# Critical depth: how it came into being and why it does not exist

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**This Paper discusses the fallacy of the concept of the critical depth. The fallacy is due to neglecting residual loads in full-scale tests and stress–scale effects at model-scales, made measured load distribution appear linear below a certain 'critical' depth. It is concluded that interpretations of results made in the light of the critical depth concept can result in unsafe designs.**

## Introduction

Current analysis of static pile load transfer is, in most areas of the world, based on effective stress theory, that is, the unit shaft resistance is equal to a value proportional to effective overburden stress plus effective cohesion. Similarly, the unit toe resistance is usually considered to be proportional to the effective overburden stress. In design of driven piles, the cohesion is usually ignored but it is normally retained for bored piles. Both the unit shaft and toe resistance values are often limited either by a maximum given numerical value or to the value calculated at a so-called critical depth. The critical depth has been given authoritative credibility by such documents as the *American Petroleum Institute Recommended Practice for Planning and Design* and the *Canadian Geotechnical Engineering Manual.*

2. The critical depth, usually assumed to be located 10–20 pile diameters deep, is the depth characterized by the fact that down to this depth the unit shaft and toe resistances follow the effective stress principle, but below this point the resistances are constant and equal to the respective value at the critical depth. The origin of the critical depth concept lies in results of full-scale pile studies published by Vesic (1964, 1970, 1977)[1–3] and Meyerhof (1964, 1976),[4,5] and in results of model tests published by Kerisel.[6] These full-scale tests were performed on instrumented piles and the shaft resistance distribution published by Vesic (1970),[2] shown in Fig. 1, is frequently referred to. Vesic's distributions actually show that the unit shaft resistance reduces below the critical depth (peak unit shaft resistance), which has been explained as the influence of the soil displacement near the pile toe associated with the pile toe's penetration. The curves also indicate a degradation of the unit shaft resistance with depth, that is, the unit shaft resistance above the critical depth becomes smaller the longer the pile. Similar results were obtained in the model tests. The critical depth was very quickly accepted and much research has been published supporting the concept, for example, Tavenas (1971)[7] Fellenius (1968).[8] However, in each of the cases of full-scale and model-scale piles, the critical depth originates in a neglect of an important aspect, which will be explained in the following.

## Full-scale behaviour

3. Forget for a moment that critical depth may exist and assume that the effective stress principles and the Coulomb relation for shear resistance is valid for the shaft resistance ($r_s = \beta\sigma'_v$) along the full length of a pile.[9] In this case, an instrumented test pile loaded to its full ultimate resistance would show a resistance distribution similar to that shown in Fig. 2 (assuming the presence of toe resistance). In reality, however, the instrumentation would most probably only register the loads applied to the pile during the test and disregard any loads present in the pile before the test. Such prior loads are called 'residual loads' and are induced in all piles, driven as well as bored, during and following installation. Put simply: residual loads are loads which are always present in a pile—even before measurements are taken or an analysis is performed—and their effect is commonly overlooked.

4. Residual loads are caused by several different phenomena, for example, wave action during driving, soil quakes along the pile and reconsolidation of the soil after the installation disturbance.[10–16] Residual loads consist usually of the sum of shear forces due to negative skin friction along the upper portion of the pile in equilibrium with the shaft and toe resistance along the rest of the pile below the point of equilibrium—the neutral plane. Very small relative movements between the pile shaft and the soil are necessary to generate shear forces between a pile and the soil. Therefore, the residual loads can be calculated assuming fully developed shear along the pile shaft. In contrast, at the pile toe larger movements are necessary to generate resistance and, even for a driven pile, the residual toe resistance is generally smaller than the ultimate toe resistance. Fig. 3 shows the distribution of residual loads representative for the assumed test pile. (Note, the dashed portion in the figure indicates that the neutral plane intersection in reality would show a curved transition from the negative to

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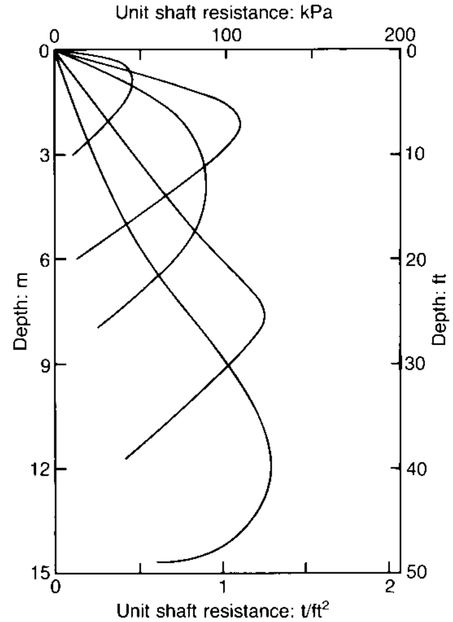


*Fig. 1. Distribution of unit shaft resistance (Vesic, 1970[2])*

positive direction of shear instead of a 'kink' or node. In Figs 4 and 5 the dashed portion of the line corresponds to the dashed portion in Fig. 3.) Ordinarily, the instrumentation would indicate zero load at the start of a static loading test, thus disregarding the residual load. Then, the 'measured' resistance distribution would not show the true shape of Fig. 2, but the false shape shown in Fig. 4 made up of the true value minus the residual load.

5. In a static loading test, one does not measure a contiguous resistance curve, only the load in the few points where the gauges are located, say in four locations as illustrated in Fig. 5. In this case, when connecting the load values, a resistance distribution results, similar to that shown in the figure. This distribution is quite different to the true distribution also shown in Fig. 5. The false shape would indicate an upper zone, where the unit shaft resistance increases progressively with depth, that is, true to the effective stress principle, followed by a zone where the resistance is essentially linear. Ergo, the critical depth is indicated.

6. Figure 6 shows the results of a static loading test on a 285 mm diameter, 15 m long, square concrete pile in homogeneous sand (see Altaee *et al.*[17–19]). The figure shows the loads in the test measured at failure by means of nine gauges embedded in the pile and the distributions of true resistance and residual load. (Note that the residual load distribution transfers gradually from increasing load to decreasing load, i.e., no kink at the neutral plane.) Obviously, had the residual load been neglected, the test would have 'proven the existence of a critical depth at an embedment of about 8 m (28 pile diameters). Fig. 7 shows the distribution of unit shaft resistance for the pile and for an identical 11 m long adjacent test pile, indicating for both piles that the shaft resistance is proportional to the effective stress (except for a zone in the immediate vicinity of the pile toe). If, on the other hand, the residual load is removed from the analysis results, the unit shaft resistance diagrams offer a very different picture, as shown in Fig. 8. Note the similarity of the two figures with those shown in Fig. 1. Note also that the apparent values of unit shaft resistance are about twice as large as the true values shown in Fig. 7.

7. It is quite clear that critical depth has no foundation in reality, but is the result of the neglect of the residual load. Critical depth is discussed further in the literature.[19–25]

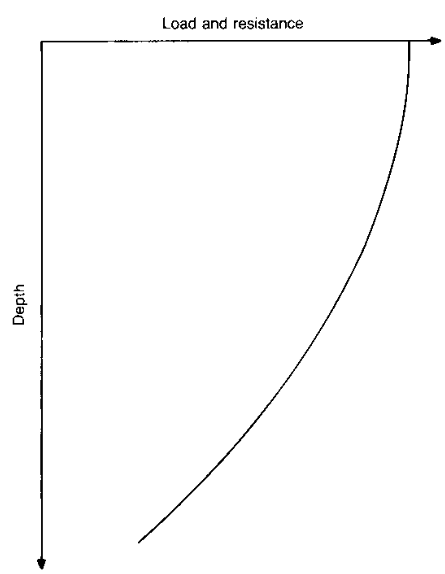


*Fig. 2. Resistance distribution at ultimate resistance in a static loading test*

## Model-scale behaviour

8. A critical depth was also found to exist in small-scale tests on model piles in sand, where residual loads are very small. The interpretation of the tests is still wrong because it neglects an additional influencing fact. The behaviour of sands subjected to a stress increase follows the principles of steady state soil mechanics (also called critical state soil mechanics), which states that at every stress level (mean stress) there is a certain (critical) void ratio, the value of which reduces with increasing stress (the function is linear if plotted as void ratio versus logarithm of mean stress). If the void ratio of the sand is at a higher value than the critical void ratio, the sand will have a tendency to contract when shear forces are induced. If at a lower value, the


*Fig. 3. Residual load present in the pile immediately before the start of the static loading test*

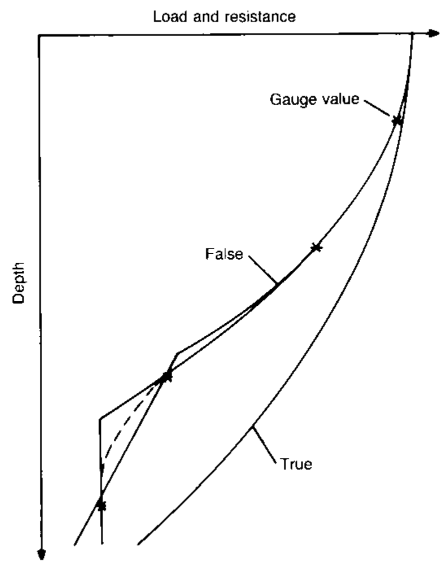


*Fig. 5. Resistance distribution as determined from four load gauges placed in the pile and 'zeroed' before the start of the test, as compared to the true distribution*

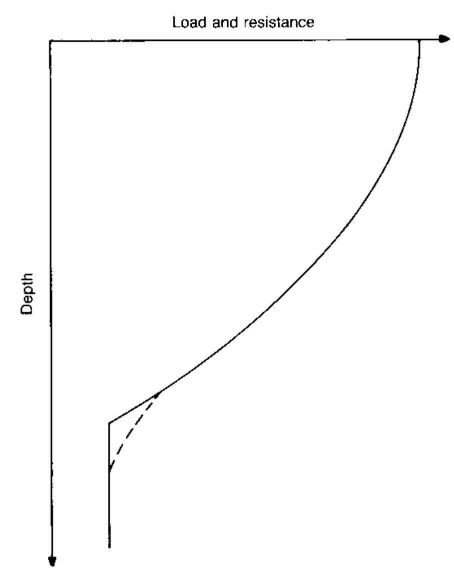


*Fig. 4. False resistance distribution appearing when ignoring the residual load*

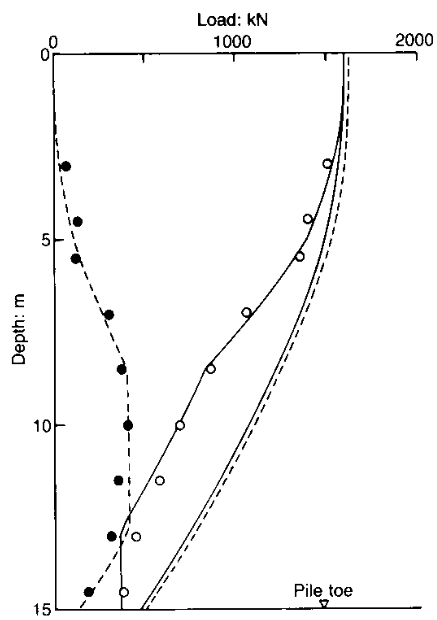


*Fig. 6. Distribution of true load, residual load, and false load in a 15 m long test pile in sand (Altaee et al., 1993*[19]*)*

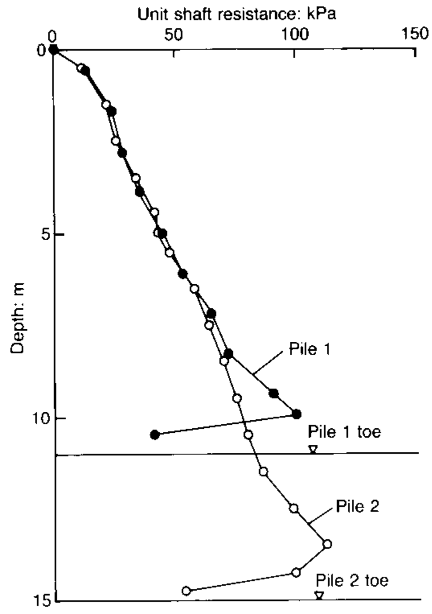


*Fig. 7. Distribution of true unit shaft resistance in two piles, 11 m and 15 m long in sand (Altaee et al., 1993[19])*

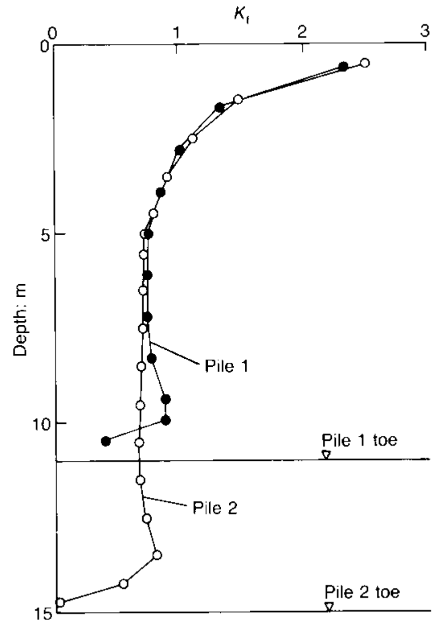


*Fig. 9. Distribution of horizontal earth pressure coefficient against the two test piles (Altaee et al., 1993[19])*

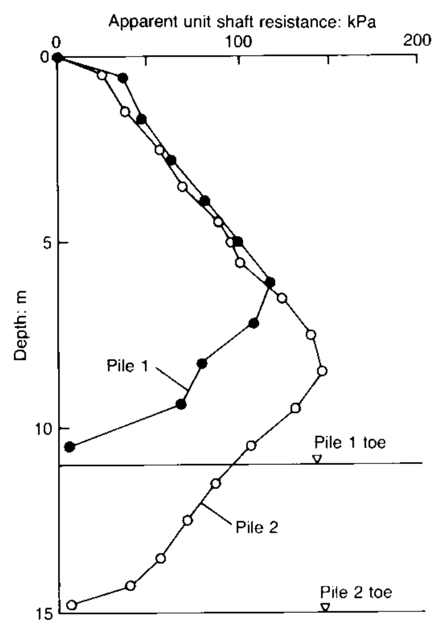


*Fig. 8. Distribution of apparent (false) shaft resistance in the two test piles (Altaee et al., 1993[19])*

sand will dilate instead. The void ratio difference is called the Upsilon value, positive above the steady state line and negative below. Sand in an initial contractible condition is very loose, and the Upsilon value is positive. Sand in an initial dilatable condition well below the steady state line is dense, and the Upsilon value is negative. Sand in an initial contractible state can easily liquefy—collapse, in fact—and pile driving near structures built in such sand can cause severe settlement damage to the structures. (Sands in a state below, but near, the steady state line are also liquefiable and may compact during pile driving in the vicinity.) The larger the initial void ratio in comparison with the critical void ratio, the greater this tendency. Correspondingly, for dense sand, the smaller the void ratio compared to the critical void ratio (the larger the distance to the steady state line), the greater the tendency to dilate, and, if restrained, the larger the horizontal stresses induced during shear.

9. Therefore, for dilatable sand, which is much more prevalent than contractible sand, near the ground surface, where the mean stress is small and the void ratio difference to the steady state line is larger, an induced movement causes the horizontal force to increase in a larger degree (i.e., develops a larger $\beta$

coefficient), as opposed to a movement induced further down in the soil.

10. This is demonstrated in Fig. 9, which presents the earth pressure coefficient, $K_f$, acting against the two test piles. The highest earth pressure coefficient is close to 3 and occurs near the ground surface from where it reduces with depth. Below a depth of about 2 m the coefficient is approximately constant and equal to about 0·7. This is independent of the diameter of the pile. The $\beta$ coefficient is, of course, proportional to the earth pressure coefficient. Shaft resistance is a function of this decreasing $\beta$ coefficient times an increasing value of overburden stress. Therefore, an instrumented model pile will, even with full consideration of residual load, show a distribution of unit shaft resistance that first increases and then becomes relatively constant. This, of course, can easily be interpreted to be the result of the presence of a critical depth.

## Conclusion

11. The critical depth is a fallacy which originates in the failure to interpret the results of full and model-scale pile tests properly. In full-scale tests, neglecting the presence of residual loads makes a measured load distribution appear linear below a certain depth, called the 'critical depth'. In model-scale piles, which are tested at shallow depths, the neglect of stress–scale effects gives a similar error of interpretation. The two independent observations seem to prove the same thing. It is not surprising, therefore, that the fallacy so rapidly gained acceptance. It is important that it be recognized as a fallacy, however, because interpreting test results in the light of the critical depth concept makes for erroneous conclusions and unsafe designs.

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# Critical depth: how it came into being and why it does not exist

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Paper 10659

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It is a pleasure to welcome Fellenius and Altaee to the legion of us who have dispelled the concept of a critical depth. Perhaps the first very important person to dispel this concept was the late A. S. Vesic, to whom the concept generally has been attributed through his 1960s research. In his definitive NCHRP 42 study in 1977,[3] this concept is not discussed because he no longer considered it correct. I had many discussions with him about this concept when he served as a consultant and advisor to a multi-million dollar (US) research project of mine during the late 1970s/early 1980s, and he always described the critical depth as a 'tentative working hypothesis', and nothing more, that he disregarded by the mid 1970s. Unfortunately, the concept was simple, attractive and only required minimal geotechnical knowledge and input. It also gave conservative answers for design beyond the so-called critical depth (10–20 diameters), not unsafe designs as stated by the authors. The design could be rather uneconomical if the 'allowed' tip and side resistances are limited to certain values that are less than the actual ones, but this practice would not be unsafe. It would instead be overly conservative. However, if load test data are interpreted with a critical depth, then some faulty design parameters could result.

13. The so-called critical depth problem has many aspects. The authors have focused on only two of them and suggest that they explain the whole problem. At full-scale, the reason is attributed to residual loads; at model-scale, the reason is attributed to dilatancy-induced stress. While residual loads, calculated by the authors assuming fully-developed shear along the pile shaft, may be an important factor with driven piles, they are of far lesser importance in drilled foundations. And the dilatancy-induced stress effect can only explain a part of the problem, and only for dilative soil and stress states.

14. Instead, attention must be focused on the in-situ soil characteristics, as suggested indirectly by Vesic in 1977.[3] As I pointed out later in my 1984 paper,[20] the dominating characteristics for the side resistance are the $K_0$ profile and its general decrease with depth and the reduction of peak friction angle with increasing stress level. For the tip resistance, the dominating characteristics are the reduction of both peak friction angle and rigidity index with increasing stress level. Kraft,[23] Randolph,[25] and others have made similar observations. An additional important factor to be addressed as well is the influence of soil structure. If a stress level or installation factor causes a collapse of the soil structure, then the stress, strength and rigidity factors can be altered, perhaps drastically.

15. To summarize, the so-called critical depth concept results from many factors, most of which are dominated by the in-situ soil characteristics. The influence of residual loads on top of these factors, particularly for driven piles, is just one further part of the explanation to dispel this concept.

## Authors' reply

The authors appreciate being 'welcomed to the legion'. Indeed, many more should join, because, contrary to Kulhawy's statement, a design based on the 'critical depth' can easily be 'unsafe'. Of course, interpreting results of a static loading test in context with the concept and applying the interpretation to piles of longer length, is conservative—although an error is still an error. However, when the interpretation is applied to piles of shorter length, the resulting design is indeed unsafe. The first author has litigation experience of such a case and the correction to the unsafe design was very dear to one of the participants in the case.

17. The authors agree with Kulhawy that Vesic has been 'credited' somewhat unfairly as being the main instigator of the hypothesis of existence of a 'critical depth'. In fact, the first author remembers having a discussion in Ottawa in early 1980 with Dr Vesic on the occasion of his trans-Canada lecture tour, when he disclaimed being the main original proponent of the 'critical depth'.

18. The authors tried to explain how the same erroneous concept could originate from different types of misinterpretation of data; that is, the 'critical depth' applies to and explains observations obtained in both field tests and model tests. In interpreting the field data, the error lies in the neglect of the residual load, while in interpreting data from model tests in sand, it lies in the ignorance of steady-state soil mechanics. The authors did not dwell on the fact that for deep embedment piles in a homogeneous deposit, the shaft resistance is not a

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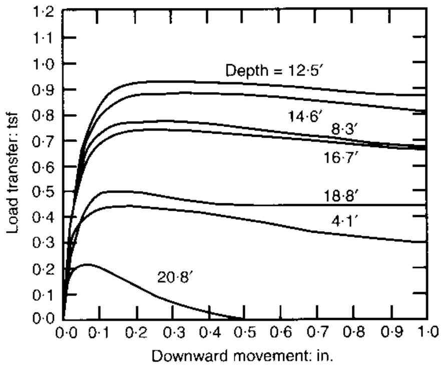


*Fig. 10. Load-movement curves at different depths (Reese et al., 1976)*[27]

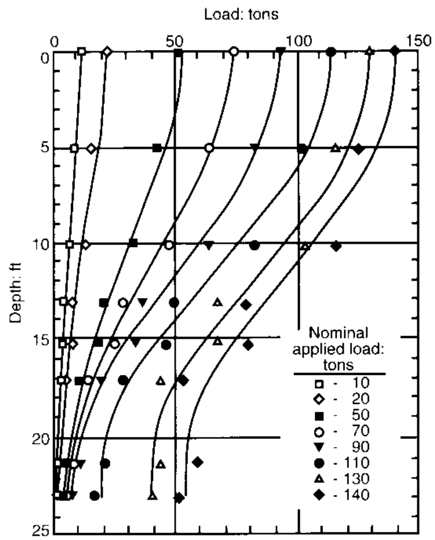


*Fig. 11. Load-transfer curves for a drilled pier (Reese et al., 1976)*[27]

constant function of the overburden stress, but decreases with depth, as Kulhawy mentions. However, the authors do not consider that explaining the fact by means of '*decrease of the $K_0$-profile with depth and reduction of peak friction angle with increasing stress level*' is useful. The authors have addressed the aspect in an earlier paper (Altaee and Fellenius, 1994),[26] noting that the steady-state soil mechanics and the void ratio 'distance' to the steady-state line (the upsilon parameter) is a more direct and useful approach. Of course, the fact is interesting, but the reduction of shaft

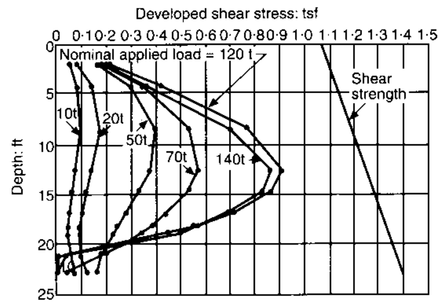


*Fig. 12. Distribution of shaft resistance (Reese et al., 1976)*[27]

resistance occurs only gradually and it does not serve well as an explanation to how the 'critical depth concept' could arise.

19. The authors do not agree with Kulhawy on the point of the residual loads being of 'far lesser importance in drilled foundations'; that is, bored piles, as opposed to driven piles. The literature contains several case histories involving bored piles, where the interpretation of the data appears to have missed that the measurements are influenced by residual load. For example, Reese *et al.* (1976)[27] presented test data on an instrumented, 30-inch (760 mm), bored pile with an embedment of 20 ft (6 m) into a sand and clay soil. Fig. 10 shows the unit shaft resistance versus movement at different depths in the pile, verifying that the full shaft resistance was mobilized along the pile. Fig. 11 shows the load-transfer distribution along the pile suggesting that the ultimate resistance reduced with depth. The similarity between the diagram and that shown in the authors' Figs 5 and 6 is striking. Fig. 12 shows the unit shaft resistance evaluated from the measurements with its typical parabolic shape so recognizable from Figs 1, 5, and 6 of the authors' article. Reese *et al.* (1976)[27] attributed the behaviour to interaction between the pile toe and the lower portion of the shaft. However, we think that the residual load, which was not considered in the evaluation of the data, is the main cause of the indicated reduction of shaft resistance with depth.

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