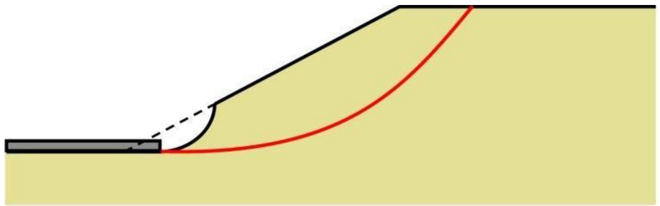
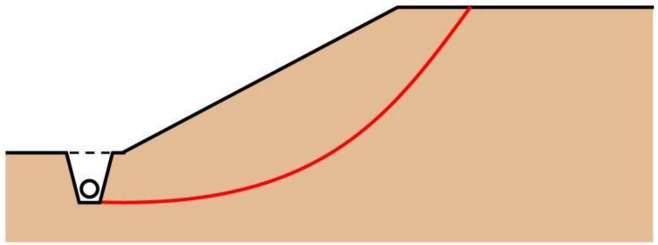
Addis Ababa University
Addis Ababa Institute of Technology
School of Civil & Environmental Engineering
Geotechnical Engineering Chair

Slope stability analysis

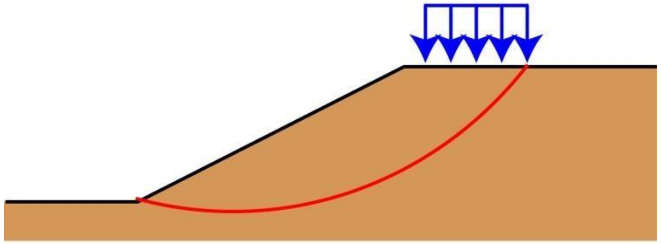
Causes of slope failure



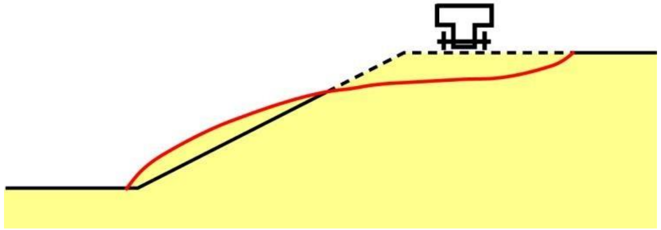
Excavation of soil at the toe of the slope, e.g. in order to increase the width of a road



Excavation of soil in front of the toe of the slope, e.g. in order to install tubes, cables, etc.



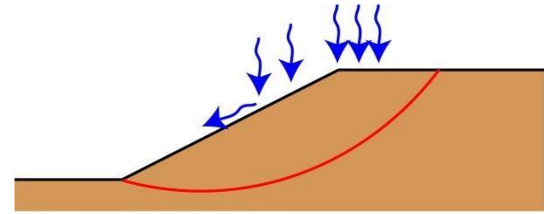
Surface loads, e.g. traffic or construction machines



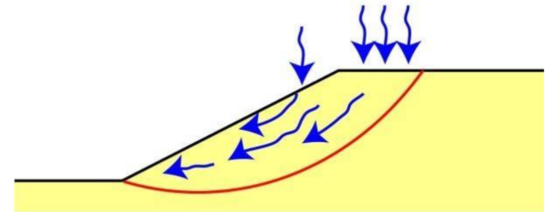
Vibration caused by traffic

Causes of slope failure

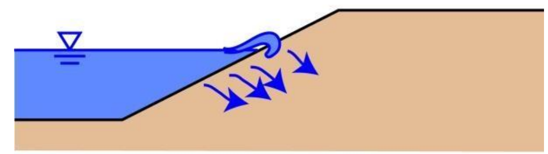
Erosion caused by surface water



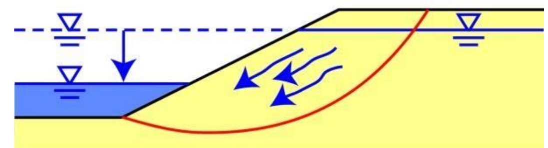
Seepage flow due to heavy rain falls



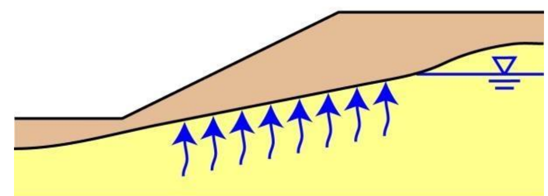
Erosion due to waves



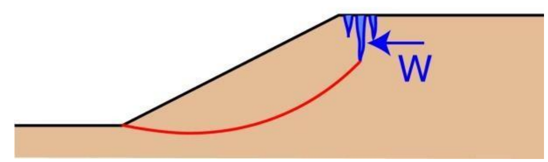
Seepage flow due to fast lowering of water level



Confined (probably artesian) groundwater

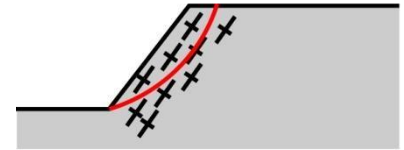


Additional horizontal load due to water pressure in tension cracks

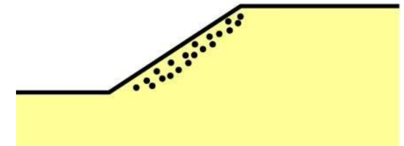


Shape of the failure surface

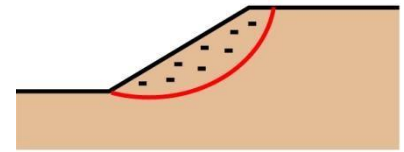
- Weathered rock or fissured clay: failure along chasms



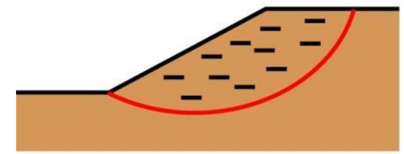
- Granular soils without cohesion (c = 0):
  Failure surface = sloping ground surface



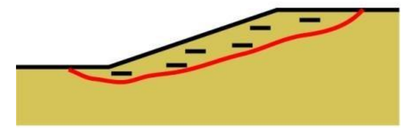
- Lightly cohesive soils: shallow slip circle



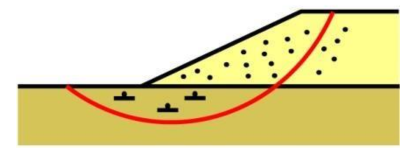
- Strongly cohesive soils: deep slip circle



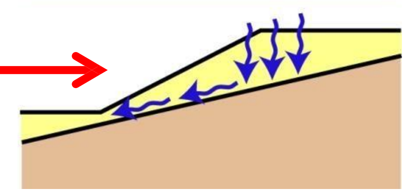
- Highly plastic clay (e.g. Montmorillonite):
  long, shallow sliding surface, slow movement of mass,
  smooth failure surfaces with very low friction (φ' = 4 – 10°)



- Organic ground: deep slip circle into this weak soil

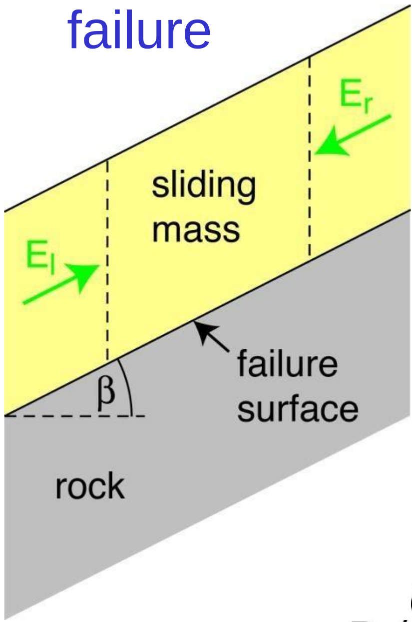


- Inclined cohesive layer in the ground: Sliding on the
  surface of this layer, interface weakened by percolating water

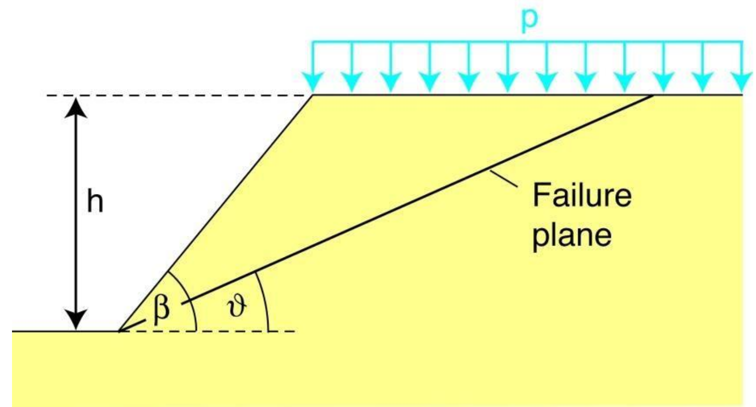


Possible slope failure mechanisms + analysis methods

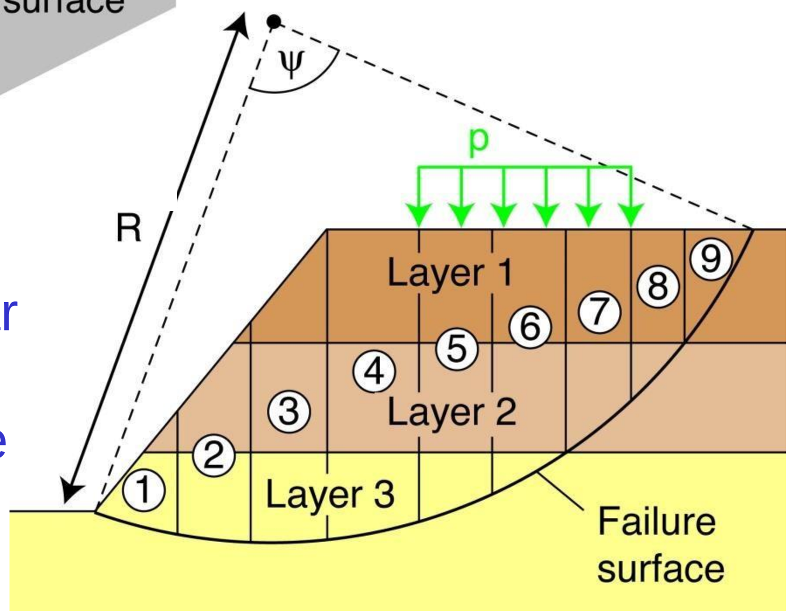
1) Infinite slope failure



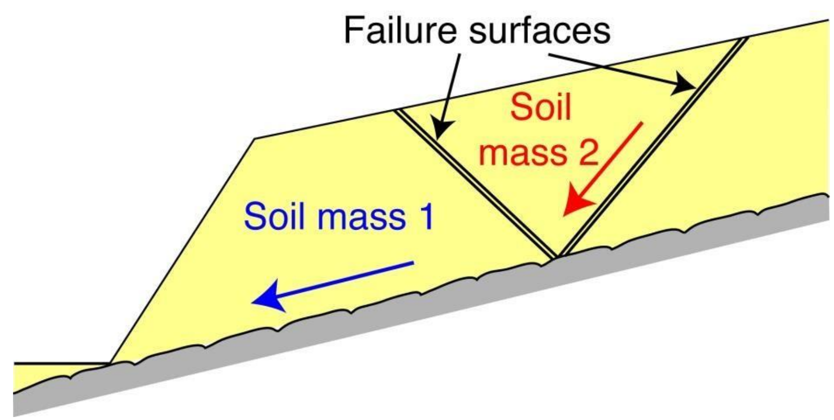
2) Planar failure (wedge analysis)



3) Circular failure surface

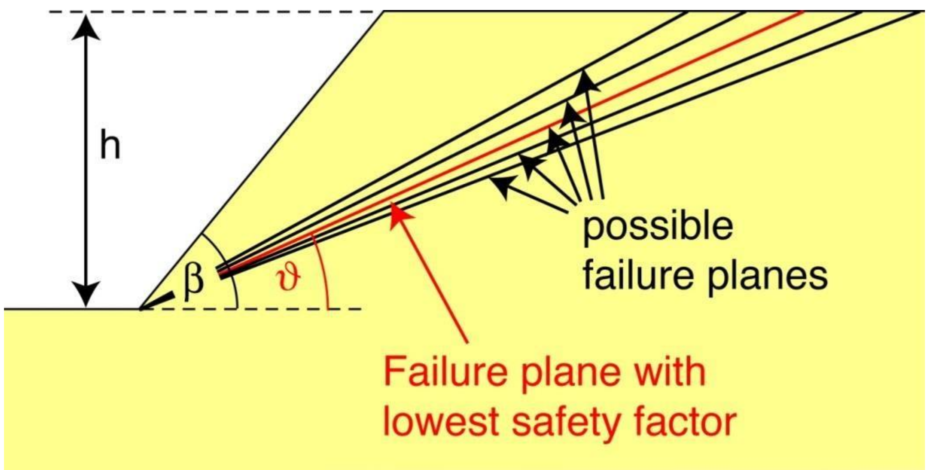


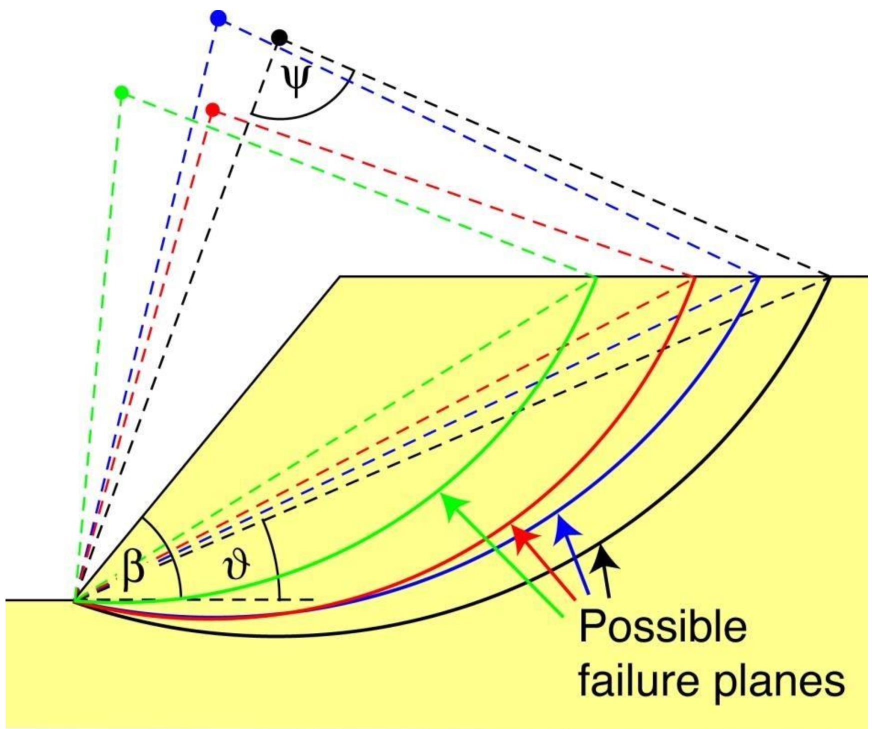
4) Combined mechanisms (two or more sliding masses)



Possible slope failure mechanisms + analysis methods

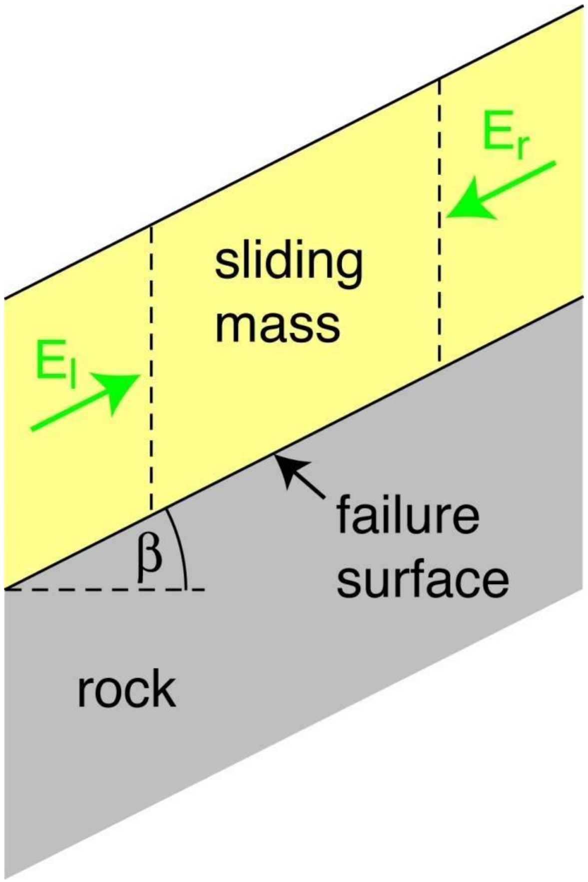
- The most unfavourable failure mechanism is usually unknown
- Several possible failure mechanisms and failure surfaces have to be inspected
- The failure mechanism with the lowest safety factor is searched for
- The slope will most likely fail with this mechanism





Infinite slope failure

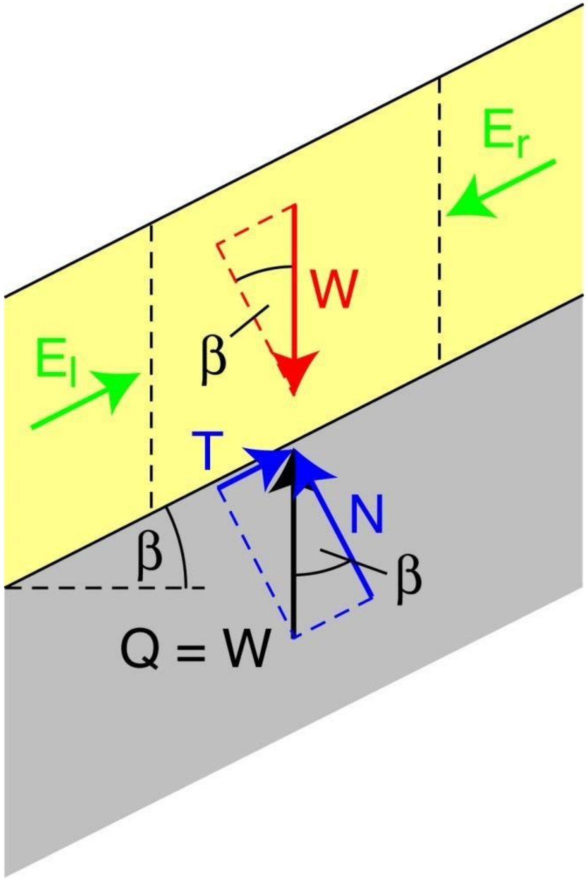
Practical relevance



- Layer of firm soil or weathered rock lies parallel to the surface of the slope at shallow depth
- Slip surface is constrained to be parallel to the slope
- If slip surface is long in comparison to depth the side forces (earth pressures) E_l and E_r can be neglected ($E_l = E_r$)
- Determination of factor of safety from the analysis of an infinite slope
- Due to spatial effects the resistance usually is greater, i.e. the analysis is somewhat conservative

Infinite slope failure

No water, soil without cohesion



- Reaction force at rock surface must be identical to self-weight W
- Reaction force can be splitted in normal component N and tangential component T
- Normal force on shear plane:

$$N = W \cdot \cos\beta$$

- Tangential force on shear plane (= driving force in direction of shear plane):

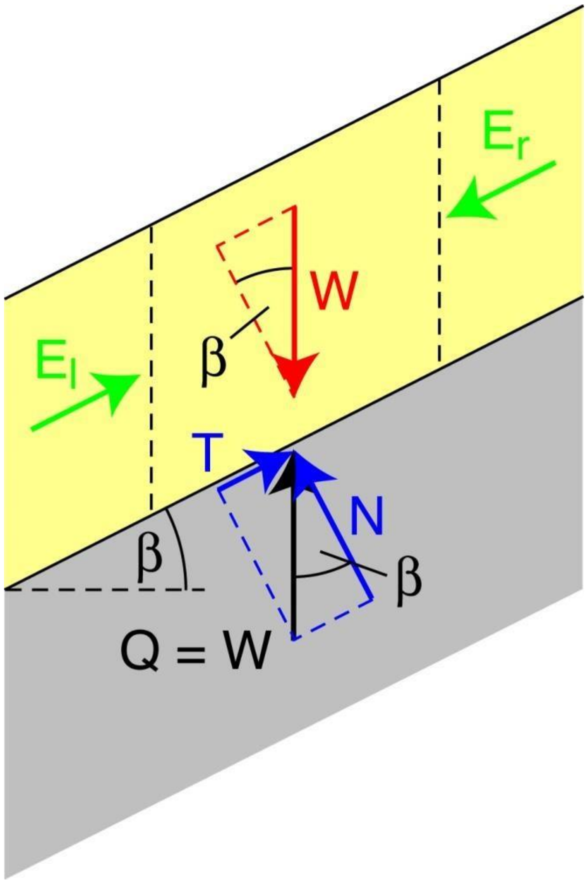
$$T = W \cdot \sin\beta$$

- Maximum tangential force that can be mobilized:

$$T_{max} = N \cdot \tan\varphi' = W \cdot \cos\beta \cdot \tan\varphi'$$

Infinite slope failure

No water, soil without cohesion



- Factor of safety (global):

$$FS = \frac{T_{max}}{T} = \frac{W \cdot \cos\beta \cdot \tan\varphi'}{W \cdot \sin\beta} = \frac{\tan\varphi'}{\tan\beta}$$

- If FS = 1 (limit equilibrium):

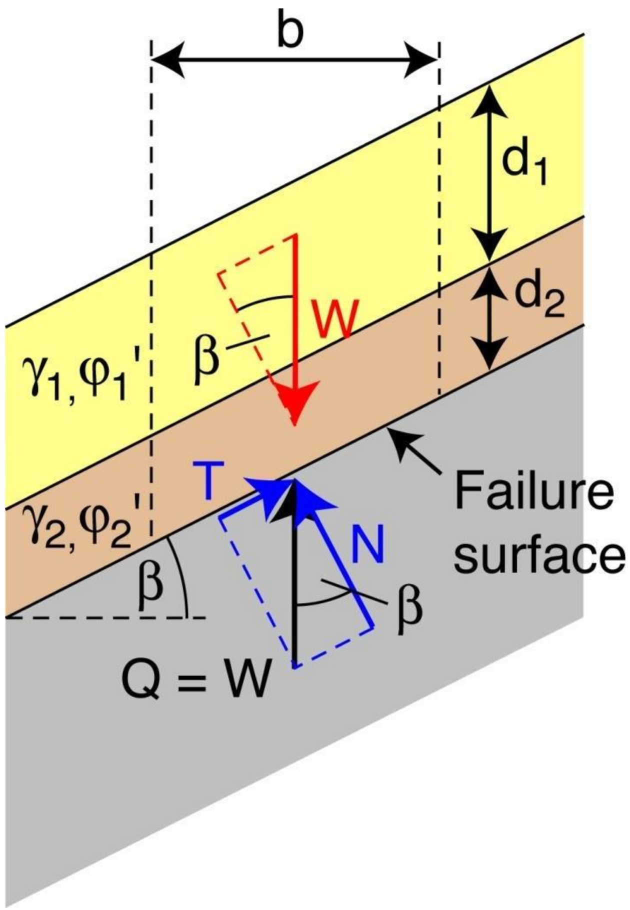
$$\tan\beta = \tan\varphi'$$

$$\beta = \varphi'$$

Maximum inclination of slope
in non-cohesive soil
= friction angle φ‘

Infinite slope failure

No water, soil without cohesion, 2 layers



- Self-weight W

$$W = \gamma_1 \cdot d_1 \cdot b + \gamma_2 \cdot d_2 \cdot b$$

- Maximum tangential force that can be mobilized:

$$T_{\max} = N \cdot \tan\varphi_2{}' = W \cdot \cos\beta \cdot \tan\varphi_2{}'$$

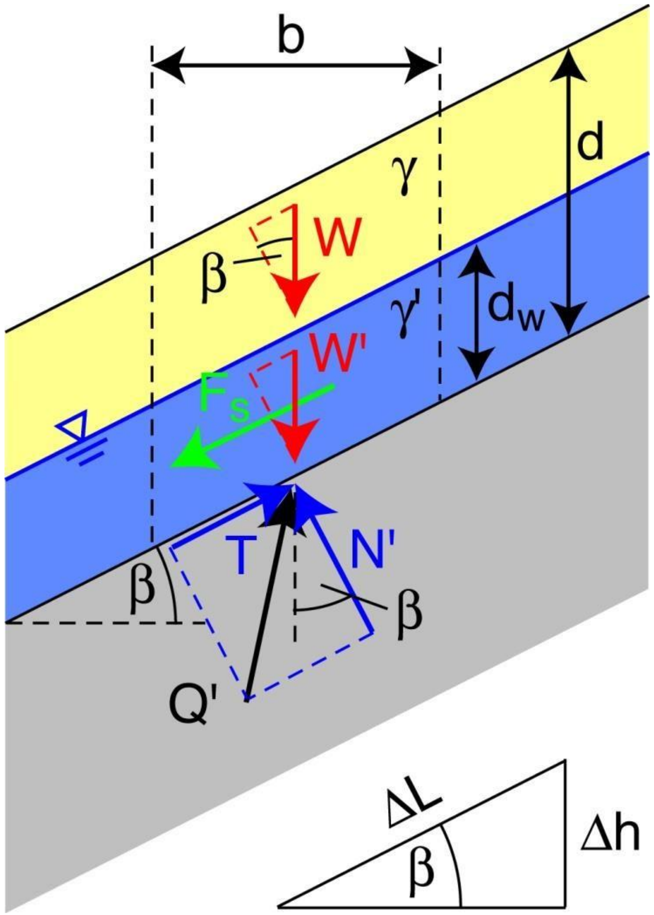
- Factor of safety (global):

$$FS = \frac{T_{\max}}{T} = \frac{W \cdot \cos\beta \cdot \tan\varphi_2{}'}{W \cdot \sin\beta} = \frac{\tan\varphi_2{}'}{\tan\beta}$$

Shear strength of the lower layer is decisive

Infinite slope failure

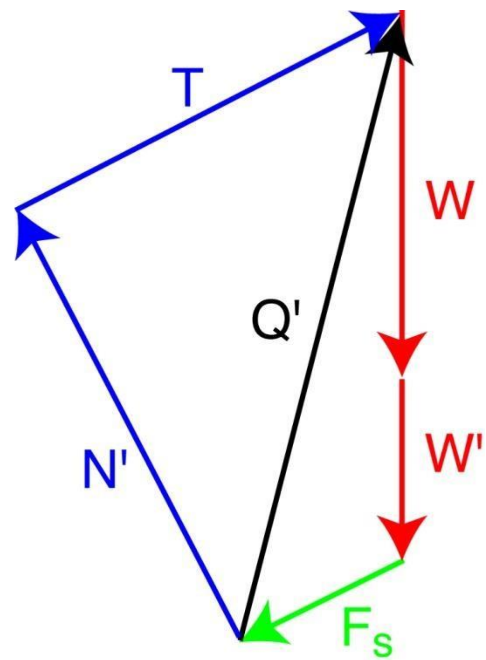
Water flow parallel to slope, soil without cohesion



Analysis with effective stresses

→ Consideration of buoyant weight W' below the ground water table and seepage force F_s

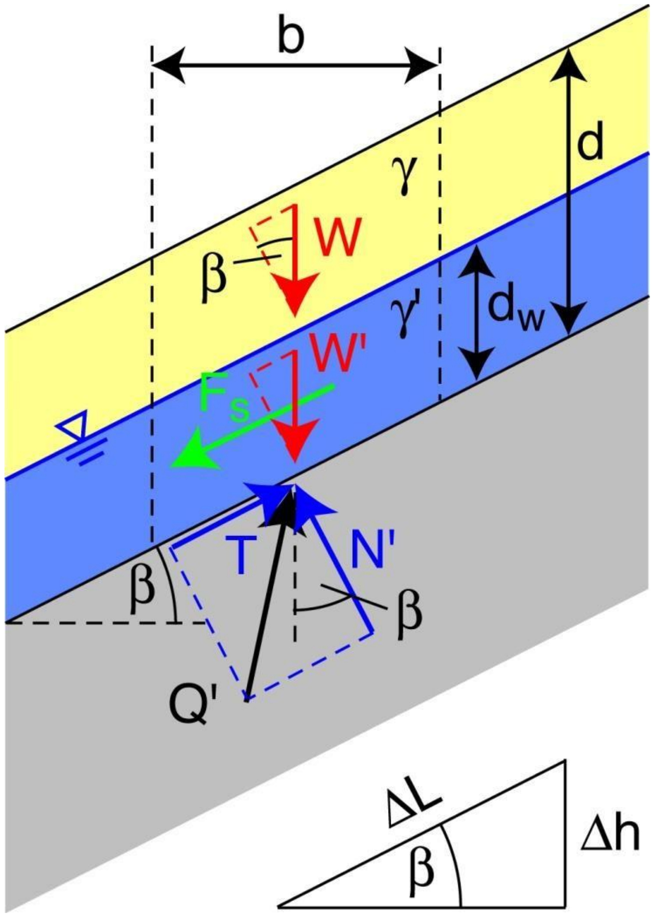
Force polygon:



Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with effective stresses



- Force equilibrium normal to failure surface

$$N' = (W + W') \cdot \cos\beta$$

- Force equilibrium parallel to failure surface

$$T = F_s + (W + W') \cdot \sin\beta$$

- Maximum tangential force that can be mobilized:

$$T_{\max} = N' \cdot \tan\varphi' = (W + W') \cdot \cos\beta \cdot \tan\varphi'$$

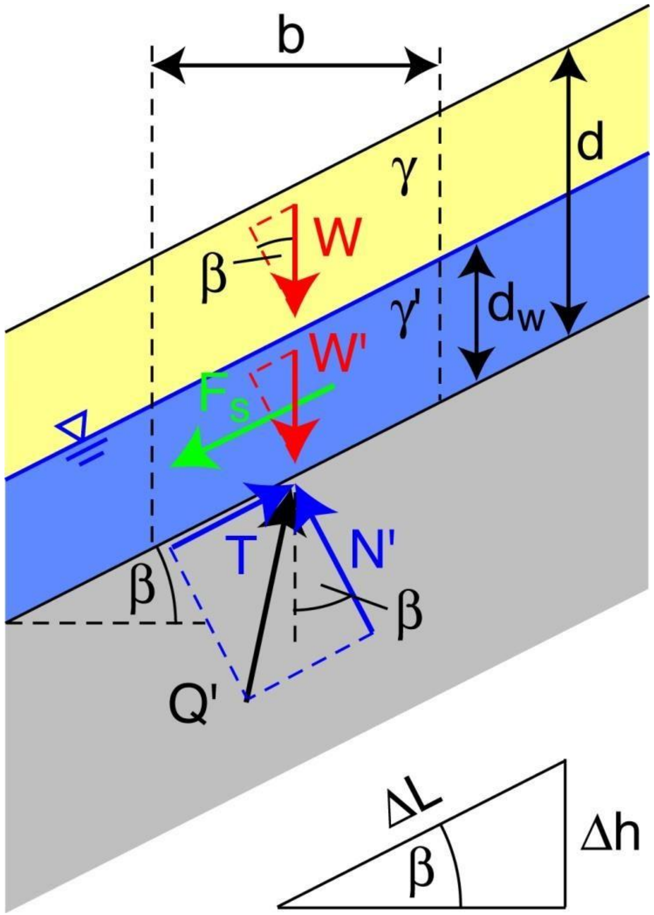
- Safety factor:

$$FS = \frac{T_{\max}}{F_s + (W + W') \cdot \sin\beta} = \frac{(W + W') \cdot \cos\beta \cdot \tan\varphi'}{F_s + (W + W') \cdot \sin\beta}$$

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with effective stresses



- Seepage force:

$$F_s = f_s \cdot V_w = f_s \cdot b \cdot d_w = \gamma_w \cdot i \cdot b \cdot d_w$$
$$= \gamma_w \cdot \sin\beta \cdot b \cdot d_w$$

- Self-weight of soil:

$$W + W' = \gamma \cdot b \cdot (d - d_w) + \gamma' \cdot b \cdot d_w$$

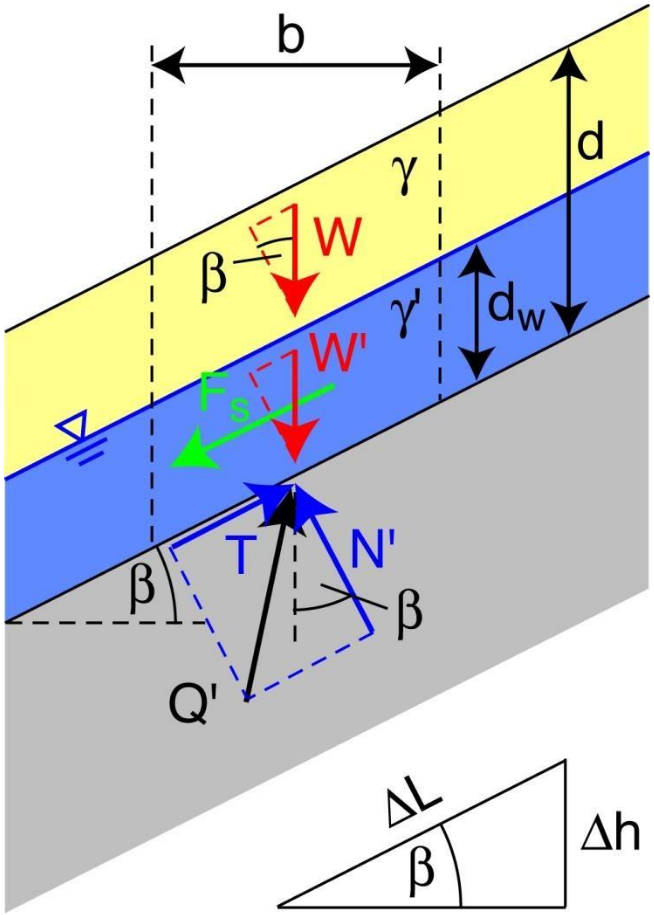
$$FS = \frac{b \cdot [\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \cos\beta \cdot \tan\varphi'}{\gamma_w \cdot \sin\beta \cdot b \cdot d_w + b \cdot [\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \sin\beta}$$

$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \tan\varphi'}{[\gamma_w \cdot d_w + \gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \tan\beta}$$

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with effective stresses



- If FS = 1 (limit equilibrium):

$$\tan\beta = \tan\varphi' \cdot \frac{\gamma\cdot(d-d_w)+\gamma'\cdot d_w}{\gamma_w\cdot d_w+\gamma\cdot(d-d_w)+\gamma'\cdot d_w}$$

$$= \tan\varphi' \cdot \frac{1}{\dfrac{1}{\dfrac{\gamma}{\gamma_w}\cdot\dfrac{d}{d_w}-\dfrac{\gamma}{\gamma_w}+\dfrac{\gamma'}{\gamma_w}}+1}$$

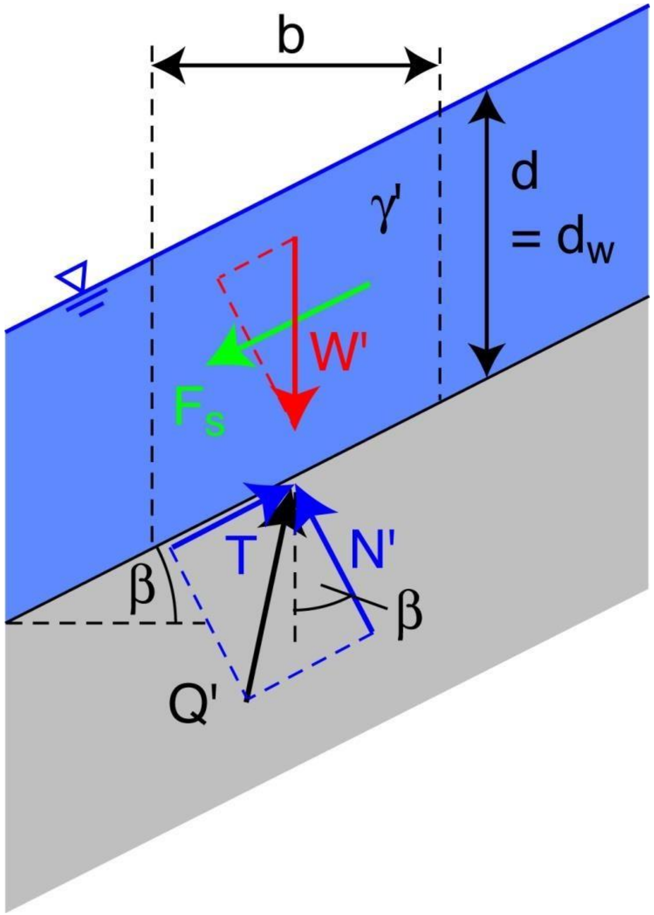
With simplified assumption $\gamma \approx \gamma_r = \gamma' + \gamma_w$

$$\tan\beta = \tan\varphi' \cdot \left[1-\frac{\gamma_w}{\gamma_r}\cdot\frac{d_w}{d}\right]$$

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with effective stresses



- Special case: water level at ground surface:

$$d_w = d$$

$$\tan\beta = \tan\varphi' \cdot \left[1 - \frac{\gamma_w}{\gamma_r} \cdot \frac{d_w}{d}\right] = \tan\varphi' \cdot \left[1 - \frac{\gamma_w}{\gamma_r}\right]$$

Considering $\gamma_r \approx 2 \cdot \gamma_w$

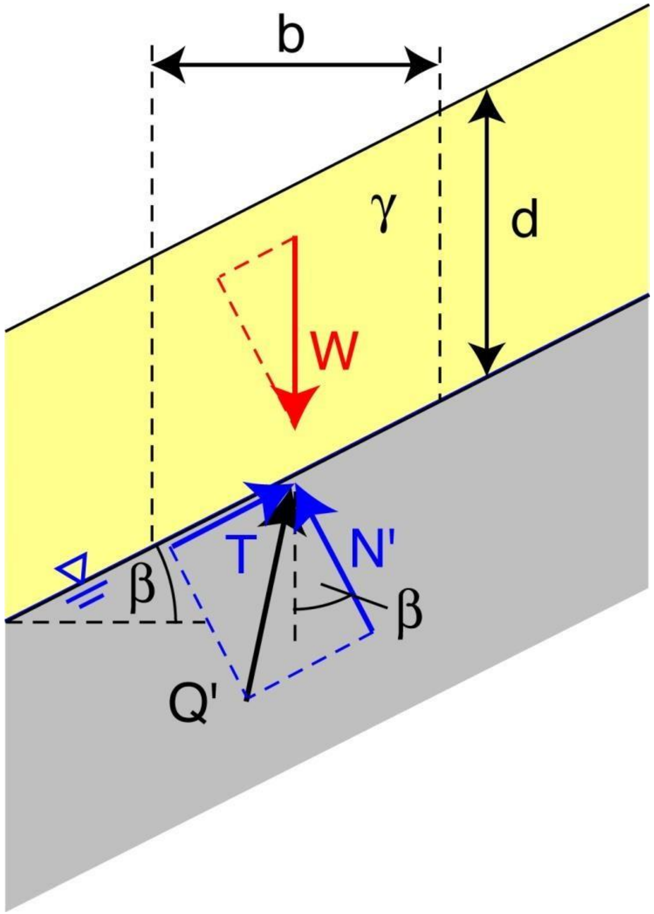
$$\tan\beta = \tan\varphi' \cdot \frac{1}{2}$$

→ Maximum slope angle β
is only half of the friction angle

Infinite slope failure

Analysis with effective stresses

Water flow parallel to slope, soil without cohesion



- Special case: no water (as already discussed above)

$$d_w = 0$$

$$\tan\beta = \tan\varphi' \cdot \left[1 - \frac{\gamma_w}{\gamma_r} \cdot \frac{d_w}{d}\right] = \tan\varphi'$$

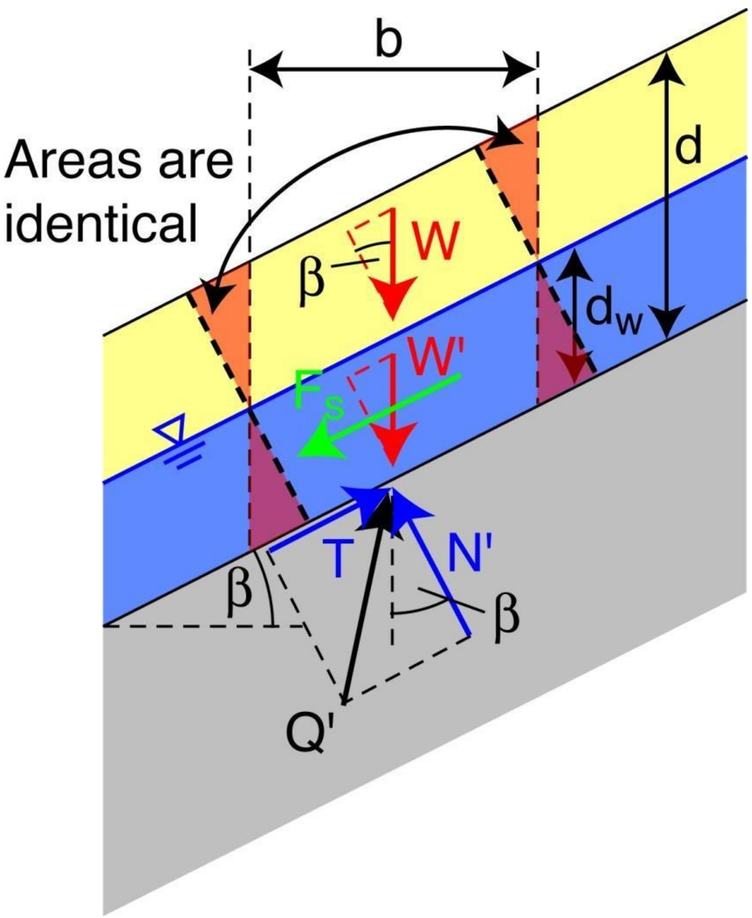
$$\beta = \varphi'$$

Maximum inclination of slope in non-cohesive soil = friction angle φ‘

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with effective stresses



Comparison of bodies with boundaries being either vertical or perpendicular to the ground surface

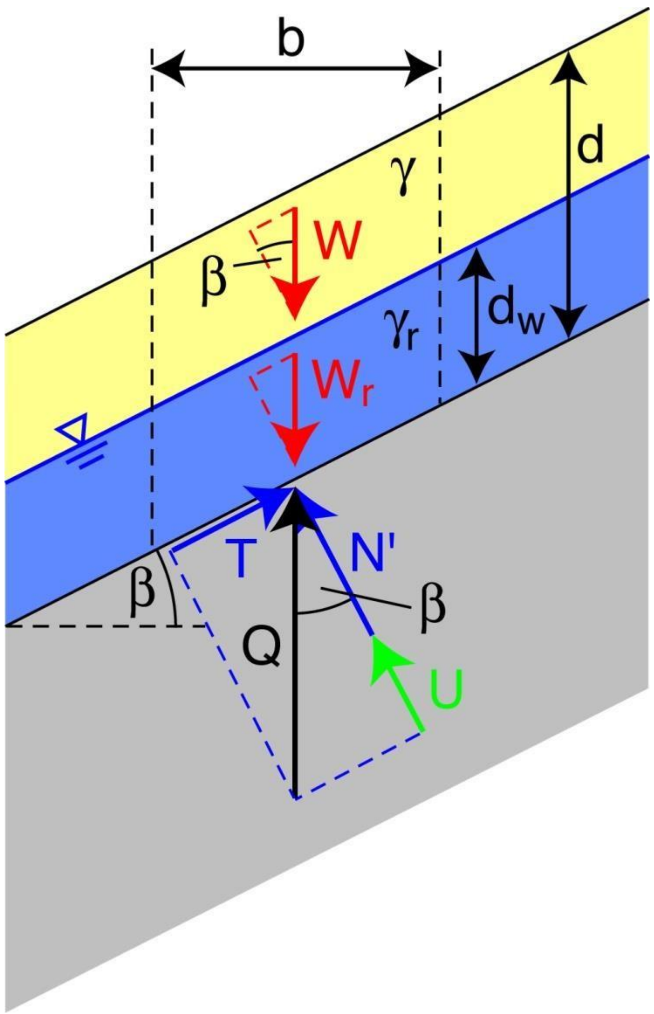
$$F_s = \gamma_w \cdot \sin\beta \cdot b \cdot d_w$$

$$W + W' = \gamma \cdot b \cdot (d - d_w) + \gamma' \cdot b \cdot d_w$$

→ Acting forces are identical
→ Solution for safety factor is identical

Infinite slope failure

Water flow parallel to slope, soil without cohesion



Alternative analysis with total stresses

W_r = weight of water-saturated soil below ground water table

U = resulting force of pore water pressure in failure surface

- Force equilibrium normal and parallel to failure surface:

$$N' + U = (W + W_r) \cdot \cos\beta$$

$$T = (W + W_r) \cdot \sin\beta$$

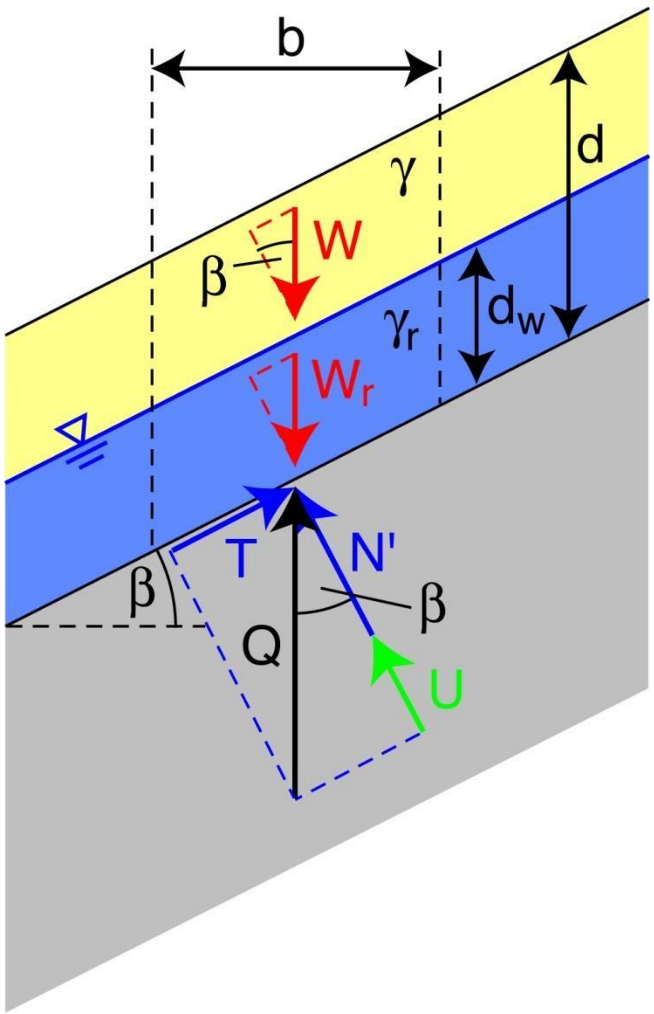
- Maximum shear force that can be mobilized:

$$T_{\max} = N' \cdot \tan\varphi' = [(W + W_r) \cdot \cos\beta - U\,] \cdot \tan\varphi'$$

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with
total stresses



- Factor of safety:

$$FS = \frac{T_{\max}}{T} = \frac{[(W + W_r) \cdot \cos\beta - U\ ] \cdot \tan\varphi'}{(W + W_r) \cdot \sin\beta}$$

- Weight of soil

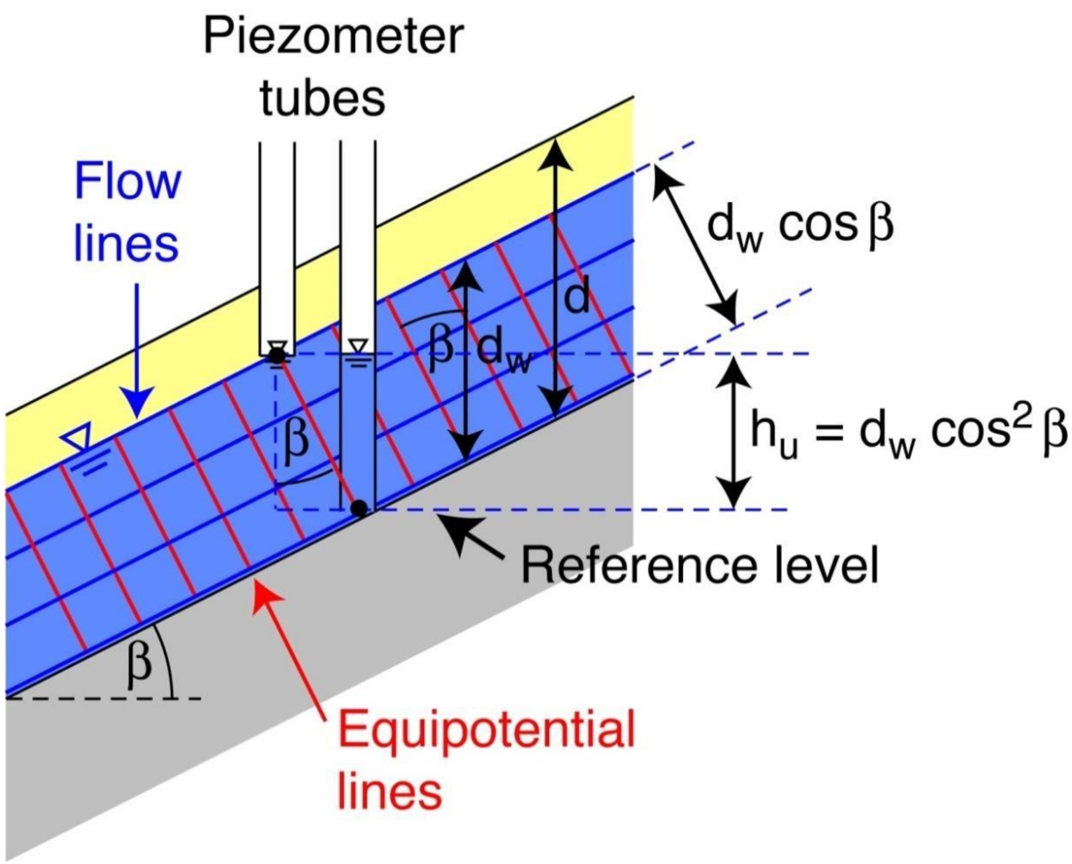
$$W + W_r = \gamma \cdot b \cdot (d - d_w) + \gamma_r \cdot b \cdot d_w$$

- Pore water pressure U?

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with total stresses



Flow net:

- Flow lines run parallel to slope
- Equipotential lines run perpendicular to the flow lines
- Hydraulic head is constant along equipotential line, i.e. piezometer tubes show same water level
- Pore water pressure u from water level in piezometer tube:

$$u = \gamma_w \cdot h_u = \gamma_w \cdot d_w \cdot \cos^2\beta$$

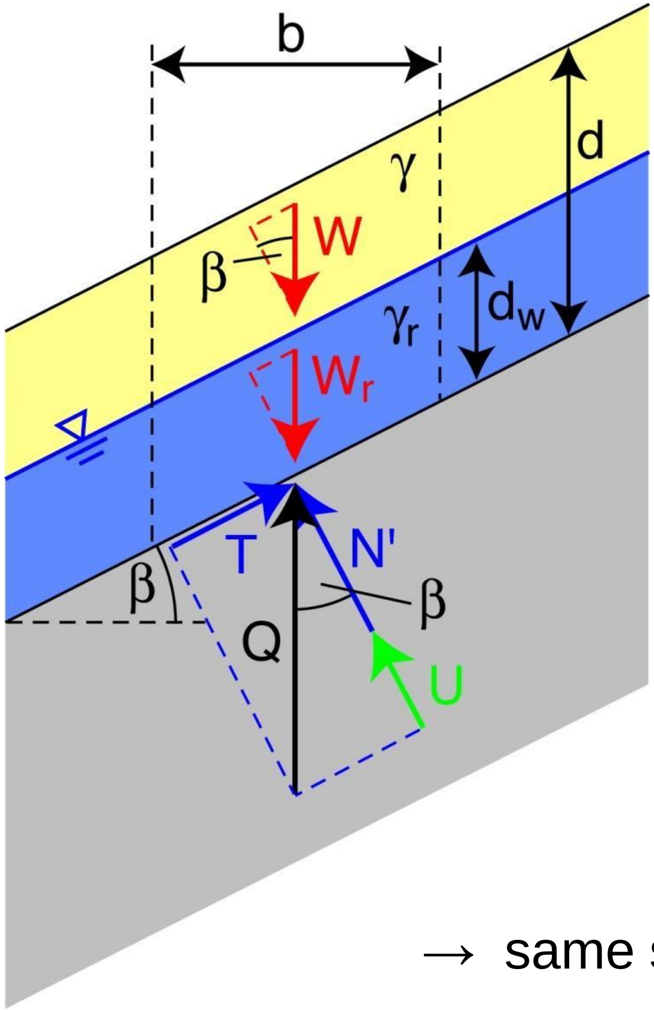
- Due to seepage it is not simply

$$u = \gamma_w \cdot d_w$$

Infinite slope failure

Water flow parallel to slope, soil without cohesion

Analysis with total stresses



- Resultant force U of pore water pressure

$$U = u \cdot \frac{b}{\cos\beta} = \gamma_w \cdot d_w \cdot \cos^2\beta \cdot \frac{b}{\cos\beta}$$
$$= \gamma_w \cdot d_w \cdot b \cdot \cos\beta$$

- Setting W + W_r and U into FS leads to:

$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma_r \cdot d_w - \gamma_w \cdot d_w] \cdot \tan\varphi'}{[\gamma \cdot (d - d_w) + \gamma_r \cdot d_w] \cdot \tan\beta}$$

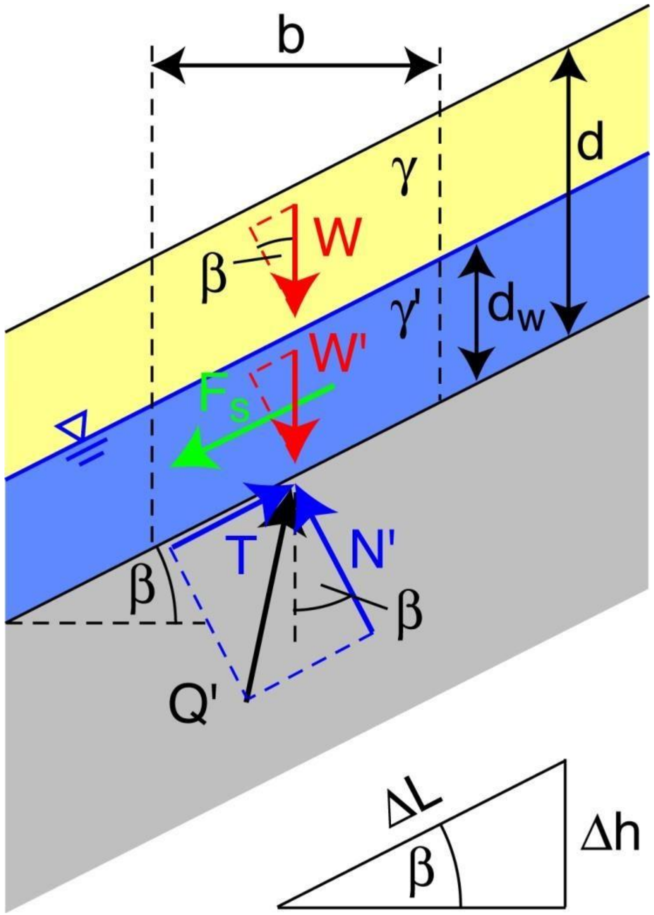
With $\gamma_r = \gamma' + \gamma_w$

$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \tan\varphi'}{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w + \gamma_w \cdot d_w] \cdot \tan\beta}$$

→ same solution as in case of analysis with effective stresses

Infinite slope failure

Water flow parallel to slope, soil with cohesion



- Maximum tangential force that can be mobilized:

$$T_{max} = N' \cdot \tan\varphi' + C'$$

- Cohesion force:

$$C' = c' \cdot L = c' \cdot \frac{b}{\cos\beta}$$

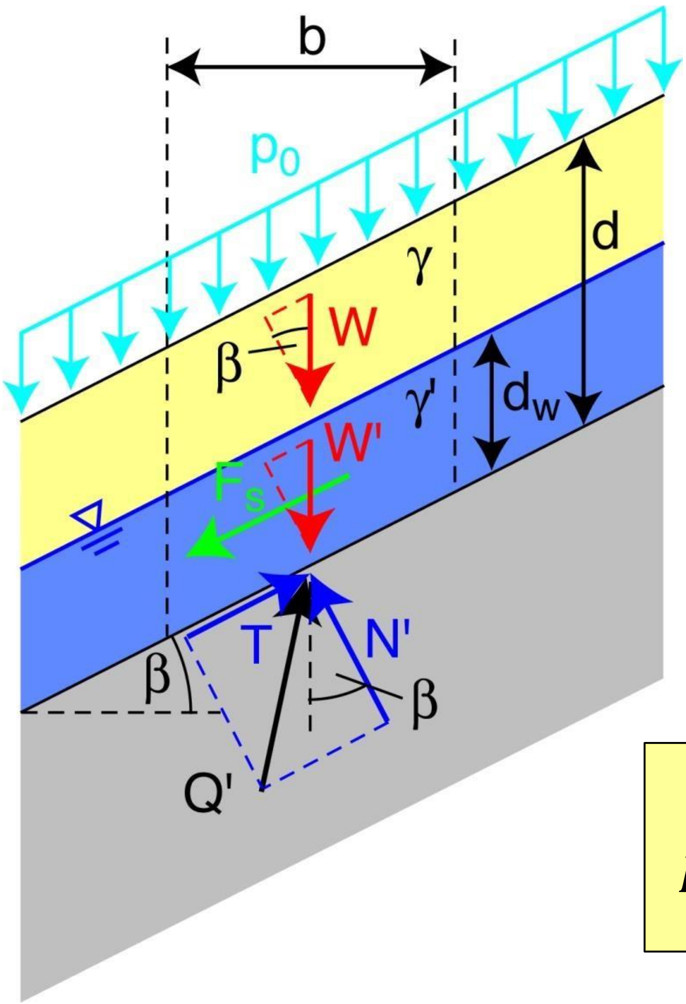
- Safety factor

$$FS = \frac{T_{max}}{T} = \frac{N' \cdot \tan\varphi' + C'}{F_s + (W + W') \cdot \sin\beta}$$

$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \tan\varphi' + c' \cdot \frac{1}{\cos^2\beta}}{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w + \gamma_w \cdot d_w] \cdot \tan\beta}$$

Infinite slope failure

Water flow parallel to slope, soil with cohesion, additional surface load



- Resulting force due to surface load

$$P_0 = p_0 \cdot b$$

- Safety factor

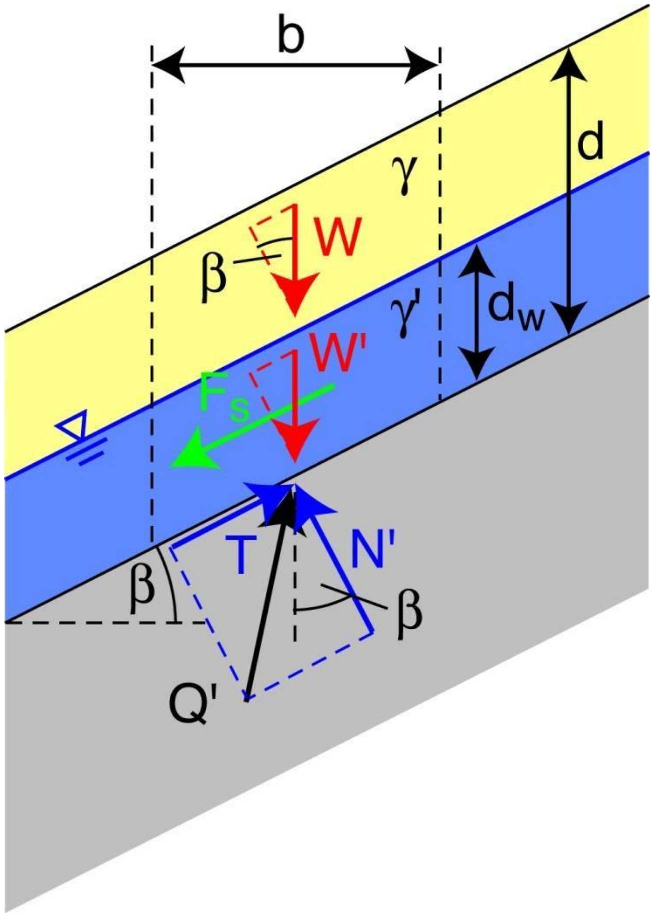
$$FS = \frac{N' \cdot \tan\varphi' + C'}{F_s + (W + W' + P_0) \cdot \sin\beta}$$

$$= \frac{(W + W' + P_0) \cdot \cos\beta \cdot \tan\varphi' + C'}{F_s + (W + W' + P_0) \cdot \sin\beta}$$

$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w + p_0] \cdot \tan\varphi' + c' \cdot \frac{1}{\cos^2\beta}}{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w + \gamma_w \cdot d_w + p_0] \cdot \tan\beta}$$

Infinite slope failure

Water flow parallel to slope, soil with friction and cohesion



$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \tan\varphi' + c' \cdot \frac{1}{\cos^2\beta}}{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w + \gamma_w \cdot d_w] \cdot \tan\beta}$$

Alternative formulation:

$$FS = A \cdot \frac{\tan\varphi'}{\tan\beta} + B \cdot \frac{c'}{\gamma_r \cdot d}$$

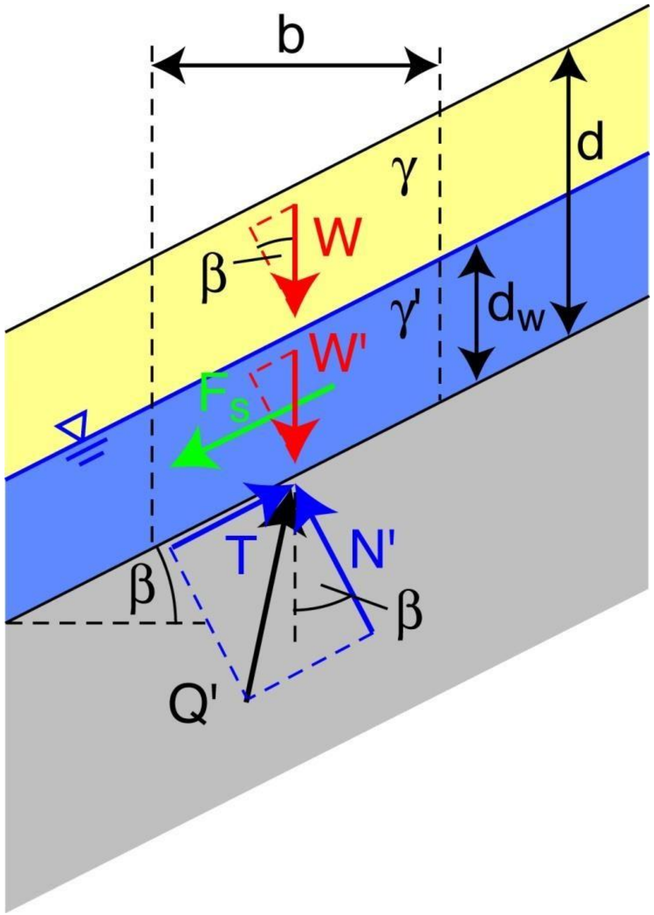
$$A = \frac{\gamma \cdot (d - d_w) + \gamma' \cdot d_w}{\gamma \cdot (d - d_w) + \gamma' \cdot d_w + \gamma_w \cdot d_w}$$

$$= \frac{\gamma \cdot (d - d_w) + (\gamma_r - \gamma_w) \cdot d_w}{\gamma \cdot (d - d_w) + \gamma_r \cdot d_w}$$

With $\gamma \approx \gamma_r$: $A = 1 - \frac{\gamma_w \cdot d_w}{\gamma_r \cdot d}$ With $\gamma_r \approx 2 \cdot \gamma_w$ and $d_w = d$: $A = 0.5$

Infinite slope failure

Water flow parallel to slope, soil with friction and cohesion



$$FS = \frac{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w] \cdot \tan\varphi' + c' \cdot \frac{1}{\cos^2\beta}}{[\gamma \cdot (d - d_w) + \gamma' \cdot d_w + \gamma_w \cdot d_w] \cdot \tan\beta}$$

Alternative formulation:

$$FS = A \cdot \frac{\tan\varphi'}{\tan\beta} + B \cdot \frac{c'}{\gamma_r \cdot d}$$

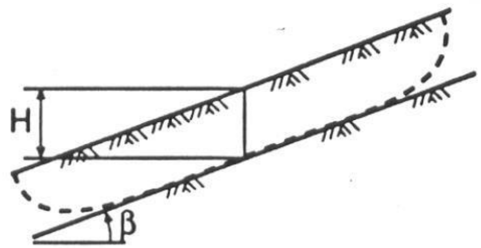
$$B = \frac{\gamma_r \cdot d}{[\gamma \cdot (d - d_w) + \gamma_r \cdot d_w] \cdot \sin\beta \cdot \cos\beta}$$

With $\gamma \approx \gamma_r$: $B = \frac{1}{\sin\beta \cdot \cos\beta}$

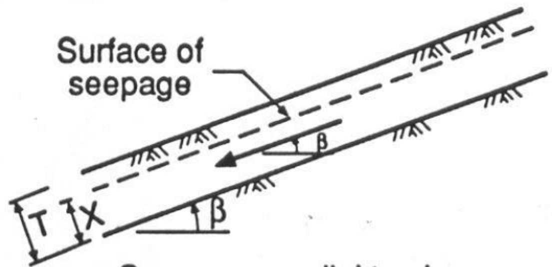
Infinite slope failure

Water flow parallel to slope, soil with φ‘ and c‘

Modified Duncan stability chart

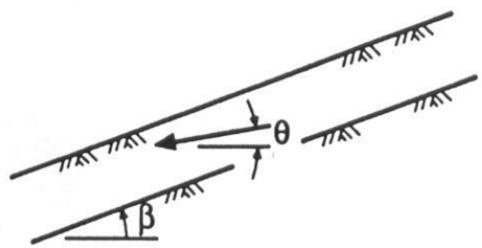


- γ = total unit weight of soil
- γ_w = unit weight of water
- c' = cohesion intercept } Effective Stress
- ϕ' = friction angle } Effective Stress
- r_u = pore pressure ratio = $\dfrac{u}{\gamma H}$
- u = pore pressure at depth H



Seepage parallel to slope

$$r_u = \frac{X}{T}\,\frac{\gamma_w}{\gamma}\cos^2\beta$$



Seepage emerging from slope

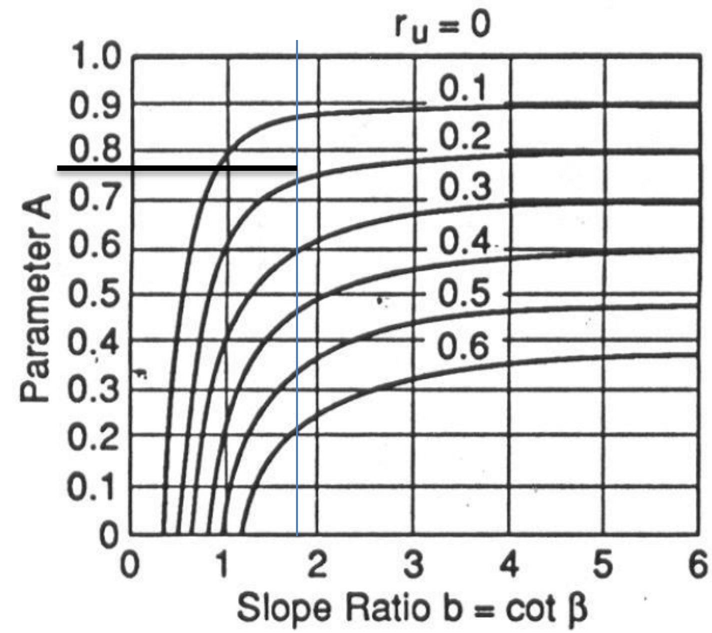
$$r_u = \frac{\gamma_w}{\gamma}\,\frac{1}{1+\tan\beta\tan\theta}$$

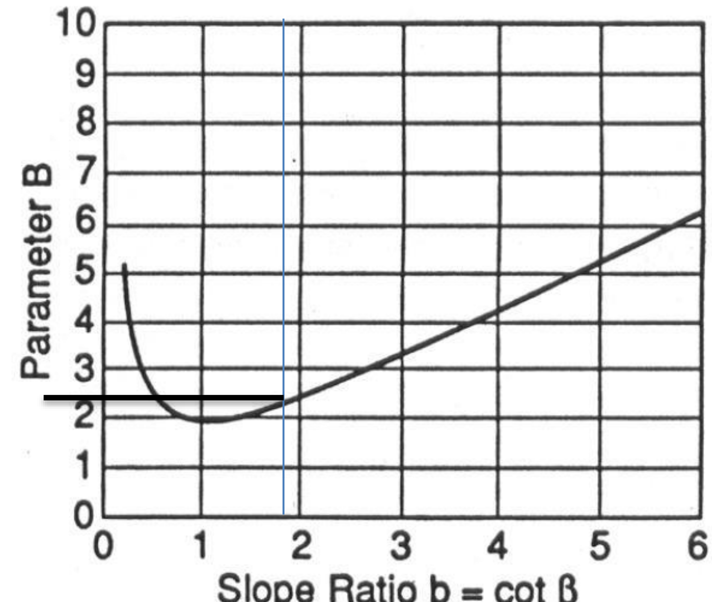
Steps:

① Determine r_u from measured pore pressures or formulas at right

② Determine A and B from charts below

③ Calculate $$FS = A\cdot\frac{\tan\varphi'}{\tan\beta} + B\cdot\frac{c'}{\gamma\cdot d}$$



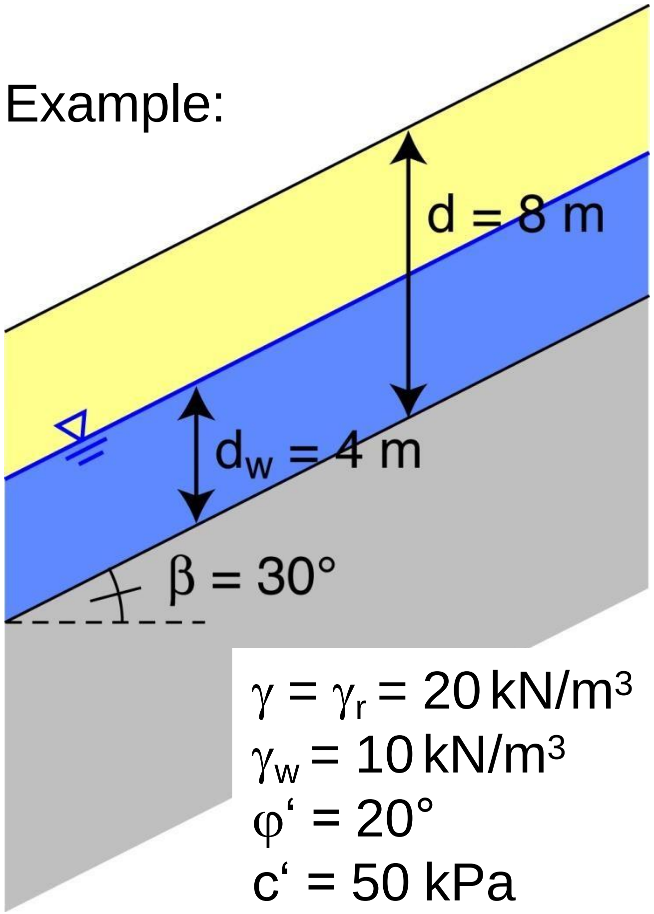


Infinite slope failure

Water flow parallel to slope, soil with φ‘ and c‘

Modified Duncan stability chart

Example:



From equations:

$$A = 1 - \frac{10 \cdot 4}{20 \cdot 8} = 0.75$$

$$B = \frac{1}{\sin(30°) \cdot \cos(30°)} = 2.31$$

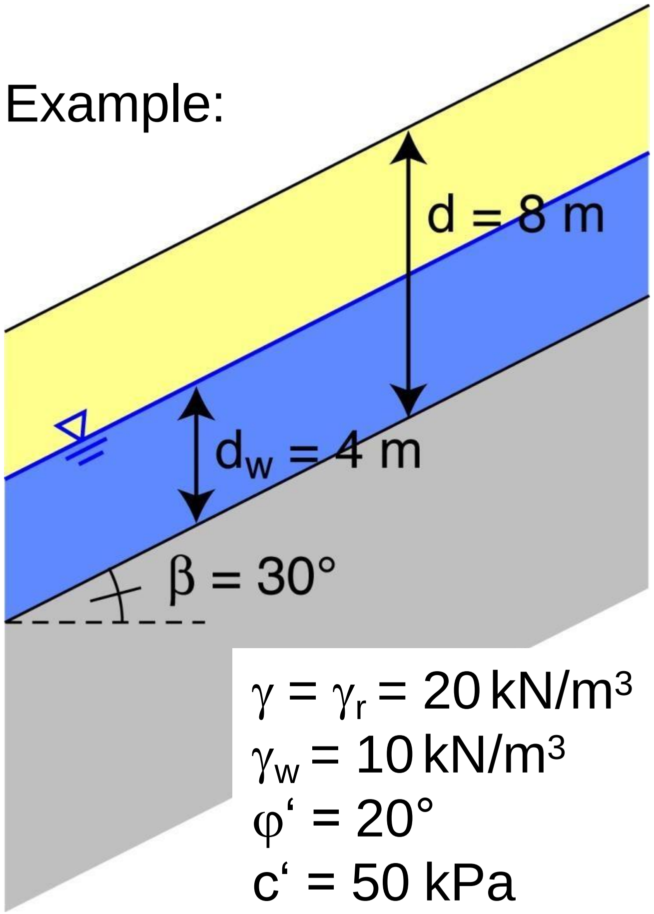
$$FS = 0.75 \cdot \frac{\tan(20°)}{\tan(30°)} + 2.31 \cdot \frac{50}{20 \cdot 8} = 1.19$$

Infinite slope failure

Water flow parallel to slope, soil with φ‘ and c‘

Modified Duncan stability chart

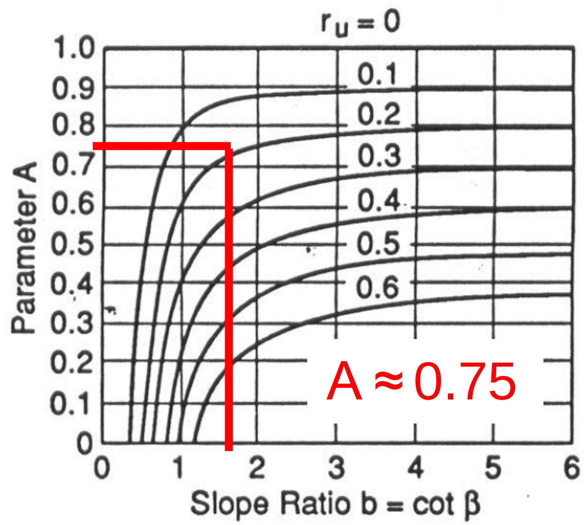
Example:

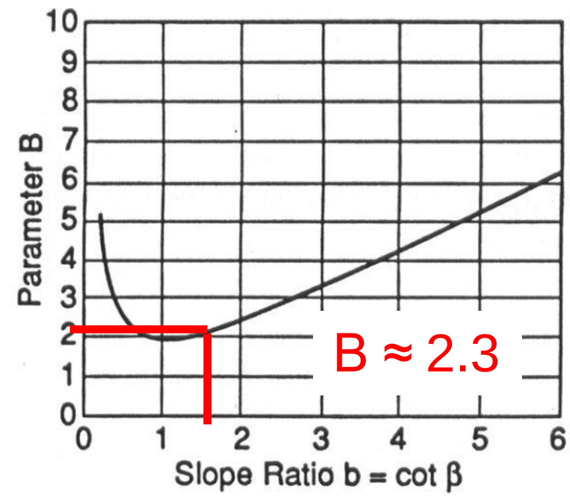


From diagrams:

$$b = \cot\beta = 1.73$$

$$r_u = \frac{X}{T}\cdot\frac{\gamma_w}{\gamma_r}\cdot\cos^2\beta = \frac{4}{8}\cdot\frac{10}{20}\cdot\cos^2(30°) = 0.19$$

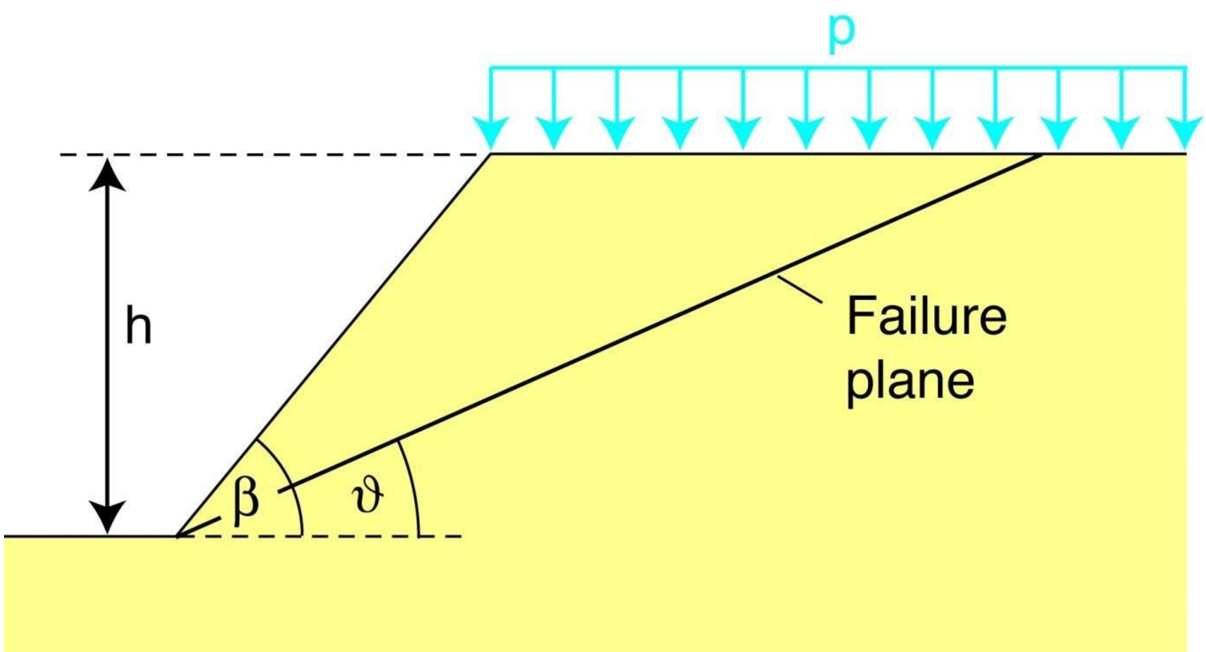




→ same solution

Plane failure – wedge analysis

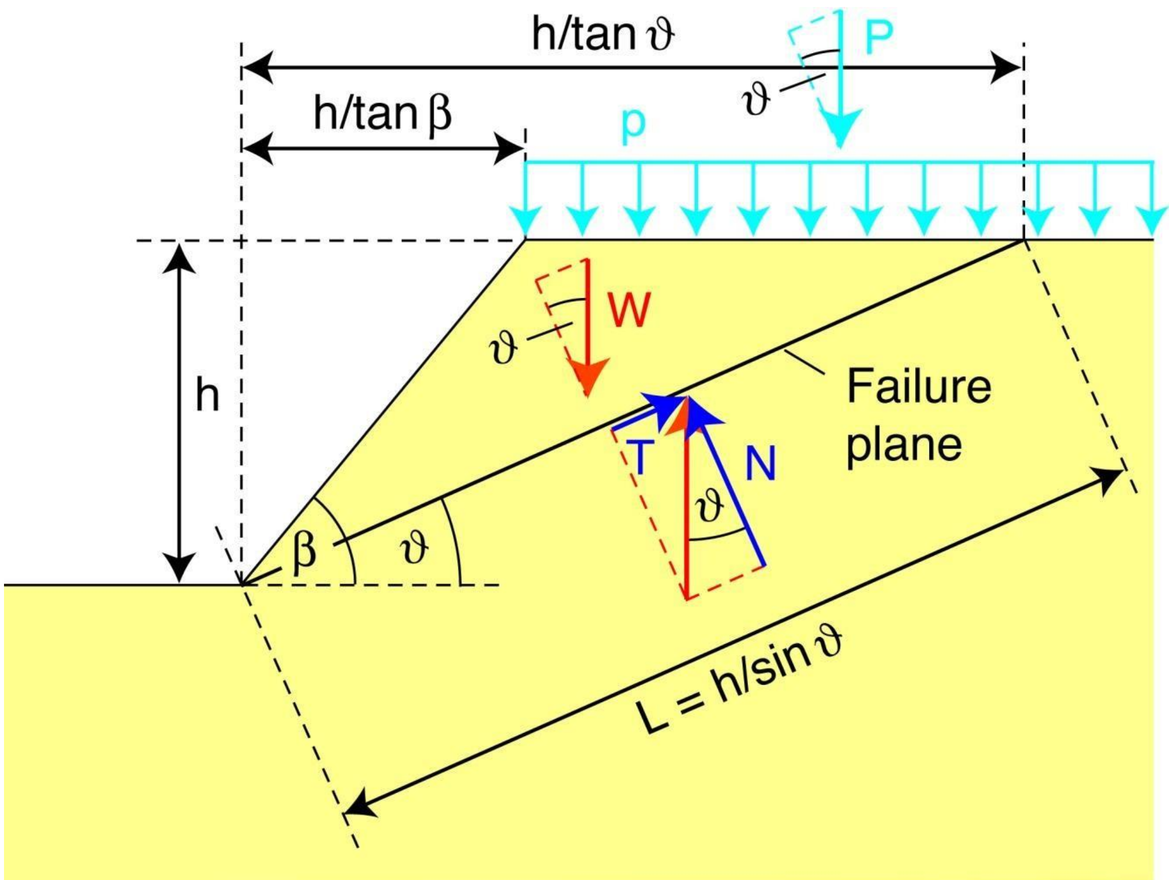
Practical relevance



Plane failure may occur in steep slopes if there are any predominant sliding planes (weak surfaces oriented in unfavourable directions)

Plane failure – wedge analysis

No water



Acting forces:

$$N = (W + P) \cdot \cos\vartheta$$

$$T = (W + P) \cdot \sin\vartheta$$

$$W = \frac{1}{2} \cdot \gamma \cdot h^2 \cdot \left(\frac{h}{\tan\vartheta} - \frac{h}{\tan\beta} \right)$$

$$= \frac{1}{2} \cdot \gamma \cdot h^2 \cdot (\cot\vartheta - \cot\beta)$$

$$P = p \cdot \left(\frac{h}{\tan\vartheta} - \frac{h}{\tan\beta} \right)$$

$$= p \cdot h \cdot (\cot\vartheta - \cot\beta)$$

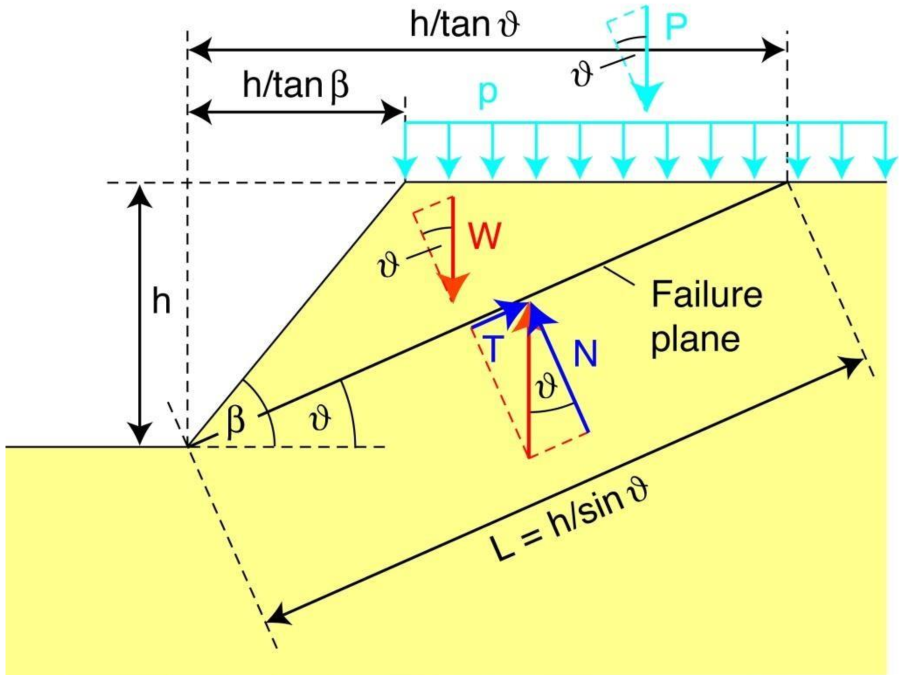
Maximum shear resistance on failure plane:

$$T_{\max} = N \cdot \tan\varphi' + C'$$

$$C' = c' \cdot L = \frac{c' \cdot h}{\sin\vartheta}$$

Plane failure – wedge analysis

No water



Factor of safety:

$$FS = \frac{T_{\max}}{T} = \frac{N \cdot \tan\varphi' + C'}{(W+P)\cdot\sin\vartheta}$$

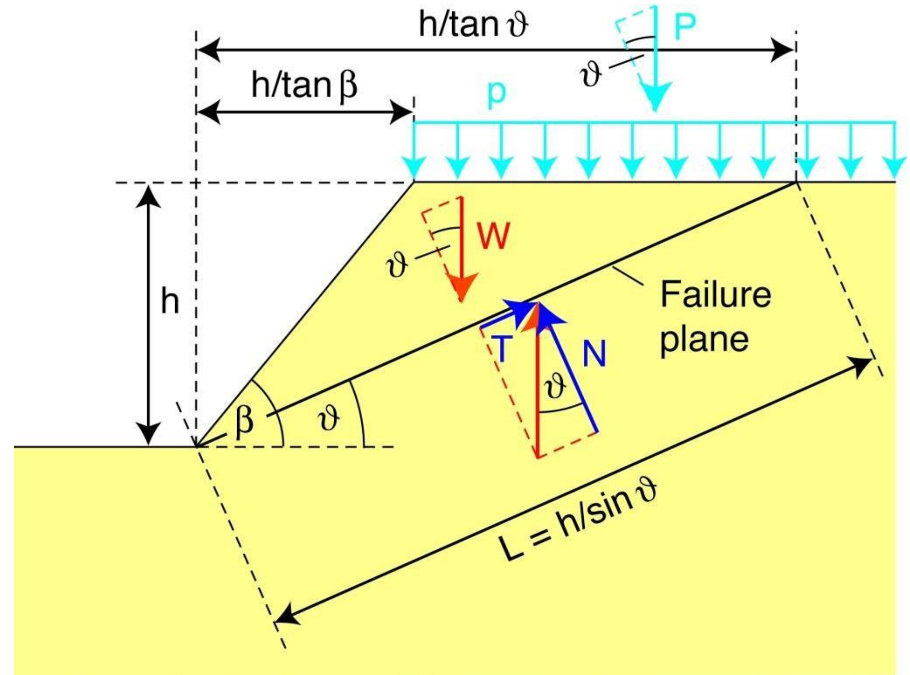
$$= \frac{(W+P)\cdot\cos\vartheta\cdot\tan\varphi' + C'}{(W+P)\cdot\sin\vartheta}$$

$$= \frac{\left[\frac{1}{2}\cdot\gamma\cdot h^2\cdot(\cot\vartheta - \cot\beta) + p\cdot h\cdot(\cot\vartheta - \cot\beta)\right]\cdot\cos\vartheta\cdot\tan\varphi' + \frac{c'\cdot h}{\sin\vartheta}}{\left[\frac{1}{2}\cdot\gamma\cdot h^2\cdot(\cot\vartheta - \cot\beta) + p\cdot h\cdot(\cot\vartheta - \cot\beta)\right]\cdot\sin\vartheta}$$

$$= \frac{\left[\frac{1}{2}\cdot\gamma\cdot h^2 + p\cdot h\right]\cdot(\cot\vartheta - \cot\beta)\cdot\cos\vartheta\cdot\tan\varphi' + \frac{c'\cdot h}{\sin\vartheta}}{\left[\frac{1}{2}\cdot\gamma\cdot h^2 + p\cdot h\right]\cdot(\cot\vartheta - \cot\beta)\cdot\sin\vartheta}$$

Plane failure – wedge analysis

No water



Special case c‘ = 0, p = 0

$$FS = \frac{\tan\varphi'}{\tan\vartheta}$$

For FS = 1:

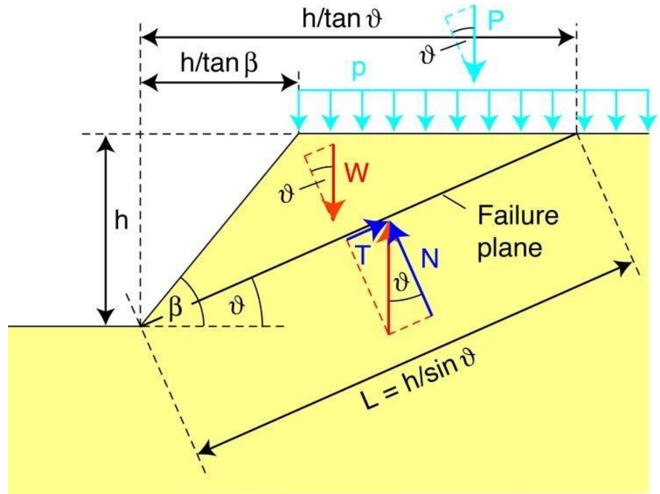
$$\tan\vartheta = \tan\varphi'$$

$$\vartheta = \varphi'$$

→ Slopes steeper than β = φ‘ are not stable

Plane failure – wedge analysis

No water



Special case c‘ ≠ 0, p = 0

$$FS = \frac{\frac{1}{2} \cdot \gamma \cdot h^2 \cdot (\cot\vartheta - \cot\beta) \cdot \cos\vartheta \cdot \tan\varphi' + \frac{c' \cdot h}{\sin\vartheta}}{\frac{1}{2} \cdot \gamma \cdot h^2 \cdot (\cot\vartheta - \cot\beta) \cdot \sin\vartheta}$$

For FS = 1:

$$c' = \frac{\gamma \cdot h}{2} \cdot (\cot\vartheta - \cot\beta) \cdot (\sin\vartheta - \cos\vartheta \cdot \tan\varphi') \cdot \sin\vartheta$$

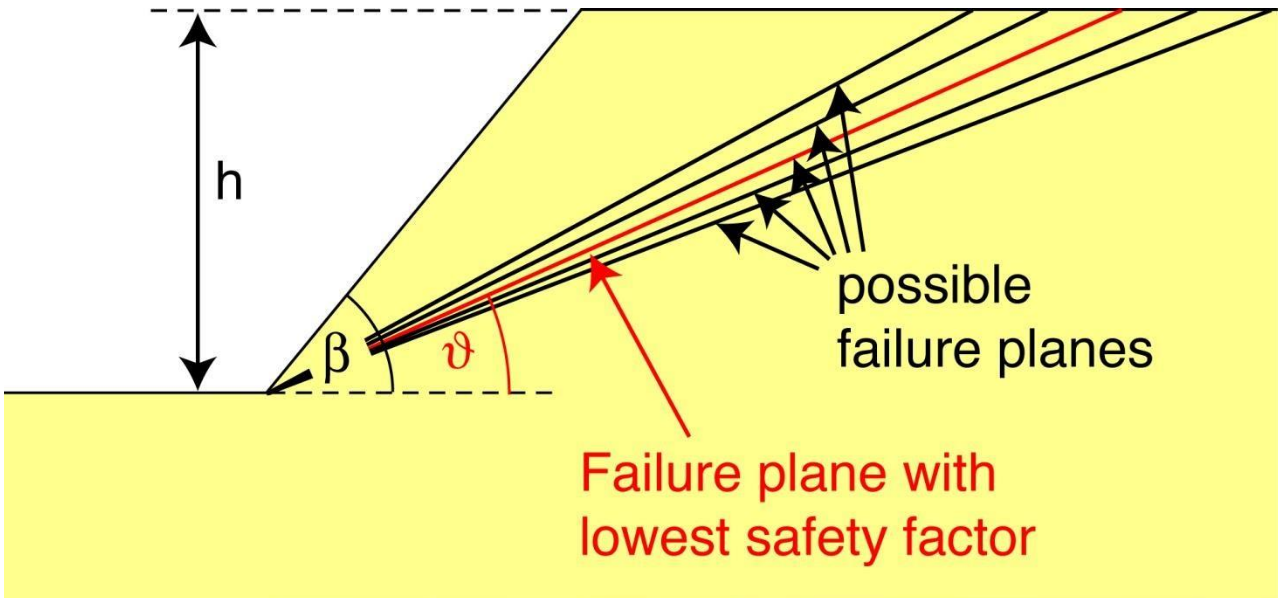
$$c' = \gamma \cdot h \cdot K_c$$

K_c = cohesion factor

$$K_c = \frac{1}{2} \cdot (\cot\vartheta - \cot\beta) \cdot (\sin\vartheta - \cos\vartheta \cdot \tan\varphi') \cdot \sin\vartheta$$

Plane failure – wedge analysis

No water



- Variation of ϑ in order to find the failure plane with the lowest safety factor
  $\rightarrow$ Slope will most likely fail under this ϑ
- In this case the minimum FS is found for

$$\vartheta_0 = \frac{\beta + \varphi'}{2}$$

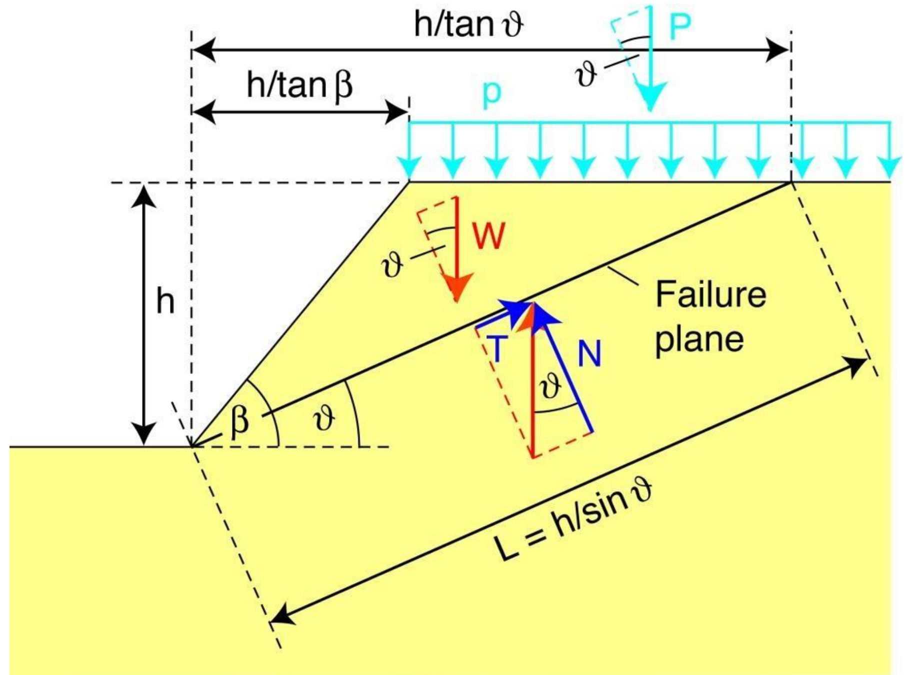
Plane failure – wedge analysis

No water

Special case c = c_u, φ = 0, p = 0

$$FS = \frac{T_{\max}}{T} = \frac{C_u}{W \cdot \sin\vartheta}$$

$$= \frac{2 \cdot c_u}{\gamma \cdot h \cdot (\cot\vartheta - \cot\beta) \cdot \sin^2\vartheta}$$



For FS = 1:

$$c_u = \frac{\gamma \cdot h}{2} \cdot (\cot\vartheta - \cot\beta) \cdot \sin^2\vartheta$$

$$c_u = \gamma \cdot h \cdot K_{cu}$$

$$K_c = \frac{1}{2} \cdot (\cot\vartheta - \cot\beta) \cdot \sin^2\vartheta$$

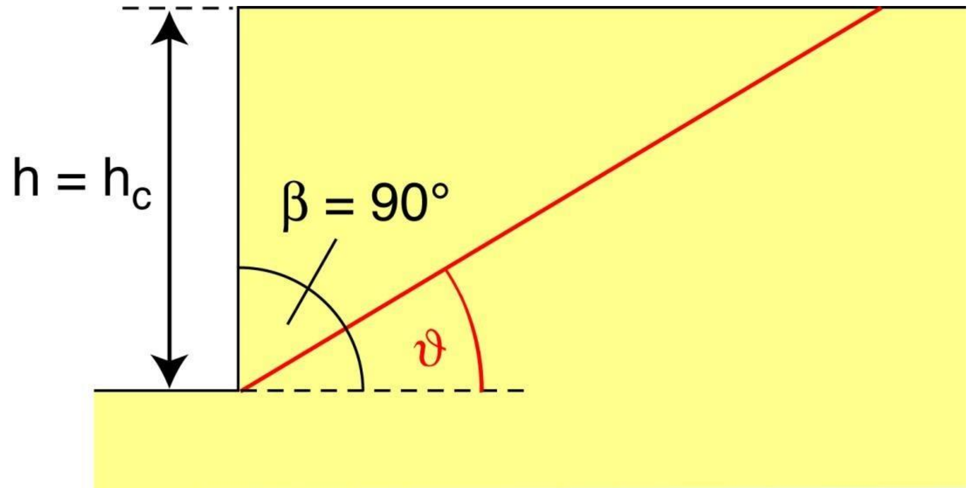
Searching ϑ for smallest FS leads to $\vartheta_0 = \frac{\beta}{2}$

Plane failure – wedge analysis

No water

Special case $c = c_u$, $\varphi = 0$, $p = 0$

Vertical excavation: $\beta = 90°$



For FS = 1 with $\vartheta_0 = \frac{\beta}{2}$:

$$c_u = \frac{\gamma \cdot h}{2} \cdot \left[\cot\left(\frac{\beta}{2}\right) - \cot\beta\right] \cdot \sin^2\left(\frac{\beta}{2}\right)$$

$$= \frac{\gamma \cdot h}{2} \cdot \left[\cot\left(\frac{90°}{2}\right) - \cot(90°)\right] \cdot \sin^2\left(\frac{90°}{2}\right)$$

$$c_u = \frac{\gamma \cdot h}{4}$$

Free standing height: $$h = h_c = \frac{4 \cdot c_u}{\gamma}$$

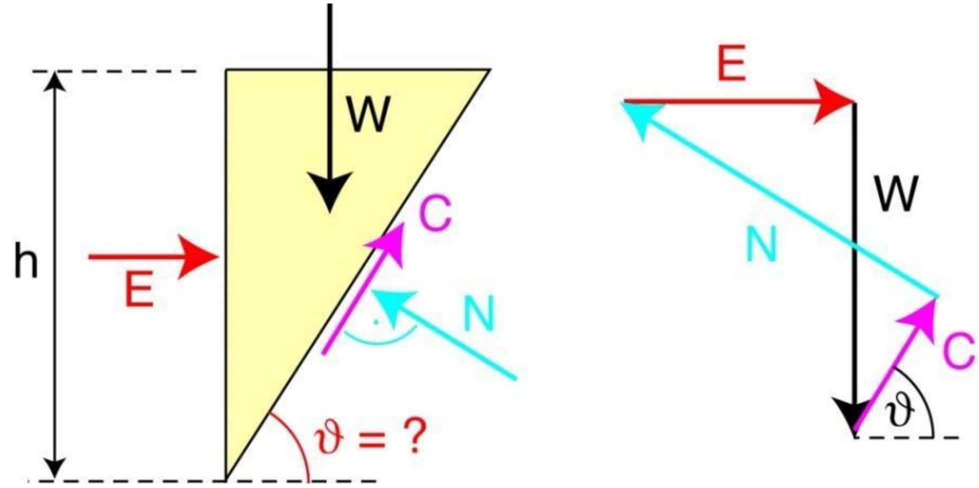
A vertical excavation can be undertaken without a wall up to this depth

Plane failure – wedge analysis

No water

Special case $c = c_u$, $\varphi = 0$, $p = 0$

Vertical excavation: $\beta = 90°$



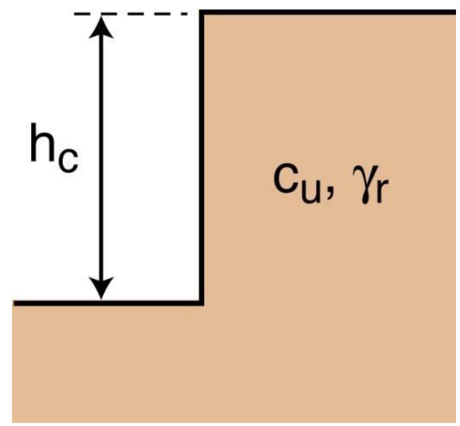
Comparison to solution from earth pressure theory (lecture Soil Mechanics)

- Maximum (= active) earth pressure obtained from $\partial E/\partial\vartheta = 0$:

$$\vartheta_a = 45° \qquad E_a = \frac{1}{2} \cdot \gamma_r \cdot h^2 - 2 \cdot c_u \cdot h$$

- Free standing height without wall from $E_a = 0$:

$$h_c = \frac{4 \cdot c_u}{\gamma_r}$$

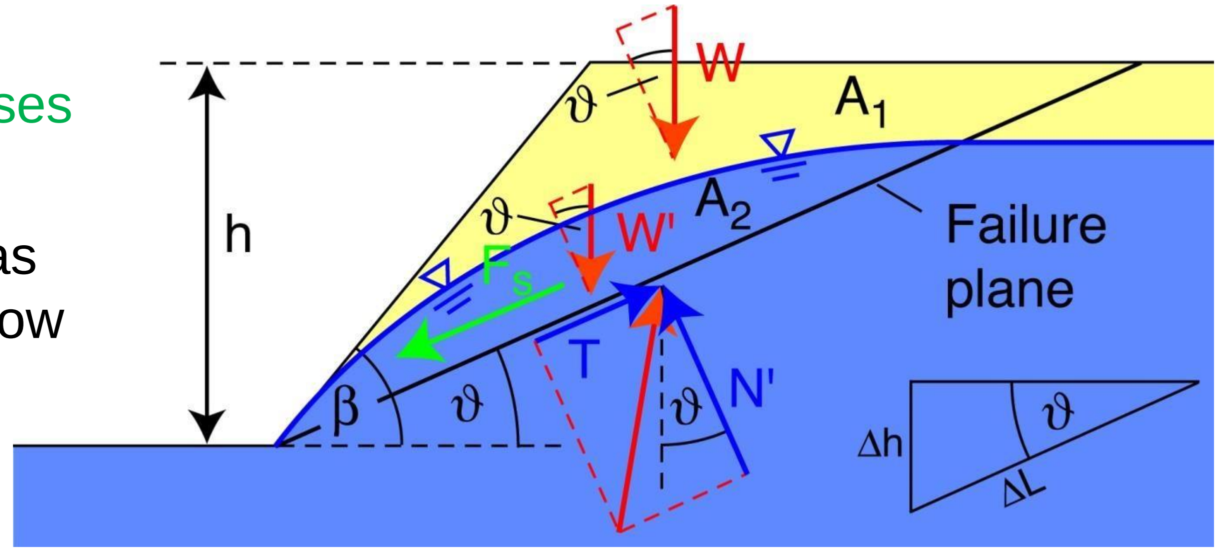


Plane failure – wedge analysis

Seepage flow within wedge

Analysis with effective stresses

A_1, A_2 = cross-sectional areas within wedge above and below ground water table



$$F_s = f_s \cdot A_2 = \gamma_w \cdot i \cdot A_2 = \gamma_w \cdot \sin\vartheta \cdot A_2$$

F_s acts approximately parallel to failure plane

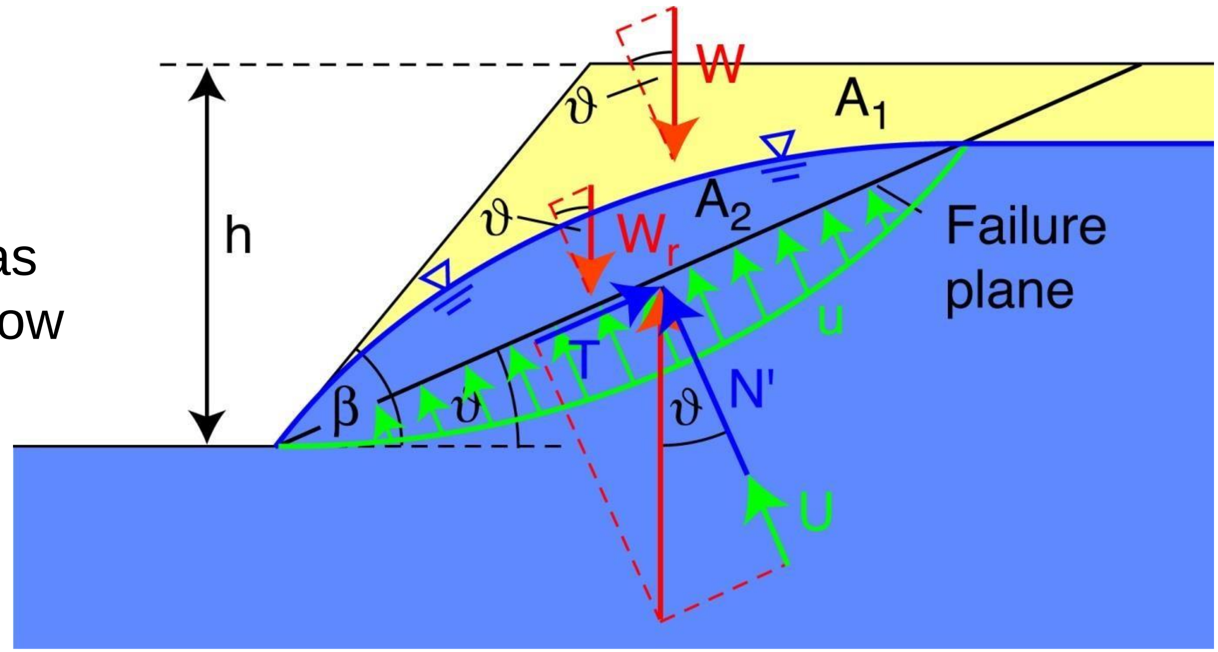
$$W = \gamma \cdot A_1 = \gamma \cdot (A - A_2) \qquad W' = \gamma' \cdot A_2$$

$$FS = \frac{T_{max}}{T} = \frac{N' \cdot \tan\varphi' + C'}{(W + W') \cdot \sin\vartheta + F_s} = \frac{(W + W') \cdot \cos\vartheta \cdot \tan\varphi' + C'}{(W + W') \cdot \sin\vartheta + F_s}$$

Plane failure – wedge analysis

Seepage flow within wedge

Analysis with total stresses



A_1, A_2 = cross-sectional areas within wedge above and below ground water table

$$U = \gamma_w \cdot A_2 \cdot \cos\vartheta$$

$$W = \gamma \cdot A_1 = \gamma \cdot (A - A_2)$$

$$W_r = \gamma_r \cdot A_2 = (\gamma' + \gamma_w) \cdot A_2 = W' + \frac{U}{\cos\vartheta} = W' + \frac{F_s}{\sin\vartheta}$$

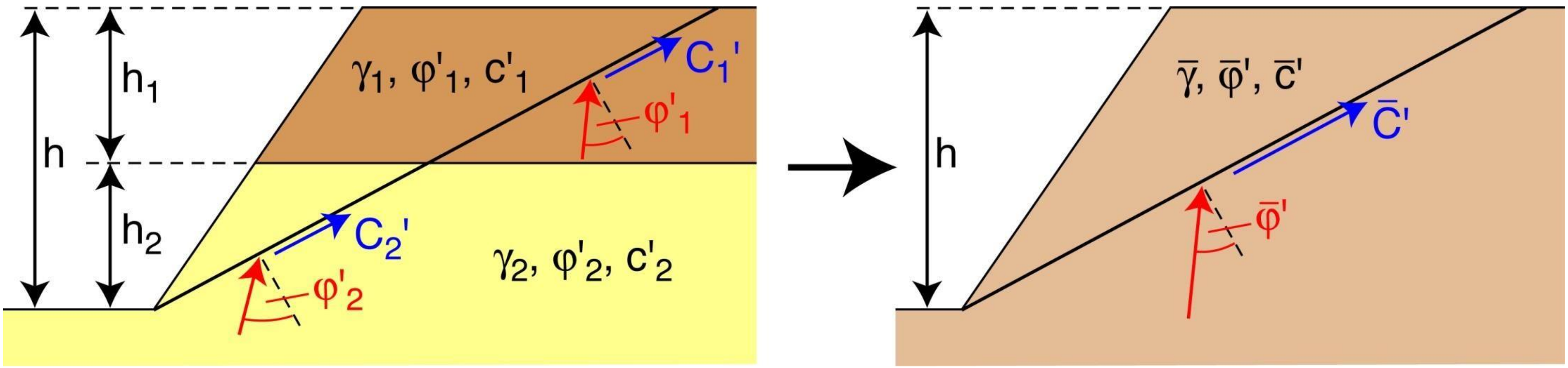
$$FS = \frac{T_{\max}}{T} = \frac{N' \cdot \tan\varphi' + C'}{(W + W_r) \cdot \sin\vartheta} = \frac{[(W + W_r) \cdot \cos\vartheta - U] \cdot \tan\varphi' + C'}{(W + W_r) \cdot \sin\vartheta}$$

$$= \frac{(W + W') \cdot \cos\vartheta \cdot \tan\varphi' + C'}{(W + W') \cdot \sin\vartheta + F_s}$$

Same solution as with effective stresses

Plane failure – wedge analysis

Homogenization of soil parameters in case of two layers



$$\gamma = \gamma_1 \cdot \left[1 - \left(\frac{h_2}{h}\right)^2\right] + \gamma_2 \cdot \left(\frac{h_2}{h}\right)^2$$

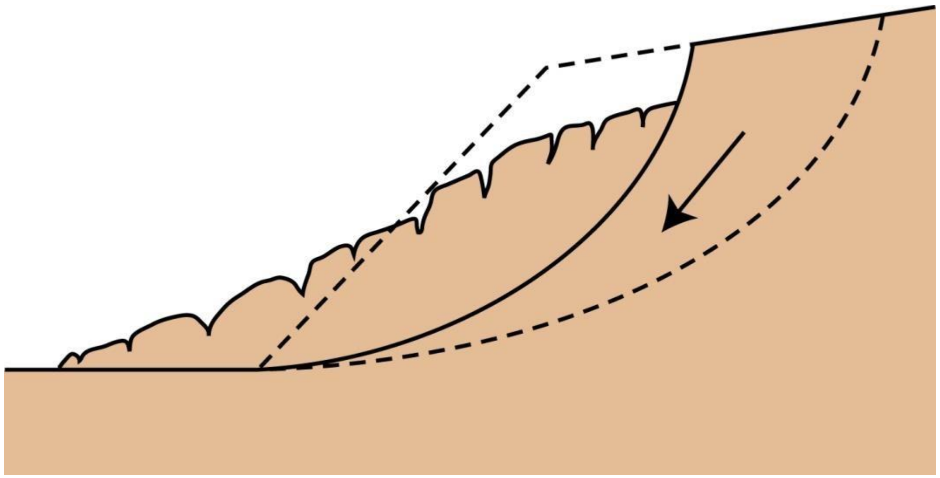
$$\tan\varphi = \tan\varphi_1 \cdot \left(\frac{h_1}{h}\right)^2 + \tan\varphi_2 \cdot \left[1 - \left(\frac{h_1}{h}\right)^2\right]$$

$$c = c_1 \cdot \left(1 - \frac{h_2}{h}\right) + c_2 \cdot \frac{h_2}{h}$$

$$\varphi \approx \varphi_1 \cdot \left(\frac{h_1}{h}\right)^2 + \varphi_2 \cdot \left[1 - \left(\frac{h_1}{h}\right)^2\right]$$

Slope stability is analyzed with these averaged parameters

Circular failure surface



- Collin (1847) observed slope failures in overconsolidated clay with curved failure surfaces
- Fellenius (1926) proposed to approximate the failure surface by a circle (so-called slip circle) passing the base point of the slope

Circular failure surface

Analysis without slices

Frictionless soil with cohesion
(e.g. $c_u \neq 0$, $\varphi_u = 0$)

- Self-weight W

$$W = m_w \cdot \gamma \cdot h^2$$

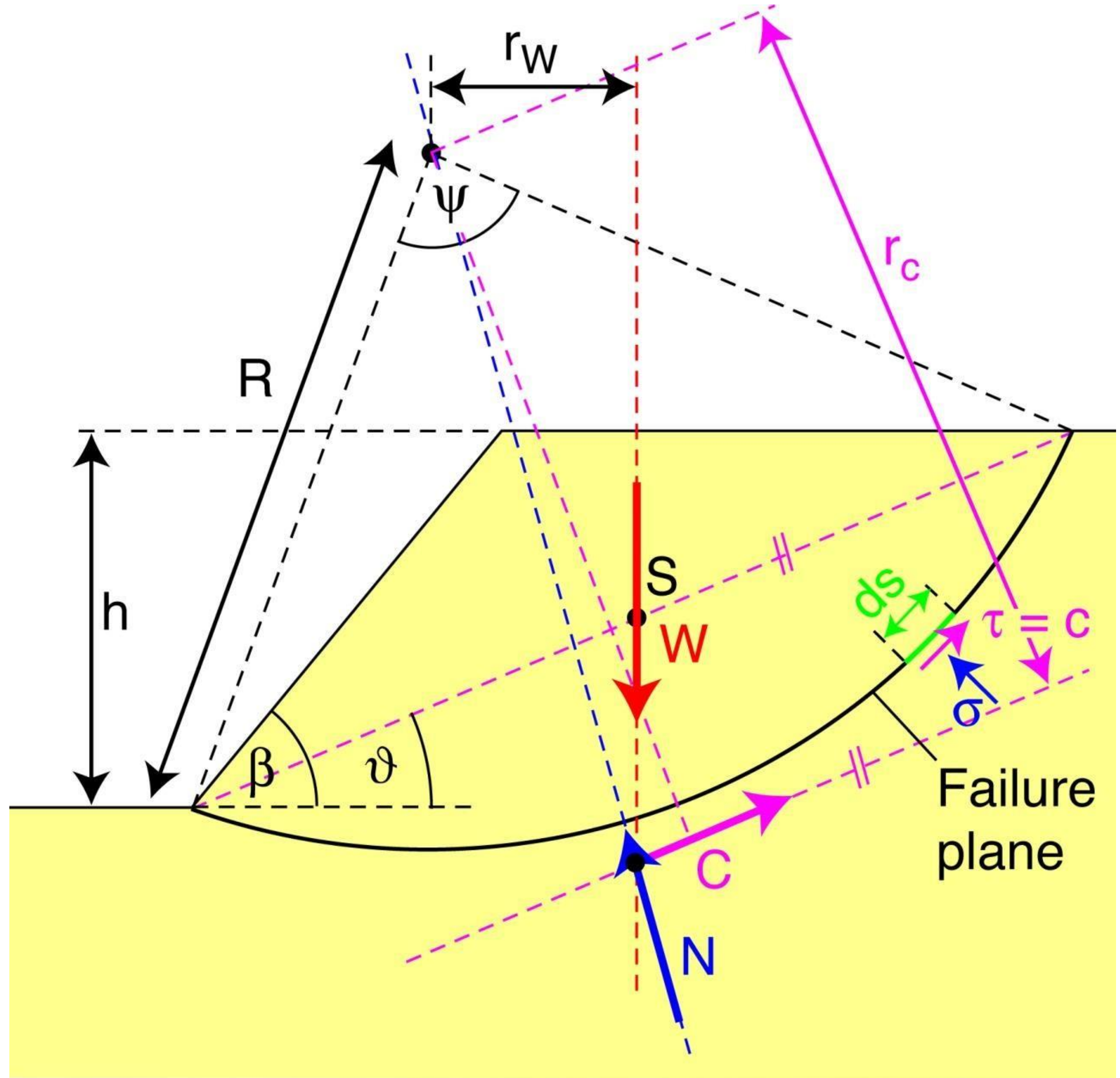
m_w = factor of geometry

$$m_w = f(\beta, \vartheta, \psi)$$

W acts in center of gravity of sliding mass, in distance r_W to center of slip circle

$$r_w = n_w \cdot h$$

$$n_w = f(\beta, \vartheta, \psi)$$



Circular failure surface

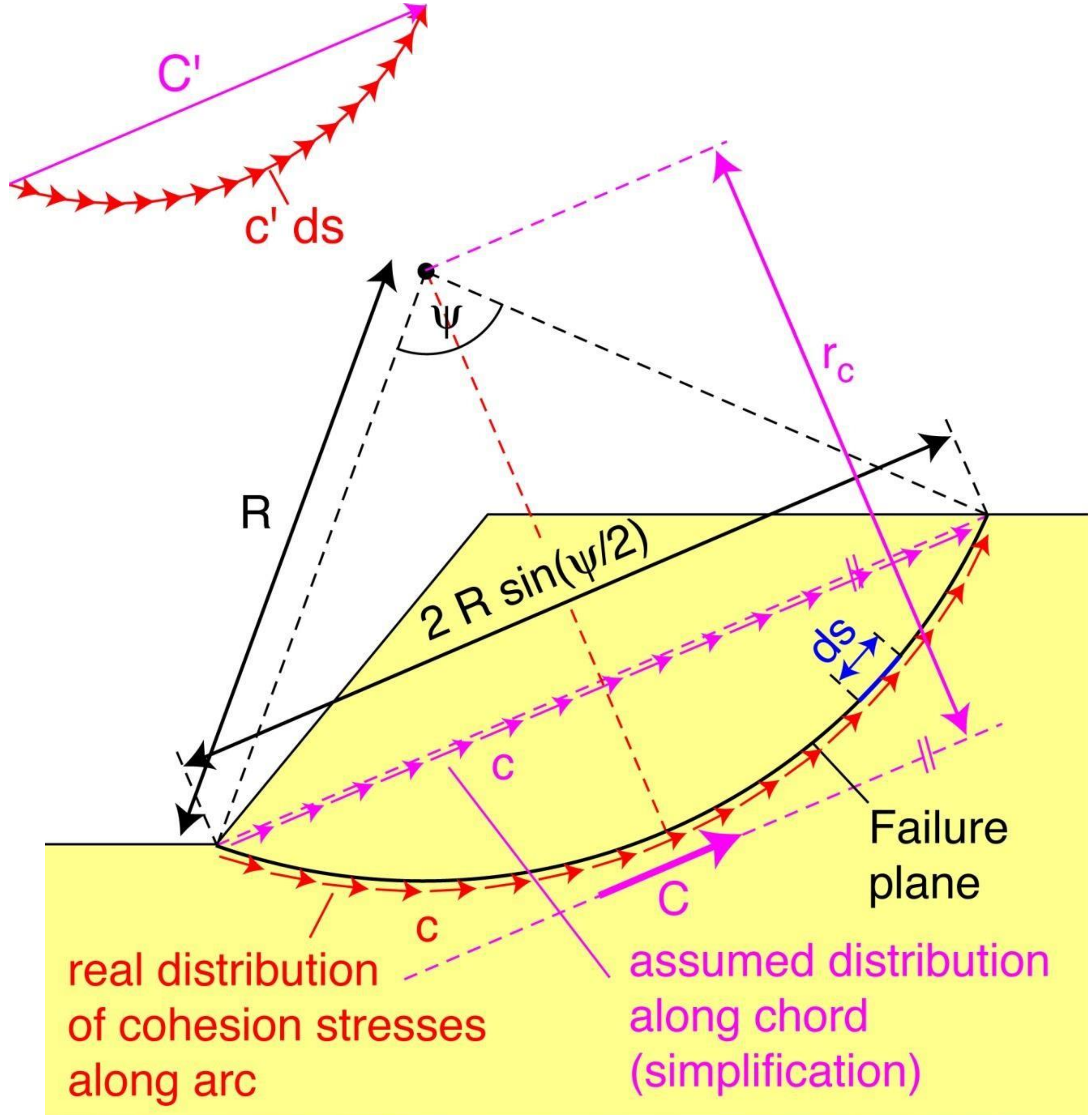
Analysis without slices

Frictionless soil with cohesion
(e.g. $c_u \neq 0$, $\varphi_u = 0$)

- Cohesion C
  - Differential cohesion forces c · ds along failure plane are added to a resulting vector C
  - The same C is obtained if c is multiplied with the length of the chord

$$C = 2 \cdot c \cdot R \cdot \sin(\psi/2)$$

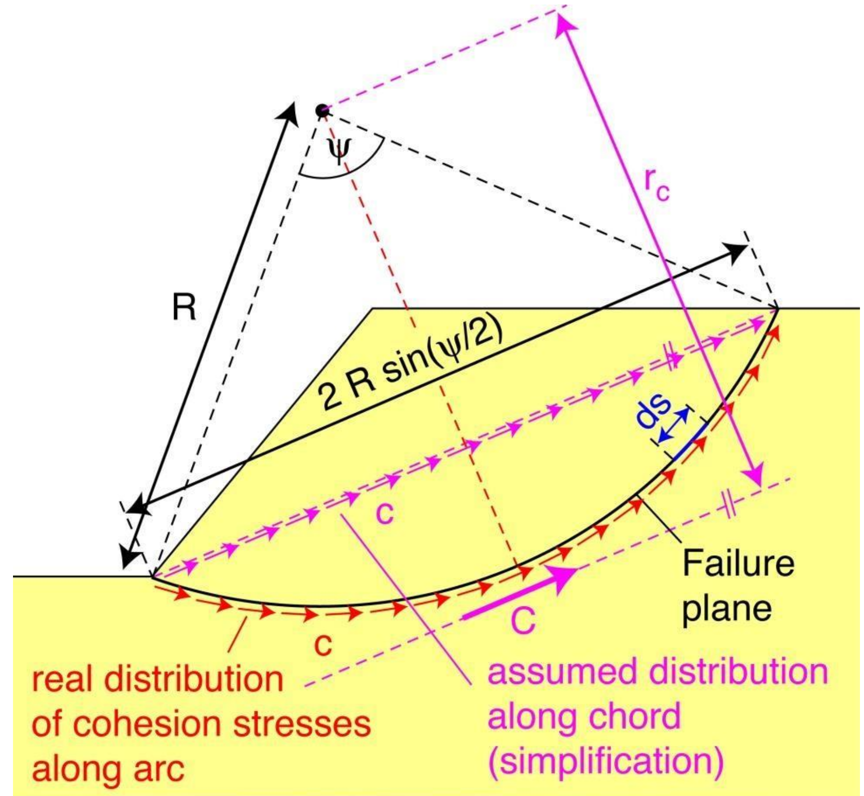
  - C acts parallel to the chord



Circular failure surface

Analysis without slices

Frictionless soil with cohesion
(e.g. $c_u \neq 0$, $\varphi_u = 0$)



- Cohesion C
  - Moment of differential cohesion forces c ds around center point of slip circle:

$$M_c = R \cdot c \cdot \mathrm{ds} = R \cdot c \cdot R \cdot \mathrm{d}\psi$$

$$= c \cdot R^2 \cdot \psi = C \cdot r_c$$

$$r_c = \frac{M_c}{C} = \frac{c \cdot R^2 \cdot \psi}{2 \cdot c \cdot R \cdot \sin(\psi/2)}$$

$$r_c = R \cdot \frac{\psi[\mathrm{rad}]}{2 \cdot \sin(\psi/2)}$$

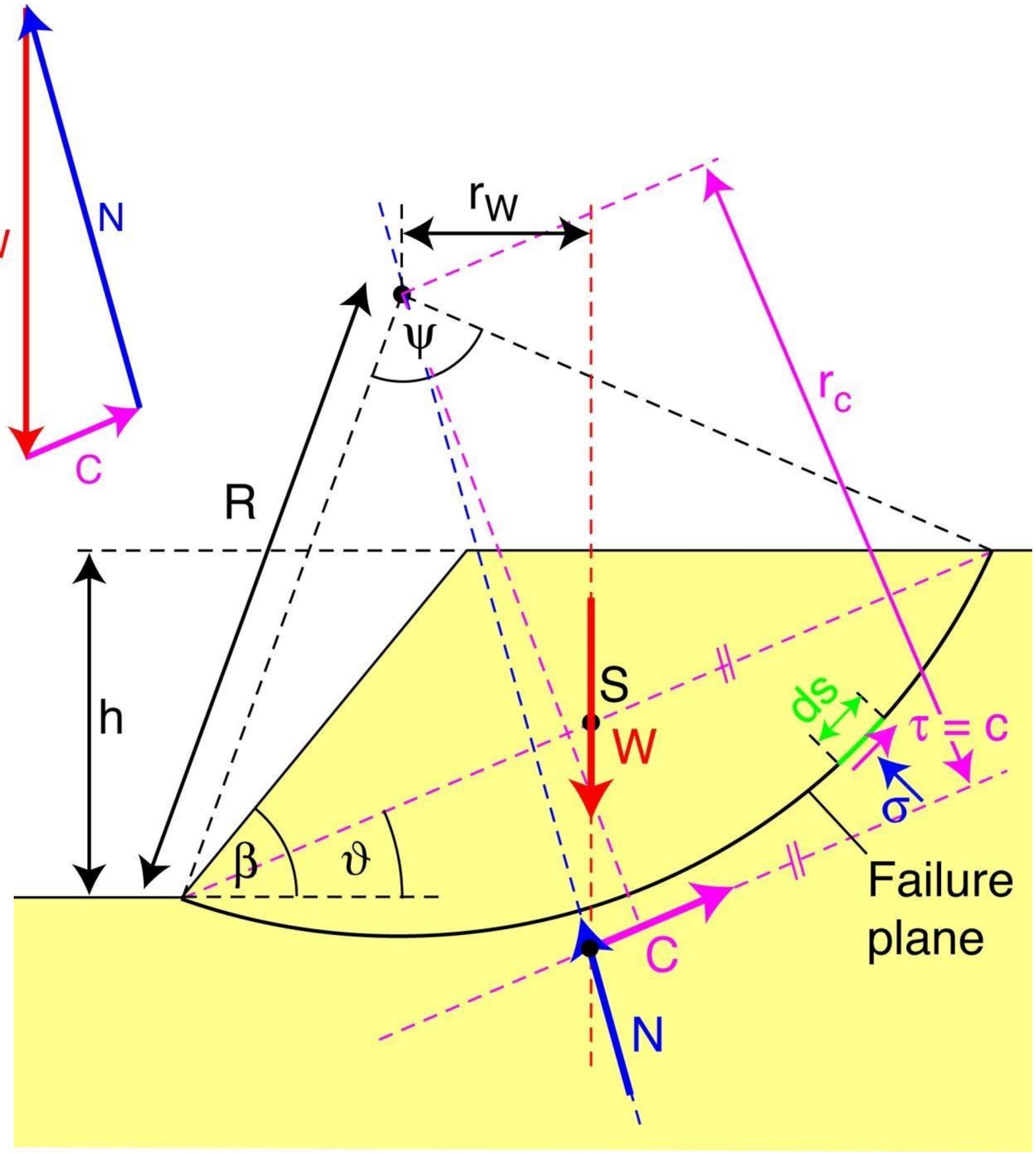
Failure analysis with slip circles is based on momentum equilibrium
→ it is important to consider the exact moment M_c

Circular failure surface

Analysis without slices

Frictionless soil with cohesion
(e.g. $c_u \neq 0$, $\varphi_u = 0$)

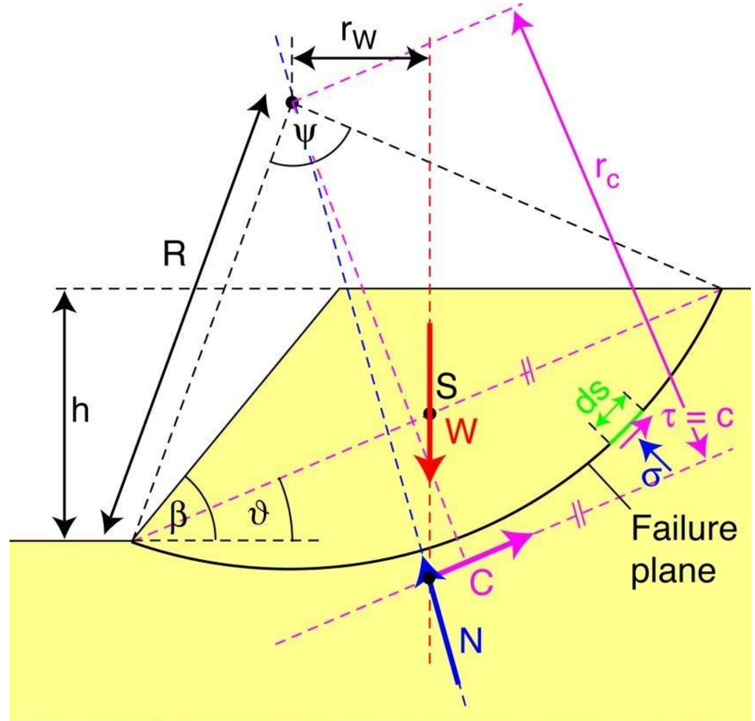
- Force N acts normal to the failure plane
- In order to fulfill equilibrium of momentum the line of application of N passes the intersection point of W and C
- Line of application of N passes the center of the slip circle, since all stresses σ act normal to failure surface, i.e. N causes no moment around center point of slip circle



Circular failure surface

Analysis without slices

Frictionless soil with cohesion
(e.g. $c_u \neq 0$, $\varphi_u = 0$)



Necessary c_{min} for preventing slope failure from either force equilibrium (force polygon) or equilibrium of momentum:

$$M_W = M_c \qquad W \cdot r_W = C \cdot r_c$$

$$m_W \cdot \gamma \cdot h^2 \cdot n_W \cdot h = c_{\min} \cdot R^2 \cdot \psi$$

$$c_{\min} = \frac{m_W \cdot n_W \cdot \gamma \cdot h^3}{R^2 \cdot \psi}$$

$$c_{\min} = K_c \cdot \gamma \cdot h$$

$$K_c = \frac{m_W \cdot n_W \cdot h^2}{R^2 \cdot \psi}$$

K_c = cohesion factor

Factor of safety:

$$FS = \frac{c}{c_{\min}} = \frac{c}{K_c \cdot \gamma \cdot h}$$

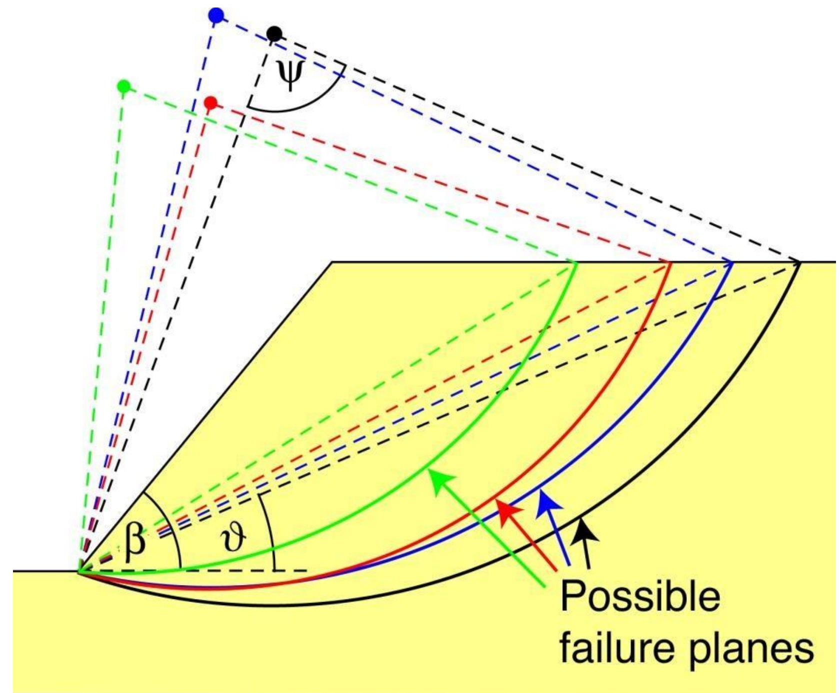
c = cohesion of soil determined from laboratory tests (e.g. c_u from UU triaxial tests)

Circular failure surface

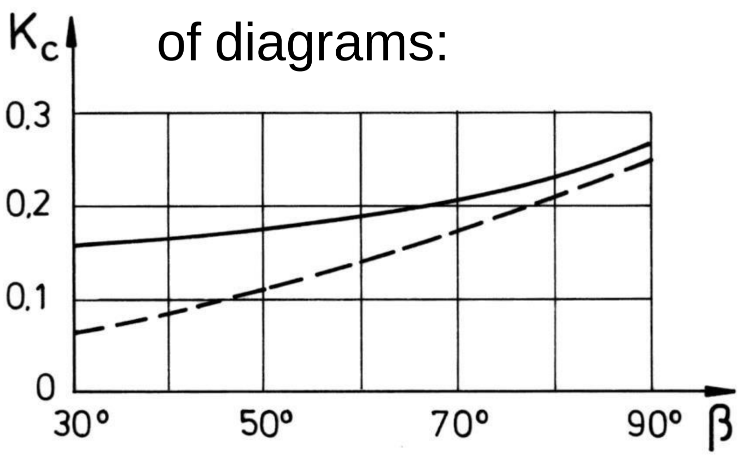
Analysis without slices

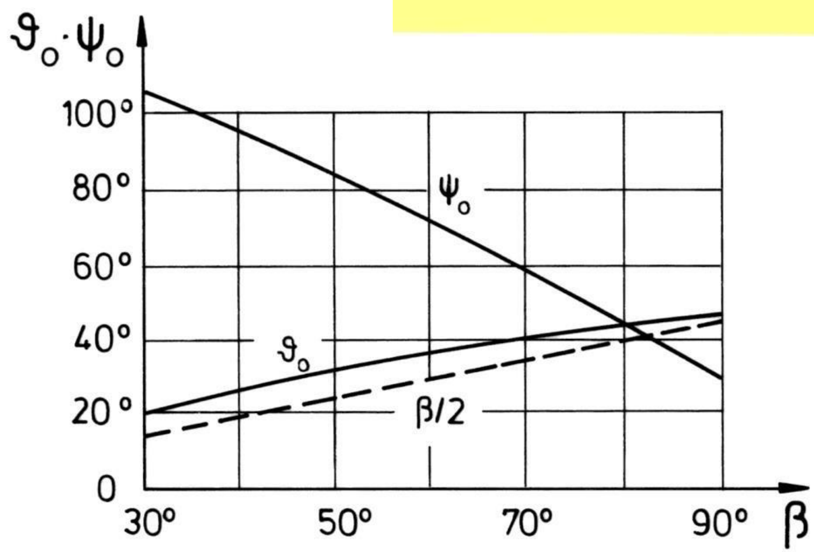
Frictionless soil with cohesion
(e.g. $c_u \neq 0$, $\varphi_u = 0$)

Variation of geometry (ϑ,ψ), until the failure circle with the lowest safety factor (highest K_c) is found → corresponding parameters ϑ_0, ψ_0



Solution in form of diagrams:





Solid lines:
= Failure circles

Dashed line
= wedge failure (overestimates FS !)

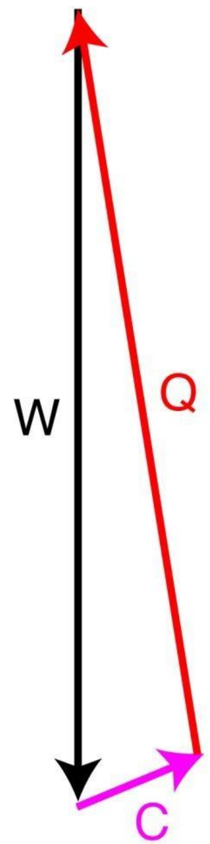
Circular failure surface

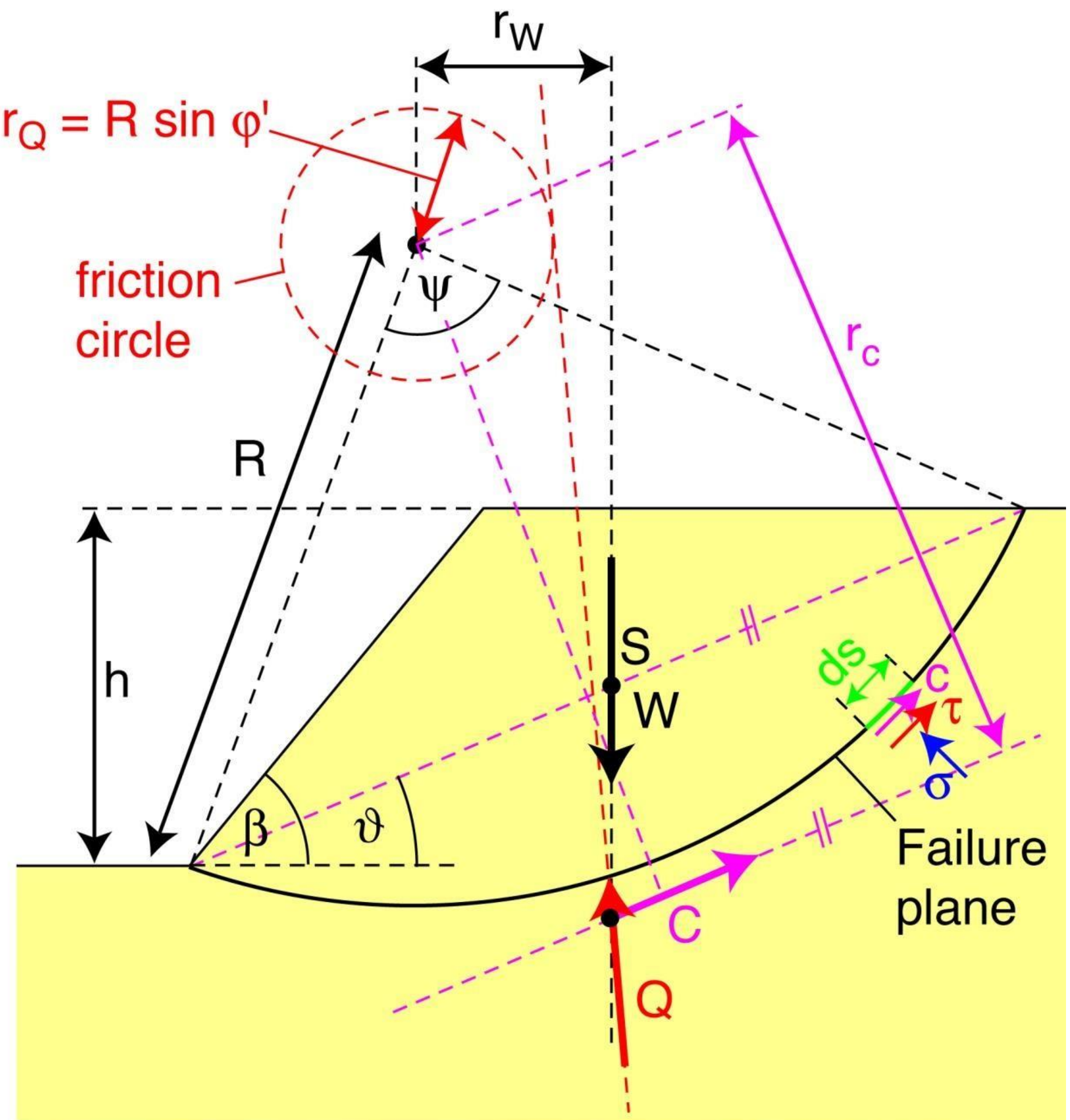
Analysis without slices

Soil with friction and cohesion
(i.e. $c' \neq 0$, $\varphi' \neq 0$)

- W and C identical to last example
- Line of application of reaction force Q passes intersection point of W and C
- Line of application of Q touches the „friction circle“ with radius

$$r_Q = R \cdot \sin \varphi'$$





Circular failure surface

Analysis without slices

Friction circle

- Incremental forces along failure surface:

$$dN' = \sigma' \cdot ds \qquad dT = \tau \cdot ds$$

$$dQ = \sqrt{(dN')^2 + (dT)^2}$$

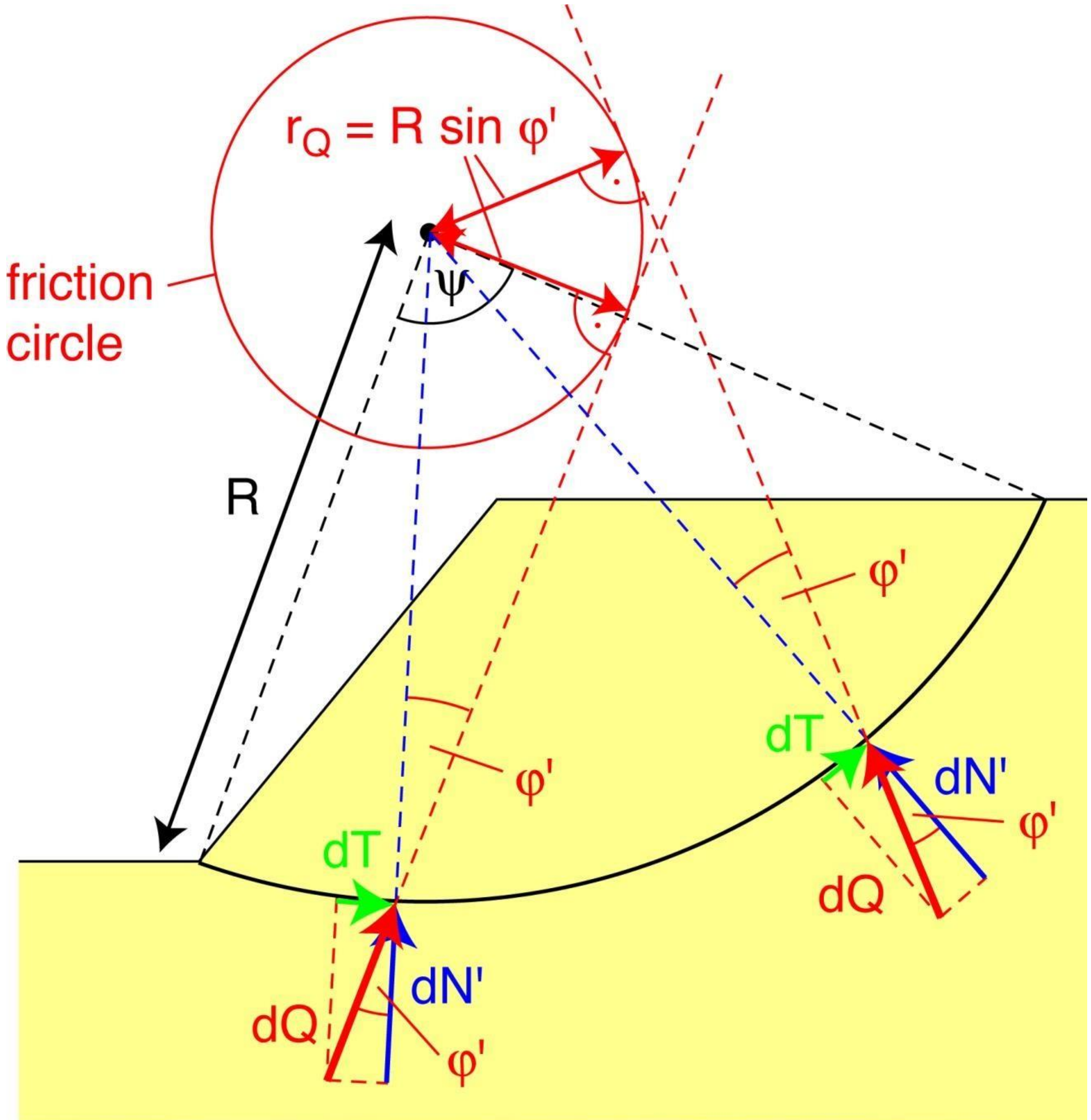
- In failure state:

$$\tau = \sigma' \cdot \tan\varphi'$$

$$dT = dN' \cdot \tan\varphi'$$

- All incremental forces dQ touch circle with radius r_Q

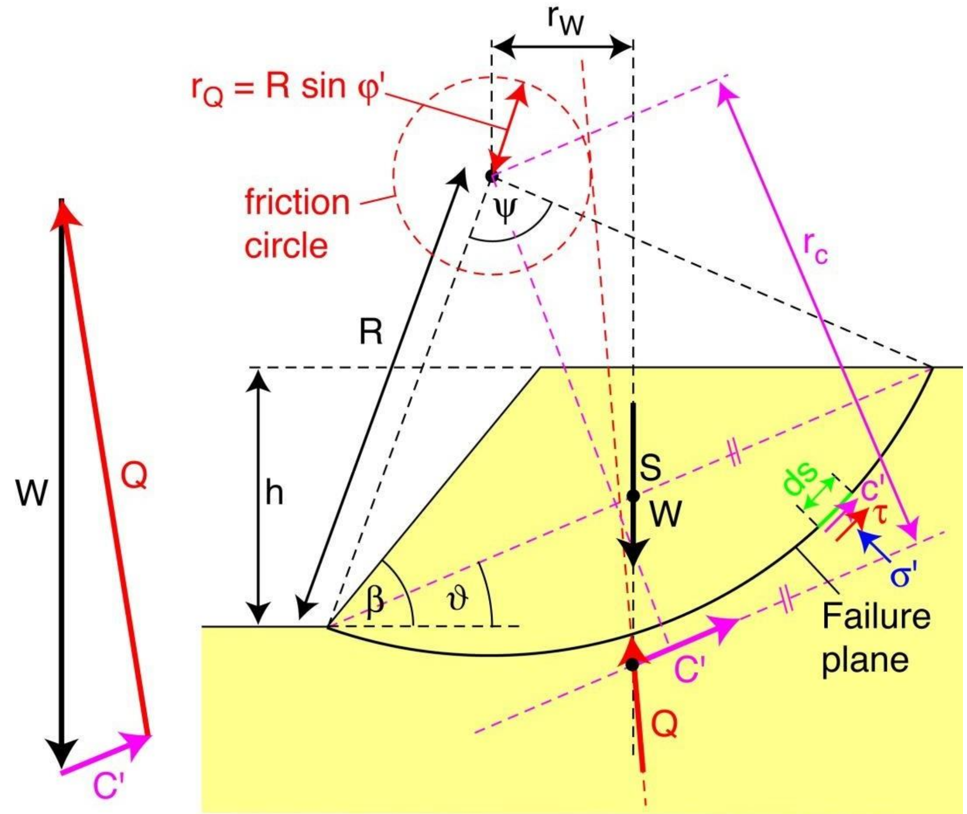
$$r_Q = R \cdot \sin\varphi'$$



Circular failure surface

Analysis without slices

Soil with friction and cohesion
(i.e. $c' \neq 0$, $\varphi' \neq 0$)



Two possibilities for global safety factor:

1. Via cohesion

- The radius r_Q is calculated with the effective friction angle φ' of the soil (e.g. from laboratory tests)
- Necessary c'_{min} for preventing slope failure from force polygon:

$$c'_{\min} = K_c \cdot \gamma \cdot h$$

- Safety factor:

$$FS = \frac{c'}{c'_{\min}} = \frac{c'}{K_c \cdot \gamma \cdot h}$$

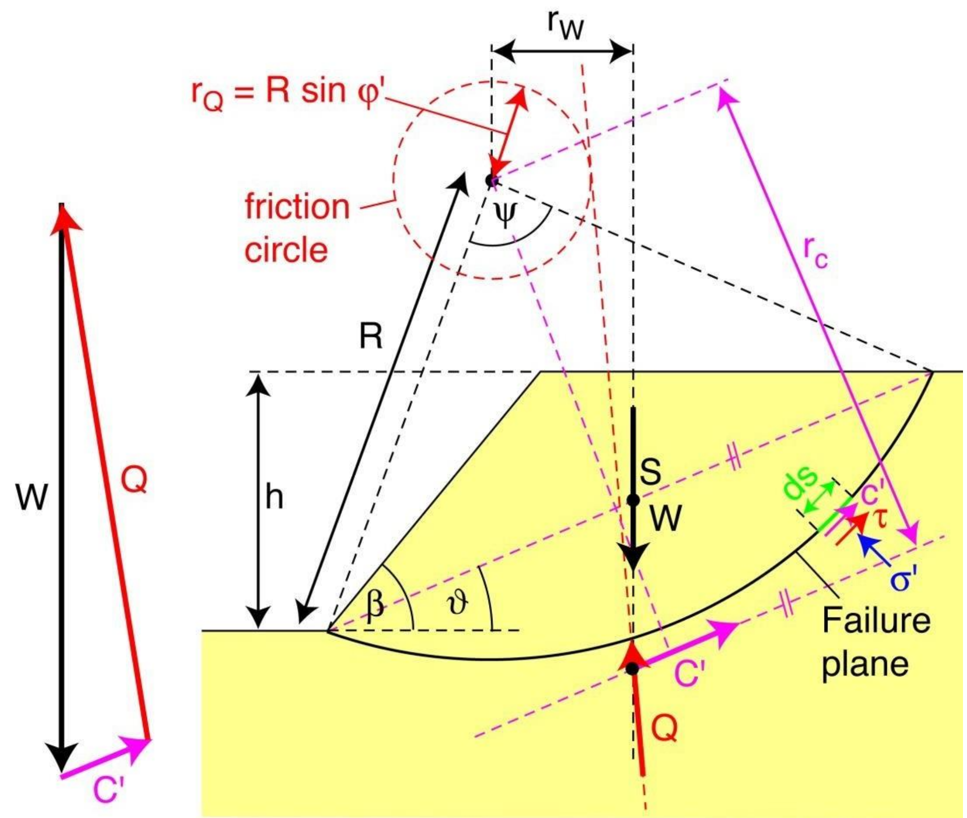
Circular failure surface

Analysis without slices

Soil with friction and cohesion
(i.e. c‘ ≠ 0, φ‘≠ 0)

Two possibilities for global safety factor:

2. Via friction angle



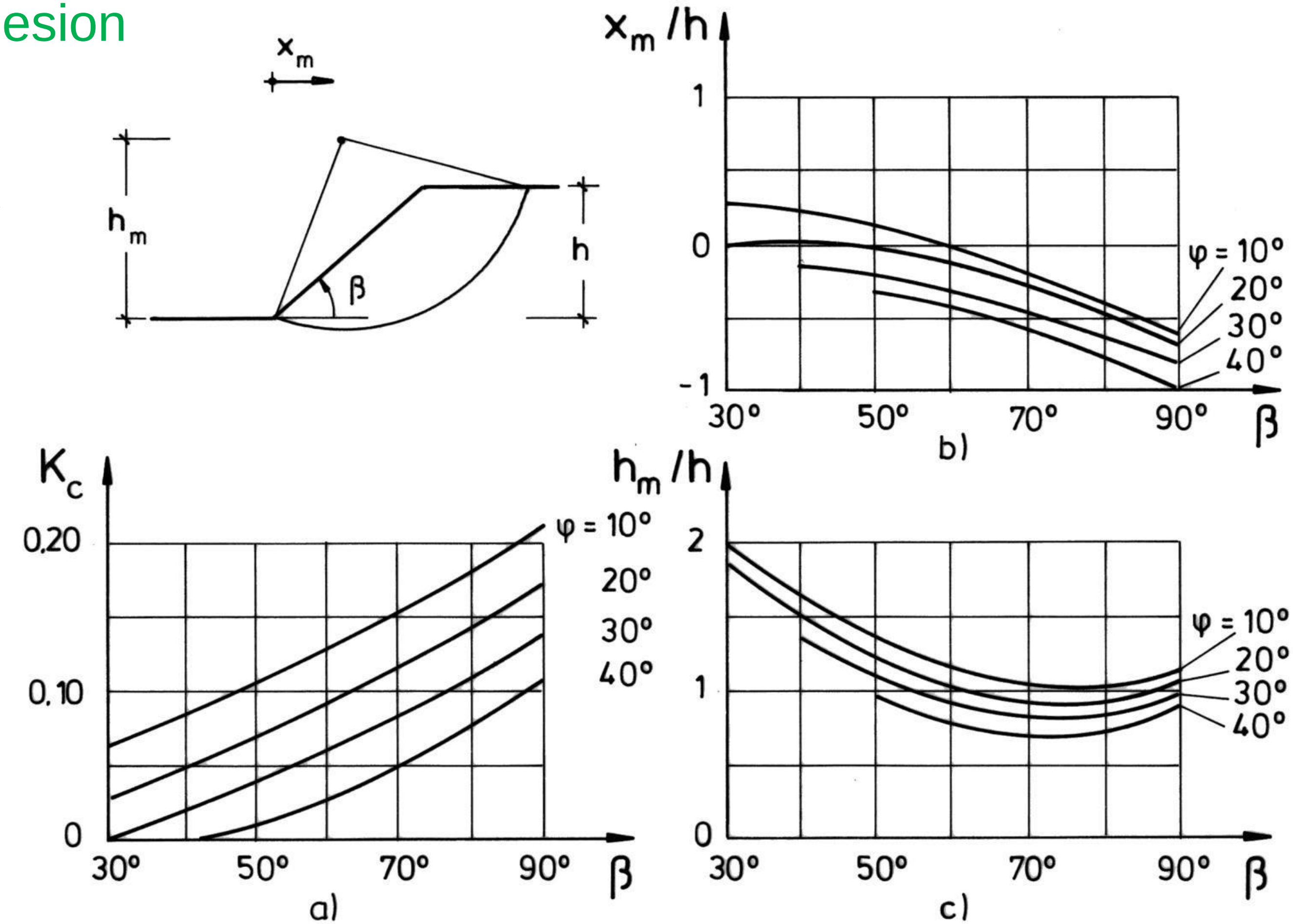
- C‘ is calculated with the cohesion c‘ of the soil (e.g. from laboratory)
- The force polygon delivers the direction of the line of application of Q
- The line of application of Q is layed through the intersection point of W and C‘
- The friction circle is constructed around the center point of the slip circle, touching the line of application of Q → radius r_Q → necessary friction angle φ'_{min}
- Safety factor: $FS = \tan(\varphi') / \tan(\varphi'_{min})$

Circular failure surface

Analysis without slices

Soil with friction and cohesion
(i.e. c‘ ≠ 0, φ‘≠ 0)

- Variation of geometry until minimum of safety factor is found
- Solution in form of diagrams:
- No cohesion necessary in case of $\beta = \varphi$‘



Circular failure surface

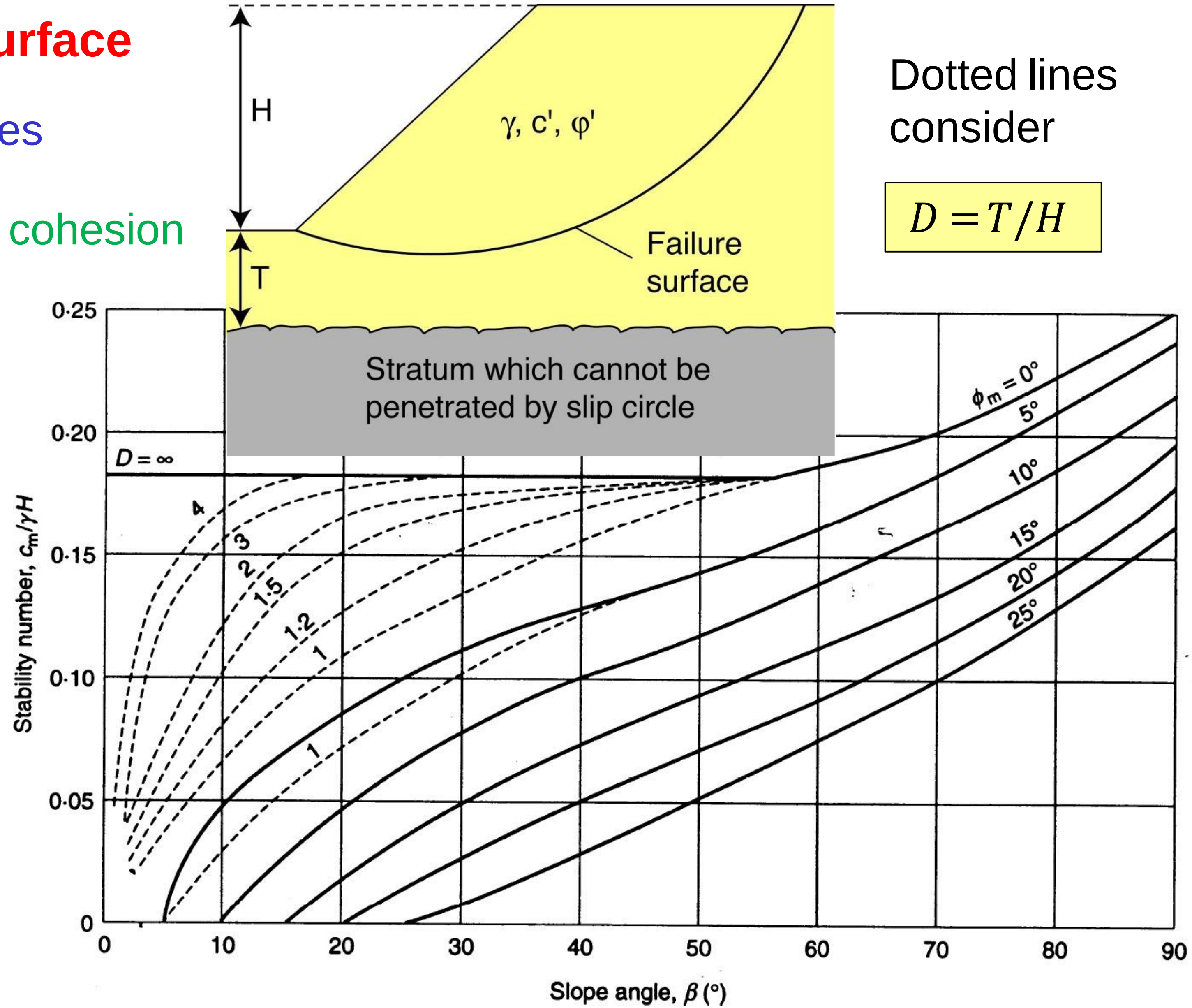
Analysis without slices

Soil with friction and cohesion
(i.e. c' ≠ 0, φ'≠ 0)

- Another representation of the same relationship by Taylor

$$K_c = \frac{c'_{min}}{\gamma \cdot H}$$

K_c is called „stability number" in this chart



Dotted lines consider

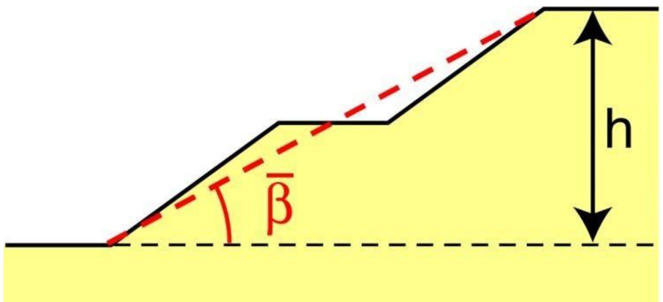
$$D = T/H$$

Circular failure surface

Analysis without slices

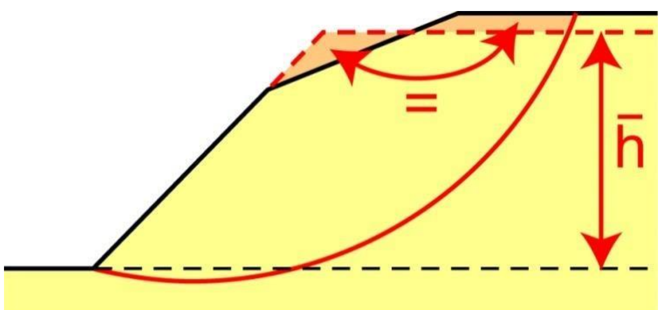
Simplifications in case of more complicated geometry

- Slope with berm:



Analysis with average inclination β

- Slope with two different inclinations



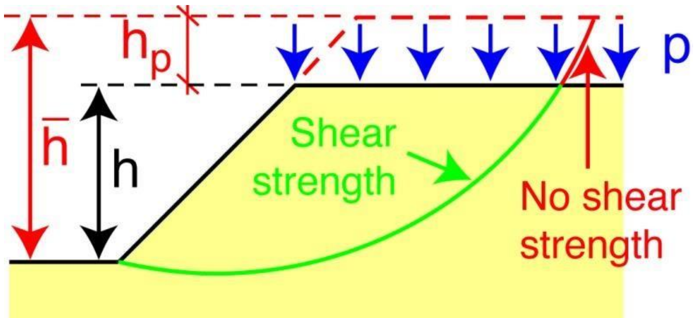
Analysis with substitutional height h,
so that the sliding mass is identical for
the original and the simplified geometry

Circular failure surface

Analysis without slices

Consideration of surface loads

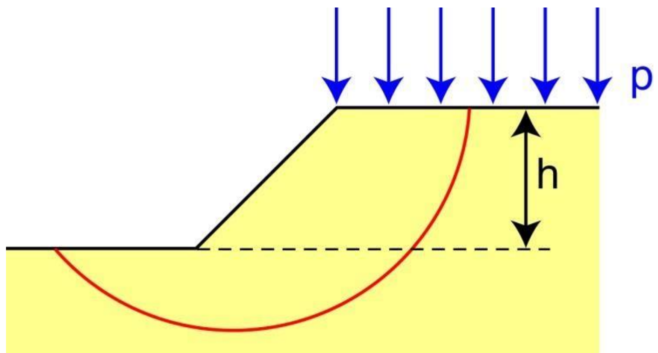
- Distributed load of magnitude $p \leq \gamma \cdot h/3$



– Replacement of p by increasing the height from *h* by $h_p = p/\gamma$ to h

– Over the height h_p there is no shear strength along the failure plane
$\rightarrow$ application of averaged shear strength parameters c, φ necessary (appropriate formulas are given later)

- Larger distributed load



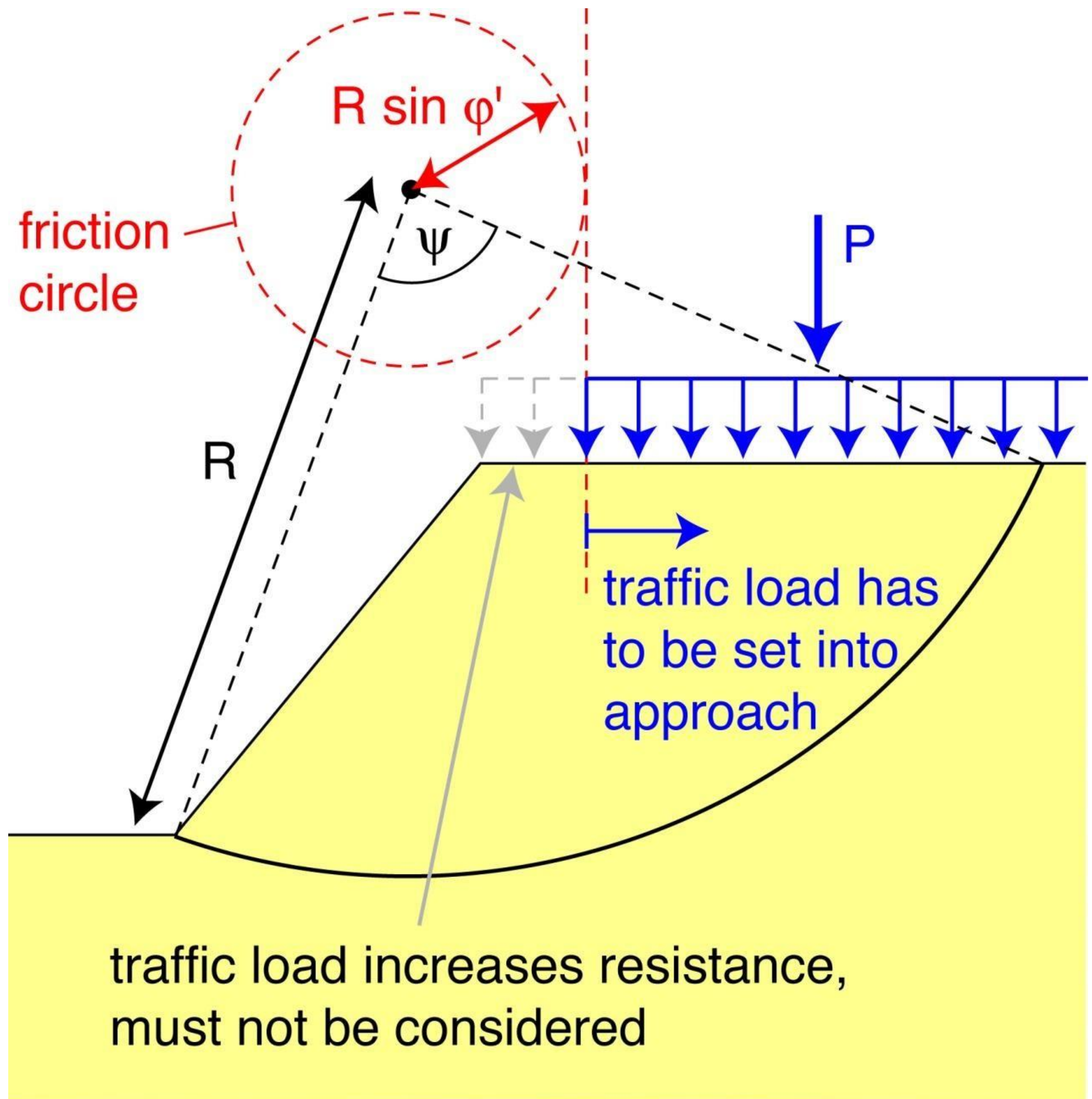
Leads to deep slip circles with toe in considerable distance to the base of the slope
$\rightarrow$ simplification not meaningful

Circular failure surface

Analysis without slices

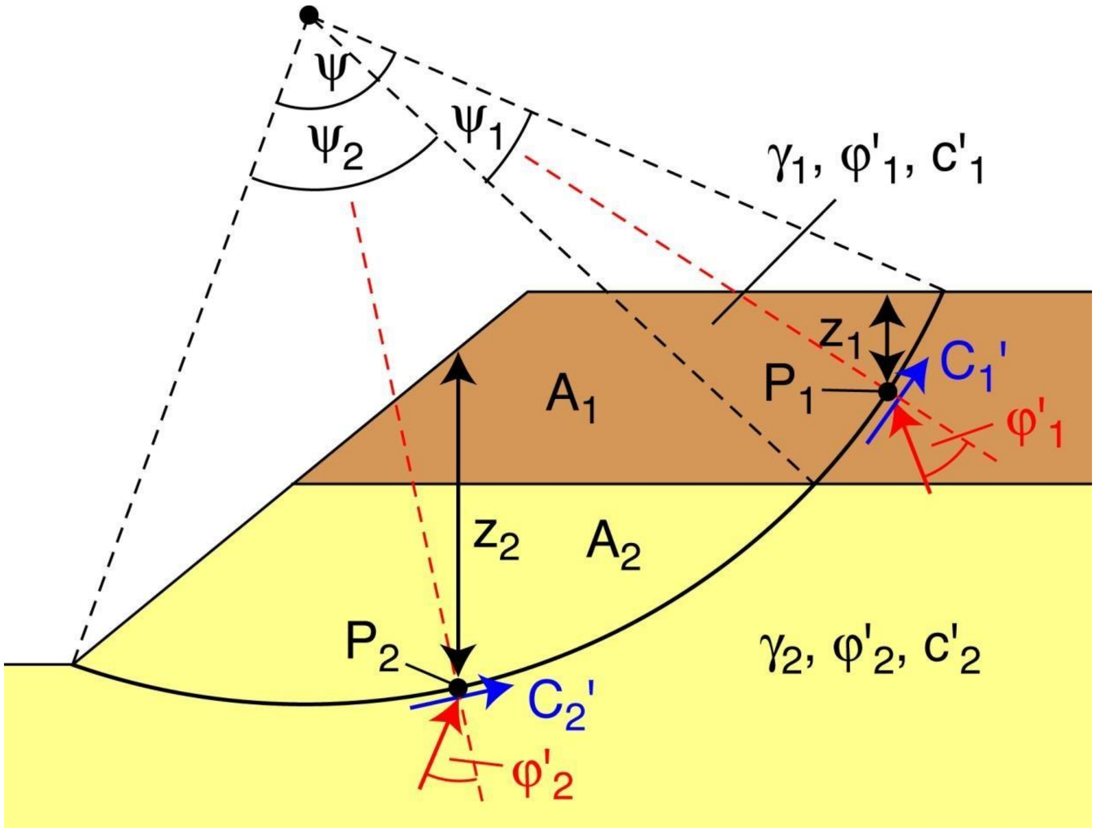
Consideration of surface loads (traffic loads)

- Traffic loads are only set into approach outside the friction circle because inside that circle they increase the resistance
- A detailed explanation is given later based on the analysis methods using slices



Circular failure surface

Homogenization of soil parameters in case of two layers



Averaged soil parameters:

- Specific weight:

$$\gamma = \frac{\gamma_1 \cdot A_1 + \gamma_2 \cdot A_2}{A_1 + A_2}$$

- Friction angle:

$$\varphi = \frac{\varphi_1 \cdot z_1 \cdot \psi_1 + \varphi_2 \cdot z_2 \cdot \psi_2}{z_1 \cdot \psi_1 + z_2 \cdot \psi_2}$$

- Cohesion:

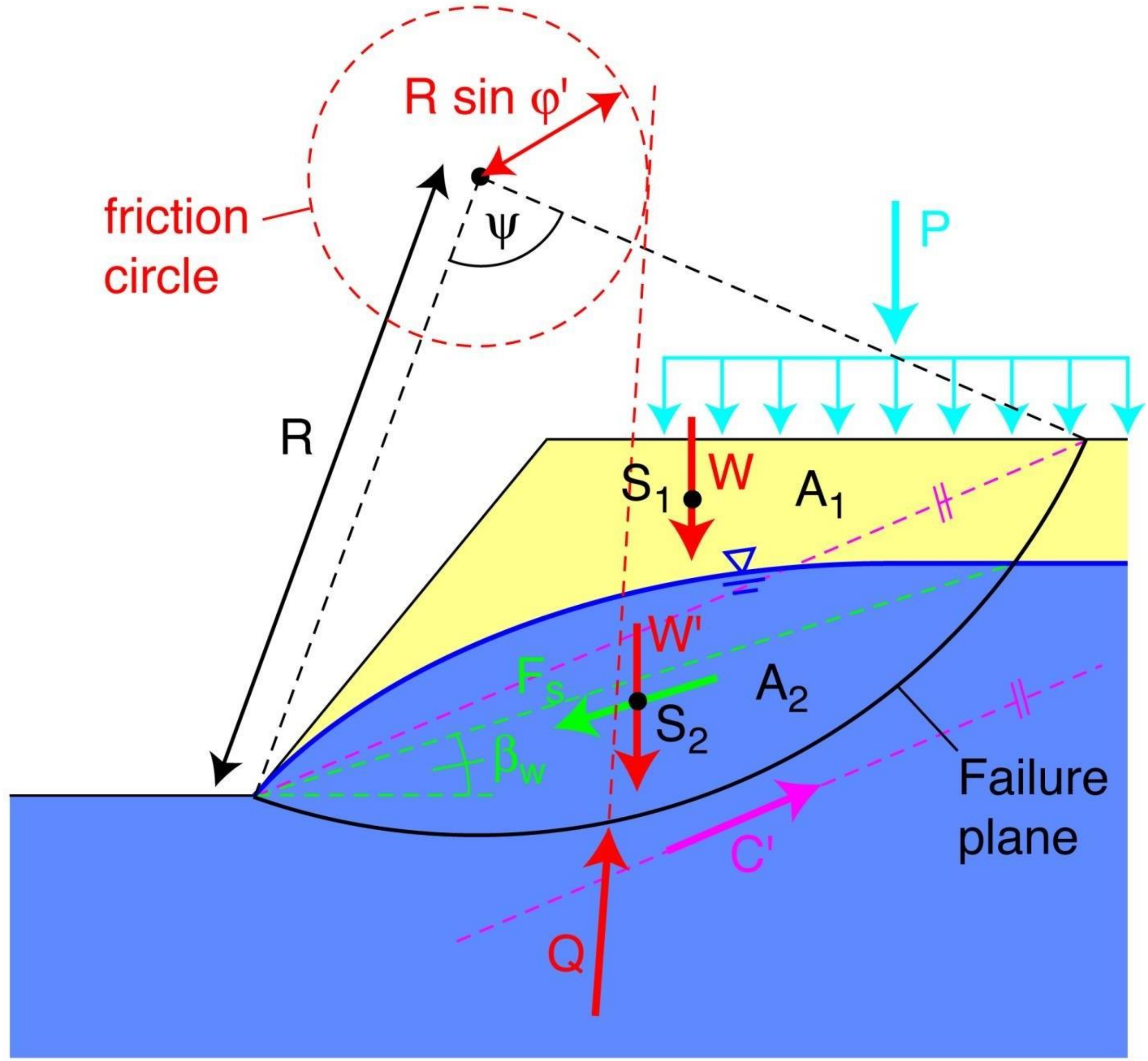
$$c = \frac{c_1 \cdot \psi_1 + c_2 \cdot \psi_2}{\psi_1 + \psi_2}$$

A_1, A_2 = cross-sectional areas of soils 1 and 2 in the sliding mass
P_1, P_2 = center points of the failure surface in soils 1 and 2

Circular failure surface

Analysis without slices

Seepage forces and surface loads



Circular failure surface

Analysis without slices

Step 1:

Summation of all vertical forces

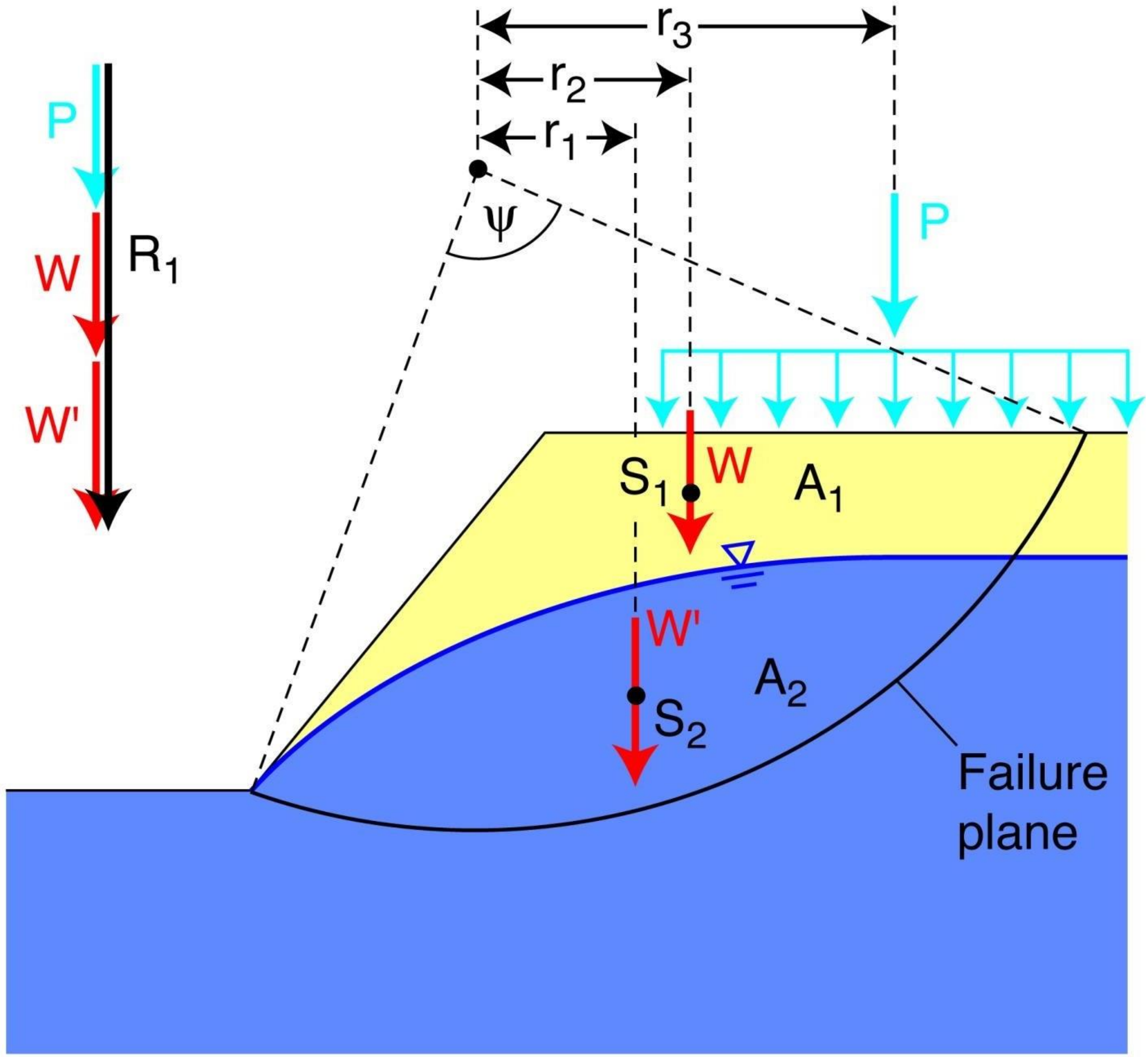
$$R_1 = P + W + W'$$

$$W = \gamma \cdot A_1$$

$$W' = \gamma' \cdot A_2$$

Distance of R_1 from center point:

$$r_{R1} = \frac{W' \cdot r_1 + W \cdot r_2 + P \cdot r_3}{P + W + W'}$$

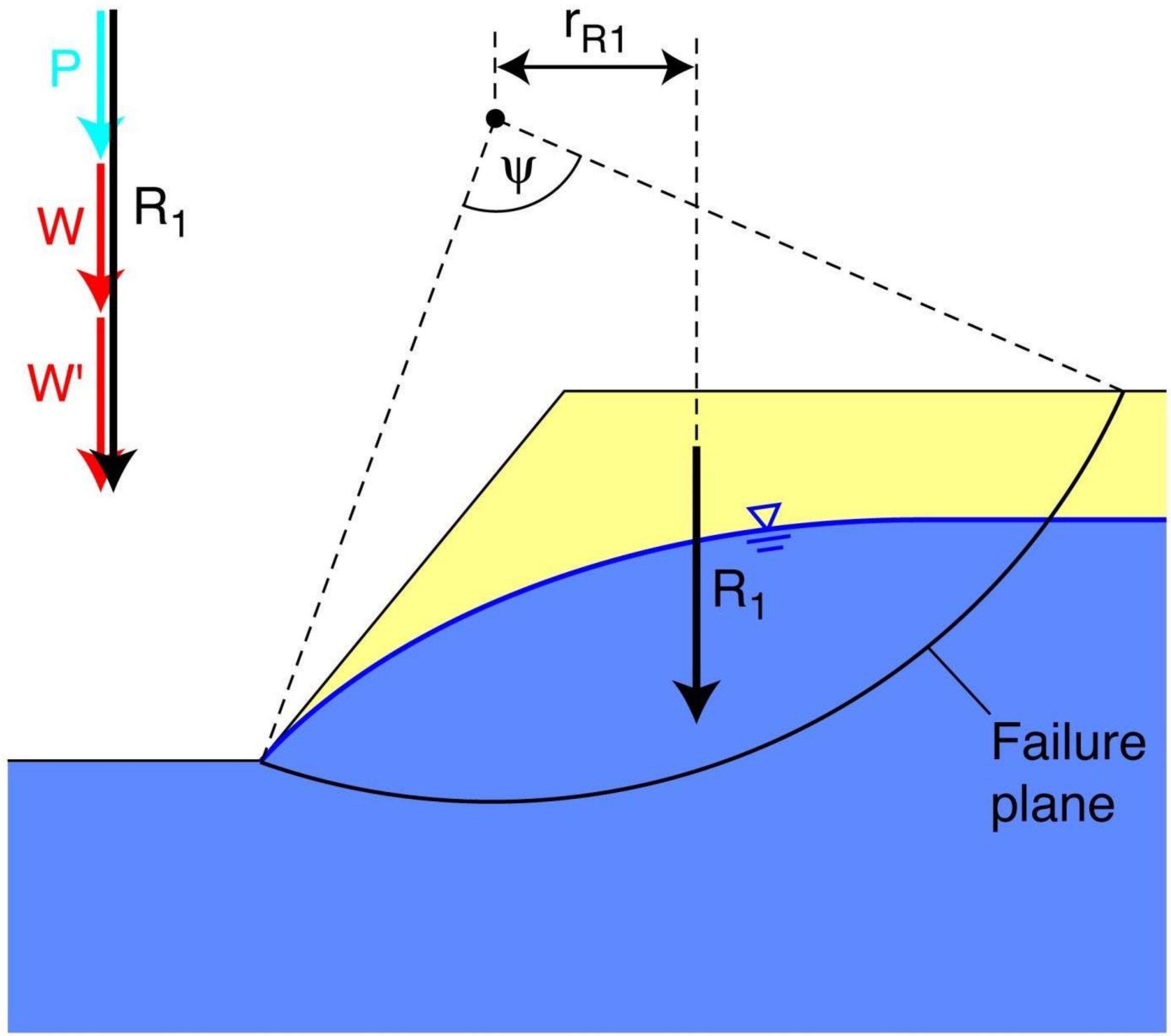


Circular failure surface

Analysis without slices

Step 1:

Summation of all vertical forces



Circular failure surface

Analysis without slices

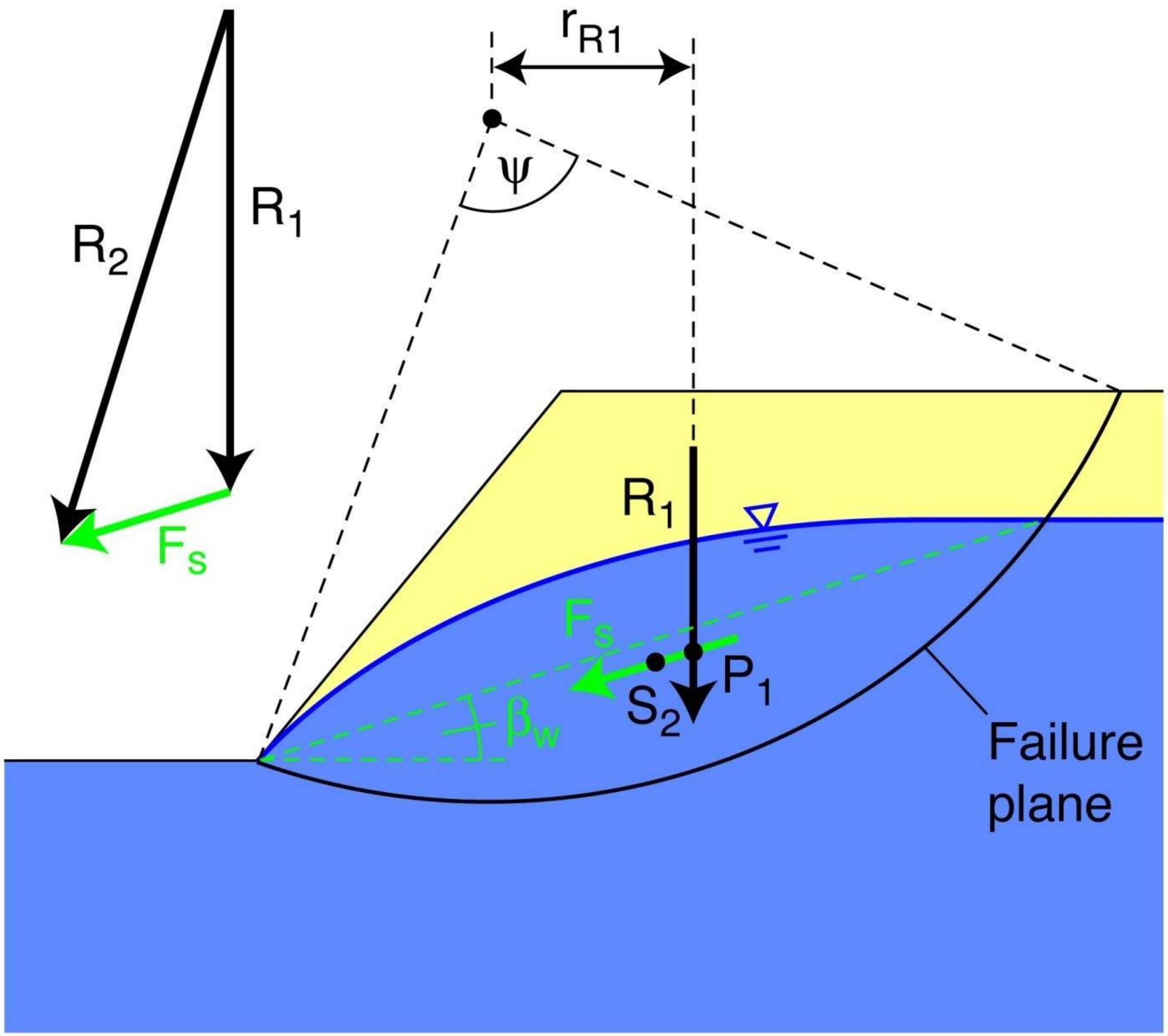
Step 2:

Summation of R_1 and seepage force F_s
→ resulting force R_2

$$F_s = f_s \cdot A_2 = \gamma_w \cdot i \cdot A_2$$
$$= \gamma_w \cdot \sin \beta_w \cdot A_2$$

F_s acts in the center of gravity S_2 of area A_2 below ground water table

Lines of application of R_1 and F_s intersect in point P_1



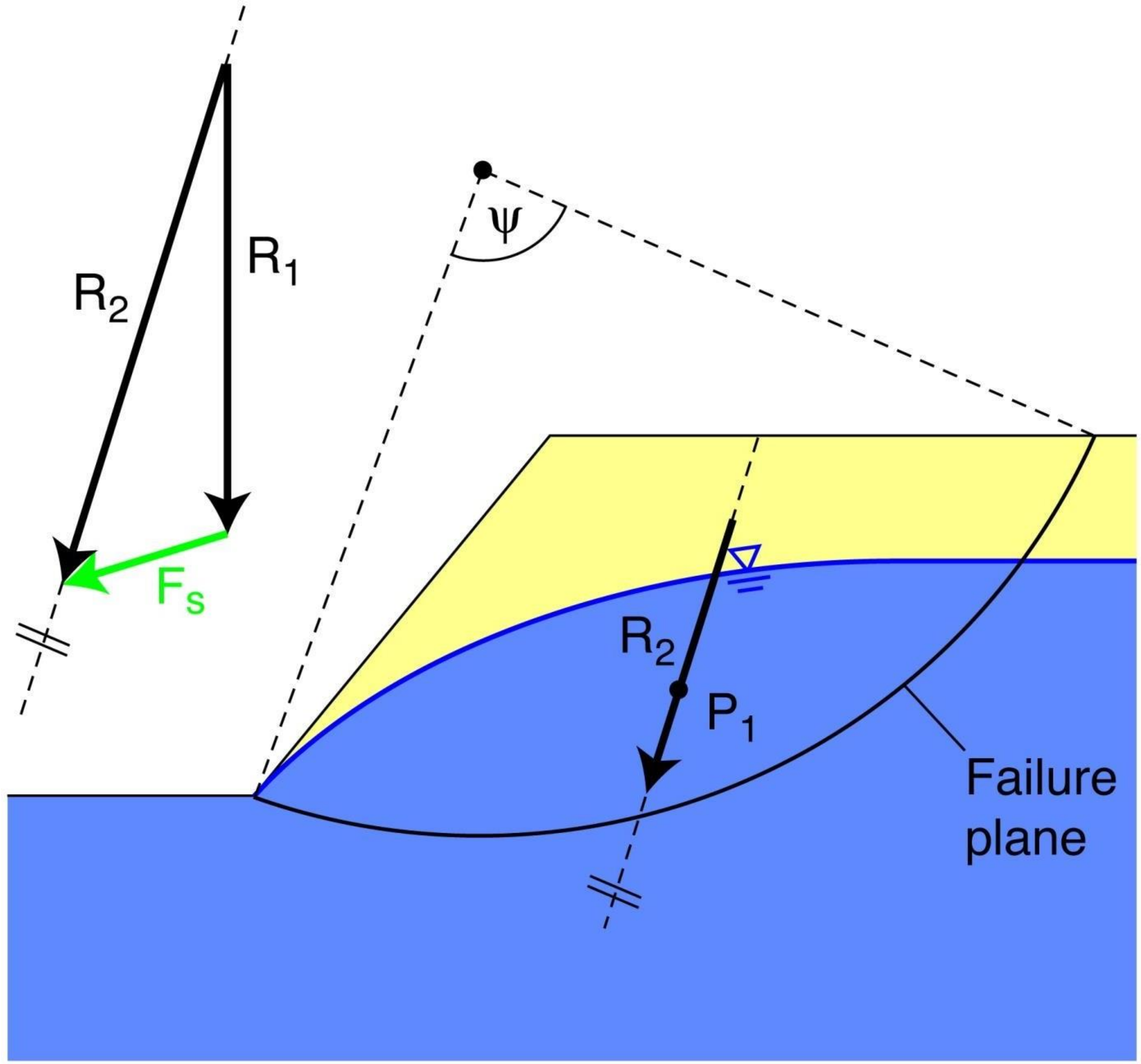
Circular failure surface

Analysis without slices

Step 2:

Summation of R_1 and seepage force F_s
→ resulting force R_2

Line of application of R_2 is obtained from the force polygon and shifted parallelly into the cross-sectional plan passing point P1



Circular failure surface

Analysis without slices

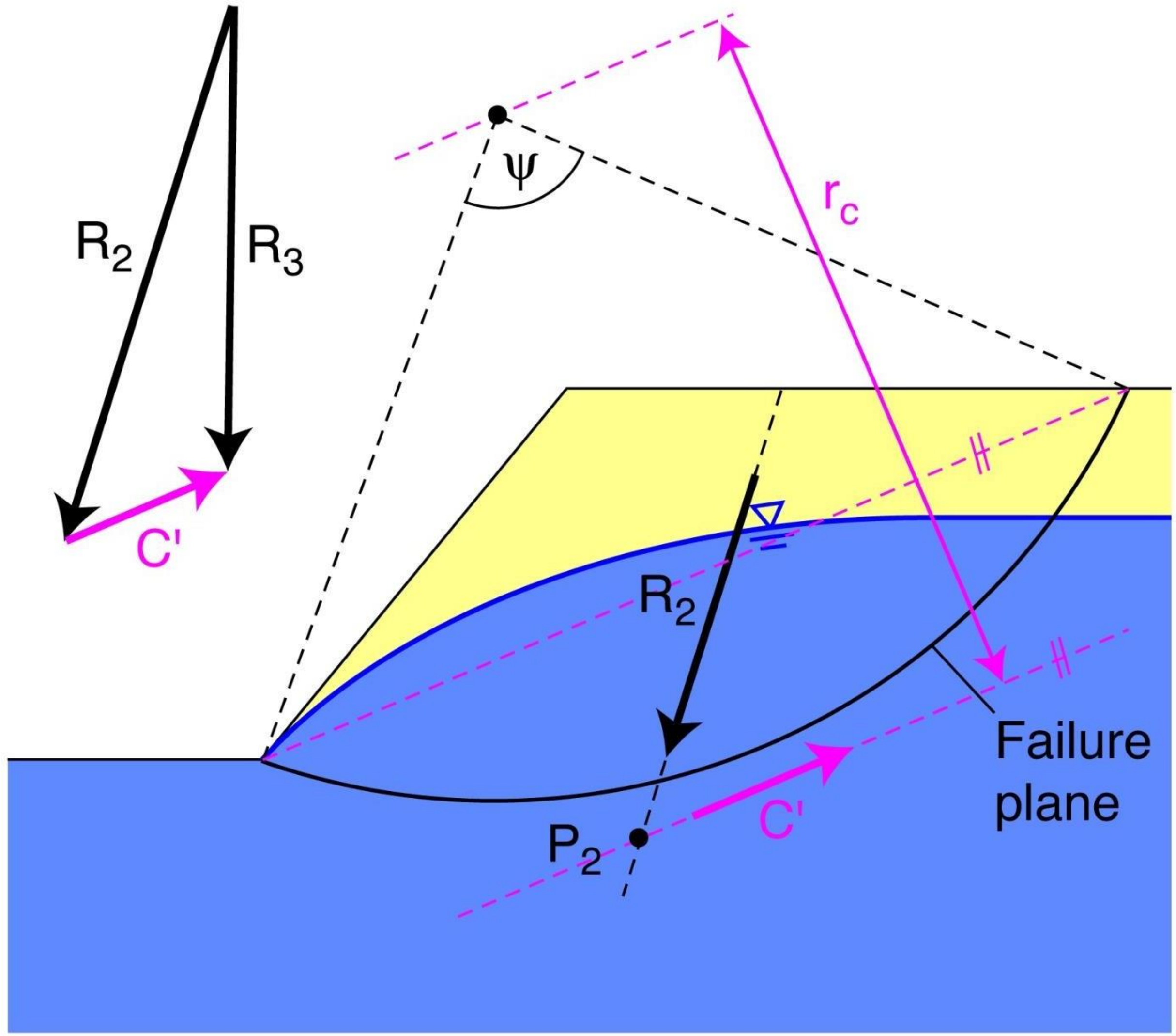
Step 3:

Summation of R_2 and effective cohesion force C‘ → resulting force R_3

$$C' = 2 \cdot c' \cdot R \cdot \sin(\psi/2)$$

$$r_c = R \cdot \frac{\psi[\text{rad}]}{2 \cdot \sin(\psi/2)}$$

Lines of application of R_2 and C‘ intersect in point P_2



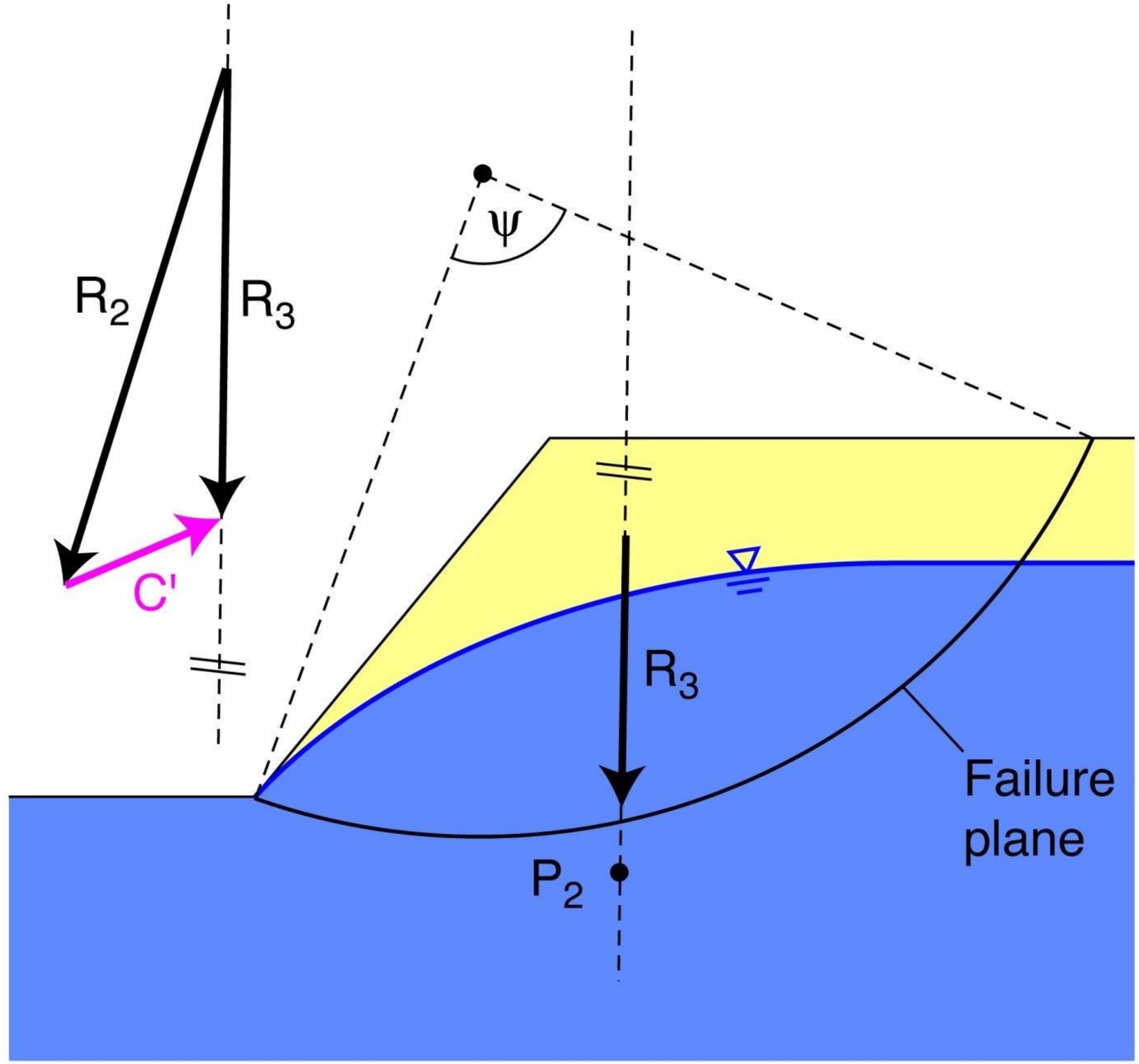
Circular failure surface

Analysis without slices

Step 3:

Summation of R_2 and effective cohesion force C‘ → resulting force R_3

Line of application of R_3 is obtained from the force polygon and shifted parallelly into the cross-sectional plan passing point P_2



Circular failure surface

Analysis without slices

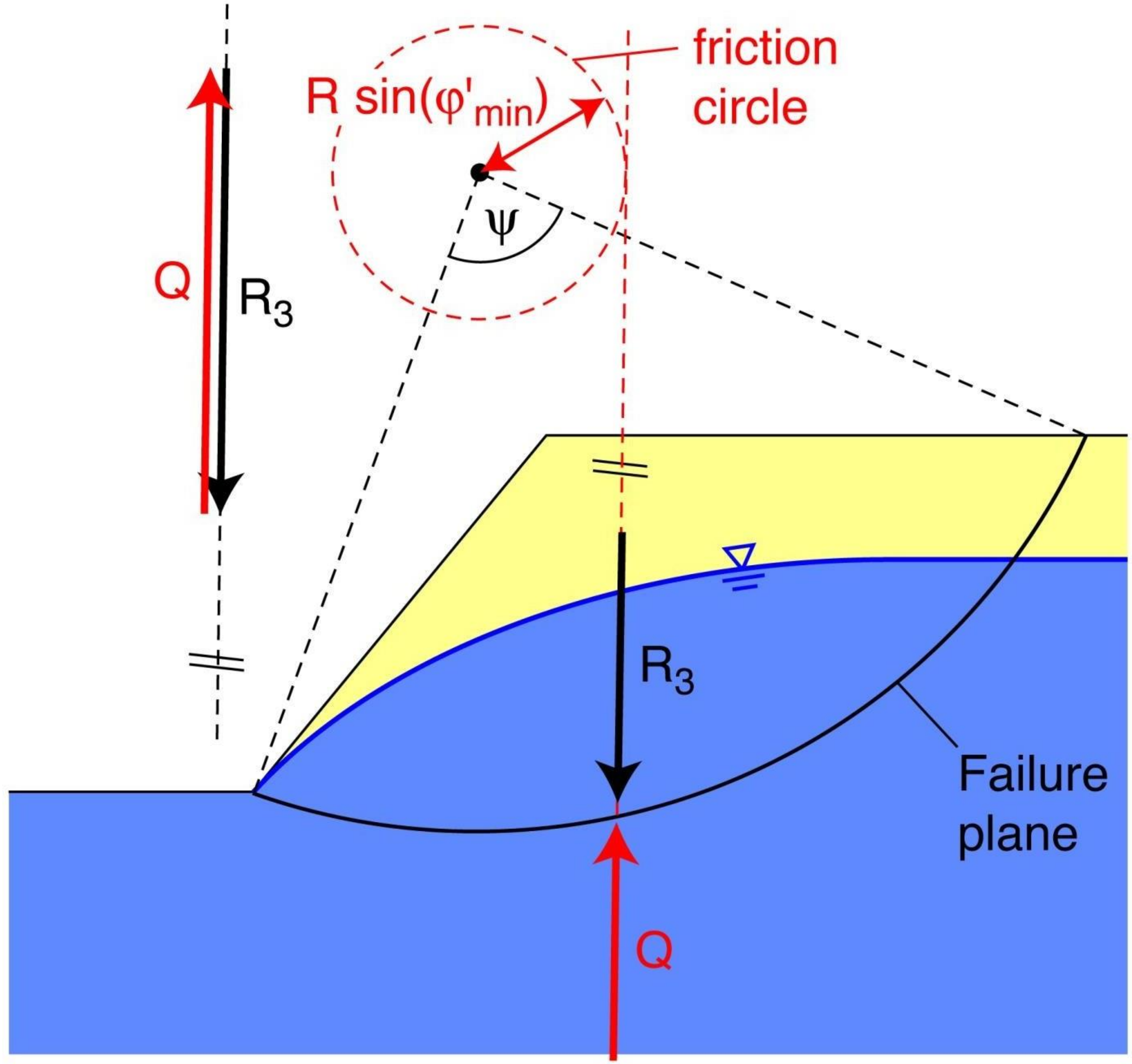
Step 4:

Determination of friction angle φ'_{min} being necessary for slope stability

- Reaction force Q in failure surface has same magnitude as R_3 but acts in opposite direction

- Line of application of Q touches the friction circle with radius

$$r_Q = R \cdot \sin(\varphi'_{min})$$



Circular failure surface

Analysis without slices

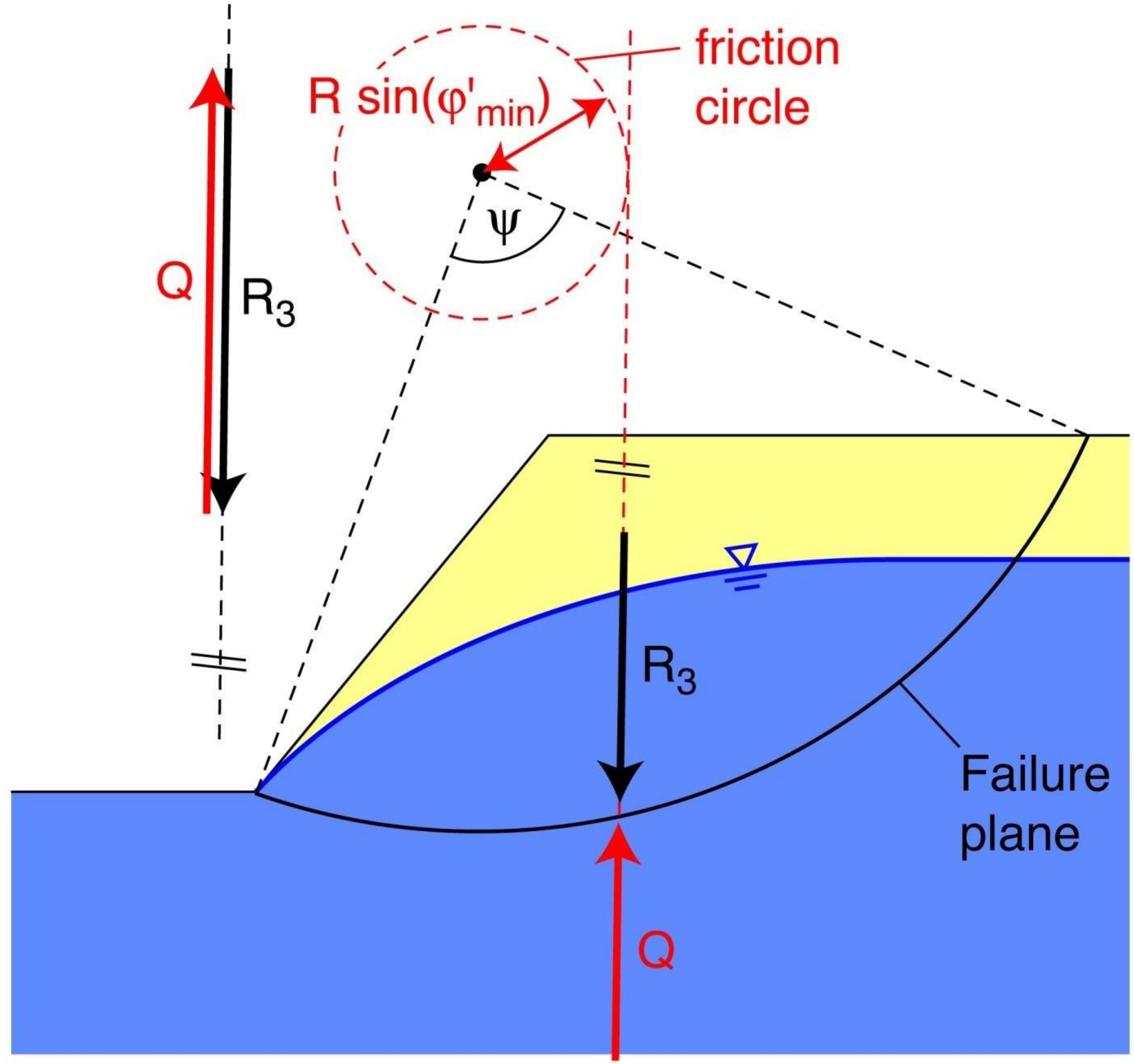
Step 5:

Calculation of
safety factor

$$FS = \frac{\tan \varphi'}{\tan(\varphi'_{min})}$$

φ' = effective friction
angle of the soil

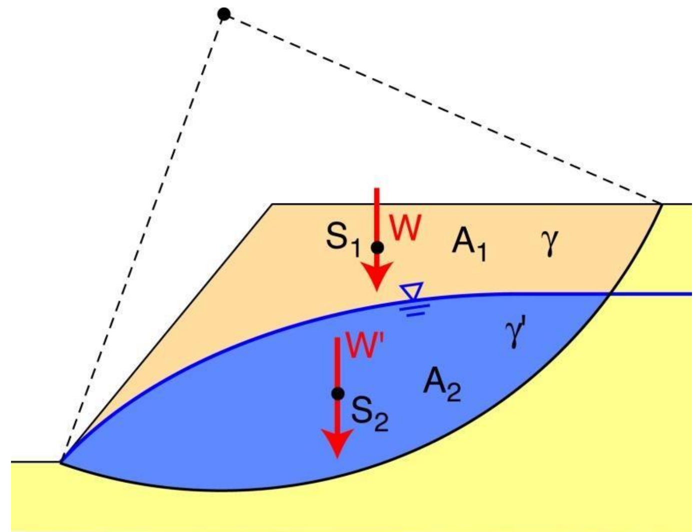
Alternative:
Safety factor via cohesion
(similar as explained above)



Circular failure surface

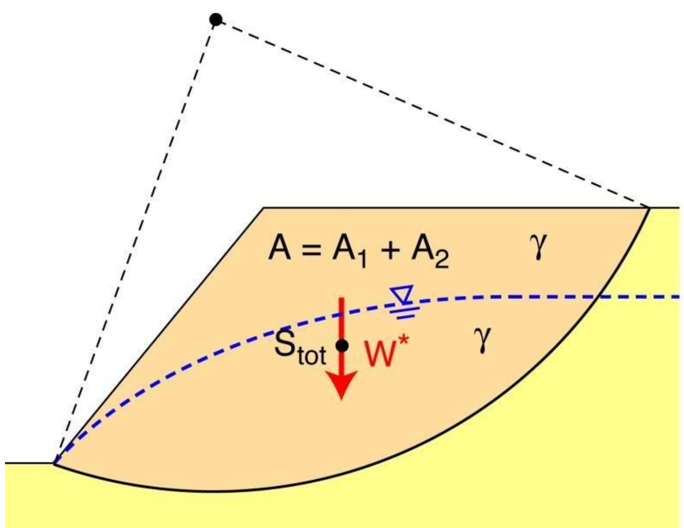
Analysis without slices

Different equivalent methods
to consider buoyant forces



Method 1: $W + W' = \gamma \cdot A_1 + \gamma' \cdot A_2$

Method 2: $W + W' = W^* - B = \gamma \cdot (A_1 + A_2) - (\gamma - \gamma') \cdot A_2$



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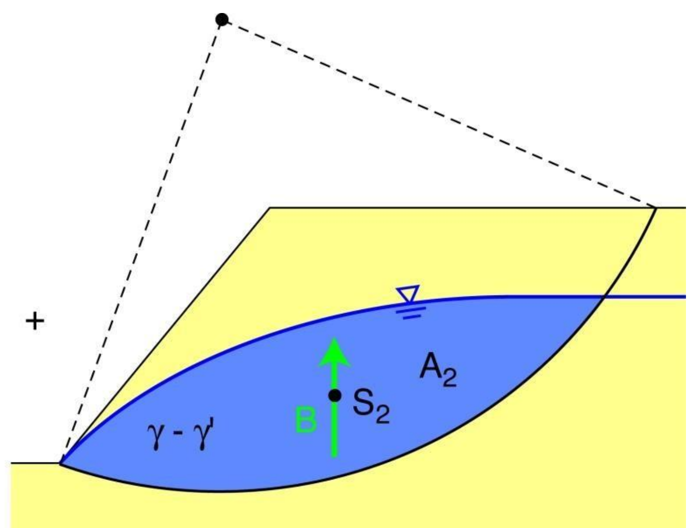
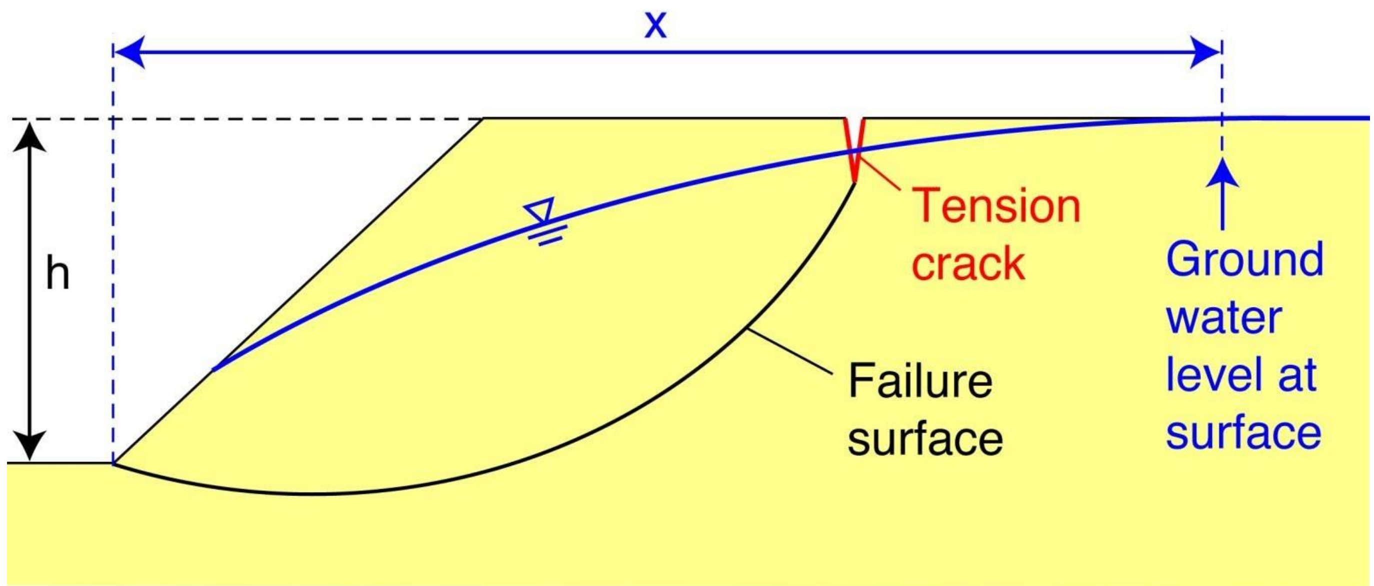


Circular failure surface

Analysis without slices

Slope stability charts of Hoek & Bray

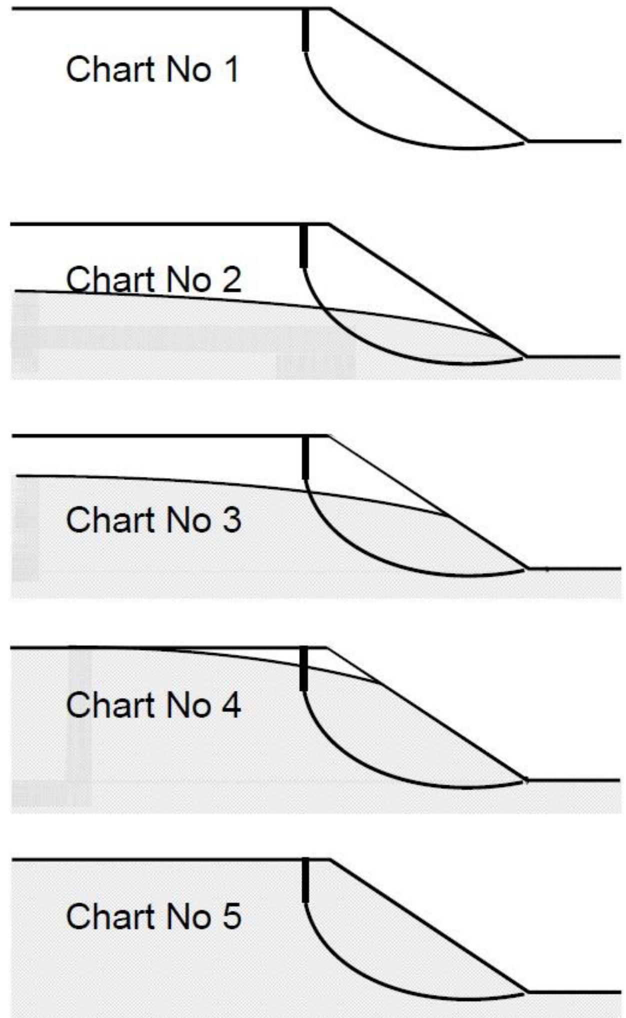


- Different charts for different conditions of ground water within the slope
- Slip circle starts from a tension crack at surface and runs through the toe of the slope
- Charts can be used for an estimation of safety factor FS or for a back analysis of the shear strength parameters from an existing slide

Circular failure surface

Analysis without slices

Slope stability charts of Hoek & Bray



Fully drained conditions

Surface water 8 x slope height behind toe of slope

x/h = 8

Surface water 4 x slope height behind toe of slope

x/h = 4

Surface water 2 x slope height behind toe of slope

x/h = 2

Saturated slope subjected to heavy surface recahrge

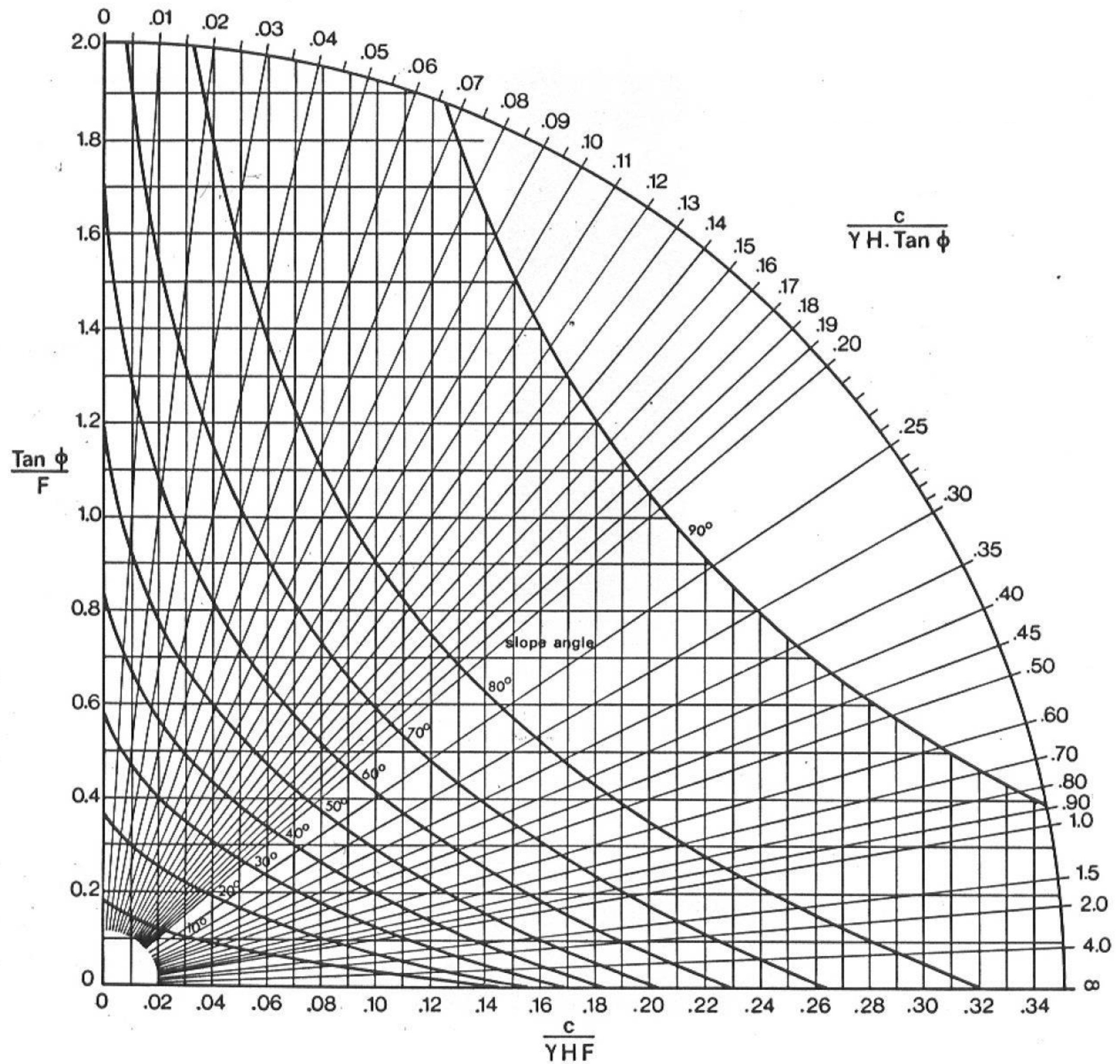
Circular failure surface

Analysis without slices

Slope stability charts of Hoek & Bray

Chart No. 1:

H = height of slope
F = safety factor

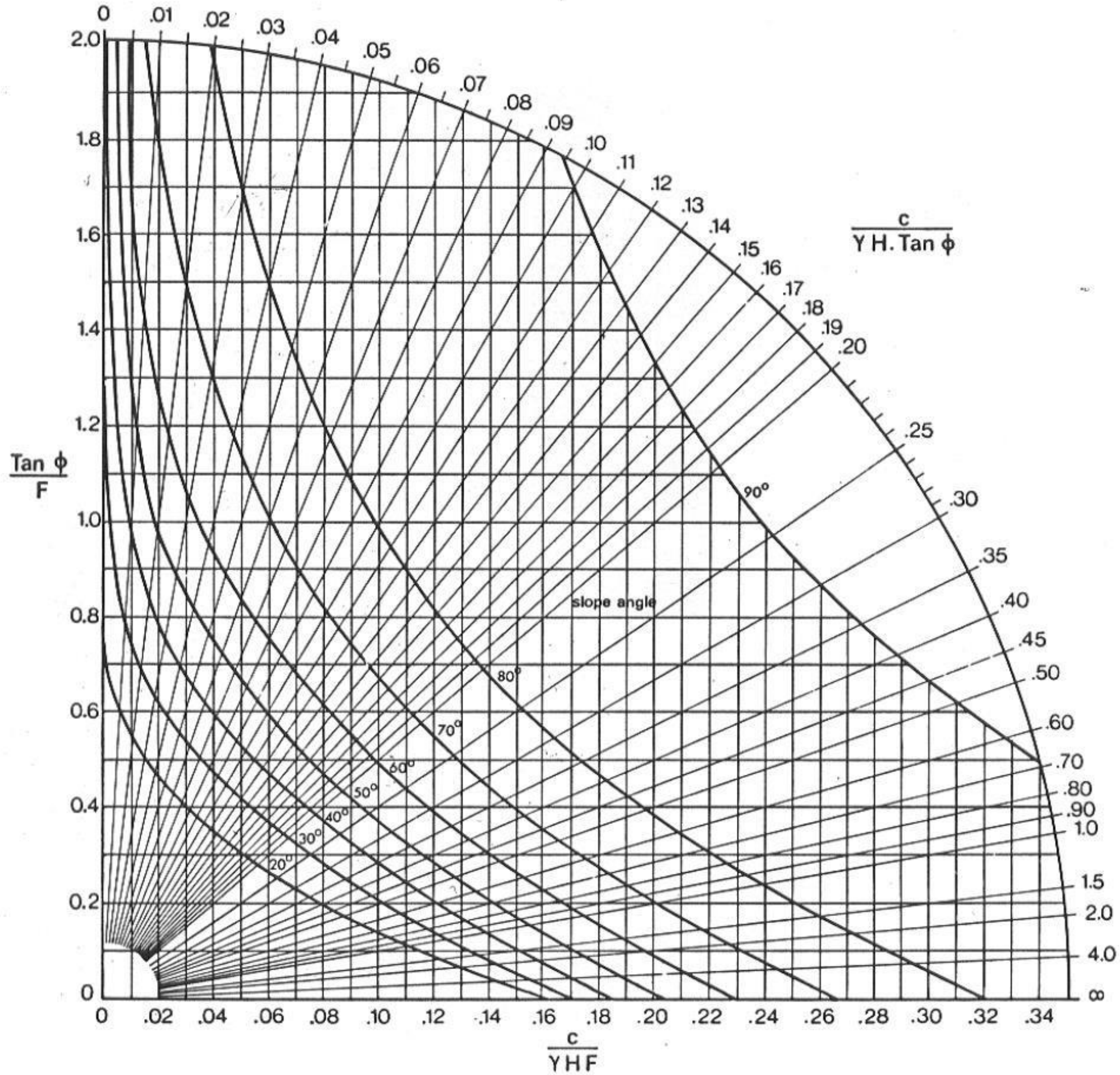


Circular failure surface

Analysis without slices

Slope stability charts of Hoek & Bray

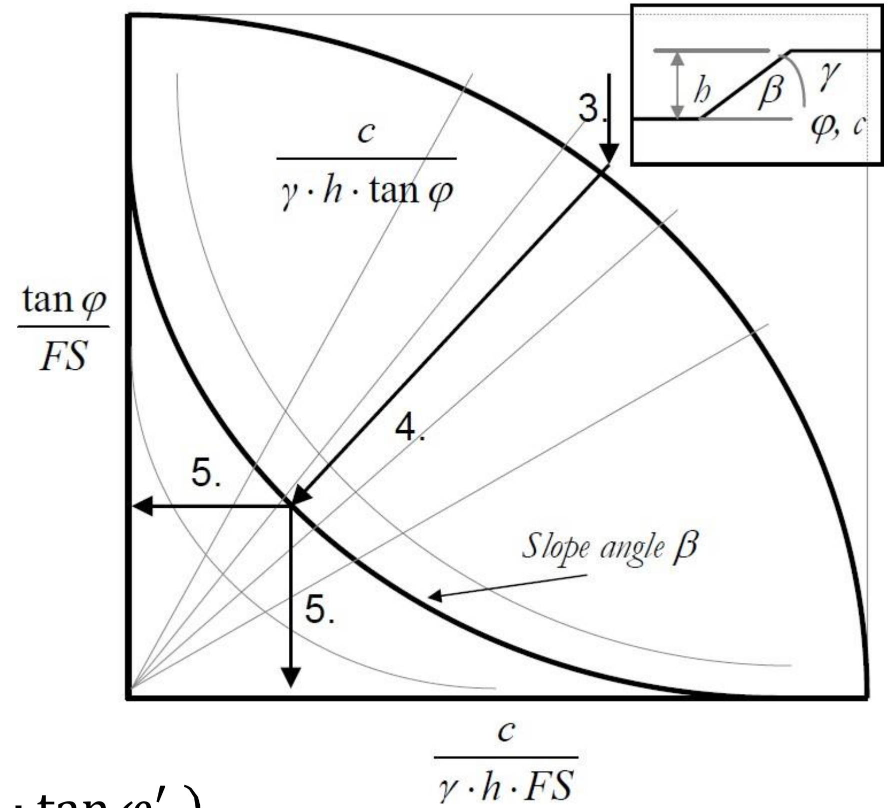
Chart No. 3:



Circular failure surface

Analysis without slices

Slope stability charts of Hoek & Bray



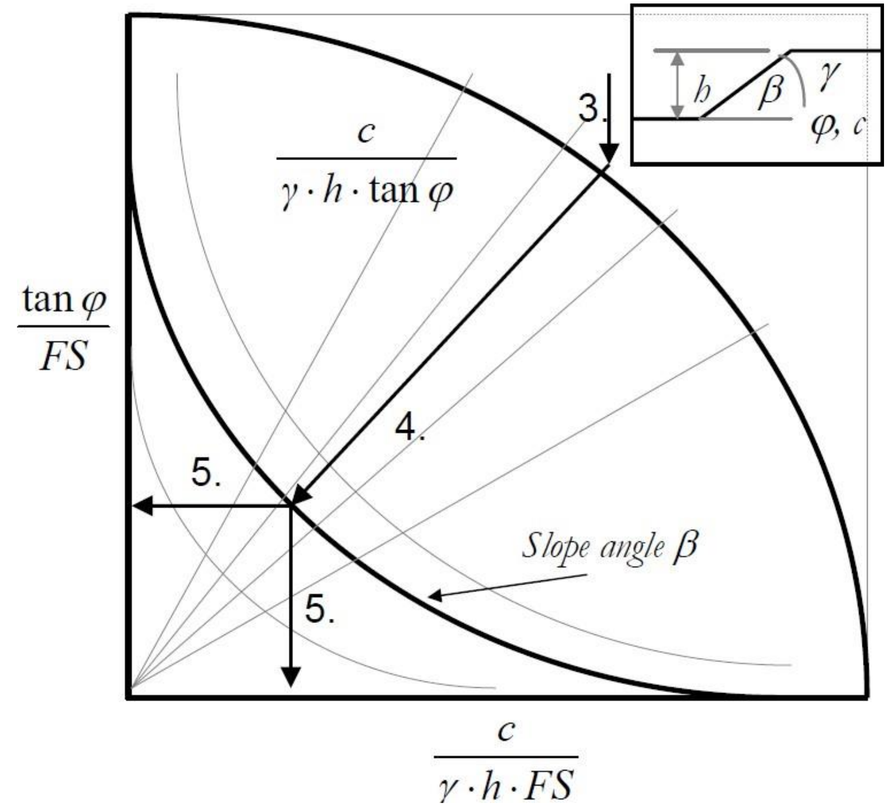
1. Decide upon the groundwater conditions which are believed to exist in the slope. Choose the corresponding chart
2. Estimate shear strength and unit weight and simplify the geometry in order to get h and β
3. Calculate the dimensionless ratio $c'/(\gamma \cdot h \cdot \tan\varphi')$ and find this ratio on the outer scale of the chart
4. Follow the radial line from the value found in step 3 to its intersection with the curve which corresponds to the slope angle β
5. Find the corresponding value of $\tan\varphi'/FS$ at the ordinate and $c'/(\gamma \cdot h \cdot FS)$ at the abscissa and calculate the factor of safety FS

Circular failure surface

Analysis without slices

Slope stability charts of Hoek & Bray

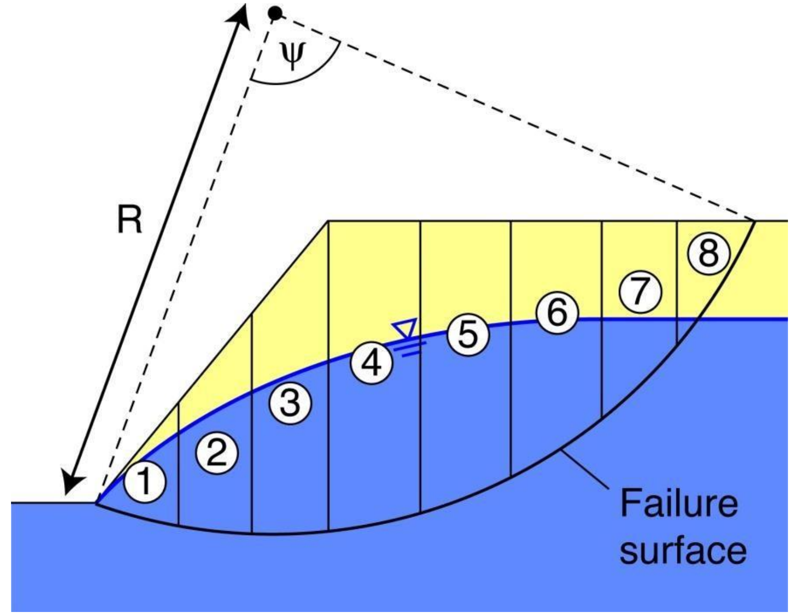
Additional charts are available for the determination of the position of the tension crack and the critical slip circle (not presented herein)

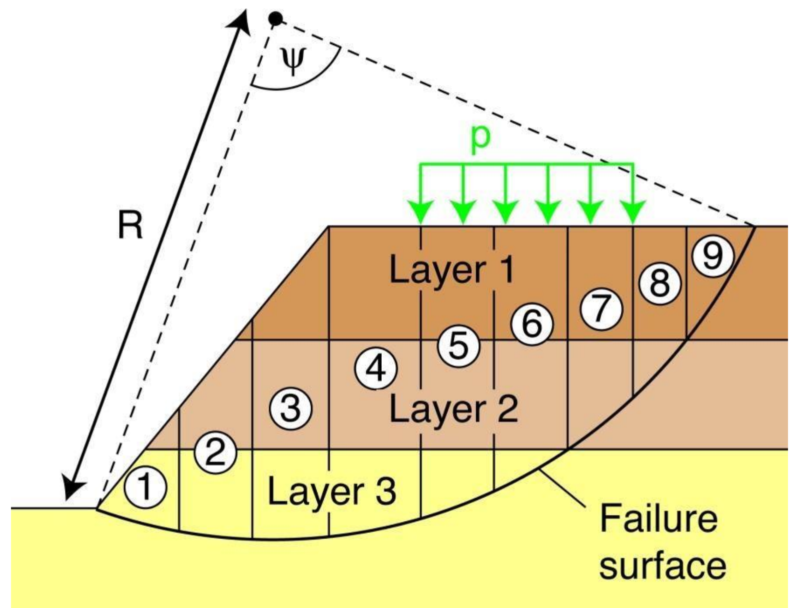


Circular failure surface

Analysis with slices

- Sliding mass is divided in several slices
- 3 to 10 slices usually are sufficient
- A larger number of slices does not lead to a higher accuracy because of the uncertainties in the shear strength parameters and the water levels / pore water pressures assumed in the analysis
- Division into slices should be adapted to the soil layers (only one type of soil in failure surface of a certain slice) and surface loads (border between two slices coincides with start and end points of distributed loads)

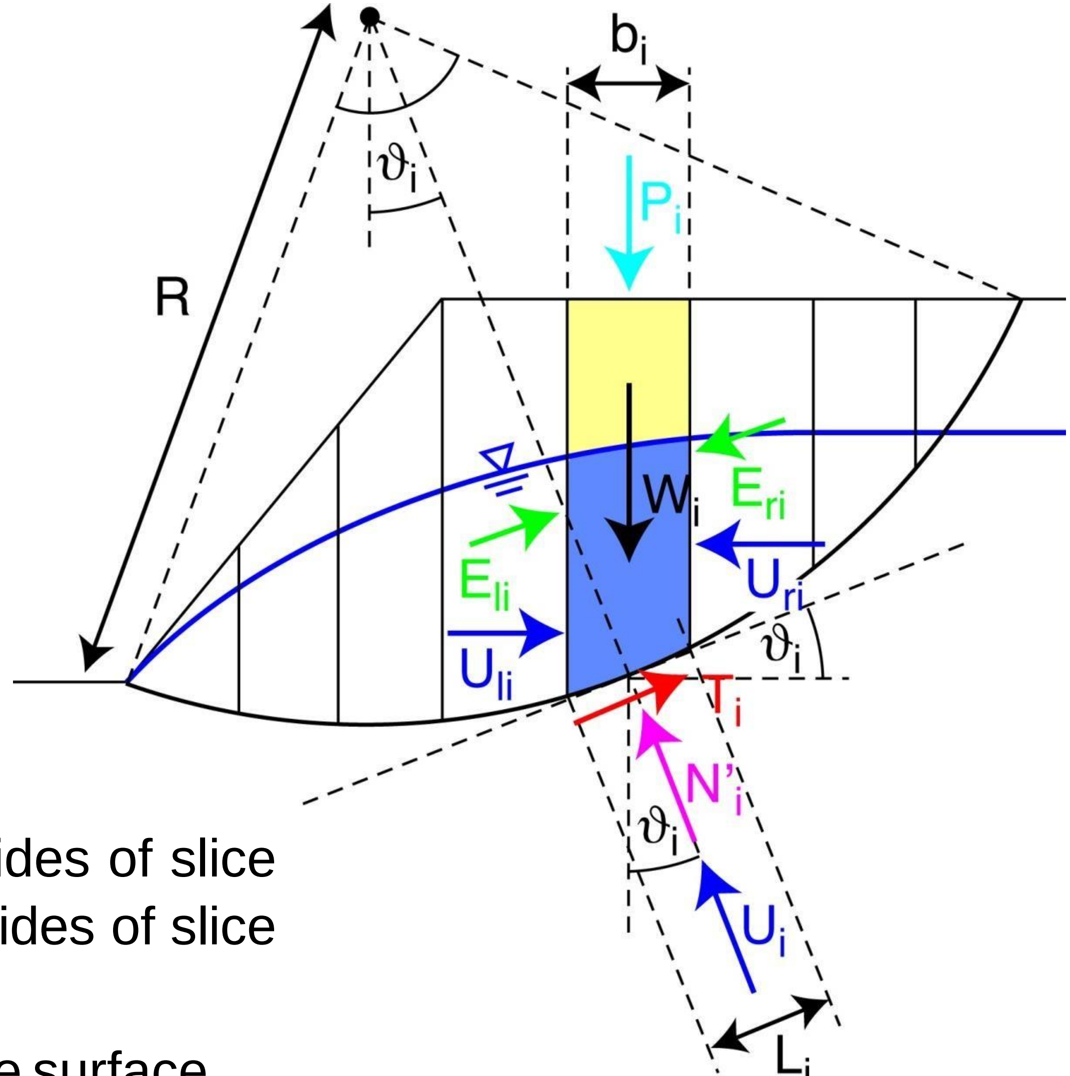




Circular failure surface

Analysis with slices

- The available methods differ with respect to the definition of safety factor and assumptions on forces acting on a slice



W_i = total weight of the slice (γ above water level, γ_r below)

P_i = surface load

E_{li}, E_{ri} = earth pressures on both sides of slice

U_{li}, U_{ri} = water pressures on both sides of slice

T_i = shear force in failure surface

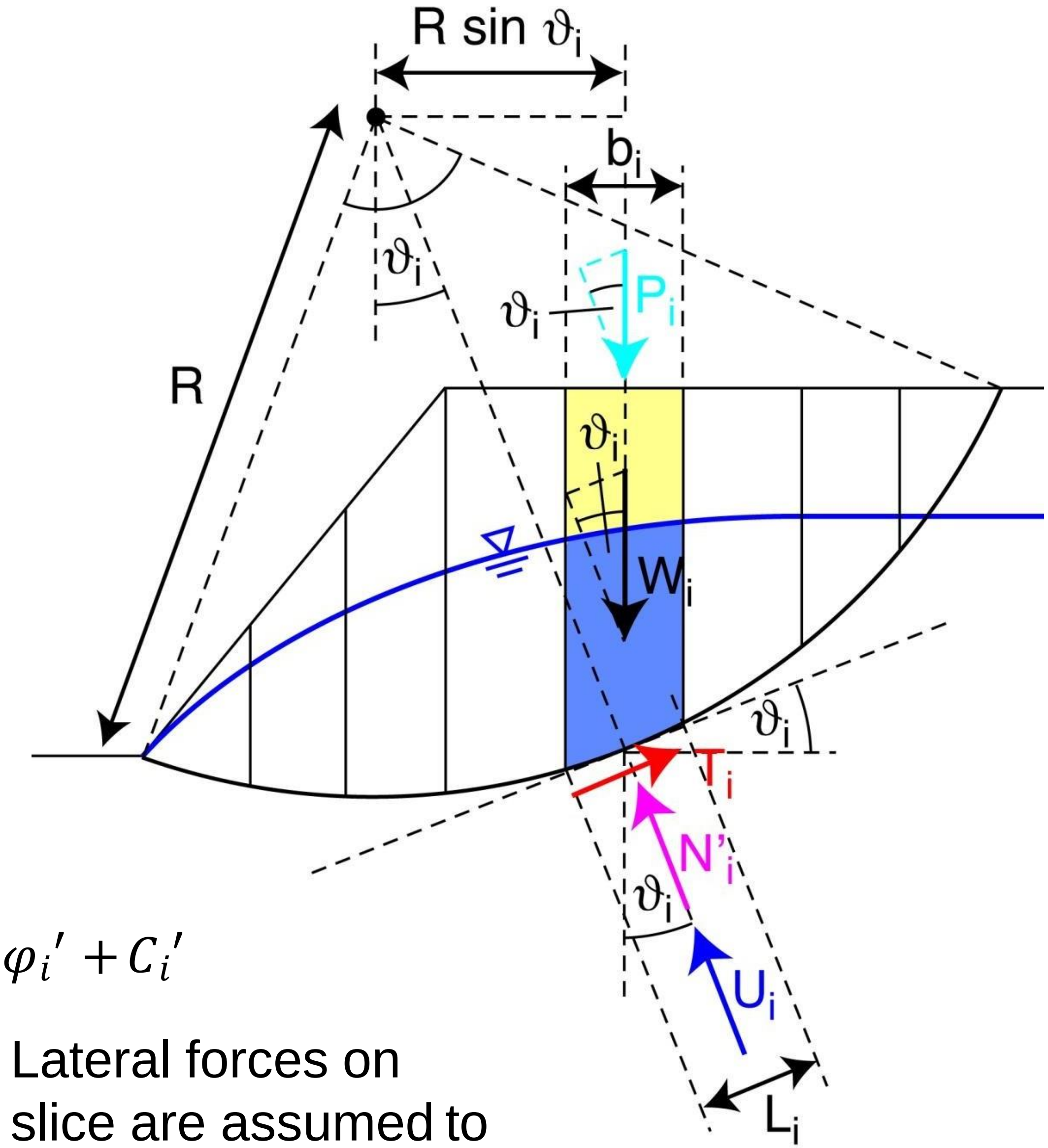
N_i‘ = effective normal force in failure surface

U_i = resultant force of pore water pressure in failure surface

Circular failure surface

Analysis with slices

Ordinary method (Fellenius, 1936)



Force equilibrium in direction perpendicular to the failure surface

$$(W_i + P_i) \cdot \cos\vartheta_i = N' + U_i$$

Maximum shear force that can be mobilized in failure surface:

$$T_{i,\max} = N' \cdot \tan\varphi'_i + C_i'$$

$$= [(W_i + P_i) \cdot \cos\vartheta_i - U_i\,] \cdot \tan\varphi_i' + C_i'$$

Total resisting moment:

$$M_{\text{res}} = T_{i,\max} \cdot R$$

Lateral forces on slice are assumed to compensate each other

Circular failure surface

Analysis with slices

Ordinary method (Fellenius, 1936)

Total driving moment:

$$M_{\text{driv}} = \sum (W_i + P_i) \cdot R \cdot \sin\vartheta_i$$

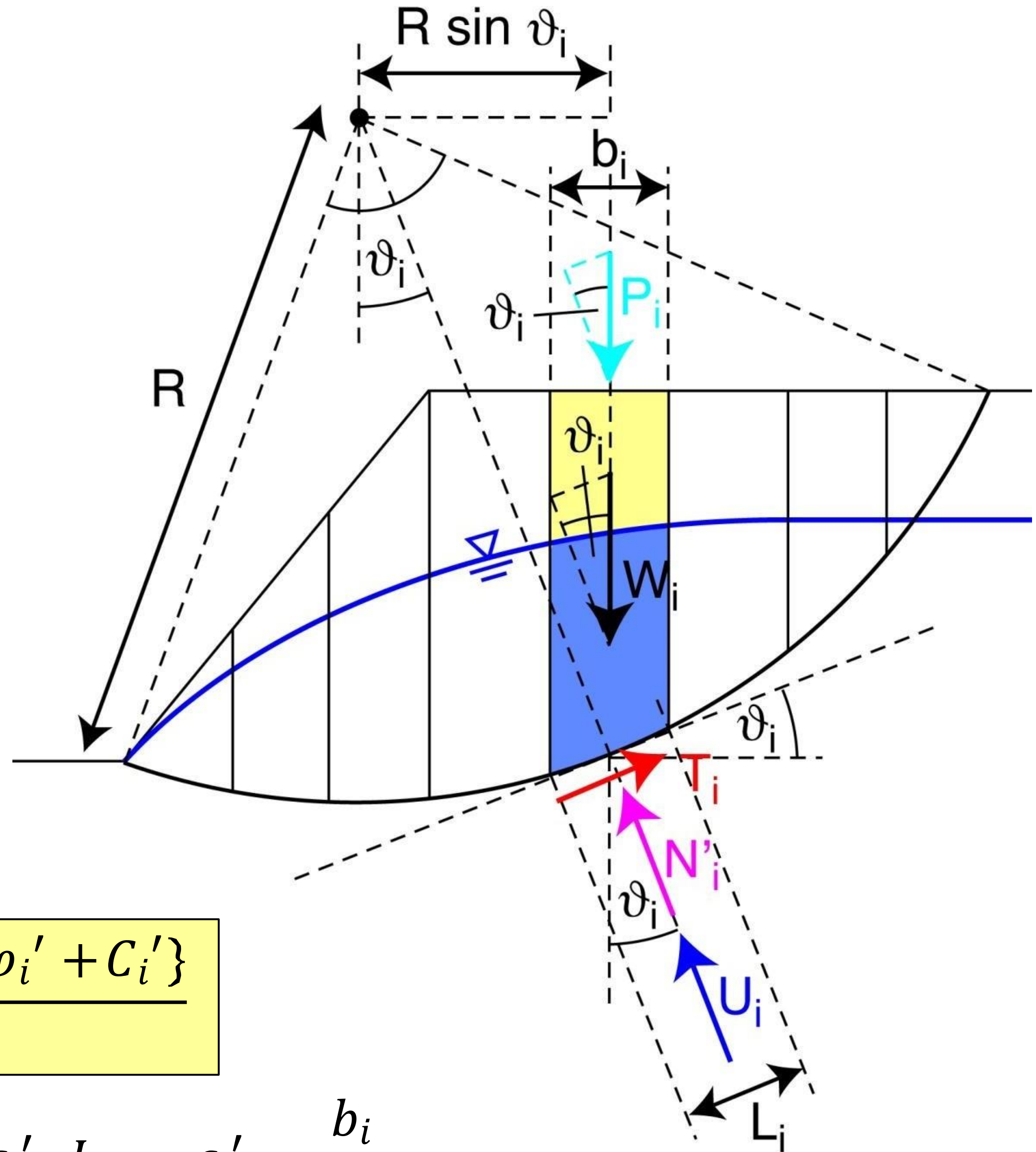
Global safety factor:

$$FS = \frac{M_{\text{res}}}{M_{\text{driv}}}$$

(Radius R can be eliminated)

$$FS = \frac{\sum \{[(W_i + P_i)\cdot\cos\vartheta_i - U_i]\cdot\tan\varphi_i' + C_i'\}}{\sum (W_i + P_i)\cdot\sin\vartheta_i}$$

$$U = u \cdot L = u \cdot \frac{b_i}{\cos\vartheta_i} \qquad C' = c' \cdot L = c' \cdot \frac{b_i}{\cos\vartheta_i}$$



Circular failure surface

Analysis with slices

Method of Krey

Force equilibrium in vertical direction

$$W_i + P_i = (N_i' + U_i) \cdot \cos\vartheta_i + T_i \cdot \sin\vartheta_i$$

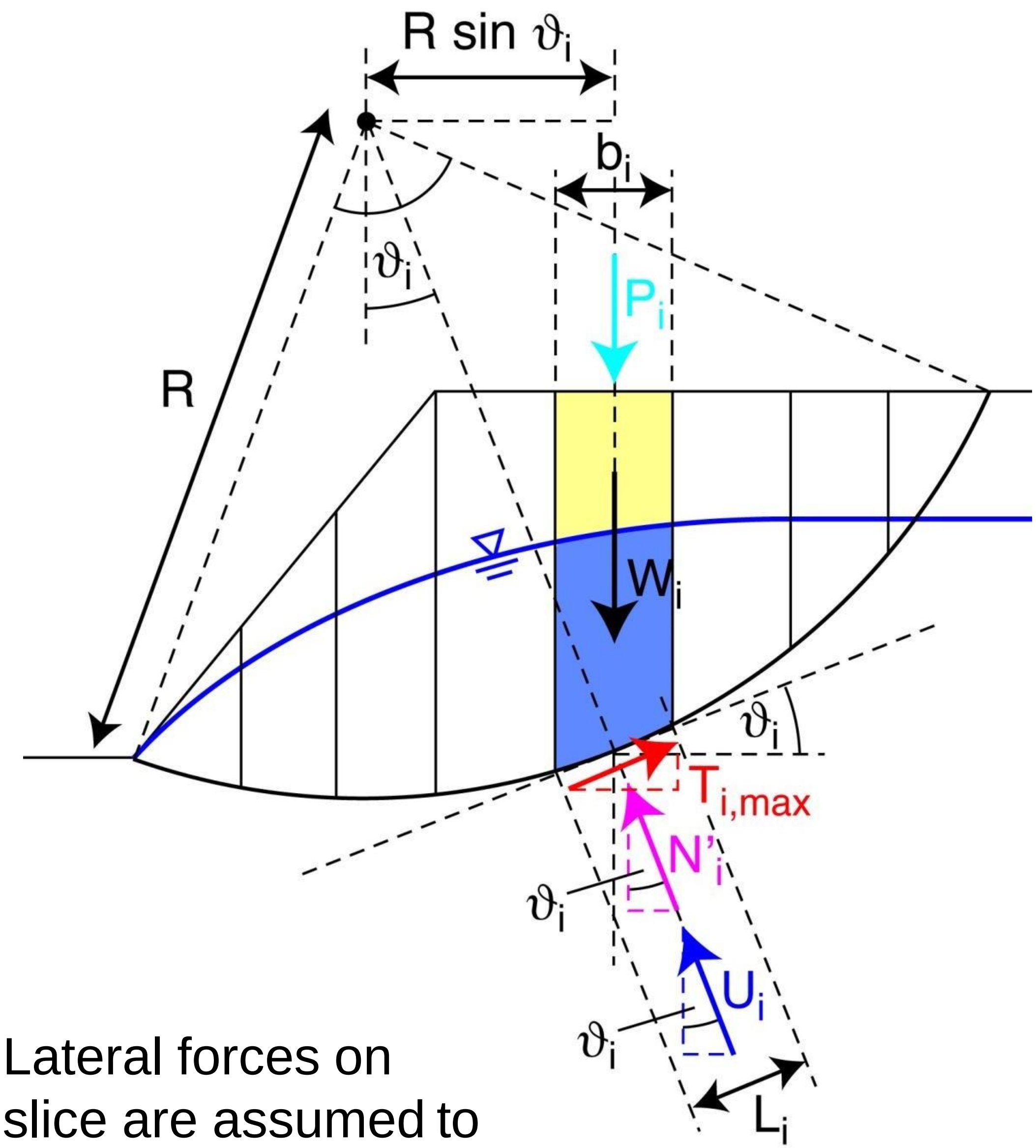
Assumption of limit equilibrium:

$$T_i = T_{i,\max}$$

Maximum shear force that can be mobilized in failure surface:

$$T_{i,\max} = N' \cdot \tan\varphi_i' + C_i'$$

$$N' = (T_{i,\max} - C_i') \cdot \cot\varphi_i'$$

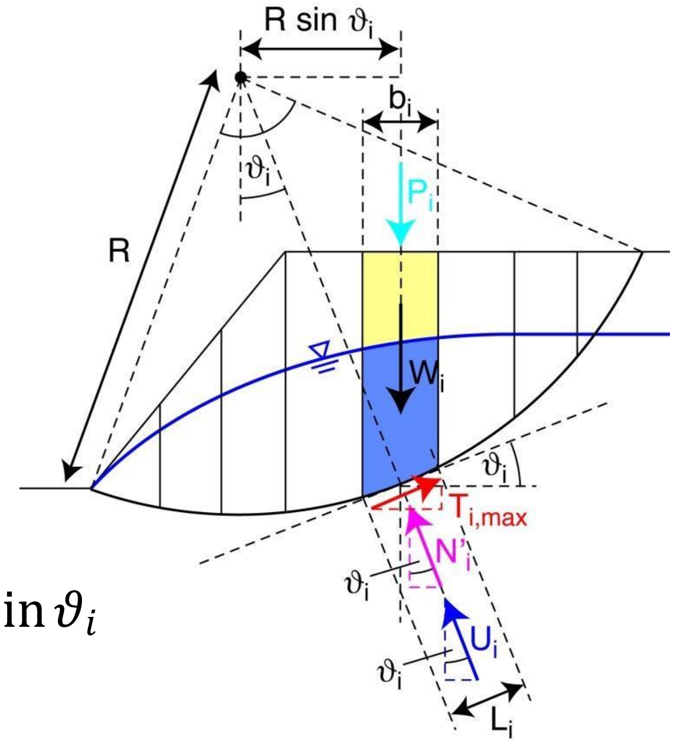


Lateral forces on slice are assumed to compensate each other

Circular failure surface

Analysis with slices

Method of Krey



Force equilibrium in vertical direction

$$W_i + P_i = (N' + U_i) \cdot \cos\vartheta_i + T_i \cdot \sin\vartheta_i$$

$$W_i + P_i = \left[\left(T_{i,\max} - C_i'\right) \cdot \cot\varphi_i' + U_i\right] \cdot \cos\vartheta_i + T_{i,\max} \cdot \sin\vartheta_i$$

$$T_{i,\max} = \frac{W_i + P_i + C' \cdot \cot\varphi_i' \cdot \cos\vartheta_i - U_i \cdot \cos\vartheta_i}{\cot\varphi_i' \cdot \cos\vartheta_i + \sin\vartheta_i}$$

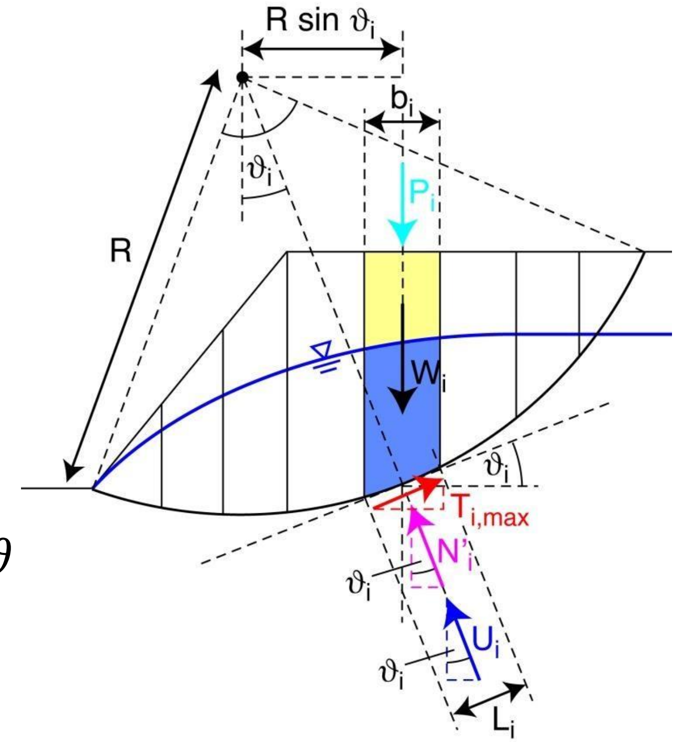
$$U_i = u_i \cdot L_i = u_i \cdot \frac{b_i}{\cos\vartheta_i} \qquad C_i' = c_i' \cdot L = c' \cdot \frac{b_i}{\cos\vartheta_i}$$

$$T_{i,\max} = \frac{W_i + P_i + c' \cdot b_i \cdot \cot\varphi_i' - u_i \cdot b_i}{\cot\varphi_i' \cdot \cos\vartheta_i + \sin\vartheta_i}$$

Circular failure surface

Analysis with slices

Method of Krey



Total resisting moment $M_{\text{res}} = T_{i,\max} \cdot R$

Total driving moment $M_{\text{driv}} = (W_i + P_i) \cdot R \cdot \sin\vartheta$

Global safety factor:

$$FS = \frac{M_{\text{res}}}{M_{\text{driv}}} = \frac{\dfrac{W_i + P_i + c' \cdot b_i \cdot \cot\varphi_i' - u_i \cdot b_i}{\cot\varphi_i' \cdot \cos\vartheta_i + \sin\vartheta_i}}{(W_i + P_i) \cdot \sin\vartheta_i}$$

(Radius R has been eliminated in enumerator and denominator)

Circular failure surface

Analysis with slices

Method of Bishop (1955)

Force equilibrium in vertical direction

$$W_i + P_i = (N_i' + U_i) \cdot \cos\vartheta_i + T_i \cdot \sin\vartheta_i$$

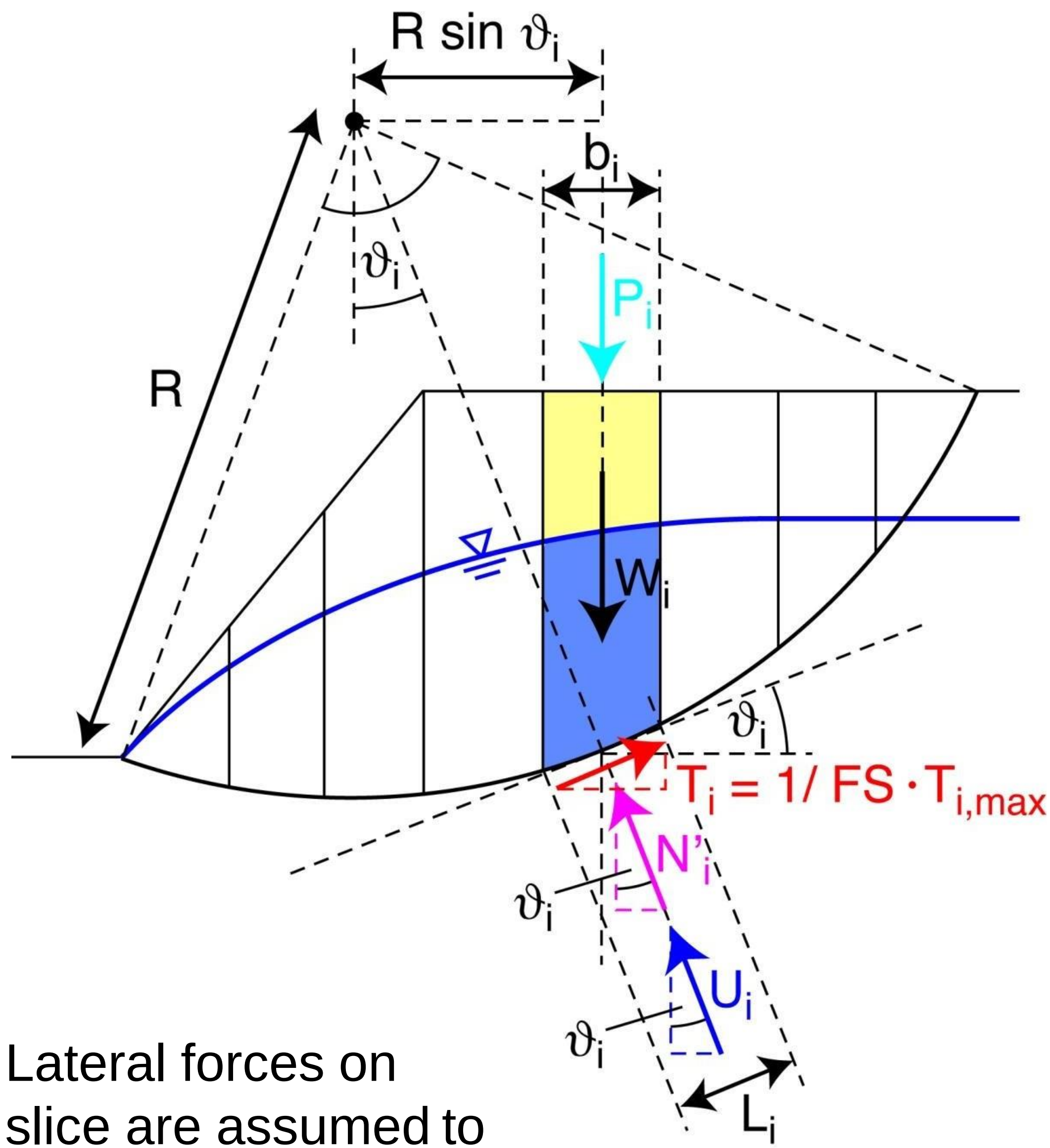
Shear strength is assumed to be only partially mobilized

$$T_i = \frac{1}{FS} \cdot T_{i,\max}$$

$$T_{i,\max} = N' \cdot \tan\varphi_i' + C_i'$$

$$N' = (T_{i,\max} - C_i') \cdot \cot\varphi_i'$$

$$N' = (FS \cdot T_i - C_i') \cdot \cot\varphi_i'$$

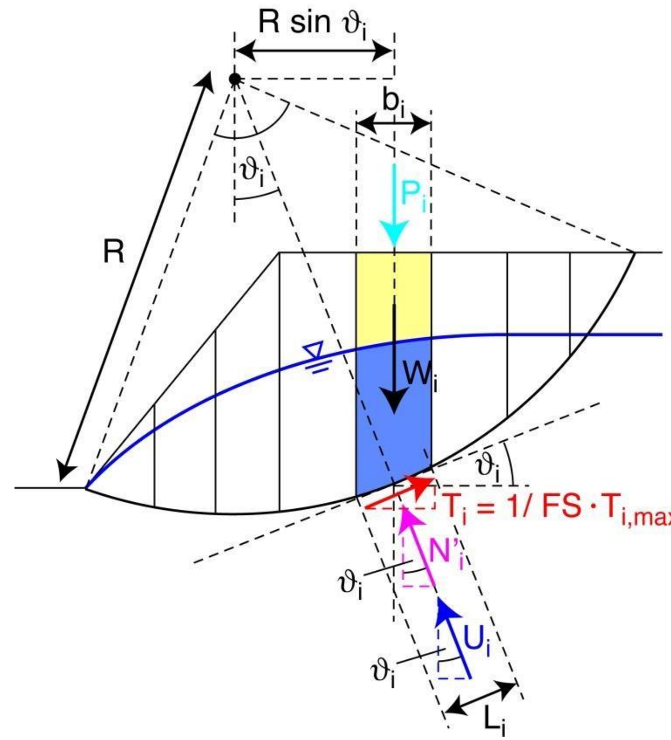


Lateral forces on slice are assumed to compensate each other

Circular failure surface

Analysis with slices

Method of Bishop (1955)



Force equilibrium in vertical direction

$$W_i + P_i = (N' + U_i) \cdot \cos\vartheta_i + T_i \cdot \sin\vartheta_i$$

$$W_i + P_i = [(FS \cdot T_i - C_i') \cdot \cot\varphi_i' + U_i] \cdot \cos\vartheta_i + T_i \cdot \sin\vartheta_i$$

$$T_i = \frac{W_i + P_i + C' \cdot \cot\varphi_i' \cdot \cos\vartheta_i - U_i \cdot \cos\vartheta_i}{FS \cdot \cot\varphi_i' \cdot \cos\vartheta_i + \sin\vartheta_i}$$

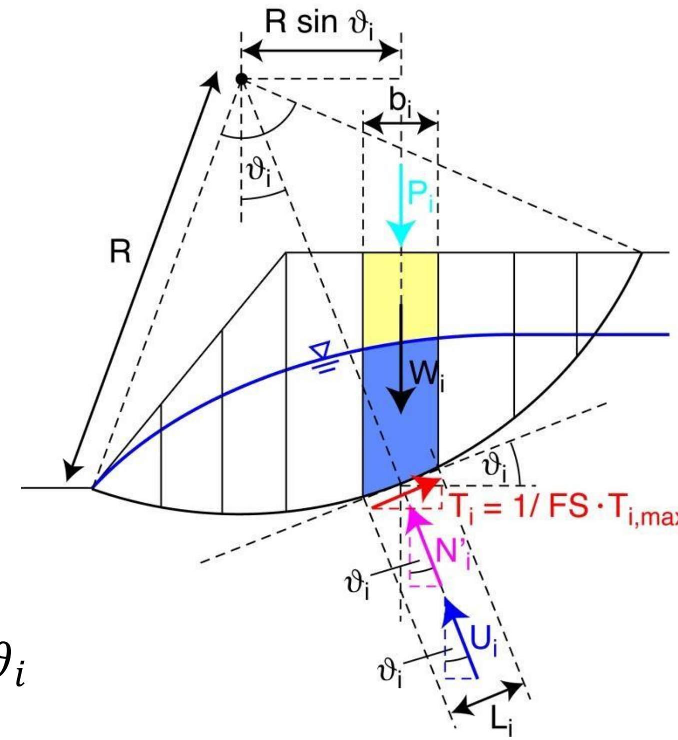
$$U_i = u_i \cdot L_i = u_i \cdot \frac{b_i}{\cos\vartheta_i} \qquad C_i' = c_i' \cdot L = c' \cdot \frac{b_i}{\cos\vartheta_i}$$

$$T_i = \frac{W_i + P_i + c' \cdot b_i \cdot \cot\varphi_i' - u_i \cdot b_i}{FS \cdot \cot\varphi_i' \cdot \cos\vartheta_i + \sin\vartheta_i} = \frac{(W_i + P_i - u_i \cdot b_i) \cdot \tan\varphi_i' + c' \cdot b_i}{FS \cdot \cos\vartheta_i + \sin\vartheta_i \cdot \tan\varphi_i'}$$

Circular failure surface

Analysis with slices

Method of Bishop (1955)



Total resisting moment

$$M_{\text{res}} = T_{i,\max} \cdot R = FS \cdot T_i \cdot R$$

Total driving moment $M_{\text{driv}} = (W_i + P_i) \cdot R \cdot \sin\vartheta_i$

Global safety factor:

$$FS = \frac{M_{\text{res}}}{M_{\text{driv}}} = \frac{\dfrac{(W_i + P_i - u_i \cdot b_i) \cdot \tan\varphi_i' + c' \cdot b_i}{\cos\vartheta_i + \dfrac{1}{FS} \cdot \sin\vartheta_i \cdot \tan\varphi_i'}}{(W_i + P_i) \cdot \sin\vartheta_i}$$

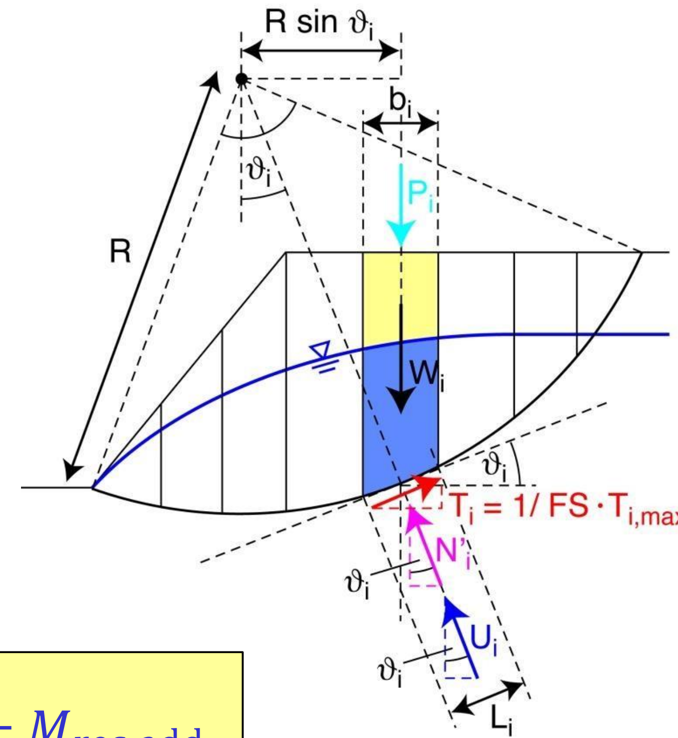
Iterative determination of FS necessary, since FS is present on both sides of equation

(Radius R has been eliminated in enumerator and denominator)

Circular failure surface

Analysis with slices

Method of Bishop (1955)



Additional external moments not captured in the forces considered so far:

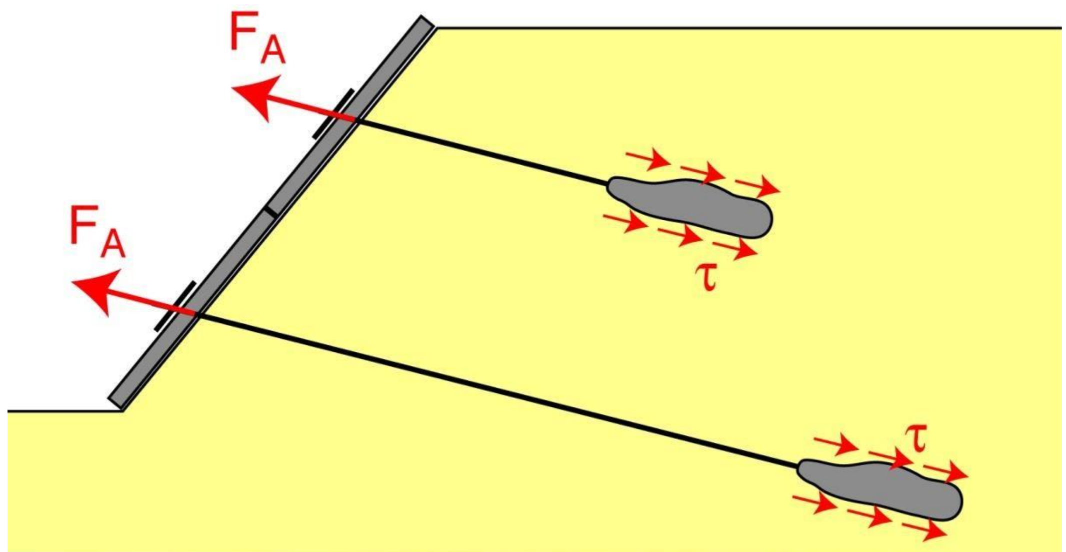
$$FS = \frac{M_{\mathrm{res}}}{M_{\mathrm{driv}}} = \frac{R \cdot \dfrac{(W_i + P_i - u_i \cdot b_i) \cdot \tan \varphi_i{}' + c' \cdot b_i}{\cos \vartheta_i + \dfrac{1}{FS} \cdot \sin \vartheta_i \cdot \tan \varphi_i{}'} \pm M_{\mathrm{res,add}}}{R \cdot (W_i + P_i) \cdot \sin \vartheta_i \pm M_{\mathrm{driv,add}}}$$

Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Anchors



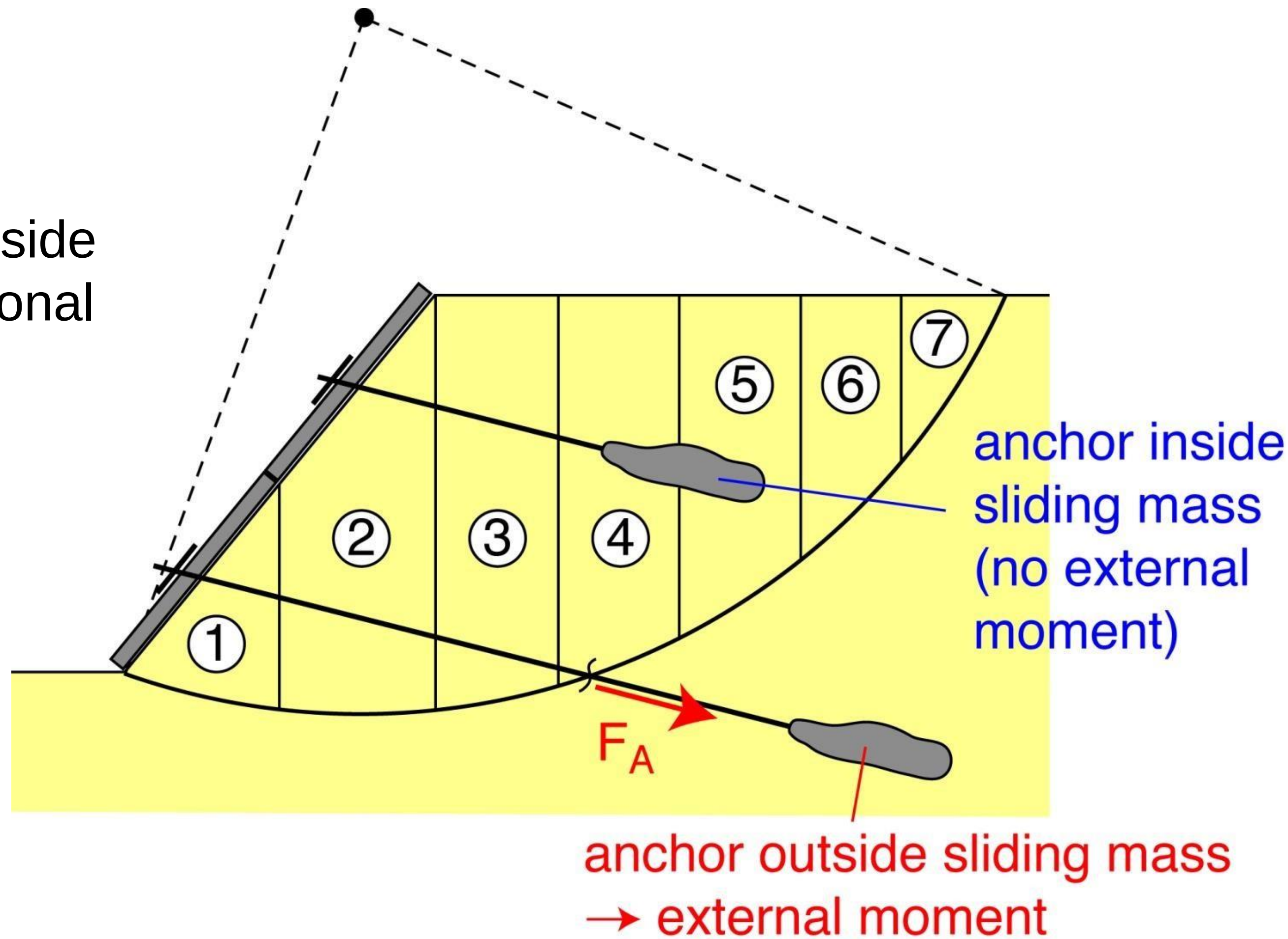
Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Anchors

- Only anchors lying outside slip circle cause additional force in failure surface and thus additional external moment
- Anchors inside slip circle are not considered, they are causing internal forces only



Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Anchors

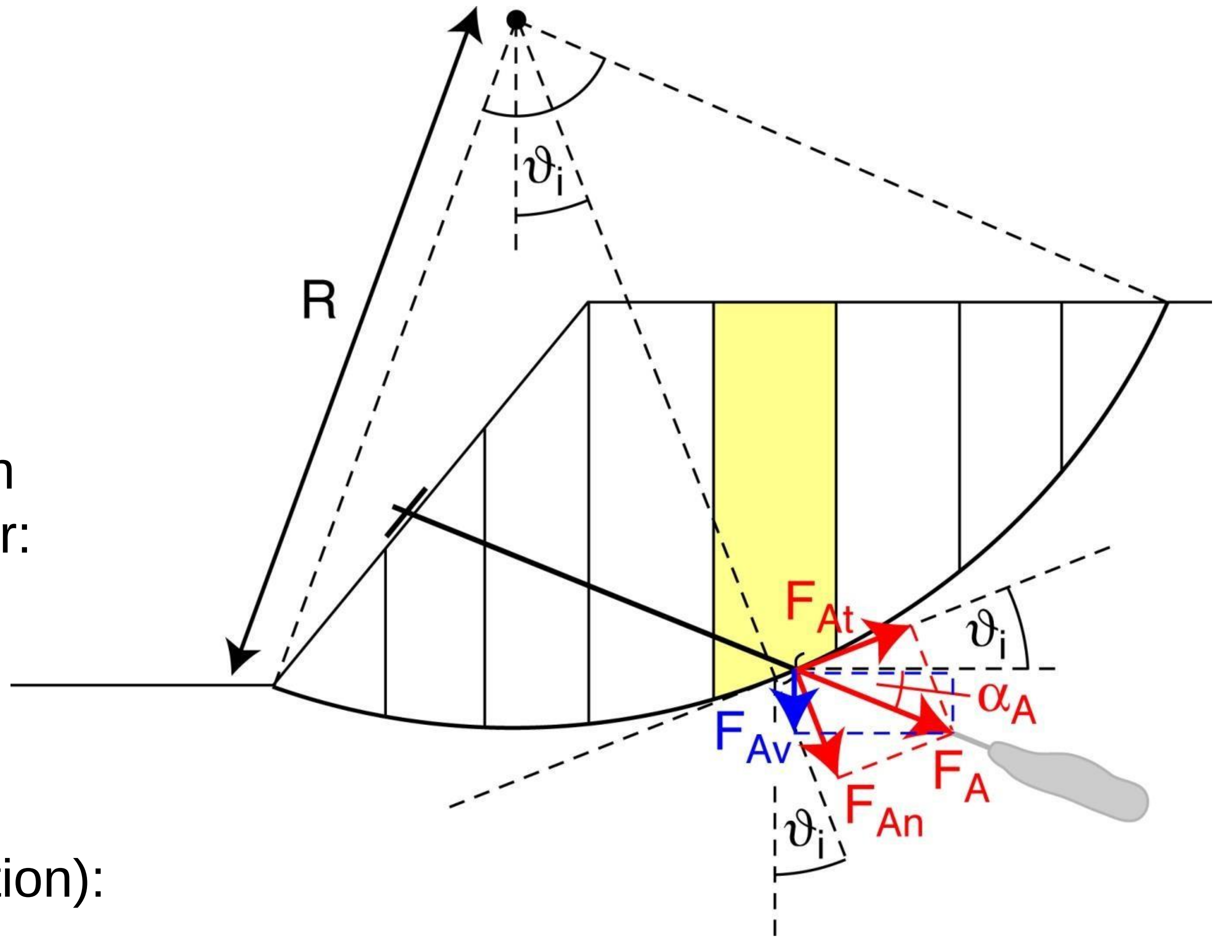
Additional tangential force in failure surface due to anchor:

$$F_{At} = F_A \cdot \cos(\vartheta_i + \alpha_A)$$

Additional moment resulting from this tangential force (acting against sliding direction):

$$M_{At} = -F_{At} \cdot R = -F_A \cdot \cos(\vartheta_i + \alpha_A) \cdot R$$

Additional force in vertical direction: $F_{Av} = F_A \cdot \sin \alpha_A$

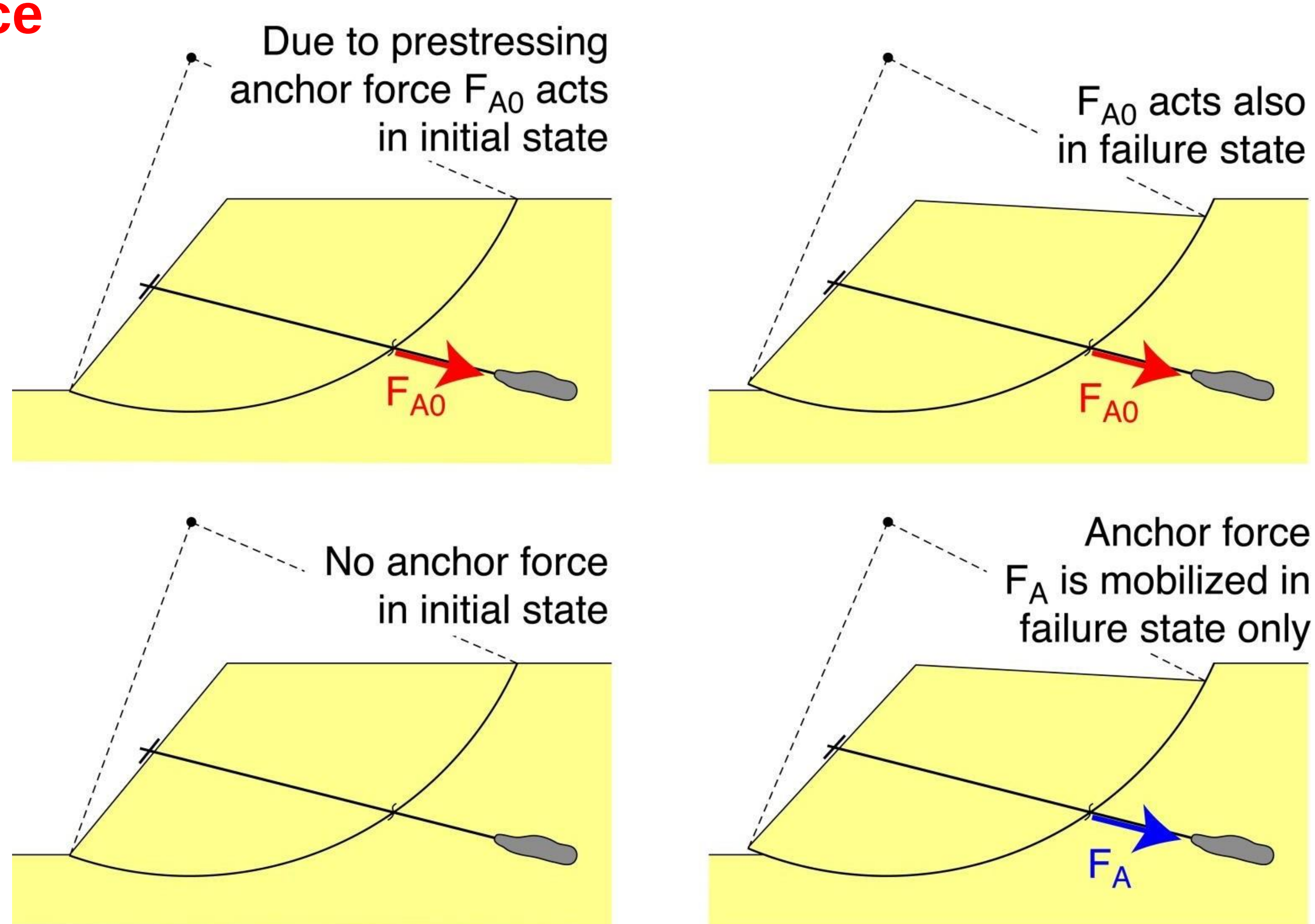


Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Anchors



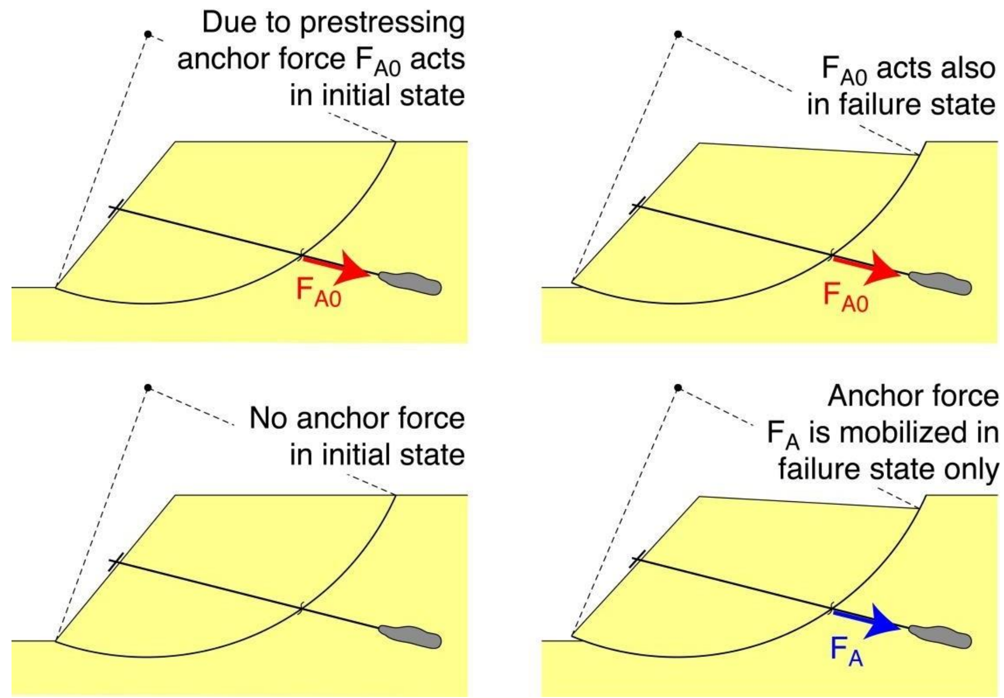
- Prestressed and non-prestressed anchors have to be distinguished
- Prestressed anchors: F_{A0} is considered on the side of driving moments (reducing)
- Non-prestressed anchors: F_A is considered on the side of resisting moments (increasing)

Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Anchors



Safety factor:

$$FS = \frac{M_{res}}{M_{driv}}$$

$$M_{res} = R \cdot \frac{\left(W_i + P_i + \frac{1}{FS} \cdot F_{Ai} \cdot \sin\alpha_{Ai} + F_{A0i} \cdot \sin\alpha_{A0i} - u_i \cdot b_i\right) \cdot \tan\varphi_i' + c' \cdot b_i}{\cos\vartheta + \frac{1}{FS} \cdot \sin\vartheta_i \cdot \tan\varphi_i'}$$

$$+R \cdot F_{Ai} \cdot \cos(\vartheta_i + \alpha_{Ai})$$

$$M_{driv} = R \cdot (W_i + P_i) \cdot \sin\vartheta_i - F_{A0i} \cdot \cos(\vartheta_i + \alpha_{A0i})$$

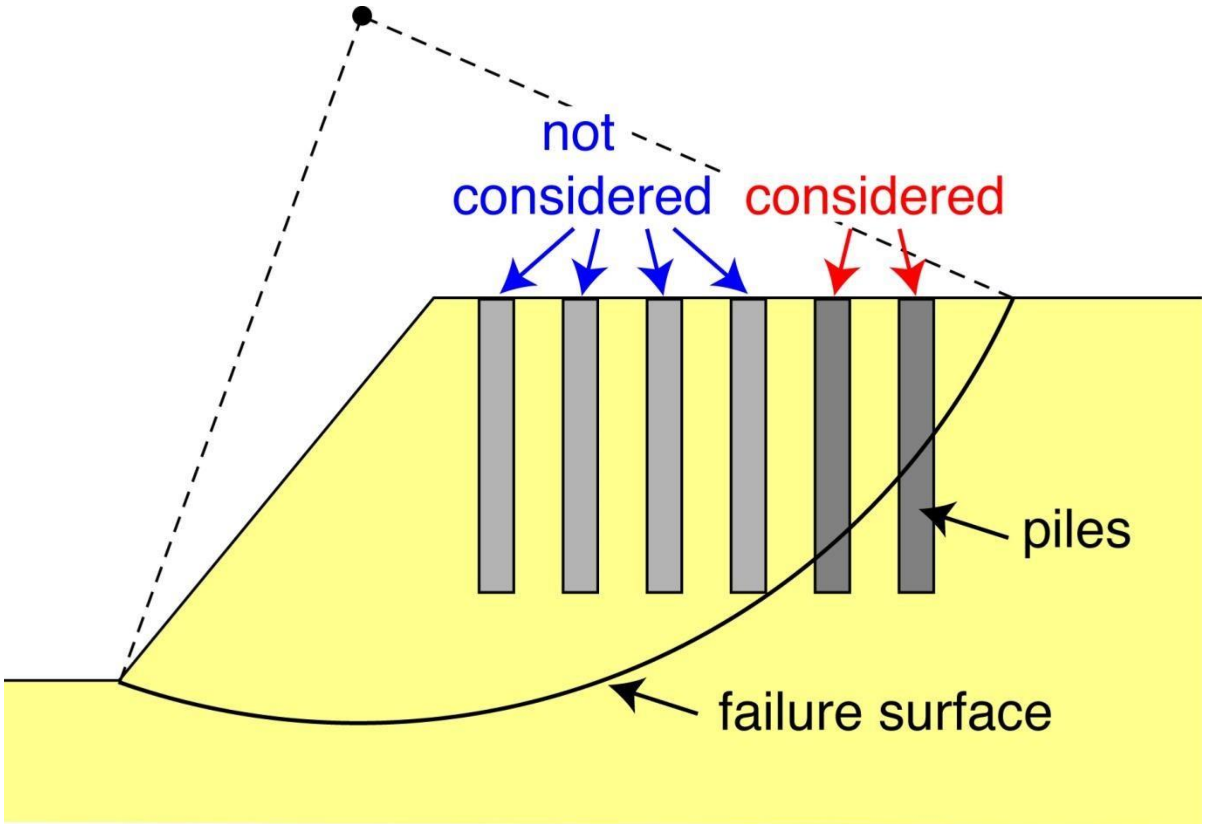
Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Piles

- Additional external force and moment due to piles cut by the failure surface only
- Piles fully lying within slip circle are considered by an increased self-weight only ($\gamma_{Concrete} = 25\,kN/m^3$)

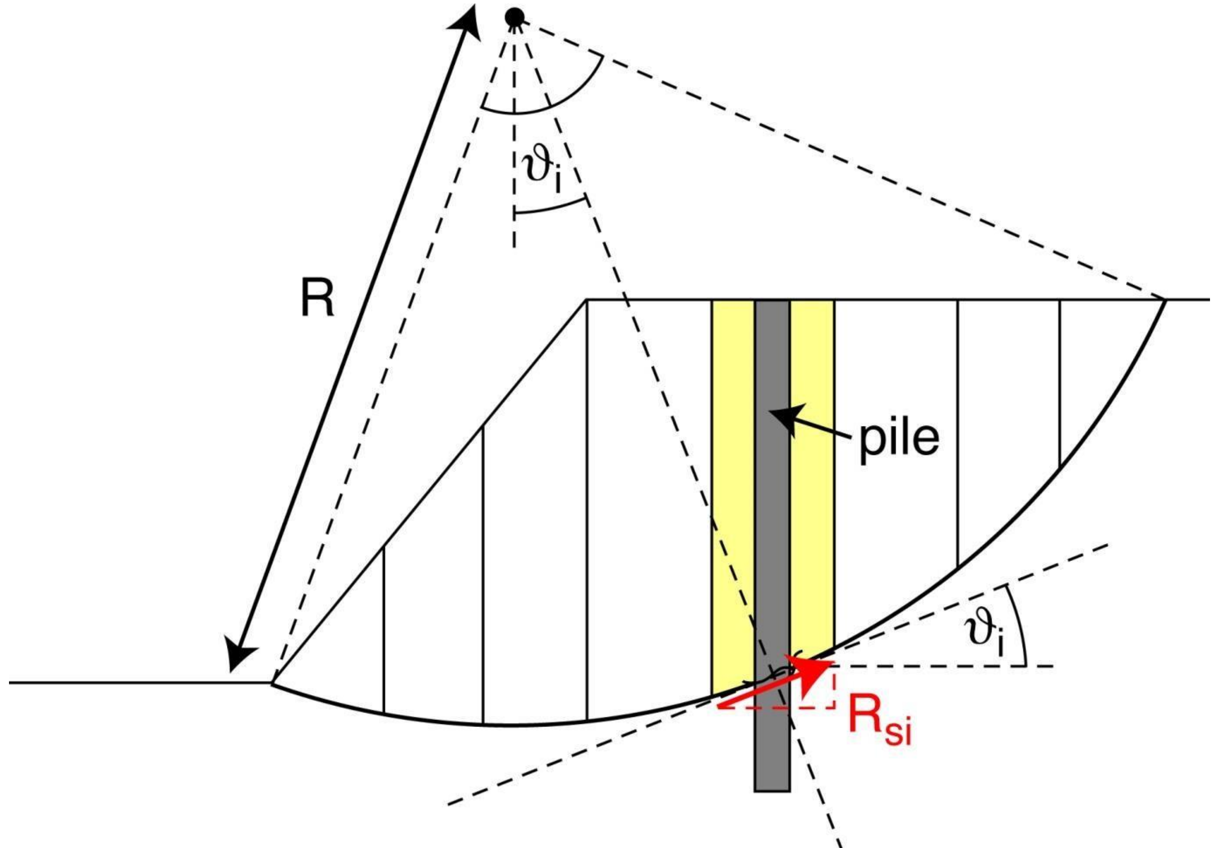


Circular failure surface

Analysis with slices

Method of Bishop (1955)

Example: Piles



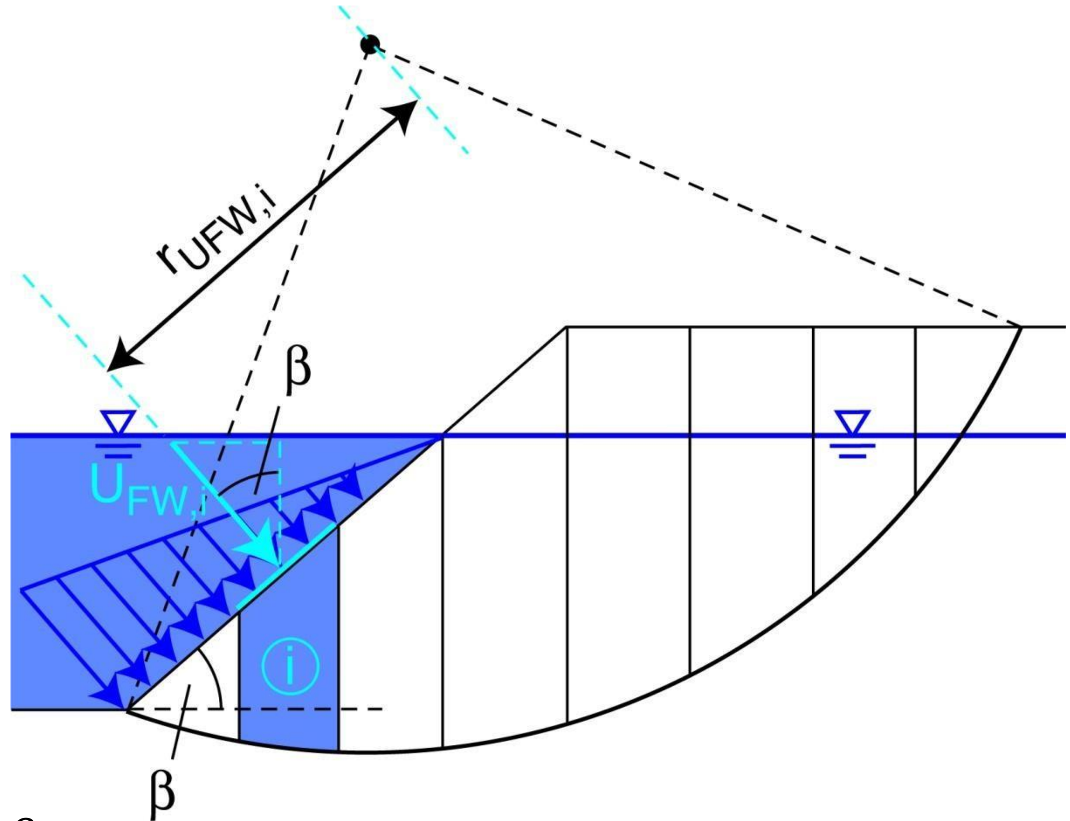
- Safety factor:

$$FS = \frac{M_{\text{res}}}{M_{\text{driv}}} = \frac{R \cdot \dfrac{(W_i + P_i - u_i \cdot b_i) \cdot \tan\varphi' + c' \cdot b_i + R_{si} \cdot \cos\vartheta_i}{\cos\vartheta_i + \dfrac{1}{FS} \cdot \sin\vartheta_i \cdot \tan\varphi'}}{R \cdot (W_i + P_i) \cdot \sin\vartheta_i}$$

Circular failure surface

Analysis with slices

Method of Bishop (1955)



Example: Free water

Additional moment
(acts against sliding direction):

$$M_{\text{add,U}} = U_{FW,i} \cdot r_{UFW,i}$$

Additional vertical force: $U_{FW,i} \cdot \cos\beta$

$$FS = \frac{M_{\text{res}}}{M_{\text{driv}}} = \frac{R \cdot \dfrac{\left(W_i + P_i - u_i \cdot b_i + U_{FW,i} \cdot \cos\beta\right) \cdot \tan\varphi' + c' \cdot b_i}{\cos\vartheta_i + \frac{1}{FS} \cdot \sin\vartheta_i \cdot \tan\varphi'}}{R \cdot (W_i + P_i) \cdot \sin\vartheta_i - M_{\text{add},U}}$$

Circular failure surface

Analysis with slices

Traffic loads

Safety factor from ordinary method with $U_i = 0$, $C'_i = 0$

$$FS = \frac{[(W_i + P_i) \cdot \cos\vartheta_i] \cdot \tan\varphi_i'}{(W_i + P_i) \cdot \sin\vartheta_i}$$

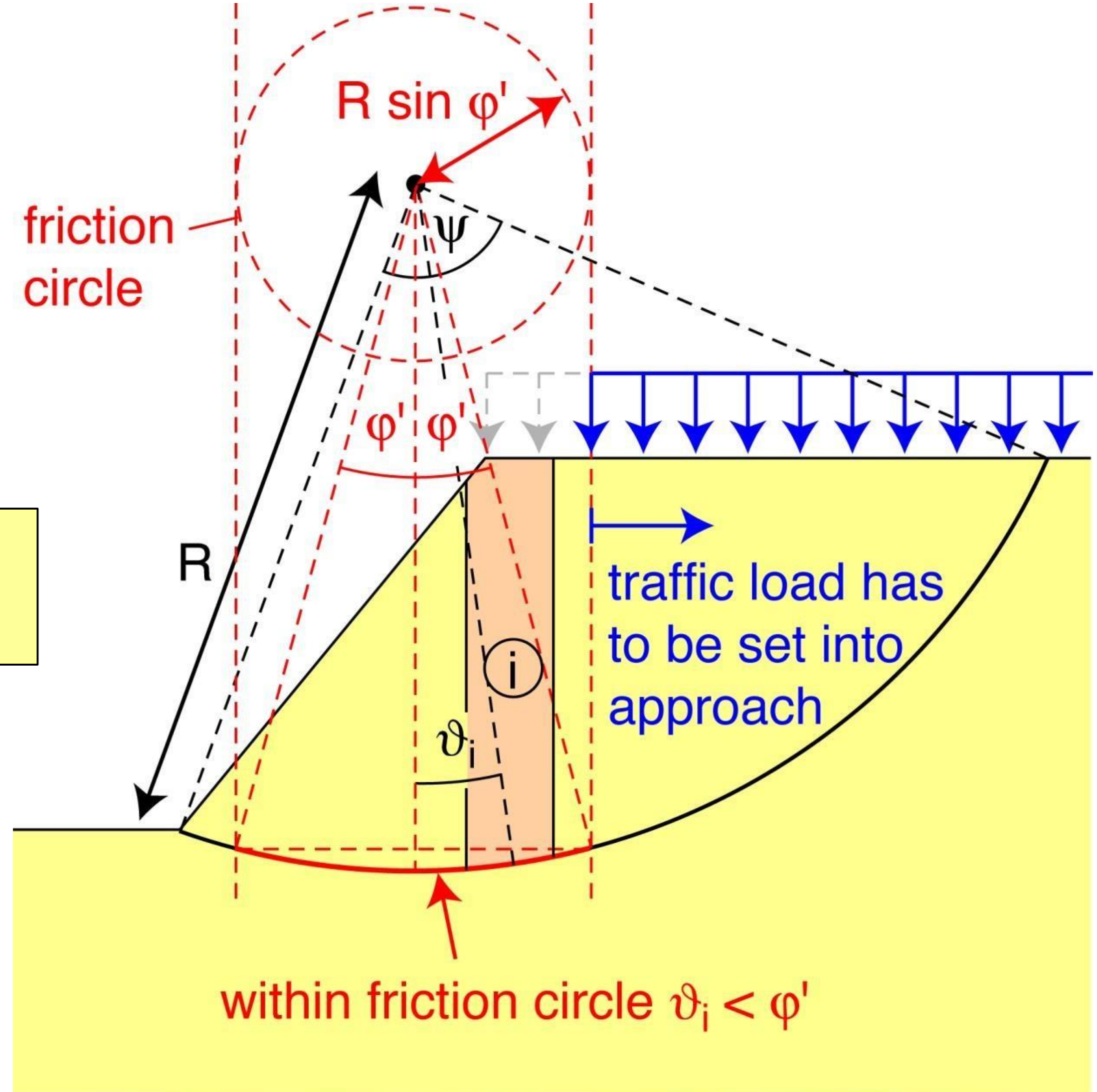
For $\vartheta_i < \varphi'$:

$$(W_i + P_i) \cdot \sin\vartheta_i$$

$$< (W_i + P_i) \cdot \cos\vartheta_i \cdot \tan\varphi'$$

$$= N_i \cdot \tan\varphi'$$

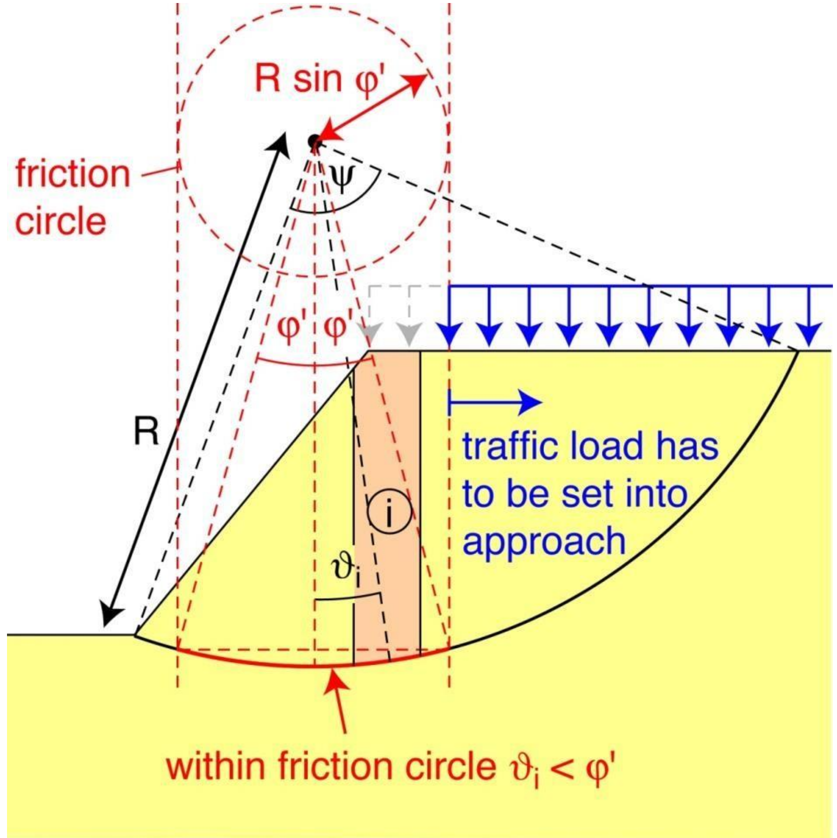
$$\sin\vartheta_i < \cos\vartheta_i \cdot \tan\varphi'$$



Circular failure surface

Analysis with slices

Traffic loads



Example: $\varphi' = 30°$

$\vartheta_i = 20°$: $\sin(20°) < \cos(20°) \cdot \tan(30°)$

$0.34 < 0.54$

$\vartheta_i = 30°$: $\sin(30°) = \cos(30°) \cdot \tan(30°)$

$0.50 = 0.50$

$\vartheta_i = 40°$: $\sin(40°) > \cos(40°) \cdot \tan(30°)$

$0.64 > 0.44$

For $\vartheta_i < \varphi$ additional driving force and moment due to P_i is smaller than additional resisting force and moment due to P_i

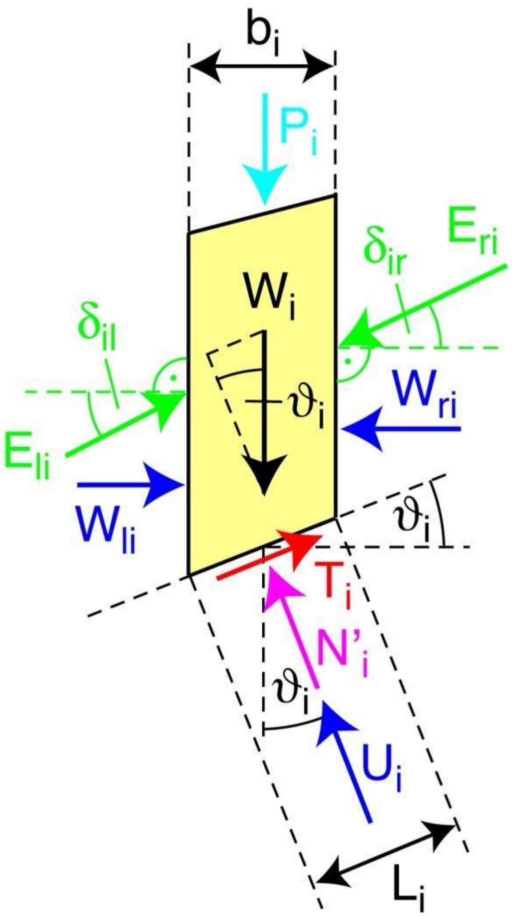
→ P_i has positive effect on slope stability

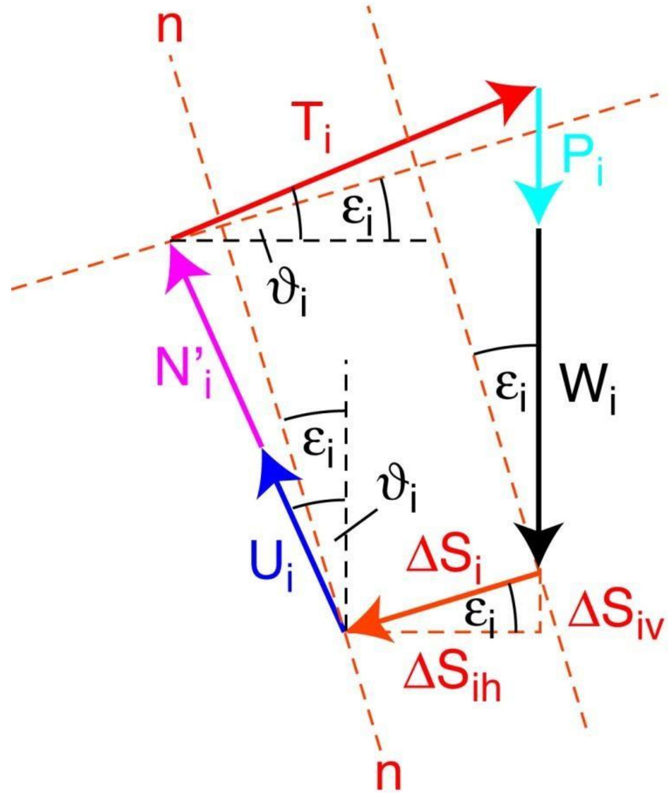
→ Traffic loads must not be set into approach inside friction circle

Circular failure surface

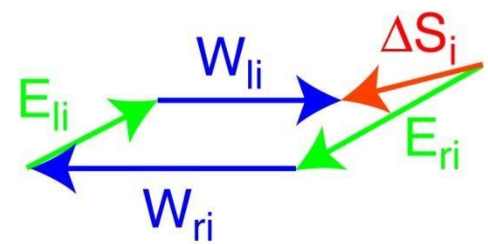
Analysis with slices

Generalized Bishop method considering lateral forces





Polygon of all lateral forces:



ΔS_i = resultant force of all lateral forces

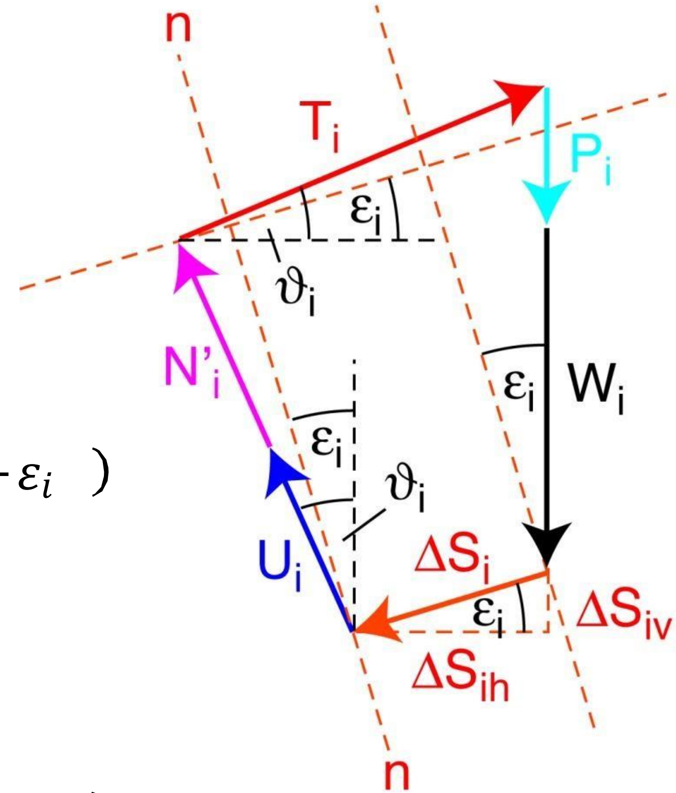
Simplified method:
$\varepsilon_i = 0$ (i.e. $E_{li} = E_{ri}$)

Circular failure surface

Analysis with slices

Generalized Bishop method considering lateral forces

Force equilibrium in direction n-n
(perpendicular to ΔS_i):



$$(W_i + P_i)\cdot\cos\varepsilon_i = (N_i' + U_i)\cdot\cos(\vartheta_i - \varepsilon_i) + T_i\cdot\sin(\vartheta_i - \varepsilon_i)$$

$$T_i = \frac{1}{FS}\cdot T_{i,\max} = \frac{1}{FS}\cdot(N_i'\cdot\tan\varphi_i' + c_i'\cdot L_i)$$

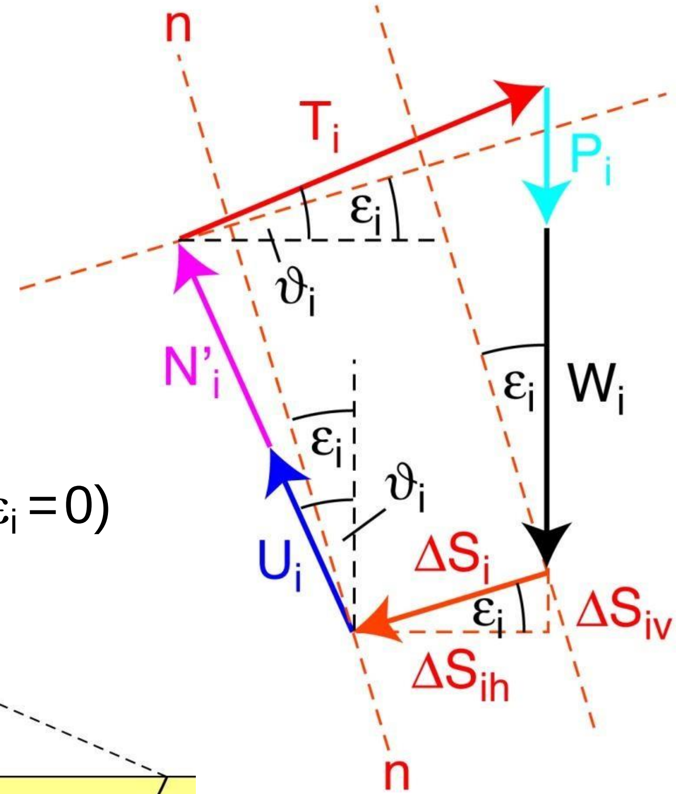
$$(W_i + P_i)\cdot\cos\varepsilon_i = (N_i' + u_i\cdot L_i)\cdot\cos(\vartheta_i - \varepsilon_i) + \frac{1}{FS}\cdot(N_i'\cdot\tan\varphi_i' + c_i'\cdot L_i)\cdot\sin(\vartheta_i - \varepsilon_i)$$

$$N_i' = \frac{(W_i + P_i)\cdot\cos\varepsilon_i - u_i\cdot L_i\cdot\cos(\vartheta_i - \varepsilon_i) - \frac{1}{FS}\cdot c_i'\cdot L_i\cdot\sin(\vartheta_i - \varepsilon_i)}{\cos(\vartheta_i - \varepsilon_i) + \frac{1}{FS}\cdot\tan\varphi_i'\cdot\sin(\vartheta_i - \varepsilon_i)}$$

Circular failure surface

Analysis with slices

Generalized Bishop method considering lateral forces



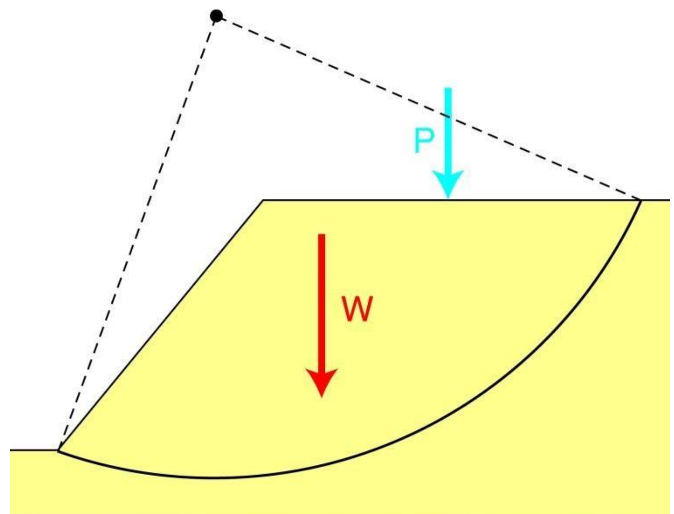
$$T_{i,\max} = N' \cdot \tan \varphi_i' + c_i' \cdot L_i$$

Driving and resisting moments:

$$M_{\mathrm{driv}} = R \cdot \quad (W_i + P_i) \cdot \sin \vartheta_i$$ (same as in case of $\varepsilon_i = 0$)

$$M_{\mathrm{res}} = R \cdot T_{i,\max}$$

$$= R \cdot N'(\cdot \tan \varphi_i' + c_i' \cdot L_i \quad)$$



Circular failure surface

Analysis with slices

Generalized Bishop method considering lateral forces

Safety factor considering inclination ε_i of resultant lateral force:

$$FS = \frac{M_{\text{res}}}{M_{\text{driv}}} = \frac{R \cdot N'(\cdot \tan \varphi_i' + c_i' \cdot L_i)}{R \cdot (W_i + P_i) \cdot \sin \vartheta_i}$$

$$= \frac{R \cdot \left[\left(\dfrac{(W_i + P_i) \cdot \cos \varepsilon_i - u_i \cdot L_i \cdot \cos(\vartheta_i - \varepsilon_i) - \frac{1}{FS} \cdot c'_i \cdot L_i \cdot \sin(\vartheta_i - \varepsilon_i)}{\cos(\vartheta_i - \varepsilon_i) + \frac{1}{FS} \cdot \tan \varphi'_i \cdot \sin(\vartheta_i - \varepsilon_i)} \right) \cdot \tan \varphi_i' + c_i' \cdot L_i \right]}{R \cdot (W_i + P_i) \cdot \sin \vartheta_i}$$

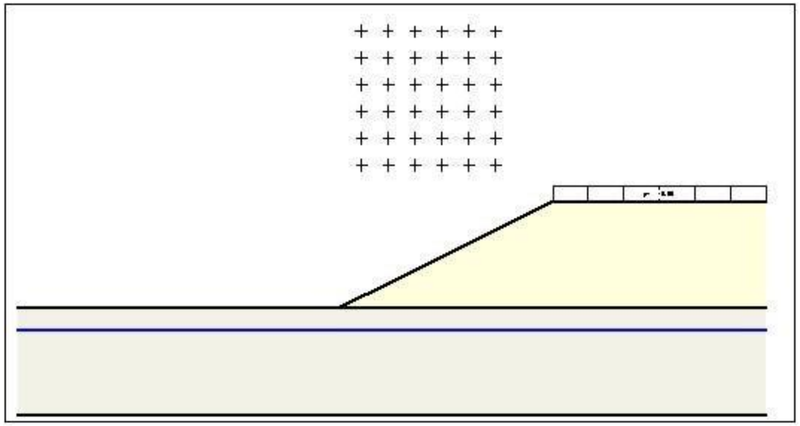
The difference (= error) in the safety factor between the generalized and the simplified method ($\varepsilon_i = 0$) of Bishop is usually less than 5 %!

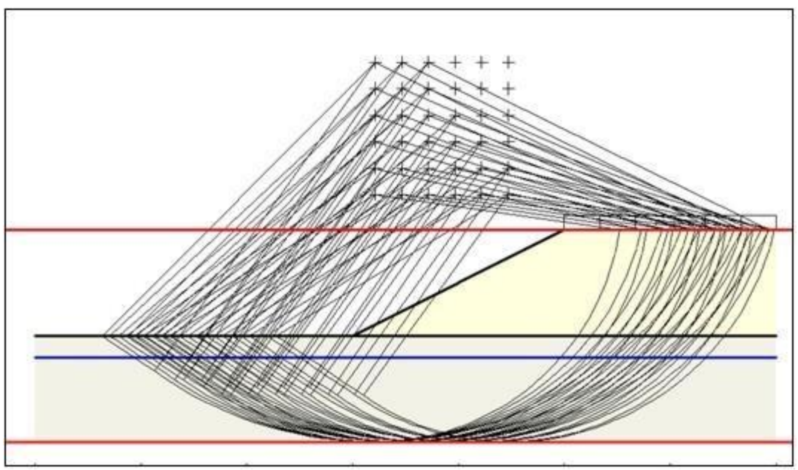
→ The simplified method of Bishop neglecting lateral forces is usually applied

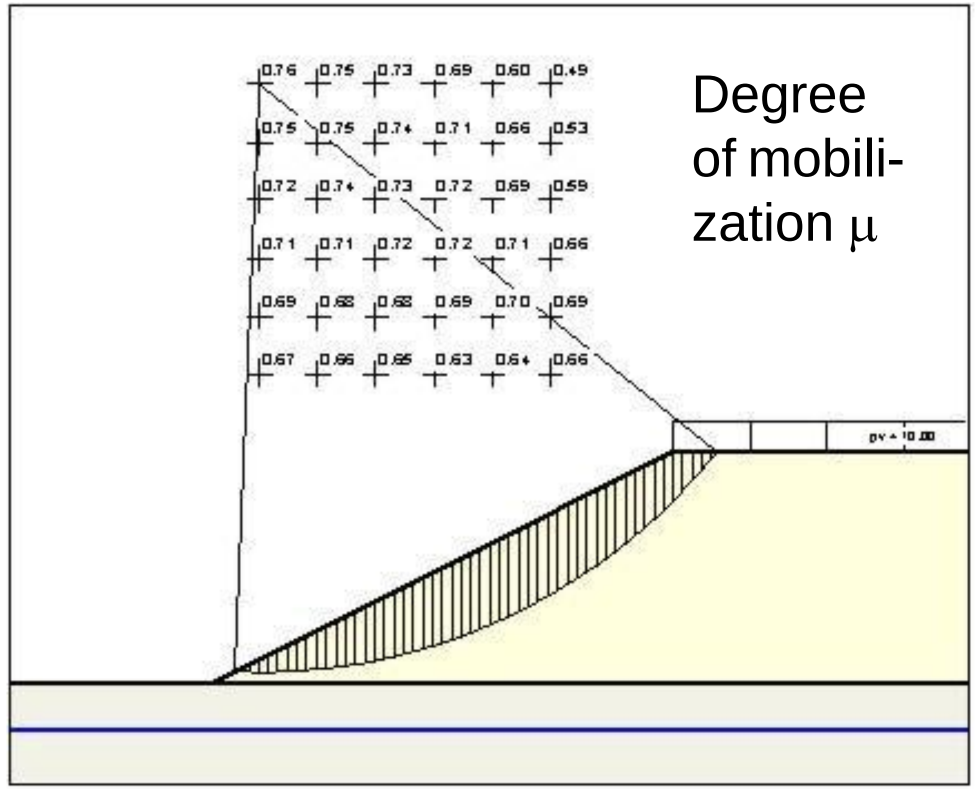
Circular failure surface

Analysis with slices

Simplified Bishop method is also used in common commercial software



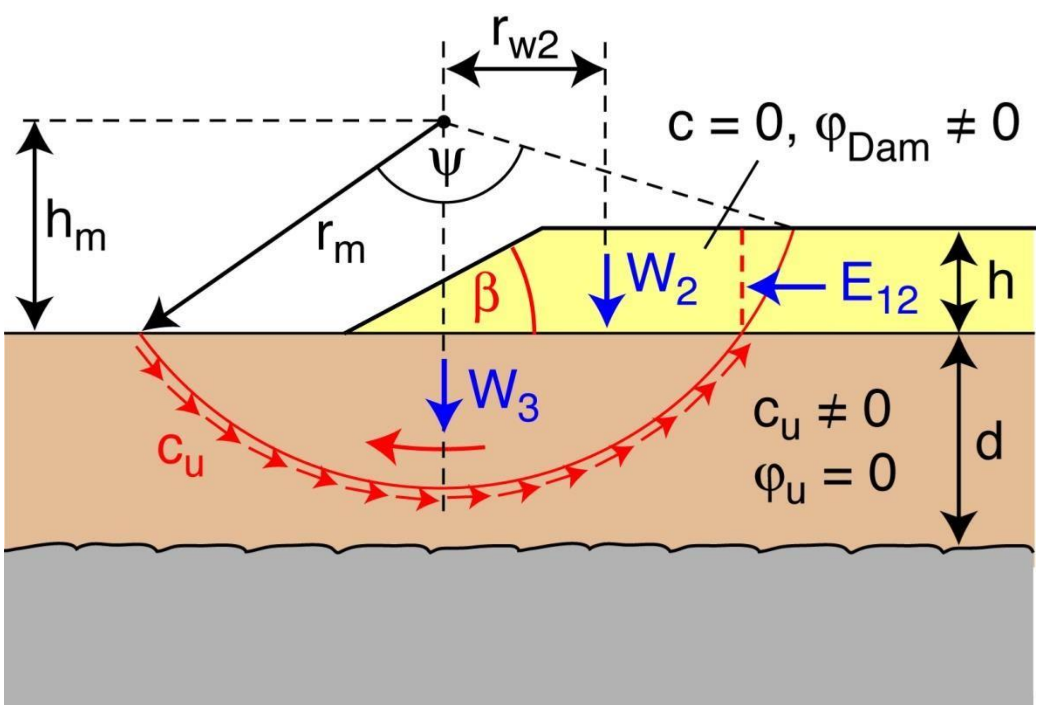




Here: GGU software

Circular failure surface

Dam on weak ground



- Failure on circular slip surface in the weak ground
- Center of slip circle is assumed to lie above the center of the slope
- Self-weight of dam:

$$W_2 = m_w \cdot \gamma_{\text{Dam}} \cdot h^2$$

$$r_{w2} = n_w \cdot h$$

$$m_w, n_w = f\left(\beta, \frac{r_m}{h}, \frac{h_m}{h}\right)$$

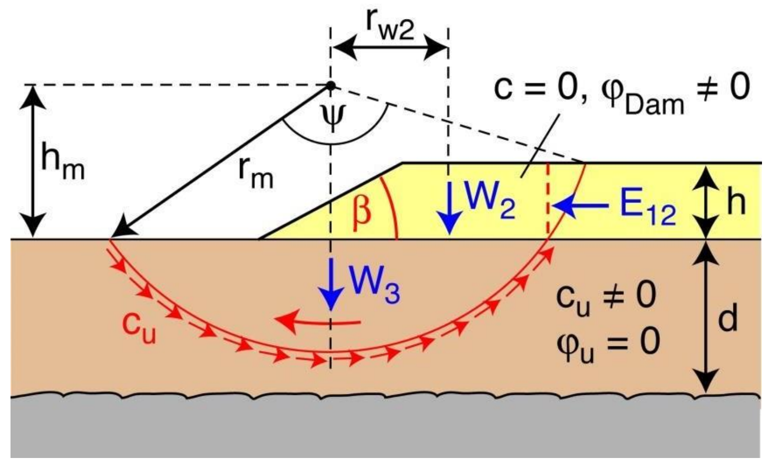
- Earth pressure:

$$E_{12} = \frac{1}{2} \cdot \gamma_{\text{Dam}} \cdot h^2 \cdot K_{ah}$$

$$K_{ah} = \tan^2\left(45° - \frac{\varphi_{\text{Dam}}}{2}\right)$$

Circular failure surface

Dam on weak ground



- Driving moment around center point:

$$M_{\text{driv}} = W_2 \cdot r_{w2} + E_{12} \cdot \left(h_m - \frac{h}{3}\right)$$

$$= m_w \cdot \gamma_{\text{Dam}} \cdot h^2 \cdot n_w \cdot h + \frac{1}{2} \cdot \gamma_{\text{Dam}} \cdot h^2 \cdot \tan^2\left(45° - \frac{\varphi_{\text{Dam}}}{2}\right) \cdot \left(h_m - \frac{h}{3}\right)$$

$$= \gamma_{\text{Dam}} \cdot h^2 \cdot \left[m_w \cdot n_w \cdot h + \frac{1}{2} \cdot \tan^2\left(45° - \frac{\varphi_{\text{Dam}}}{2}\right) \cdot \left(h_m - \frac{h}{3}\right)\right]$$

- Resisting moment due to cohesion:

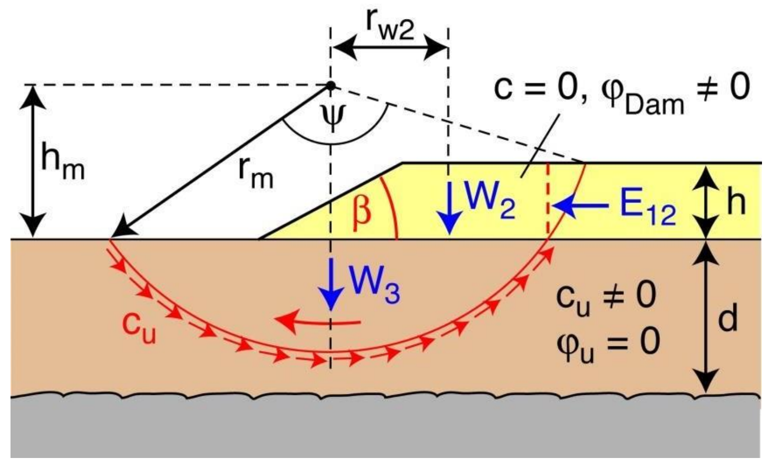
$$M_{\text{res}} = M_c = c_{u,\min} \cdot R^2 \cdot \psi \qquad \cos\left(\frac{\psi}{2}\right) = \frac{h_m}{r_m} \qquad \psi = 2 \cdot \arccos\left(\frac{h_m}{r_m}\right)$$

$$= c_{u,\min} \cdot R^2 \cdot \left[2 \cdot \arccos\left(\frac{h_m}{r_m}\right)\right]$$

- Equilibrium of momentum: $M_{\text{driv}} = M_{\text{res}}$

Circular failure surface

Dam on weak ground



- Equilibrium of momentum leads to undrained cohesion necessary for slope stability:

$$c_{u,\min} = \frac{\gamma_{\text{Dam}} \cdot h^2}{R^2 \cdot \psi} \cdot \left[m_w \cdot n_w \cdot h + \frac{1}{2} \cdot \tan^2\left(45° - \frac{\varphi_{\text{Dam}}}{2}\right) \cdot \left(h_m - \frac{h}{3}\right)\right]$$

$$= K_c \cdot \gamma_{\text{Dam}} \cdot h$$

K_c = cohesion factor

$$K_c = \frac{h}{R^2 \cdot \psi} \cdot \left[m_w \cdot n_w \cdot h + \frac{1}{2} \cdot \tan^2\left(45° - \frac{\varphi_{\text{Dam}}}{2}\right) \cdot \left(h_m - \frac{h}{3}\right)\right]$$

$$= f\left(\beta, \varphi_{\text{Dam}}, \frac{r_m}{h}, \frac{h_m}{h}\right)$$

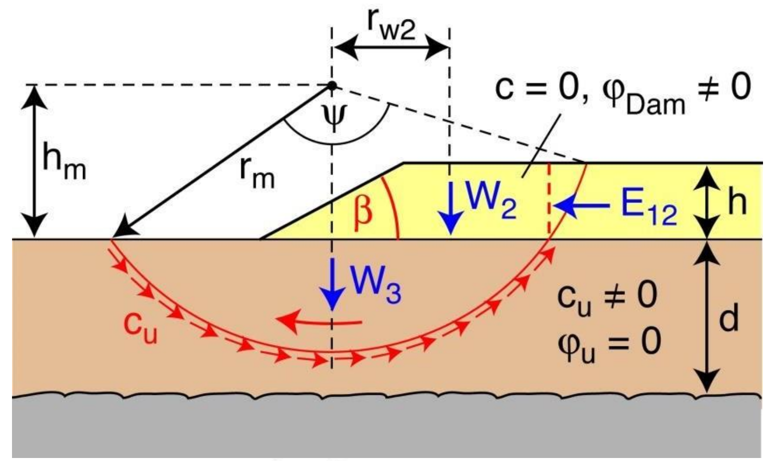
- Safety factor

$$FS = \frac{c_u}{c_{u,\min}} = \frac{c_u}{K_c \cdot \gamma_{\text{Dam}} \cdot h}$$

Circular failure surface

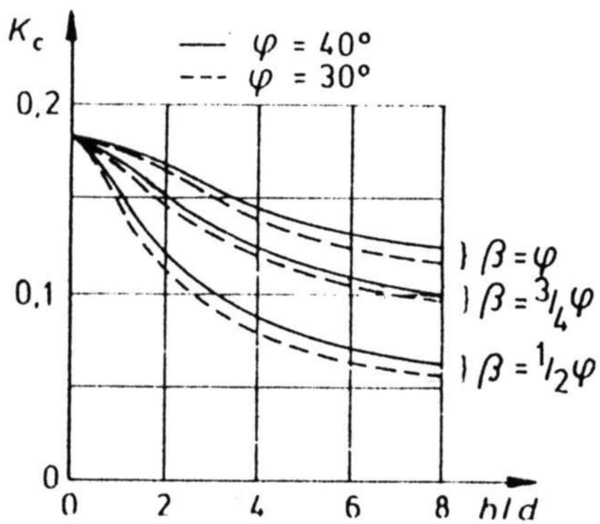
Dam on weak ground

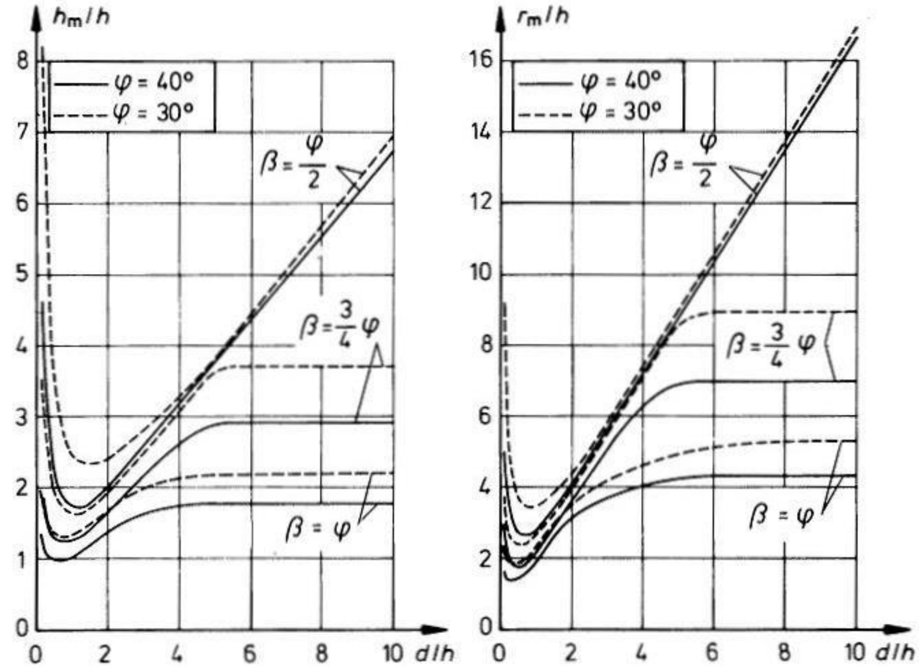
- Variation of r_m/h and h_m/h, until slip circle with lowest safety factor (largest K_c) is found



- Solution in form of diagrams:

$$c_{u,\min} = K_c \cdot \gamma_{\text{Dam}} \cdot h$$

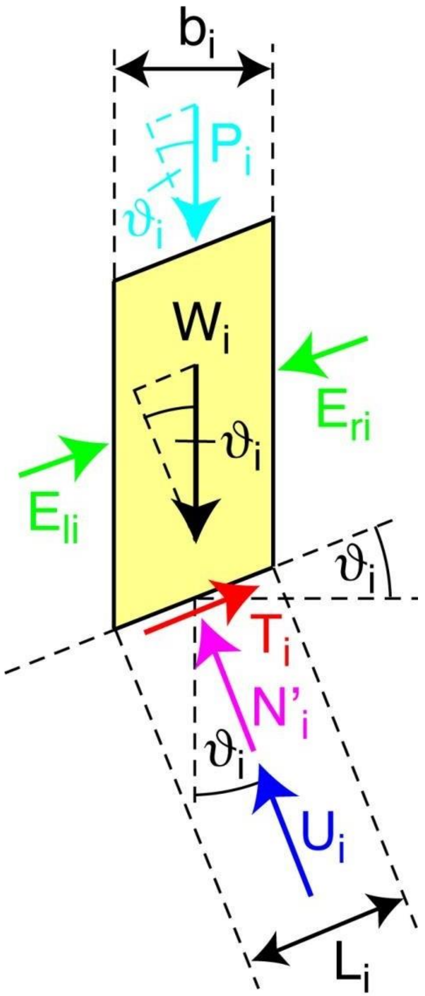


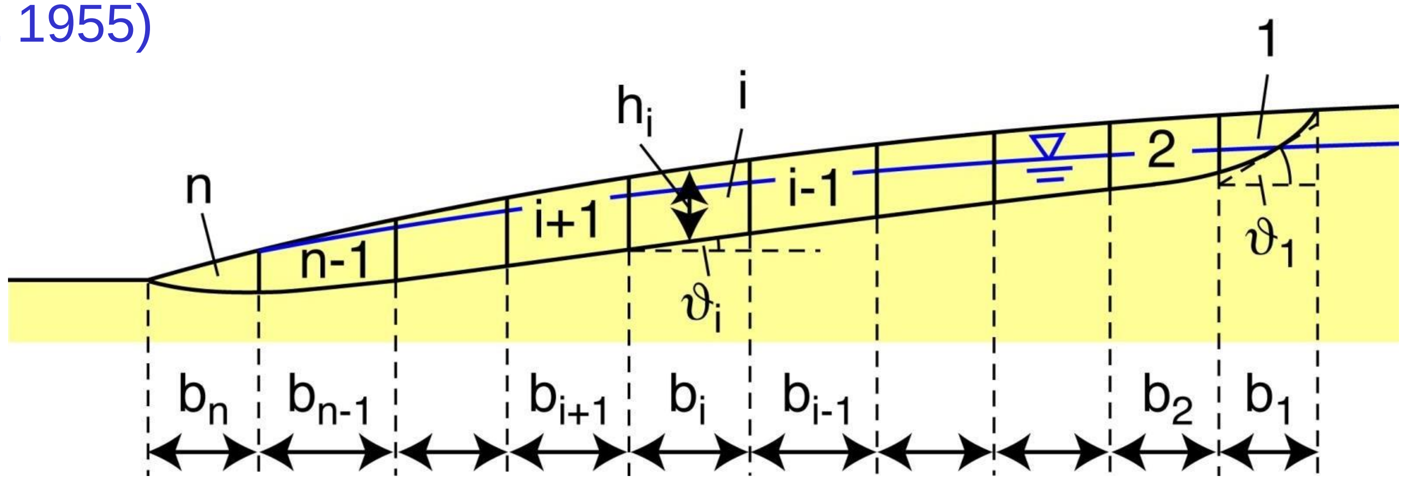


- Can be used to estimate the maximum possible height h of embankment

Slices method for failure surface parallel to the sloping ground

(according to Janbu, 1955)



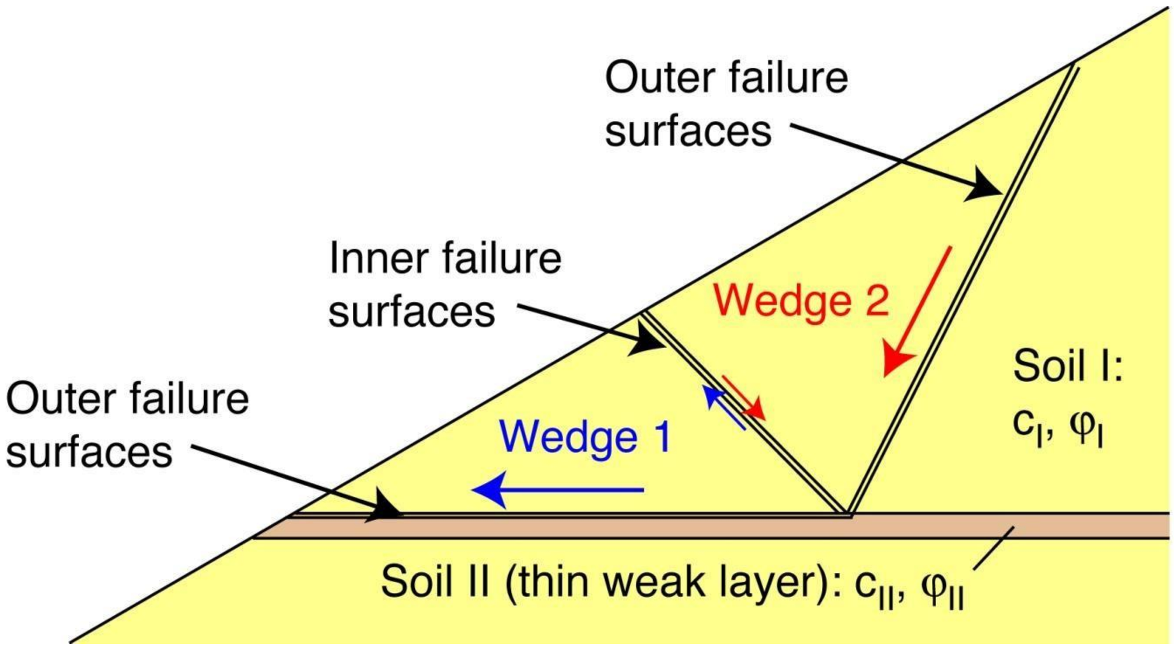


$$\frac{T_i}{\cos\vartheta_i} = \frac{(W_i + P_i - u_i \cdot b_i) \cdot \tan\varphi_i' + c' \cdot b_i}{\cos^2\vartheta + \frac{1}{FS} \cdot \tan\vartheta_i \cdot \tan\varphi_i'}$$

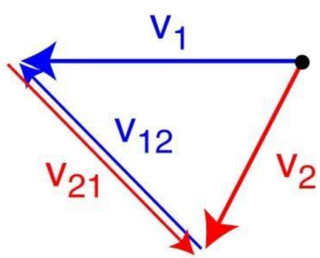
$$FS = \frac{F_{h,res}}{F_{h,driv}} = \frac{\dfrac{(W_i + P_i - u_i \cdot b_i) \cdot \tan\varphi_i' + c' \cdot b_i}{\cos^2\vartheta_i + \frac{1}{FS} \cdot \tan\vartheta_i \cdot \tan\varphi_i'}}{(W_i + P_i) \cdot \tan\vartheta_i}$$

Failure mechanisms with multiple sliding masses

Example 1: Two wedges on weak soil layer



Hodograph:



Relative displacements / velocities of soil masses from hodograph:

- Draw velocities v_1, v_2 in the directions of sliding of both soil masses on outer failure surfaces, starting from same point
- v_{21} = velocity of wedge 2 relative to wedge 1: tip of vector v_1 to tip of vector v_2

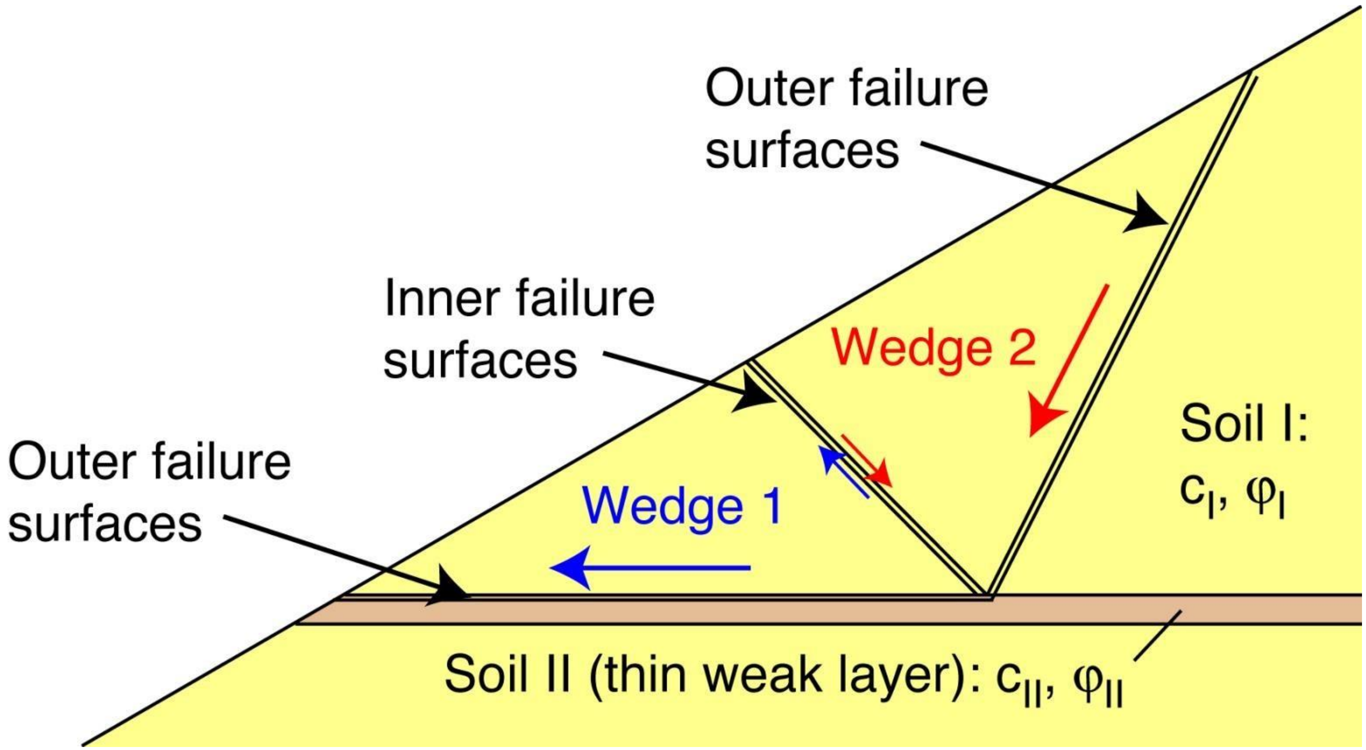
Failure mechanisms with multiple sliding masses

Example 1: Two wedges on weak soil layer

1. Estimate safety factor FS
2. Reduce shear strength parameters by 1/FS

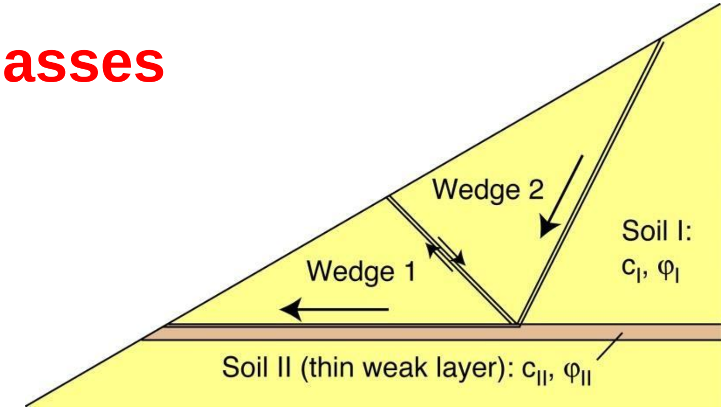
$$c^* = \frac{c}{FS}$$

$$\tan \varphi^* = \frac{\tan\varphi}{FS}$$



Failure mechanisms with multiple sliding masses

Example 1: Two wedges on weak soil layer

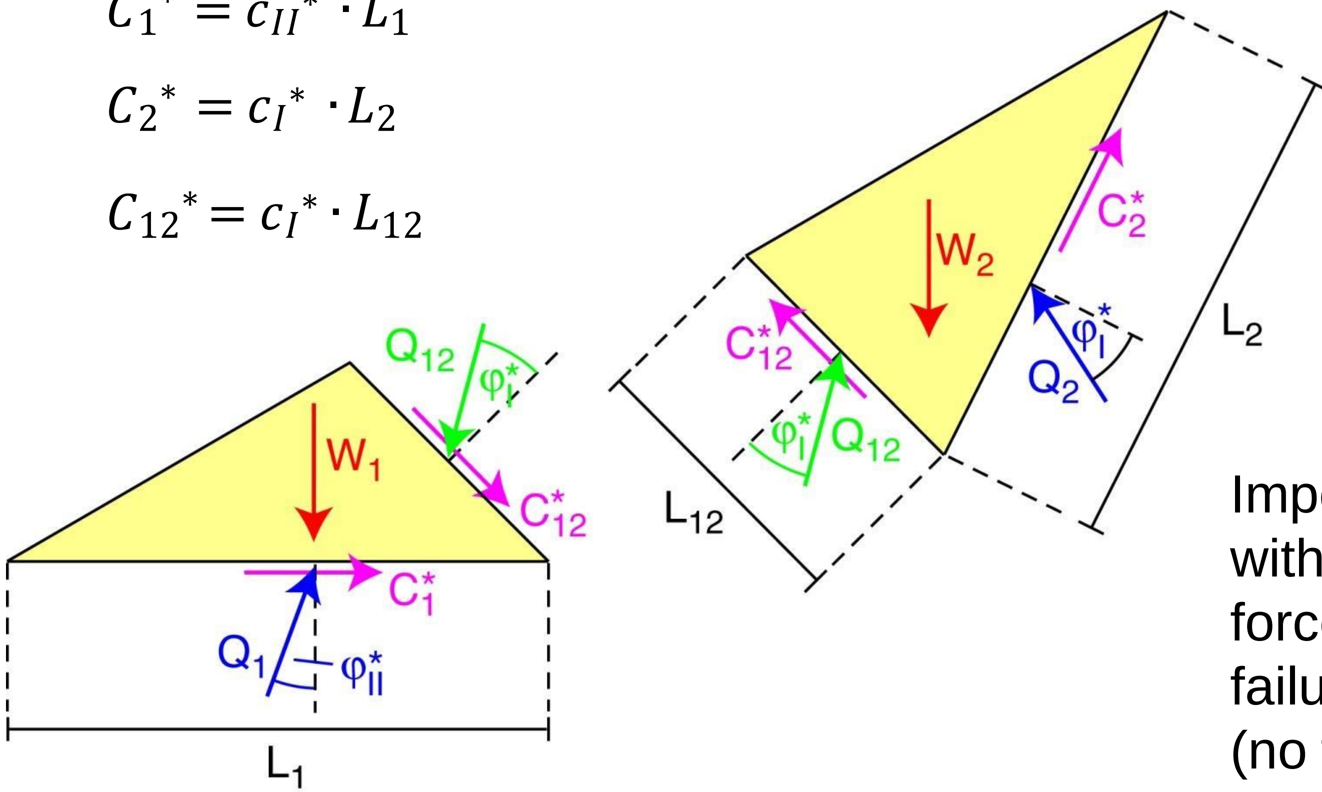


3. Calculate forces with reduced shear strength parameters

$$C_1^* = c_{II}^* \cdot L_1$$

$$C_2^* = c_I^* \cdot L_2$$

$$C_{12}^* = c_I^* \cdot L_{12}$$

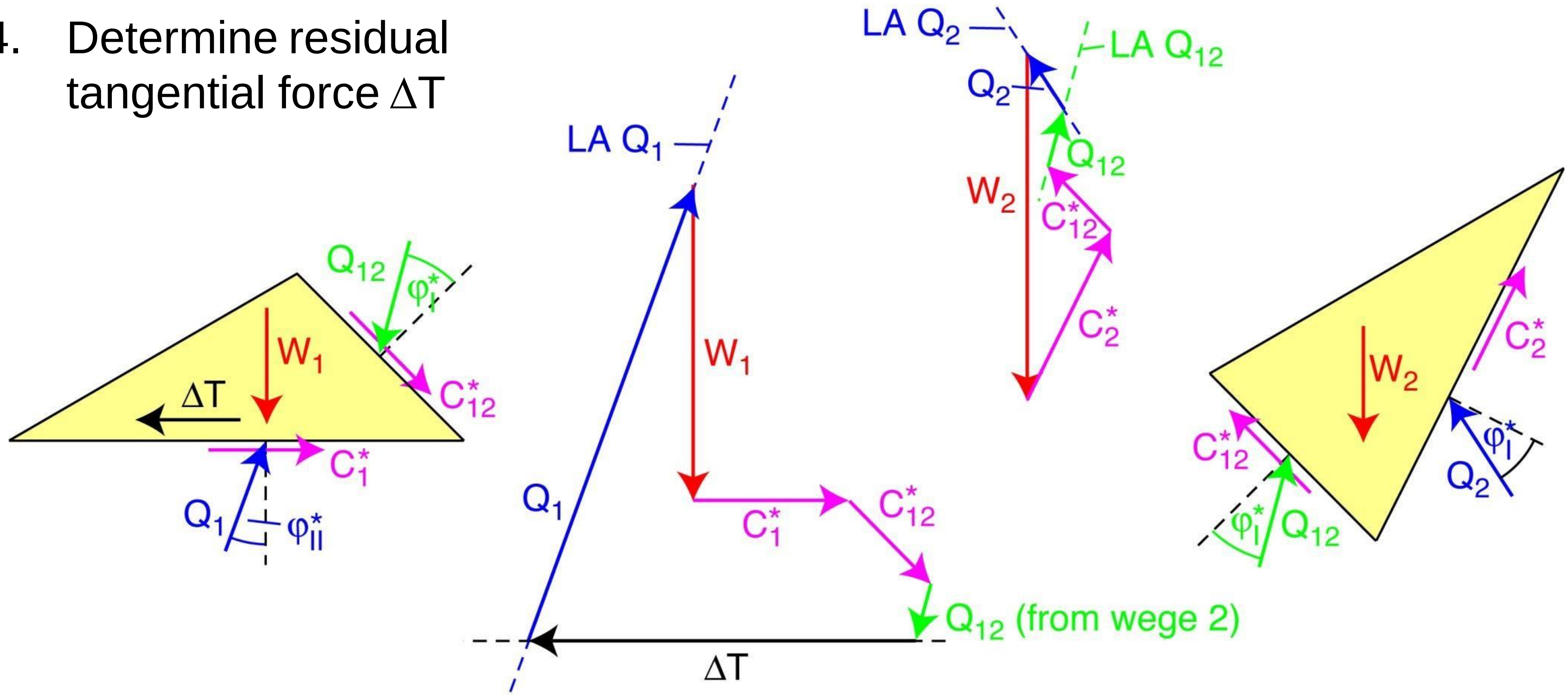


Important: only mechanisms with compressive normal forces on all inner and outer failure surfaces are allowed (no tension)!

Failure mechanisms with multiple sliding masses

Example 1: Two wedges on weak soil layer

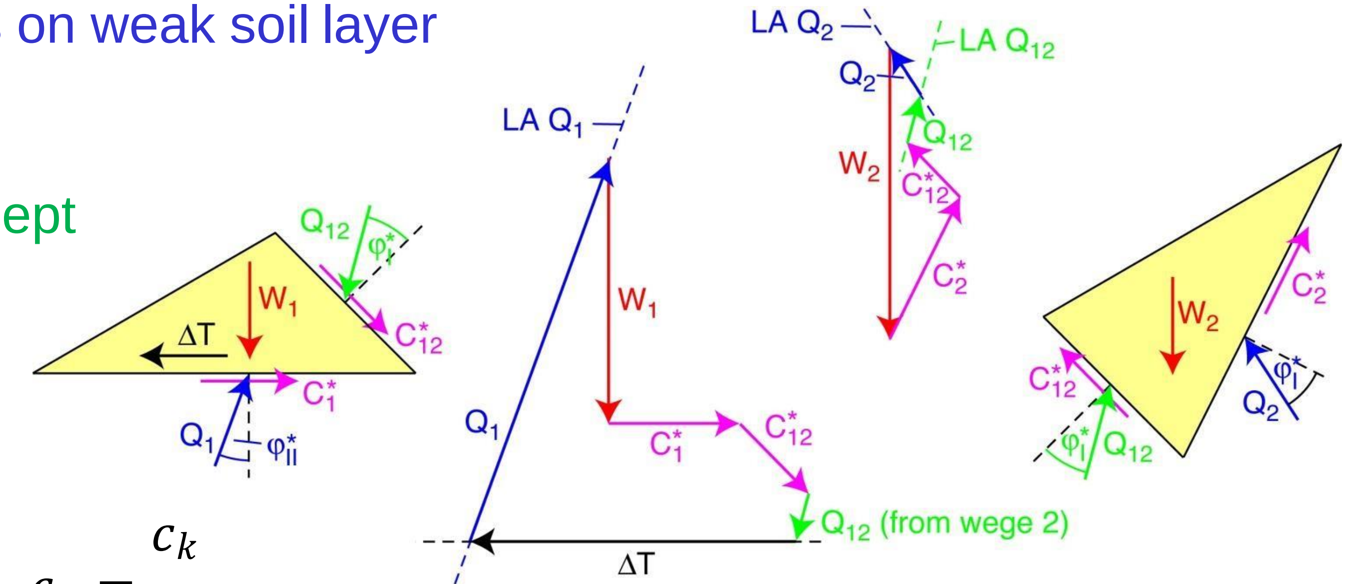
4. Determine residual tangential force ΔT



If ΔT is acting in sliding direction → safety is higher than estimated FS (additional ΔT is necessary to cause failure) → next iteration with higher FS until $\Delta T \approx 0$

Failure mechanisms with multiple sliding masses

Example 1: Two wedges on weak soil layer



Procedure in case of
partial safety factor concept

- Calculate design values of shear strength parameters

$$\tan\varphi_d = \frac{\tan\varphi_k}{\gamma_\varphi} \qquad c_d = \frac{c_k}{\gamma_c}$$

- Multiplication of the shear strength parameters with the degree of mobilization μ

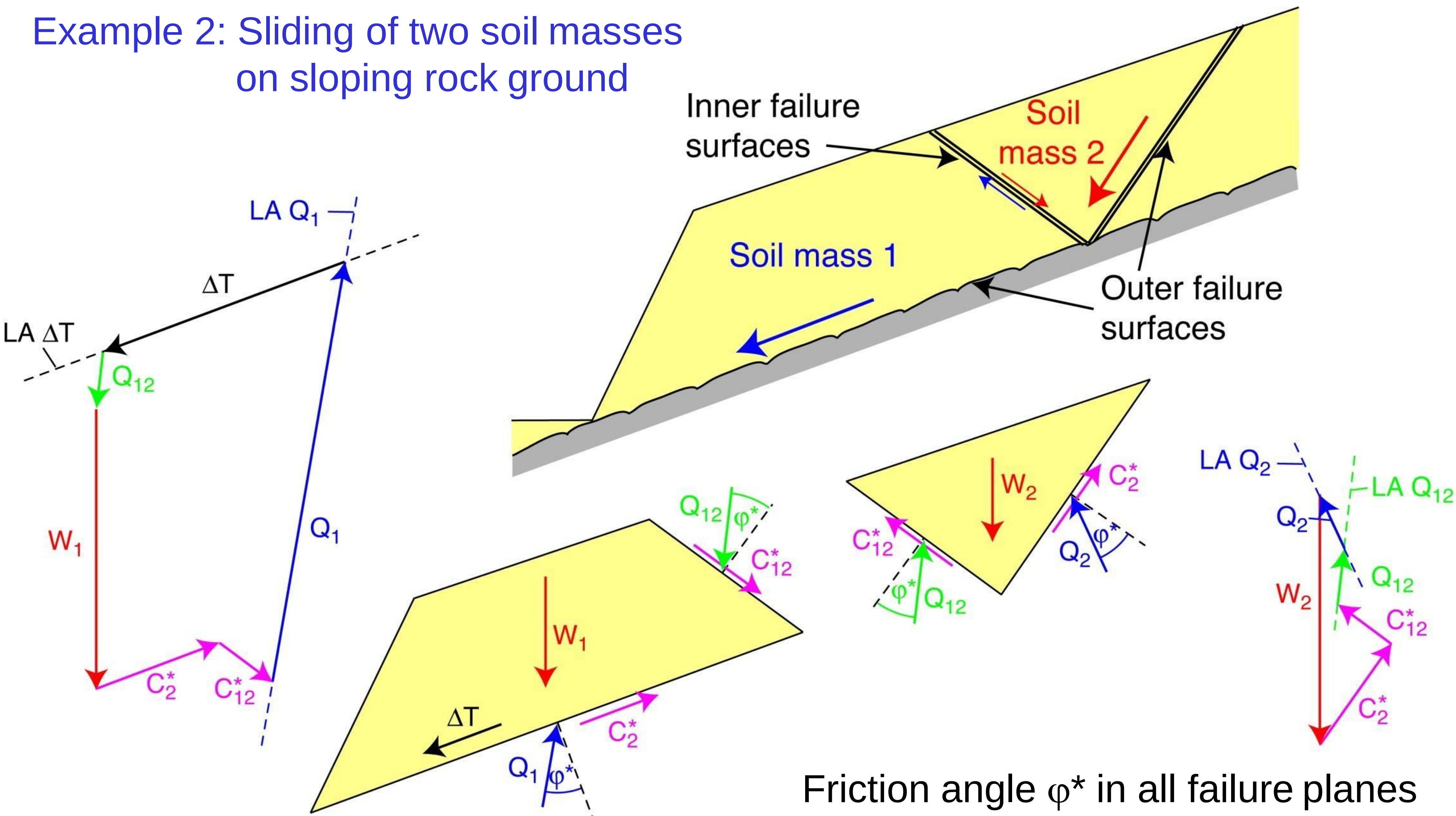
$$\tan\varphi_d{}^* = \mu \cdot \tan\varphi_d \qquad c_d{}^* = \mu \cdot c_d$$

- Iteration of μ until $\Delta T \approx 0$

- Necessary criterion for slope stability: $\mu < 1$

Failure mechanisms with multiple sliding masses

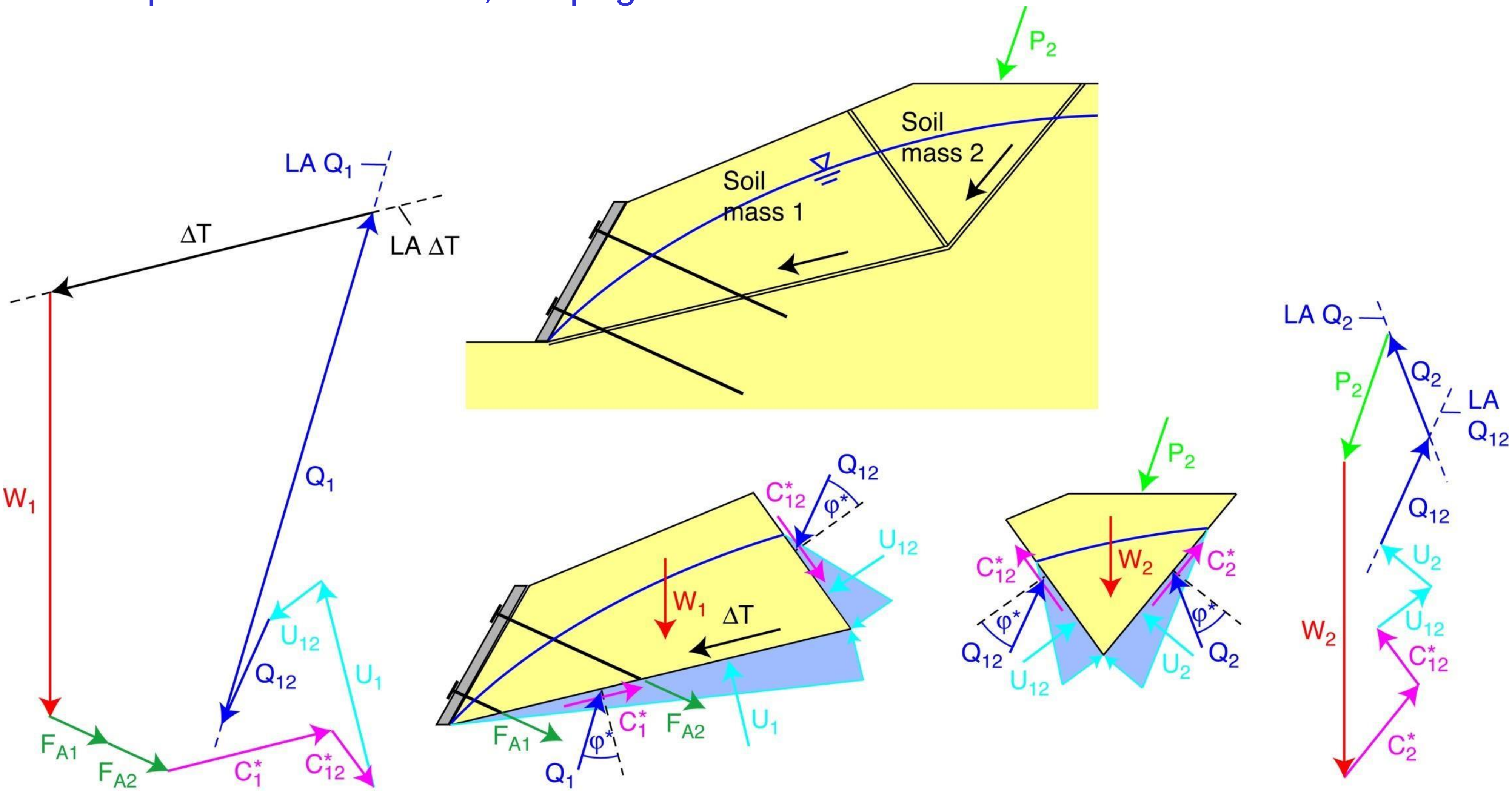
Example 2: Sliding of two soil masses on sloping rock ground



Friction angle φ* in all failure planes

Failure mechanisms with multiple sliding masses

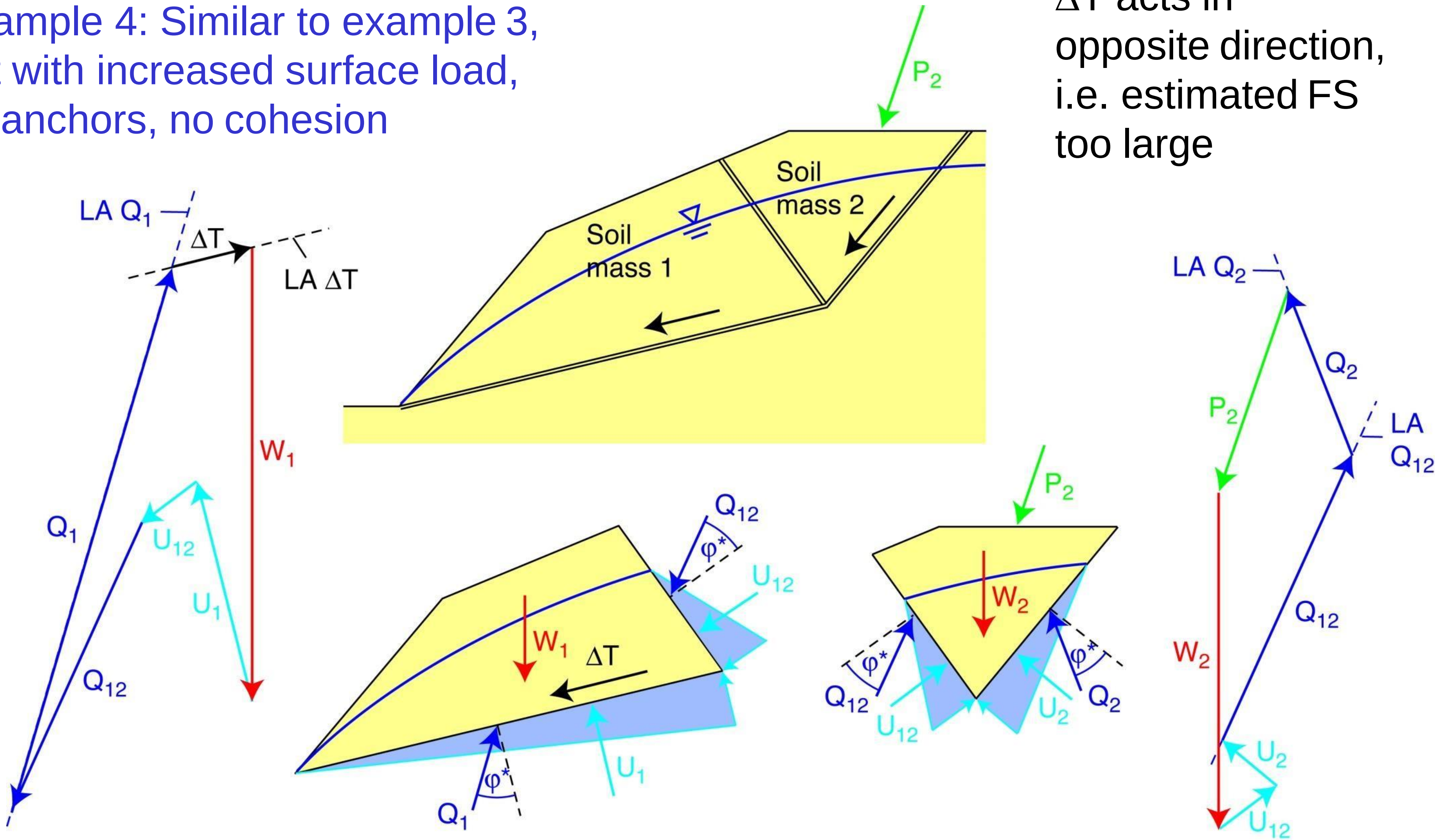
Example 3: Surface load, seepage and anchors



Failure mechanisms with multiple sliding masses

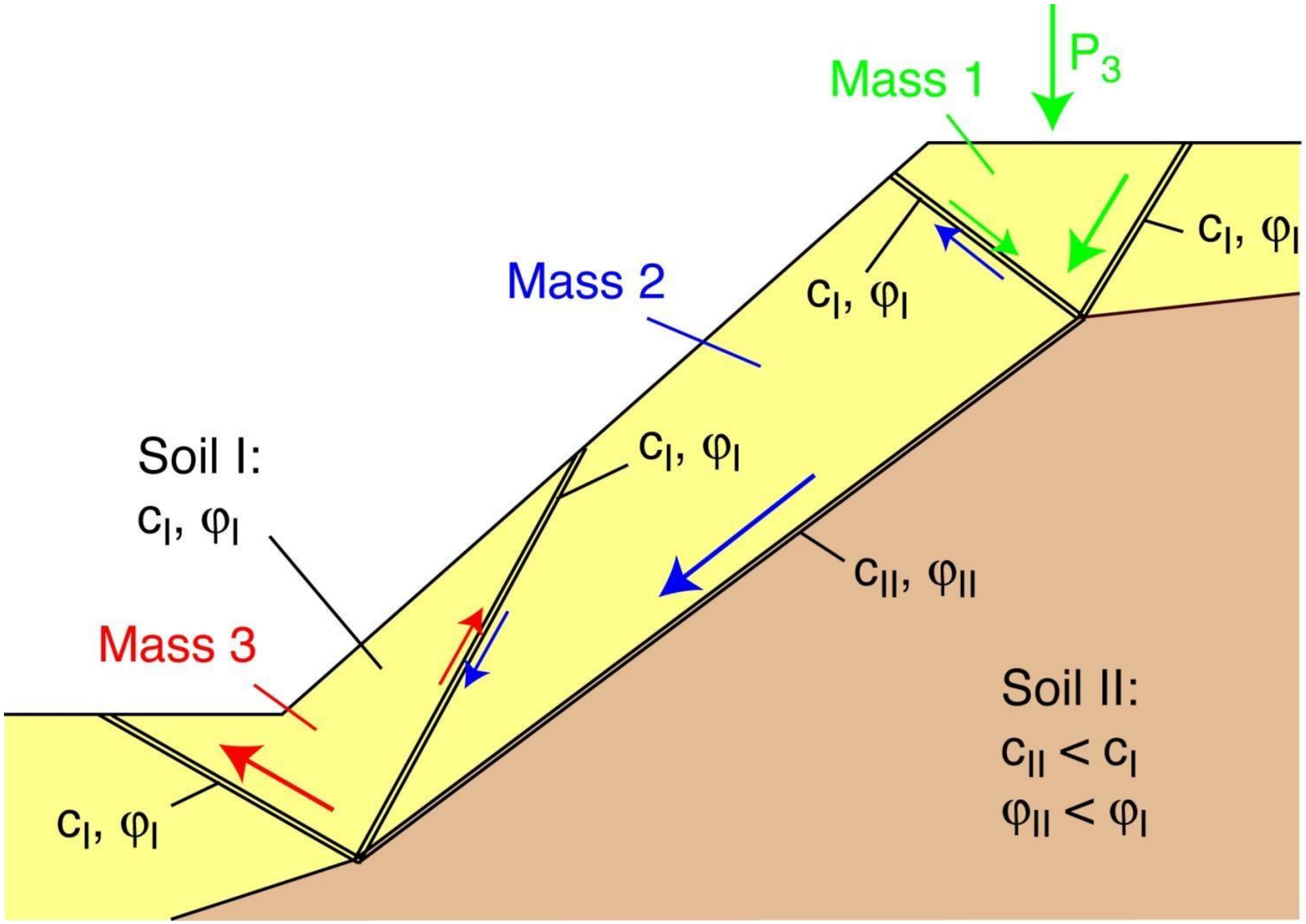
Example 4: Similar to example 3, but with increased surface load, no anchors, no cohesion

ΔT acts in opposite direction, i.e. estimated FS too large

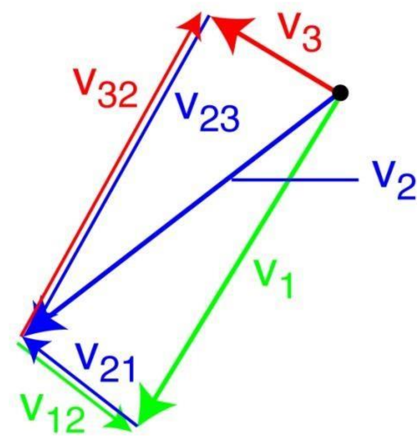


Failure mechanisms with multiple sliding masses

Example 5:
Three soil masses

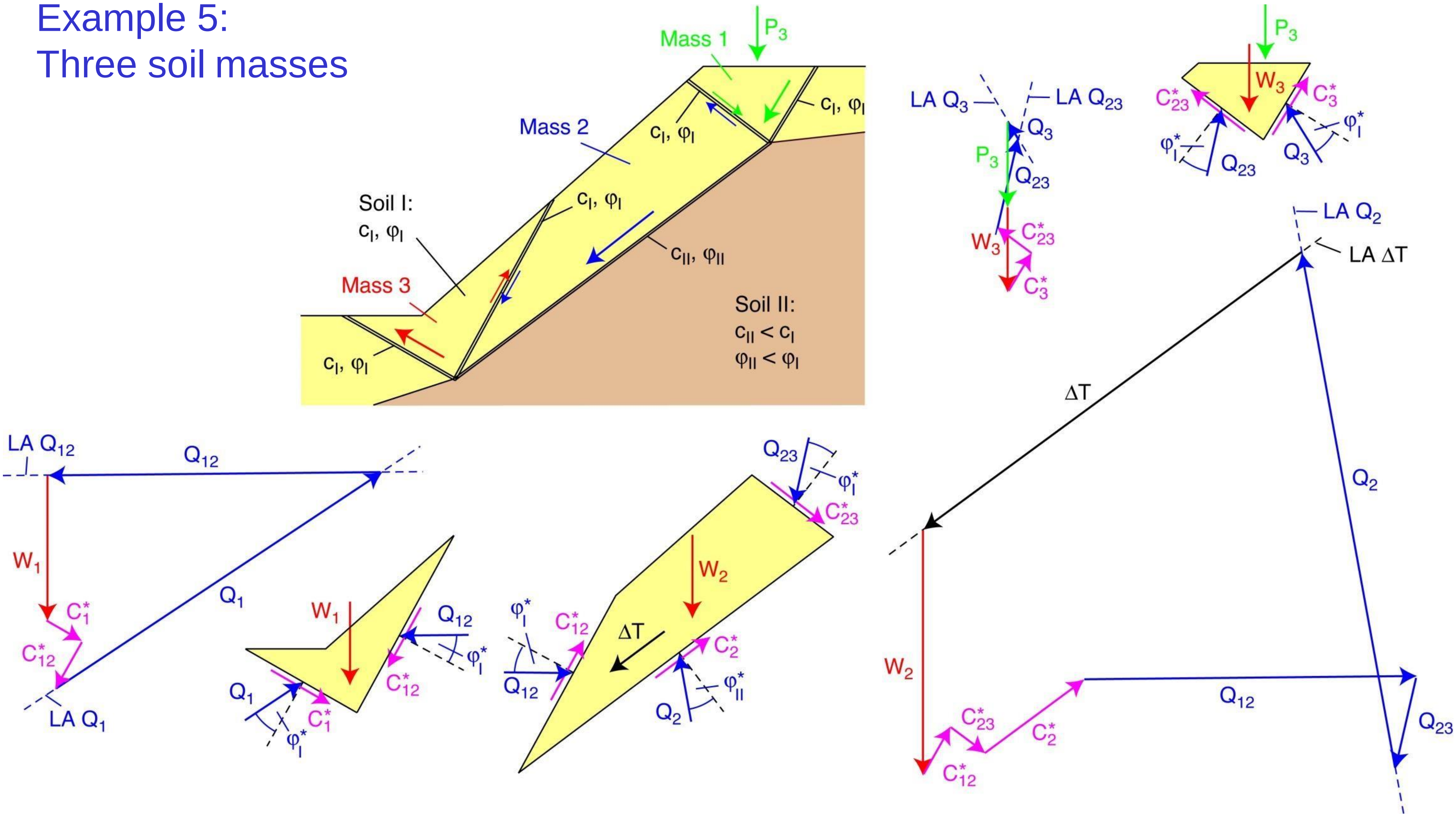


Hodograph:



Failure mechanisms with multiple sliding masses

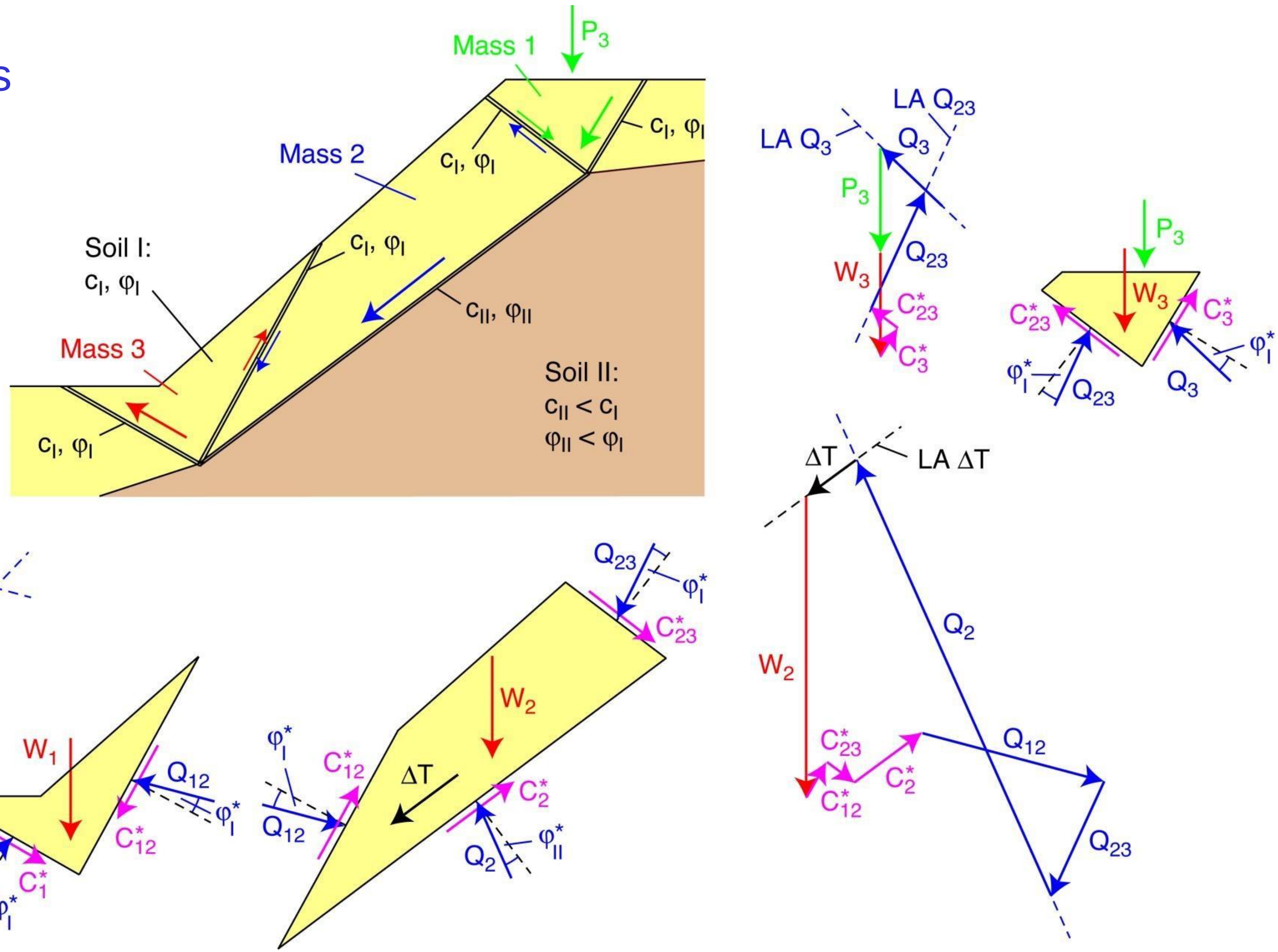
Example 5:
Three soil masses

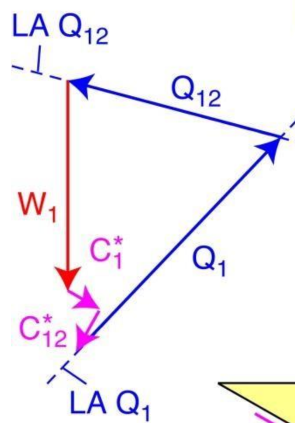


Failure mechanisms with multiple sliding masses

Example 5:
Three soil masses

After reduction
of shear strength
→ smaller ΔT

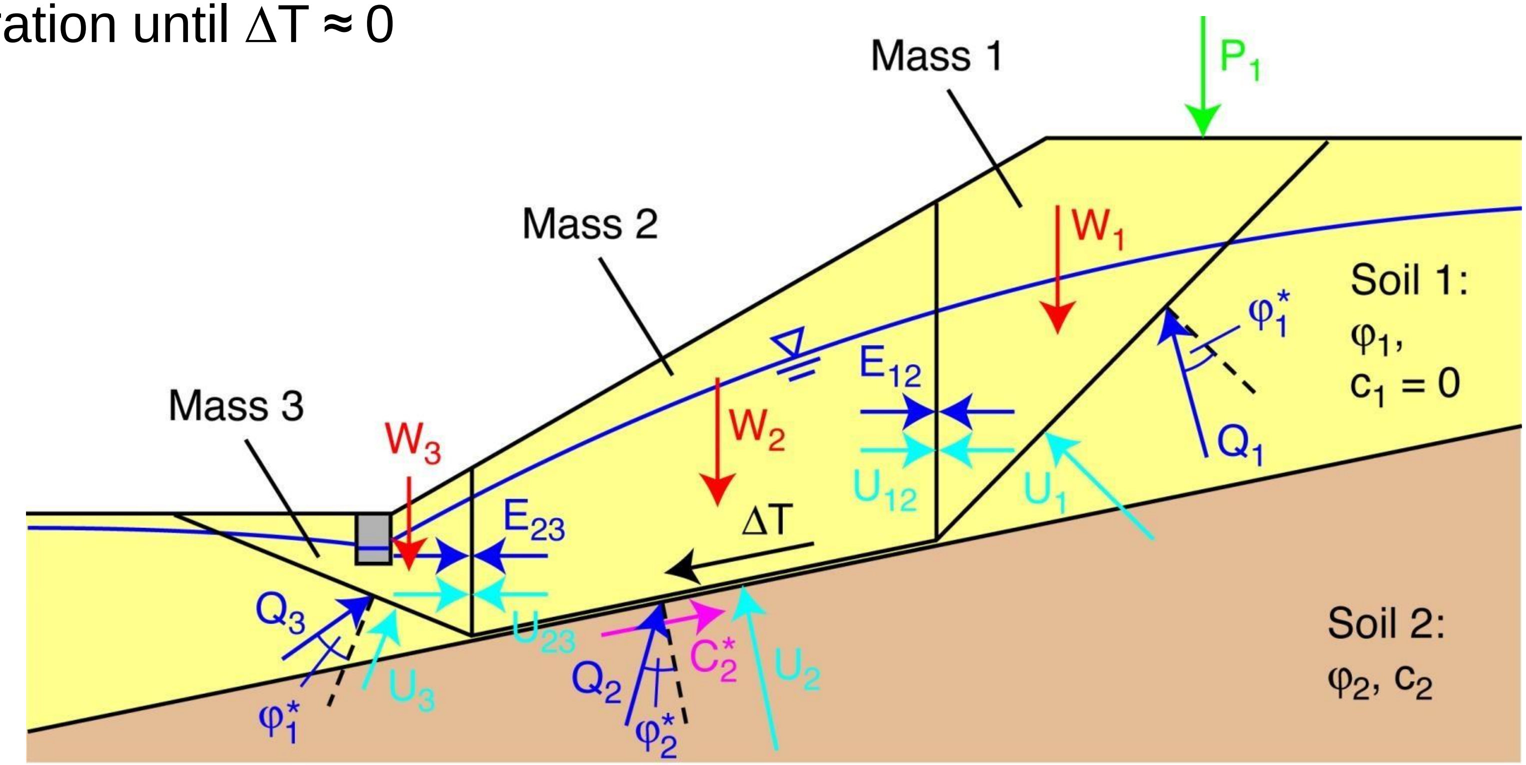




Failure mechanisms with multiple sliding masses

Special case: Block sliding method

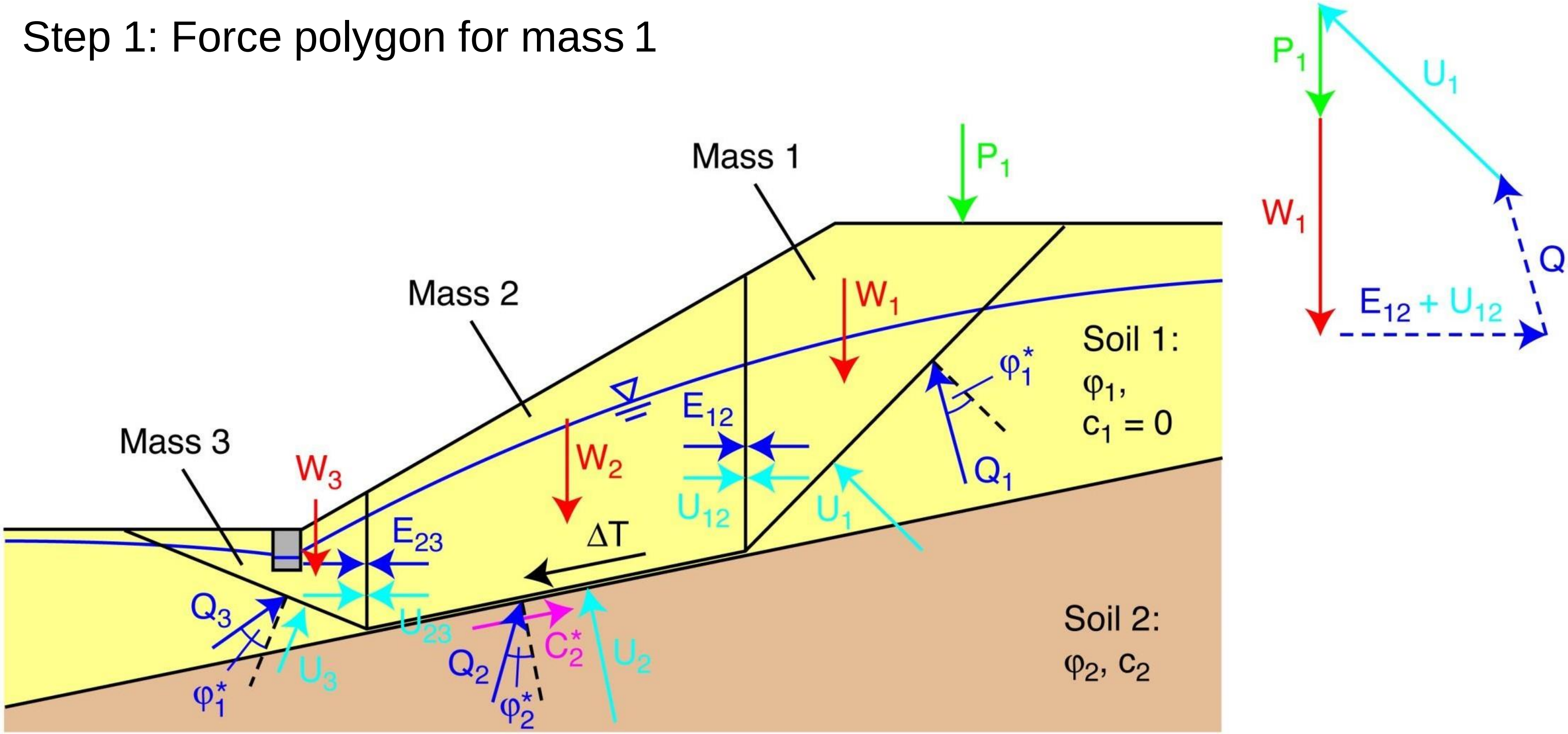
- Vertical inner sliding planes (similar to slices methods), 3 to 5 blocks
- Horizontal earth and water pressure forces between individual masses
- Estimation of safety factor FS, reduction of shear strength parameters with 1/FS
- Iteration until $\Delta T \approx 0$



Failure mechanisms with multiple sliding masses

Special case: Block sliding method

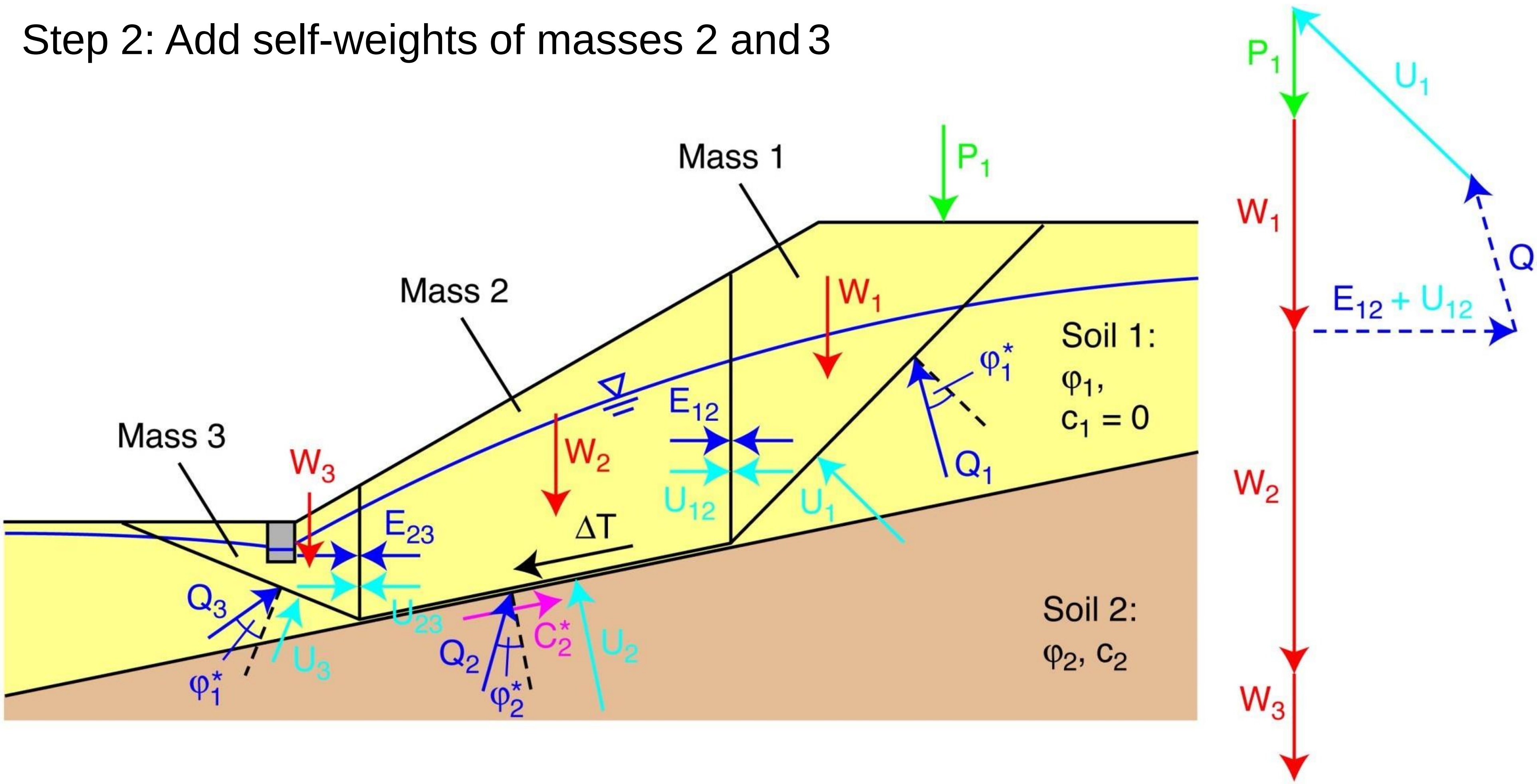
Step 1: Force polygon for mass 1



Failure mechanisms with multiple sliding masses

Special case: Block sliding method

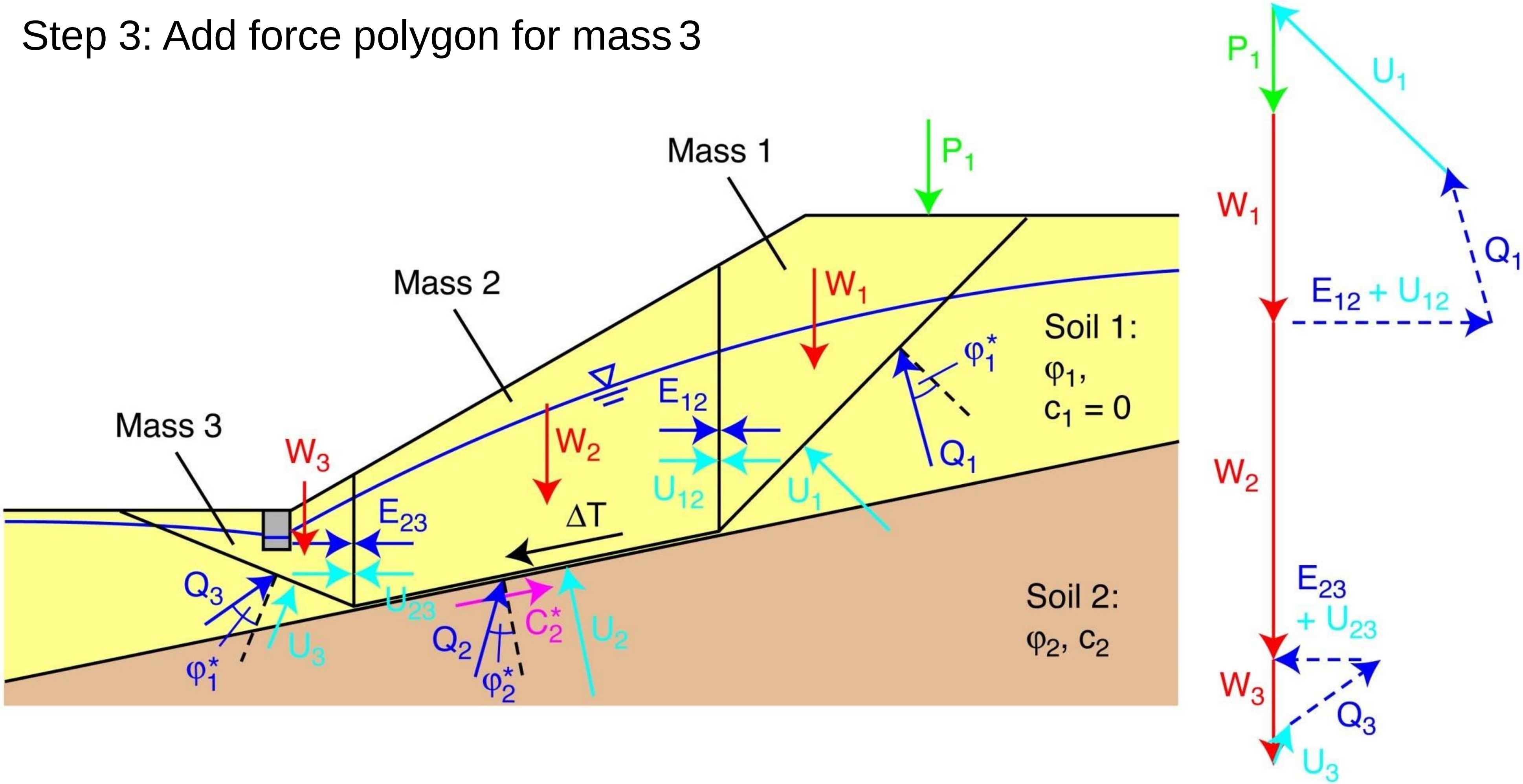
Step 2: Add self-weights of masses 2 and 3



Failure mechanisms with multiple sliding masses

Special case: Block sliding method

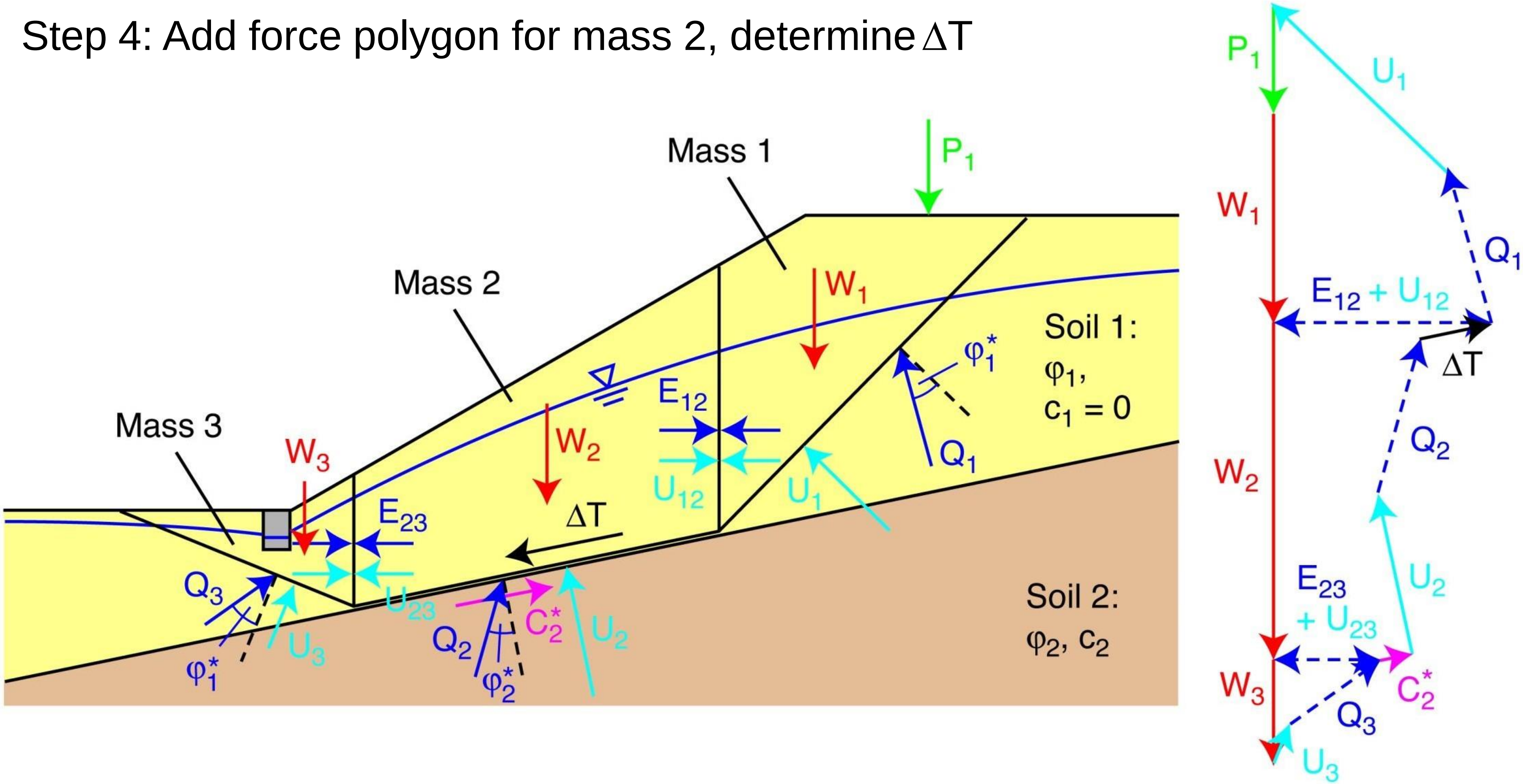
Step 3: Add force polygon for mass 3



Failure mechanisms with multiple sliding masses

Special case: Block sliding method

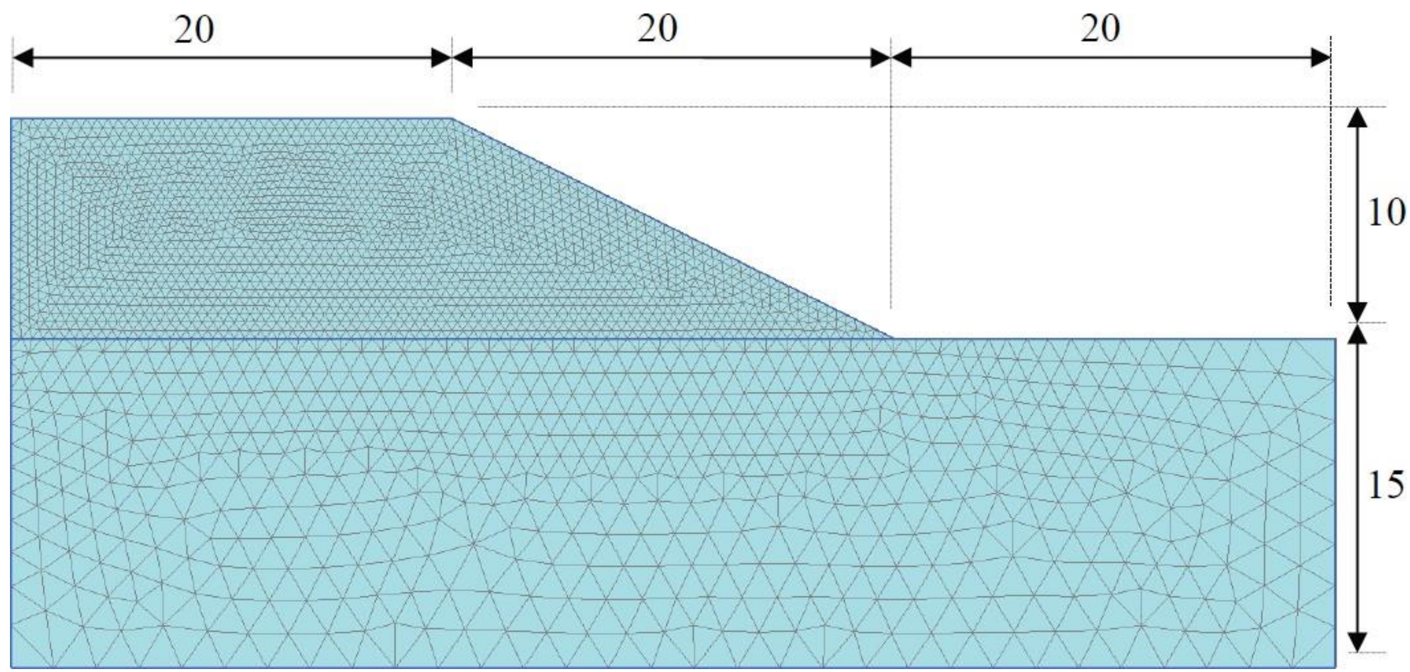
Step 4: Add force polygon for mass 2, determine ΔT



Slope stability analysis with numerical methods

c-φ reduction method

- FE model of a slope
- Mohr-Coulomb constitutive model for the soil with shear strength parameters effective cohesion c' and effective friction angle φ'



Slope stability analysis with numerical methods

c-φ reduction method

- Starting from the real values of c‘ and φ‘ of the soil (determined from laboratory tests) both values are decreased in proportional steps, e.g. c‘ and φ‘ are simultaneously decreased in steps of 0.1 % of the original value, until failure occurs, visible by failure surface with localized strains
- Shear strength parameters at failure: c'_f, φ'_f

- Factor of safety:

$$FS = \frac{c'}{c'_f} \quad \text{or} \quad FS = \frac{\tan\varphi'}{\tan\varphi'_f}$$

Both definitions deliver same value of FS