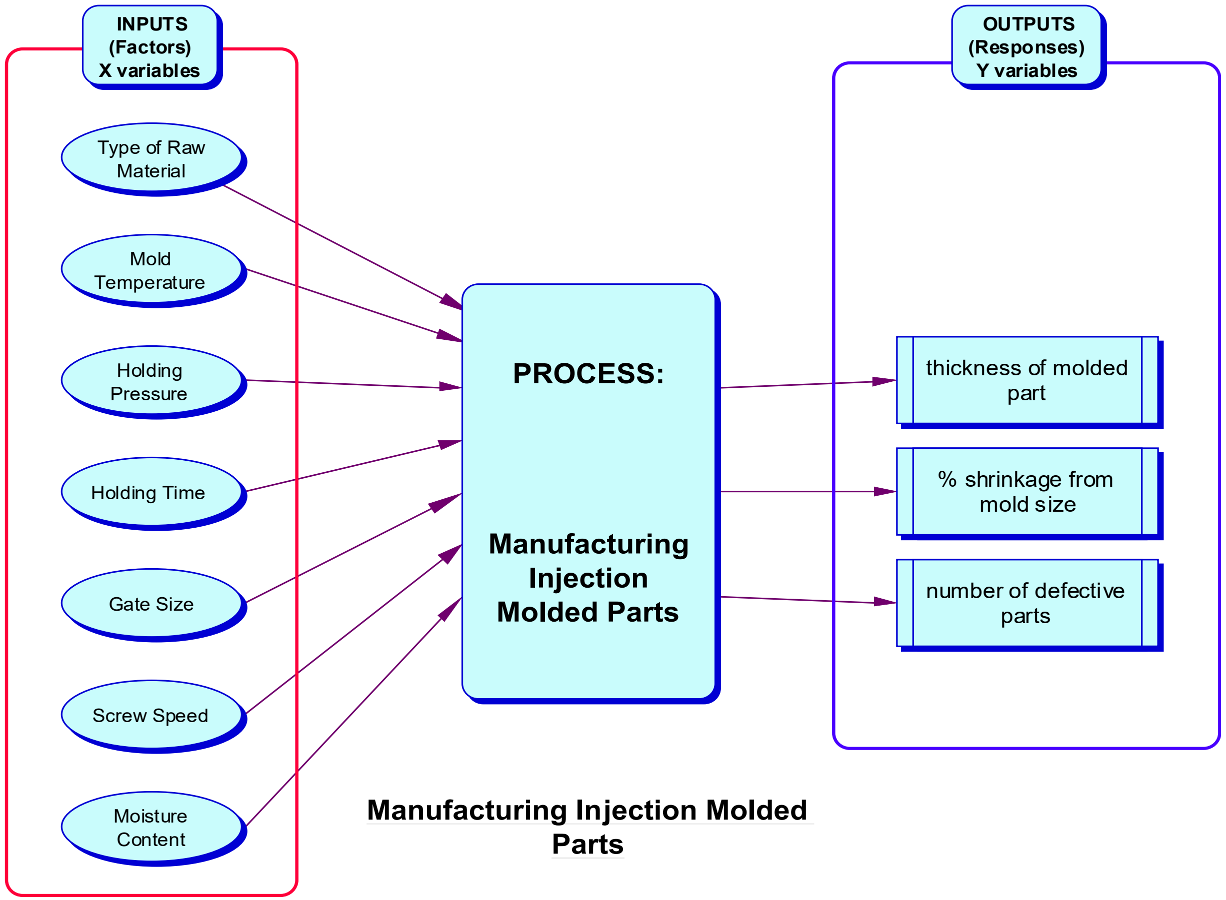
- ❖DOE is a methodology for systematically applying statistics to experimentation.
- ❖DOE lets experimenters develop a mathematical model that predicts how input variables *interact* to create output variables or responses in a process or system.
- ❖DOE can be used for a wide range of experiments for various purposes including nearly all fields of engineering and even in business marketing.
- ❖Use of statistics is very important in DOE and the basics are covered in a first course in an engineering program.

Statistical Design of Experiments

- In general, by using DOE, we can:
  - Learn about the process we are investigating
  - Screen important variables
  - Build a mathematical model
  - Obtain prediction equations
  - Optimize the response (if required)

- Statistical significance is tested using **ANOVA**, and the prediction model is obtained using **regression analysis.**

Example



Manufacturing Injection Molded Parts

Basic Principles

❖Statistical design of experiments (DOE)

– the process of planning experiments so that appropriate data can be analyzed by statistical methods that results in valid, objective, and meaningful conclusions from the data

– involves two aspects: design and statistical analysis

Basic Principles

- Every experiment involves a sequence of activities:
  - Conjecture - hypothesis that motivates the experiment
  - Experiment - the test performed to investigate the conjecture
  - Analysis - the statistical analysis of the data from the experiment
  - Conclusion - what has been learned about the original conjecture from the experiment.

Three basic principles of Statistical DOE

- **Replication**
  - allows an estimate of experimental error
  - allows for a more precise estimate of the sample mean value
- **Randomization**
  - cornerstone of all statistical methods
  - "average out" effects of extraneous factors
  - reduce bias and systematic errors
- **Blocking**
  - increases precision of experiment
  - "factor out" variable not studied

Guidelines for Designing Experiments

- ❖ **Recognition of and statement of the problem**
  - – need to develop all ideas about the objectives of the experiment - get input from everybody - use team approach.
- ❖ **Choice of factors, levels, ranges, and response variables.**
  - – Need to use engineering judgment or prior test results.
- ❖ **Choice of experimental design**
  - – sample size, replicates, run order, randomization, software to use, design of data collection forms.

Guidelines for Designing Experiments

- **Performing the experiment**
  - vital to monitor the process carefully. Easy to underestimate logistical and planning aspects in a complex R and D environment.
- **Statistical analysis of data**
  - provides objective conclusions - use simple graphics whenever possible.
- **Conclusion and recommendations**
  - follow-up test runs and confirmation testing to validate the conclusions from the experiment.
- Do we need to add or drop factors, change ranges, levels, new responses, etc.. ???

Using Statistical Techniques in Experimentation - things to keep in mind

- **Use non-statistical knowledge of the problem**
  - physical laws, background knowledge
- **Keep the design and analysis as simple as possible**
  - Don't use complex, sophisticated statistical techniques
  - If design is good, analysis is relatively straightforward
  - If design is bad - even the most complex and elegant statistics cannot save the situation
- **Recognize the difference between practical and statistical significance**
  - statistical significance ≠ practically significance

Using Statistical Techniques in Experimentation - things to keep in mind

- **Experiments are usually iterative**
  - unwise to design a comprehensive experiment at the start of the study
  - may need modification of factor levels, factors, responses, etc.. - too early to know whether experiment would work
  - use a sequential or iterative approach
  - should not invest more than **25%** of resources in the initial design.
  - Use initial design as learning experiences to accomplish the final objectives of the experiment.

Factorial v.s. OFAT

- Factorial design - experimental trials or runs are performed at all possible combinations of factor levels in **contrast** to OFAT experiments.

- **Factorial** and **fractional factorial** experiments are among the most useful **multi-factor experiments** for engineering and scientific investigations.

Factorial v.s. OFAT

- The ability to gain competitive advantage requires extreme care in the design and conduct of experiments. **Special attention** must be paid to **joint effects** and estimates of variability that are provided by factorial experiments.

- **Full** and **fractional** experiments can be conducted using a **variety of statistical designs**. The design selected can be chosen according to **specific requirements** and **restrictions of the investigation**.

Special Terminology : Design of Experiments

- **Experiment:** Process of collecting sample data
- **Design of Experiment:** Plan for collecting the sample
- **Response Variable:** Variable measured in experiment (outcome, y)
- **Experimental Unit:** Object upon which the response y is measured
- **Factors:** Independent Variables
- **Level:** The value assumed by a factor in an experiment
- **Treatment:** A particular combination of levels of the factors in an experiment

Special Terminology : Design of Experiments

- **Replication:** Completely re-run experiment with same input levels Used to determine impact of measurement error
- **Interaction:** Effect of one input factor depends on level of another input factor

Volume and "Noise"

- **Volume:** quantity of information in an experiment
  - *Increase* with larger sample size, selection of treatments such that the observed values (*y*) provide information on the parameters of interest
- **Noise:** experimental error
  - *Reduce* by assigning treatments to experimental units

Factorial Designs

❖ In a factorial experiment, **all possible combinations** of factor levels are tested

One-factor-at-a-time experiments (OFAT)

- ❖ OFAT is a prevalent, but potentially disastrous type of experimentation commonly used by many engineers and scientists in both industry and academia.
- ❖ Tests are conducted by systematically changing the levels of one factor while holding the levels of all other factors fixed. The "optimal" level of the first factor is then selected.
- ❖ Subsequently, each factor in turn is varied and its "optimal" level selected while the other factors are held fixed.

One-factor-at-a-time experiments (OFAT)

- OFAT experiments are regarded as easier to implement, more easily understood, and more economical than factorial experiments. Better than trial and error.
- OFAT experiments are believed to provide the optimum combinations of the factor levels.
- Unfortunately, each of these presumptions can generally be shown to be false except under very special circumstances.
- The key reasons why OFAT should not be conducted except under very special circumstances are:
  - ***Do not provide adequate information on interactions***
  - ***Do not provide efficient estimates of the effects***

Factorial vs OFAT (2-levels only)

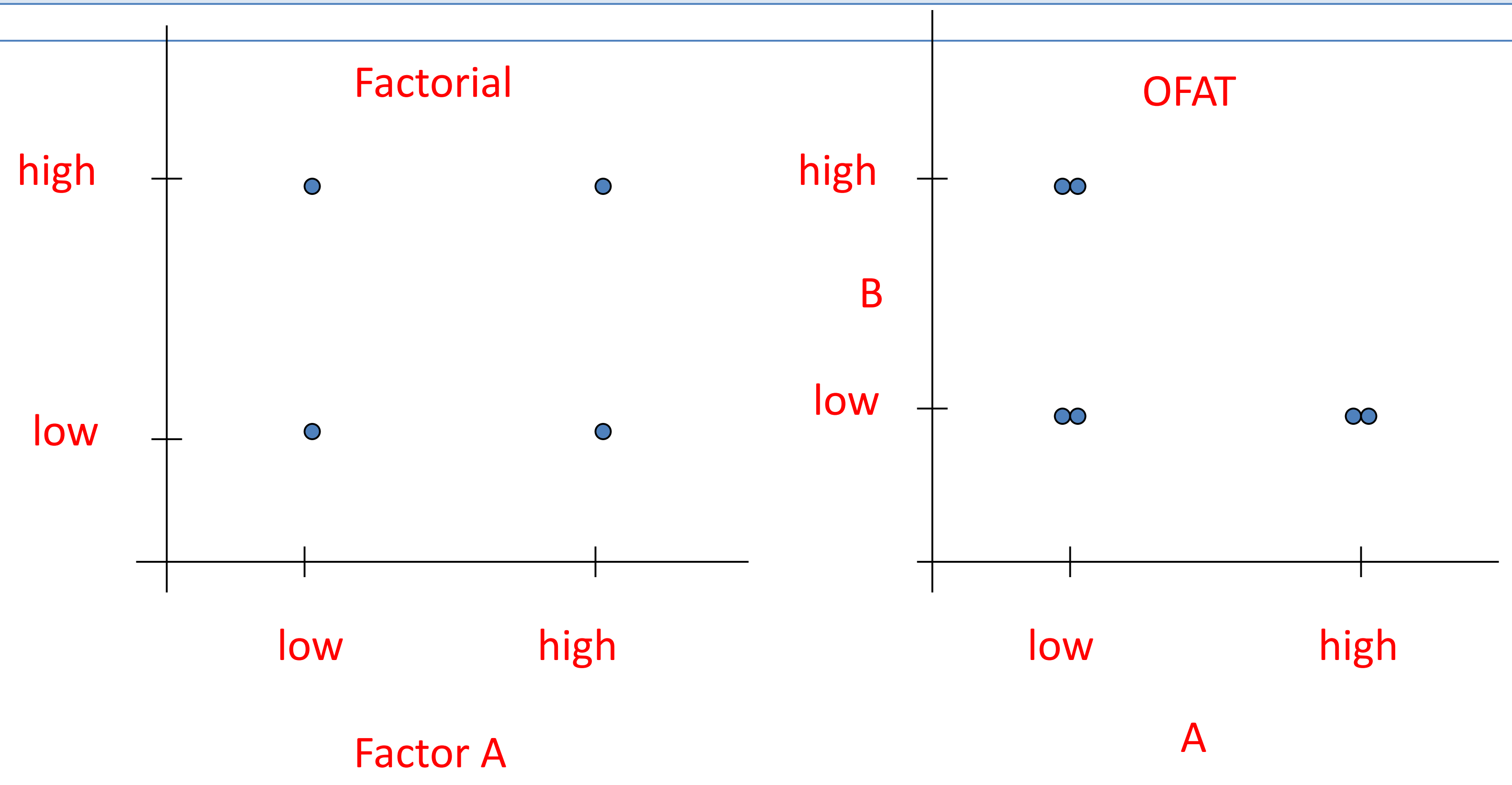
Factorial

- 2 factors: 4 runs
  - 3 effects
- 3 factors: 8 runs
  - 7 effects
- 5 factors: 32 or 16 runs
  - 31 or 15 effects
- 7 factors: 128 or 64 runs
  - 127 or 63 effects

OFAT

- 2 factors: 6 runs
  - 2 effects
- 3 factors: 16 runs
  - 3 effects
- 5 factors: 96 runs
  - 5 effects
- 7 factors: 512 runs
  - 7 effects

Example: Factorial vs OFAT



E.g. Factor A: Reynold's number, Factor B: k/D

Research Methods and Experimental Design

Lecture 6: Experimental Design: Brief Introduction-Probability, Sampling, descriptive statistics

Probability

Sample Spaces and Events

❖ Example

If we measure the current in a thin copper wire, we are conducting an experiment.

+++day-today repetitions of the measurement can differ slightly

+++small variations in variables that are not controlled in our experiment

changes in ambient temperatures, slight variations in gauge and small impurities in the chemical composition of the wire, and current source drift

Sample Spaces and Events…

- this experiment (as well as many we conduct) is said to have a **random** component

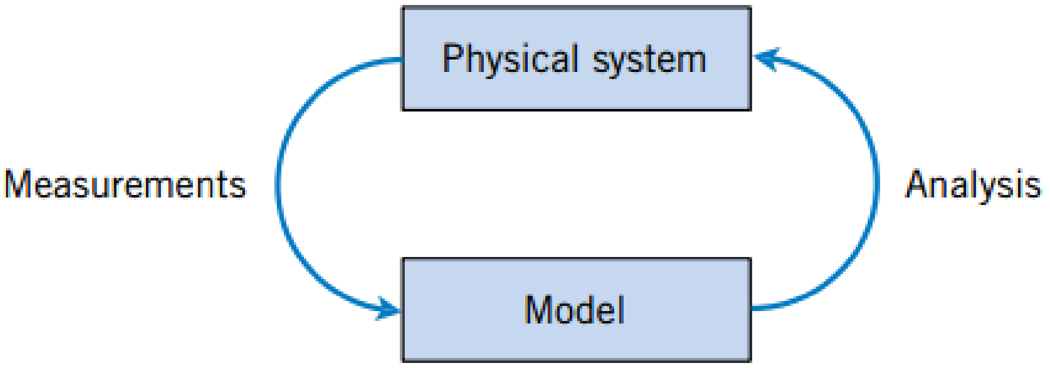
no matter how carefully our experiment is designed and conducted, the variation is almost always present, and its magnitude can be large enough that the important conclusions from our experiment are not obvious

modelling and analysing experimental results

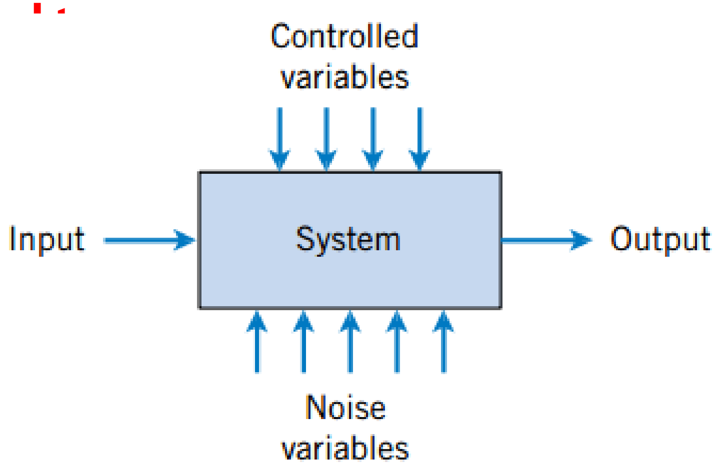
❖ Our goal is to

– understand, quantify, and model the type of variations that we often encounter

– Make informed judgments from our res



Continuous iteration between model and physical system.



Noise variables affect the transformation of inputs to outputs.

Definitions

❖Random Experiment

An experiment that can result in different outcomes, even though it is repeated in the same manner every time, is called a **random experiment.**

❖Sample Spaces

The set of all possible outcomes of a random experiment is called the **sample space** of the experiment. The sample space is denoted as S.

❖Discrete and Continuous Sample Spaces

A sample space is **discrete** if it consists of a finite or countable infinite set of outcomes. A sample space is **continuous** if it contains an interval (either finite or infinite) of real numbers.

Continuous Random Variables and Probability Distributions

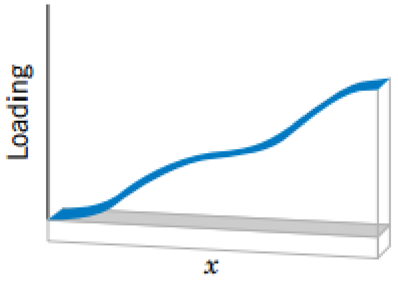
- Continuous random variable is a random variable with an interval (either finite or infinite) of real numbers for its range
- Example
- dimensional length is measured on a manufactured part selected from a day's production
  - the number of possible values of X (random variable) is uncountably infinite

PROBABILITY DISTRIBUTIONS AND PROBABILITY DENSITY FUNCTIONS

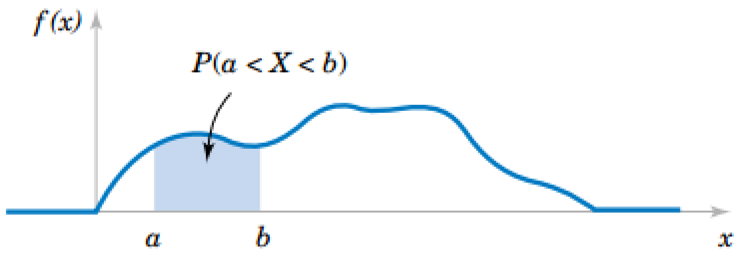
- ❖Density functions are commonly used in engineering to describe physical systems

For example, consider the density of a loading on a long, thin beam. For any point *x* along the beam, the density can be described by a function (in grams/cm). Intervals with large loadings correspond to large values for the function. The total loading between points *a* and *b* is determined as the integral of the density function from *a* to *b*. This integral is the area under the density function over this interval, and it can be loosely interpreted as the sum of all the loadings over this interval.

Density function of a loading on a long, thin beam



Probability determined from the area under f(x)



Probability Density Function

For a continuous random variable X, a **probability density function** is a function such that

(1) $f(x) \geq 0$

(2) $\displaystyle\int_{-\infty}^{\infty} f(x)\, dx = 1$

(3) $\displaystyle P(a \leq X \leq b) = \int_{a}^{b} f(x)\, dx =$ area under $f(x)$ from a to b

for any a and b

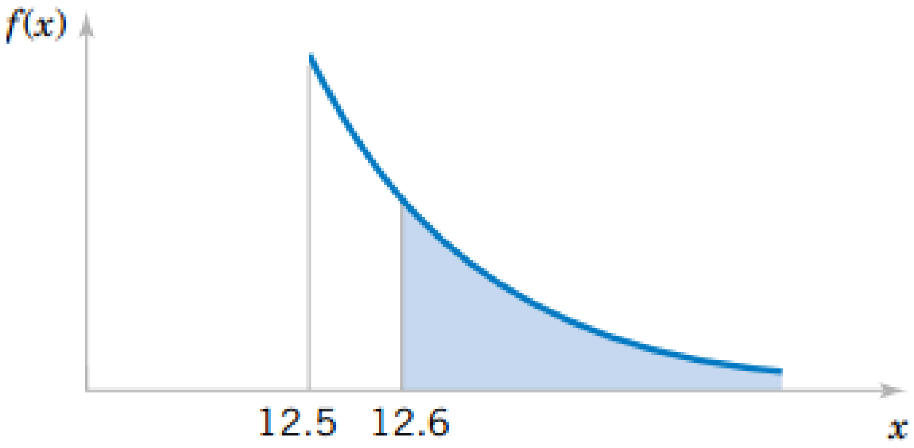
Example

- Let the continuous random variable *X* denote the diameter of a hole drilled in a sheet metal component. The target diameter is 12.5 millimeters. Most random disturbances to the process result in larger diameters. Historical data show that the distribution of *X* can be modeled by a probability density function
- If a part with a diameter larger than 12.60 millimeters is scrapped, what proportion of parts is scrapped?

Solution

$$P(X > 12.60) = \int_{12.6} f(x)\,dx = \int_{12.6} 20e^{-20(x-12.5)}\,dx$$

$$= -e^{-20(x-12.5)}\Big|_{12.6}^{\infty} = 0.135$$



Probability density function

MEAN AND VARIANCE OF A CONTINUOUS RANDOM VARIABLE

Suppose X is a continuous random variable with probability density function $f(x)$. The **mean** or **expected value** of X, denoted as μ or $E(X)$, is

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

The **variance** of X, denoted as $V(X)$ or σ^2, is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

The **standard deviation** of X is $\sigma = \sqrt{\sigma^2}$.

Previous Example

$$E(X) = \int_{12.5}^{\infty} xf(x)\,dx = \int_{12.5}^{\infty} x\,20e^{-20(x-12.5)}\,dx$$

Integration by parts can be used to show that

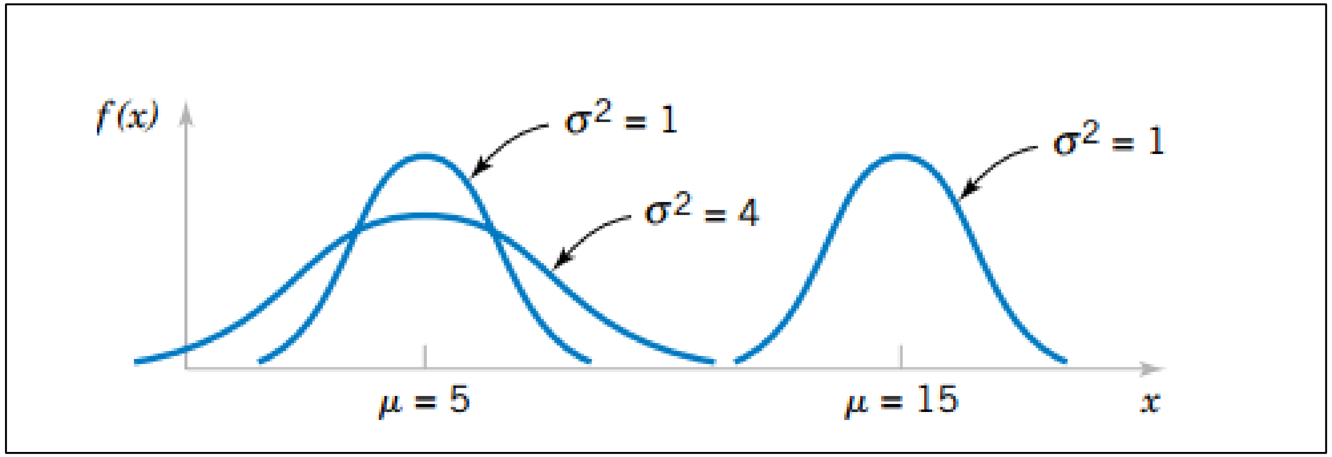
$$E(X) = -xe^{-20(x-12.5)} - \frac{e^{-20(x-12.5)}}{20}\Bigg|_{12.5}^{\infty} = 12.5 + 0.05$$

$$= 12.55$$

NORMAL DISTRIBUTION

- the most widely used model for the distribution of a random variable is a **normal distribution.**
- Whenever a random experiment is replicated, the random variable that equals the average (or total) result over the replicates tends to have a normal distribution as the number of replicates becomes large.
- a normal distribution is also referred to as a **Gaussian distribution**

NORMAL DISTRIBUTION



Normal probability density functions for selected values of the parameters μ and σ^2

Random variables with different means and variances can be modeled by normal probability density functions with appropriate choices of the center and width of the curve. The value of $E(X)=\mu$ determines the center of the probability density function and the value of $V(X)=\sigma^2$ determines the width

Normal Distribution

A random variable X with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty$$

is a **normal random variable** with parameters μ, where $-\infty < \mu < \infty$, and $\sigma > 0$. Also,

$$E(X) = \mu \quad \text{and} \quad V(X) = \sigma^2$$

and the notation $N(\mu, \sigma^2)$ is used to denote the distribution.

EXAMPLE

- Assume that the current measurements in a strip of wire follow a normal distribution with a mean of 10 milliamperes and a variance of 4 $(\text{milliamperes})^2$. What is the probability that a measurement exceeds 13 milliamperes?

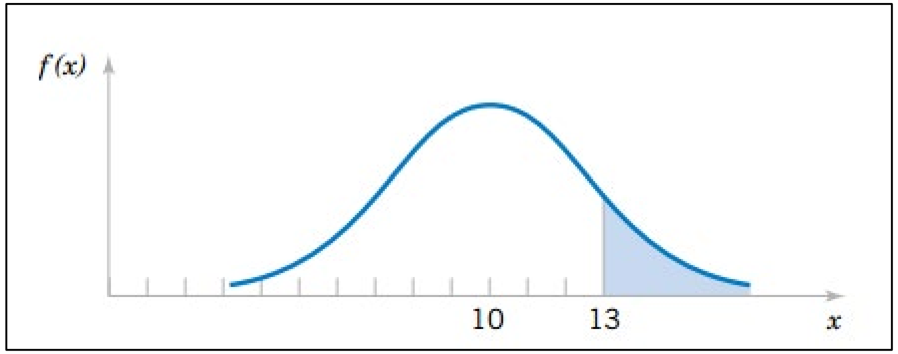
- Let X denote the current in milliamperes. The requested probability can be represented as This probability $P(X > 13)$.

Unfortunately, there is no closed-form expression for the integral of a normal probability density function, and probabilities based on the normal distribution are typically found numerically or from a table (that we will later introduce)

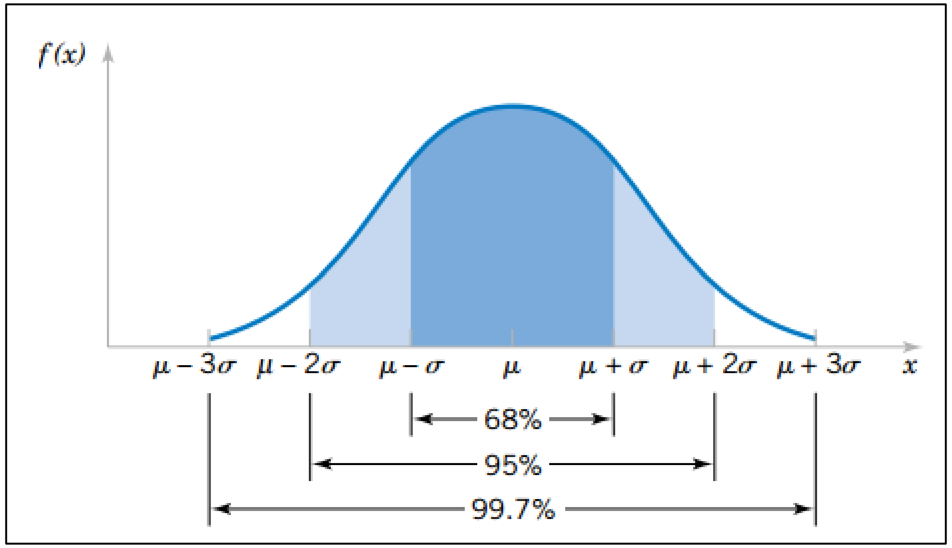
Normal distribution

$$P(\mu - \sigma < X < \mu + \sigma) = 0.6827$$
$$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9545$$
$$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$$

from the symmetry of Because $f(x)$ is positive for all x, this model assigns some probability to each interval of the real line.



Probability that $X > 13$ for a normal random variable with $\mu = 10$ and $\sigma^2 = 4$



Probabilities associated with a normal distribution.

Normal Distribution

- probability density function decreases as x moves farther from μ .
- Consequently, the probability that a measurement falls far from μ is small, and at some distance from μ the probability of an interval can be approximated as zero.
- The area under a normal probability density function beyond 3 σ from the mean is quite small.

Standard Normal Random Variable

A normal random variable with

$$\mu = 0 \quad \text{and} \quad \sigma^2 = 1$$

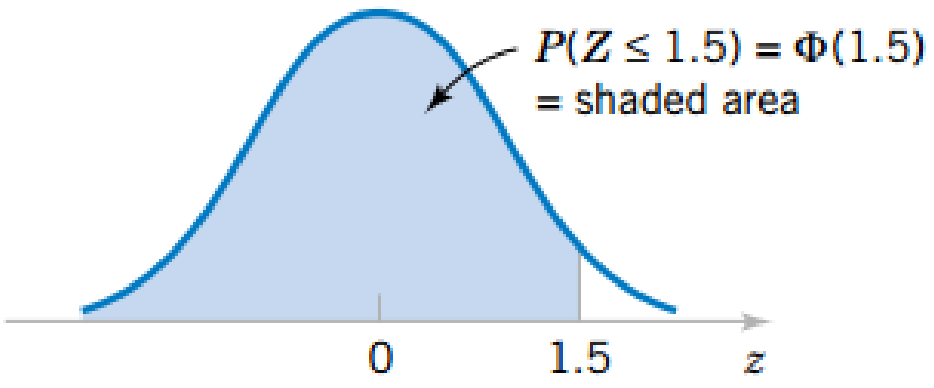
is called a **standard normal random variable** and is denoted as Z. The cumulative distribution function of a standard normal random variable is denoted as

$$\Phi(z) = P(Z \leq z)$$

EXAMPLE

- $P(Z \leq 1.5) = 0.93319$

- $P(Z \leq 1.53) = 0.93699$



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50398	0.51197
⋮		⋮		
1.5	0.93319	0.93448	0.93574	0.93699

practice

$P(Z > 1.26) = 1 - P(Z \le 1.26) = 1 - 0.89616$
$= 0.10384$

$P(Z < -0.86) = 0.19490.$

$P(Z > -1.37) = P(Z < 1.37) = 0.91465$

$P(-1.25 < Z < 0.37)$. This probability can be found from the difference of two areas, $P(Z < 0.37) - P(Z < -1.25)$. Now,

$P(Z < 0.37) = 0.64431$

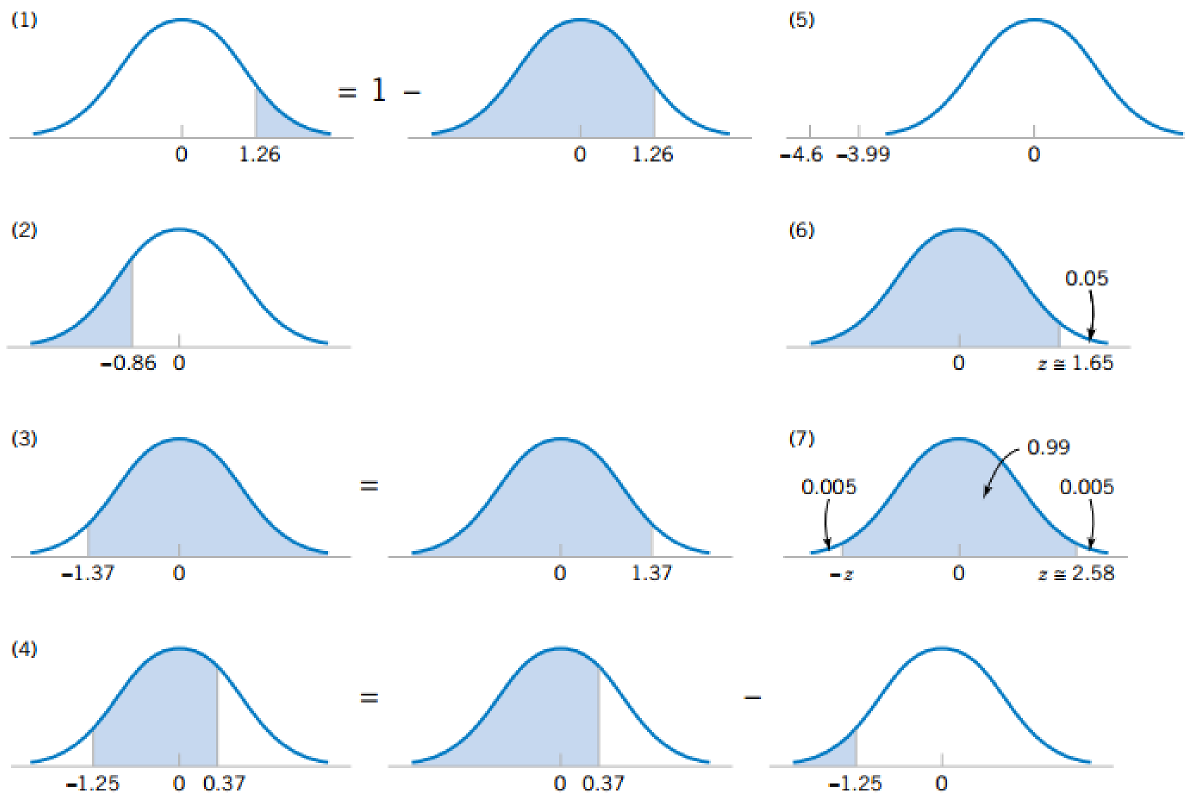
and

$P(Z < -1.25) = 0.10565$

Therefore,

$P(-1.25 < Z < 0.37) = 0.64431 - 0.10565$
$= 0.53866$

Graphical displays for standard normal distributions.



Standardizing a Normal Random Variable

If X is a normal random variable with $E(X) = \mu$ and $V(X) = \sigma^2$, the random variable

$$Z = \frac{X - \mu}{\sigma}$$

is a normal random variable with $E(Z) = 0$ and $V(Z) = 1$. That is, Z is a standard normal random variable.

Creating a new random variable by this transformation is referred to as **standardizing.**

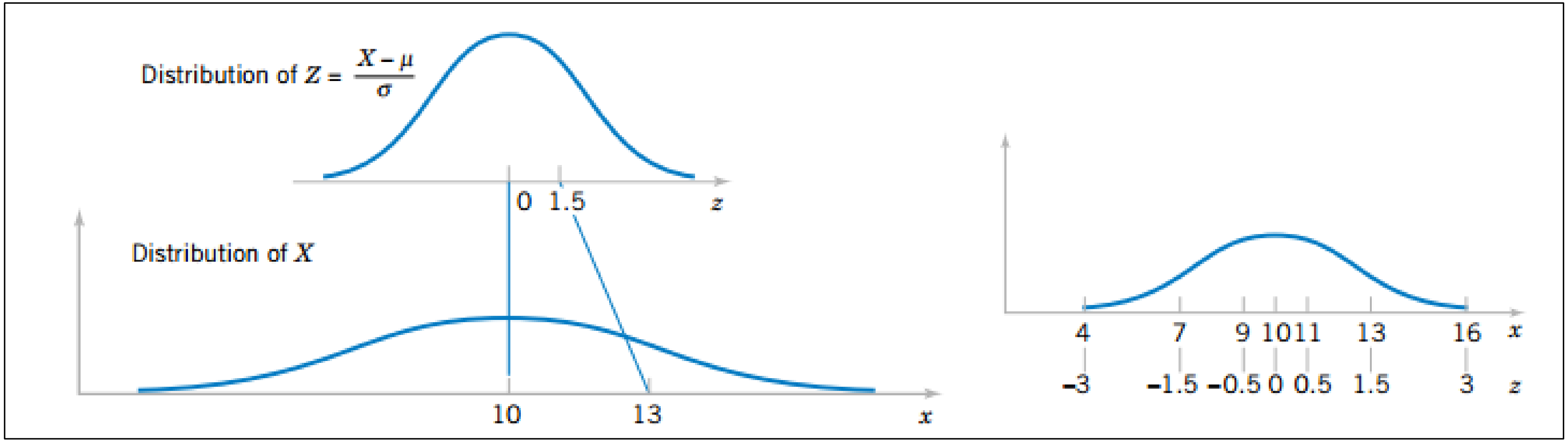
EXAMPLE

- Suppose the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 (milliamperes)^2. What is the
  proba
  millia

$$P(X > 13) = P(Z > 1.5) = 1 - P(Z \le 1.5) = 1 - 0.93319$$
$$= 0.06681$$

$$P(X > 13) = P\left(\frac{(X - 10)}{2} > \frac{(13 - 10)}{2}\right) = P(Z > 1.5)$$
$$= 0.06681$$

Standardizing a normal random variable.



Standardizing to Calculate a Probability

Suppose X is a normal random variable with mean μ and variance σ^2. Then,

$$P(X \leq x) = P\left(\frac{X - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}\right) = P(Z \leq z)$$

where Z is a **standard normal random variable,** and $z = \dfrac{(x - \mu)}{\sigma}$ is the **z-value** obtained by **standardizing** X. The probability is obtained by using Appendix Table III with $z = (x - \mu)/\sigma$.

EXAMPLE

- what is the probability that a current measurement is between 9 and 11 milliamperes?

$$
\begin{aligned}
P(9 < X < 11) &= P((9 - 10)/2 < (X - 10)/2 \\
&\quad < (11 - 10)/2) \\
&= P(-0.5 < Z < 0.5) = P(Z < 0.5) \\
&\quad - P(Z < -0.5) \\
&= 0.69146 - 0.30854 = 0.38292
\end{aligned}
$$

- Determine the value for which the probability that a current measurement is below this value is 0.98.

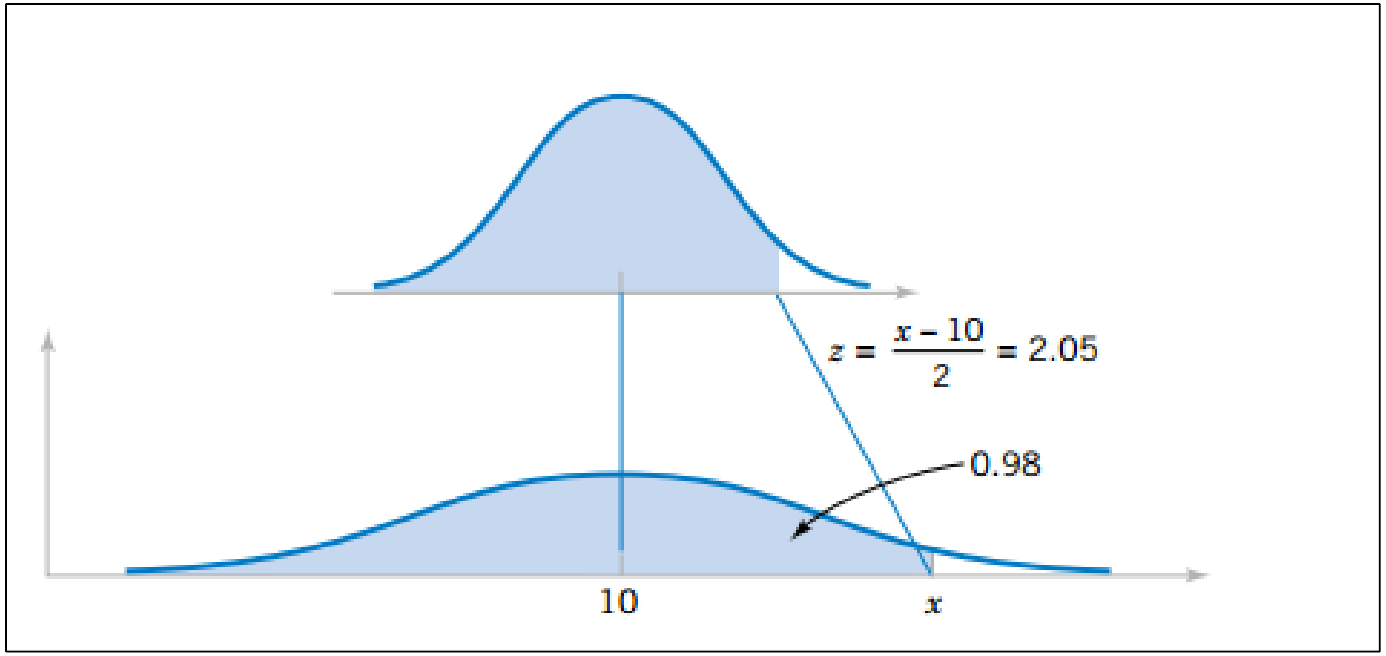
$$
\begin{aligned}
P(X < x) &= P((X - 10)/2 < (x - 10)/2) \\
&= P(Z < (x - 10)/2) \\
&= 0.98
\end{aligned}
$$

$$P(Z < 2.05) = 0.97982$$

Therefore, $(x - 10)/2 = 2.05$, and the standardizing transformation is used in reverse to solve for x. The result is

$$x = 2(2.05) + 10 = 14.1 \text{ milliamperes}$$

Example



Determining the value of *x* to meet a specified probability.

Exercise

- The compressive strength of samples of cement can be modelled by a normal distribution with a mean of 6000 kilograms per square centimetre and a standard deviation of 100 kilograms per square centimetre.
- (a) What is the probability that a sample's strength is less than 6250 Kg/cm^2?
- (b) What is the probability that a sample's strength is between 5800 and 5900 Kg/cm^2?
- (c) What strength is exceeded by 95% of the samples?

EXCEL EXAMPLE

- NORM.DIST
- NORM.S.DIST

.DIST gets you probability or height

- NORM.INV
- NORM.S.INV

.INV gets you a particular value

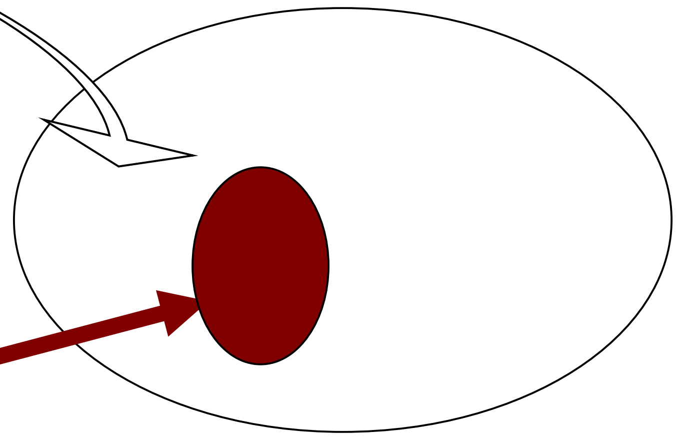
Sampling

Important statistical terms

Population:

a set which includes all measurements of interest to the researcher

(The collection of ***all*** responses, measurements, or counts that are of interest)



Sample:

A subset of the population

Why sampling?

Get information about large populations

- ❖Less costs
- ❖Less field time
- ❖More accuracy i.e. Can Do A Better Job of Data Collection
- ❖When it's impossible to study the whole population

Sampling

Target Population:

The population to be studied/ to which the investigator wants to generalize his results

Sampling Unit:

smallest unit from which sample can be selected

Sampling frame

List of all the sampling units from which sample is drawn

Sampling scheme

Method of selecting sampling units from sampling frame

Types of Sampling

- ❖Simple Random Sampling
- ❖Stratified Random Sampling
- ❖Cluster Sampling
- ❖Systematic Sampling
- ❖Representative Sampling (Can be stratified random or quota sampling)
- ❖Convenience or Haphazard Sampling
- ❖Sampling with Replacement vs. Sampling without Replacement

Representative Sample

- Sample should be representative of the target population
  - so you can generalize to population
- Random sampling
  - All members of pop have equal chance of being selected
  - Roll dice, flip coin, draw from hat

Errors in sample

- ❖Systematic error (or bias) Inaccurate response (information bias) Selection bias

- ❖Sampling error (random error)

Type 1 error

- The probability of finding a difference with our sample compared to population, and there really isn't one….
- Known as the α (or "type 1 error")
- Usually set at 5% (or 0.05)

Type 2 error

- The probability of not finding a difference that actually exists between our sample compared to the population…

- Known as the ß (or "type 2 error")

- Power is (1- ß) and is usually 80%

Sample size

❖ Sample size

Quantitative

$$n = \frac{Z^2 \sigma^2}{D^2}$$

Qualitative

$$n = \frac{Z^2 \pi(1-\pi)}{D^2}$$

EXAMPLE

A study is to be performed to determine a certain parameter in a community. From a previous study a sd of 46 was obtained.

If a sample error of up to 4 is to be accepted. How many subjects should be included in this study at 99% level of confidence?

Answer

$$n = \frac{Z^2 \sigma^2}{D^2}$$

$$n = \frac{2.58^2 \ \text{x} \ 46^2}{4^2} = 880.3 \sim 881$$

Types of Variables

- Quantitative
  - Measured in amounts
  - Ht, Wt, Test score
- Qualitative
  - Measured in categories
  - Gender, race, diagnosis

- Discrete:
  - separate categories
  - Letter grade
- Continuous:
  - infinite values in between
  - GPA

Types of Statistics

- Descriptive statistics:
  - Organize and summarize scores from samples
- Inferential statistics:
  - Infer information about the population based on what we know from sample data
  - Decide if an experimental manipulation has had an effect

Descriptive Statistics

Introduction

- ❖Presenting, Organizing, and summarizing data
- ❖The values that describe the characteristics of a sample or population
- ❖Determine if the sample is normally distributed (bell curve).Most statistical tests required the sample to have normally distributed.
- ❖Determine if the sample can be compared larger population
- ❖Are display as tables, charts, percentages, frequency distribution, and reported as measures of central tendency

Introduction

- Descriptive statistics are used to summarize data from individual respondents, etc.
- They help to make sense of large numbers of individual responses, to communicate the essence of those responses to others
- They focus on typical or average scores, the dispersion of scores over the available responses, and the shape of the response curve

Information about the sample

- Central tendency-the sample mean (average), median (midpoint), mode (most frequent occurring numbers)
- Measures of variability-range (the difference between the largest and smallest variables), variance (how far the numbers are spread-out), and standard deviation (how much variation exists from the average/mean)
- Skewness (how symmetrical the distribution of variables)
- Kurtosis (peakedness or flatness of the distribution)
- Shape (includes modality, outliers)

Measuring central tendency

Sample mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Population mean

$$\mu = \frac{\sum_{i=1}^{n} x_i}{N}$$

The mean is sensitive to extreme scores (outliers) in the sample

Measuring central tendency

When data are listed in order, the median is the point at which 50% of the cases are above and 50% below it.

The median is the better representative of the sample When scores are extrem. The median is not sensitive to extreme scores (outliers) in the sample

variance

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2 / n}{n-1}$$

standard deviation

- square root of variance:

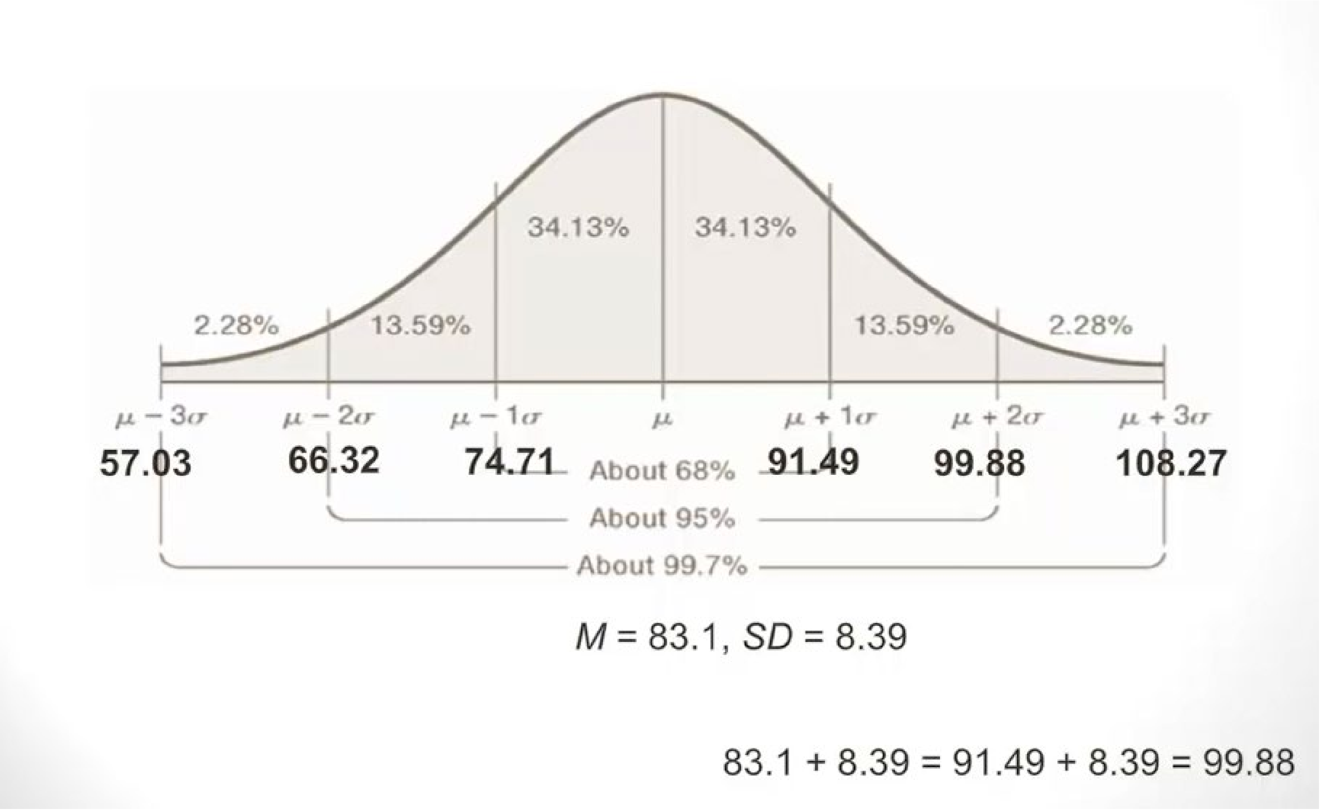
$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \qquad s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 / n}{n-1}}$$

Exampel

- List of chemical concentration taken from a reactor

69,77,77,77,84,85,85,87,92,98

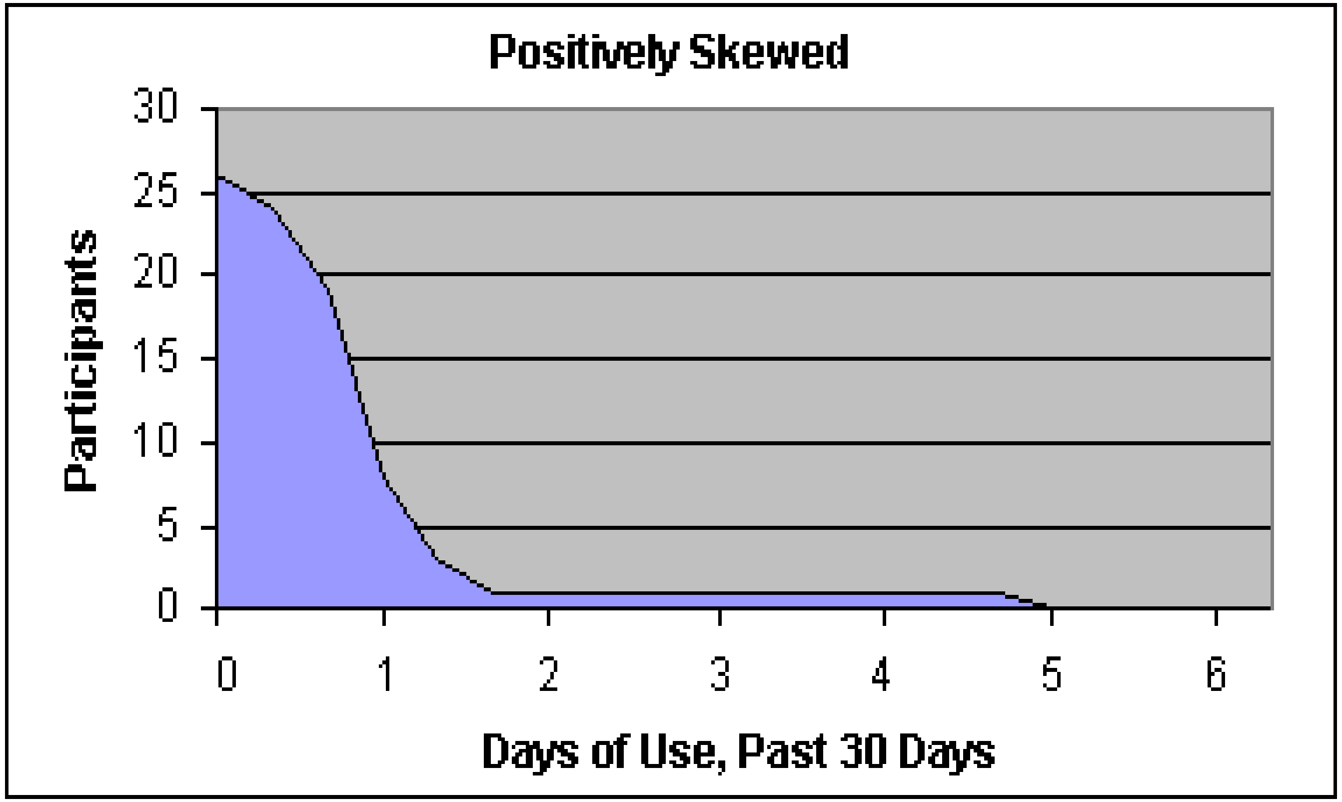
Normal Distribution



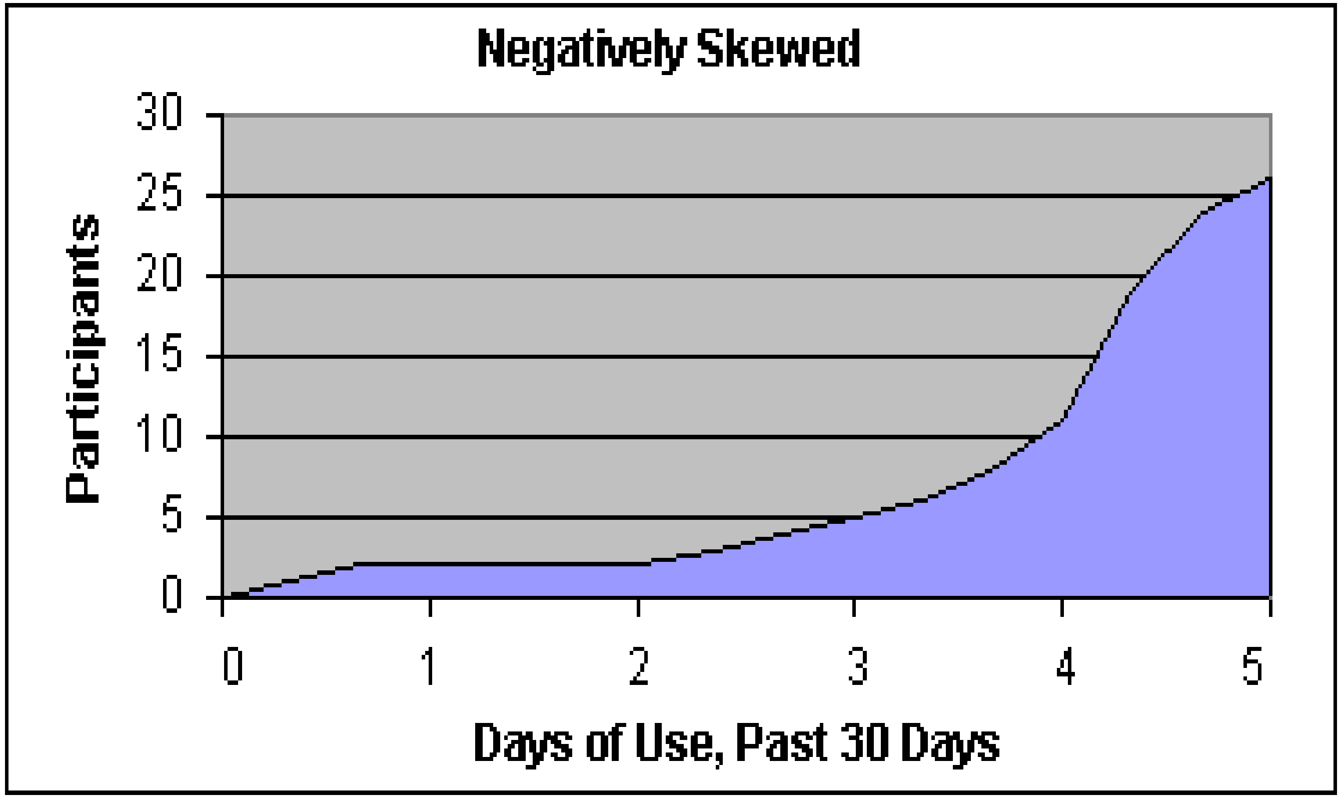
Skewness of distributions

- Measures look at how lopsided distributions are—how far from the ideal of the normal curve they are
- When the median and the mean are different, the distribution is skewed. The greater the difference, the greater the skew.
- Distributions that trail away to the left are negatively skewed and those that trail away to the right are positively skewed
- If the skewness is extreme, the researcher should either transform the data to make them better resemble a normal curve or else use a different set of statistics—nonparametric statistics—to carry out the analysis

Positively skewed

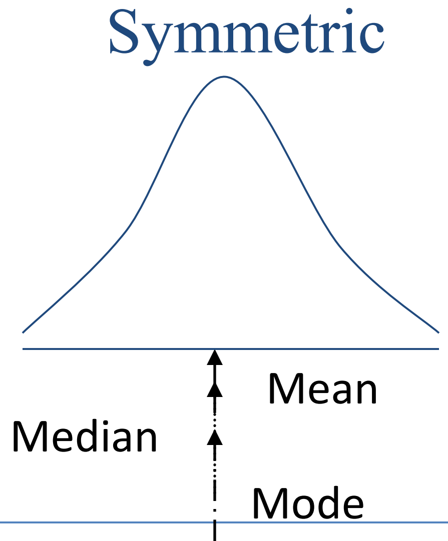


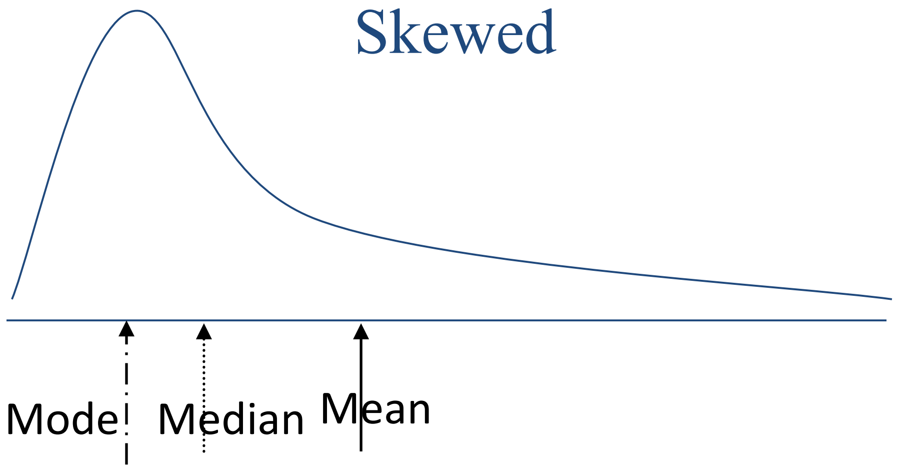
Negatively skewed



Mode

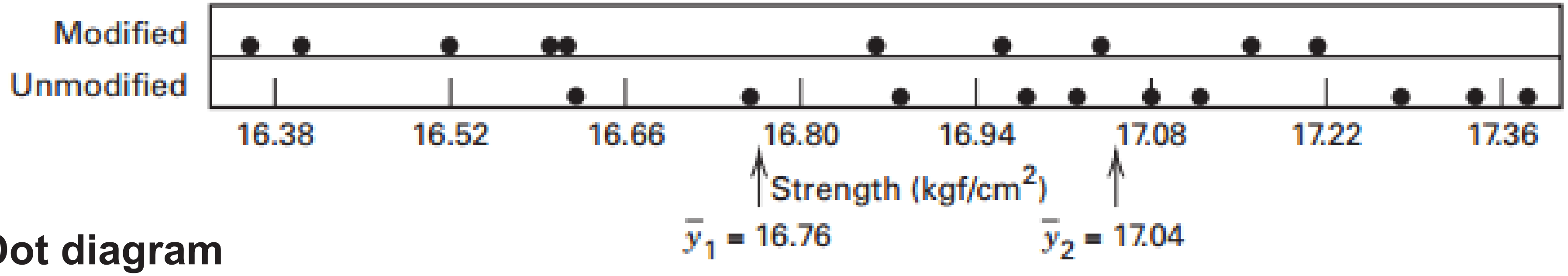
1. It may give you the most likely experience rather than the “typical” or “central” experience.
2. In symmetric distributions, the mean, median, and mode are the same.
3. In skewed data, the mean and median lie further toward the skew than the mode.





Graphical Description of Variability

- simple graphical methods often assist in analysing the data from an experiment.
- The **dot diagram**, illustrated in Figure below a very useful device for displaying a small body of data (say up to about 20 observations).



Dot diagram

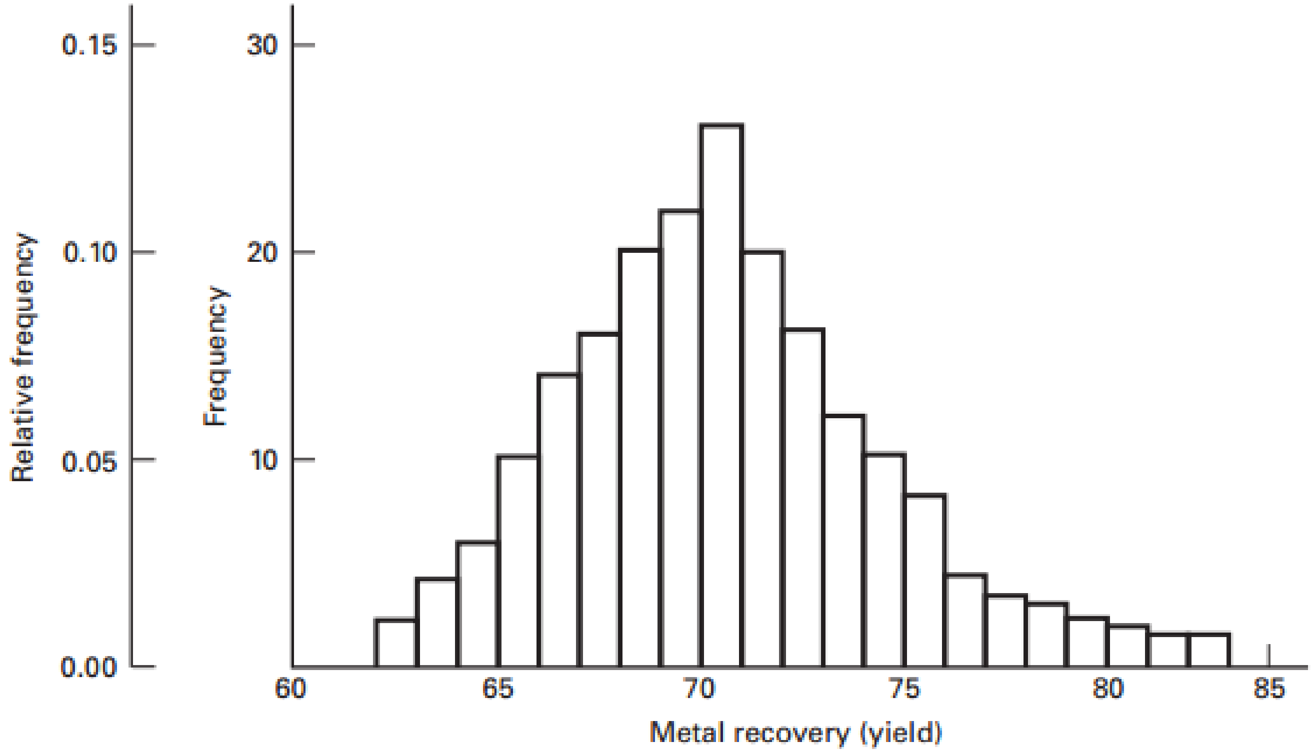
Graphical Description of Variability

- The dot diagram enables the experimenter to see quickly the general **location** or **central tendency** of the observations and their **spread** or **variability**.
- If the data are fairly numerous, the dots in a dot diagram become difficult to distinguish and a **histogram** may be preferable.
- Figure presents a histogram for 200 observations on the metal recovery, or yield, from a smelting process.

Graphical Description of Variability

- ❖The histogram shows the **central tendency**, **spread**, and **general shape** of the distribution of the data.
- ❖The histogram is a large-sample tool. When the sample size is small the shape of the histogram can be very sensitive to the number of bins, the width of the bins, and the starting value for the first bin.
- ❖Histograms should not be used with fewer than 75-100 observations.

Graphical Description of Variability

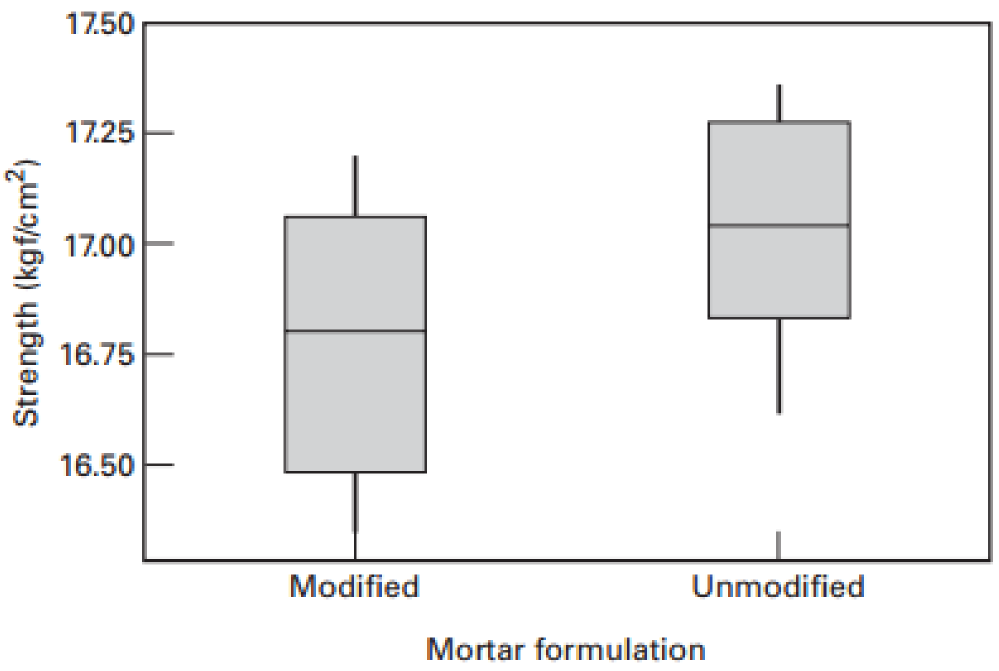


Histogram for 200 observations

Graphical Description of Variability

- The **box plot** (or **box-and-whisker plot**) is a very useful way to display data. A box plot displays the minimum, the maximum, the lower and upper quartiles (the 25th percentile and the 75th percentile, respectively), and the median (the 50th percentile) on a rectangular box aligned either horizontally or vertically.
- The box extends from the lower quartile to the upper quartile, and a line is drawn through the box at the median. Lines (or whiskers) extend from the ends of the box to (typically) the minimum and maximum values.

Graphical Description of Variability



Box plots

Confidence interval Using z distribution (a population mean – when σ known)

- Population mean μ

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Where: $\bar{x}$ is sample mean; z standard normal random variable ; σ population standard deviation; n is sample size

Research Methods and Experimental Design

Lecture 7: Experimental Design: Inferential Statistics

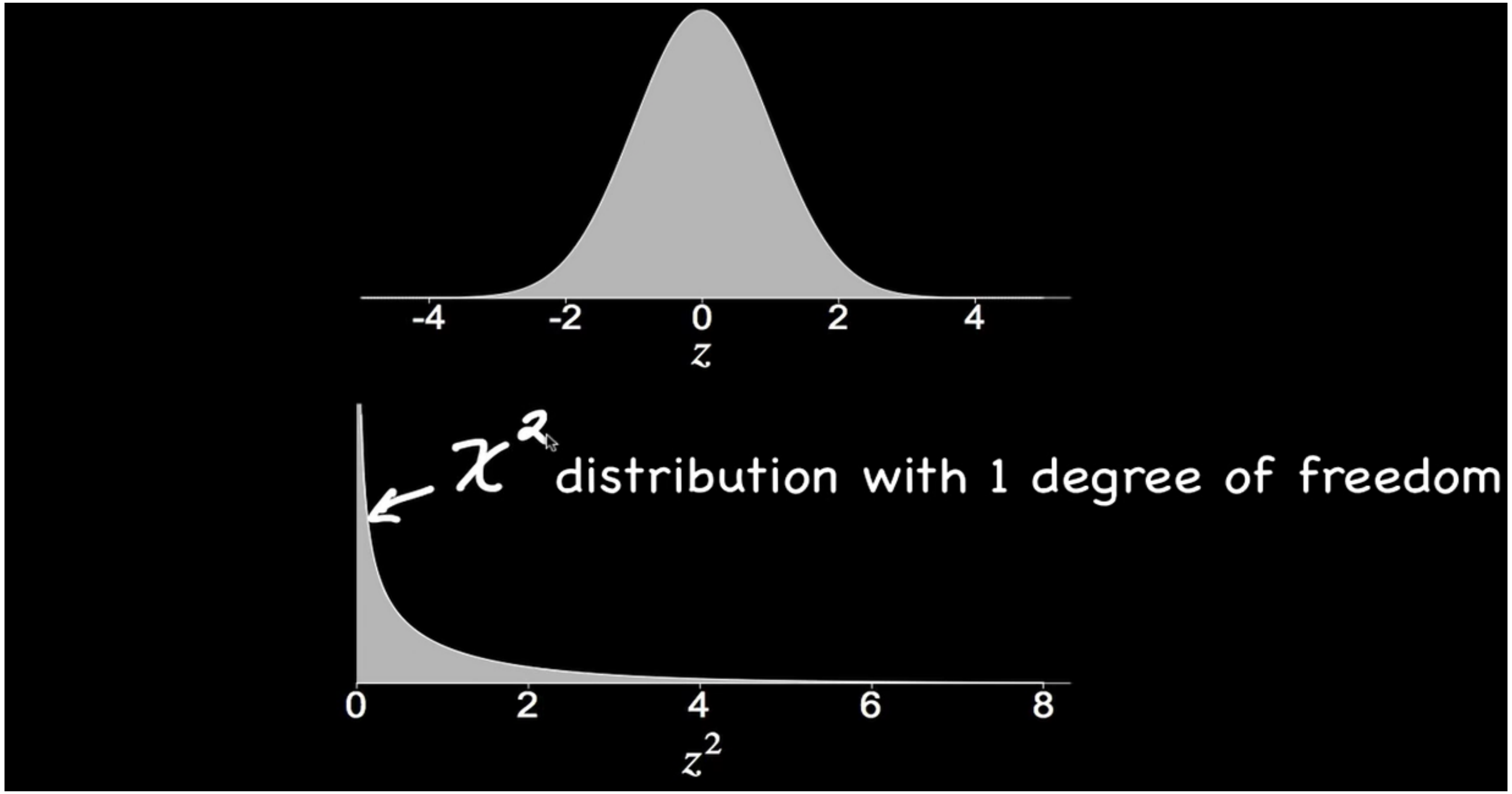
Chi-square distribution

- Used to test How well theoretical distribution explained observed one.
- How good it fit the observed results for theoretical distributions
- $X \sim N(0,1),\ E(X) = 0, V(X) = 1$
- Take another random variable Q sampling from standard normal distribution squaring what ever number you got

Chi-square distribution

- $Q_1 = X^2$
- The distribution for this random variable Q is going to b an example of the chi-square
- Depends on how many sums we have
- $Q \sim x_1^2$ (1 is our degree of freedom)
- $Q_1 = X_1^2 + X_2^2$
- $Q \sim x_2^2$ (1 is our degree of freedom)

Normal distribution curve



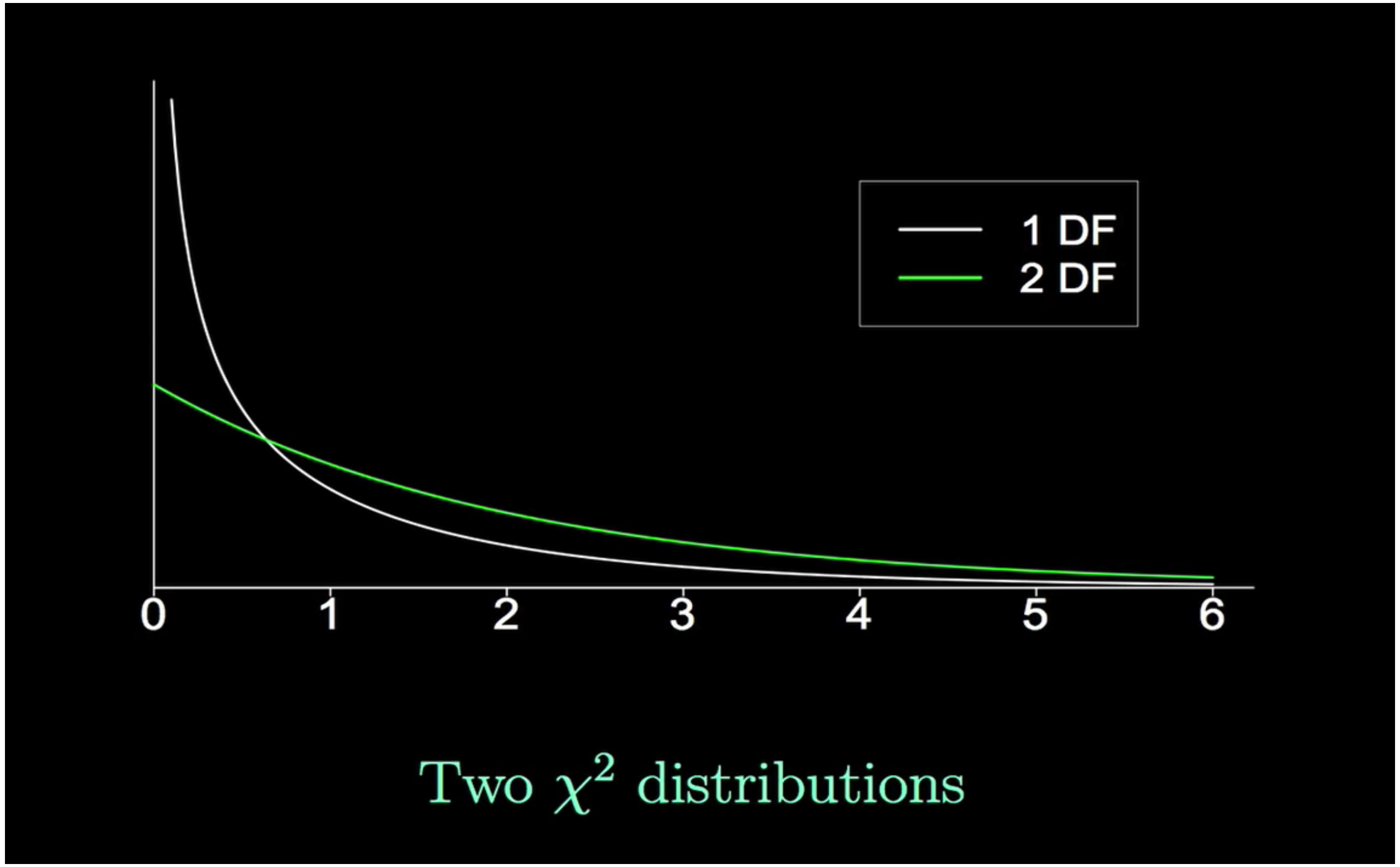
Chi-square distribution

❖The pdf of the x^2 distribution with k degree of freedom

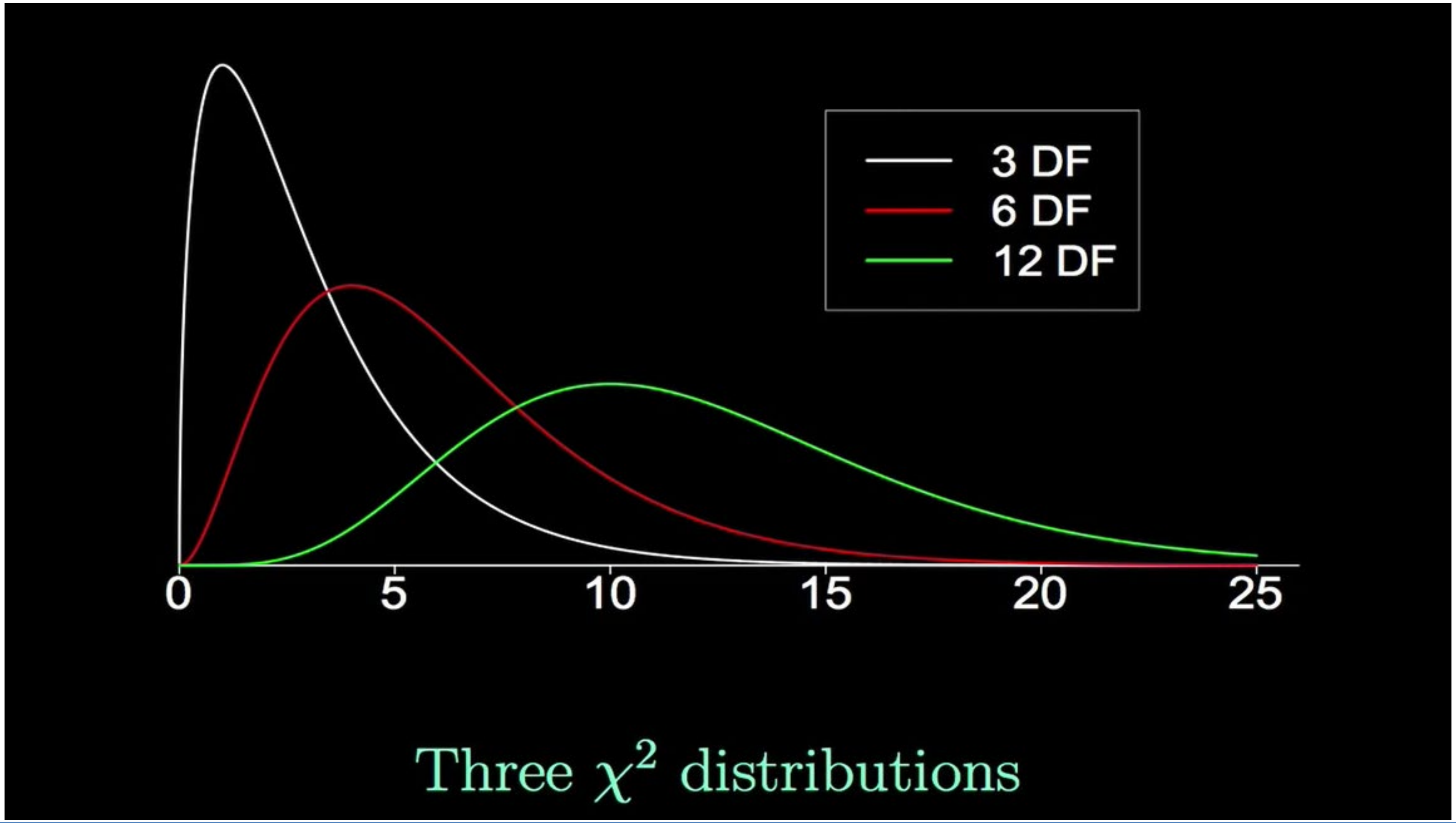
$$f(x) = \frac{x^{\frac{k}{2}-1} e^{-x/2}}{2^{\frac{k}{2}} \Gamma(\frac{k}{2})} \text{ for x} > 0$$

$$\mu = k$$
$$\sigma^2 = 2k$$

Chi-square plot



Chi-square plot



Example

observed

#	1	2	3	4	5	6
Freq.	22	24	38	30	46	44

expected

#	1	2	3	4	5	6
Freq.						

Hypothesis testing steps

- State null(H_o) and alternative (H_1) hypothesis
- Choses the level of significance
- Find the critical values
- Find the test statistics
- Draw your conclusion

solution

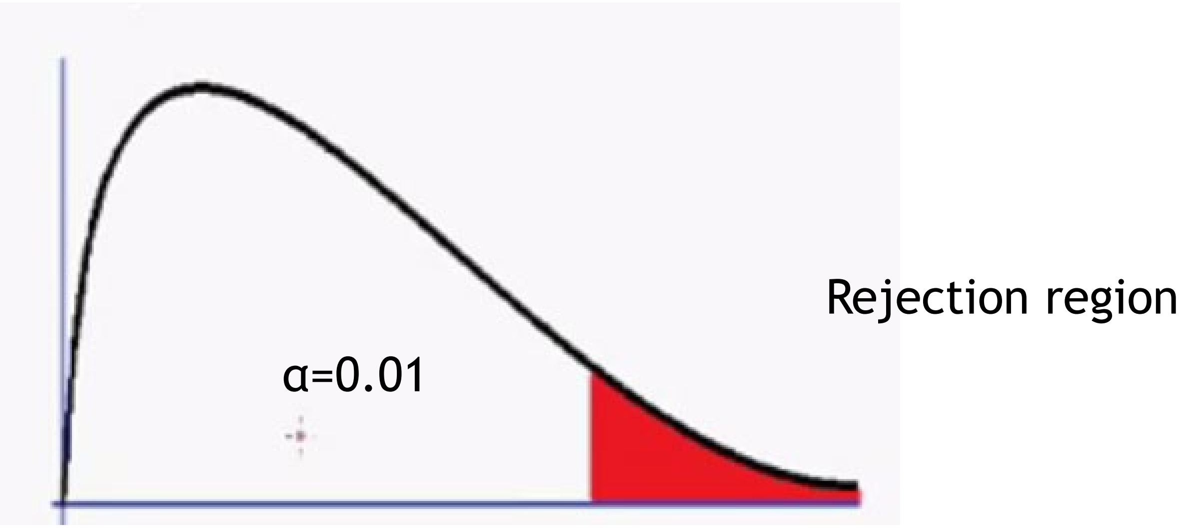
❖Total is 24

#	1	2	3	4	5	6
Freq.	22	24	38	30	46	44

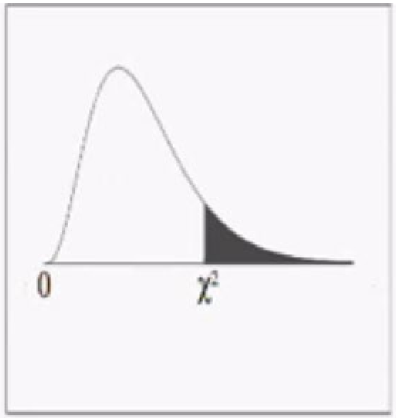
❖Expected value for every outcomes can be calculated by 1/6*204=34

Solution

- $H_o: \mu\ die\ is\ fair$
- $H_o: \mu\ die\ is\ unfair$



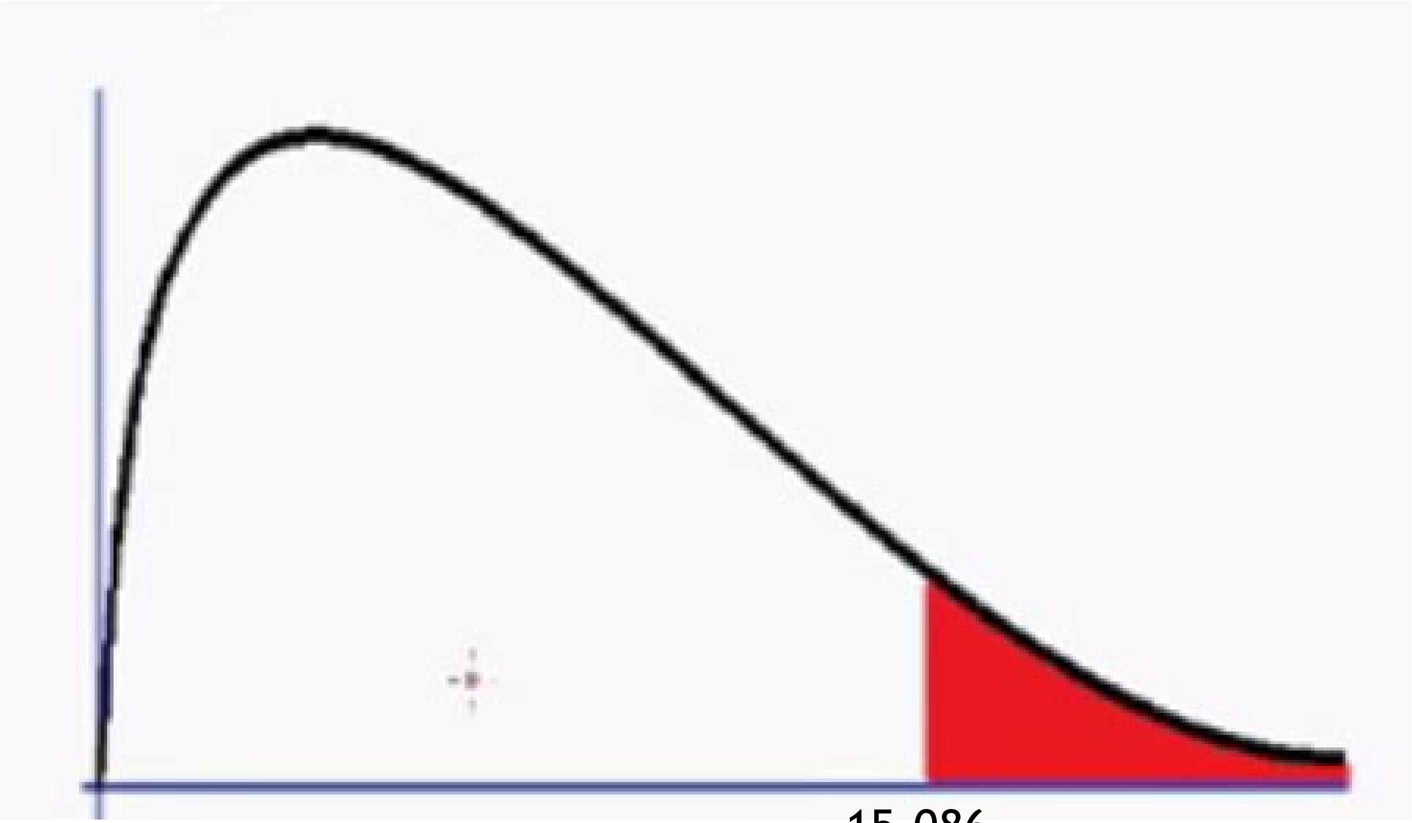
solution



The shaded area is equal to α for $\chi^2 = \chi^2_\alpha$.

A critical value=15.086

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750



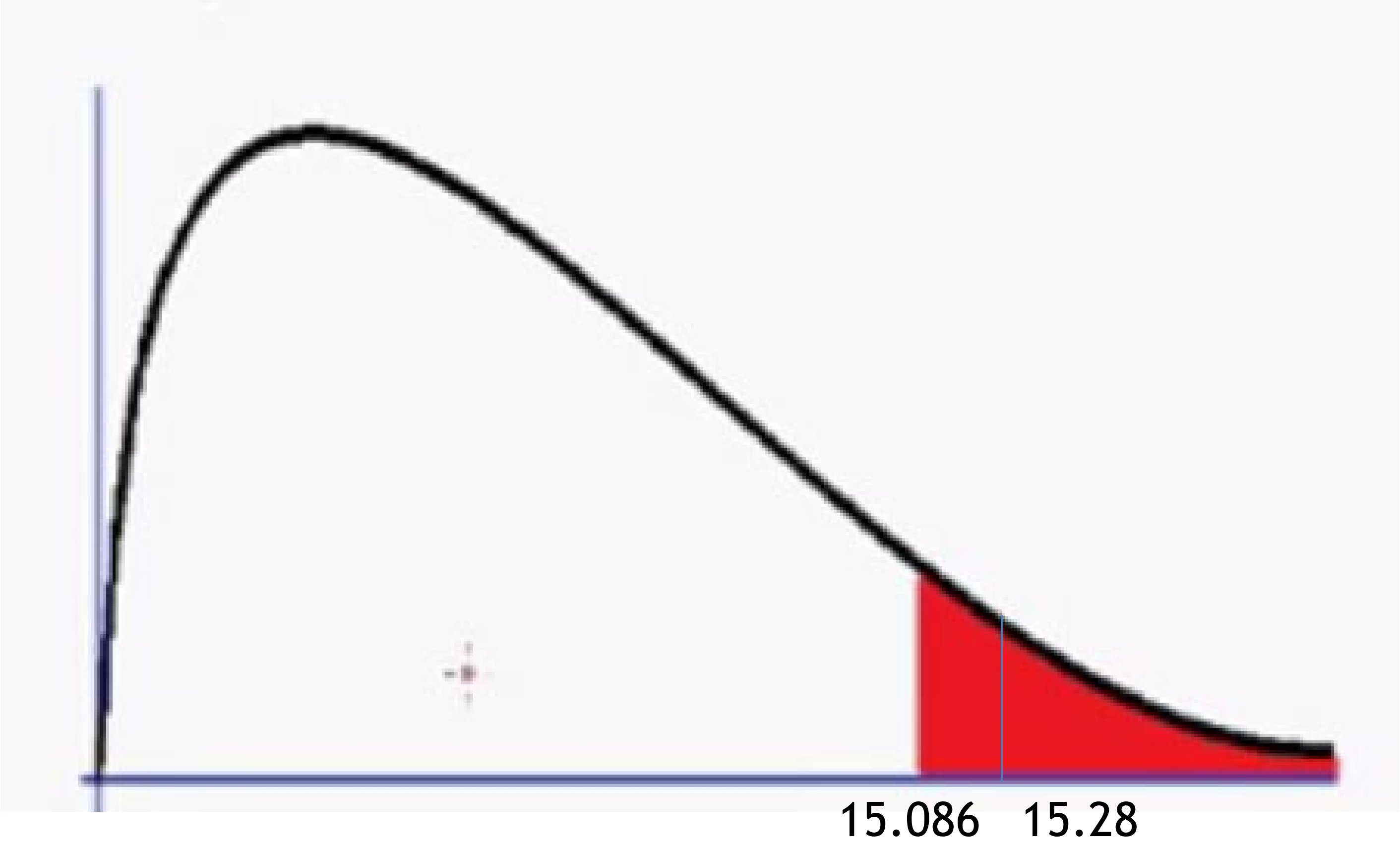
15.086

Solution

$$X^2 = \sum_{i=1}^{n} \frac{(O-E)^2}{E}$$

❖ $X^2 = \frac{(22-34)^2}{34} + \frac{(24-34)^2}{34} + \frac{(38-34)^2}{34} + \frac{(30-34)^2}{34} + \frac{(46-34)^2}{34} + \frac{(44-34)^2}{34} =$ 15.28 (test statistic)

solution



Conclusion

- We reject the null hypothesis and we accept the alternative hypothesis

The χ^2 Statistic for nominal data

Presume you observe 100 people to see who deposits garbage in the can and who litters. You want to see if there is a difference based on gender.

A person can fall in one of four categories:

- Male, deposits garbage
- Male, litters
- Female, deposits garbage
- Female, litters

	Deposit	Litter	
Females	18	7	25
Males	42	33	75
	60	40	100

To answer this question, you have to figure out what numbers you might expect if everything were left to chance; if H_0 were true—that there is no difference based on gender.

	Deposit	Litter	
Females	18 15	7	25
Males	42	33	75
	60	40	100

Since 60 people deposited their garbage, and 25% of them were female, you'd expect 15 (25% of 60) females to be the value in the upper left cell, if there's an equal distribution—no effect of gender.

	Deposit	Litter	
Females	18 15	7	25
Males	42	33 30	75
	60	40	100

Since 40 people littered, and 75% of them were male, you'd expect 30 (75% of 40) males to be the value in the lower right cell if there is no gender effect.

	Deposit	Litter	
Females	18 15	7 10	25
Males	42 45	33 30	75
	60	40	100

Working in a similar method, you can fill in all the expected values.

The further the observed values are from the expected values, the more likely that there really *is* a significant difference.

	Deposit	Litter	
Females	18 15	7 10	25
Males	42 45	33 30	75
	60	40	100

The formula for χ^2 is:

$$\sum \frac{(O-E)^2}{E}$$

Where O is the observed value and E is the expected value for each cell.

	Deposit	Litter	
Females	18 15	7 10	25
Males	42 45	33 30	75
	60	40	100

In this case, that works out to:

$$\frac{(18-15)^2}{15}+\frac{(7-10)^2}{10}+\frac{(42-45)^2}{45}+\frac{(33-30)^2}{30}$$
$$=\frac{9}{15}+\frac{9}{10}+\frac{9}{45}+\frac{9}{30}$$
$$=.6+.9+.2+.3$$
$$=2.0$$

	Deposit	Litter	
Females	18 15	7 10	25
Males	42 45	33 30	75
	60	40	100

Looking up the value 2.0 in the χ^2 table for 1 degree of freedom, you find the probability of this result is 0.16, so you retain H_0; there's no significant difference based on gender.

Degree of freedom

In general: the number of degrees of freedom for χ^2 is (number of rows – 1) times (number of columns – 1). In this case, that's 1 times 2, or 2.

Confidence interval

- Constructed form sample data in the range of values that is likely include the population parameter at some specified confidence level.
- The confidence interval for Population mean μ is determined by:

$$\mu = \bar{x} \pm E$$

- Where: $\bar{x}$ is sample mean (the point estimate); E margine of error

Confidence interval

❖If σ is known E is can be determined from

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

❖Command In excel: Confidence (α, σ, n)

❖α is a significance level α=1-CL (Confidence level)

❖Cl=1- α

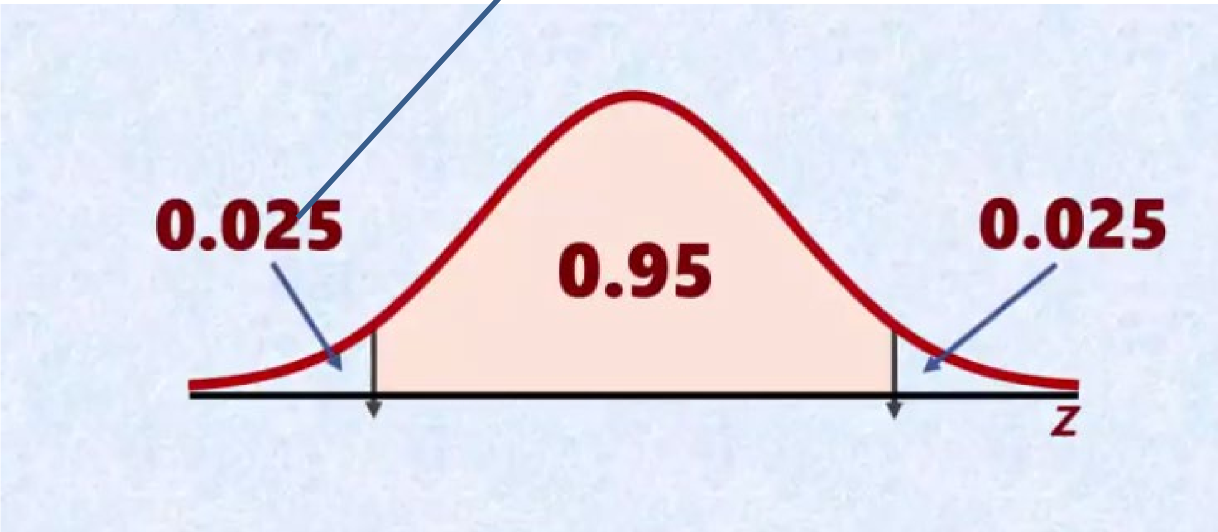
Confidence interval

❖Example

CL=95%; α=1-0.95=0.05

The critical value ($z_{\alpha/2}$) is can be found from normal tables

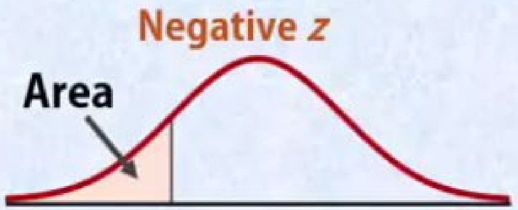
$\alpha/2$



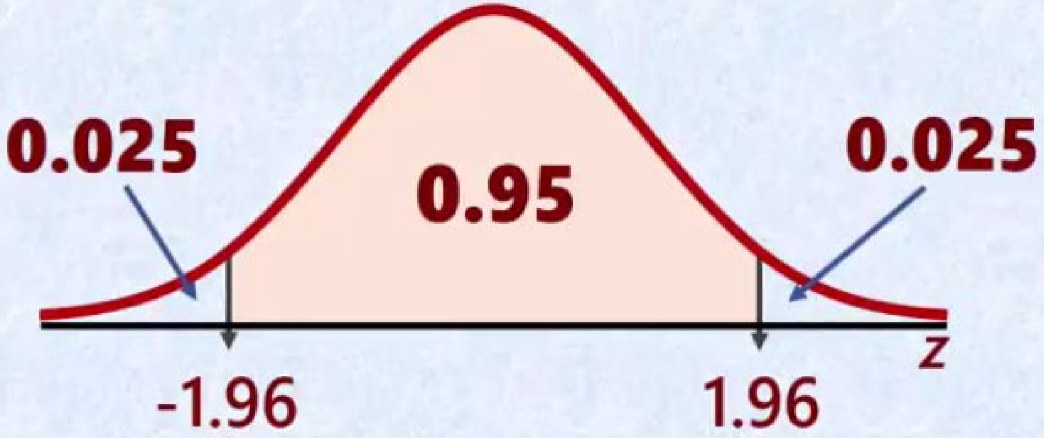
Confidence interval

$$CL = 95\% \qquad \alpha = 1 - 0.95 = 0.05$$

$$z_{\alpha/2} = z_{0.025} = 1.96$$



z	0.00	...	0.04	0.05	0.06	0.07
⋮	⋮	⋮	⋮	⋮	⋮	⋮
-2.0	0.0228	...	0.0207	0.0202	0.0197	0.0192
-1.9	0.0287	...	0.0262	0.0256	0.0250	0.0244
-1.8	0.0359	...	0.0329	0.0322	0.0314	0.0307
-1.7	0.0446	...	0.0409	0.0401	0.0392	0.0384
-1.6	0.0548	...	0.0505	0.0495	0.0485	0.0475
-1.5	0.0668	...	0.0618	0.0606	0.0594	0.0582
...	...	...	...	...	...	...



Example

- ❖Suppose the moisture content measurements in a coffee bines are assumed to follow a normal distribution with population standard deviation of 5.6%. A random sample of 40 test has a mean of 32%.
- ❖Estimate the population mean with

1. 80% confidence
2. 90% confidence
3. 98% confidence

Solution

- ❖ $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
- ❖ σ=5.6; $\bar{x}$=32; n=40

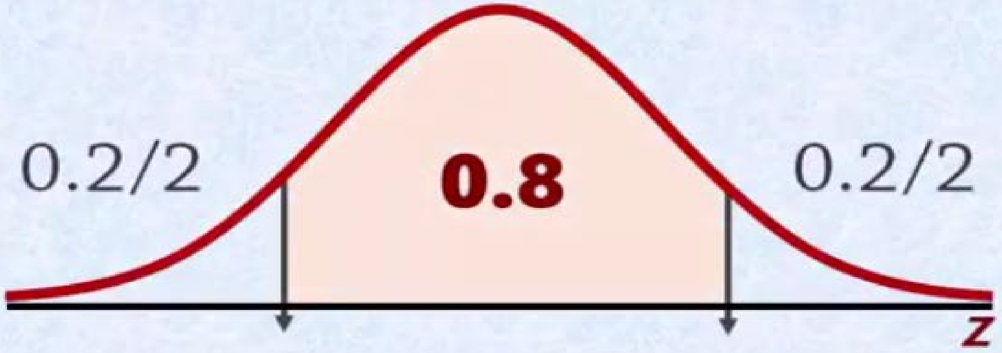
$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = z_{\alpha/2} \frac{5.6}{\sqrt{40}}$$

Solution

$\sigma = 5.6 \qquad n = 40 \qquad \bar{x} = 32$

1) **80% confidence**
2) 90% confidence
3) 98% confidence

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$



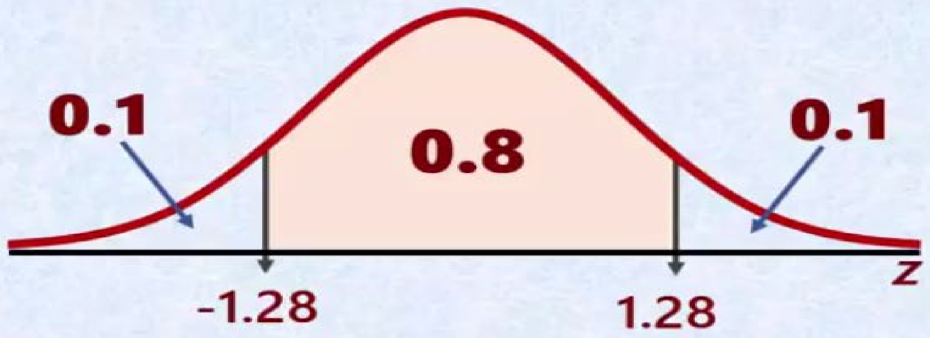
$$\alpha = 1 - 0.8 = 0.2$$

Solution

$\sigma = 5.6 \qquad n = 40 \qquad \bar{x} = 32 \qquad z_{0.1} = 1.28$

1) **80% confidence**
2) 90% confidence
3) 98% confidence

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$



z	0.00	...	0.05	0.06	0.07	0.08	0.09
.	.	.	.	.	.	.	.
-1.4	0.0808	...	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	...	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	...	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	...	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	...	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	...	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	...	0.1977	0.1949	0.1922	0.1894	0.1867
...	...	...	...	...	...	...	...

Solution

$\sigma = 5.6$ $n = 40$ $\bar{x} = 32$ $z_{0.1} = 1.28$

1) **80% confidence**
2) 90% confidence
3) 98% confidence

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 1.28\frac{5.6}{\sqrt{40}}$$

$$E = 1.13$$

Excel: CONFIDENCE(0.2,5.6,40)=1.13

Solution

$$\text{Lower Limit}: \quad \bar{x} - E = 32 - 1.13 = 30.87$$

$$\text{Upper Limit}: \quad \bar{x} + E = 32 + 1.13 = 33.13$$

$$30.87 < \mu < 33.13$$

We are 80% confident that the population mean test is between 30.87% and 33.13%

Exercise

- Repeat the steps for the 90% and 98% confidence level
- Compare the Critical values, Marginal Error and Confidence interval with the given confidence level

(support you answer with table)

t-test

- ❖ Consider experiments to compare two conditions
- ❖ Simple comparative experiments
- ❖ Example:
  - – The strength of Portland cement mortar
  - – Two different formulations: modified v.s. unmodified
  - – Collect 10 observations for each formulations
  - – Formulations = Treatments (levels)

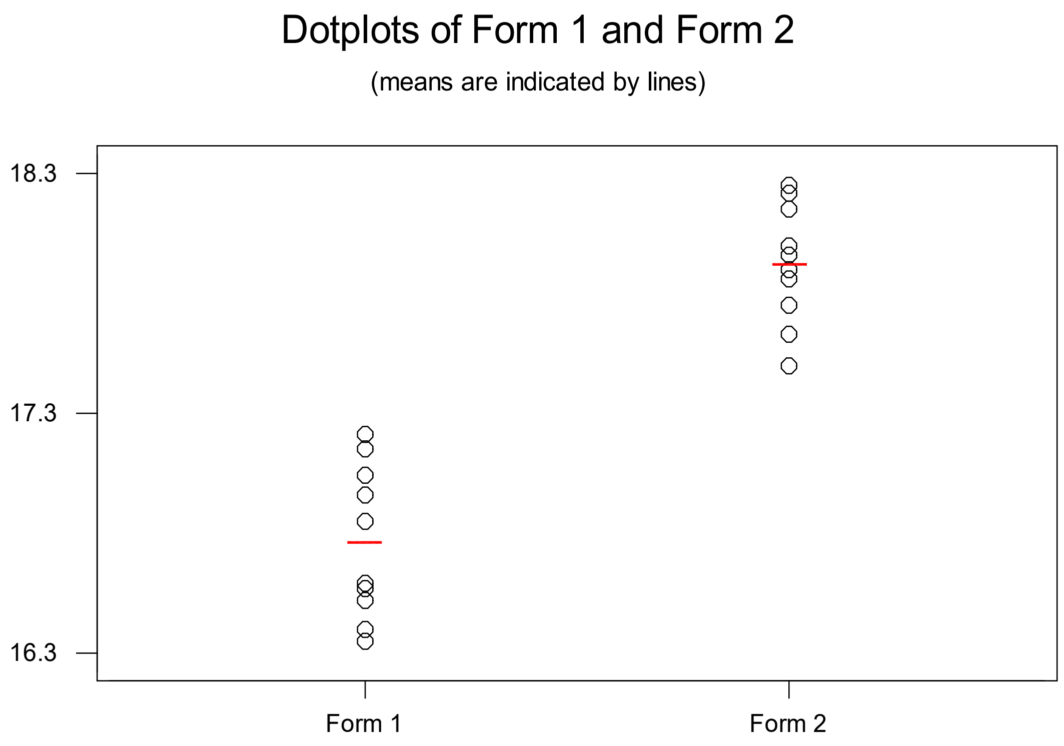
t-test

Observation (sample), *j*	Modified Mortar (Formulation 1)	Unmodified Mortar (Formulation 2)
1	16.85	17.50
2	16.40	17.63
3	17.21	18.25
4	16.35	18.00
5	16.52	17.86
6	17.04	17.75
7	16.96	18.22
8	17.15	17.90
9	16.59	17.96
10	16.57	18.15

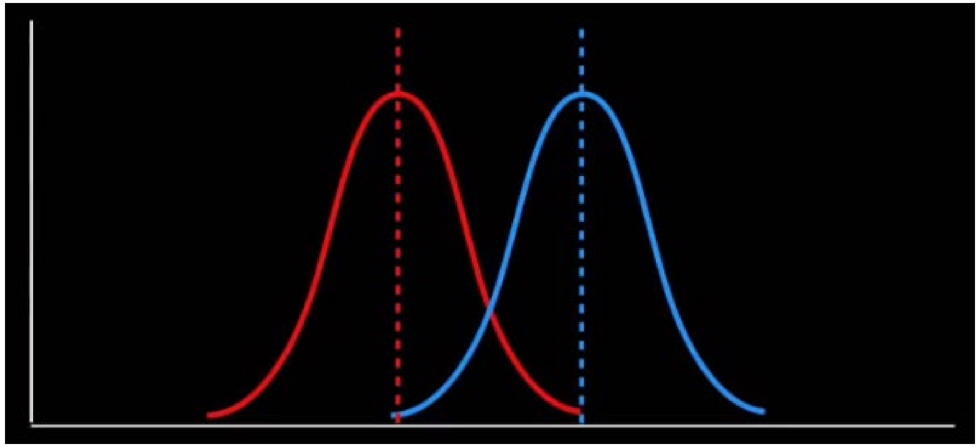
t-test

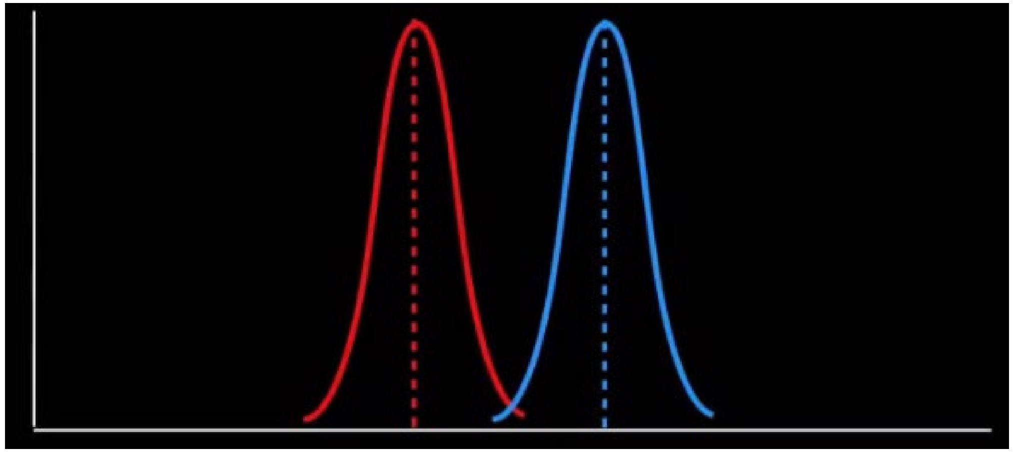
- Which one of this has a higher strength?
- We can figure out the mean/average of each smaple
- Dot diagram: Form 1 (modified) v.s. Form 2 (unmodified)
- unmodified (17.92) > modified (16.76)
- Look like the average of unmodified is greater than modified
- It is only part of the picture. The means do not tell us so much

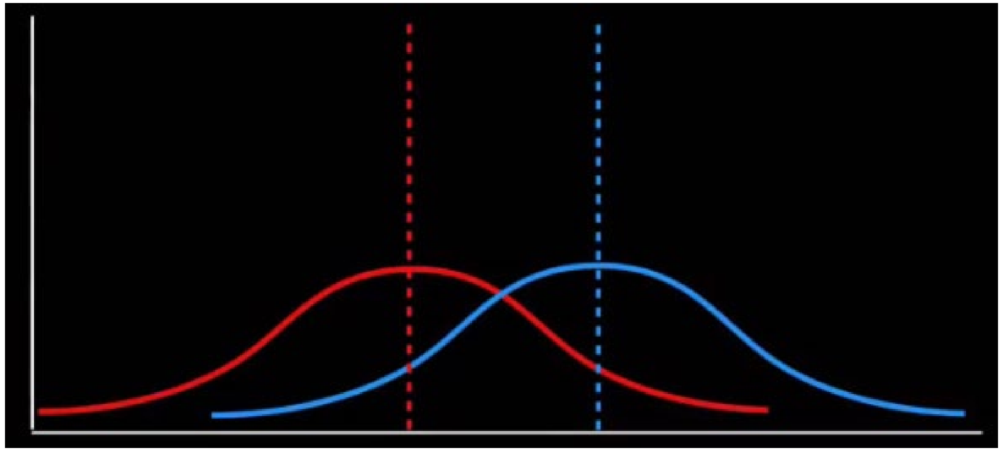
t-test


Dotplots of Form 1 and Form 2
(means are indicated by lines)
18.3
17.3
16.3
Form 1
Form 2

t-test







t-test

- The variance with in that samples statically significance difference between the to or not.
- That is why the t-value is comes in handy
- It is a ratio of

$$\frac{signal}{noise} = \frac{difference\ between\ group\ means}{variability\ of\ groups} = \frac{|\overline{X_1} - \overline{X_2}|}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

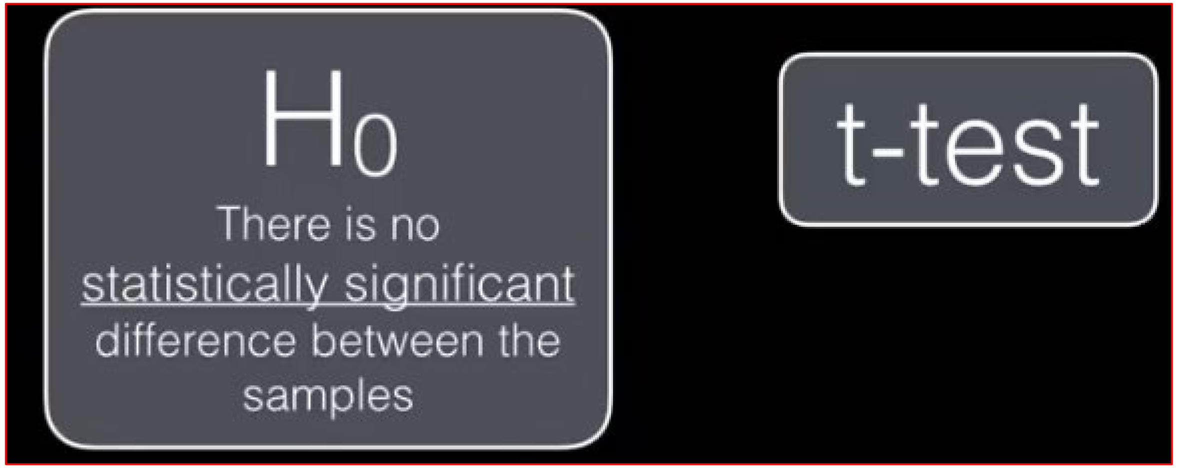
- Signal is numbers indicate the difference between two samples
- Noise is variability

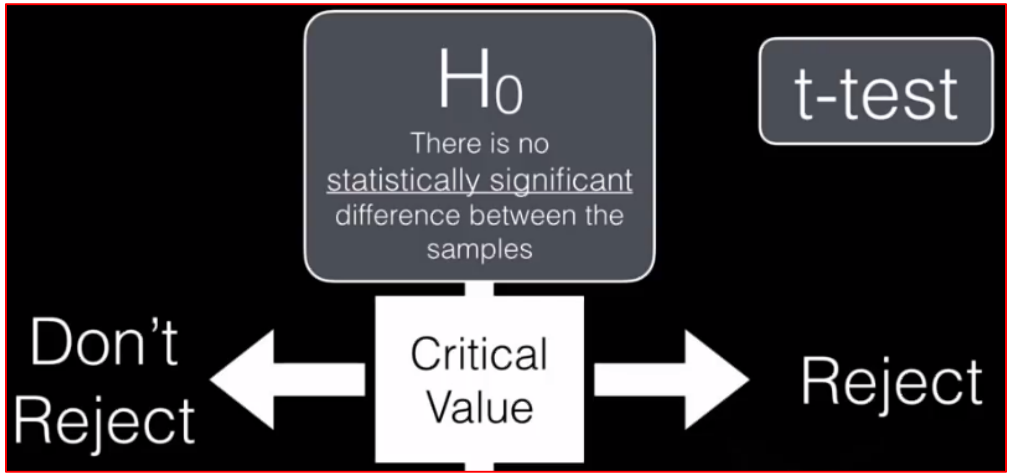
Example

		Field 1	Field 2
2		15.2	15.9
3		15.3	15.9
4		16.0	15.2
5		15.8	16.6
6		15.6	15.2
7		14.9	15.8
8		15.0	15.8
9		15.4	16.2
10		15.6	15.6
11		15.7	15.6
12		15.5	15.8
13		15.2	15.5
14		15.5	15.5
15		15.1	15.5
16		15.3	14.9
17		15.0	15.9
18	Mean	15.38125	15.68125
19	StDev	0.31245	0.40697
20	Variance	0.097625	0.165625
21	n	16.00000	16.00000

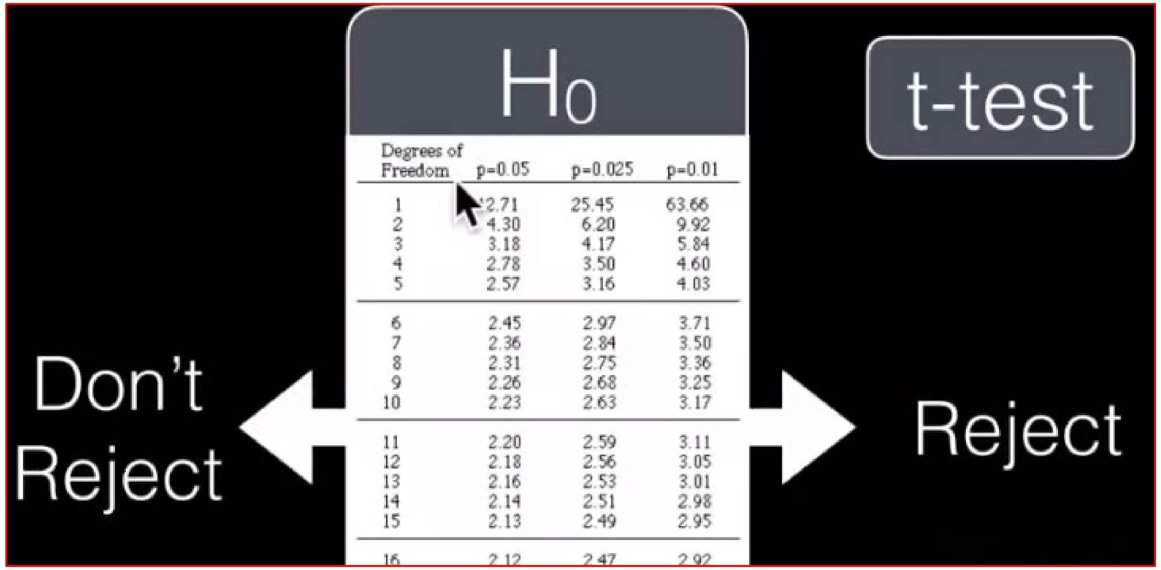
$$\text{t-value} = \frac{15.38 - 15.68}{\sqrt{\frac{.097}{16} + \frac{.165}{16}}} = \frac{0.30}{.13} = \boxed{2.3}$$

Example

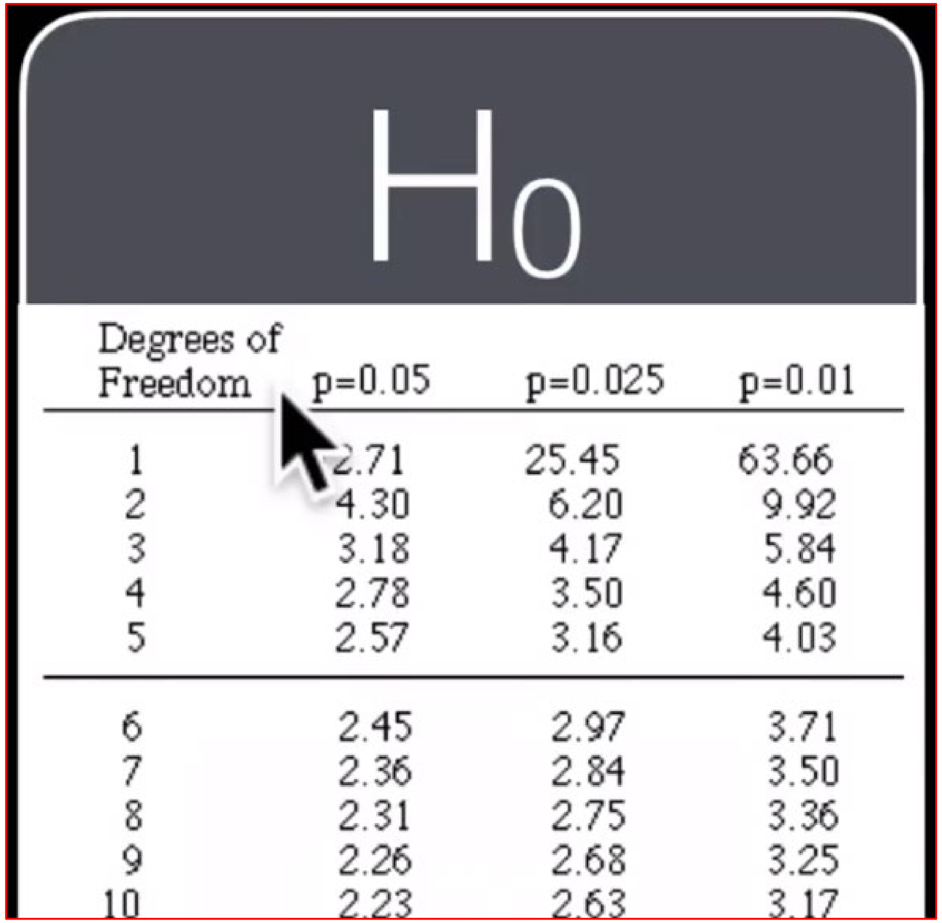

H0
There is no
statistically significant
difference between the
samples
t-test


H0
There is no
statistically significant
difference between the
samples
t-test
Don't
Reject
Critical
Value
Reject

Example



Degrees of Freedom	p=0.05	p=0.025	p=0.01
1	2.71	25.45	63.66
2	4.30	6.20	9.92
3	3.18	4.17	5.84
4	2.78	3.50	4.60
5	2.57	3.16	4.03
6	2.45	2.97	3.71
7	2.36	2.84	3.50
8	2.31	2.75	3.36
9	2.26	2.68	3.25
10	2.23	2.63	3.17
11	2.20	2.59	3.11
12	2.18	2.56	3.05
13	2.16	2.53	3.01
14	2.14	2.51	2.98
15	2.13	2.49	2.95
16	2.12	2.47	2.92



Degrees of Freedom	p=0.05	p=0.025	p=0.01
1	2.71	25.45	63.66
2	4.30	6.20	9.92
3	3.18	4.17	5.84
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5	2.57	3.16	4.03
6	2.45	2.97	3.71
7	2.36	2.84	3.50
8	2.31	2.75	3.36
9	2.26	2.68	3.25
10	2.23	2.63	3.17

H_0

Degrees of Freedom	p=0.05	p=0.025	p=0.01
1	12.71	25.45	63.66
2	4.30	6.20	9.92
3	3.18	4.17	5.84
4	2.78	3.50	4.60
5	2.57	3.16	4.03
6	2.45	2.97	3.71
7	2.36	2.84	3.50
8	2.31	2.75	3.36
9	2.26	2.68	3.25
10	2.23	2.63	3.17
11	2.20	2.59	3.11
12	2.18	2.56	3.05
13	2.16	2.53	3.01
14	2.14	2.51	2.98
15	2.13	2.49	2.95
16	2.12	2.47	2.92
17	2.11	2.46	2.90
18	2.10	2.44	2.88
19	2.09	2.43	2.86
20	2.09	2.42	2.84
21	2.08	2.41	2.83
22	2.07	2.41	2.82
23	2.07	2.40	2.81
24	2.06	2.39	2.80
25	2.06	2.38	2.79
26	2.06	2.38	2.78
27	2.05	2.37	2.77
28	2.05	2.37	2.76
29	2.04	2.36	2.76
30	2.04	2.36	2.75
40	2.02	2.33	2.70

$$df = n_1 + n_2 - 2$$

6	2.45	2.97	3.71
7	2.36	2.84	3.50
8	2.31	2.75	3.36
9	2.26	2.68	3.25
10	2.23	2.63	3.17
11	2.20	2.59	3.11
12	2.18	2.56	3.05
13	2.16	2.53	3.01
14	2.14	2.51	2.98
15	2.13	2.49	2.95
16	2.12	2.47	2.92
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18	2.10	2.44	2.88
19	2.09	2.43	2.86
20	2.09	2.42	2.84
21	2.08	2.41	2.83
22	2.07	2.41	2.82
23	2.07	2.40	2.81
24	2.06	2.39	2.80
25	2.06	2.38	2.79
26	2.06	2.38	2.78
27	2.05	2.37	2.77
28	2.05	2.37	2.76
29	2.04	2.36	2.76
30	2.04	2.36	2.75
40	2.02	2.33	2.70
60	2.00	2.30	2.66
120	1.98	2.27	2.62

2		15.2	15.9
3		15.3	15.9
4		16.0	15.2
5		15.8	16.6
6		15.6	15.2
7		14.9	15.8
8		15.0	15.8
9		15.4	16.2
10		15.6	15.6
11		15.7	15.6
12		15.5	15.8
13		15.2	15.5
14		15.5	15.5
15		15.1	15.5
16		15.3	14.9
17		15.0	15.9
18	**Mean**	**15.38125**	**15.68125**
19	**StDev**	**0.31245**	**0.40697**
20	**Variance**	**0.097625**	**0.165625**
21	**n**	**16.00000**	**16.00000**
22			
23	**t-test**	**0.0261981**	
24			

Degrees of Freedom	p=0.05	p=0.025	p=0.01
1	12.71	[illegible].45	63.66
2	4.30	6.20	9.92
3	3.18	4.17	5.84
4	2.78	3.50	4.60
5	2.57	3.16	4.03
6	2.45	2.97	3.71
7	2.36	2.84	3.50
8	2.31	2.75	3.36
9	2.26	2.68	3.25
10	2.23	2.63	3.17
11	2.20	2.59	3.11
12	2.18	2.56	3.05
13	2.16	2.53	3.01
14	2.14	2.51	2.98
15	2.13	2.49	2.95
16	2.12	2.47	2.92
17	2.11	2.46	2.90
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19	2.09	2.43	2.86
20	2.09	2.42	2.84
21	2.08	2.41	2.83
22	2.07	2.41	2.82
23	2.07	2.40	2.81
24	2.06	2.39	2.80
25	2.06	2.38	2.79
26	2.06	2.38	2.78
27	2.05	2.37	2.77
28	2.05	2.37	2.76
29	2.04	2.36	2.76
30	2.04	2.36	2.75

Conclusion

- There is statistically significance between these two samples.

Exercise

- Perform t-test on strength of Portland cement mortar problem indicated in slide # 258

Confidence interval Using t distribution (a population mean - σ unknown)

❖When to use t-value

1. σ is unknown
2. Sample size is less than 30

Confidence interval

- Constructed form sample data in the range of values that is likely include the population parameter at some specified confidence level.
- The confidence interval for Population mean μ is determined by:

$$\mu = \bar{x} \pm E$$

- Where: $\bar{x}$ is sample mean (the point estimate); E margine of error

Confidence interval Using t distribution (a population mean - σ unknown)

❖Population mean μ

$$\bar{x} \pm t * \frac{s}{\sqrt{n}}$$

Where: $\bar{x}$ is sample mean; t distribution value; s sample standard deviation; n is sample size

Confidence interval

- If σ is unknown E is can be determined from

$$E = t * \frac{s}{\sqrt{n}}$$

- Command In excel:T.INV($probability$,degree of freedom)

Example

- A Biological Oxygen Demand (BOD) concentration measurement of 9 randomly selected wastewater samples are

83	73	62	63	71	77	77	59	92

- Compute the 99% confidence interval of the true mean

Solution

- Sample mean $\bar{x}$=73
- Sample standard deviation s=10.69
- t-value=3.355
- n sample size 9

Table V Percentage Points $t_{\alpha,\nu}$ of the t Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819

Solution

- $\mu = 73 \pm 2.896 * \frac{10.69}{\sqrt{9}}$=11.95
- μ= (61.05,84.95)

We are 99% confident that the population mean test is between 61.05% and 84.95%

- Command In excel:CONFIDENCE.T ($alpha$,,degree of freedom, size)

Exercise

- Repeat the above problem with 90% and 95% confidence interval.

Hypothesis Testing

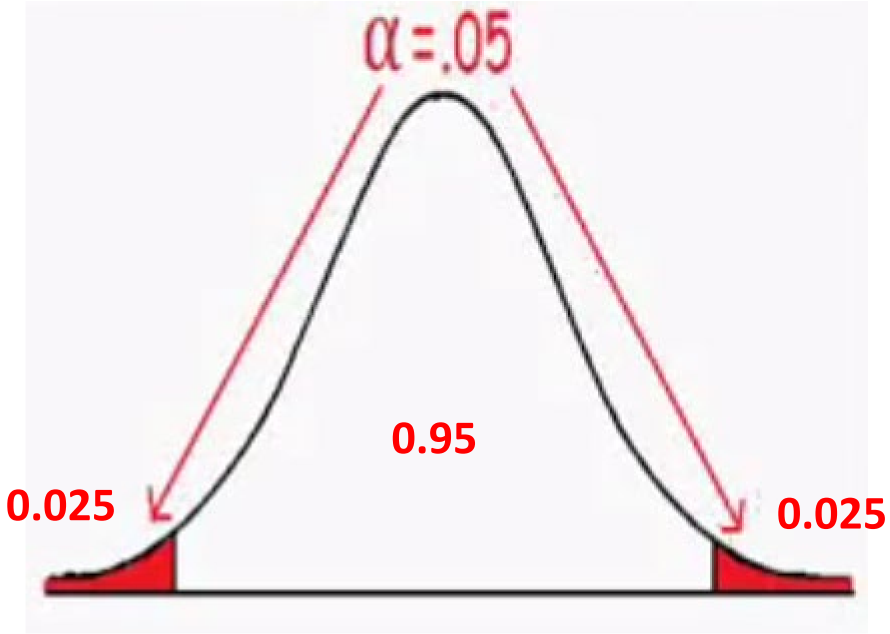
- The average protein content for soybean 100 mg/g with standard deviation of 15 mg/g. A researcher believes this value has changed. The researcher decides to test the protein content of 75 random samples. The average protein content of the sample is 105 mg/g. is there enough evidence to suggest the average protein content is changed?

Solution

- Hypothesis testing steps
  - State null(H_o) and alternative (H_1) hypothesis
  - Choses the level of significance
  - Find the critical values
  - Find the test statistics
  - Draw your conclusion

Solution

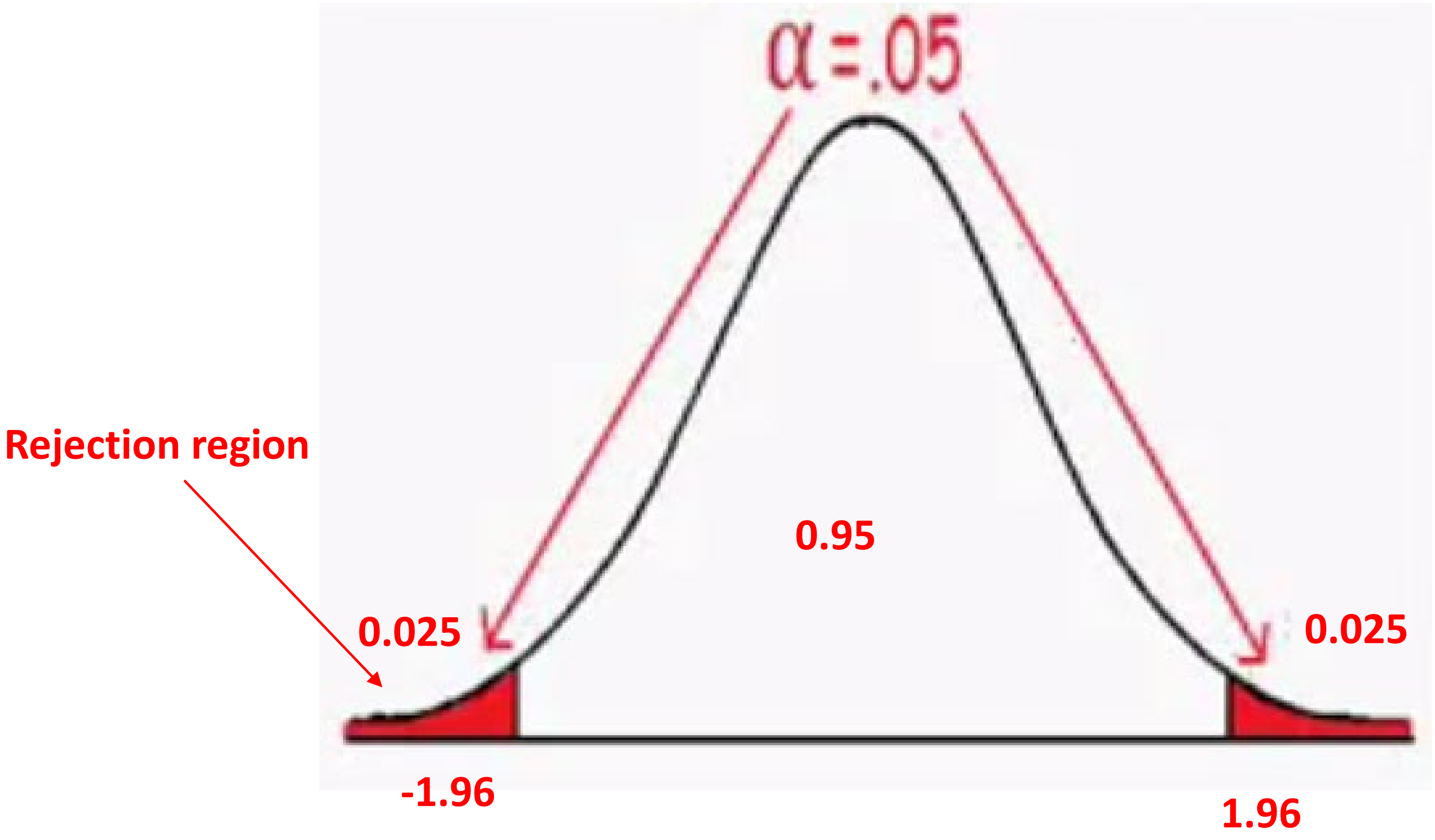
- $H_o: \mu = 100$
- $H_o: \mu \neq 100$
- we will follow the 2 tailed test



Solution

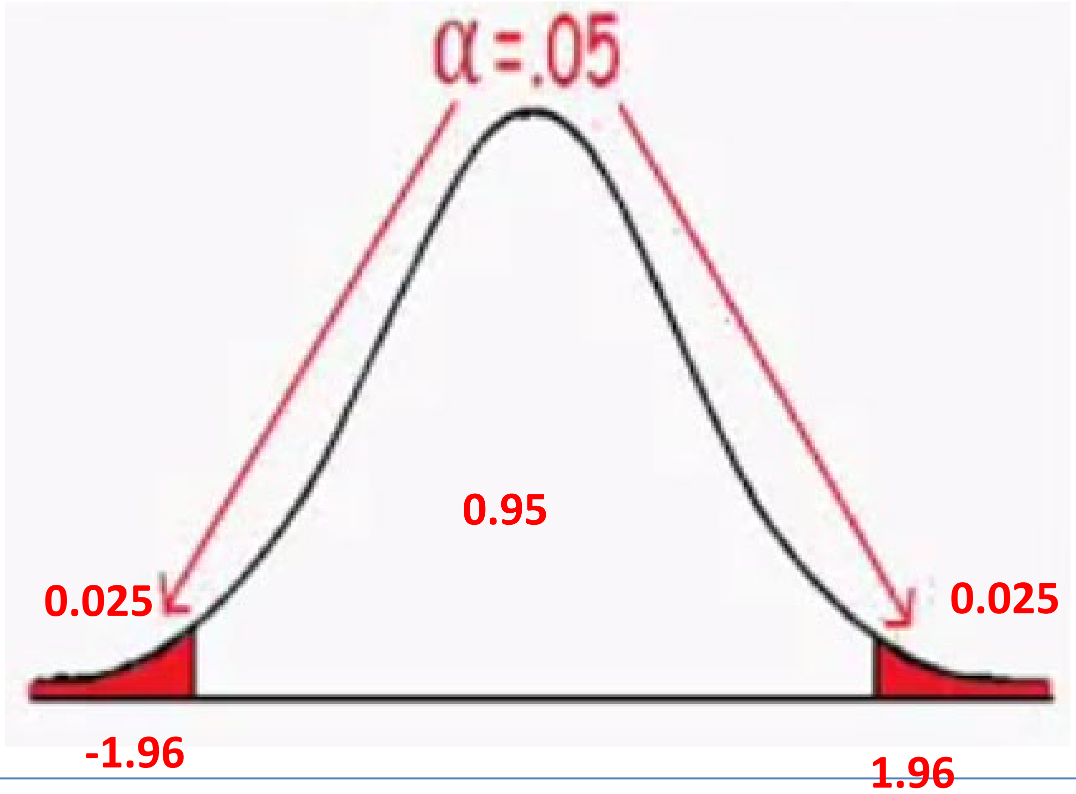
Confidence Level	Area between 0 and z-score	Area in one tail (alpha/2)	z-score
50%	0.2500	0.2500	0.674
80%	0.4000	0.1000	1.282
90%	0.4500	0.0500	1.645
95%	0.4750	0.0250	1.960
98%	0.4900	0.0100	2.326
99%	0.4950	0.0050	2.576

solution



solution

- $t = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{105 - 100}{\frac{15}{\sqrt{75}}} = 2.89\ (test\ statistics)$



Solution

- It is in rejection region
- There for we reject null(H_o) and accept alternative (H_1) hypothesis

Conclusion

- Yes! There is enough evidence to suggest the average protein content is changed with 95 % confidence.

Hypothesis Testing - one tailed 't' distribution

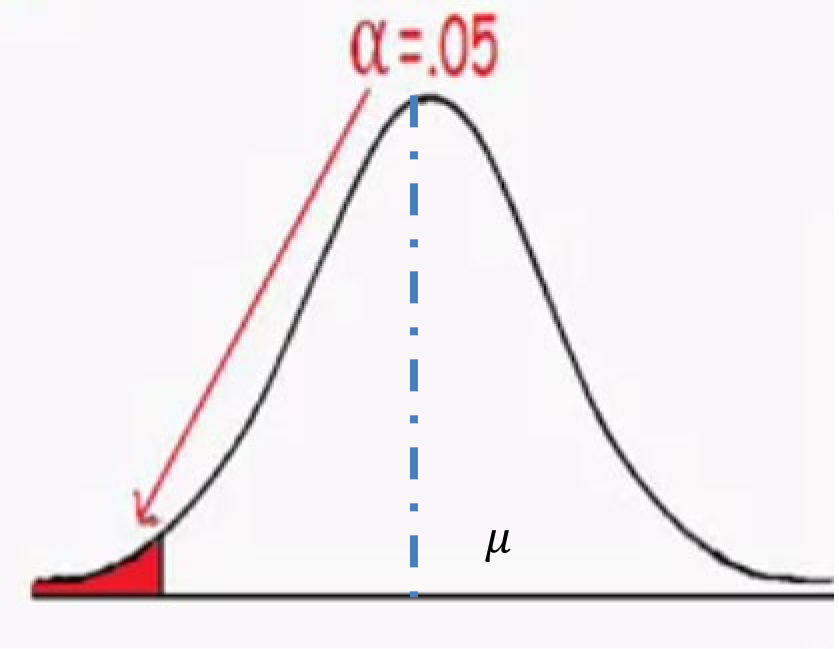
- The average compressive strength of a PVC pipe is 100 psi. A customer believes the average strength of the pipe is lower. A random sample of 5 pipes are tested and scored:69,79,89,99,109. (s.d=15.81)
- Is there enough evidence to suggest the average strength is lower?

solution

- Hypothesis testing steps
  - State null(H_o) and alternative (H_1) hypothesis
  - Choses the level of significance
  - Find the critical values
  - Find the test statistics
  - Draw your conclusion

Solution

- $H_o: \mu = 100$
- $H_o: \mu < 100$
- we will follow the one tailed test



solution

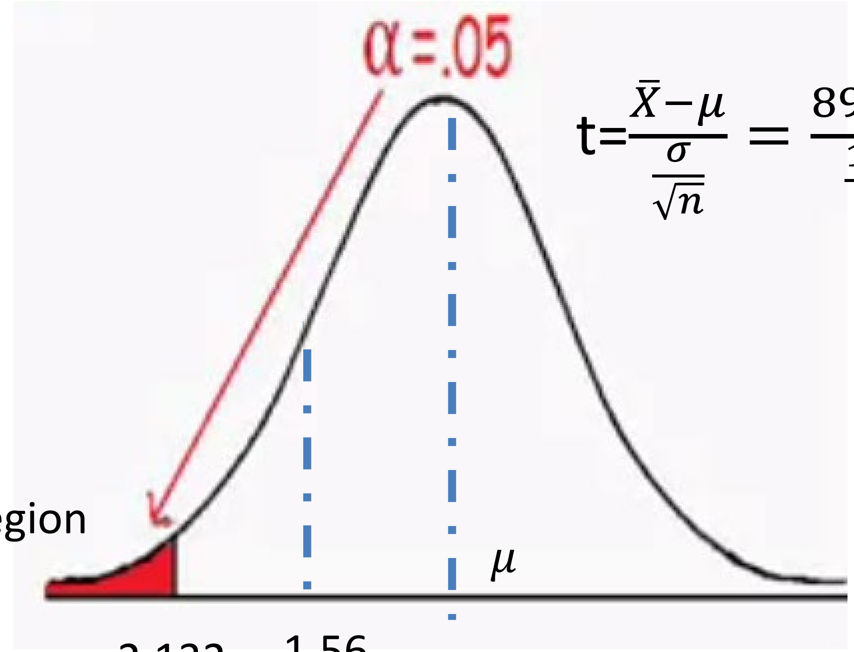
- ❖ When to use t-value
  1. σ is unknown
  2. Sample size is less than 30

Soution

area

one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05
df							
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571

Solution



$$t=\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}=\frac{89-100}{\frac{15.81}{\sqrt{5}}}=-1.56\ (test\ statistics)$$

Conclusion

- We fail to reject the null hypothesis. Which means the customer is wrong on the strength of the pipe

The F-test

- ❖Tells us if the standard deviations from two sets of data are statistically different.
- ❖Example
- ❖If we have two type of method to calculate the percentage of zinc in wastewater. We can compare weather there is statistical significant difference between this two method
- ❖F can be calculated by $F = \frac{S_1^2}{S_2^2}$

Example

	Method 1	Method 2
$Mean \pm standard\ deviation\ (\%\ Zn)$	$96.8\% \pm 0.5\%$	$97.9\% \pm 0.8\%$
$number\ of\ measurement\ (n)$	10	5

$$F = \frac{0.8^2}{0.5^2} = 2.56$$

solution

3.63

Table 4-3 Critical values of $F = s_1^2/s_2^2$ at 95% confidence level

Degrees of freedom for s_2	Degrees of freedom for s_1													
	2	3	4	5	6	7	8	9	10	12	15	20	30	∞
2	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5
3	9.55	9.28	9.12	9.01	8.94	8.89	8.84	8.81	8.79	8.74	8.70	8.66	8.62	8.53
4	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.63
5	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.36
6	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.67
7	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.58	3.51	3.44	3.38	3.23
8	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	2.93
9	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.71
10	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.70	2.54
11	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.57	2.40
12	3.88	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.30
15	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.07
20	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.84
30	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.62
∞	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.00

Conclusion

- If F calculated is smaller than F from the table then we can say 95% confident that any difference in standard deviation is due to random error. There for these two set of data is statistically similar.

Introduction to Analysis of Variance (ANOVA)

- **Purpose:** To compare the differences of more than two populations means.
- Also known as factorial experiments.
- The approach that allows us to use sample data to see if the values of more than two populations means are likely to be different from each other.
- This name is derived from the fact that in order to test for statistical significance between means, we are actually comparing variances. (so F-distribution will be used).

Cont…

❖ There are two-types of ANOVA:

- One-way ANOVA – involve one factor
- Two-way ANOVA – involve two factors

Example

a) Will **three different levels of a chemical concentration** have different effect on **an electroplating process**?

- H_0 : *The mean effect of electroplating process is the same for all three* concentration levels.
- One-way ANOVA

❖b) Will **three different levels of a chemical concentration have different effect** on **four electroplating processes**?

Cont..

H_0 : Different levels of a chemical concentration have no effect on the electroplating processes.

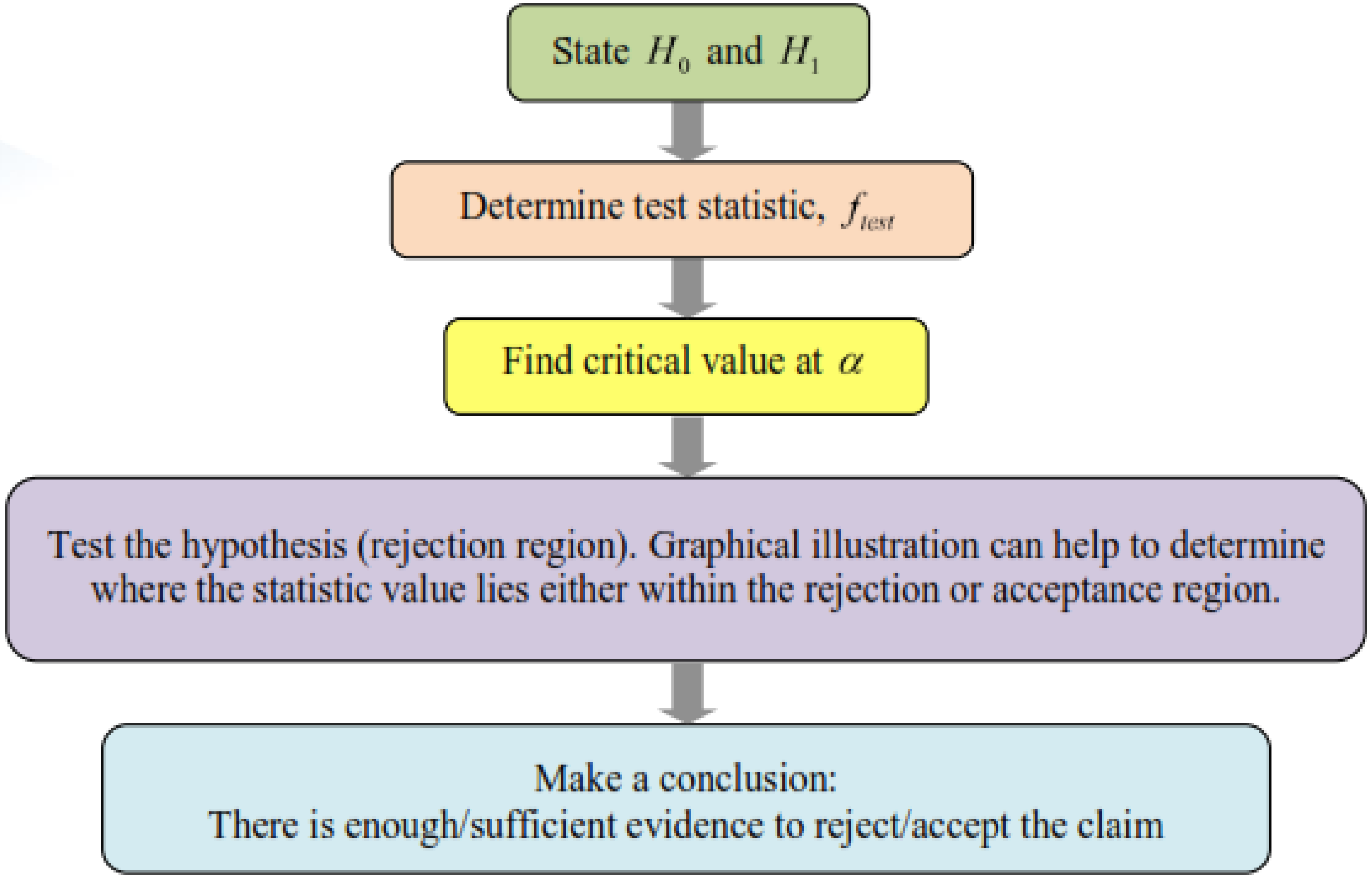
(There is no interaction effect between chemical concentration and electroplating processes)

-Two-way ANOVA

The Procedural Steps for an ANOVA Test

1. State the Null (*H_0) and Alternative (H_1) hypotheses.*
2. Determine the test statistic to be used - *F_{test}.*
3. Establish the test criterion by determining the critical value (point) and rejection region, based on significance level, a – *$F_{critical}$.*
4. Make a decision to reject or fail to reject the null hypothesis based on the comparison between test statistic and critical values. **The test is right tailed only. Graphical illustration can** help to determine where the statistic value lies either within the rejection or acceptance region.
5. Draw a conclusion to reject or not to reject the claim.

General procedure of ANOVA



ONE-WAY ANOVA

Assumptions for one-way ANOVA:

❖ To use the one-way ANOVA test, the following assumptions of data should be considered:

1. The populations under study follow normal distribution.
2. The samples are drawn randomly, and each sample is independent of the other samples.
3. All the populations from which the samples values are obtained, have the same unknown population variances, that is for *k number of populations,*

$$\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$$

Tabulation of Data for One-Way ANOVA

	Replicates						Total
Treatment/The level of factor (variable) considered under study/Number of population	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1n}	$x_{1.}$
	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2n}	$x_{2.}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{in}	$x_{i.}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	x_{k1}	x_{k2}	$\cdots$	x_{kj}	$\cdots$	x_{kn}	$x_{k.}$
Total	$x_{.1}$	$x_{.2}$	$\cdots$	$x_{.j}$	$\cdots$	$x_{.n}$	$x_{..}$

where

x_{ij} : the jth observation from the ith treatment

$x_{i.}$: the total of all observations from the ith treatment

$x_{.j}$: the total of all treatments from the jth observation

$x_{..}$: the total of all observations

x_{kn} : the nth observation from the kth treatment

Reminder: The roles of i and j can be interchanged.

Model for one-way ANOVA

$$x_{ij} = \mu + \alpha_i + \varepsilon_{ij} \qquad , i = 1,2,\ldots k \qquad j = 1,2,\ldots n$$

where

x_{ij} : the jth observation from the ith treatment

μ : the overall mean

α_i : the ith treatment effect

ε_{ij} : the random error

k : number of populations (treatments)

n_i : sample size from ith population

Two rules should be fulfilled which are:

a) Assume $\varepsilon_{ij} \sim NID(0, \sigma^2)$
where *NID* is abbreviated for Normally Identically Distributed.

b) $\sum_{i=1}^{k} \alpha_i = 0$, all treatment effects are the same.

The Null and Alternative hypotheses

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

- All population means are equal
  OR
- No treatment effect (no variation in means among groups)

$$H_1 : \mu_i \neq \mu_j \text{ for at least one } i \neq j$$

- At least one population mean is different
  OR
- There are differences between the population means
  OR
- Not all population means are equal
  OR
- There is a treatment effect between treatment *i* and treatment *j*

One-way ANOVA Table

Source of Variation	Sum Squares (SS)	Degrees of Freedom	Mean Squares (MS)	f_{test}
Treatment	$SSTr = \sum_{i=1}^{k} \frac{x_{i.}^2}{n} - \frac{x_{..}^2}{N}$	$k-1$	$MSTr = \frac{SSTr}{k-1}$	$f_{test} = \frac{MSTr}{MSE}$
Error	$SSE = SST - SSTr$	$N-k$	$MSE = \frac{SSE}{N-k}$	
Total	$SST = \sum_{i=1}^{k}\sum_{j=1}^{n} x_{ij}^2 - \frac{x_{..}^2}{N}$	$N-1$		

Reject H_0 if $f_{test} > f_{\alpha,k-1,N-k}$

Example

❖ An experiment was performed to determine whether the annealing temperature of ductile iron affects its tensile strength. Five specimens were annealed at each of four temperatures. The tensile strength (in kilo-pounds per square inch, ksi) was measured for each temperature. The results are presented in the following table.

Temperature (°C)	Tensile strength (in ksi)				
750	19.72	20.88	19.63	18.68	17.89
800	16.01	20.04	18.10	20.28	20.53
850	16.66	17.38	14.49	18.21	15.58
900	16.93	14.49	16.15	15.53	13.25

solution

a) Write down the model, assumptions and rules of the one-way ANOVA for the given data.

Model: $x_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, $i = 1,2,3,4,$ $j = 1,2,3,4,5.$

Assumption:

1. The populations under study follow normal distribution.
2. The samples are drawn randomly, and each sample is independent of the other samples.
3. $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

Rules: Assume $\varepsilon_{ij} \sim NID(0, \sigma^2)$ and $\sum_{i=1}^{k} \alpha_i = 0$

Cont…

(b) Perform the ANOVA and test the hypothesis at 0.01 level of significance that the mean data provided by the tensile strength is the same for all four temperatures.

- From the model above, clearly can be seen that thus the table above can be explained as k=4 and n=5

Temperature (ºC)	Sample Values					Total
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	
750, $i=1$	19.72	20.88	19.63	18.68	17.89	$x_{1.}=96.80$
800, $i=2$	16.01	20.04	18.10	20.28	20.53	$x_{2.}=94.96$
850, $i=3$	16.66	17.38	14.49	18.21	15.58	$x_{3.}=82.32$
900, $i=4$	16.93	14.49	16.15	15.53	13.25	$x_{4.}=76.35$
Total						$x_{..}=350.43$

Cont…

$\sum_{i=1}^{4} \frac{x_{i.}^2}{5}$

$$SSTrt = \frac{96.80^2}{5} + \frac{94.96^2}{5} + \frac{82.32^2}{5} + \frac{76.35^2}{5} - \frac{350.43^2}{20} = 58.6501$$

$\sum_{j=1}^{5} \sum_{i=1}^{4} x_{ij}^2$

$$SST = 19.72^2 + 20.88^2 + \ldots + 13.25^2 - \frac{350.43^2}{20} = 95.4875$$

$$SSE = 95.4875 - 58.6501 = 36.8374$$

Source of Variation	Sum of Squares (SS)	Degrees of Freedom	Mean of Squares (MS)	f_{test}
Treatment	58.6501	3	$\frac{58.6501}{3} = 19.5500$	$f_{test} = \frac{19.5500}{2.3023} = 8.4915$
Error	36.83.74	16	$\frac{36.8374}{16} = 2.3023$	
Total	95.4875	19		

Cont...

The hypothesis is:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_1 : \mu_i \neq \mu_j, \text{ for at least one } i \neq j$$

From statistical table, test statistic $f_{0.01,3,16} = 5.2922$.

Clearly that

$$\left(f_{test} = 8.4915\right) > \left(f_{0.01,3,16} = 5.2922\right)$$

Thus, H_0 is rejected.

At $\alpha = 0.01$, at least one of population means of temperature is different

Solve one-way ANOVA using Microsoft EXCEL

1. Excel - Key in data:

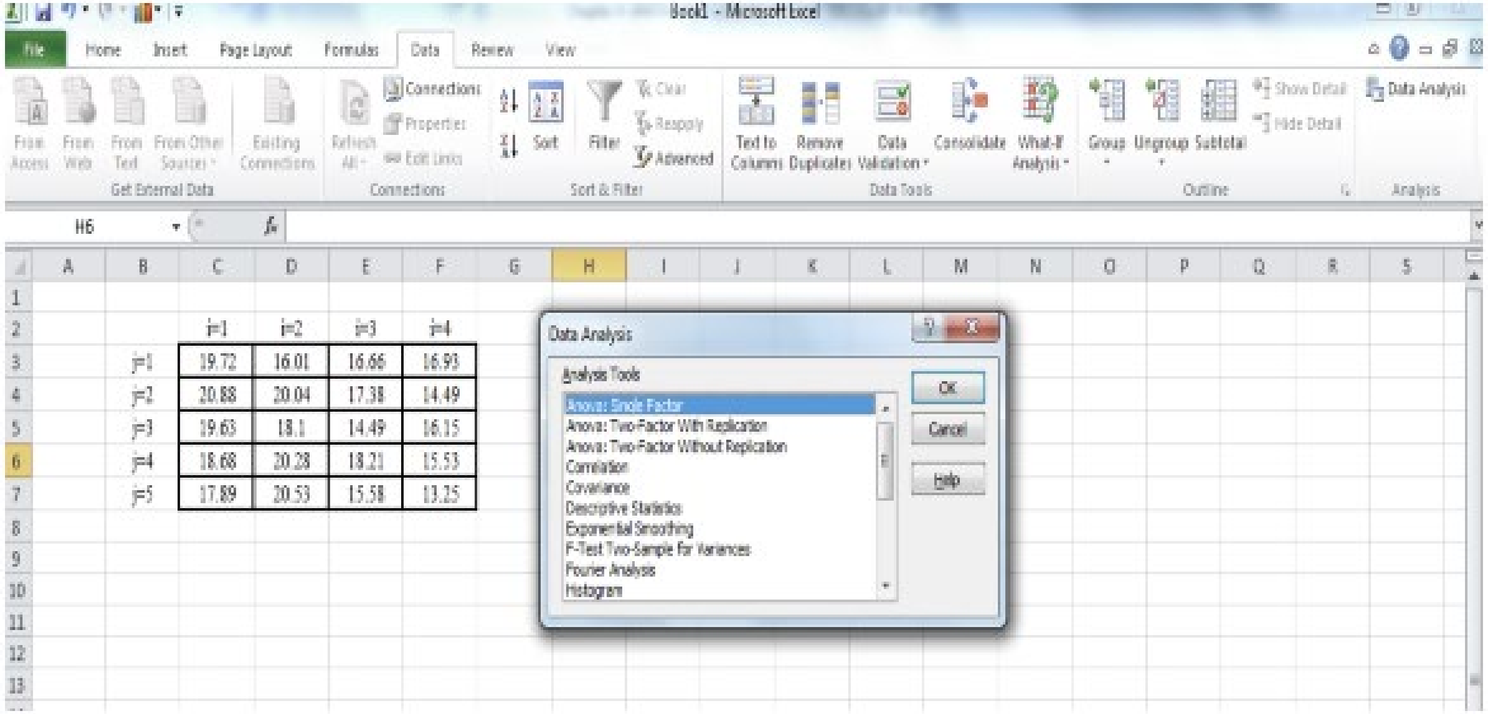
Temperature (°C)	Sample Values				
750	19.72	20.88	19.63	18.68	17.89
800	16.01	20.04	18.10	20.28	20.53
850	16.66	17.38	14.49	18.21	15.58
900	16.93	14.49	16.15	15.53	13.25

	i=1	i=2	i=3	i=4
j=1	19.72	16.01	16.66	16.93
j=2	20.88	20.04	17.38	14.49
j=3	19.63	18.1	14.49	16.15
j=4	18.68	20.28	18.21	15.53
j=5	17.98	20.53	15.53	13.25

Cont…

2. Follow the steps below:

Click menu: Data → Data Analysis → ANOVA single factor → enter the data range → set a value of a → OK



Cont…

3. Output:

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	58.65006	3	19.55002	8.491378	0.001327	5.292214
Within Groups	36.8374	16	2.302338			
Total	95.48746	19				

Reject H_0 if *P*-value ≤ α or $F > F\ crit$

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_1: \mu_i \neq \mu_j$, for at least one $i \neq j$

Since $(P-value = 0.0013) < (\alpha = 0.01)$, thus, H_0 is rejected.

At $\alpha = 0.01$, at least one of population means of temperature is different

The Two-Factor Factorial Design

- The simplest types of factorial designs involve only two factors or sets of treatments. There are *a* levels of factor *A* and *b* levels of factor *B*, and these are arranged in a factorial design; that is, each replicate of the experiment contains all *ab* treatment combinations. In general, there are *n* replicates.

Example

- an engineer is designing a battery for use in a device that will be subjected to some extreme variations in temperature. The only design parameter that he can select at this point is the plate material for the battery, and he has three possible choices. When the device is manufactured and is shipped to the field, the engineer has no control over the temperature extremes that the device will encounter, and he knows from experience that temperature will probably affect the effective battery life. However, temperature can be controlled in the product development laboratory for the purposes of a test.

Example

- The engineer decides to test all three plate materials at three temperature levels—15, 70, and 125°F—because these temperature levels are consistent with the product end-use environment. Because there are two factors at three levels, this design is sometimes called a **3^2 factorial design**. Four batteries are tested at each combination of plate material and temperature, and all 36 tests are run in random order. The experiment and the resulting observed battery life data are given in Table 5.1.

Example

■ TABLE 5.1

Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F)					
	15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

Example

- In this problem the engineer wants to answer the following questions:
  1. What effects do material type and temperature have on the life of the battery?
  2. Is there a choice of material that would give *uniformly long life regardless of temperature*?

Example

- This last question is particularly important. It may be possible to find a material alternative that is not greatly affected by temperature. If this is so, the engineer can make the battery **robust** to temperature variation in the field. This is an example of using statistical experimental design for **robust product design**, a very important engineering problem

Exampel

- This design is a specific example of the general case of a two-factor factorial. To pass to the general case, let *yijk* be the observed response when factor *A* is at the *i*th level (*i* 1, 2, . . . , *a*) and factor *B* is at the *j*th level (*j* 1, 2, . . . , *b*) for the *k*th replicate (*k* 1, 2, . . . , *n*).

- In general, a two-factor factorial experiment will appear as in Table 5.2. The order in which the *abn* observations are taken is selected at random so that this design is a **completely randomized design**.

Example

The observations in a factorial experiment can be described by a model. There are several ways to write the model for a factorial experiment. The **effects model** is

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \quad \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \\ k = 1, 2, \ldots, n \end{cases} \qquad \textbf{(5.1)}$$

where μ is the overall mean effect, τ_i is the effect of the ith level of the row factor A, β_j is the effect of the jth level of column factor B, $(\tau\beta)_{ij}$ is the effect of the interaction between τ_i and β_j, and ϵ_{ijk} is a random error component. Both factors are assumed to be **fixed**, and the treatment effects are defined as deviations from the overall mean, so $\sum_{i=1}^{a} \tau_i = 0$ and $\sum_{j=1}^{b} \beta_j = 0$. Similarly, the interaction effects are fixed and are defined such that $\sum_{i=1}^{a} (\tau\beta)_{ij} = \sum_{j=1}^{b} (\tau\beta)_{ij} = 0$. Because there are n replicates of the experiment, there are abn total observations.

General Arrangement for a Two-Factor Factorial Design

		Factor B			
		1	2	. . .	b
Factor A	1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
	2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
	$\vdots$				
	a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

Exampel

Another possible model for a factorial experiment is the **means model**

$$y_{ijk} = \mu_{ij} + \epsilon_{ijk} \quad \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \\ k = 1, 2, \ldots, n \end{cases}$$

where the mean of the *ij*th cell is

$$\mu_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij}$$

We could also use a **regression model** as in Section 5.1. Regression models are particularly useful when one or more of the factors in the experiment are quantitative. Throughout most of this chapter we will use the effects model (Equation 5.1) with an illustration of the regression model in Section 5.5.

In the two-factor factorial, both row and column factors (or treatments), A and B, are of equal interest. Specifically, we are interested in **testing hypotheses** about the equality of row treatment effects, say

$$H_0: \tau_1 = \tau_2 = \cdots = \tau_a = 0$$
$$H_1: \text{at least one } \tau_i \neq 0 \qquad \textbf{(5.2a)}$$

and the equality of column treatment effects, say

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_b = 0$$
$$H_1: \text{at least one } \beta_j \neq 0 \qquad \textbf{(5.2b)}$$

We are also interested in determining whether row and column treatments *interact*. Thus, we also wish to test

$$H_0: (\tau\beta)_{ij} = 0 \qquad \text{for all } i, j$$
$$H_1: \text{at least one } (\tau\beta)_{ij} \neq 0 \qquad \textbf{(5.2c)}$$

We now discuss how these hypotheses are tested using a **two-factor analysis of variance**.

Let $y_{i..}$ denote the total of all observations under the ith level of factor A, $y_{.j.}$ denote the total of all observations under the jth level of factor B, $y_{ij.}$ denote the total of all observations in the ijth cell, and $y_{...}$ denote the grand total of all the observations. Define $\bar{y}_{i..}$, $\bar{y}_{.j.}$, $\bar{y}_{ij.}$, and $\bar{y}_{...}$ as the corresponding row, column, cell, and grand averages. Expressed mathematically,

$$y_{i..} = \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{i..} = \frac{y_{i..}}{bn} \qquad i = 1, 2, \ldots, a$$

$$y_{.j.} = \sum_{i=1}^{a} \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{.j.} = \frac{y_{.j.}}{an} \qquad j = 1, 2, \ldots, b$$

$$y_{ij.} = \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{ij.} = \frac{y_{ij.}}{n} \qquad \begin{matrix} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{matrix}$$

$$y_{...} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{...} = \frac{y_{...}}{abn} \qquad (5.3)$$

The **total corrected sum of squares** may be written as

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{...})^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}[(\bar{y}_{i..}-\bar{y}_{...})+(\bar{y}_{.j.}-\bar{y}_{...})$$
$$+(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})+(y_{ijk}-\bar{y}_{ij.})]^2$$
$$= bn\sum_{i=1}^{a}(\bar{y}_{i..}-\bar{y}_{...})^2+an\sum_{j=1}^{b}(\bar{y}_{.j.}-\bar{y}_{...})^2$$
$$+ n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2$$
$$+\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{ij.})^2 \qquad \textbf{(5.4)}$$

because the six cross products on the right-hand side are zero. Notice that the total sum of squares has been partitioned into a sum of squares due to "rows," or factor A, (SS_A); a sum of squares due to "columns," or factor B, (SS_B); a sum of squares due to the interaction between A and B, (SS_{AB}); and a sum of squares due to error, (SS_E). This is the fundamental ANOVA equation for the two-factor factorial. From the last component on the right-hand side of Equation 5.4, we see that there must be at least two replicates ($n \geq 2$) to obtain an error sum of squares.

We may write Equation 5.4 symbolically as

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E \tag{5.5}$$

The number of degrees of freedom associated with each sum of squares is

Effect	Degrees of Freedom
A	$a - 1$
B	$b - 1$
AB interaction	$(a - 1)(b - 1)$
Error	$ab(n - 1)$
Total	$abn - 1$

We may justify this allocation of the $abn - 1$ total degrees of freedom to the sums of squares as follows: The main effects A and B have a and b levels, respectively; therefore they have $a - 1$ and $b - 1$ degrees of freedom as shown. The interaction degrees of freedom are simply the number of degrees of freedom for cells (which is $ab - 1$) minus the number of degrees of freedom for the two main effects A and B; that is, $ab - 1 - (a - 1) - (b - 1) =$

$(a - 1)(b - 1)$. Within each of the ab cells, there are $n - 1$ degrees of freedom between the n replicates; thus there are $ab(n - 1)$ degrees of freedom for error. Note that the number of degrees of freedom on the right-hand side of Equation 5.5 adds to the total number of degrees of freedom.

Each sum of squares divided by its degrees of freedom is a **mean square**. The expected values of the mean squares are

$$E(MS_A) = E\left(\frac{SS_A}{a - 1}\right) = \sigma^2 + \frac{bn \sum_{i=1}^{a} \tau_i^2}{a - 1}$$

$$E(MS_B) = E\left(\frac{SS_B}{b - 1}\right) = \sigma^2 + \frac{an \sum_{j=1}^{b} \beta_j^2}{b - 1}$$

$$E(MS_{AB}) = E\left(\frac{SS_{AB}}{(a - 1)(b - 1)}\right) = \sigma^2 + \frac{n \sum_{i=1}^{a} \sum_{j=1}^{b} (\tau\beta)_{ij}^2}{(a - 1)(b - 1)}$$

and

$$E(MS_E) = E\left(\frac{SS_E}{ab(n - 1)}\right) = \sigma^2$$

The total sum of squares

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

The sums of squares for the main effects are

$$SS_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn} \tag{5.7}$$

and

$$SS_B = \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn} \tag{5.8}$$

It is convenient to obtain the SS_{AB} in two stages. First we compute the sum of squares between the ab cell totals, which is called the sum of squares due to "subtotals":

$$SS_{\text{Subtotals}} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn}$$

This sum of squares also contains SS_A and SS_B. Therefore, the second step is to compute SS_{AB} as

$$SS_{AB} = SS_{\text{Subtotals}} - SS_A - SS_B \tag{5.9}$$

We may compute SS_E by subtraction as

$$SS_E = SS_T - SS_{AB} - SS_A - SS_B \tag{5.10}$$

or

$$SS_E = SS_T - SS_{\text{Subtotals}}$$

The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a - 1$	$MS_A = \frac{SS_A}{a - 1}$	$F_0 = \frac{MS_A}{MS_E}$
B treatments	SS_B	$b - 1$	$MS_B = \frac{SS_B}{b - 1}$	$F_0 = \frac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a - 1)(b - 1)$	$MS_{AB} = \frac{SS_{AB}}{(a - 1)(b - 1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n - 1)$	$MS_E = \frac{SS_E}{ab(n - 1)}$	
Total	SS_T	$abn - 1$		

Solution

■ **TABLE 5.4**

Life Data (in hours) for the Battery Design Experiment

Material Type	Temperature (°F) 15			70			125			$y_{i..}$
	130	155		34	40		20	70		
1	74	180	(539)	80	75	(229)	82	58	(230)	998
	150	188		136	122		25	70		
2	159	126	(623)	106	115	(479)	58	45	(198)	1300
	138	110		174	120		96	104		
3	168	160	(576)	150	139	(583)	82	60	(342)	1501
$y_{.j.}$		1738			1291			770		$3799 = y_{...}$

Solution

Using Equations 5.6 through 5.10, the sums of squares are computed as follows:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$= (130)^2 + (155)^2 + (74)^2 + \cdots$$

$$+ (60)^2 - \frac{(3799)^2}{36} = 77{,}646.97$$

$$SS_{\text{Material}} = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn}$$

and

$$SS_{\text{Interaction}} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_{\text{Material}}$$

$$- SS_{\text{Temperature}}$$

$$= \frac{1}{4}[(539)^2 + (229)^2 + \cdots + (342)^2]$$

$$- \frac{(3799)^2}{36} - 10{,}683.72$$

$$- 39{,}118.72 = 9613.78$$

Solution

$$= \frac{1}{(3)(4)}[(998)^2 + (1300)^2 + (1501)^2] - \frac{(3799)^2}{36} = 10{,}683.72$$

$$SS_{\text{Temperature}} = \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn}$$

$$= \frac{1}{(3)(4)}[(1738)^2 + (1291)^2 + (770)^2] - \frac{(3799)^2}{36} = 39{,}118.72$$

$$SS_E = SS_T - SS_{\text{Material}} - SS_{\text{Temperature}} - SS_{\text{Interaction}}$$

$$= 77{,}646.97 - 10{,}683.72 - 39{,}118.72 - 9613.78 = 18{,}230.75$$

The ANOVA is shown in Table 5.5. Because $F_{0.05,4,27} = 2.73$, we conclude that there is a significant interaction between material types and temperature. Furthermore, $F_{0.05,2,27} = 3.35$, so the main effects of material type and temperature are also significant. Table 5.5 also shows the *P*-values for the test statistics.

To assist in interpreting the results of this experiment, it is helpful to construct a graph of the average responses at

Solution

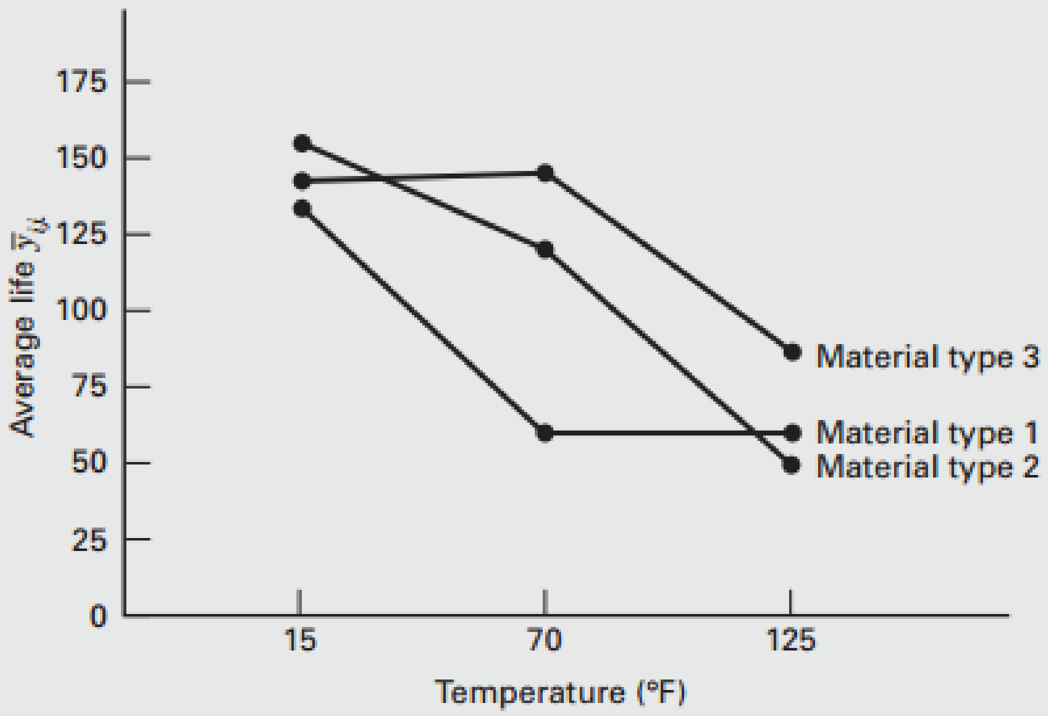
■ **TABLE 5.5**

Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
Material types	10,683.72	2	5,341.86	7.91	0.0020
Temperature	39,118.72	2	19,559.36	28.97	< 0.0001
Interaction	9,613.78	4	2,403.44	3.56	0.0186
Error	18,230.75	27	675.21		
Total	77,646.97	35			

each treatment combination. This graph is shown in Figure 5.9. The significant interaction is indicated by the lack of parallelism of the lines. In general, longer life is attained at low temperature, regardless of material type. Changing from low to intermediate temperature, battery life with material type 3 may actually increase, whereas it decreases for types 1 and 2. From intermediate to high temperature, battery life decreases for material types 2 and 3 and is essentially unchanged for type 1. Material type 3 seems to give the best results if we want less loss of effective life as the temperature changes.

Solution



■ **FIGURE 5.9** **Material type–temperature plot for Example 5.1**

Solution

$$\begin{aligned} SS_{\text{Model}} &= SS_{\text{Material}} + SS_{\text{Temperature}} + SS_{\text{Interaction}} \\ &= 10{,}683.72 + 39{,}118.72 + 9613.78 \\ &= 59{,}416.22 \end{aligned}$$

$$R^2 = \frac{SS_{\text{Model}}}{SS_{\text{Total}}} = \frac{59{,}416.22}{77{,}646.97} = 0.7652$$

Example-2

- A chemical engineer is studying the effects of various reagents and catalyst on the yield of a certain process. Yield is expressed as a percentage of a theoretical maximum. 2 runs of the process were made for each combination of 3 reagents and 4 catalysts.

Catalyst	Reagent		
	1	2	3
A	86.8 82.4	93.4 85.2	77.9 89.6
B	71.9 72.1	74.5 87.1	87.5 82.7
C	65.5 72.4	66.7 77.1	72.7 77.8
D	63.9 70.4	73.7 81.6	79.8 75.7

a) Construct an ANOVA table.
b) Test is there an interaction effect between reagents and catalyst. Use $\alpha = 0.05$.
c) Do we need to test whether there is an effect that is due to reagents or catalyst? Why? If Yes, test is there an effect from reagents or catalyst.

Solution

a) How many treatments involved in this experiment?

Factor A has 4 levels, factor B has 3 levels and each sample has 2 replicates.

Thus, $a = 4$*,* $b = 3$*, and* $r = 2$*. Total treatment = 12.*

b) ANOVA

Catalyst	Reagent			Total
	1	2	3	
A	$x_{11.} = 169.2$	$x_{12.} = 178.6$	$x_{13.} = 167.5$	$x_{1..} = 515.3$
B	$x_{21.} = 144.0$	$x_{22.} = 161.6$	$x_{23.} = 170.2$	$x_{2..} = 475.8$
C	$x_{31.} = 137.9$	$x_{32.} = 143.8$	$x_{33.} = 150.5$	$x_{3..} = 432.2$
D	$x_{41.} = 134.3$	$x_{42.} = 155.3$	$x_{43.} = 155.5$	$x_{4..} = 445.1$
Total	$x_{.1.} = 585.4$	$x_{.2.} = 639.3$	$x_{.3.} = 643.7$	$x_{...} = 1868.4$

Cont…

❖Calculate sum of square values:

$\sum_{i=1}^{4}\sum_{j=1}^{3}\sum_{k=1}^{2}x_{ijk}^2$

$$SST = 86.8^2 + 82.4^2 + \ldots + 75.7^2 - \frac{1868.4^2}{(4)(3)(2)} = 1440.0400$$

$\sum_{i=1}^{4}x_{i..}^2$

$$SSA = \frac{1}{(3)(2)}\left(515.3^2 + 475.8^2 + 432.2^2 + 445.1^2\right) - \frac{1868.4^2}{(4)(3)(2)} = 683.4900$$

$\sum_{j=1}^{3}x_{.j.}^2$

$$SSB = \frac{1}{(4)(2)}\left(585.4^2 + 639.3^2 + 643.7^2\right) - \frac{1868.4^2}{(4)(3)(2)} = 263.4775$$

$\sum_{i=1}^{4}\sum_{j=1}^{3}x_{ij.}^2$

$$SSAB = \frac{1}{2}\left(169.2^2 + 178.6^2 + \ldots + 155.5^2\right) - \frac{1868.4^2}{(4)(3)(2)} - 683.4900 - 263.4775 = 138.7825$$

$$SSE = 1440.0400 - 683.4900 - 263.4775 - 138.7825 = 354.29$$

Cont…

ANOVA table:

Source	Sum Squares	Degree of Freedom	Mean Squares	f_{test}
A (row effect)	683.4900	3	$MSA = \frac{683.4900}{3} = 227.8300$	$f_{testA} = \frac{227.8300}{29.5242} = 7.7167$
B (column effect)	263.4775	2	$MSB = \frac{263.4775}{2} = 131.7388$	$f_{testB} = \frac{131.7388}{29.5242} = 4.4621$
AB (interaction effect)	138.7825	6	$MSAB = \frac{138.7825}{6} = 23.1304$	$f_{testAB} = \frac{23.1304}{29.5242} = 0.7834$
Error	354.2900	12	$MSE = \frac{354.2900}{12} = 29.5242$	
Total	1440.0400	23		

(d) Test if there is an interaction effect between reagents and catalysts on the yield of a chemical process at 5% significance level

H_{0AB} : There is no interaction effect between reagent and catalyst

H_{1AB} : There is an interaction effect between reagent and catalyst

From statistical table, $f_{0.05,6,12} = 2.9961$.

Clearly that, $\left(f_{testAB} = 0.7834\right) < \left(f_{0.05,6,12} = 2.9961\right)$. Thus, do not reject H_0.

At $\alpha = 0.05$, there is no interaction effect between catalyst and reagent on the yield of a chemical process

Since there is no interaction effect, test of row effect and column effect should be conducted as follows.

Cont…

Row effect

H_{0A} : There is no effect of catalysts
H_{1A} : There is an effect of catalyst

From statistical table, $f_{0.05,3,12} = 3.4903$.

Clearly that, $(f_{testA} = 7.7167) > (f_{0.05,3,12} = 3.4903)$. Thus, reject H_0

At $\alpha = 0.05$, there is an effect of catalyst on the yield of a chemical process .

Column effect

H_{0B} : There is no effect of reagents
H_{1B} : There is an effect of reagents

From statistical table, $f_{0.05,2,12} = 3.8853$.

Clearly that, $(f_{testB} = 4.4621) > (f_{0.05,2,12} = 3.8853)$. Thus, reject H_0

At $\alpha = 0.05$, there is an effect of reagents on the yield of a chemical process.

Solve two-way ANOVA by using Microsoft EXCEL

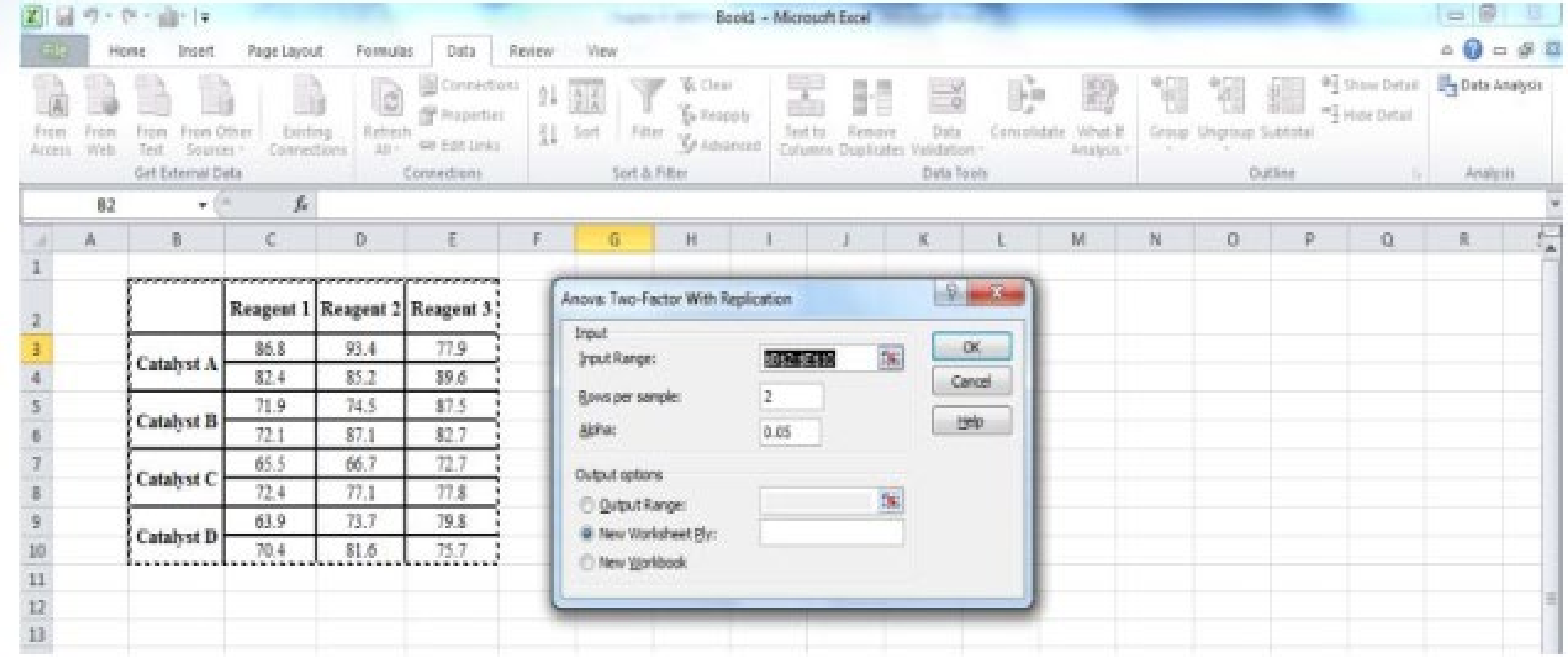
1. Excel – Key in data

	Reagent 1	Reagent 2	Reagent 3
Catalyst A	86.8	93.4	77.9
	82.4	85.2	89.6
Catalyst B	71.9	74.5	87.5
	72.1	87.1	82.7
Catalyst C	65.5	66.7	72.7
	72.4	77.1	77.8
Catalyst D	63.9	73.7	79.8
	70.4	81.6	75.7

Cont…

2. Follow the steps below:

❖ Data-Data Analysis → ANOVA two factor with replication → enter the data range → set a value for *a* → *OK*



Cont...

3. ANOVA table:

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Sample	683.49	3	227.83	7.716729	0.003903	3.490295
Columns	263.4775	2	131.7388	4.462065	0.03558	3.885294
Interaction	138.7825	6	23.13042	0.78344	0.598995	2.99612
Within	354.29	12	29.52417			

Reject H_0 if *P*-value ≤ *α* or *F* > *F crit*

H_{0AB} : There is no interaction effect between reagent and catalyst

H_{1AB} : There is an interaction effect between reagent and catalyst

Clearly that, $(Pvalue = 0.5990) > (\alpha = 0.05)$. Thus, do not reject H_0.

At $\alpha = 0.05$, there is no interaction effect between catalyst and reagent on the yield of a chemical process. Since there is no interaction effect, test of row effect and column effect should be conducted.

Cont…

3. ANOVA table:

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
catalyst → Sample	683.49	3	227.83	7.716729	0.003903	3.490295
reagents → Columns	263.4775	2	131.7388	4.462065	0.03558	3.885294
Interaction	138.7825	6	23.13042	0.78344	0.598995	2.99612
Within	354.29	12	29.52417			

H_{0A} : There is no effect of catalysts
H_{1A} : There is an effect of catalyst

H_{0B} : There is no effect of reagents
H_{1B} : There is an effect of reagents

ROW EFFECT:

$(P\text{-value} = 0.0039) < (\alpha = 0.05)$, the decision is reject H_0.

At $\alpha = 0.05$, there is an effect of catalyst on the yield of a chemical process .

COLUMN EFFECT:

$(P\text{-value} = 0.0356) < (\alpha = 0.05)$, the decision is reject H_0

At $\alpha = 0.05$, there is an effect of reagents on the yield of a chemical process.

Exercise

- A study was done to determine the effects of two factors on the lather ability of soap. The two factors were type of water and glycerol. The outcome measured was the amount of foam produced in mL. The experiment was repeated 3 times for each combination of factors. The result are presented in the following table..

Water type	Glycerol	Foam (mL)
De-ionized	Absent	168 178 168
De-ionized	Present	160 197 200
Tap	Absent	152 142 142
Tap	Present	139 160 160

Construct an ANOVA table and test is there an interaction effect between factors. Use $\alpha = 0.05$.

3 WAY ANOVA

introduction

- Three way ANOVA means three variables,each have two or more than two levels.
- If there are ‘n’ levels of one variable,’m’ levels of second variables, and ‘l’ variables of third variable,then it can be called ‘nxmxl’ factorial design ANOVA.

A machine is used to fill 5-gallon metal containers with soft drink syrup. The variable of interest is the amount of syrup loss due to frothing. Three factors are thought to influence frothing: the nozzle design (A), the filling speed (B), and the operating pressure (C). Three nozzles, three filling speeds, and three pressures are chosen, and two replicates of a 3^3 factorial experiment are run. The coded data are shown in Table 9.1.

The analysis of variance for the syrup loss data is shown in Table 9.2. The sums of squares have been computed by the usual methods. We see that the filling speed and operating pressure are statistically significant. All three two-factor interactions are also significant. The two-factor interactions are analyzed graphically in Figure 9.4. The middle level of speed gives the best performance, nozzle types 2 and 3, and either the low (10 psi) or high (20 psi) pressure seems most effective in reducing syrup loss.

■ **TABLE 9.1**

Syrup Loss Data for Example 9.1 (units are cubic centimeters −70)

	Nozzle Type (A)								
	1			2			3		
	Speed (in RPM) (B)								
Pressure (in psi) (C)	100	120	140	100	120	140	100	120	140
10	−35	−45	−40	17	−65	20	−39	−55	15
	−25	−60	15	24	−58	4	−35	−67	−30
15	110	−10	80	55	−55	110	90	−28	110
	75	30	54	120	−44	44	113	−26	135
20	4	−40	31	−23	−64	−20	−30	−61	54
	5	−30	36	−5	−62	−31	−55	−52	4

Solution

source	SS	df	MS	F
A	[A]-[T]	a-1	SS / df	MS / MSE
B	[B]-[T]	b-1		
C	[C]-[T]	c-1		
A × B	[AB]-[A]-[B]+[T]	(a-1)(b-1)		
A × C	[AC]-[A]-[C]+[T]	(a-1)(c-1)	SS / df	
B × C	[BC]-[B]-[C]+[T]	(b-1)(c-1)		
A × B × C	[ABC]-[AB]-[AC] - [BC]+[A]+[B]+[C]-[T]	(a-1)(b-1)(c-1)		
S/ABC	[Y]-[ABC]	abc(n-1)	SS / df	
Total	[Y]-[T]	abcn-1		


[A]
[B]
[C]
[AB]
[AC]
[BC]
[ABC]
[Y]
[T]

Research Methods and Experimental Design

Lecture 8: Experimental Design: Fractional factorial design

How to handle large number of factorial

- ❖When testing or developing a new process, we do not know which factors influencing it and to what extent.

- ❖Hence we need to include a large number of factors so that no potential influential factor is excluded.

How to handle large number of factorial

- However, even a 2^n factorial design involving all these factor require considerable manpower and investment.

- Hence, we may do a sequential study wherein a subset of (called as the fraction) of the overall 2^n design if first carried out and those results are analyzed.

Closer look at 2^n design

- The 2^n design models the response in terms of main factors, two-factor interaction, three factor interactions etc.

- Further, the level of significance of the interaction may decline with increasing order.(i.e. binary >tertiary >quarterly etc.)

- When repeated are not carried out, the higher order interaction effects may be clubbed and their total sum of sqared may be used for mean square error estimation.

Closer look at 2^n design

- Among n factors,we have
- n-main factors
- nC_2-two factorial interaction,
- nC_3-three factorial interaction

$$nC_n = \frac{n!}{(n-r)!\,r!}$$

Closer look at 2^n design

- Hence in a 2^5 design, we have

- 5 main factorial
- 10 two factorial interaction
- 10 three factorial interaction
- 5 four factorial interaction
- 1 five factor interaction

Closer look at 2^n design

- Hence, more than 50% of the total number of factors are three factors interaction and above

- This fact may be exploited while running fractional factorial design involving many factors.

Construction of the fractions in the 2^n design

- An allowable fraction may be (1/2) or (1/4) or (1/8) fraction of the overall design depending on the value of n.
- For example in 2^4 design we have a half (1/2) fraction comprising of 8 runs or a quarter (1/4) fraction comprising 4 runs.

Construction of the fractions in the 2^n design

- A fraction may be denoted as follow

$$\frac{1}{2^p}2^n = 2^{n-p} \; where, n > p$$

- The table of contrasts may be used to set up the different fractions
- It is evident that while dealing with individual fractions there could be a potential loss of information

Identification of fractions

- Most important is to ensure that atleast the main effects (single factor) are identified without them getting aliased with other main factors.

- Even interactions are important ,sometimes more so.

Identification of fractions

- Let us work with the fairly large 2^4 design
- A full set comprising of 16 experiments
- Let us start with a 2^{4-1} design i.e. constructing two half fractions of 2^3 runs each from the full 2^4 design

Identification of fractions

- We may look at the highest order interaction factors ABCD and split the overall design into two fractions according to the “+1” and “-1” sign in the ABCD column

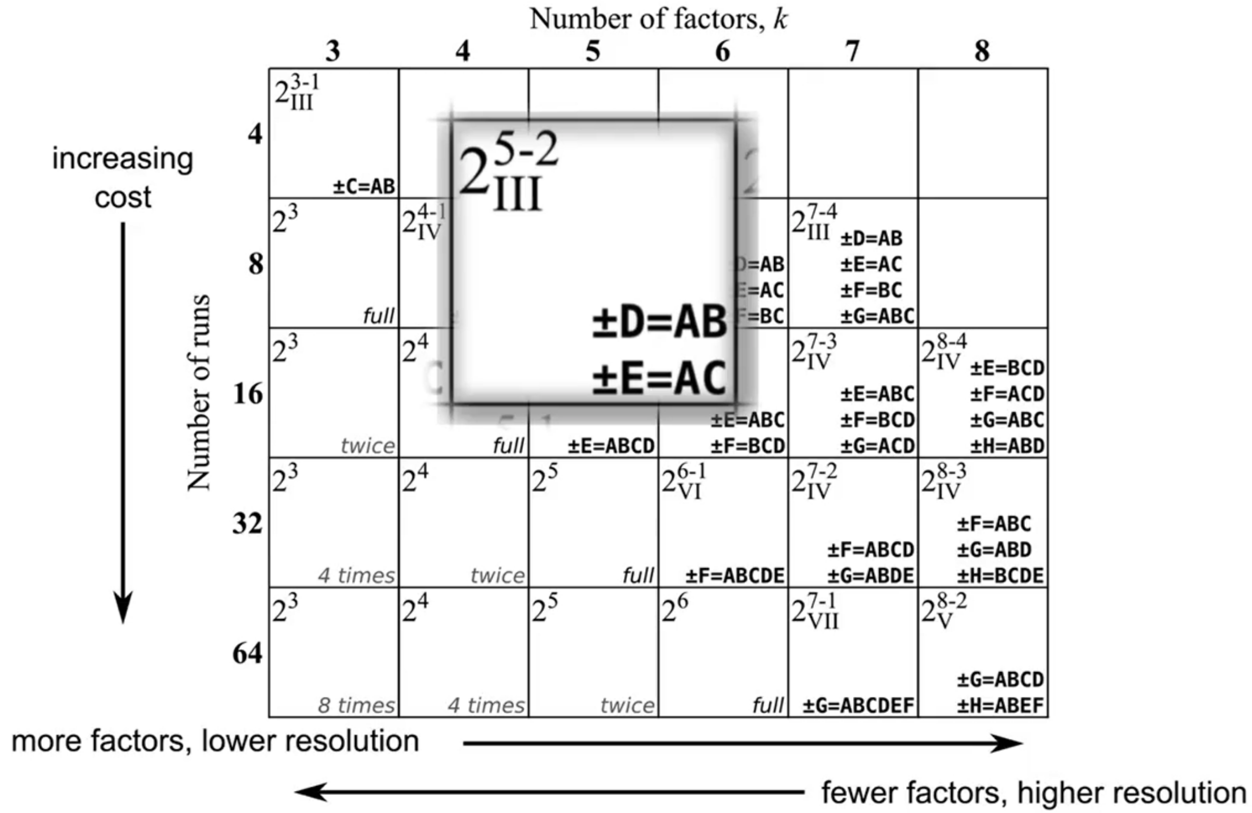
Identification of fractions

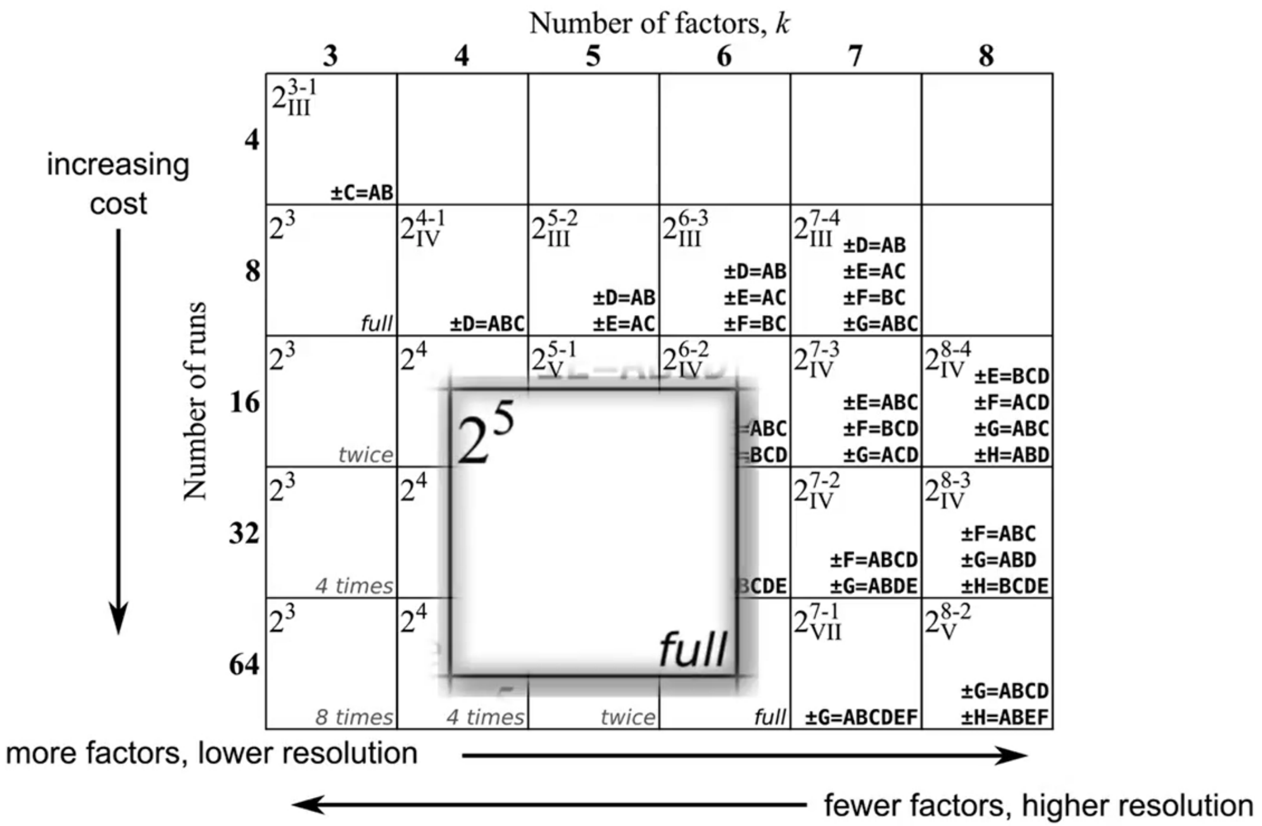
increasing cost

Number of runs \ Number of factors, *k*	3	4	5	6	7	8
4	2^{3-1}_{III} ±C=AB					
8	2^{3} *full*	2^{4-1}_{IV} ±D=ABC	2^{5-2}_{III} ±D=AB ±E=AC	2^{6-3}_{III} ±D=AB ±E=AC ±F=BC	2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC	
16	2^{3} *twice*	2^{4} *full*	2^{5-1}_{V} ±E=ABCD	2^{6-2}_{IV} ±E=ABC ±F=BCD	2^{7-3}_{IV} ±E=ABC ±F=BCD ±G=ACD	2^{8-4}_{IV} ±E=BCD ±F=ACD ±G=ABC ±H=ABD
32	2^{3} *4 times*	2^{4} *twice*	2^{5} *full*	2^{6-1}_{VI} ±F=ABCDE	2^{7-2}_{IV} ±F=ABCD ±G=ABDE	2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=BCDE
64	2^{3} *8 times*	2^{4} *4 times*	2^{5} *twice*	2^{6} *full*	2^{7-1}_{VII} ±G=ABCDEF	2^{8-2}_{V} ±G=ABCD ±H=ABEF

more factors, lower resolution

fewer factors, higher resolution


Number of factors, k
3
4
5
6
7
8
Number of runs
4
8
16
32
64
increasing cost
±C=AB
full
twice
4 times
8 times
±D=AB
±E=AC
±F=BC
±G=ABC
±E=ABCD
±E=ABC
±F=BCD
±G=ACD
±E=BCD
±F=ACD
±G=ABC
±H=ABD
±F=ABCDE
±F=ABCD
±G=ABDE
±F=ABC
±G=ABD
±H=BCDE
±G=ABCDEF
±G=ABCD
±H=ABEF
more factors, lower resolution
fewer factors, higher resolution



Number of factors, k
3
4
5
6
7
8
Number of runs
4
8
16
32
64
increasing cost
2^{3-1}_{III}
±C=AB
2^3
full
2^{4-1}_{IV}
±D=ABC
2^{5-2}_{III}
±D=AB
±E=AC
2^{6-3}_{III}
±D=AB
±E=AC
±F=BC
2^{7-4}_{III}
±D=AB
±E=AC
±F=BC
±G=ABC
2^3
twice
2^4
2^{5-1}_{V}
2^{6-2}_{IV}
2^{7-3}_{IV}
±E=ABC
±F=BCD
±G=ACD
2^{8-4}_{IV}
±E=BCD
±F=ACD
±G=ABC
±H=ABD
2^3
4 times
2^4
2^5
full
2^{7-2}_{IV}
±F=ABCD
±G=ABDE
2^{8-3}_{IV}
±F=ABC
±G=ABD
±H=BCDE
2^3
8 times
2^4
4 times
twice
full
2^{7-1}_{VII}
±G=ABCDEF
2^{8-2}_{V}
±G=ABCD
±H=ABEF
more factors, lower resolution
fewer factors, higher resolution

Number of factors, *k*

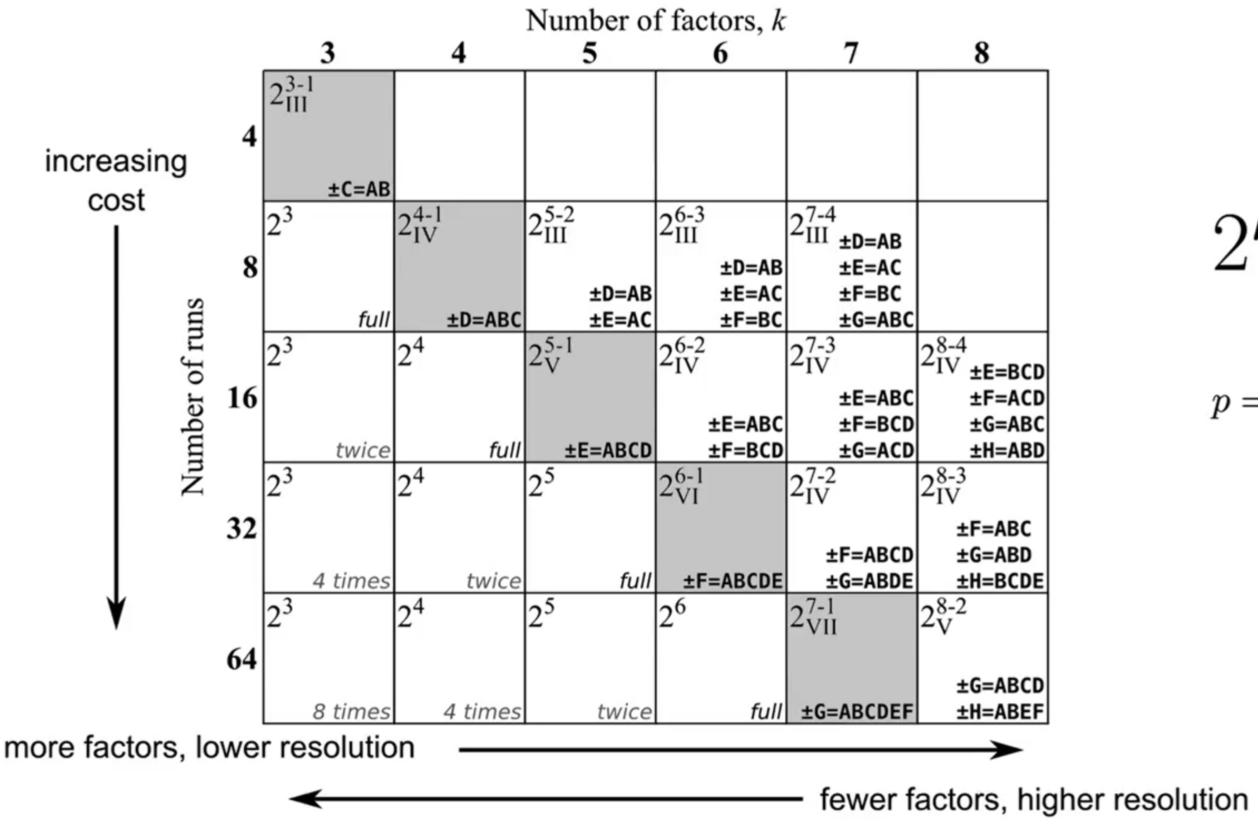
Number of runs	3	4	5	6	7	8
4	2^{3-1}_{III} ±C=AB					
8	2^3 *full*	2^{4-1}_{IV} ±D=ABC	2^{5-2}_{III} ±D=AB ±E=AC	2^{6-3}_{III} ±D=AB ±E=AC ±F=BC	2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC	
16	2^3 *twice*	2^4 *full*	2^{5-1}_{V} ±E=ABCD	2^{6-2}_{IV} ±E=ABC ±F=BCD	2^{7-3}_{IV} ±E=ABC ±F=BCD ±G=ACD	2^{8-4}_{IV} ±E=BCD ±F=ACD ±G=ABC ±H=ABD
32	2^3 *4 times*	2^4 *twice*	2^5 *full*	2^{6-1}_{VI} ±F=ABCDE	2^{7-2}_{IV} ±F=ABCD ±G=ABDE	2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=BCDE
64	2^3 *8 times*	2^4 *4 times*	2^5 *twice*	2^6 *full*	2^{7-1}_{VII} ±G=ABCDEF	2^{8-2}_{V} ±G=ABCD ±H=ABEF

increasing cost ↓

more factors, lower resolution →

← fewer factors, higher resolution

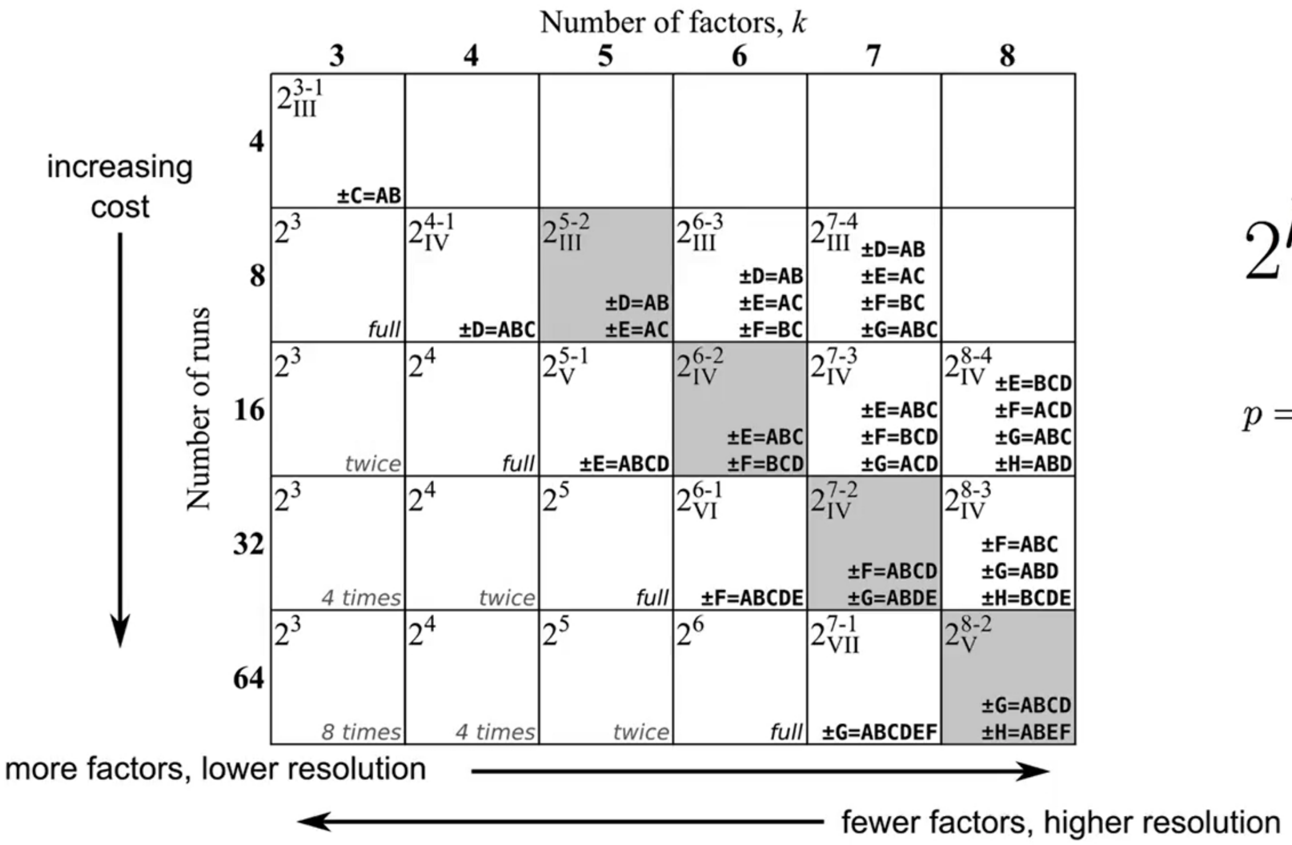
$$2^{k-p}$$



Number of factors, k
3
4
5
6
7
8
Number of runs
4
8
16
32
64
increasing cost
2^{3-1}_{III} ±C=AB
2^3 full
2^{4-1}_{IV} ±D=ABC
2^{5-2}_{III} ±D=AB ±E=AC
2^{6-3}_{III} ±D=AB ±E=AC ±F=BC
2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC
2^3 twice
2^4 full
2^{5-1}_{V} ±E=ABCD
2^{6-2}_{IV} ±E=ABC ±F=BCD
2^{7-3}_{IV} ±E=ABC ±F=BCD ±G=ACD
2^{8-4}_{IV} ±E=BCD ±F=ACD ±G=ABC ±H=ABD
2^3 4 times
2^4 twice
2^5 full
2^{6-1}_{VI} ±F=ABCDE
2^{7-2}_{IV} ±F=ABCD ±G=ABDE
2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=BCDE
2^3 8 times
2^4 4 times
2^5 twice
2^6 full
2^{7-1}_{VII} ±G=ABCDEF
2^{8-2}_{V} ±G=ABCD ±H=ABEF
more factors, lower resolution
fewer factors, higher resolution

$$2^{k-p}$$

$$p = 1$$


Number of factors, k
3
4
5
6
7
8
increasing cost
Number of runs
4
8
16
32
64
±C=AB
full
±D=ABC
±D=AB
±E=AC
±D=AB
±E=AC
±F=BC
±D=AB
±E=AC
±F=BC
±G=ABC
twice
full
±E=ABCD
±E=ABC
±F=BCD
±E=ABC
±F=BCD
±G=ACD
±E=BCD
±F=ACD
±G=ABC
±H=ABD
4 times
twice
full
±F=ABCDE
±F=ABCD
±G=ABDE
±F=ABC
±G=ABD
±H=BCDE
8 times
4 times
twice
full
±G=ABCDEF
±G=ABCD
±H=ABEF
more factors, lower resolution
fewer factors, higher resolution

$$2^{k-p}$$

$p = 2$

Fractional Factorial For 2-Level Designs

Run	A	B	C	AB	AC	BC	ABC
1	-1	-1	-1	+1	+1	+1	-1
2	-1	-1	+1	+1	-1	-1	+1
3	-1	+1	-1	-1	+1	-1	+1
4	-1	+1	+1	-1	-1	+1	-1
5	+1	-1	-1	-1	-1	+1	+1
6	+1	-1	+1	-1	+1	-1	-1
7	+1	+1	-1	+1	-1	-1	-1
8	+1	+1	+1	+1	+1	+1	+1

Fractional Factorial For 2-Level Designs

Run	BCD	ACD	ABD	CD	BD	AD	D
	A	B	C	AB	AC	BC	ABC
1	-1	-1	-1	+1	+1	+1	-1
2	-1	-1	+1	+1	-1	-1	+1
3	-1	+1	-1	-1	+1	-1	+1
4	-1	+1	+1	-1	-1	+1	-1
5	+1	-1	-1	-1	-1	+1	+1
6	+1	-1	+1	-1	+1	-1	-1
7	+1	+1	-1	+1	-1	-1	-1
8	+1	+1	+1	+1	+1	+1	+1

Confounding or Aliasing

X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3
+	−	−	−	−	+
−	−	+	+	−	−
−	+	−	−	+	−
+	+	+	+	+	+

X3 = X1X2 → X1X3 = X2 and X2X3 = X1
(main effects aliased with two-factor interactions) – Resolution III design

6 factors with 16 runs?

¼ fraction of $2^6 = 2^{6-2}$ FFD

Place x5 Here → x2x3x4

Place x6 Here → x1x2x3x4

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	+	-	+	+	-	+	-	+	-	-	+	-	-	-
-	-	+	+	+	-	-	-	-	+	+	+	-	-	+
+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X2*X3*X4; X6 = X1*X2*X3*X4; → X5*X6 = X1

2^{6-2} Experiment

Place x5 Here ↓ (x1x2x3)

Place x6 Here ↓ (x2x3x4)

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	+	-	+	+	-	+	-	+	-	-	+	-	-	-
-	-	+	+	+	-	-	-	-	+	+	+	-	-	+
+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

$$X5 = X1*X2*X3; \quad X6 = X2*X3*X4 \rightarrow X5*X6 = X1*X4$$

Aliasing Relationships

I = 1235 = 2346 = 1456

Main-effects:

1=235=456=2346; 2=135=346=1456; 3=125=246=1456; 4=...

15-possible 2-factor interactions:

12=35

13=25

14=56

15=23=46

16=45

24=36

26=34

¼ fraction of $2^6 = 2^{6-2}$ FFD

Place x5 Here → x2x3x4

Place x6 Here → x1x2x3x4

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	+	-	+	+	-	+	-	+	-	-	+	-	-	-
-	-	+	+	+	-	-	-	-	+	+	+	-	-	+
+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X2*X3*X4; X6 = X1*X2*X3*X4; → X5*X6 = X1

or I = 2345 = 12346 = 156 → Resolution III design

2^{6-2} Experiment

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3 (Place x5 Here)	x1x2x4	x1x3x4	x2x3x4 (Place x6 Here)	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
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+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4

or I = 1235 = 2346 = 1456 → Resolution IV design

Design Generators and Resolution

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4

5 = 123; 6 = 234; 56 = 14 →

Generators: I = 1235 = 2346 = 1456

Resolution: Length of the shortest "word"
in the generator set → resolution IV here

Resolution

Resolution III: (1+2)

Main effect aliased with 2-order interactions

Resolution IV: (1+3 or 2+2)

Main effect aliased with 3-order interactions and

2-factor interactions aliased with other 2-factor …

Resolution V: (1+4 or 2+3)

Main effect aliased with 4-order interactions and

2-factor interactions aliased with 3-factor interactions

Selected 2^k Fractional Designs

Design	Runs	Design Generator	Resolution
2^{3-1}	4	C = AB	III
2^{4-1}	8	D = ABC	IV
2^{5-1}	16	E = ABCD	V
2^{5-2}	8	D = AB, E = AC	III
2^{6-1}	32	F = ABCDE	VI
2^{6-2}	16	E = ABC, F = ACD	IV
2^{6-3}	8	D = AB, E = AC, F = BC	III

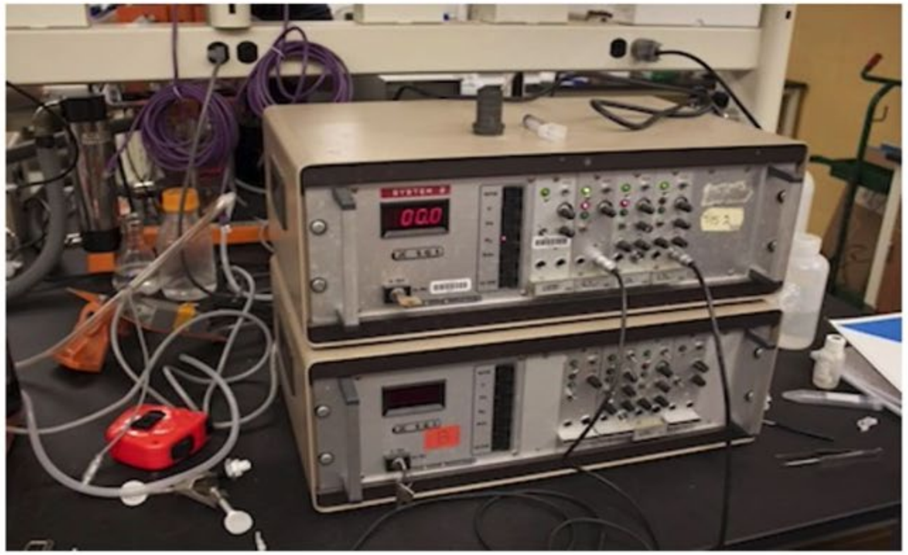
Resolution tells us which terms are confounded

Example

Cell-culture example: long duration runs; and many factors are possible

Laboratory equipment to control the culture:

1. **T**: the temperature profile
2. **D**: dissolved oxygen
3. **A**: agitation rate
4. **P**: pH
5. **S**: substrate type (A or B)



[Flickr: mjanicki]

3 months available: that corresponds to 9 experiments.

Research Methods and Experimental Design

Lecture 8: Experimental Design: Response surface method (RSM)

Introduction: Response surface method (RSM)

- Response surface method (RSM) is one of the powerful statistical experimental design techniques that is applied t **build models** and **investigate individual** and **interaction effects** of the selected **operating condition** on the give response in a given experiment.

- It is a very effective approach for **optimization** of **complex processes** in a more convenient way resulting in **saving time**, **labor**, and **cost**.

Introduction: Response surface method (RSM)

- Response-surface methodology comprises a body of methods for exploring for **optimum operating conditions** through **experimental methods**.

- Typically, this involves doing several experiments, using the results of one experiment to provide **direction** for what to do **next**.

Introduction…

- Response surface methodology (RSM) is a collection of **mathematical** and **statistical techniques** for empirical model building.
- By careful design of experiments, the objective is to **optimize** a **response** (output variable) which is influenced by several **independent variables** (input variables).
- An experiment is a series of tests, **called runs**, in which changes are made in the **input variables** in order to **identify** the **reasons** for **changes** in the **output response**.

Introduction…

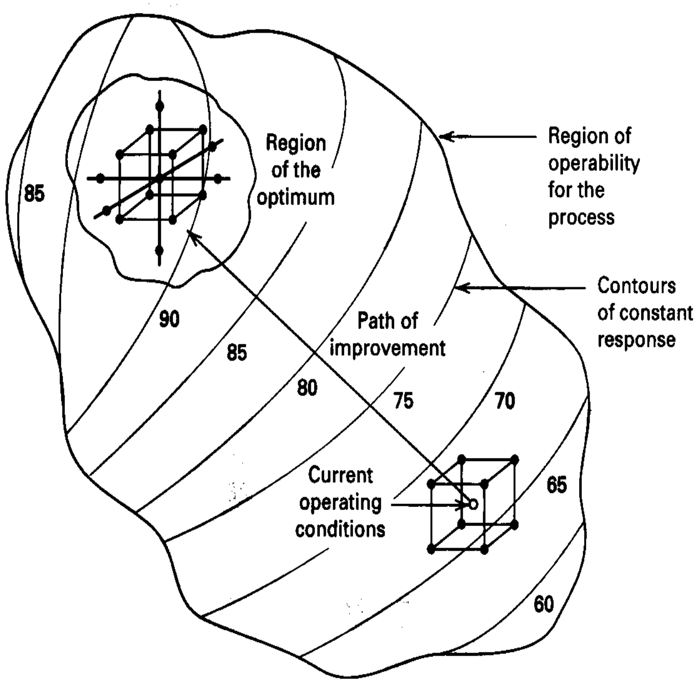


Figure 11-3 The sequential nature of RSM.

- A sequential procedure
- The objective is to lead the experimenter rapidly and efficiently along a path of improvement toward the general vicinity of the optimum.
- First-order model => Second-order model
- Climb a hill

Introduction…

- **Models are simple polynomials**
- **Include terms for interaction and curvature**
- **Coefficients are usually established by regression analysis with a computer program**
- **Insignificant terms are discarded**

RESPONSE SURFACE MODEL FOR TWO FACTORS

Response Surface Model for two factors X_1 and X_2 and measured response Y (Regardless of number of levels):

$Y = \beta_0$	constant
$+ \beta_1 X_1 + \beta_2 X_2$	main effects
$+ \beta_3 X_1^2 + \beta_4 X_2^2$	curvature
$+ \beta_5 X_1 X_2$	interaction
$+ \varepsilon$	error

RESPONSE SURFACE MODEL FOR THREE FACTORS TWO LEVELS

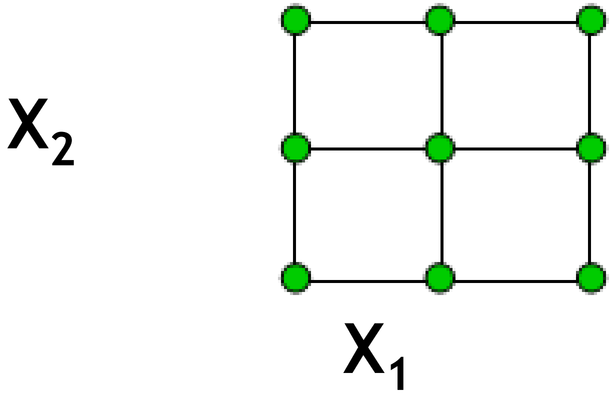
$Y = \beta_0$	constant
$+ \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$	main effects
$+ \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2$	curvature
$+ \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3$	interactions
$+ \varepsilon$	error

(Note that higher order interactions are not included.)

THREE LEVEL FACTORIAL EXPERIMENTS FOR TWO FACTORS

- **3^2 Factorial Experiments**

  - **Geometric Presentation**



  - **Mathematical Model**

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1^2 + \beta_4 X_2^2 + \beta_5 X_1 X_2 + \beta_6 X_1^2 X_2 + \beta_7 X_1 X_2^2 + \beta_8 X_1^2 X_2^2 + \varepsilon$$

RESPONSE SURFACE MODEL FOR TWO FACTOR EXPERIMENT

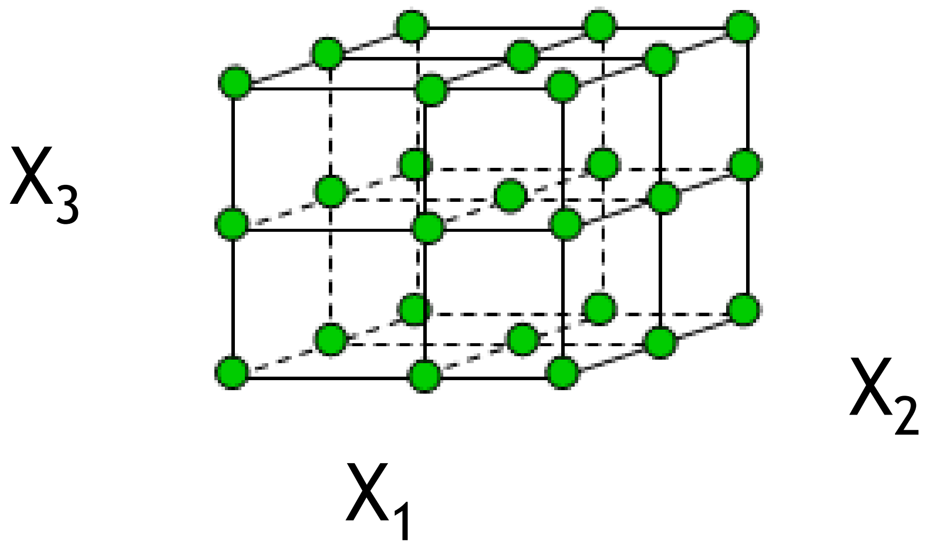
$Y = \beta_0$	constant
$+ \beta_1 X_1 + \beta_2 X_2$	main effects
$+ \beta_3 X_1^2 + \beta_4 X_2^2$	curvature
$+ \beta_5 X_1 X_2$	interaction
$+ \varepsilon$	error

All the other terms are dropped into the error term.

THREE LEVEL FACTORIAL EXPERIMENTS FOR THREE FACTORS

❖ 3^3 FACTORIAL EXPERIMENT

– Geometric Presentation



THREE LEVEL FACTORIAL FOR THREE FACTOR EXPERIMENT

❖ **Mathematical Model**

$$Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4 X_1X_2 + \beta_5X_1X_3 + \beta_6X_2X_3$$
$$+ \beta_7X_1^2 + \beta_8X_2^2 + \beta_9X_3^2 + \beta_{10}X_1^2X_2 + \beta_{11}X_1^2X_3$$
$$+ \beta_{12}X_1X_2^2 + \beta_{13}X_2^2X_3 + \beta_{14}X_1X_3^2 + \beta_{15}X_2X_3^2$$
$$+ \beta_{16}X_1^2X_2^2 + \beta_{17}X_1^2X_3^2 + \beta_{18}X_2^2X_3^2 + \beta_{19}X_1X_2X_3$$
$$+ \beta_{20}X_1^2X_2X_3 + \beta_{21}X_1X_2^2X_3 + \beta_{22}X_1X_2X_3^2 + \beta_{23}X_1^2X_2^2X_3$$
$$+ \beta_{24}X_1^2X_2X_3^2 + \beta_{25}X_1X_2^2X_3^2 + \beta_{26}X_1^2X_2^2X_3^2 + \varepsilon$$

RESPONSE SURFACE MODEL FOR THREE FACTOR EXPERIMENT

$Y = \beta_0$	constant
$+ \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$	main effects
$+ \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2$	curvature
$+ \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3$	interaction
$+ \varepsilon$	error

All the other terms are dropped into the error term.

NUMBER OF RUNS FOR A 3k FACTORIAL EXPERIMENT

Number of factors (k)	Number of runs
2	9
3	27
4	81
5	243 [81]
6	729 [243]
7	2189 [729]

- ❖The number inside [brackets] is the number of runs needed for a third replicate of the full 3^k factorial experiment

Types of RSM

- ***Central composite design (CCD)***

- **Box-Behnken design (BBD)**

CCD

- The most popular response surface method (RSM) design is the central composite design (CCD).
- A CCD has three groups of design points:

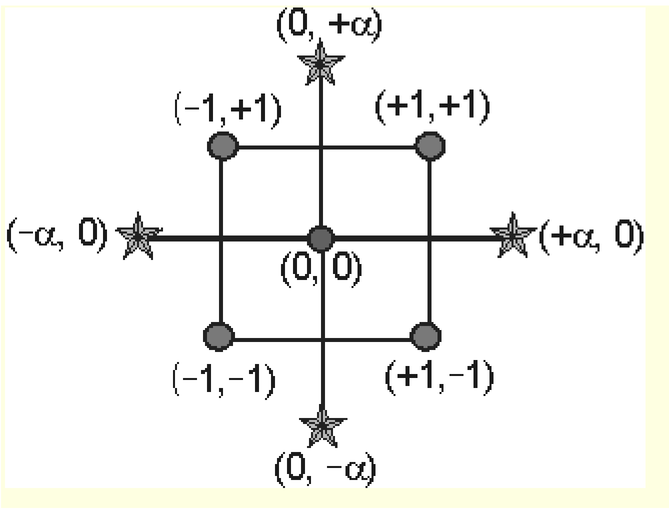
(a) ***two-level*** factorial or fractional factorial design points

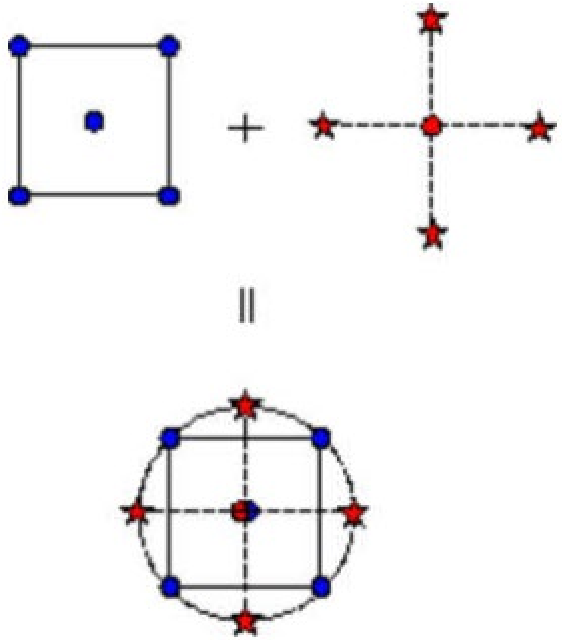
(b) axial points (sometimes called "star" points)

(c) centre points

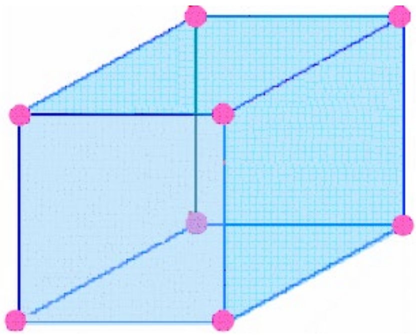
CCD

- CCD's are **designed to estimate the coefficients of a quadratic model**. All point descriptions will be in terms of coded values of the factors.





3 FACTOR CENTRAL COMPOSITE DESIGNS



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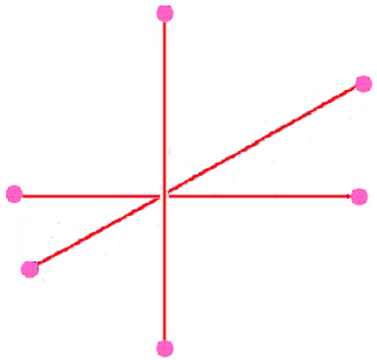
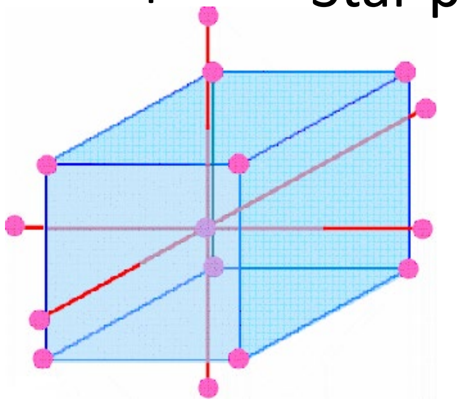


Factorial + Star points

=



CCD

Determining in Central Composite Designs

- ❖the value of depends on the number of experimental runs in the factorial portion of the central composite design:

$$\alpha = [number\ of\ factorial\ runs]^{1/4}$$

If the factorial is a full factorial, then

$$\alpha = \left[2^k\right]^{1/4}$$

Upper and Lower Limits

- **Upper star level**

High star value =(Average of factor level at (-1) and (+1)) + Alpha((range between the (-1) and (+1) level)/2)

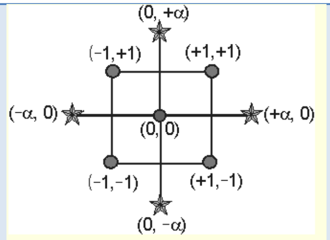
- **Lower star level**

High star value =(Average of factor level at (-1) and (+1)) - Alpha((range between the (-1) and (+1) level)/2)

Determining for Rotatability

Number of Factors	**Factorial Portion**	**Scaled Value for α Relative to ±1**
2	2^2	$2^{2/4} = 1.414$
3	2^3	$2^{3/4} = 1.682$
4	2^4	$2^{4/4} = 2.000$
5	2^{5-1}	$2^{4/4} = 2.000$
5	2^5	$2^{5/4} = 2.378$
6	2^{6-1}	$2^{5/4} = 2.378$
6	2^6	$2^{6/4} = 2.828$

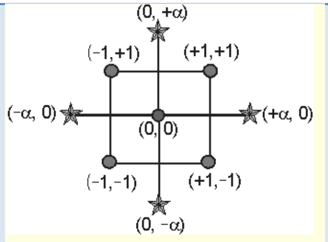
1. Factorial Points



- The **two-level factorial** part of the design consists of all possible combinations of the +1 and -1 levels of the factors. For the two factor case there are four design points:

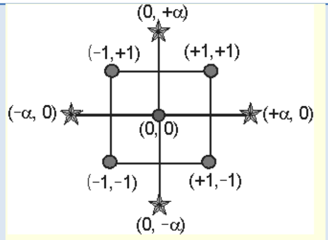
- (-1, -1) (+1, -1) (-1, +1) (+1, +1)

2. Star or Axial Points



- ❖The star points have all of the factors set to 0, the midpoint, except one factor, which has the value +/- Alpha. For a two factor problem, the star points are:
- ❖(-Alpha, 0) (+Alpha, 0) (0, -Alpha) (0, +Alpha)
- ❖The value for Alpha is calculated in each design for both **rotatability** and **orthogonality** of blocks. The experimenter can choose between these values or enter a different one. The default value is set to the rotatable value.

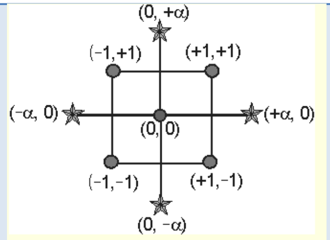
2. Star or Axial Points



- Another position for the star points is at the face of the cube portion on the design. This is commonly referred to as a **face-centered central composite design**.

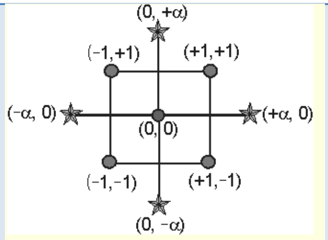
- You can create this by **setting the alpha value equal to one**, or choosing the **Face Centered option**. This design only requires **three levels for each factor**.

3. Center Points



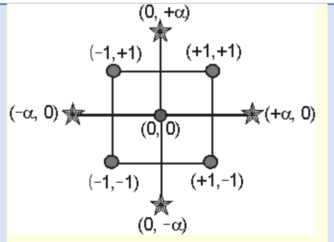
- Center points, as implied by the name, are points with all levels set to coded level 0 - the midpoint of each factor range:
- (0, 0)
- Center points are usually **repeated 4-6 times** to get a **good estimate of experimental error (pure error)**. For example, with **two factors the design** will be created with **five center** points by default. These runs can be identified in the design layout by doing a right mouse click on the Block column heading and choosing Show Point Type.

3. Center Points



- To summarize, central composite designs require 5 levels of each factor: **-Alpha, -1, 0, 1, and +Alpha**.
- One of the commendable attributes of the central composite design is that its structure lends itself to sequential experimentation.
- Central composite designs can be carried out in blocks.

Categorical Factors



- You may also add **categorical factors** to this design.
- This will cause the number of runs generated to be multiplied by the number of combinations of the **categorical factor levels.**

Response model

- If the response is well modeled by a linear function of the independent variables, then the approximating function is the first-order model (linear):
- $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon$

- This model can be obtained from a 2^k or 2^{k-p} design.

- If there is curvature in the system, then a polynomial of higher degree must be used, such as the second-order model:
- $Y = \beta_0 + \Sigma\beta_i x_i + \Sigma\beta_{ii} x^2_i + \Sigma\Sigma\beta_{ij} x_i x_j + \varepsilon$

- This model has linear + interaction + quadratic terms.

COMPARISON OF CCD WITH 3^k FACTORIAL EXPERIMENTS

- ❖ Are as efficient as 3^k factorial experiments

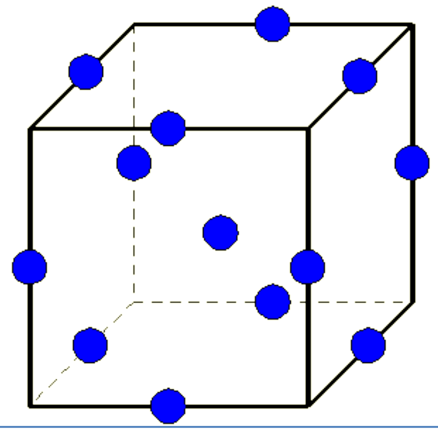
  - minimum number of trials for estimating main effects and quadratic terms

- ❖ Require less runs than 3^k factorial experiments

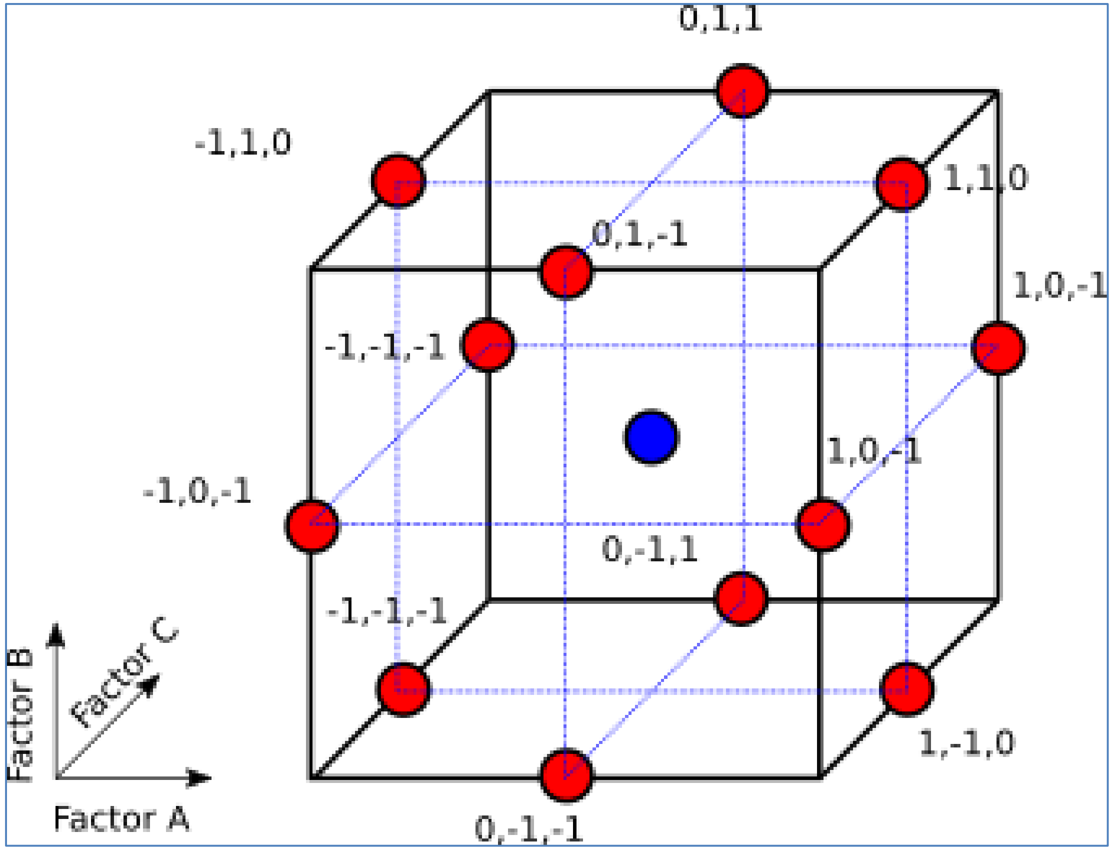
- ❖ Allow sequential experimentation, which provides flexibility in running the experiment

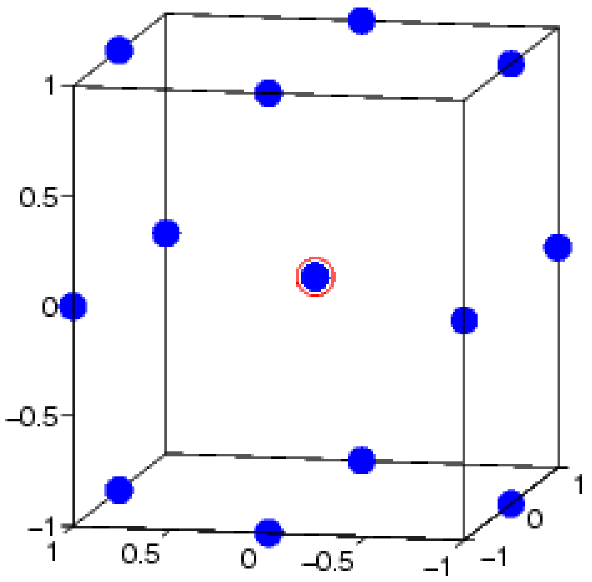
BBD

- The Box-Behnken design is an independent quadratic design in that it does **not contain** an **embedded factorial** or **fractional factorial design**.
- In this design the treatment combinations are at the **midpoints** of edges of the **process space** and at **the centre**.



A Box-Behnken Design for Three Factors





BBD

- ❖is **one kind of RSM**, which helps to design a **second-order response model**.
- ❖can provide a **maximum amount of complex information** with **minimum experimental time**.
- ❖In comparison with other statistical methods, such as full factorial design, it **requires a few number of runs**.
- ❖**it avoids the analyses at their extreme combinations** (such as at **highest** and **lowest** levels) for which unsatisfactory results might occur

BBD

- the total number of experiment can be calculated as

$$N = k^2 + k + C_p$$

- where k is a number of factors, and Cp is a central replication point

Comparisons of response surface designs

CCC (CCI)				CCF				Box-Behnken			
Rep	X_1	X_2	X_3	Rep	X_1	X_2	X_3	Rep	X_1	X_2	X_3
1	-1	-1	-1	1	-1	-1	-1	1	-1	-1	0
1	+1	-1	-1	1	+1	-1	-1	1	+1	-1	0
1	-1	+1	-1	1	-1	+1	-1	1	-1	+1	0
1	+1	+1	-1	1	+1	+1	-1	1	+1	+1	0
1	-1	-1	+1	1	-1	-1	+1	1	-1	0	-1
1	+1	-1	+1	1	+1	-1	+1	1	+1	0	-1
1	-1	+1	+1	1	-1	+1	+1	1	-1	0	+1
1	+1	+1	+1	1	+1	+1	+1	1	+1	0	+1
1	-1.682	0	0	1	-1	0	0	1	0	-1	-1
1	1.682	0	0	1	+1	0	0	1	0	+1	-1
1	0	-1.682	0	1	0	-1	0	1	0	-1	+1
1	0	1.682	0	1	0	+1	0	1	0	+1	+1
1	0	0	-1.682	1	0	0	-1	3	0	0	0
1	0	0	1.682	1	0	0	+1				
6	0	0	0	6	0	0	0				
Total Runs = 20				Total Runs = 20				Total Runs = 15			

Number of Runs Required by Central Composite and Box-Behnken Designs

Number of Factors	Central Composite	Box-Behnken
2	13 (5 center points)	-
3	20 (6 centerpoint runs)	15
4	30 (6 centerpoint runs)	27
5	33 (fractional factorial) or 52 (full factorial)	46
6	54 (fractional factorial) or 91 (full factorial)	54

Summary of Properties of Classical Response Surface Designs

Design Type	Comment
CCC	CCC designs provide high quality predictions over the entire design space, but require factor settings outside the range of the factors in the factorial part. **Note:** When the possibility of running a CCC design is recognized before starting a factorial experiment, factor spacings can be reduced to ensure that $\pm\alpha$ for each coded factor corresponds to feasible (reasonable) levels. Requires 5 levels for each factor.
CCI	CCI designs use only points within the factor ranges originally specified, but do not provide the same high quality prediction over the entire space compared to the CCC. Requires 5 levels of each factor.
CCF	CCF designs provide relatively high quality predictions over the entire design space and do not require using points outside the original factor range. However, they give poor precision for estimating pure quadratic coefficients. Requires 3 levels for each factor.
Box-Behnken	These designs require fewer treatment combinations than a central composite design in cases involving 3 or 4 factors. The Box-Behnken design is rotatable (or nearly so) but it contains regions of poor prediction quality like the CCI. Its "missing corners" may be useful when the experimenter should avoid combined factor extremes. This property prevents a potential loss of data in those cases. Requires 3 levels for each factor.

Exercise

- There will be class exercise on three way anova, RSM (CCD&BBD),Fractional Factorial using different software packages

Such as: Excel, Design expert, Mintab, Reliasoft, JMP, SPSS

- Case studies will be selected respected to each stream
  - ✓ Food Engineering
  - ✓ Process Engineering
  - ✓ Environmental Engineering
  - ✓ Material Engineering
- Students can make themselves familiar with the listed software specially with Design expert form different tutorial from internate (YouTube)

“There is no end to learning, but there are many beginnings”

Tim Johnson