Process Dynamics and Control

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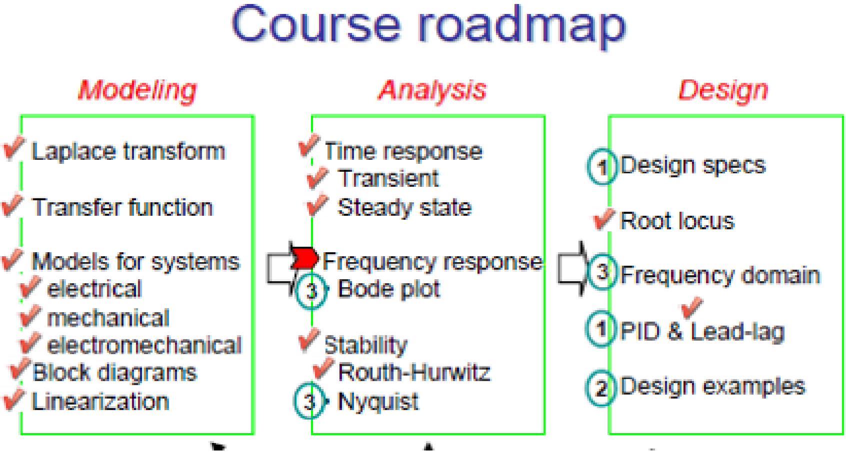
Chapter Three

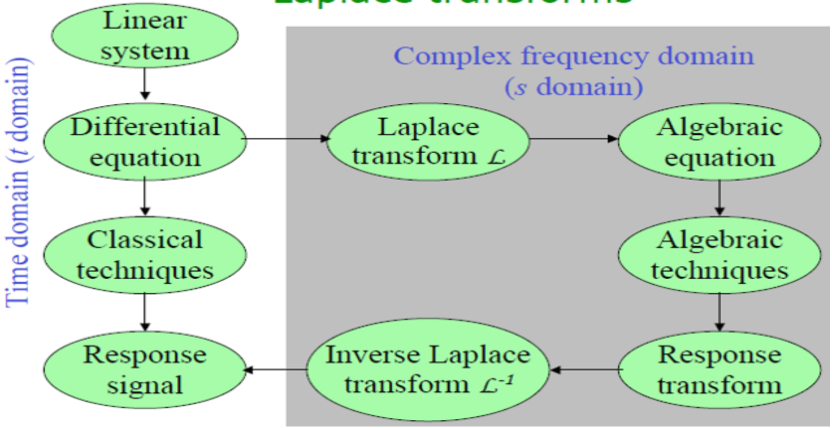
Dynamics Behavior of Process



Laplace Transform

- Transforms - a mathematical conversion from one way of thinking to another to make a problem easier to solve.
- Use of Laplace transforms are:
  - Offers a very simple method of solving linear or linearized ordinary differential equations.
  - Simple development of input-output models which are useful for control process.
  - Straight forward qualitative analysis of how chemical processes react to various external influences.

Laplace transforms



- Definition of Laplace transform is:

$$\mathscr{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt$$

- Laplace transformation is a transformation of a function from the time domain (where t is the independent variable) to the s-domain (where s the independent variable).
- s is a variable defined in the complex plane) i.e. `s = a + ib`.

Table : Laplace Transforms of some commonly functions

f(t)	**F(s)**
1	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$sin(at)$	$\frac{a}{s^2+a^2}$
$cos(at)$	$\frac{s}{s^2+a^2}$
$e^{-bt}sin(at)$	$\frac{a}{(s+b)^2+a^2}$
$e^{-bt}cos(at)$	$\frac{s+b}{(s+b)^2+a^2}$
t^n	$\frac{n!}{s^{n+1}}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$

Transfer Function (TF)

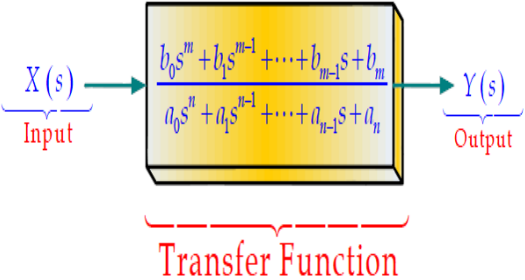
- The TF is the ratio of the output (response function) to the input (driving function) under the assumption that all initial conditions are zero.
- If the TF of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.
- If the TF of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system.

- Consider the system defined by the differential equation:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_o y = b_m \frac{d^m u}{dt^m} + b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \cdots + b_1 \frac{du}{dt} + b_o u$$

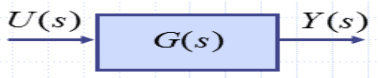
$$\text{Transfer function} = G(s) = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_o}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_o}$$



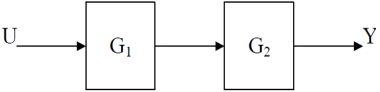
Block diagram representation of a transfer function

- We will consider only two classes of dynamic process models
  - state-space models
  - input-output models
- State-space models : can be derived directly from the general conservation equation: $\Rightarrow$ Accumulation $=$ (Inlet Outlet) $+$ (Generation Consumption)
- They are written in terms of differential equations relating process states to time. $\Rightarrow$ They occur in the time domain
- Input-output models : completely disregard the process states.
- They only give a relationship between process inputs and process outputs. $\Rightarrow$ They occur in the s domain

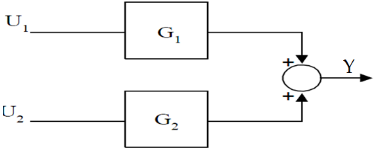


$$Y(s) = G(s)U(s)$$

G (s) is called transfer function of the process



Multiplicative rule: $Y(s) = G_1(s)G_2(s)U(s)$



Additive rule: $Y(s) = G_1(s)U_1(s) + G_2(s)U_2(s)$

Linearization of Nonlinear Models

- Linearization is the process by which non-linear systems is approximated with linear ones.
- In the time domain, a linear system is modeled by a linear differential equation.

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_1 \frac{dy}{dt} + a_0 y = bu(t)$$

- The s-domain representation is possible only for linear (or linearized) systems.
- Linear system or equations are equations containing variables only to the first power in any one term of the equation.

- If square roots, squares, exponents, product of variables etc, appear in the equation, it is non-linear.

The following are the steps to drive transfer functions:

1) Write the dynamic model describing the system
2) Linearize equations using Taylor series expansion
3) Express the equations in dynamic equation
4) Express the equations in steady state equation
5) Express the equations in deviation equation
6) Operate Laplace transform on the deviation equations
7) Obtain the ratio of Laplace transform output to input

- The material and energy balance models that describe the behavior of chemical processes are generally nonlinear, while commonly used control strategies are based on linear systems theory.
- It is important, then, to be able to linearize nonlinear models for control system design and analysis purposes.
- The method that we use to form linear models is based on a Taylor series approximation to the nonlinear model.
- The Taylor series approximation is based on the steady state operating point of the process.

- Consider the non-linear of one variable: $\frac{dy}{dt} = f(x)$ expand at x_o

$$f(x) = f(x_o) + f'(x_o)(\frac{x - x_o}{1!}) + f''(x_o)(\frac{(x - x_o)^2}{2!}) + ...$$

$$\Rightarrow f(x) = f(x_o) + f'(x_o)(x - x_o) + \epsilon$$

$$\Rightarrow f(x) \simeq f(x_o) + f'(x_o)(x - x_o)$$

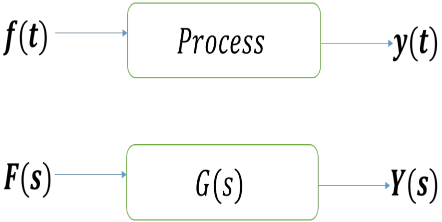
$$\Rightarrow \frac{dy}{dt} \simeq f(x_o) + f'(x_o)(x - x_o)$$

- Consider the non-linear of two variables: $\frac{dy}{dt} = f(x, y)$ expand at x_o

$$\Rightarrow \frac{dy}{dt} \simeq f(x_o, y_o) + f'(x_o, y_o)(x - x_o) + f'(x_o, y_o)(y - y_o)$$

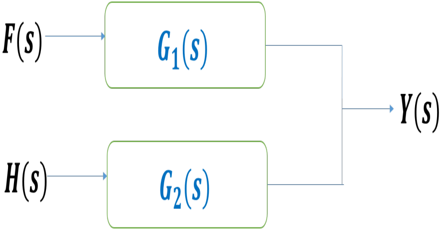
Developing Transfer Function

- Consider a process with one output y(t) and one input f(t)



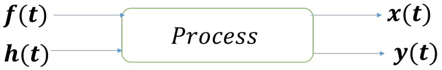
$$Y(s) = G(s)F(s)$$

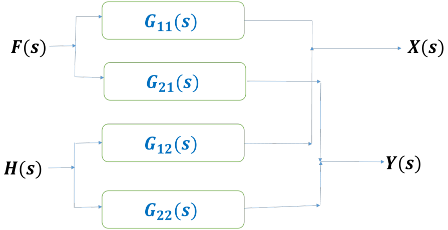
- Consider a process with one output y(t) and two inputs f(t) and h(t)



$$Y(s) = G_1(s)F(s) + G_2(s)H(s)$$

- Consider a process with two outputs x(t) and y(t) and two inputs f(t) and h(t)





$$X(s) = G_{11}(s)F(s) + G_{12}(s)H(s)$$

$$Y(s) = G_{21}(s)F(s) + G_{22}(s)H(s)$$

Example 1: Find the transfer function for the mathematical model for gravitation flow system. $A\frac{dh}{dt} + \alpha\sqrt{h} = F_i$.

Example 2: Find the transfer function for the mathematical model for a stirred-tank heating process with constant holdup.

$\frac{dh}{dt} = \frac{F}{V}(T_i - T) + \frac{Q}{V\rho C_p}$, Where Ti and Q are inputs, T is output and F, V, CP and are parameters.

Example 3: A stirred-tank blending system initially is full of water and is being fed pure water at a constant flow rate, q. At a particular time, an operator adds caustic solution at the same volumetric flow rate q but concentration C_t. If the liquid volume V is constant, the dynamic model for this process is: $\frac{VdC}{dt} + qC = qC_i$ with c(0)=0. What is the concentration response of the reactor effluent stream, c(t)?

Data: $V = 2m^3, q = 0.4m^3/min, C_i = 50kg/m$.