



# CHAPTER FOUR

# WATER QUALITY ENGINEERING



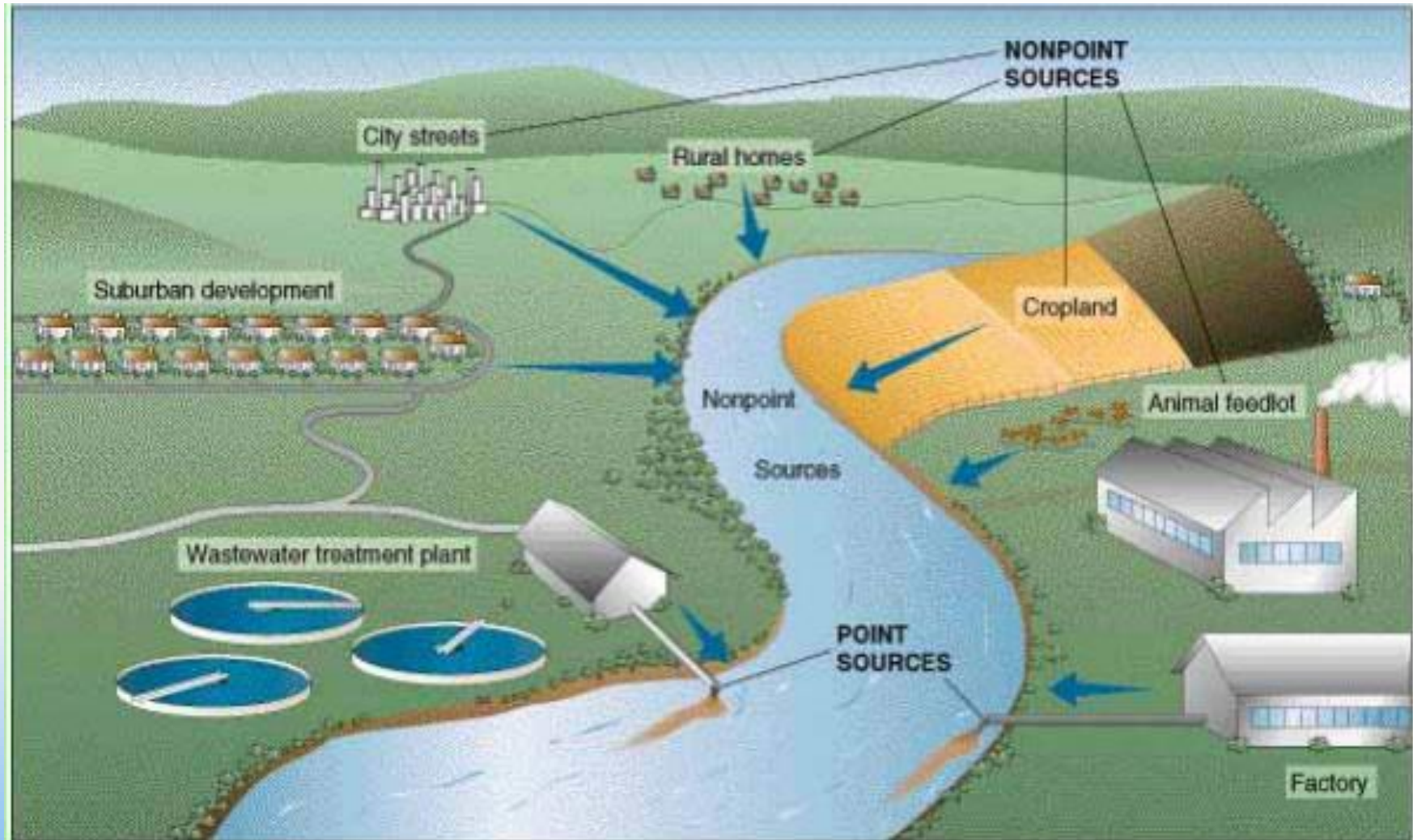
# Water Quality

- Water has different use
  - Agricultural
  - Industrial
  - Domestic
- According to the use we need different scale of quality for water
- Mainly the quality of water is affected by human activities
- Water quality management is concerned with the control of pollution from the human activities so that the water is not degraded to the point that it is no longer suitable for intended uses.

# ||| Physico -Chemical and Biological water pollutant

- the addition of toxic substances;
- the addition of suspended solids;
- deoxygenation;
- the addition of non-toxic salts;
- heating the water;
- the effect on the buffering system;
- the addition of human, animal and plant pathogens.

# SOURCES OF POLLUTANT



# Water pollutants and their sources


Pollutant category	Point sources		Non-point sources	
	Domestic sewage	Industrial wastes	Agricultural runoff	Urban runoff
Oxygen-demanding material	X	X	X	X
Nutrients	X	X	X	X
Pathogens	X	X	X	X
Suspended solids/sediments	X	X	X	X
Salts		X	X	X
Toxic metals		X		X
Toxic organic chemicals		X	X	
Endocrine-disrupting chemicals	X	X	X	
Heat		X		

# ||| River Water Quality Management

- **The objective:**
  - to control the discharge of pollutants so that water quality is not degraded to an unacceptable extent below the natural background level.
- To control we need to:
  - measure the pollutants
  - predict the impact of the pollutants
  - determine the background water quality, and
  - decide the acceptable levels



# Effect of pollutants on river

- The effect of pollutants on river communities depends on:
    - The type of pollutant
    - Its concentration in the water
    - The length of exposure to the community
    - The characteristics of the river (volume and speed water, river's depth)
- 

# Total maximum daily loads (TMDL)

- A TMDL specifies the **maximum** amount of pollutant that a water body can **receive** and **still meet water quality standards**.
  - Use since 1972 after the federal Clean Water Act (CWA) **America**
  - **its goal was to protect and improve the quality** of America's water.
- As a result of recent litigation,
  - current interpretation of the Clean Water Act has extended the use of TMDLs to include nonpoint source pollution.
- Computed on a pollutant-by-pollutant basis

$$\text{TMDL} = \Sigma \text{WLA} + \Sigma \text{LA} + \text{MOS}$$

Where

WLA = waste load allocation for point sources

LA = waste load allocation for nonpoint sources

MOS = margin of safety

Example: TMDL for Standard Domestic use : 250 mg/L



# Effects of Oxygen-Demanding Wastes on Rivers

- Depletes the dissolved oxygen in the water which poses a threat to fish and other higher forms of aquatic life.
- To predict the extent of oxygen depletion
  - How much waste is being discharged and
  - How much oxygen will be required to degrade the waste
- Measured by determining the amount of oxygen consumed during degradation

# Effects of Oxygen-Demanding Wastes on Rivers

As oxygen demanding (organic/degradable) wastes discharged to rivers microorganisms in the river decompose these materials.

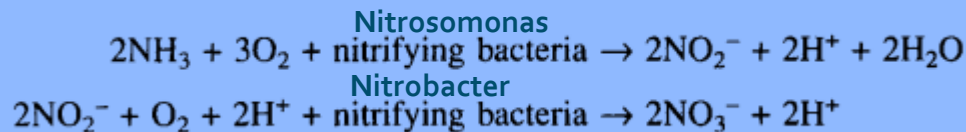


Oxygen taken from the stream

$$1g \text{ glucose} = 192/180g \text{ O}_2$$

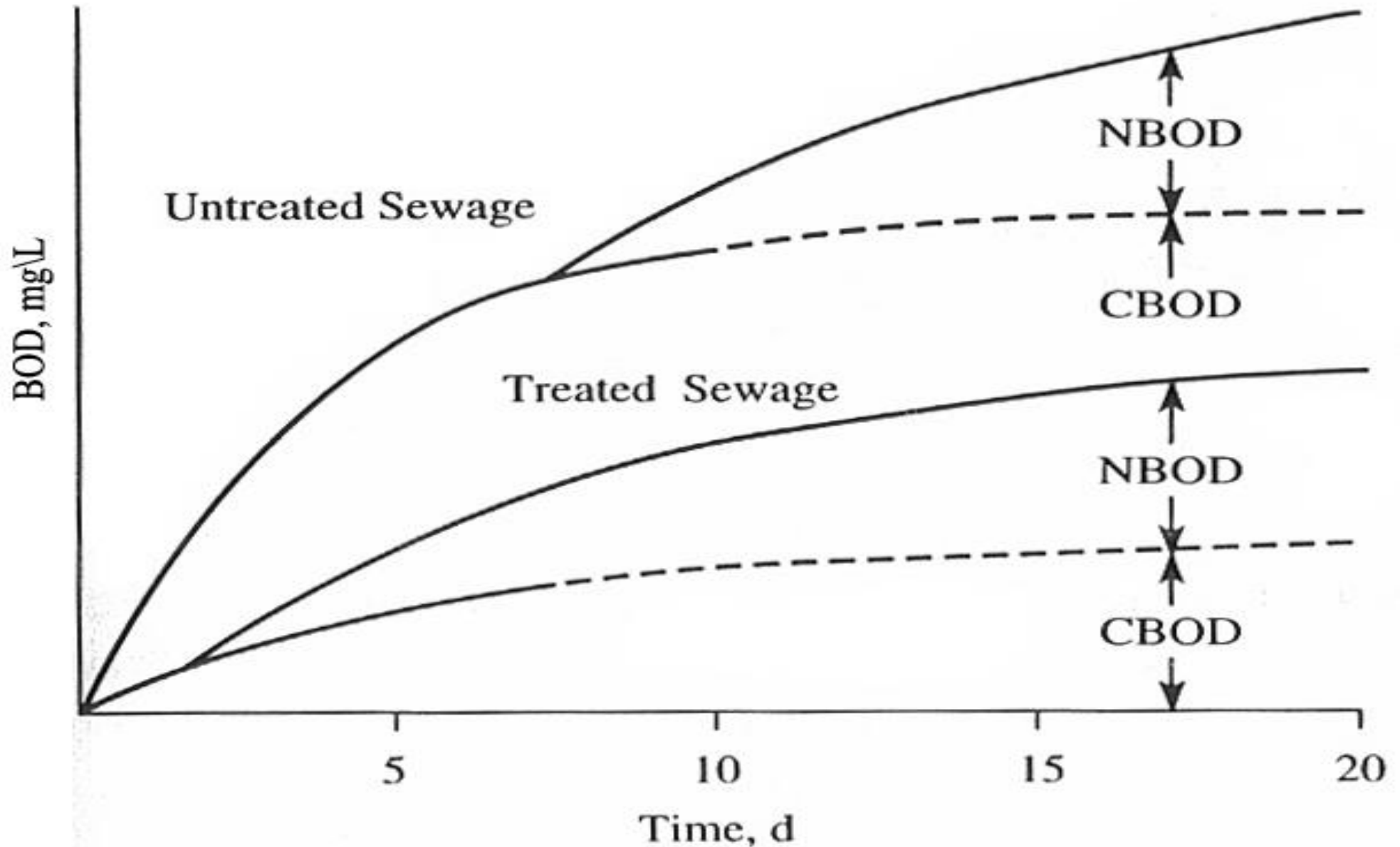
$$1g \text{ glucose consumes } 1.07g \text{ O}_2$$

- In the case of more complex compounds such as protein in addition to the oxygen required to stabilize carbonaceous matter, there is also a considerable oxygen demand during the nitrification of nitrogenous compounds



$$NBOD = \frac{\text{grams of oxygen used}}{\text{grams of nitrogen oxidized}} = \frac{4 \times 16}{14} = 4.57 \frac{g \text{ O}_2}{g \text{ N}}$$

# Oxygen Consumption Curves



# Effects of Oxygen-Demanding Wastes on Rivers cont.

These two demand of oxygen cause depletion of dissolved oxygen (DO) which is a threat for fishes & higher aquatic lives.

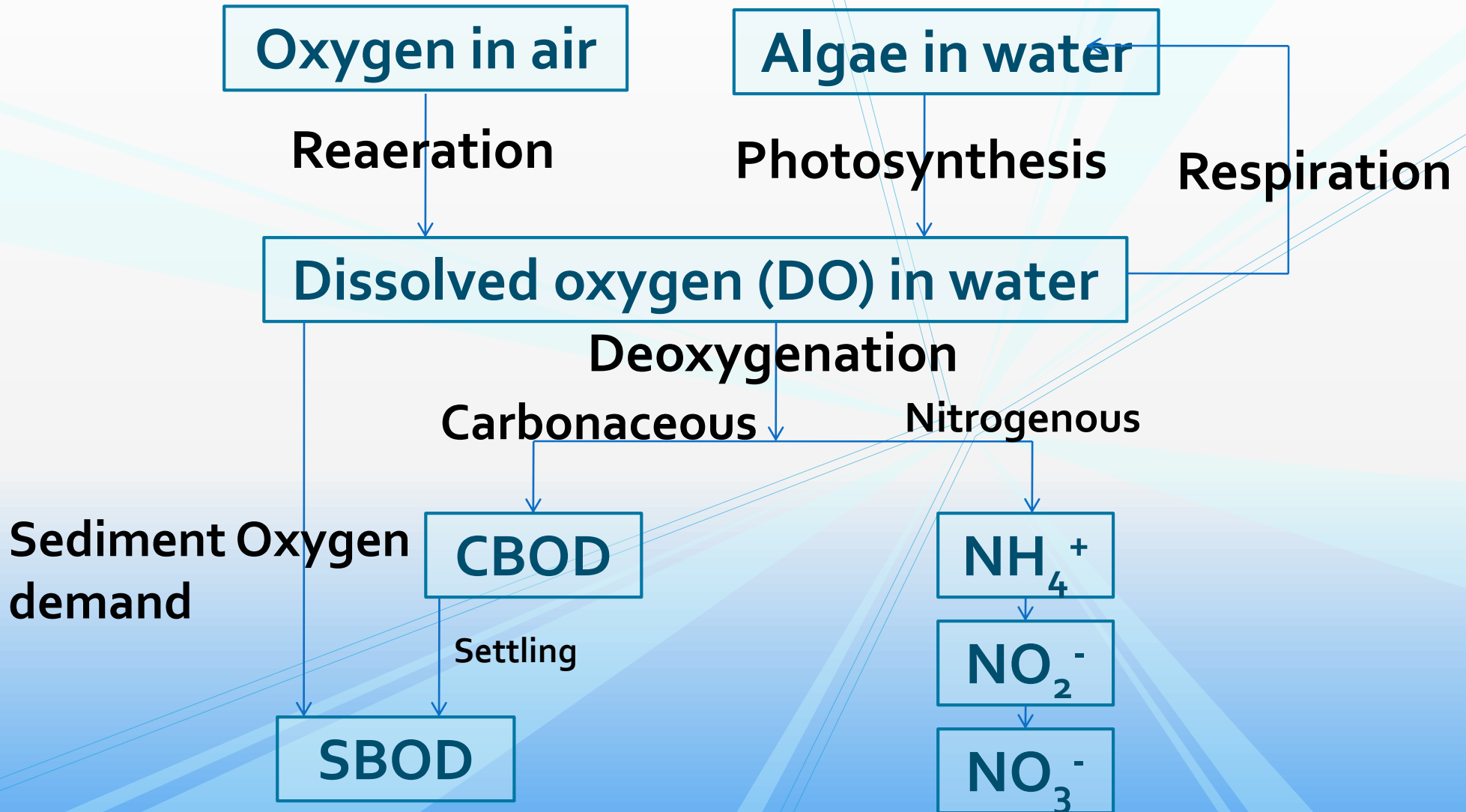
DO level (mg/L)	Qualitative effect
6 – 10	OK
4 – 6	Stressed
2 – 4	Choking
1 – 2	Dying
0 – 1	Dead



The dissolved oxygen is replenished by

- Re-aeration from the atmosphere and
- Photosynthesis by algae and aquatic plants

# Factor Affecting DO in Rivers



# Measurement of Oxygen demand

## A. Theoretical oxygen demand (ThOD)

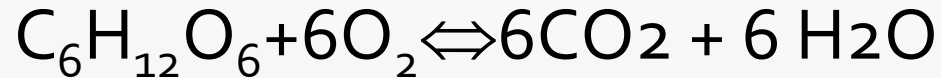
- The amount of oxygen required to oxidize a substance to carbon dioxide and water may be calculated by **stoichiometry** if the chemical composition of the substance is known.
- Unlike BOD & COD tests the theoretical oxygen demand (ThOD) is **not a laboratory test**. Because it basis of a complete chemical analysis

# Example

Compute the ThOD of 108.75 mg/L of glucose ( $C_6H_{12}O_6$ ).

## Solution

- Write balanced equation for the reaction



- Compute the molecular weight
- Glucose ( $6C + 12H + 6O$ ) =  $72 + 12 + 96 = 180$
- Oxygen ( $12O$ ) = 192
- Thus, it takes 192 g of oxygen to oxidize 180 g of glucose
- The ThOD of 108.75 mg/L of glucose is

$$(108.75 \text{ mg} / \text{L}) \left( \frac{192 \text{ g } O_2}{180 \text{ g glucose}} \right) = 116 \text{ mg} / \text{L } O_2$$

# Measurement of Oxygen demand

## B. Chemical Oxygen Demand (COD)

- The COD test is used to determine the oxygen equivalent of the organic matter that can be oxidized by a strong chemical oxidizing agent (potassium dichromate)
- In the COD test, a strong chemical oxidizing agent (chromic acid) is mixed with a water sample and then boiled.
- The difference between the amount of oxidizing agent at the beginning of the test and that remaining at the end of the test is used to calculate the COD.



# Chemical Oxygen Demand (COD) cont.

## Advantage

- ✓ Used to measure the oxidizable matter in industrial and municipal wastes containing compounds that are **toxic to biological life**
- ✓ Completed within only 3 hours

## Disadvantage

- ✓ no indication of whether or not the substance is **degradable biologically**.
- ✓ Don't indicate the **rate** at which biological oxidation would proceed and hence the rate at which oxygen would be required in a biological system

# Measurement of Oxygen demand cont.

## C. Biochemical Oxygen Demand (BOD)

- The BOD test measures the oxygen consumed by bacteria while oxidizing organic matter under aerobic conditions.

### Test Procedure

1. a small sample of the wastewater to be tested is placed along with dilution water in a BOD bottle (300 ml).
2. The dissolved oxygen concentration of the mixture in the bottle is measured.
3. The bottle is incubated for 5 days at 20°C and the dissolved oxygen concentration is measured again.
4. The BOD of the sample is the decrease in the dissolved oxygen concentration values, expressed in mg/l; divided by the decimal fraction of the sample used.

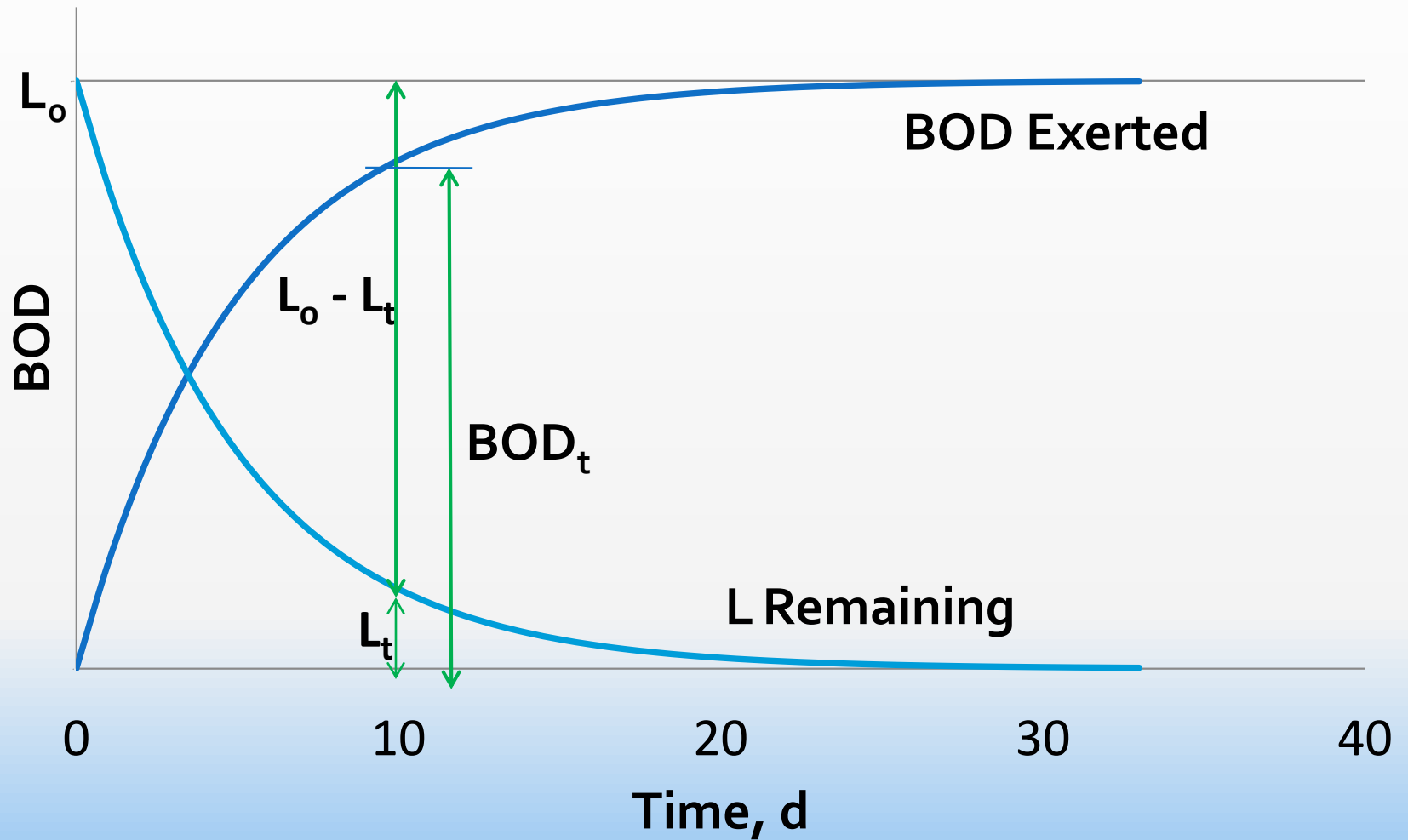
# Measurement of Oxygen demand of

## C. Biochemical Oxygen Demand

### Limitations of BOD test:

1. A minimum DO depletion of 2 mg/l is desirable
2. The final DO should never be 0 mg/l (as it is impossible to know when the entire DO content got fully depleted i.e., within 1, 2, 3, 4 or 5 days) and preferably it should not be less than 1 mg/l

# BOD Curve



$$BOD_t = L_o - L_t$$

# Biochemical Oxygen Demand (BOD)

**cont.** In a BOD test, the rate at which **organics are utilized** by microorganisms is assumed to be a **first order reaction**.

- The rate at which organics utilized is proportional to the amount of oxidizable organic matter available.

$$\frac{dL}{L} = -k^1 dt$$

$$\int_L^{L_t} \frac{dL}{L} = -k^1 \int_0^t dt$$

$$\log_e \frac{L_t}{L_o} = -k^1 t$$

$$L_t = L_o e^{-k^1 t}$$

**BOD exerted = Ultimate BOD – BOD remaining**

$$\begin{aligned} BOD_t &= L_o - L_t \\ &= L_o - (L_o e^{-kt}) \end{aligned}$$

$$BOD_t = L_o (1 - e^{-kt})$$

# ||| Biochemical Oxygen Demand (BOD)

## cont.

### Example

- If the 3 day BOD ( $BOD_3$ ) of a waste is 75mg/l and the BOD decay constant,  $k$ , is  $0.345d^{-1}$ . What is the ultimate BOD?

# ||| Biochemical Oxygen Demand (BOD)

cont.

- Ultimate BOD best expresses the concentration of degradable organic matter
- But by itself it does not indicate how rapidly oxygen will be depleted.
- Oxygen depletion is related to ultimate BOD and BOD rate constant ( $k$ )
- The rate constant is dependent on the following
  - The nature of the waste
  - The ability of the organisms in the system to use the waste
  - The temperature

# Biochemical Oxygen Demand (BOD) cont.

## Typical Values for the BOD Rate Constant

Sample	K (20°C) (day <sup>-1</sup> )
Raw sewage	0.35-0.70
Well treated sewage	0.12-0.23
Polluted river water	0.13-0.23

$$k_T = k_{20}(\theta)^{T-20}$$

where  $T$  = temperature of interest (in °C)

$k_T$  = BOD rate constant at the temperature of interest (in days<sup>-1</sup>)

$k_{20}$  = BOD rate constant determined at 20°C (in days<sup>-1</sup>)

$\theta$  = temperature coefficient. For typical domestic wastewater, this has a value of 1.135 for temperatures between 4 and 20°C and 1.056 for temperatures between 20 and 30°C. (Schroepfer, Robins, and Susag, 1964)



# ||| Biochemical Oxygen Demand (BOD)

## cont.

### Example

- A waste is being discharged into a river that has a temperature of  $10^{\circ}\text{C}$ . What fraction of the maximum oxygen consumption has occurred in 4 days if the BOD rate constant,  $k$ , determined in the laboratory under standard conditions is  $0.115 \text{ day}^{-1}$ ?

*(Note: All rate constants are base  $e$ .)*

# Correlation of ThOD, COD & BOD

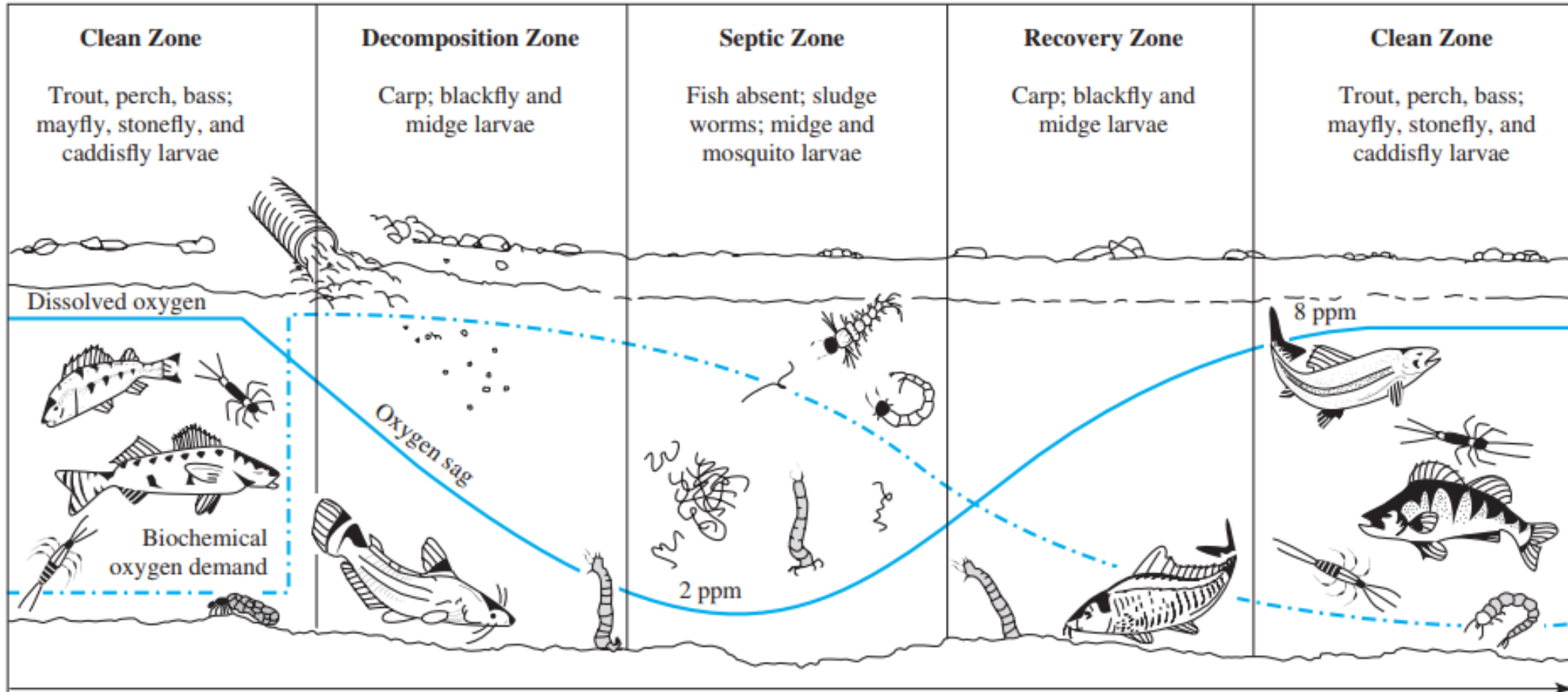
- ThoD give the greatest value of all there.
- This is because Thod measurement doesn't account the carbon used for new bacteria cell synthesis.
- BOD and COD are correlated by Biodegradability Index .
- Biodegradability Index is the ratio of BOD<sub>5</sub> & COD and it varies from 0.4 to 0.8 for domestic wastewaters.
- If BOD/COD is  $> 0.6$  then the waste is fairly biodegradable and can be effectively treated biologically.
- If BOD/COD ratio is between 0.3 and 0.6, then seeding is required to treat it biologically.
- If BOD/COD is  $< 0.3$  then it cannot be treated biologically.

# DO Sag Curve

- To assess the capability of a river to absorb the waste load the profile of DO concentration downstream of the waste discharge is determined.
- This profile is called the DO sag curve.
- DO sag curve is the profile of DO concentration downstream of the waste disposal site.
- DO sag equation is developed using O<sub>2</sub> deficit rather than DO concentration to make it easier to solve the integral equation

# DO Sag Curve

Oxygen sag downstream of an organic source.



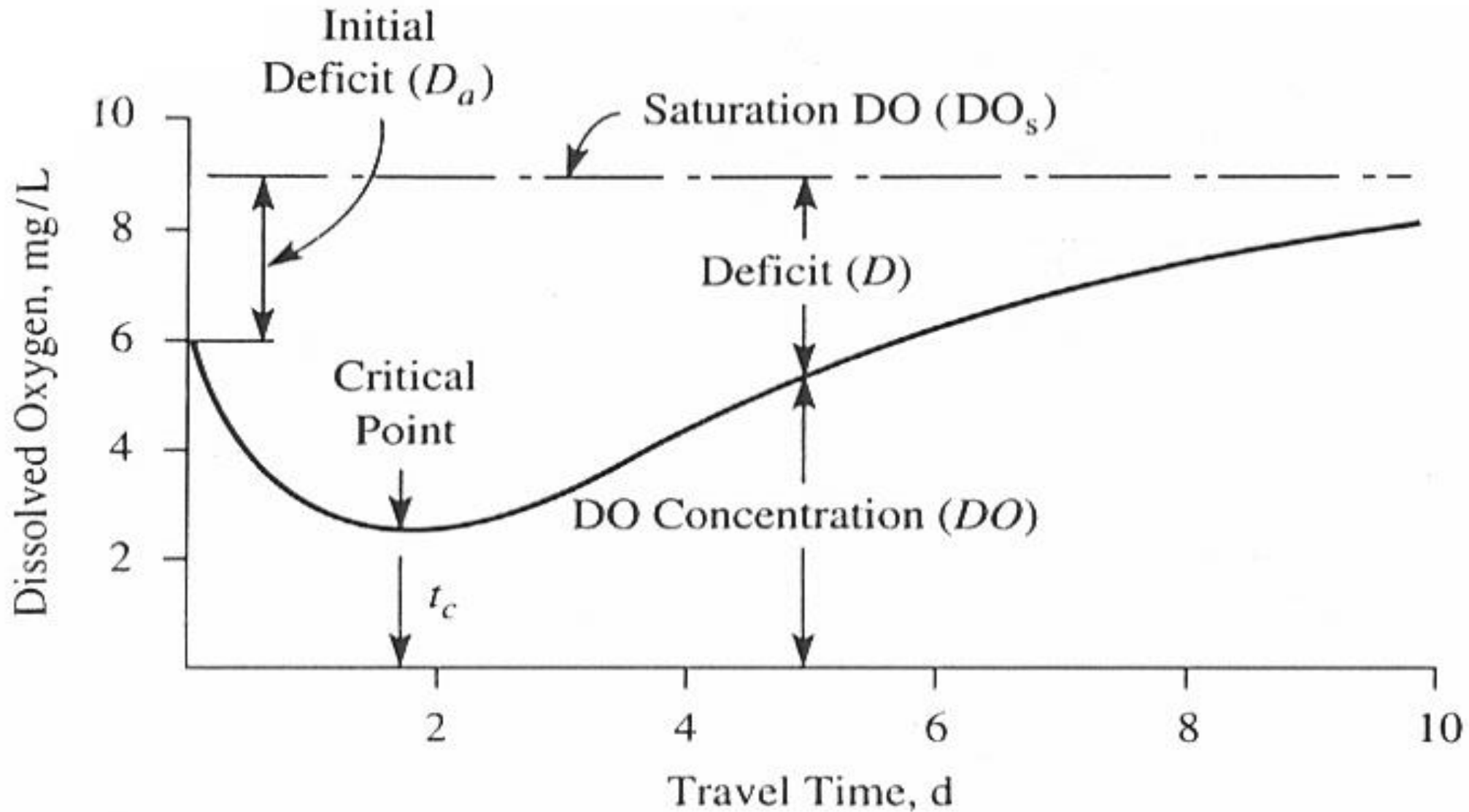
the sources of oxygen  
the factors affecting oxygen depletion must be quantified.

# DO Sag Curve (Mass balance approach)

## Steps in Developing the DO Sag Curve

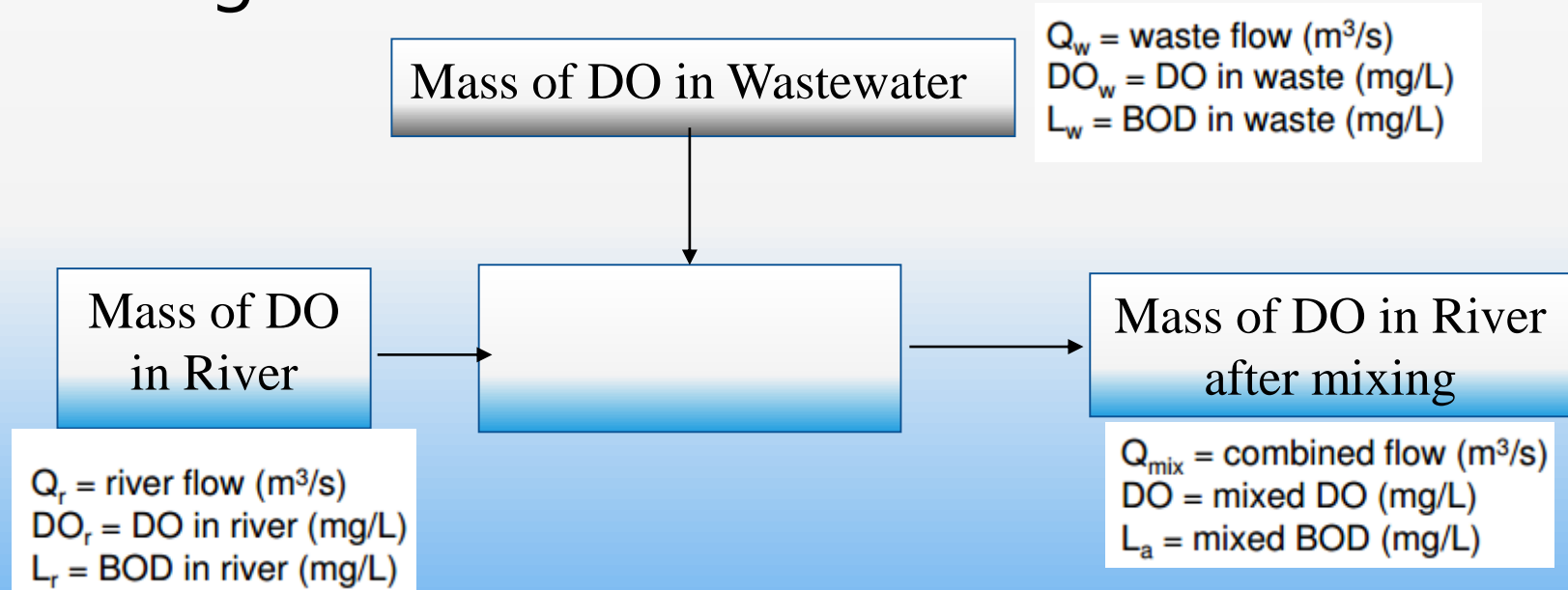
1. Determine the initial conditions
2. Determine the de-oxygenation rate from BOD test and stream geometry
3. Determine the re-aeration rate from stream geometry
4. Calculate the DO deficit as a function of time
5. Calculate the time and deficit at the critical point (worst conditions)

# DO Sag Curve



# Step 1 Mass-Balance Approach for Initial Condition

- Three conservative mass balances may be used to account for initial mixing of the waste stream and the river.
- DO, CBOD, and temperature all change as the result of mixing of the waste stream and the river.



# DO and BOD after mixing

$$DO = \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r} \quad L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

Where  $Q_w$  = volumetric flow rate of wastewater,  $m^3/s$

$Q_r$  = volumetric flow rate of river,  $m^3/s$

$DO_w$  = dissolved oxygen concentration in the wastewater,  $g/m^3$

$DO_r$  = dissolved oxygen concentration in the river,  $g/m^3$

$L_w$  = ultimate BOD of the wastewater,  $mg/L$

$L_r$  = ultimate BOD of the river,  $mg/L$

$L_a$  = initial ultimate BOD after mixing  $mg/L$



# ■ The temperature after mixing

- For temperature, we must consider a heat balance rather than a mass balance.

Loss of heat by hot bodies = gain of heat by cold bodies

$$(m_w)(4.19)(\Delta T_w) = (m_r)(4.19)(\Delta T_r)$$

The final river temperature after mixing is thus calculated as:

$$T_f = \frac{Q_w T_w + Q_r T_r}{Q_w + Q_r}$$

## Example

A certain college discharges  $17,360 \text{ m}^3/\text{d}$  of treated wastewater into a nearby river. The treated wastewater has a  $\text{BOD}_5$  of  $12 \text{ mg/L}$  and  $k$  of  $0.12 \text{ d}^{-1}$  at  $20^\circ\text{C}$ . The river has a flow rate of  $0.43 \text{ m}^3/\text{s}$  and an ultimate BOD of  $5.0 \text{ mg/L}$ . The DO of the river is  $6.5 \text{ mg/L}$  and the DO of the wastewater is  $1.0 \text{ mg/L}$ . Compute the DO and initial ultimate BOD after mixing.

**Ans**  $\text{Do}_{\text{mix}}$  **4.75mg/l**  
**La = 11.86mg/l**

# Oxygen Deficit

- The oxygen deficit is the amount by which the actual dissolved oxygen concentration is less than the saturated value with respect to oxygen in the air:

- $D = DO_s - DO$

$$D_a = DO_s - \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r}$$

Where  $D$  = oxygen deficit, mg/L

$DO_s$  = saturation concentration of dissolved oxygen at the temperature of the river after mixing, mg/L

$DO$  = actual dissolved oxygen concentration, mg/L

## DO Saturated

- The saturation value of dissolved oxygen is heavily dependent on water temperature.
- It decreases as the temperature increases.
- Saturation values of dissolved oxygen in freshwater exposed to a saturated atmosphere containing 20.9% oxygen under a pressure of 101.325kPa are given in the table below.

Temperature (°C)	Dissolved oxygen (mg/l)
0	14.62
5	12.8
10	11.33
15	10.15
20	9.15
25	8.38
30	7.63

## Example

Calculate the initial deficit of the river after mixing with the wastewater from the college (previous example). The stream temperature is  $10^{\circ}\text{C}$  and the wastewater temperature is  $20^{\circ}\text{C}$ .

# ||| Solution

- Saturation dissolved oxygen can be read from table.  
Thus at 10°C  $DO_s = 11.33$  mg/L
- From the previous example  $DO_{mix} = 4.75$  mg/L
- Therefore, initial deficit DO,  $D_a = 11.33 - 4.75$
- $D_a = 6.58$  mg/L

## Step 2 Determine De-oxygenation Rate

$$\text{rate of de-oxygenation} = k_d L_t$$

$$k_d = k + \frac{v}{h} \eta$$

Where

k=BOD rate constant (at 20°C)

v=average velocity of stream flow

h=average depth of stream

$\eta$ =bed activity coefficient (ranges from 0.1 for stagnant or deep water to 0.6 for rapidly flowing water)

$$L_t = L_0 e^{-k_d t}$$

Where

$L_t$ = ultimate BOD remaining at time

$L_0$ =Ultimate BOD

## Step 3 Determine Re-aeration Rate

$$\text{rate of re-aeration} = k_r D$$

$$k_r = \frac{3.9v^{0.5}}{H^{1.5}}$$

$k_r$  = re-aeration constant depends on the degree of turbulent mixing, which is related to stream velocity, and on the amount of water surface exposed to the atmosphere compared to the volume of water in the river

$v$  = average stream velocity, m/s

$H$  = average depth, m

**$D$  = dissolved oxygen deficit (DOs-DO)**

DOs = saturated value of oxygen

DO = actual dissolved oxygen at a given location downstream



## Example

Determine the deoxygenation rate constant for the reach of the river (For the previous examples) below the wastewater outfall (discharge pipe). The average speed of the stream flow in the river is 0.03 m/s. The depth is 5.0 m and the bed-activity coefficient is 0.35.

# ■ Solution

- From the previous examples we have  $k = 0.12d^{-1}$
- The deoxygenation constant at  $20^{\circ}\text{C}$  is

$$k_d = 0.12d^{-1} + \frac{0.03m/s}{5.0m} (0.35) = 0.1221d^{-1}$$

- The deoxygenation constant is at  $20^{\circ}\text{C}$ , but the Stream temperature is  $10^{\circ}\text{C}$ . Thus we must correct it

$$k_d \text{ at } 10^{\circ}\text{C} = (0.1221d^{-1})(1.135)^{10-20} = 0.034d^{-1}$$

## Step 4 DO deficit as a function of time (Streeter Phelps equation)

- Taking a mass balance of DO as a reactive substance expressed as the deficit we get the Streeter-Phelps oxygen sag curve

Rate of increase  
Of the deficit

=

Rate of  
deoxygenation

-

Rate of  
oxygenation

$$\frac{dD}{dt} = k_d L - k_r D$$

$dD/dt$  = the change in oxygen deficit (D) per unit time, mg/L.d

$k_d$  = deoxygenation rate constant,  $d^{-1}$

$k_r$  = reaeration rate constant,  $d^{-1}$

L = ultimate BOD of river water, mg/L

D = oxygen deficit in river water, mg/L

# DO Sag Equation

- By integrating the previous Eq., and using the initial conditions (at  $t = 0$ ,  $D = D_a$  and  $L=L_a$ ) we obtain the DO sag equation:

$$D = \frac{k_d L_a}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_a (e^{-k_r t})$$

$D$  = oxygen deficit in river water after exertion of BOD for time,  $t$ , mg/L

$L_a$  = initial ultimate BOD after river and wastewater have mixed, mg/L

$t$  = time of travel of wastewater discharge downstream, d

# Step 5 Time and deficit at the critical point

- The lowest point on the DO sag curve, which is called the critical point, indicates the worst conditions in the river.
- The time to the critical point ( $t_c$ ) can be calculated by differentiating and setting DO sag equation to zero

$$t_c = \frac{1}{k_r - k_d} \ln \left[ \frac{k_r}{k_d} \left( 1 - D_a \frac{k_r - k_d}{k_d L_a} \right) \right] \quad D_c = \frac{k_d L_a}{k_r - k_d} (e^{-k_d t_c} - e^{-k_r t_c}) + D_a e^{-k_r t_c}$$

- when  $k_d = k_r$ : 
$$t_c = \frac{1}{k_d} \left( 1 - \frac{D_a}{L_a} \right)$$

# Example

A wastewater of  $5.0 \text{ m}^3/\text{sec}$  is discharged into a river of flow  $50 \text{ m}^3/\text{sec}$ . The ultimate BOD of wastewater is  $200 \text{ mg/l}$  and DO is  $1.5 \text{ mg/l}$ . The river water has a BOD of  $3 \text{ mg/l}$  and DO of  $7 \text{ mg/l}$ . The reaeration coefficient of the river water is  $0.2/\text{day}$  and BOD decay coefficient is  $0.4/\text{day}$ . The river has a cross-sectional area of  $200 \text{ m}^2$  and the saturated DO concentration of the river water is  $8 \text{ mg/l}$ .

- (a) At a downstream point of  $10 \text{ km}$  calculate the DO of the mixture.
- (b) At which point the DO is a bare minimum.
- (c) If a minimum DO needed in the river is  $4 \text{ mg/l}$ , do you allow the wastewater to be discharged without treatment?

# Solution

	wastewater	stream	After disposal
Flow (m <sup>3</sup> /sec)	5	50	55
Dissolved oxygen, mg/l	1.5	7	$=\frac{1.5*5+7*50}{5+50}=6.5$
Ultimate BOD, mg/l	200	3	$=\frac{200*5+3*50}{5+50}=20.91$
Kd	0.2		0.2
Kr		0.3	0.3

Initial DO deficit (Da)=8-6.5

=1.5mg/l

Initial ultimate BOD(La)=20.91

$$\text{Velocity of flow} = \frac{\text{Rate of flow}}{\text{Area of cross-section}} = \frac{50 + 5}{200} = 0.275 \text{ m/s}$$

Length of flow = 10 km = 10000 m

$$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{10000}{0.275} = 36363.63 \text{ s} = 0.42 \text{ d}$$

# Solution

- a. Using Streeter Phelps equation the deficit at 0.42d can be found.

$$D = \frac{k_d L_a}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_a (e^{-k_r t})$$
$$= 2.9 \text{ mg/l}$$

DO = DO saturated - deficit

$$= 8 - 2.9$$

$$= 5.1 \text{ mg/l}$$

b.

$$t_c = \frac{1}{k_r - k_d} \ln \left[ \frac{k_r}{k_d} \left( 1 - D_a \frac{k_r - k_d}{k_d L_a} \right) \right]$$

$$= 3.69 \text{ days}$$

$$\text{distance} = \text{velocity} * t_c = 0.275 \text{ m/sec} * 3.69 * 24 * 3600$$

$$= 87.67 \text{ km}$$



# Solution

C. 
$$D_c = \frac{k_d L_a}{k_r - k_a} (e^{-k_d t_c} - e^{-k_r t_c}) + D_a e^{-k_r t_c}$$

$$= 6.67 \text{ mg/l}$$

DO = DO saturated - deficit

$$= 8 - 6.67$$

$$= 1.33 \text{ mg/l} < 4 \text{ mg/l}$$

Hence the wastewater should be treated before it is discharged to the river.

# Nitrogenous BOD in DO sag equation

- only carbonaceous BOD has been considered
- Nitrogenous BOD can be incorporated into the DO sag curve by adding an additional term

$$D = \frac{k_d L_a}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_a (e^{-k_r t}) + \frac{k_n L_n}{k_r - k_n} (e^{-k_n t} - e^{-k_r t})$$

where

$k_n$  = the nitrogenous deoxygenation coefficient (in day<sup>-1</sup>)

$L_n$  = ultimate nitrogenous BOD after waste and river have mixed (in mg /l)

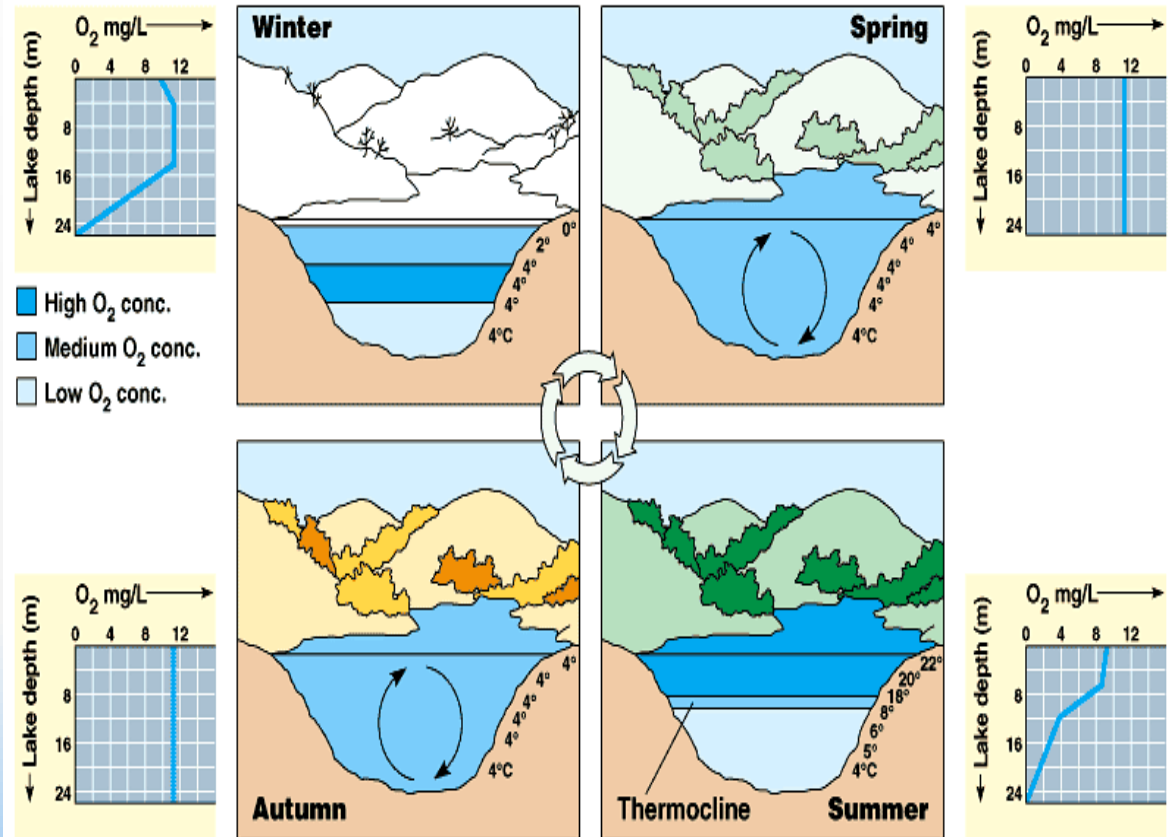
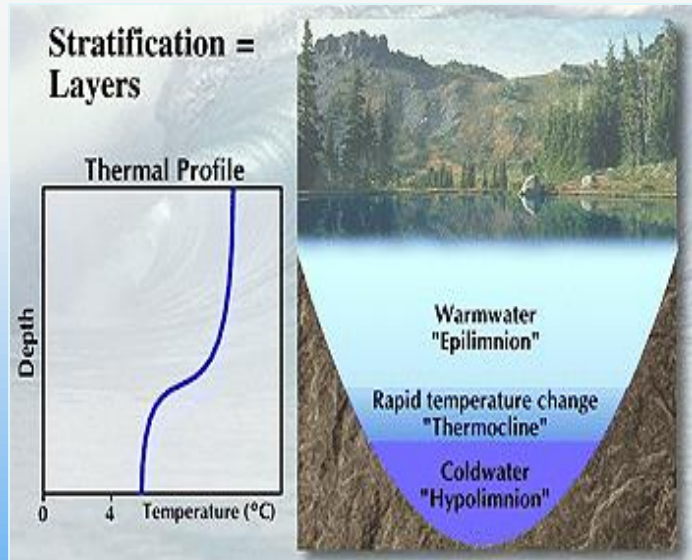
# Water Quality in Lakes

The study of lakes is called limnology



# Stratification and Turn over

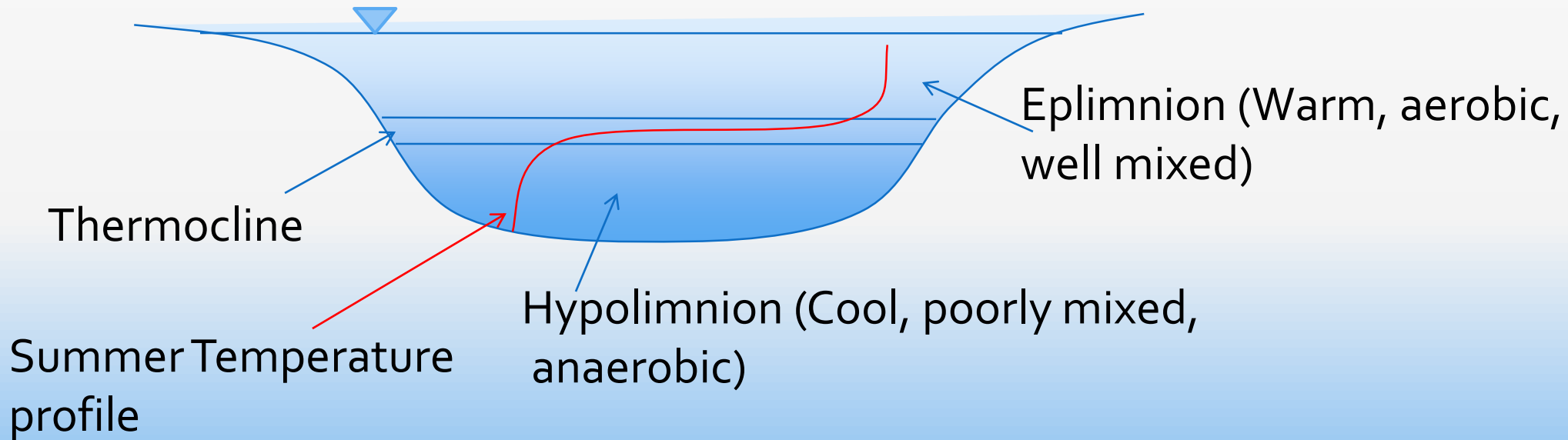
- Vertical stratification in lakes
  - Epilimnion
  - Thermocline
  - Hypolimnion



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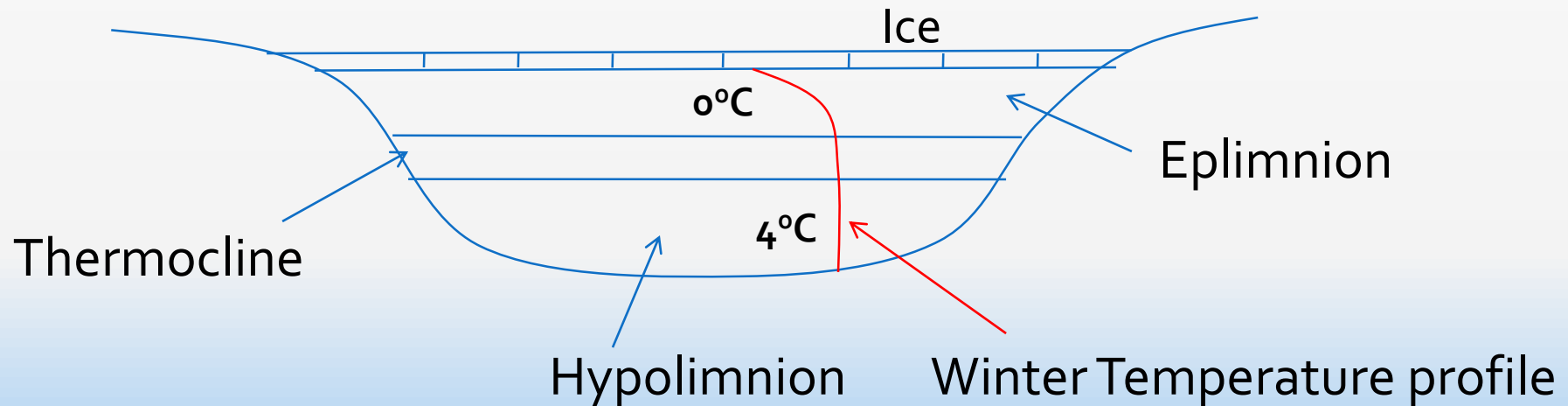
# Stratification and Turn over

- Lakes in temperate zone become stratified during summer and overturn (turnover) in the fall.



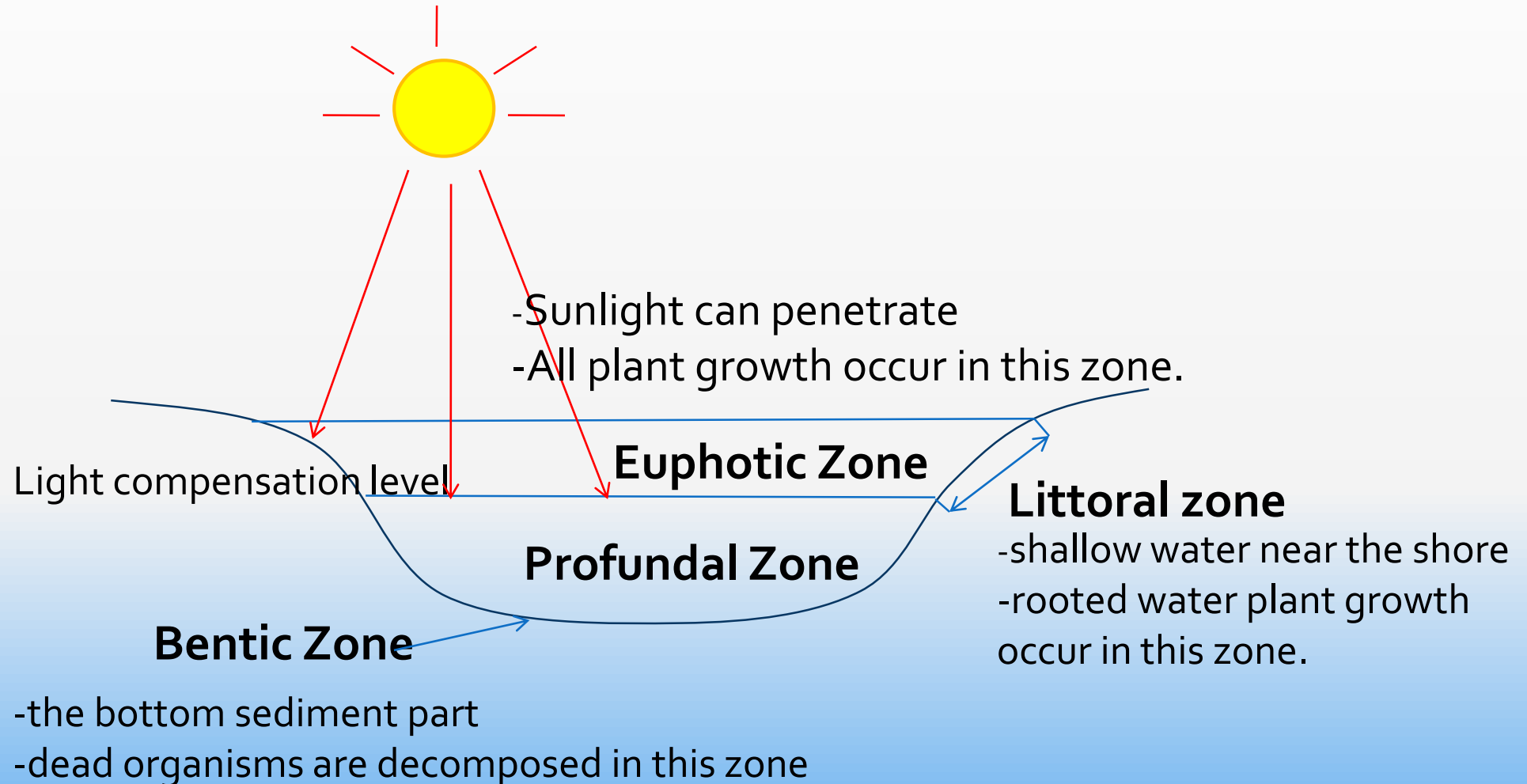
# Stratification and Turn over

- During winter there is turnover at  $4^{\circ}\text{C}$  since water is most dense at  $4^{\circ}\text{C}$ .



# Biological Zones

- Determined by availability of oxygen and light



# ||| Lake Productivity

- Lake productivity is a measure of its ability to support a food web.
- Generally measured by algal growth
- Increased productivity reduces water quality
- Based on productivity we can classify lakes as
  - **Oligotrophic**: low level of productivity
  - **Mesotrophic**: intermediate productivity
  - **Eutrophic**: high level of productivity
  - **Senescent**: old and shallow lakes with thick organic sediment and rooted plant



# Eutrophication

**Eutrophication** is a natural process in which lakes gradually become shallower and more productive through the introduction and cycling of nutrients.

Oligotrophic

Mesotrophic

Eutrophic

Senescent

**Cultural Eutrophication** is caused by human activity by speeding up the natural process

# Algal Growth Requirement

- **Carbon**: from dissolved  $\text{CO}_2$  in water
- **Nitrogen**: usually as  $\text{NO}_3^-$ , comes from atmosphere or industrial waste
- **Phosphorus**: from external sources as inorganic form ( $\text{PO}_4^{3-}$ )
- Trace elements

# ||| The limiting Nutrient

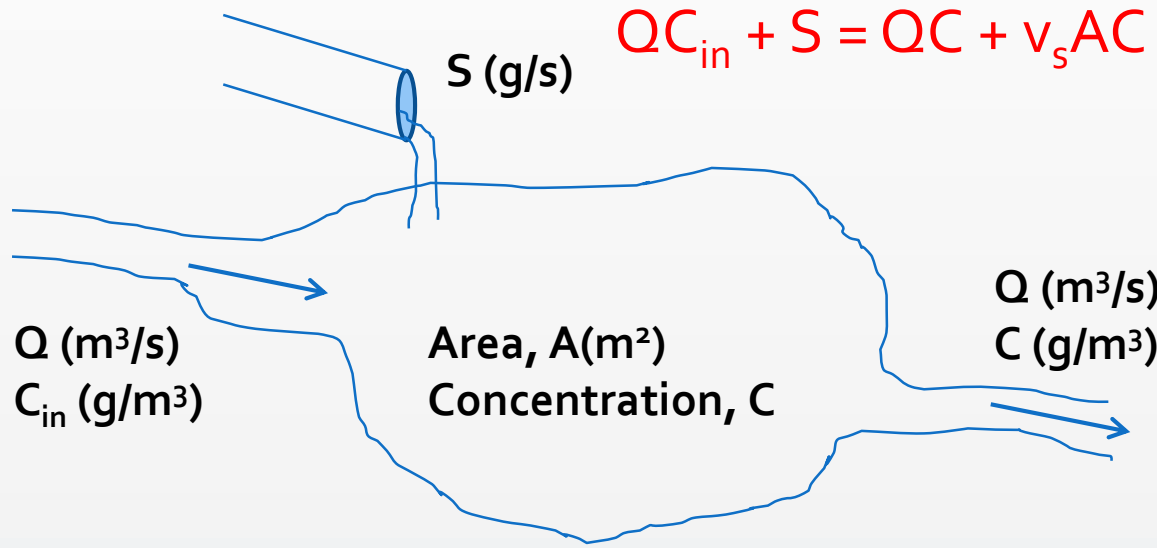
- Justin Liebig's law of the minimum: "*growth of a plant is dependent on the amount of foodstuff that is presented to it in minimum quantity.*"
- Phosphorus is the limiting nutrient in lakes.
- Should be below 0.010 to 0.015 mg/L to limit algae.

# Control of Phosphorus

- Reducing the input from the source
  - Rock weathering
  - Human activity
- Precipitating with alum
- Removing phosphorus-rich sediments by dredging.

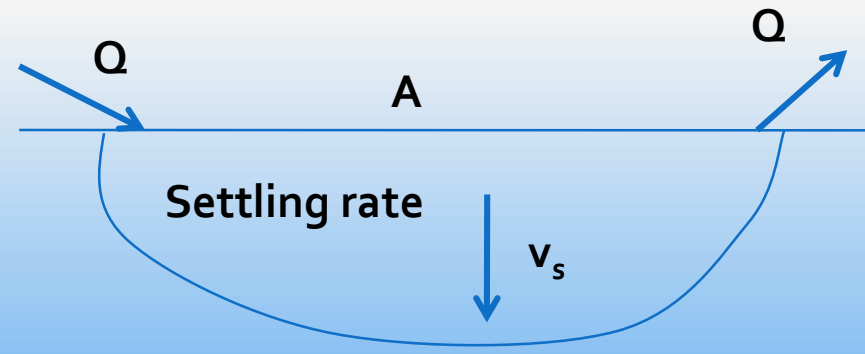
# A simple phosphorus model

- Rate of addition of P = Rate of removal of P



$$QC_{in} + S = QC + v_s AC$$

Well-mixed lake phosphorus mass balance



# ■ A simple phosphorus model

$$QC_{in} + S = QC + v_s AC$$

The steady-state concentration becomes

$$C = \frac{QC_{in} + S}{Q + v_s A}$$

The settling rate,  $v_s$ , is empirically determined quantity. Usually it is around 10 to 16 m/yr.

# Example: Phosphorus loading

A phosphorus-limiting lake with surface area equal to  $80 \times 10^6 \text{ m}^2$  is fed by a  $15.0 \text{ m}^3/\text{s}$  stream that has a phosphorus concentration of  $0.010 \text{ mg/L}$ . In addition, effluent from a point source adds  $1 \text{ g/s}$  of phosphorus. The phosphorus settling rate is estimated at  $10 \text{ m/yr}$ .

- a. Estimate the average total phosphorus concentration.
- b. What rate of phosphorus removal at the wastewater treatment plant would be required to keep the concentration of phosphorus in the lake at acceptable level of  $0.010 \text{ mg/L}$ ?

# ■ Solution

a. Phosphorus loading from the incoming stream

$$QC_{in} = 15 \text{ m}^3/\text{s} \times 0.01 \text{ mg/l} \times (1\text{g/L})/(\text{mg/L}) = 0.15 \text{ g/s}$$

The estimated settling rate:

$$v_s = \frac{10 \text{ m / yr}}{365 \text{ d / yr} \times 24 \text{ hr / d} \times 3600 \text{ s / hr}} = 3.17 \times 10^{-7} \text{ m / s}$$

The steady-state concentration of phosphorus:

$$C = \frac{QC_{in} + S}{Q + v_s A} = \frac{(0.15 + 1.0) \text{ g / s}}{15 \text{ m}^3/\text{s} + 3.17 \times 10^{-7} \text{ m / s} \times 80 \times 10^6 \text{ m}^2}$$
$$= 0.028 \text{ g/m}^3 = 0.028 \text{ mg/L}$$



# ||| Solution

- b. To reach 0.010 mg/L, the phosphorus loading from the point source must be

$$\begin{aligned} S &= C(Q + v_s A) - QC_{in} \\ &= 0.010 \text{ g/m}^3 (15 \text{ m}^3/\text{s} + 3.17 \times 10^{-7} \text{ m/s} \times 80 \\ &\quad \times 10^6 \text{ m}^2) \\ &\quad - 15 \text{ m}^3/\text{s} \times 0.010 \text{ g/m}^3 \\ &= 0.25 \text{ g/s} \end{aligned}$$

The point source efficiency currently supplies 1.0 g/s, so 75 % removal of phosphorus is needed.