Fundamentals of Geotechnical Engineering – III

Chapter 2 Shear Strength of Soils



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General Outline

- Introduction
- *** Failure Criteria**
- Determination of Shear Strength
- Dilatancy & Liquefaction
- Sensitivity & Thixotrophy



- Definition & Implication
- Basics of Shearing Resistance
- Mohr's Circle: A Recap
- Material Responses to Loading
- Typical Response of Soils to Shearing
 Loading Conditions

Some terms

Stress, or intensity of loading, is the load per unit area.

-The fundamental definition of stress is the ratio of the force ΔP acting on a plane ΔS to the area of the plane ΔS when ΔS tends to zero; Δ denotes a small quantity.

Stress (strain) state at a point is a set of stress (strain) vectors corresponding to all planes passing through that point.

-Mohr's circle is used to graphically represent stress (strain) state for two-dimensional bodies.

Isotropic means the material properties are the same in all directions, and also the loadings are the same in all directions.

Anisotropic means the material properties are different in different directions, and also the loadings are different in different directions.

Shear Strength

- maximum internal resistance to applied shearing forces
- maximum resistance to shear stresses just before the failure.
- maximum or ultimate stress the material can sustain against the force of landslide, failure, etc.
- principal engineering property which controls the stability of a soil mass under loads.
- governs the bearing capacity of soils, stability of slopes in soils, earth pressure against retaining structures and many other problems.

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Shear Strength

□Soil derives its shearing strength from the following:

resistance due to interlocking of particles

 frictional resistance between individual soil grains (sliding friction or rolling friction or both)

adhesion or cohesion

Sources of Cohesion

Source	Range	Remark
Electrostatic forces (in stiff, OC clays)	5-24 kPa	may be lost through weathering
Cementing by Fe ₂ O ₃ , CaCO ₃ , NaCl etc	\leq 500 kPa	may be lost through solution
Root cohesion	2-20 kPa	may be lost through logging or fire
Apparent cohesion (in unsat. soils)		Will disappear upon wetting



Shear Strength Parameters

- > Cohesion (c) sticking together of like materials.
- Internal Friction Angle (Ø) stress-dependent component which is similar to sliding friction of two or more soil particles

Alternatively

Attraction (a) and angle of internal friction are used.

COHESIVE [c-Ø] Soils	COHESIONLESS [Granular] Soils
Has cohesion (c)	Only has internal friction angle (\emptyset); c = 0
Drive shear strength from friction and adhesion/cohesion	Drive shear strength from grain interlocking & friction
Example : Clay, Silt	Example : Sand, Gravel

NB. Highly plastic soils drive their shear capacity on from adhesion/cohesion





Factors affecting Shear Strength

Soil composition (basic soil material): mineralogy, grain size and grain size distribution, shape of particles, pore fluid type and content, ions on grain and in pore fluid.

State (initial): initial void ratio, effective normal stress and shear stress (stress history).

Description: loose, dense, overconsolidated, normally consolidated, stiff, soft, contractive, dilative,

Factors affecting Shear Strength ... cntd

Structure: arrangement of particles within the soil mass; the manner the particles are packed or distributed.

Description: undisturbed, disturbed, remolded, compacted, cemented; flocculent, honey-combed, single-grained; flocculated, deflocculated; stratified, layered, laminated; isotropic and anisotropic.

Loading conditions: drained, and undrained; and type of loading, magnitude, rate (static, dynamic), and time history (monotonic, cyclic)).

- Shear stresses develop when the soil is subjected to direct compression.
- Shear failure of a soil mass occurs when the shear stresses induced due to the applied compressive loads exceed the shear strength of the soil.
- Failure occurs by relative movement of particles not by breaking of the particles.





failure surface

The soil grains slide over each other along the failure surface.

No crushing of individual grains.

At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).



Friction between Solid Bodies

Consider a prismatic block resting on the surface MN. The block is subjected to two forces: force Pn acting at right angle to surface MN and force Fa acting tangentially to the plane MN



- Let Pn remain constant, Fa increases gradually until sliding starts.
- If Fa is relatively small, the block will remain at rest and the resisting force can be written as Fr=Pn tanδ. This resisting force is due to surface roughness between block and plane.
- As Fa increases, Fr also increases such that Fa=Fr. The block will start sliding when angle of obliquity δ reaches a maximum value, δ m.
- If the block and surface MN are the same material, $\delta m = \phi$ ($\phi = angle$ of internal friction) and thus tan $\phi = \mu$ ($\mu = coefficient$ of friction)

If the block and surface MN are different materials, then δm = angle of skin (wall) friction.





EXERCISE 2.1.1 – BASICS OF SHEARING RESISTANCE

A block of mass 2.1 kg rests on a horizontal table. The coeeficient of friction is 0.2. What horizontal force H will cause the block to start moving?



 $\tan \phi' = F_f / F_n$



Internal Friction within Soil Masses

□In granular or cohesionless soil masses, the resistance to sliding on any plane through the point with in the mass involves the movement of one particle relative to another.

The angle of internal friction, which is a limiting angle of obliquity and hence the primary criterion for slip or failure to occur in a certain plane, varies appreciably for given sand with the density, since the degree of interlocking is known to be directly dependent upon the density.

□This angle also varies somewhat with the normal stress. However, the angle of internal friction is mostly considered constant, since it is almost so for a given sand at a given density.



Stresses and Strains – A Recap of the Fundamentals

Normal Stresses and Strains

Consider a cube of dimensions x = y = z that is subjected to forces

 $P_x = P_y = P_z$, normal to three adjacent sides.





The normal stresses:

$$\sigma_z = \frac{P_z}{xy}$$
, $\sigma_x = \frac{P_x}{yz}$, $\sigma_y = \frac{P_y}{xz}$

Consider a cube of dimensions x = y = z that is subjected to forces $P_x = P_y = P_z$, normal to three adjacent sides.

Let us assume that under these forces the cube compressed by Δx , Δy , Δz in the X, Y, and Z directions.

The strains in these directions, assuming they are small (infinitesimal):

$$\varepsilon_z = \frac{\Delta z}{z}$$
, $\varepsilon_x = \frac{\Delta x}{x}$, $\varepsilon_y = \frac{\Delta y}{y}$

Volumeteric strain:

$$\varepsilon_p = \varepsilon_x + \varepsilon_y + \varepsilon_z$$



Shear Stresses and Shear Strains

Let us consider, for simplicity, the XZ plane and apply a force F that causes the square to distort into a parallelogram, as shown below.



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Introduction

The force *F* is a shearing force, and the shear stress is

Simple shear strain is a measure of the angular distortion of a body by shearing forces. If the horizontal displacement is
$$\Delta x$$
, the shear strain or simple shear strain, γ_{zx} , is

$$\gamma_{zx} = \tan^{-1} \frac{\Delta x}{z}$$

For small strains, $\tan \gamma_{zx} = \gamma_{zx}$, and therefore

$$\gamma_{zx} = \frac{\Delta x}{z}$$

$$\tau = \frac{F}{xy}$$



Principal Planes, Principal Stresses & Mohr's Circle

The most general state of stress at a point may be represented by 6 components, $\sigma_x, \sigma_y, \sigma_z$ normal stresses

 $\tau_{xy}, \tau_{yz}, \tau_{zx}$ shearing stresses

(Note: $\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz}$)

Same state of stress is represented by a different set of components if axes are rotated.



Principal Planes

Plane Stress - state of stress in which two facesof the cubic element are free of stress.For the illustrated example,the state of stress is defined by

$$\sigma_x, \sigma_y, \tau_{xy}$$
 and $\sigma_z = \tau_{zx} = \tau_{zy} = 0.$

State of plane stress occurs in a thin plate subjected to forces acting in the midplane of the plate. It also occurs on the free surface of a structural element or machine component, i.e., at any point of the surface not subjected to an external force.



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Principal Planes - planes which do not have shear stresses

Since failure or slip within a soil mass cannot be restricted to any specific plane, it is necessary to understand the relationships that exist between the stresses on different planes passing through a point, as a prerequisite for further consideration of shearing strength of soils.



Principal Stresses

Consider the conditions for equilibrium of a prismatic element with faces perpendicular to the x, y, and x' axes.

$$\sum F_{x'} = 0 = \sigma_{x'} \Delta A - \sigma_x (\Delta A \cos \theta) \cos \theta - \tau_{xy} (\Delta A \cos \theta) \sin \theta - \sigma_y (\Delta A \sin \theta) \sin \theta - \tau_{xy} (\Delta A \sin \theta) \cos \theta$$
$$\sum F_{y'} = 0 = \tau_{x'y'} \Delta A + \sigma_x (\Delta A \cos \theta) \sin \theta - \tau_{xy} (\Delta A \cos \theta) \cos \theta - \sigma_y (\Delta A \sin \theta) \cos \theta + \tau_{xy} (\Delta A \sin \theta) \sin \theta$$

The equations may be rewritten to yield $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$ $\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$ $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$



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Principal Stresses

The previous equations are combined to yield parametric equations for a circle

$$(\sigma_{x'}-\sigma_{ave})^2+\tau_{x'y'}^2=R^2$$

where

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal stresses occur on the *principal planes of stress* with zero shearing stresses.





Note : defines two angles separated by 90°



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Principal Stresses

Maximum shearing stress occurs for

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note : defines two angles separated by 90° and

offset from θ_p by 45°

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$



Mohr's Circle

For a known state of plane stress plot the points *X* and *Y* and construct the circle centered at *C*.

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The principal stresses are obtained at *A* and *B*. $\sigma_{\max,\min} = \sigma_{ave} \pm R$ $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$ The direction of rotation of *Ox* to *Oa*

is the same as CX to CA.

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Mohr's Circle - Some Notes

- 1. The maximum shear stress is numerically equal to $(\sigma_1 \sigma_3)/2$ and it occurs on a plane inclined a 45 degrees to principal planes.
- 2. Point D on the Mohr circle represents the stresses on a plane make an angle θ with the major principal plane.
- 3. The maximum angle of obliquity β max is obtained by drawing a tangent through to the circle from the origin O.



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Mohr's Circle - Some Notes

- 4. The shear stress on the plane of maximum obliquity is less than the maximum shear stress.
- 5. Shear stresses on planes at right angle to each other are numerically equal but are of opposite sign.
- 6. As the Mohr circle is symmetrical about σ -axis, it is usual practice to draw only the top half circle for convenience.
- 7. There is no need to be rigid about sign convention for plotting the shear stresses in Mohr's circle. These can be plotted either upward or downwards. Numerical results wont be affected.

NB Mohr's circle is a graphical method for the determination of stresses on a plane inclined to the principal planes.

It represents all possible combinations of shear and normal stresses at the stressed point.

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Material Responses to Normal Loading / Unloading

If we apply an incremental vertical load, ΔP , to a deformable cylinder of cross-sectional area A, the cylinder will compress by, say, Δz and the radius will increase by Δr .

The loading condition we apply here is called uniaxial loading.

The change in vertical stress is $\Delta \sigma_z = \frac{\Delta P}{A}$ The vertical and radial strains are, respectively, $\Delta \varepsilon_z = \frac{\Delta z}{H_o}$ Original configuration $\Delta \varepsilon_z = \frac{\Delta r}{H_o}$ Deformed configuration

Material Responses to Normal Loading / Unloading

The ratio of the radial (or lateral) strain to the vertical strain is called Poisson's ratio, v, defined as

$$\upsilon = \frac{-\Delta \varepsilon_r}{\Delta \varepsilon_z}$$

Typical Values of Poisson's Ratio TABLE νa Soil type Description Clay Soft 0.35 - 0.40Medium 0.30-0.35 Stiff 0.20-0.30 Sand Loose 0.15 - 0.25Medium 0.25-0.30 Dense 0.25-0.35

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^aThese values are effective values, ν' .

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Linear elastic materials

- For equal increments of ΔP , we get the same value of Δz .
- If at some stress point, say, at A, we unload the cylinder and it returns to its original configuration.

Non-linear elastic materials

- For equal increments of ΔP , we get the different value of Δz .
- If at some stress point, say, at A, we unload the cylinder and it returns to its original configuration.

Elasto-plastic materials

- do not return to their original configurations after unloading.
- The strains that occur during loading consist an elastic or recoverable part and a plastic or unrecoverable part.

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Moduli

- The elastic modulus or initial tangent elastic modulus (E) is the slope of the stress–strain line for linear isotropic material.
- The tangent elastic modulus (Et) is the slope of the tangent to the stress—strain point under consideration.
- The secant elastic modulus (Es) is the slope of the line joining the origin (0, 0) to some desired stress– strain point.





Material Responses to Shear Forces

-Shear forces distort materials.







Typical Respons of Soils to Shearing Forces



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Type I soils—loose sands, normally consolidated and lightly overconsolidated clays (OCR ≤ 2)

• Show gradual increase in shear stresses as the shear strain increases (strain-hardens) until an approximately constant shear stress, which we will call the critical state shear stress, τ_{cs} , is attained.

• Compress, that is, they become denser until a constant void ratio, which we will call the critical ratio, e_{cs} , is reached.


Type II soils—dense sands and heavily overconsolidated clays (OCR > 2)

• Show a rapid increase in shear stress reaching a peak value, τ_p , at low shear strains (compared to Type I soils) and then show a decrease in shear stress with increasing shear strain (strain-softens), ultimately attaining a critical state shear stress.

• Compress initially (attributed to particle adjustment) and then expand, that is, they become looser until a critical void ratio (the same void ratio as in Type I soils) is attained.

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Effects of increasing normal effective stresses on the response of soils



Effects of OCR on peak strength and volume expansion



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Undrained Loading Condition

-relevant for loading on cohesive soils (clays and fine silts) when the condition may be assessed as undrained.

>occur when excess pore water pressure can't drain, at least quickly from the soil.

>Volume of the soil remains constant.

In an idealized model, a **sudden** change in total mean stress does not result in change in mean inter granular stress, but in an instantaneous pore pressure change only.

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Drained Loading Condition

-a change in stressleads to a change in effective stresses.

- For a cohesive soil, this requires that the load case is a long term condition, allowing pore pressure changes to dissipate.
- For granular materials it will always be required to use effective stresses in a proper evaluation of shear strength.

-There will however be cases where mixed materials of sand, silt and clay under rapid.



Drained Condition		Undrained Condition	
 occurs when there is no change in pore water pressure due to external loading 		-occurs when the pore water is unable to drain out of the soil	
-pore water can drain out of the soil easily		-increased pore water pressure since most of the external loading is taken by the pore water.	
-volumetric strains occur in the	soil	- tendency of suppressed	soil to change in volume is
Condition	Drained		Undrained
Excess porewater pressure	- 0		
	~0		Not zero; could be positive or negative
Volume change	~o Compression Expansion		Not zero; could be positive or negative Positive excess porewater pressure Negative excess porewater pressure
Volume change Consolidation	Compression Expansion Yes, fine-grair	ned soils	Not zero; could be positive or negative Positive excess porewater pressure Negative excess porewater pressure No
Volume change Consolidation Compression	Compression Expansion Yes, fine-grair Yes	ned soils	Not zero; could be positive or negative Positive excess porewater pressure Negative excess porewater pressure No Yes, but lateral expansion must occur so that the volume change is zero
Volume change Consolidation Compression Analysis	Compression Expansion Yes, fine-grain Yes Effective stres	ned soils ss	Not zero; could be positive or negative Positive excess porewater pressure Negative excess porewater pressure No Yes, but lateral expansion must occur so that the volume change is zero Total stress

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- The existence of these conditions in a soil depends on:
 (i)soil types → saturated coarse grained soils experience drained conditions and saturated fine-grained soils experience undrained conditions for a rate of loading associated with a normal construction activity
- (ii) geological formation and
- (iii) rate of loading → If the rate of loading is fast enough (e.g. during an earthquake), even coarse-grained soils can experience undrained loading, often resulting in liquefaction.



Effects of drained and undrained conditions on volume changes





- Introduction
- Coulomb Failure Criterion
- Taylor Failure Criterion
- Mohr-Coulomb Failure Criterion
- Tresca Failure Criterion
- Practical Implications

Introduction

A failure criterion is a mathematical equation for the strength of a material. Such criteria are expressed in terms of stress components and material properties.

Soil models

- help us interpret the shear strength of soils.
- > idealized representation of the soil to allow us to understand its response to loading and other external events.
- > not expected to capture all the intricacies of soil behavior by definition.

Each soil model may have a different set of assumptions and may only represent one or more aspects of soil behavior.

Coulomb's Criterion

Soils, in particular granular soils, are endowed by nature with slip planes. Each contact of one soil particle with another is a potential micro slip plane.

Loadings can cause a number of these micro slip planes to align in the direction of least resistance. Thus, we can speculate that a possible mode of soil failure is slip on a plane of least resistance.

In 1976 Coulomb observed that if the thrust of a soil against a retaining wall caused the wall to move forward slightly, an essentially straight slip plane formed in the retained soil.

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Coulomb's Model



A slip plane in a soil mass



Coulomb's law requires the existence or the development of a critical sliding plane, also called slip plane. In the case of the wooden block on the table, the slip plane is the horizontal plane at the interface between the wooden block and the table.

Unlike the wooden block, we do not know where the sliding plane is located in soils.

In terms of stresses, Coulomb's law is expressed as $\tau_f = (\sigma'_n)_f \tan \phi'$

where τ_f (=*T*/*A*, where *T* is the shear force at impending slip and *A* is the area of the plane parallel to *T*)

 τ_f is the shear stress when slip is initiated, and $(\sigma'_n)_f$ is the normal effective stress on the plane on which slip is initiated.

- The subscript *f* denotes failure, which, according to Coulomb's law, occurs when rigid body movement of one body relative to another is initiated.
- Failure does not necessarily mean collapse, but is the impeding movement of one rigid body relative to another.



Coulomb postulated that the maximum resistance to shear,

on the failure plane is given by $\tau_f = c + \sigma_f \tan \phi$

Where: σ is the total stress normal to the failure plane

 ϕ is the angle of internal friction; c is the cohesion of the soil

The use of Coulomb's equation didn't always result in successful design of soil structures.

Why? \rightarrow Water cannot sustain shear stress; the shear resistance of a soil must result solely from the frictional resistance arising at the particle contact points, the magnitude of which depends solely on the magnitude of the effective stress carried by the soil skeleton.

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Failure Criteria on Total Stress Basis – TRESCA

The failure criterion on total stress basis is defined by the undrained shear strength, s_u , being equal to the maximum shear stress at failure

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \tau_f = s_u$$

Drawing this failure criterion in a Mohr-diagram gives ---- \rightarrow

The critical soil element shows that failure also occurs along planes with an angle of 45 deg. the failure criterion is a constant maximum shear stress limitation defined by the maximum radius of the Mohr circle.



Failure Criteria on Total Stress Basis – [TRESCA(1864)]

-drawn in the a principal stress diagram with major and minor principal stresses as axes, begets the picture shown.

The failure criterion shows as a line With inclination 45, intercepting the σ_1 -axis at a stress level $2s_u$.

This interception describes a situation where $\sigma_3 = 0$, a situation that can be found in the uniaxial stress apparatus used to find the undrained strength of soil.



Failure Criteria on Effective Stress Basis

Mohr-Coulomb Failure Criterion : states that a material fails because of a critical combination of normal stress and shear stress, and not from their either maximum normal or shear stress alone.



Mohr-Coulomb Failure Criterion



Mohr-Coulomb Failure Criterion



Mohr-Coulomb Failure Criterion

As loading progresses, Mohr circle becomes larger...



 σ_{c} .. and finally failure occurs when Mohr circle touches the envelope 58

Mohr-Coulomb Failure Criterion







Name	Failure criteria	Soil treated as	Best used for	Test data interpretation*
Coulomb	Failure occurs by impending, frictional sliding on a slip plane.	Rigid, frictional material	Layered or fissured overconsolidated soils or a soil where a prefailure plane exists	Direct shear
Mohr– Coulomb	Failure occurs by impending, frictional sliding on the plane of maximum principal effective stress obliquity.	Rigid, frictional material	Long-term (drained condition) strength of overconsolidated fine- grained and dense coarse-grained soils	Triaxial
Tresca	Failure occurs when one-half the maximum principal stress difference is achieved.	Homogeneous solid	Short-term (undrained condition) strength of fine-grained soils	Triaxial
Taylor	Failure occurs from sliding (frictional strength) and interlocking of soil particles.	Deformable, frictional solid	Short-term and long- term strength of homogeneous soils	Direct simple shear

The Mohr Coulomb

criterion is the by far

the most important

criterion for the

strength of soils.

Mohr-Coulomb's Criterion

$$\sin\phi' = \frac{OB}{OA} = \frac{\frac{(\phi_1')_f - (\sigma_3')_f}{2}}{\frac{(\sigma_1')_f + (\sigma_3')_f}{2}}$$



3. Determination of Shear Strength

Laboratory Tests

- Fall Cone Test
- Unconfined Compression Test
- Shear Box Test
- Triaxial Test
- ≻In-situ Tests
 - Vane Shear Test
 - Standard Penetration Test
 - Cone Penetrometer Test

Empirical Relationships

Determination of shear strength of soil involves plotting of failure envelopes and evaluation of shear strength parameters for the necessary conditions.

Field conditions



 $\sigma_{\rm vc} + \Delta \sigma$ $\sigma_{\rm hc} \rightarrow \sigma_{\rm hc}$ $\sigma_{\rm hc} \rightarrow \sigma_{\rm hc}$ $\sigma_{\rm vc} + \Delta \sigma$ After and during

Before construction

Undrained Shear Strength – Su

- may be found by use of a variety of test methods.

In the laboratory:

- ✓● fall cone
- ✓● uniaxial test
- shear box test
- ✓ triaxial test

Characterization of clay based on undrained shear strength, $s_{\rm w}$

	s_u [kPa]
Soft	< 25
Medium firm	25 - 50
Firm	> 50

Fall cone Test

This test is normally a part of the routine investigations and is a rapid and simple method for obtaining information on the undrained strength of the material at hand.

The soil specimens are cut clean at

The end surfaces and mounted in the

fall cone apparatus as shown.

A properly selected fall cone piston

is lowered until it touches the soil surface,

the piston is released and penetrates into

the soil by its own weight.

The penetration is measured.



Undrained shear strength

Undrained Shear Strength – Su

This procedure is repeated three times at slightly different places at

the surface of the same specimen

and the average penetration is calculated.

The average penetration is related to the undrained

shear strength through

calibration curves

(or rather tables)

as shown in figure.



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Uniaxial (unconfined compression) test

In the uniaxial test, a 10 cm long test specimen is taken directly from the sampling cylinder into a compression apparatus able to apply axial deformation at constant speed.

The specimen is only loaded at the ends and the axial stress therefore becomes the principal stress ($\sigma_a = \sigma_1$). The radial stress is the minor principal stress and we have

 $\sigma_r = \sigma_2 = \sigma_3 = 0.$



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Unconfined compression test

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$$\sigma_{1} = \sigma_{VC} + \Delta \sigma$$

$$\sigma_{3} = 0$$

$$\sigma_{3} = 0$$

$$\sigma_{1} = \sigma_{VC} + \Delta \sigma f$$

$$\sigma_{1} = \sigma_{VC} + \Delta \sigma f$$

$$\sigma_{3} = 0$$



Undrained Shear Strength – Su UC test

The axial load and compression is measured and the result is plotted as

a load deformation curve that is interpreted with respect to maximum load and corresponding strain.

The undrained shear strength from uniaxial testing is found by:

$$s_u = \frac{P}{2A_0} \left(1 - \varepsilon_f\right)$$

where P is axial load at failure, A_o is the specimen area (23.2 cm²) at start of testing, and ε_f is the axial strain at failure

 $\mathcal{E}_f = \mathbf{O}/n_o$



 σ_{n}

Direct shear

Shear box test

Shear Box Test



"Simple shear"

 σ_n

$$\gamma = \frac{\delta_h}{H}$$

where δ_h is the shearing displacement of the specimen and H is the thickness.





Torsion

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Undrained Shear Strength – Su Shear Box Test

The result drawn as shear stress versus shear strain, τh versus γ , may look like the curve shown, and one may interpret the shear stress capacity of the soil under the given test conditions.



Laboratory TestscntdShear Box TestDirect Shear Test



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Shear Box TestDirect Shear TestPreparation of a sand specimen





Components of the shear box

Preparation of a sand specimen

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Shear Box Test

Direct Shear Test

Preparation of a sand specimen



Leveling the top surface of specimen

Specimen preparation completed

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Shear Box Test

Direct Shear Test



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Direct Shear Test

Shear Stress-controlled	Strain-controlled
Shearing stress is applied at a constant rate or by equal increments.	Shear displacement is applied at a constant rate by means of a screw.
The shear displacement is measured by a dial gauge	The shearing force necessary to overcome the resistance within the soil is automatically developed. The shearing force is measured with the aid of a proving ring.
	Mostly used because both peak and critical shear force can be obtained.





Direct Shear Test

Advantages	Drawbacks	
Easy, inexpensive and quick test for sands and gravels	Drainage conditions cannot be controlled.	
Large deformations can be achieved by reversing shear direction. This is useful for	The failure plane is predetermined and this may not be the weakest plane.	
determining the residual strength of a soil.	The area of the sliding surface is not constant- better to account it.	
Large samples may be tested in large shear boxes. Small samples may give misleading results due to imperfections (fractures and	There is no means of estimating pore pressures so effective stresses cannot be determined from undrained tests.	
fissures) or the lack of them.	The effect of lateral restraints by the side walls of the shear box is likely to affect results.	
Samples may be sheared along predetermined planes. This is useful when the shear strengths along fissures or other selected planes are required.	Undrained strengths are unreliable because it is impossible to prevent localized drainage without high shearing rates.	

Laboratory Tests Triaxial Test



- appropriate for most soil and rock materials, given an apparatus of ample capacity to bring the actual soil to failure.

At use on clay, a laboratory sample is taken from the sampling cylinder or the block sample, trimmed to a predefined dimension and installed in the apparatus.

In traditional triaxial testing the axial deformation is controlled, and an outside pressure is applying a radial stress on the specimen. This is obtained by filling the space inside the cell with a fluid enabling an accurate control of the pressure. To avoid the cell fluid to enter the specimen a rubber membrane is surrounding the soil.

Laboratory Tests Triaxial Test



At the ends of the specimen a filter is applied and by thin tubes these filters are communicating with the outside environment, facilitating measurement of the amount of pore water extorted from the specimen (to a burette) or the pressure of the pore water inside the sample. Before the rubber membrane is placed on the sample, the surface is covered by a filter paper.

By this, water extorted from the specimen is flowing radially and transported to the end filters along the outside of the sample. This is the shortest drainage path for water flow out of the soil.



Triaxial specimen before the outside cell is mounted







Triaxial Test

Advantage	Drawbacks		
There is complete control over drainage conditions.	The apparatus is elaborate, costly and bulky.		
Pore pressure changes and volumetric changes can be measure directly.	The drained test takes a longer period as compared with that in a direct shear test.		
The stress distribution on the failure plane is uniform.	The consolidation of the specimen is is isotropic, whereas in the field, the		
The specimen is free fail on the weakest plane.	consolidation is generally anisotropic		
The state of stress at all intermediate stages up to failure is known. The Mohr circle can be drawn at any stage of shear.	The test simulates axis-symmetric problems. In the field, the problem is generally 3-D. A general test in which		
The test is suitable for accurate research work.	all the three stresses are varied would be more useful.		



UU Triaxial Tests

Initial specimen condition

 $\sigma_{c} = \sigma_{3}$ No drainage

Initial volume of the sample = $A_0 \times H_0$

Volume of the sample during shearing = A \times H

Since the test is conducted under undrained condition,

 $A \times H = A_0 \times H_0$

 $A \times (H_0 - \Delta H) = A_0 \times H_0$

 $A \times (1 - \Delta H/H_0) = A_0$



Specimen condition during shearing



$$A = \frac{A_0}{1 - \varepsilon_z}$$



Note: If soil is fully saturated, then B = 1 (hence, $\Delta u_c = \Delta \sigma_3$)

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 $\Delta u = B \left[\Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3) \right]$

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UU Triaxial Tests



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UU Triaxial Tests



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Some Practical Applications of UU Triaxial Tests

1. Embankment constructed rapidly over a soft clay deposit







Some Practical Applications of UU Triaxial Tests

2. Large earth dam constructed rapidly with no change in water content of soft clay



 τ = Undrained shear strength of clay core



Some Practical Applications of UU Triaxial Tests

3. Footing placed rapidly on clay deposit



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Laboratory Tests EXERCISE 2.3.1 – SHEAR BOX TEST



The data recorded during a shear box test on a sand sample, 10 cm X 10 cm X 3 cm, at a constant vertical force of 1200 N are shown in the table below. A negative sign denotes vertical expansion.

(a) Plot graphs of (1) horizontal forces versus horizontal displacements, and (2) vertical displacements versus horizontal displacements.

(b) Would you characterize the behavior of this sand as that of a dense or a loose sand? Explain your answer.

(c) Determine (1) the maximum or peak shear stress, (2) the critical state shear stress,

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Horizontal displacement (mm)	Horizontal force (N)	Vertical displacement (mm)	Horizontal displacement (mm)	Horizontal force (N)	Vertical displacement (mm)
0.00	0.00	0.00	6.10	988.29	-0.40
0.25	82.40	0.00	6.22	988.29	-0.41
0.51	157.67	0.00	6.48	993.68	-0.45
0.76	249.94	0.00	6.60	998.86	-0.46
1.02	354.31	0.00	6.86	991.52	-0.49
1.27	425.72	0.01	7.11	999.76	-0.51
1.52	488.90	0.00	7.37	1005.26	-0.53
1.78	538.33	0.00	7.75	1002.51	-0.57
2.03	571.29	-0.01	7.87	994.27	-0.57
2.41	631.62	-0.03	8.13	944.83	-0.58
2.67	663.54	-0.05	8.26	878.91	-0.58
3.30	759.29	-0.09	8.51	807.50	-0.58
3.68	807.17	-0.12	8.64	791.02	-0.59
4.06	844.47	-0.16	8.89	774.54	-0.59
4.45	884.41	-0.21	9.14	766.30	-0.60
4.97	928.35	-0.28	9.40	760.81	-0.59
5.25	939.34	-0.31	9.65	760.81	-0.59
5.58	950.32	-0.34	9.91	758.06	-0.60
5.72	977.72	-0.37	10.16	758.06	-0.59
5.84	982.91	-0.37	10.41	758.06	-0.59
5.97	988.29	-0.40	10.67	755.32	-0.59

Laboratory Tests EXERCISE 2.3.2 UC Test



An unconfined compression test was carried out on a saturated clay sample. The maximum (peak) load the claysustained was 127 N and the vertical displacement was 0.8 mm. The size of the sample was 38 mm diameter 3 76 mm long.

Determine the undrained shear strength.

Draw Mohr's circle of stress for the test and locate su.

Laboratory Tests EXERCISE 2.3.3 CD Triaxial Test

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The results of three CD tests on 38-mm-diameter and 76-mm-long samples of a soil at failure are as follows:

Test number	σ'₃ (kPa)	Deviatoric stress (kPa)	Δ <i>z</i> (mm)	ΔV (cm ³)	Axial load = $P_z(N)$
1	100	247.8 (peak)	0	0.00	0.0
2	180	362.0 (peak)	0.152	0.02	61.1
3	300	564 (no peak observed)	0.228	0.03	94.3
(a) Determine the friction			0.38	-0.09	124.0
			0.76	-0.50	201.5
angle for each test.			1.52	- 1.29	257.5
			2.28	- 1.98	292.9
			2.66	- 2.24	298.9
(b) Dete	ermine $ au$	$_p, au_{cs}, E'\&E'_s$	3.04 3.8	-2.41 -2.55	298.0 279.2
at peak	shear st	ress for Test 1.	4.56 5.32 6.08	-2.59 -2.67 -2.62	268.4 252.5 238.0
(c) Determine ϕ_{cs}^{\prime} .			6.84	-2.64	229.5
			7.6	-2.66	223.2
(d) Dete	ermine <i>a</i>	T_p for Test 1.	8.36	-2.63	224.3

Laboratory Tests EXERCISE 2.3.4 CU Triaxial Test



A CU test was conducted on a saturated clay soil by isotropically consolidating the soil using a cell pressure of 150 kPa and then incrementally applying loads on the plunger while keeping the cell pressure constant.

At large axial strains (<15%), the axial stress exerted by the plunger was approximately constant at 160 kPa and the porewaterpressure recorded was constant at 54 kPa.

Determine (a) su and (b) ϕ_{cs} .

Illustrate your answer by plotting Mohr's circle for total and effective stresses.

Laboratory Tests EXERCISE 2.3.5 UU Triaxial Test



A UU test was conducted on saturated clay.

The cell pressure was 200 kPa and the peak deviatoric stress was 220 kPa.

Determine the undrained shear strength.

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In-situ Tests

Introduction

The extraction, transportation and testing procedures of a soil sample may greatly affect the behavior of the soil thus the test results.

It might not be feasible to duplicate the field condition under investigation in the laboratory.

■Therefore there are a number of field tests that are used to estimate the shear strength of a soil.

Vane shear test

Standard penetration test

Cone penetrometer test



Vane Shear Test [Field Vane]

-used to measure the undrained shear strength in soft and saturated clays where undisturbed specimen is difficult to obtain.



In-situ Tests

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Field vane procedures

>The field vane is penetrated to desired depth before testing.

> At testing level, the vane is pushed out of the installation device and into undisturbed soil.

>Thereafter a torque is applied to the push rod and the correlation between torque and rotation is measured.

>At failure, the failure surface will be cylindrical with circular ends, and the measured moment is used to calculate the shear stresses at failure.

>From this and empirical corrections the undrained shear strength, s_u is found.

In-situ Tests

Field vane interpretation of undrained shear strength, su



Resisting moment = cylindrical surface resistance + two circular end face resistance

$$T = 2\pi r L (C_u r) + 2[\pi r^2 C_u (2/3r)] = 2\pi r^2 C_u (L+2/3r)$$

$$\Rightarrow C_u = \frac{T}{2\pi r^2 (L+\frac{3}{3}r)} \qquad \Rightarrow C_u = \frac{3T}{28\pi r^3} \text{ if } L = 4r \text{ (commonly used ratio)}$$

Resisting moment = cylindrical surface resistance + one circular end face resistance $T = 2\pi r L (C_u r) + \pi r^2 C_u (2/3r) = 2\pi r^2 C_u (L+1/3r)$ $\Rightarrow C_u = \frac{T}{2\pi r^2 (L+\frac{1}{3}r)} \qquad \Rightarrow C_u = \frac{3T}{26\pi r^3} \text{ if } L = 4r$
In-situ Tests

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Standard Penetration Test (SPT)

The number of blows required to drive the split-spoon sampler
30cm into the soil is the standard penetration value, N.



Sampler attached to a drill rod and driven into the soil by blows from a hammer (64kg) falling from a height of 76cm. Initially 15cm then 30cm in two stages.





Standard Penetration Test (SPT)

- The standard penetration number has been correlated to several soil parameters.
- The test is suitable for cohesionless soils.

Condition	Ν	φ (degree)	D _r (%)
Very loose	0-4	<20	0-15
Loose	2-4	28-30	15-35
Medium	10-30	30-36	35-65
Dense	30-50	36-42	65-85
Very dense	>50	>42	>85

Consistency	N	q _u (KPa)
Very soft	0-2	<25
Soft	2-4	25-50
Medium	4-8	50-100
Stiff	8-15	100-200
Very stiff	15-30	200-400
Hard	>30	>400

In-situ Tests

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Cone Penetration Test (CPT)

- An instrumented cone is placed at the point of a rod system and forced downwards into the soil at constant speed by a rig. The standard systems may penetrate to a depth of 40 to 50m.
- During penetration the point resistance and the sleeve friction on the cone intrument are recoreded more or less continously.
- Recent versions (CPTu) may also record the pore pressure against the cone tip. One of the advantages of the method is that it is very sensitive and is able to detect rather thin soil layers and display these layers by plotting for instance the point resistance versus depth.
- The recordings from the CPT(U) test are normally plotted versus depth.

In-situ Tests

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In situ determination of s_u by Cone Penetration Test (CPT)

The undrained strength may be determined from the CPT data through the following relation: $s_u = \frac{(q_T - \sigma_{\rm v0})}{_{N}}$ where q_{τ} is point resistance, σ_{vo} is in situ total vertical overburden, and N is a *bearing* capacity factor. N may according to classical theories take the values of 6-9, but N-values in the order of 10-20 are

versus *su-results from* laboratory tests (i.e. triaxial tests).





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In situ determination of su by CPT

In many literatures point resistance is referred to q_c and the cone factor as N_k .

 $N_{\rm k}$: depends on the geometry of the cone and the rate of penetration.



Empirical Relationships



Soil type	Equation	Reference
Normally consolidated clays	$\left(\frac{s_u}{\sigma'_z}\right)_{nc} = 0.11 + 0.0037 \mathrm{Pl}$	Skempton (1957)
	$\left(\frac{s_u}{\sigma'_{zo}}\right) = 0.22$	Mesri (1975)
Overconsolidated clays	$\frac{(s_u/\sigma_z')_{oc}}{(s_u/\sigma_z')_{nc}} = (\text{OCR})^{0.8} \text{ See Note 1.}$	Ladd et al. (1977)
	$\frac{s_u}{\sigma'_z} = (0.23 \pm 0.04) \text{OCR}^{0.8}$ See Note 1.	Jamiolkowski et al. (1985)
Clean quartz sand	$\phi'_p = \phi'_{cs} + 3D_r (10 - \ln p'_f) - 3$, where p'_f is the mean effective stress at failure (in kPa) and D_r is relative density. This equation should only be used if $12 > (\phi'_p - \phi'_{cs}) > 0$.	Bolton (1986)



DilatancyLiquefaction

Dilatancy & Liquefaction Dilatancy

■Soil material tends to increase or decrease its volume when subjected to shear stresses (and shear strains).

In an undrained condition, the volume is not allowed to change and therefore this effect will lead to a change in pore pressure.

The triaxial cell is superior to control these effects as it gives control over the shear stress, the mean stress and the pore pressure.

- The remarkable phenomenon of the coupling between volume and shape changes observed qualitatively and termed granular dilatancy by Osborne Reynolds has influenced many a concept in granular media and soil mechanics.
- Reynolds (1902) demonstrated the granular dilatancy with two rubber balloons, each full of colored water that his audience saw standing in a tube above each balloon in turn.
- One balloon contained only water. The other contained a fully saturated dense aggregate.

- When he squeezed each balloon in turn, the water level rose in the tube from the water-filled balloon whereas the water level lowered down in the other tube.
- He explained this surprise result by using the dilatancy phenomenon that when the dense sand is sheared it tends to dilate and enlarge its voids.
- If there is a water supply at the moment, the enlarging voids draw the water from the supply, leading to the fall in water level.

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- One of the earliest attempts to account for the increased shear strength due to dilatancy in dense sand was by D.W.Taylor (1948).
- Taylor used the term interlocking to describe the effects of dilatancy.
- He calculated the power at peak strength for some direct shear-box data and found that the energy input is partly dissipated by a critical state friction component and partly by the work needed to increase the volume.



Video: Dilation and contraction



Liquefaction

- When loading is rapidly applied and large enough such that it does not flow out in time before the next cycle of load is applied, the water pressure may build to an extent where they exceed the contact stresses between the grains of soil that keep them in contact with each other.
- These contact between grains are the means by which the weight of the buildings and overlying soil layers are transferred from the ground surface to layers of soil or rock at greater depth.

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This loss of soil structure causes it to lose all of its strength and it may be observed to flow like a liquid.

- If loose saturated sand is subjected to ground vibration, it tends to compact and decrease in volume; if drainage is ceased, the tendency to decrease in volume leads to increase in pore water pressure.
- If the pore water pressure builds to the point at which it becomes equal to the overburden pressure, the sand loses its strength completely, and attains a liquefied state.

Dilatancy & Liquefaction Liquefaction

- Although the term liquefaction was first used by Hazen (1920) to explain the mechanism of flow failure of the hydraulic-filled Calaveras Dam in California it has now been used to describe a number of different, though related phenomena.
- The generation of excess pore water pressure under undrained loading conditions is a hallmark of all liquefaction phenomena.

6. Sensitivity & Thixotrophy



SensitivityThixotrophy

Sensitivity & Thixotrophy

Sensitivity

The unconfined compression strength of many naturally deposited clay soils greatly reduce when the soils are

- tested after remolding with out any change in the
- moisture content.
- This property of clay is called sensitivity.

$$S_t = \frac{S_{u(undisturbed)}}{S_{u(remolded)}}$$

Sensitivity & Thixotrophy



$$S_t = \frac{q_{u(\text{undisturbed})}}{q_{u(\text{remolded})}}$$

Sensitivity, Sf	Classification
1	Insensitivity
1-2	Low
2-4	Medium
4-8	Sensitive
8-16	Extra-sensitive
>16	Quick (Sf can be upto 150)

Sensitivity & Thixotrophy



Thixotrophy

The loss of strength from remolding is primarily caused by the destruction of the clay particle structure that was developed during the original process of sedimentation.

- If the soil is kept in an undisturbed state after remolding, it will continue to gain strength with time.
- This phenomenon is called Thixotropy.
- Thixotropy is a time-dependent and reversible process.

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THANK