Chapter 3
Lateral Earth Pressure

AAiT
Addis Ababa Institute of Technology
Addis Ababa University

Geotechnical Engineering Chair
School of Civil & Environmental Engineering

General Outline

- Introduction
- Earth Pressure Theories
- Additional Considerations
- Graphical Methods

1. Introduction

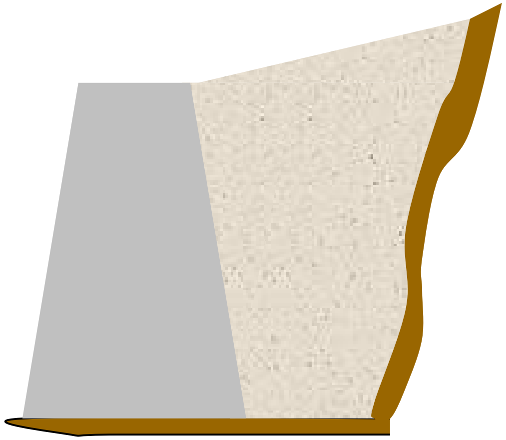
- Definition & Implication
- Basics of Earth Pressure
- Retaining Walls – An Overview
- At-Rest Earth Pressure

Introduction

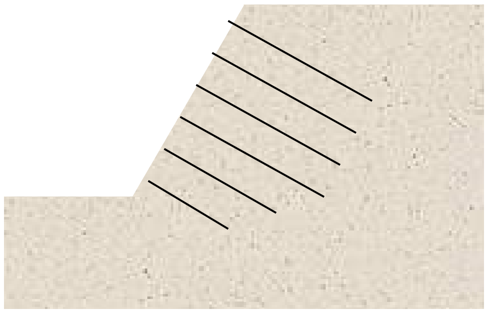
Earth pressure is the common expression for the stress components (normal and shear) that act in the interface between soil and structure.

- Soil is neither liquid nor solid but exhibits the characteristics of both.
- Similar to a liquid, soil exerts a lateral pressure against any object in contact.
- In geotechnical engineering, it is often necessary to prevent lateral soil movements.
- We have to estimate the lateral soil pressures acting on retaining structures, to be able to design them.

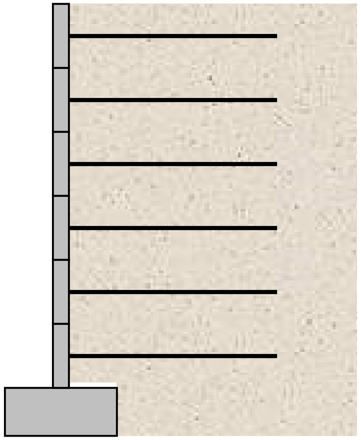
Introduction cntd



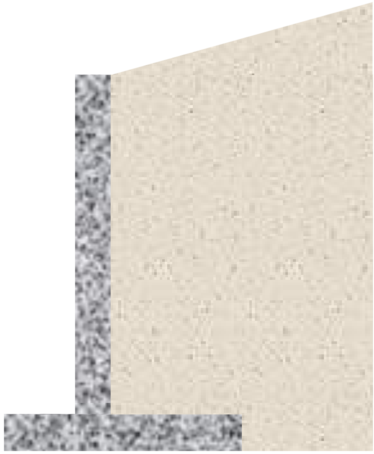
Gravity Retaining wall



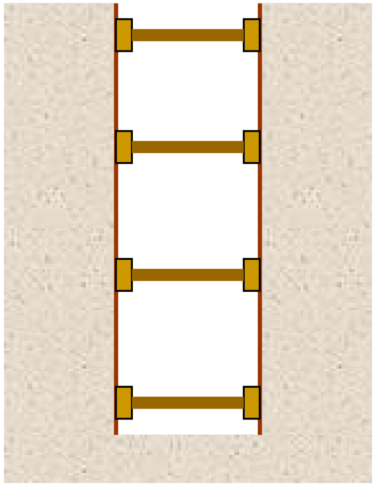
Soil nailing



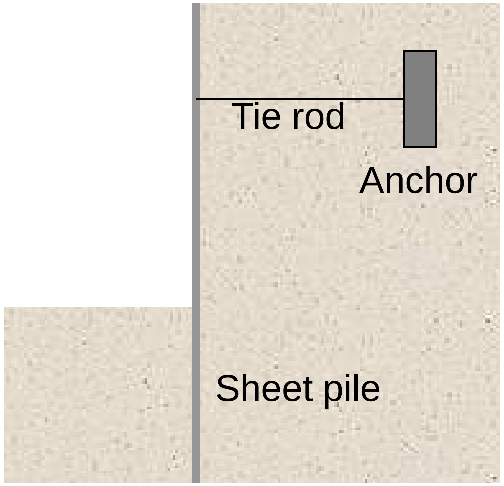
Reinforced earth wall



Cantilever retaining wall



Braced excavation



Anchored sheet pile

Introduction cntd

Introduction cntd

Basics

Vertical pressure at a point below ground surface is normally induced in four ways:

- **Geostatic Pressure/Overburden:** from self weight of soil; increases linearly with depth
- **Hydrostatic Pressure:** from ground water; increases linearly with depth
- **Additional stress**: from an imposed structural load; decreases with depth in a non-linear manner
- **Surcharge load**: a load of infinite extent on the surface; does not vary with depth.

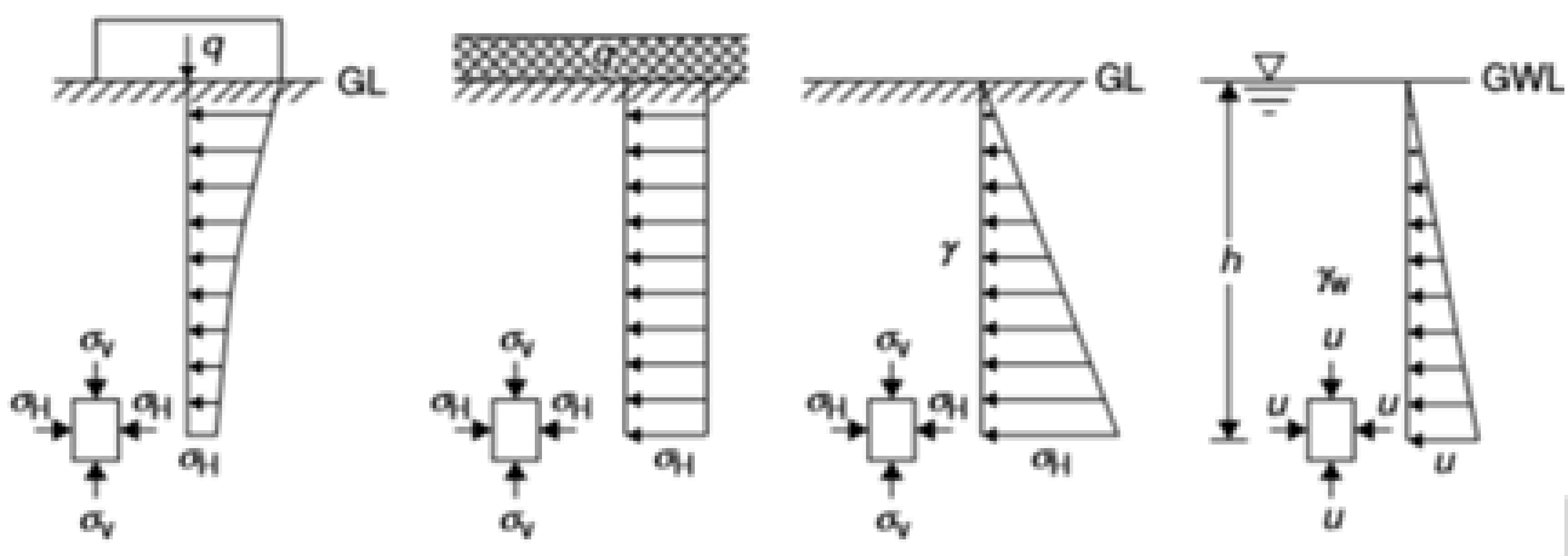
Introduction cntd

Basics

Each of these induces horizontal pressure at the point considered.

The magnitude is governed by constant of proportionality (K) normally referred to as the ‘coefficient of lateral earth pressure.

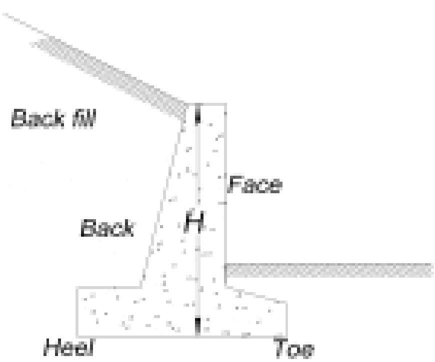
$\sigma_H \propto \sigma_v$ that is $\boxed{\sigma_H = K\sigma_v}$



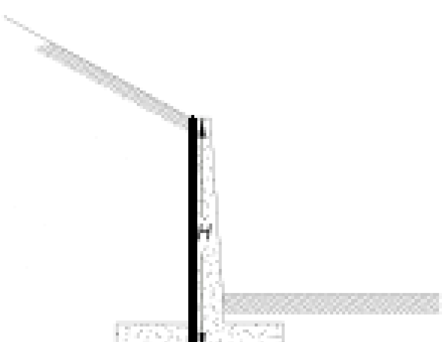
Introduction cntd

Retaining Wall

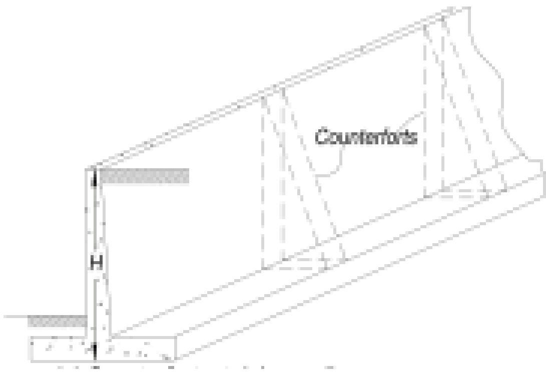
- A structure that is used to support a vertical or near vertical slopes of soil.
- Any structure which retains soil need consideration for lateral pressures (lateral earth pressure) while designing.
- Thus to determine the magnitude of the lateral earth pressure, an engineer must know the strength parameters (cohesion, angle of friction and unit weight) for the soil retained behind the wall.



(a) Gravity wall



(b) Cantilever wall

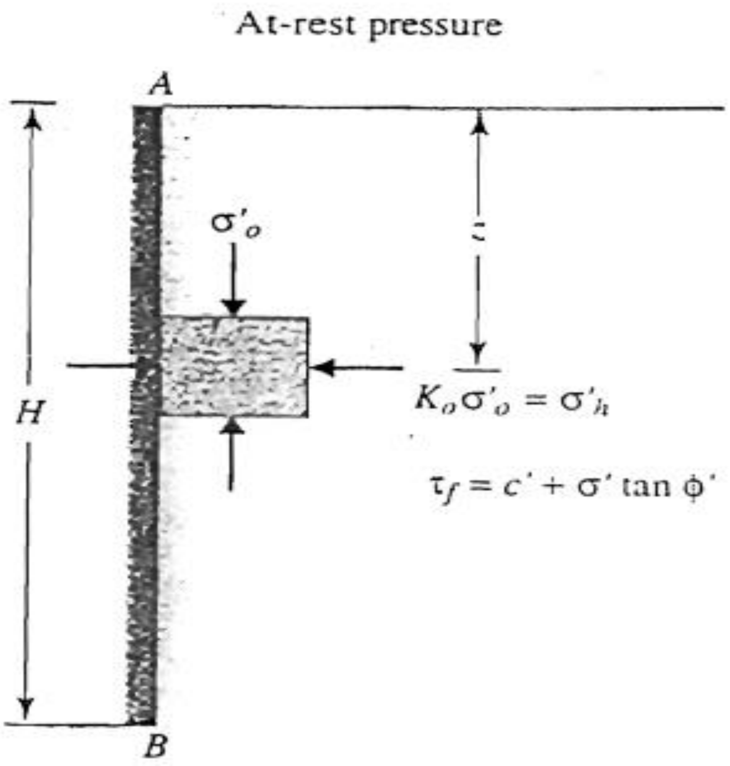


Introduction cntd

- When considering a retaining wall with back fill, three possible cases may arise.

Case1: At-rest Earth Pressure

- if the wall is static, the soil mass will be in a state of static equilibrium. σ‘h is referred to as **at-rest earth pressure**.



$$K = K_o = \frac{\sigma'_h}{\sigma'_o}$$

K_o = at-rest earth pressure coefficient.

Introduction cntd

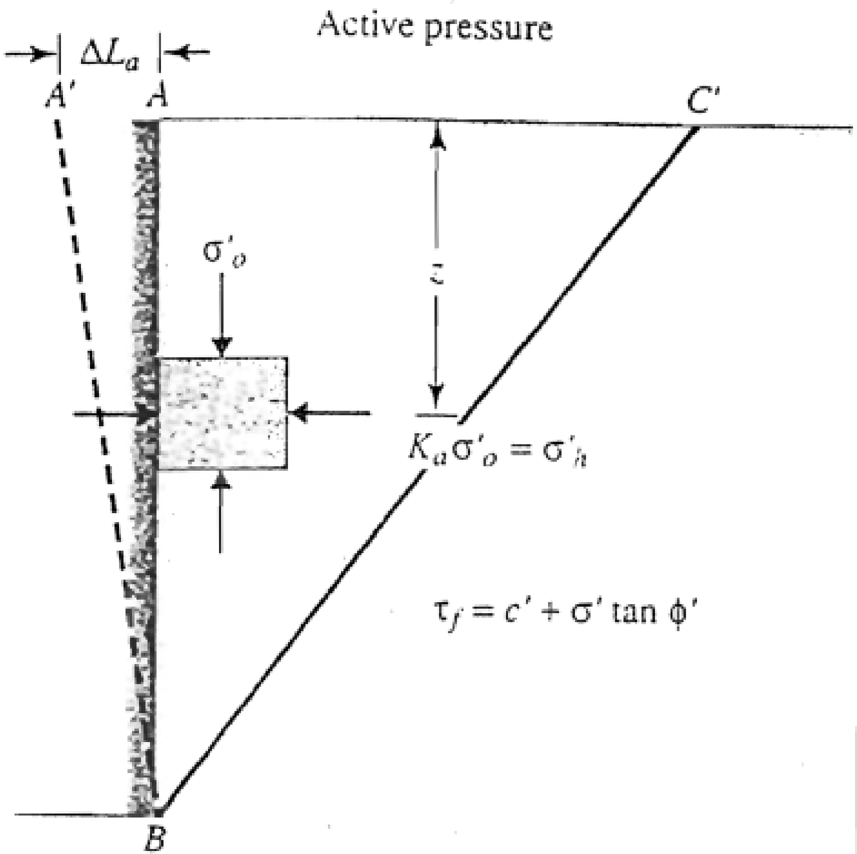
Case2: Active Pressure

- If the wall rotates about its bottom to a position A'B, a triangular soil mass ABC' adjacent to the wall will fail sliding down the plane BC'. The horizontal effective stress will be referred as ***active pressure.***

- Soil exerts a push against the wall b/c of its tendency to slip laterally and seek its natural slope (the soil is the actuating element) Pa > Pp

$$K = K_a = \frac{\sigma'_h}{\sigma'_o} = \frac{\sigma'_a}{\sigma'_o}$$

K_a = active earth pressure coefficient.

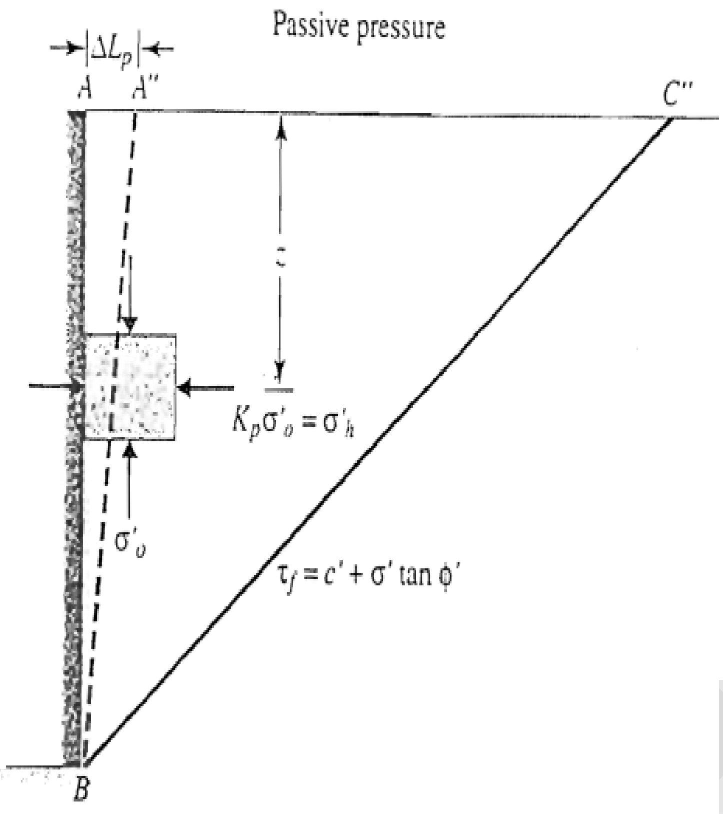


Introduction cntd

Case3: Passive Pressure

- If the wall rotates about its bottom to a position of A’’B, a triangular soil mass ABC’’ will reach a state of plastic equilibrium and will fail sliding upward along the plane BC’’. The horizontal effective stress will be referred as ***passive pressure***.

- Retaining wall is caused to move toward the soil. Soil provides the resistance for maintaining stability (the retaining wall is the actuating element) Pa < Pp



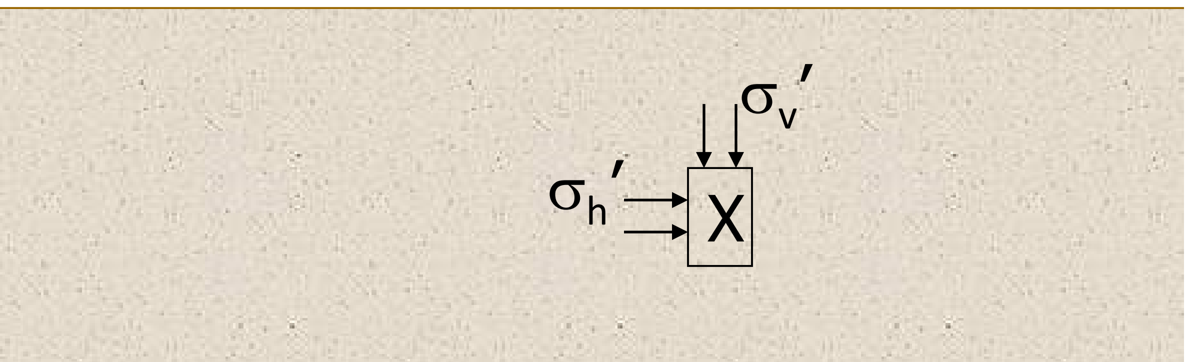
$$K = K_p = \frac{\sigma'_h}{\sigma'_o} = \frac{\sigma'_p}{\sigma'_o}$$

K_p = passive earth pressure coefficient

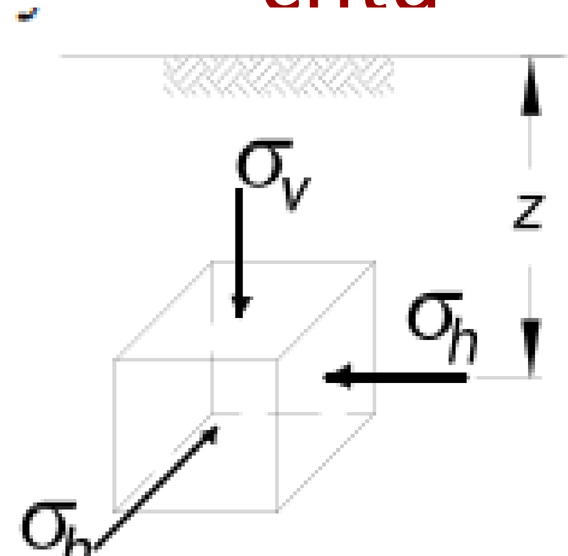
Introduction cntd

At-rest Earth Pressure

In a homogeneous natural soil deposit,



GL



(a) Stresses on element of soil at depth z

the ratio σ_h'/σ_v' is a constant known

as coefficient of earth pressure at rest (K_0).

Importantly, at K_0 state, there are no lateral strains. The soil deforms vertically under its self weight but is prevented to deform laterally.

Introduction cntd

At-rest Earth Pressure

To determine the earth pressure at rest we assume that the principal stresses act on an element of soil at a depth z which is semi-infinite, elastic, homogeneous and isotropic.

$$,\varepsilon_h = \frac{\sigma_h}{E} - \nu\left(\frac{\sigma_v}{E} + \frac{\sigma_h}{E}\right) = 0 \qquad \frac{\sigma_h}{\sigma_v} = \frac{\nu}{1-\nu} = K_o$$

For normally consolidated clays & granular soils, $K_0 = 1 - \sin\phi'$

For overconsolidated clays,

$$K_{0,\text{overconsolidated}} = K_{0,\text{normally consolidated}}\ OCR^{0.5}$$

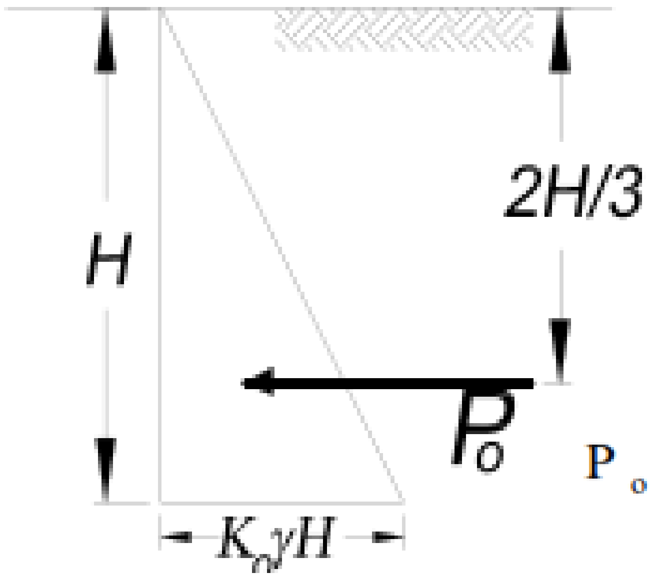
From elastic analysis

$$K_0 = \frac{\upsilon}{1-\upsilon}$$

Poisson's ratio

Introduction cntd

- The distribution of the earth pressure at rest with depth is linear for constant properties of E, v, and ϒ



$K_o = (1-\sin\phi')$	[Jaky, 1944]
$K_o = 0.9\,(1-\sin\phi')$	[Fraser, 1957]
$K_o = 0.19 + 0.233\log I_p$	[Kenney, 1959]
$K_o = [1+(2/3)\sin\phi']\,[(1-\sin\phi)/(1+\sin\phi)]$	[Kezdi,1962]
$K_o = (0.95-\sin\phi')$	[Booker and Ireland, 1965]

$$P_o = \int_0^H \sigma_h \, dz$$

$$= \int_0^H K_o . \gamma z . dz$$

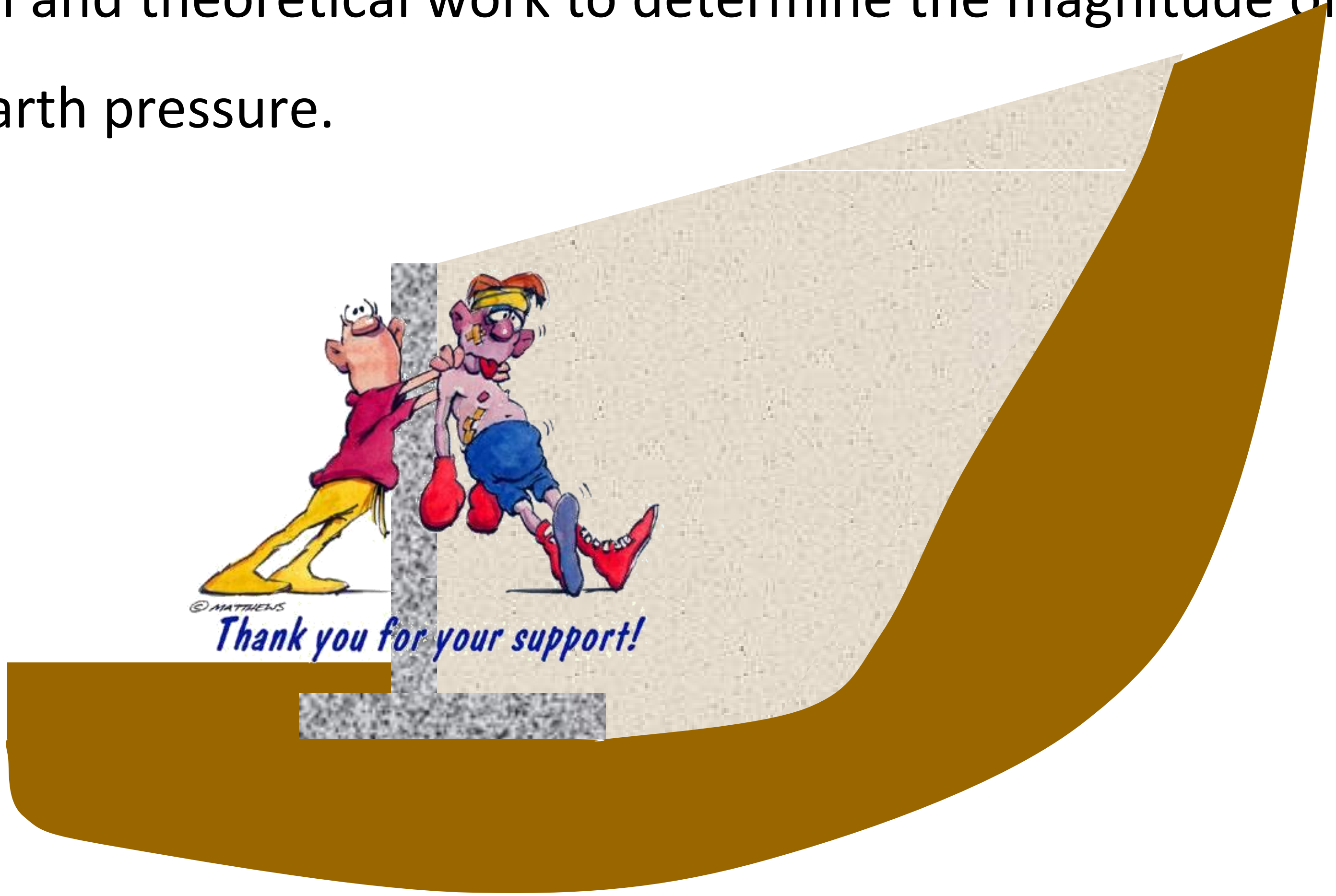
$$= \frac{1}{2} K_o . \gamma . H^2$$

2. Earth Pressure Theories

- Rankine's Theory
  - For cohesionless soil
  - For c- ϕ soil
- Coulomb's wedge theory

Earth Pressure Theories

Many earth pressure theories has been proposed after a lot of experimental and theoretical work to determine the magnitude of the lateral earth pressure.



Earth Pressure Theories

Rankine's Theory

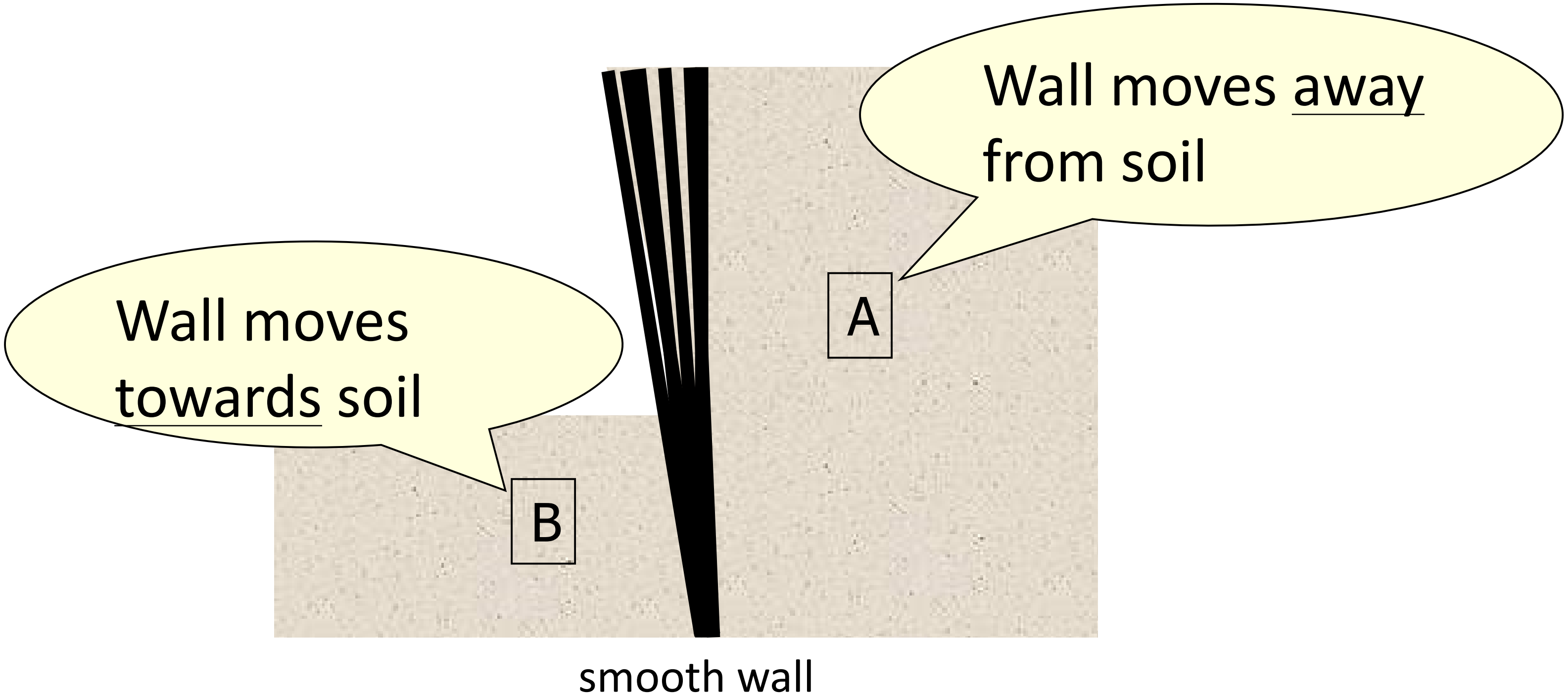
- Proposed by Rankine (1857) for homogeneous ϕ-soil having horizontal surface.
- The basis of the proposal was that the soil changes from a state of elastic equilibrium into a plastic one, when the entire mass is on the point of imminent shear failure.

Assumptions

- Soil mass is semi-infinite, homogeneous, dry, cohessionless.
- The face of the wall in contact with the backfill is vertical and smooth.

Earth Pressure Theories

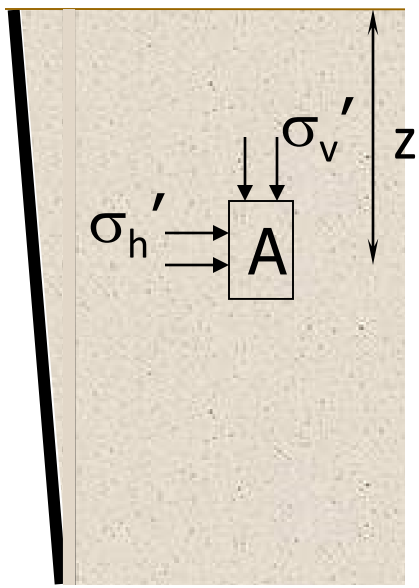
Rankine's Theory



Let's look at the soil elements A and B during the wall movement.

Earth Pressure Theories

Rankine's Theory - Active Pressure



$\sigma_v' = \gamma z$

Initially, there is no lateral movement.

$\therefore \sigma_h' = K_0 \sigma_v' = K_0 \gamma z$

As the wall moves away from the soil,

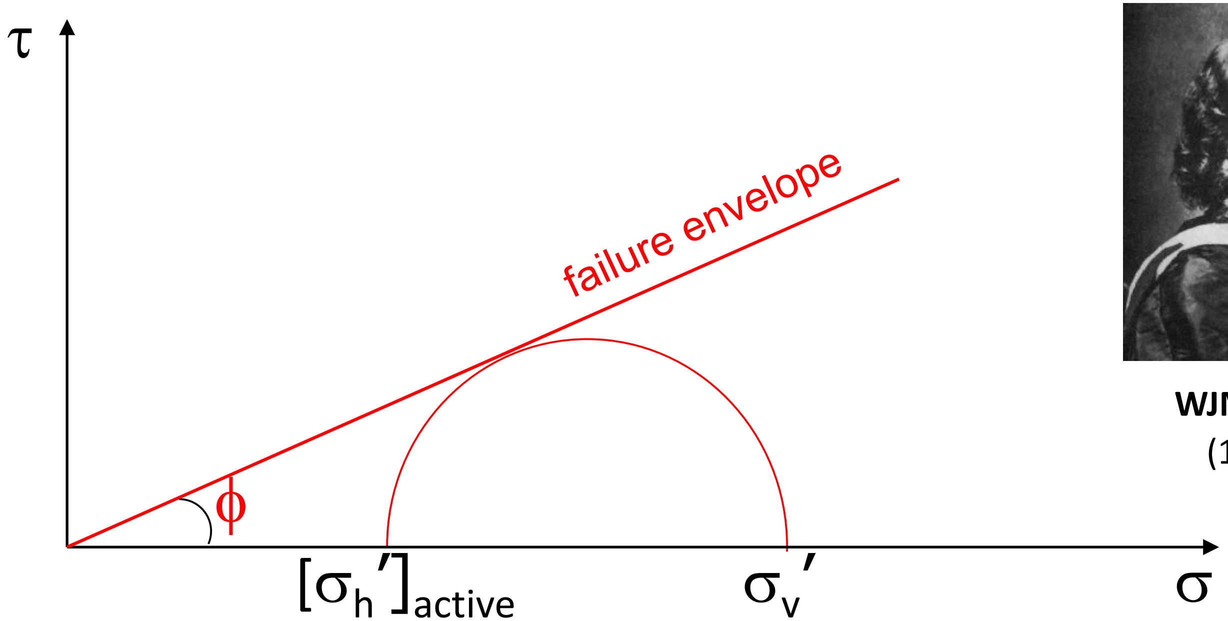
σ_v' remains the same; and

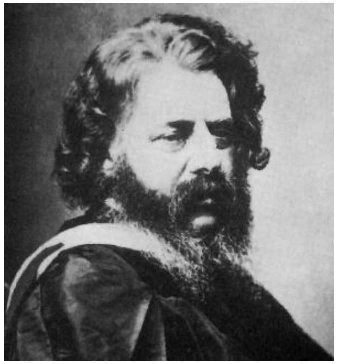
σ_h' decreases till failure occurs.

Active state

Earth Pressure Theories

Rankine's Theory - Active Pressure





WJM Rankine
(1820-1872)

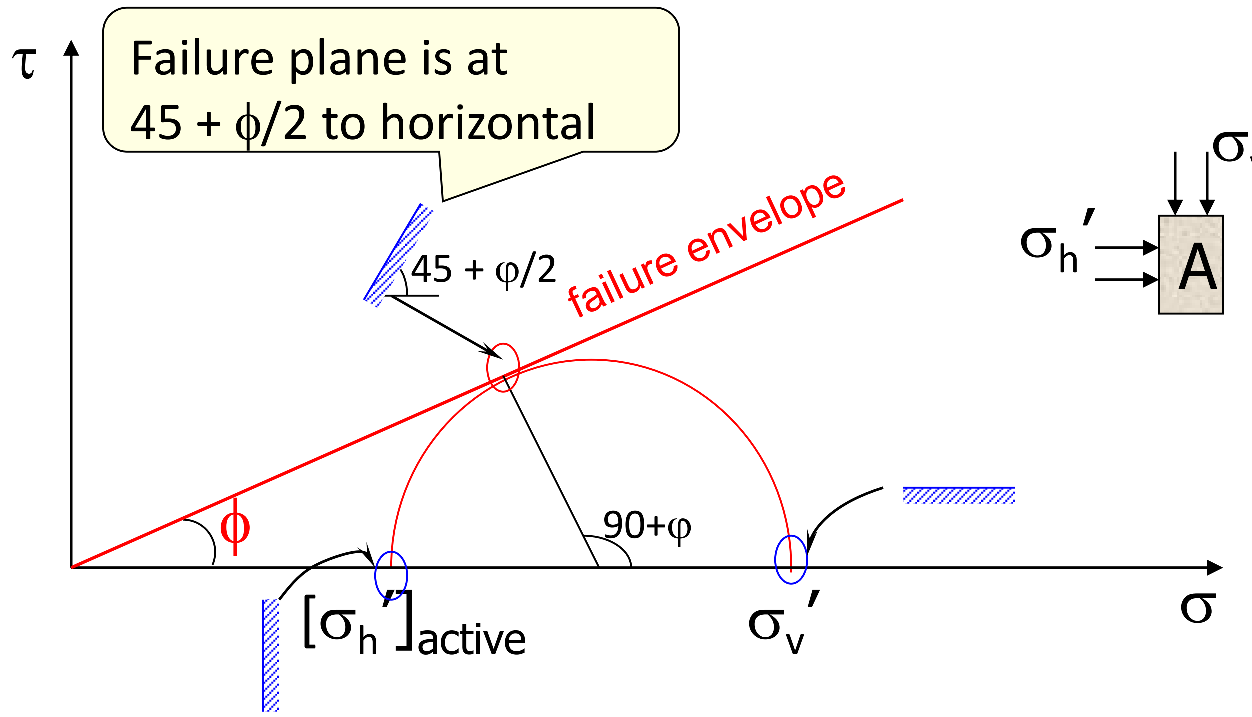
$$[\sigma_h']_{active} = K_A \sigma_v'$$

Rankine's coefficient of active earth pressure

$$K_A = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2(45 - \phi/2)$$

Earth Pressure Theories

Rankine's Theory - Active Pressure

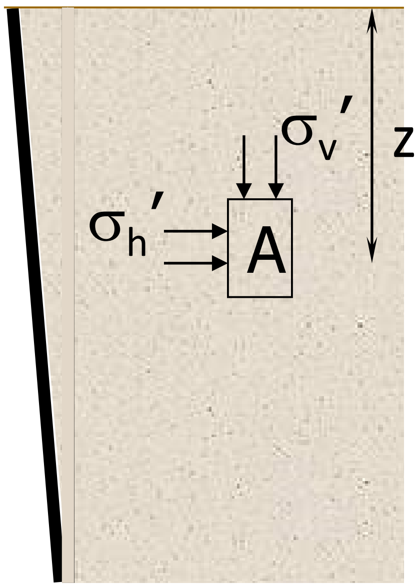


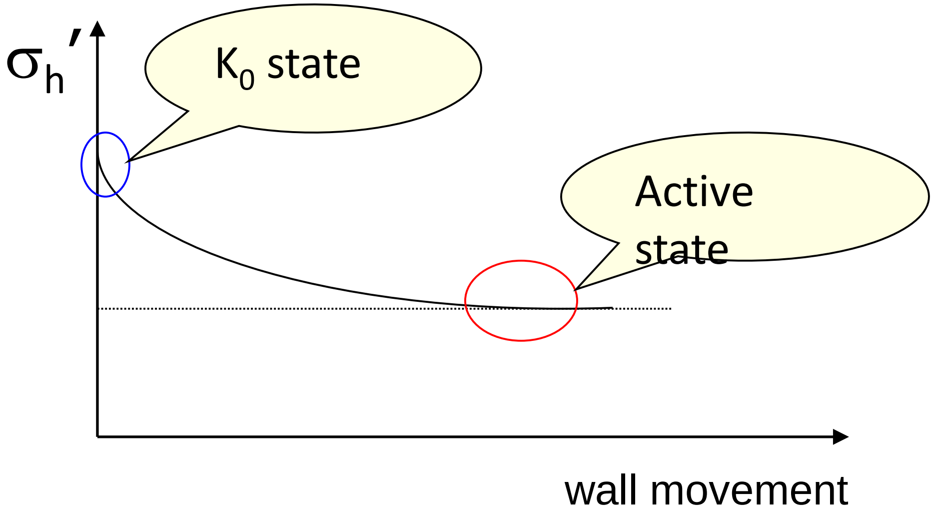
Earth Pressure Theories

Rankine's Theory - Active Pressure

As the wall moves away from the soil,

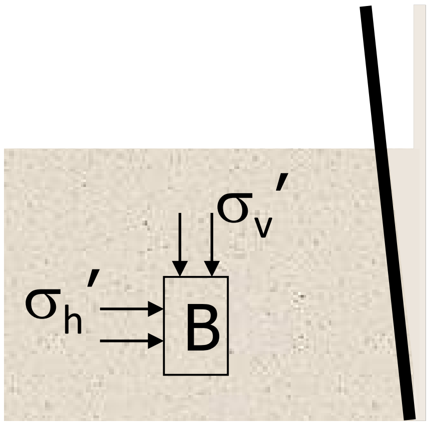
σ_h' decreases till failure occurs.





Earth Pressure Theories

Rankine's Theory - Passive Pressure



Initially, soil is in K_0 state.

As the wall moves towards the soil,

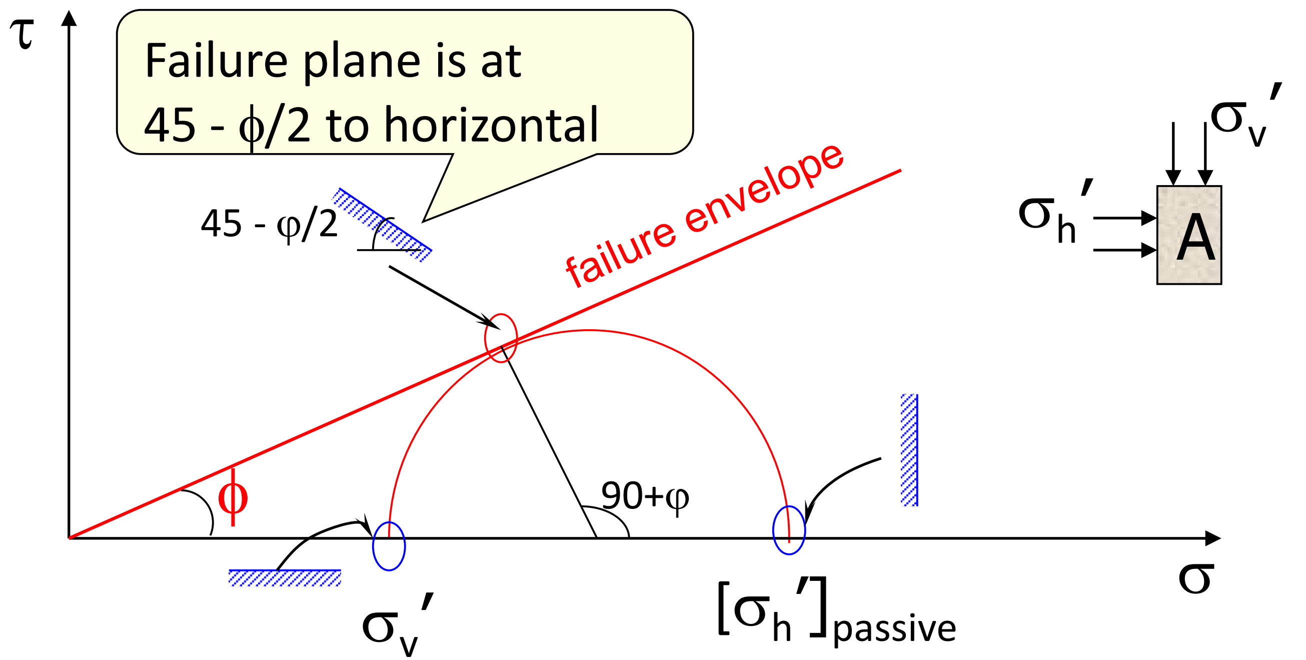
σ_v' remains the same, and

σ_h' increases till failure occurs.

Passive state

Earth Pressure Theories

Rankine's Theory - Passive Pressure

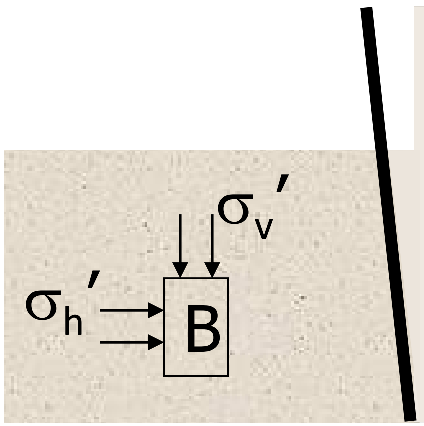


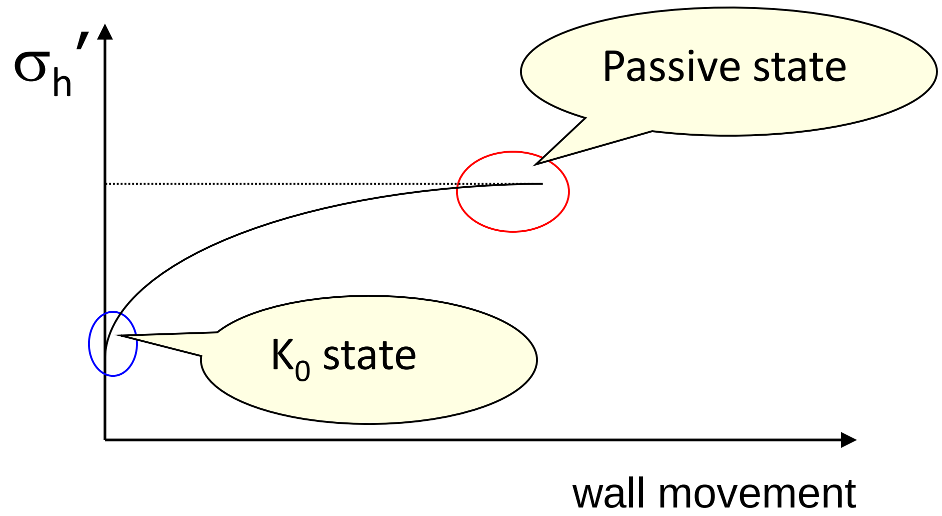
Earth Pressure Theories

Rankine's Theory - Passive Pressure

As the wall moves towards the soil,

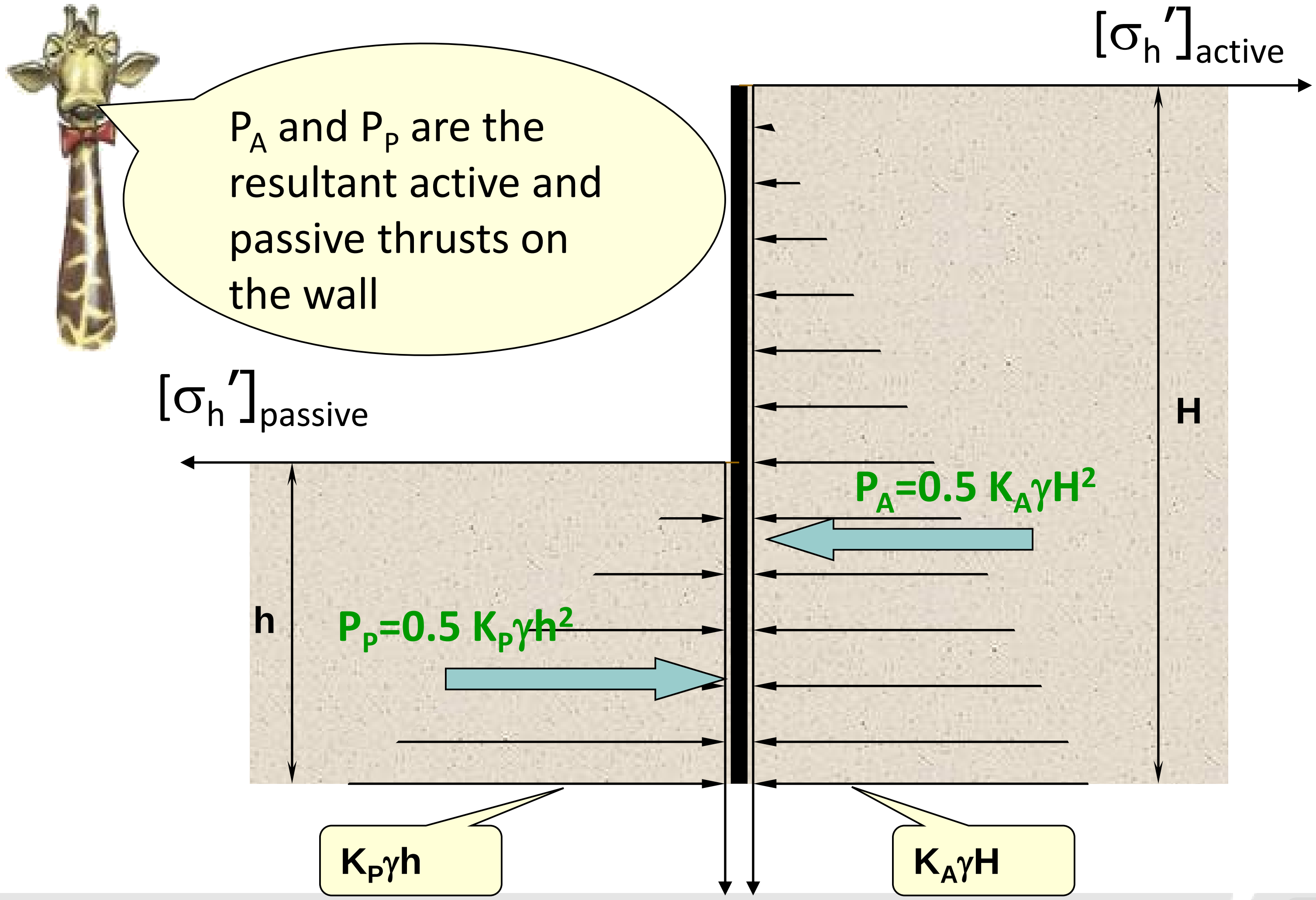
σ_h' increases till failure occurs.





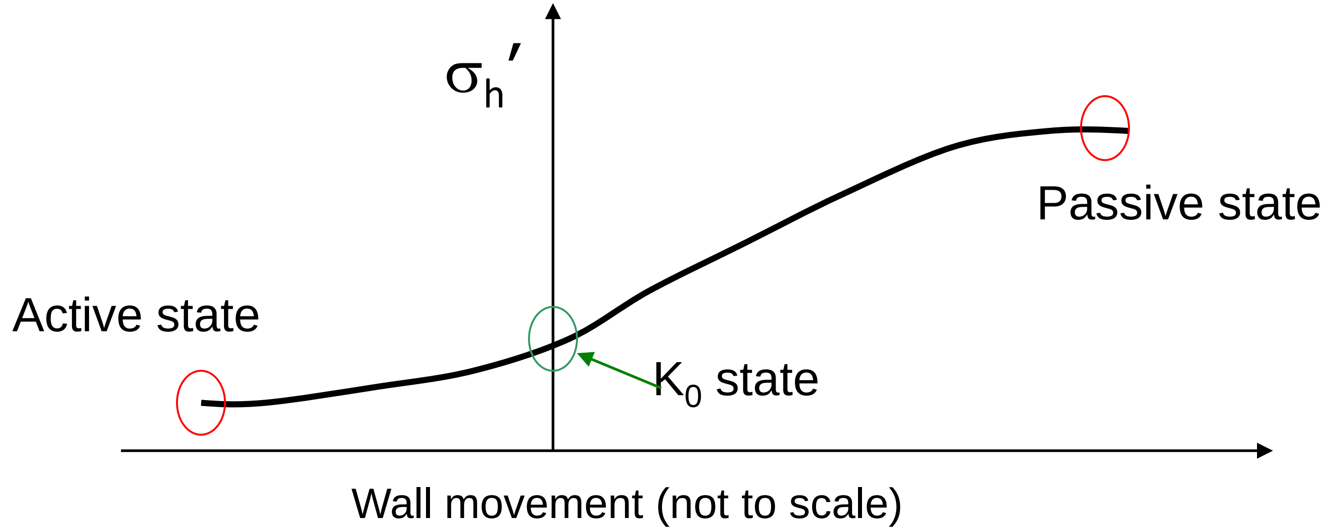
Earth Pressure Theories

Rankine's Theory



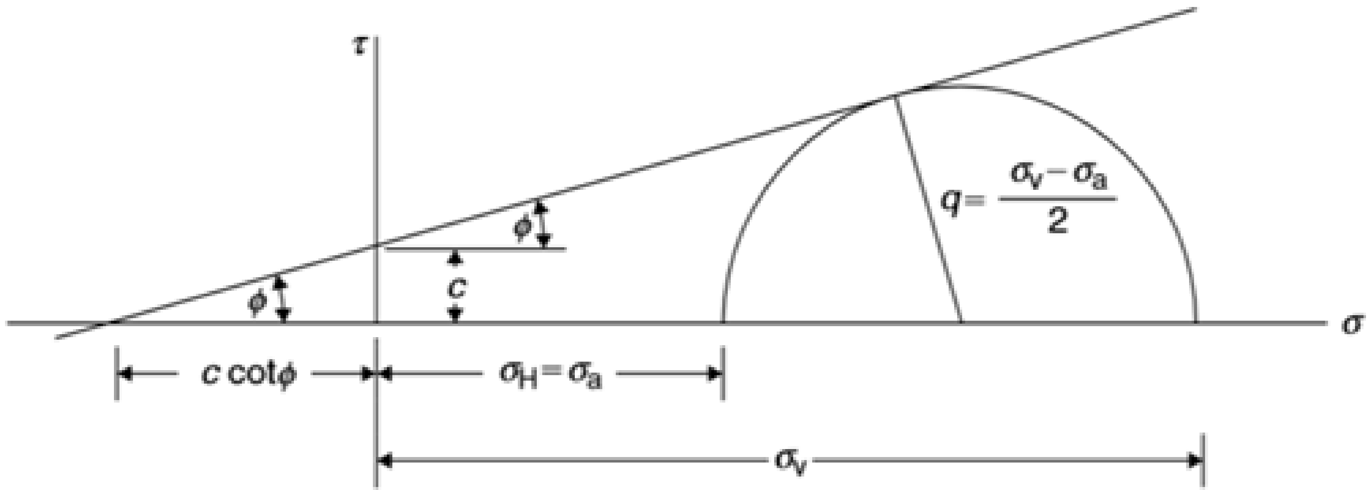
Earth Pressure Theories

Rankine's Theory



Earth Pressure Theories

Rankine-Bell Theory for c-ϕ soil



$$\sin\phi = \frac{\frac{1}{2}(\sigma_v - \sigma_a)}{c \times \cot\phi + \sigma_a + \frac{1}{2}(\sigma_v - \sigma_a)}$$

$$\sigma_a = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)\sigma_v - \frac{2c - \cos\phi}{1+\sin\phi}$$

$$\sigma_a = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)\sigma_v - 2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

$$\sigma_a = K_a\sigma_v - 2c\sqrt{K_a}$$

$$\sigma_p = K_p\sigma_v + 2c\sqrt{K_p}$$

Earth Pressure Theories

Rankine's Theory - Summary

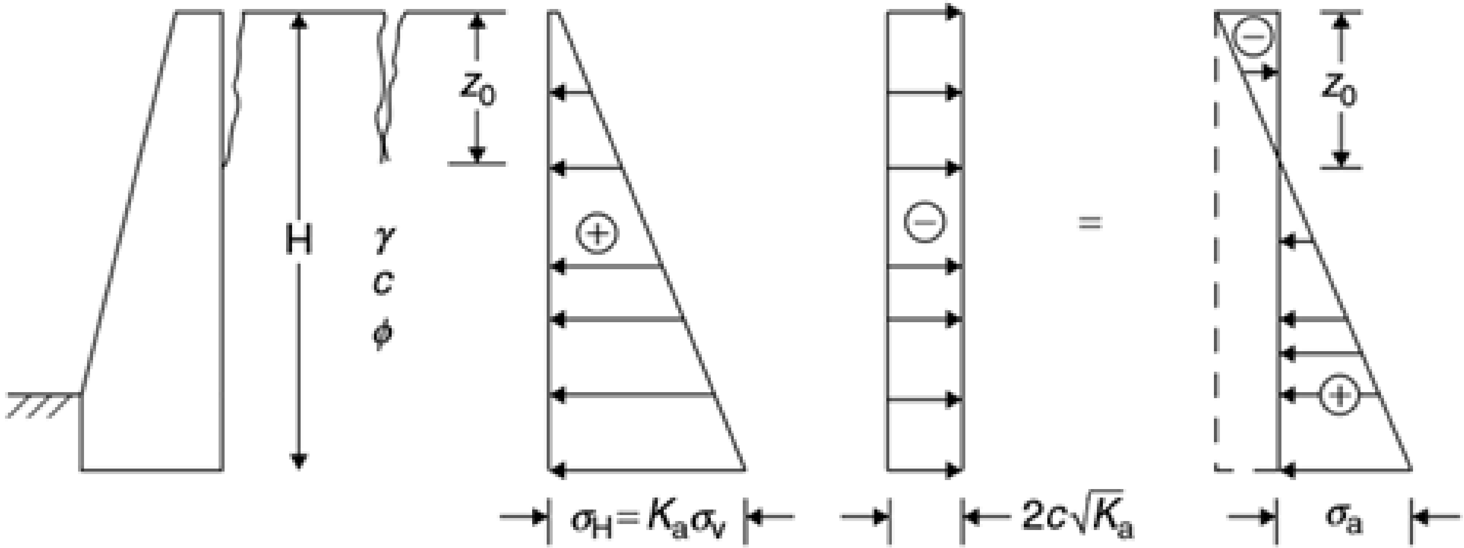
$$[\sigma_h']_{active} = K_A \sigma_v' - 2c\sqrt{K_A}$$

$$[\sigma_h']_{passive} = K_P \sigma_v' + 2c\sqrt{K_P}$$

- Assumes smooth wall
- Applicable only on vertical walls

Earth Pressure Theories

Tension Cracks



When $\sigma_a = 0$ then $K_a\sigma_v - 2c\sqrt{K_a} = 0$

But $\sigma_v = z_0\gamma$

$$K_a z_0 \gamma = 2c\sqrt{K_a}$$

$$z_0 = \frac{2c}{\gamma}\frac{\sqrt{K_a}}{K_a}$$

$$= \frac{2c}{\gamma}\sqrt{\frac{K_a}{K_a^2}}$$

$$z_0 = \frac{2c}{\gamma\sqrt{K_a}}$$

For pure clay: $\phi = 0$ and $K_a = 1$

$$z_0 = \frac{2c}{\gamma}$$

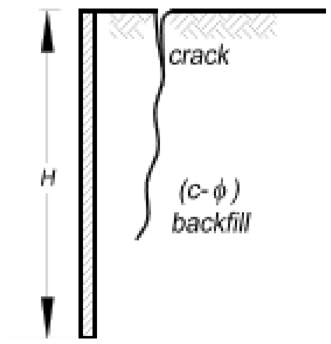
Earth Pressure Theories

Unsupported Cuts in C-φ Soils

Unsupported excavation would theoretically be possible in c-ø soils if the lateral pressure (σ3 active case) would not exceed the strength of the soil.

$$\sigma_3 = \gamma h K_a - 2c\sqrt{K_a} \qquad h=0 \qquad \sigma_3 = -2c\sqrt{K_a}$$

The negative sign implies that the formation of crack.

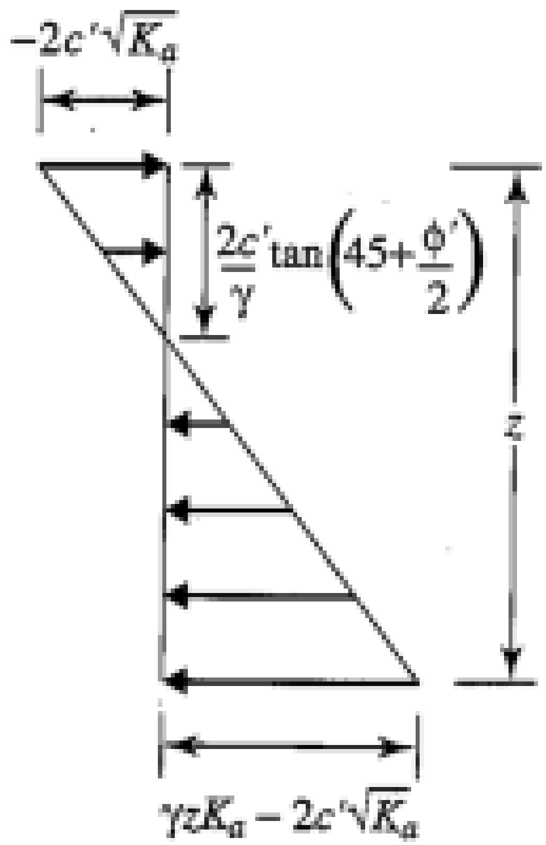


Earth Pressure Theories

Unsupported Cuts in C-φ Soils

To calculate the theoretical depth of crack, ht, notice that σ3 is 0 at the bottom of the crack.

The theoretical maximum depth Hc of unsupported excavation can be calculated as the point where the tension forces equal the cohesive strength.



$$0 = \gamma h K_a - 2c\sqrt{K_a}$$

$$\Rightarrow h_t = \frac{2c}{\gamma\sqrt{K_a}}$$

$$H_c = \frac{4c}{\gamma\sqrt{K_a}} = 2h_t$$

Earth Pressure Theories

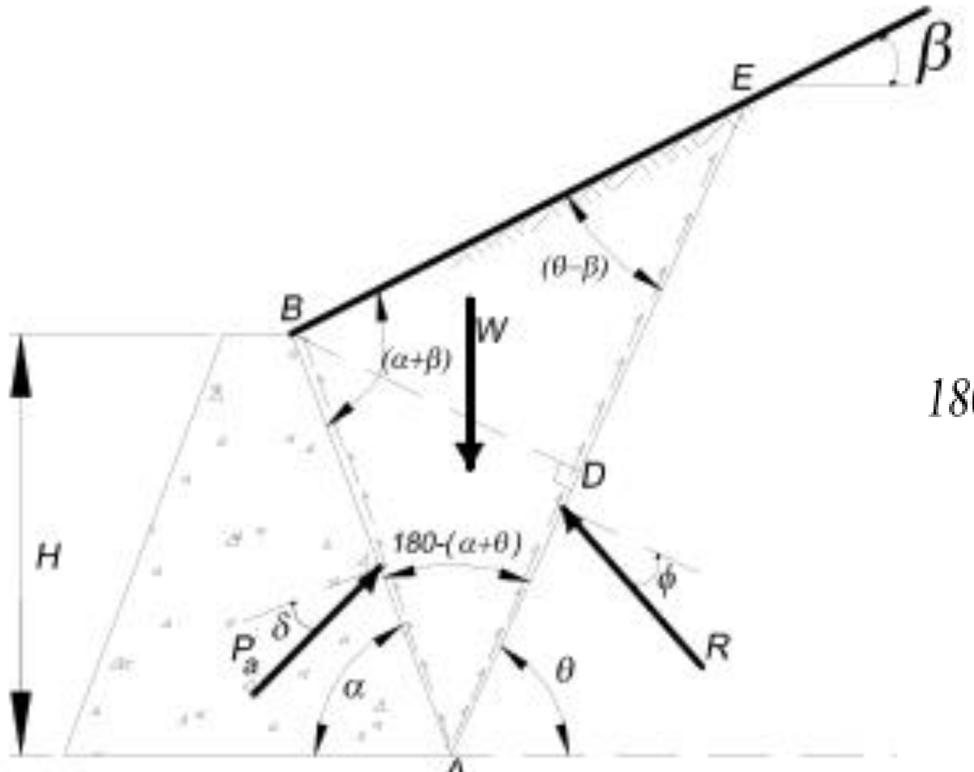
Coulomb's Theory

Assumption

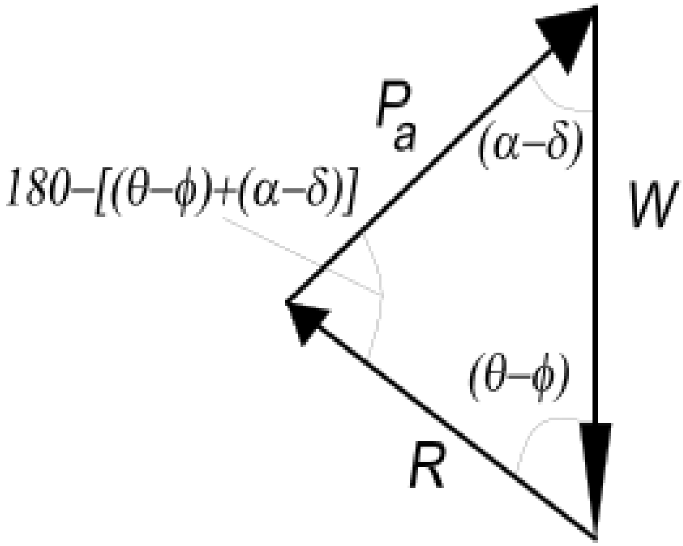
- The soil is isotropic and homogeneous and posses both cohesion and internal friction
- The failure surface is plane, the backfill surface is planner
- Frictional forces are distributed uniformly along the plane failure surface
- The failure wedge is a rigid body undergoing translation
- There is a wall friction which develops as the sliding wedge moves along the back of the wall

Earth Pressure Theories

Coulomb's Theory



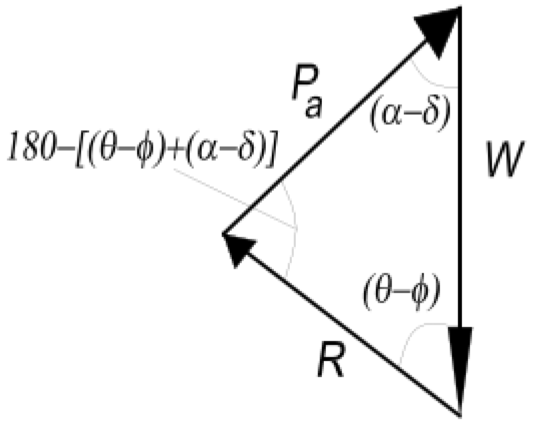
(a) Coulomb's sliding wedge



(b) Force polygon

Earth Pressure Theories

Coulomb's Theory - Active Case



(b) Force polygon

$$\frac{P_a}{\sin(\theta-\phi)} = \frac{W}{\sin[180 - \alpha - \theta + \phi + \delta)}$$

$$W = \gamma * area(\Delta ABE) = \frac{1}{2}\frac{\gamma H^2}{\sin^2\alpha}\left[\sin(\alpha+\theta)\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}\right]$$

$$P_a = \left[\frac{1}{2}\frac{\gamma H^2}{\sin^2\alpha}\left[\sin(\alpha+\theta)\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}\right]\right]\left[\frac{\sin(\theta-\phi)}{\sin[180 - \alpha - \theta + \phi + \delta)}\right]$$

To determine the orientation of the failure plane that produces the maximum Pa, set $\frac{\partial P_a}{\partial \theta} = 0$.

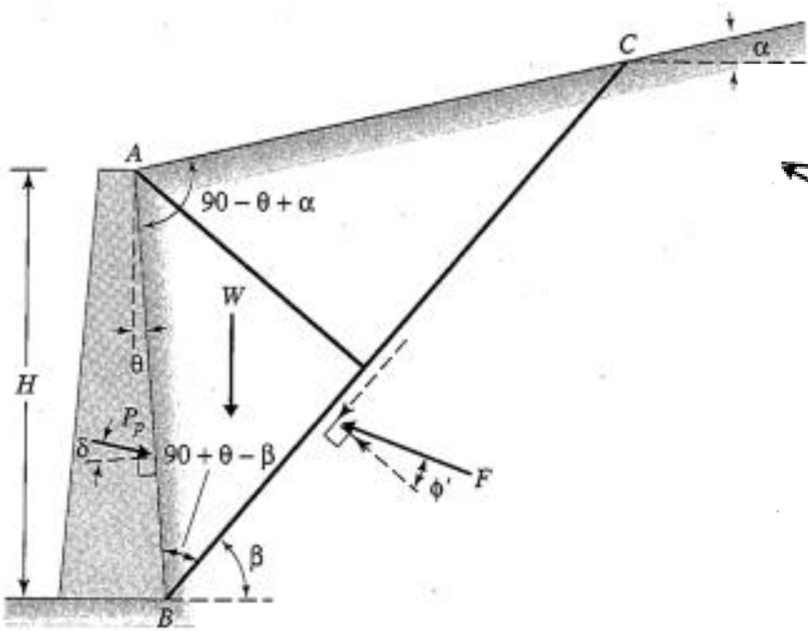
After we determine θcr, substitute in Pa.

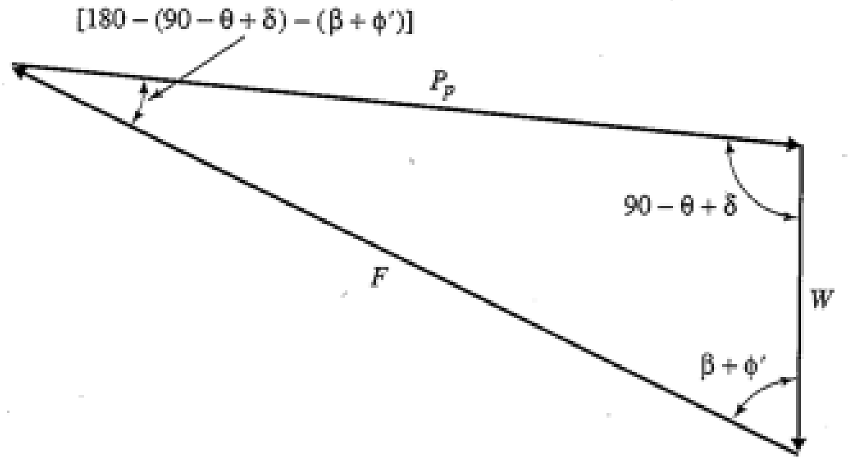
$$P_a = \frac{\gamma H^2}{2} K_a$$

$$K_a = \frac{\sin^2(\alpha+\phi)}{(\sin^2\alpha)\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

Earth Pressure Theories

Coulomb's Theory - Passive Case





$$P_p = \frac{\gamma H^2}{2} K_p$$

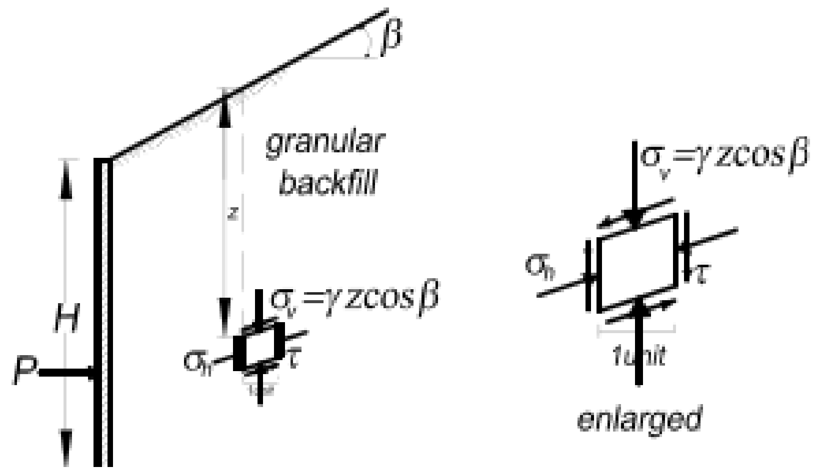
$$K_p = \frac{\sin^2(\alpha - \phi)}{(\sin^2\alpha)\sin(\alpha + \delta)\left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^2}$$

3. **Additional Considerations**

- Inclined Backfill
- Uniform Surcharge
- Submergence
- Soil Layering

Additional Considerations

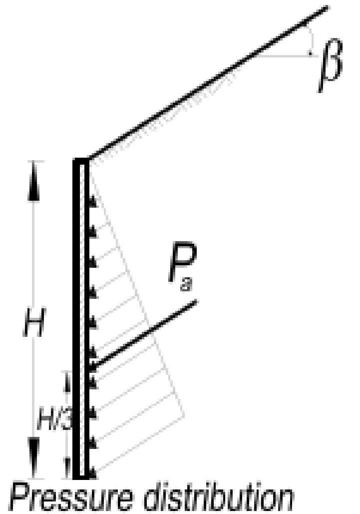
Inclined Backfill



$$K_a = \frac{\sigma_h}{\sigma_v} = \frac{\left[\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}\right]}{\left[\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}\right]}$$

$$\sigma_h = \sigma_v K_a = \gamma z \cos\beta \, K_a$$

$$P_a = \frac{1}{2}\gamma H^2 \cos\beta \, K_a$$



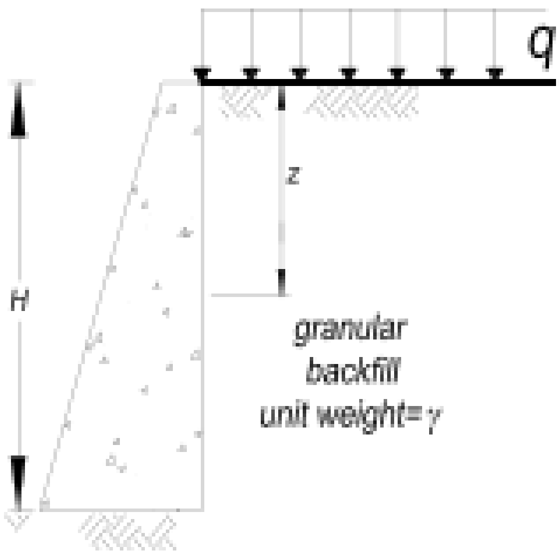
$$K_p = \frac{\sigma_h}{\sigma_v} = \frac{\left[\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}\right]}{\left[\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}\right]}$$

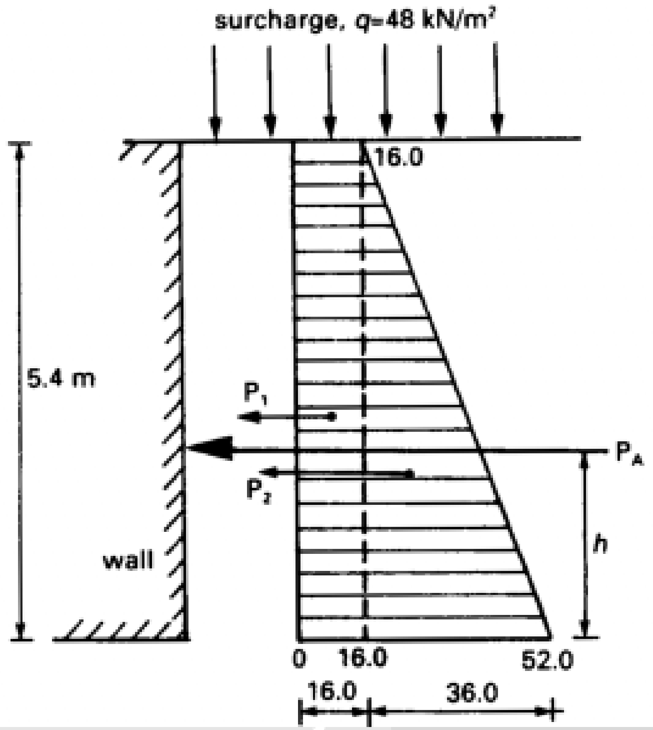
$$P_p = \frac{1}{2}\gamma H^2 \cos\beta \, K_p$$

Additional Considerations

Effect of Uniform Surcharge

- The extra loading carried by the retaining structure is known as surcharge.
- If there is a uniform surcharge of intensity q per unit area, the vertical stress at every elevation in the backfill increases by q.

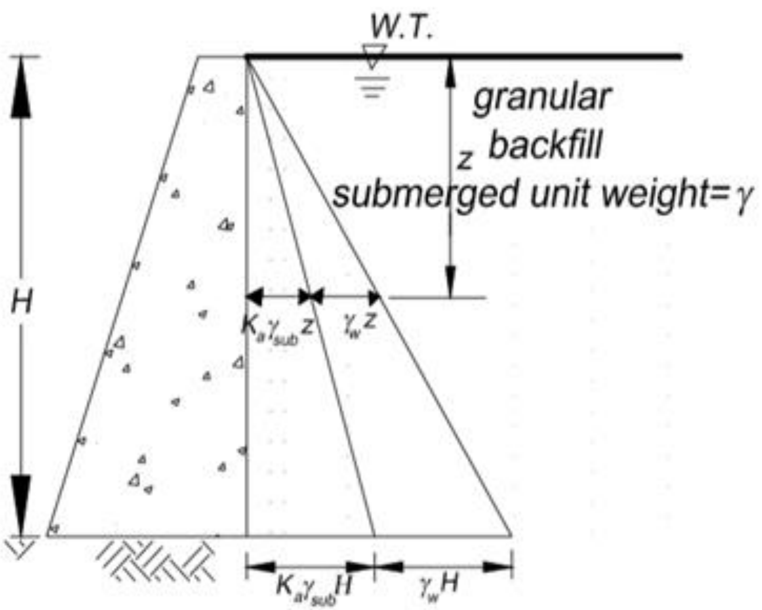




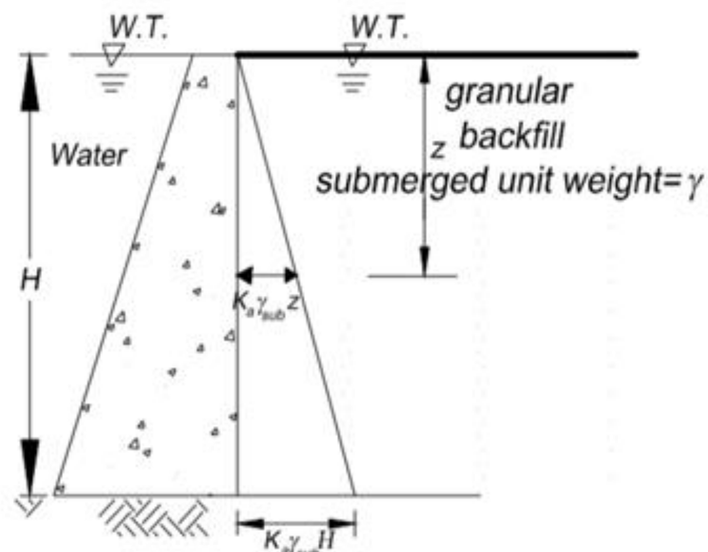
Additional Considerations

Effect of Submergence

- When the backfill is fully saturated or submerged, the lateral pressure will be due to two component;
  - Lateral earth pressure due to submerged unit weight of the backfill soil
  - Lateral pressure due to pore water



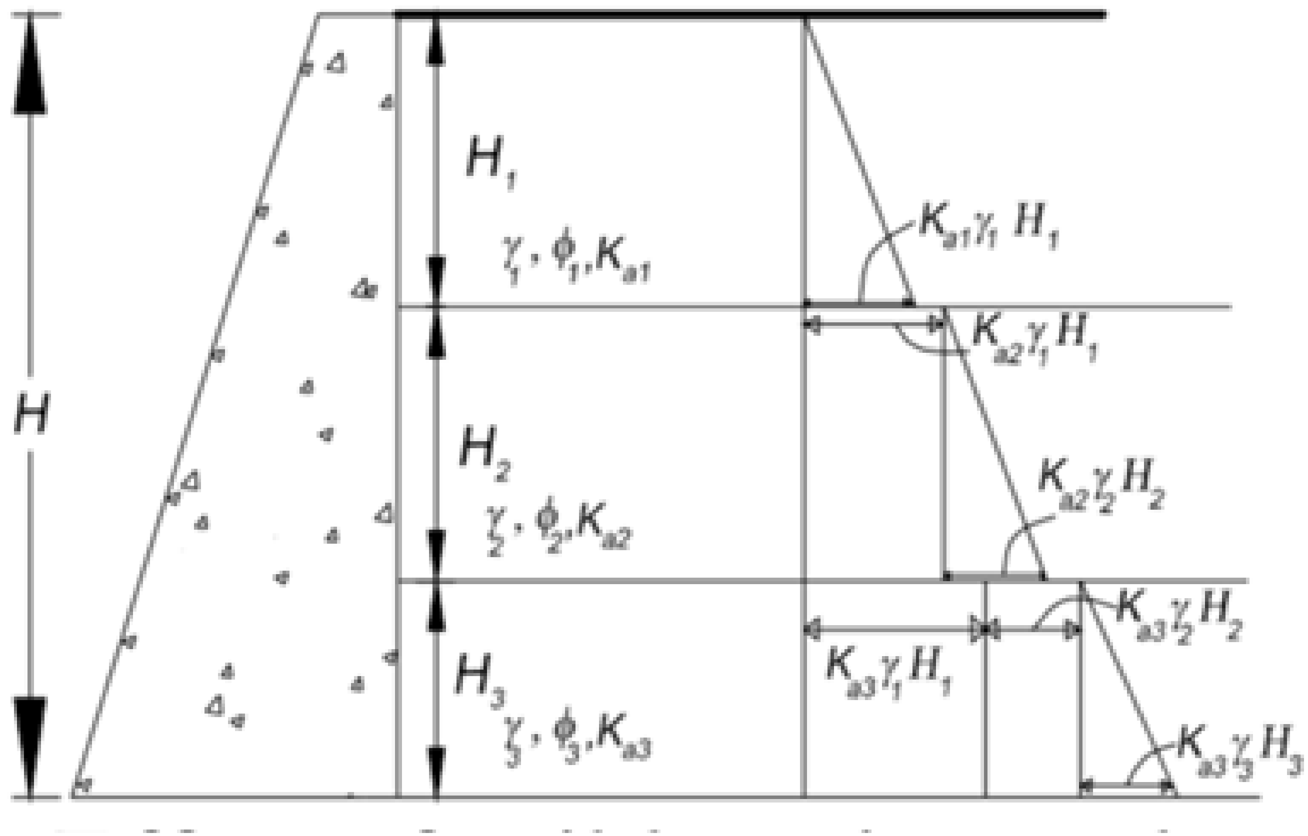
(a) Submerged backfill



(b) Wall with submerged backfill and water on the other side

Additional Considerations

Soil Layering



4. Graphical Methods

- Rebhann's Graphical Method
- Culmann's Graphical Method

Reading Assignment

አመሰግናለሁ!

Galatoma!