CHAPTER THREE

Material and Energy Balance

AAiT-SCEE

Materials and Energy Balances

- Tools for quantitative understanding of the behavior of environmental systems.

- For accounting of the flow of energy and materials into and out of the environmental systems.

Materials and Energy Balances

Material Balance

Energy Balance

Pollutant

Energy

modeling

production, transport, and fate

Materials and Energy Balances

Example of Mass balance

- Prediction of rainwater runoff
- Oxygen balance in a stream

Example of Energy balance

- Efficiency estimation of thermal process
- Temperature rise prediction in a stream from the discharge of cooling water from a power plant

Unifying Theories

Conservation of Matter

- The *law of conservation of matter* states that (without nuclear reaction) matter can neither be created nor destroyed.
- We ought to be able to account for the “matter” at any point in time.
- The mathematical representation of this accounting system is called a *materials balance or mass balance.*

Unifying Theories

Conservation of Energy

- The *law of conservation of energy* states that energy cannot be created or destroyed.
- Meaning that we should be able to account for the "energy" at any point in time. (we should be able to trace the **energy of food through a series of organisms** from green plants through animals)

- The mathematical representation of this accounting system we use to trace energy is called an *energy balance.*

Material Balances

Important steps to follow

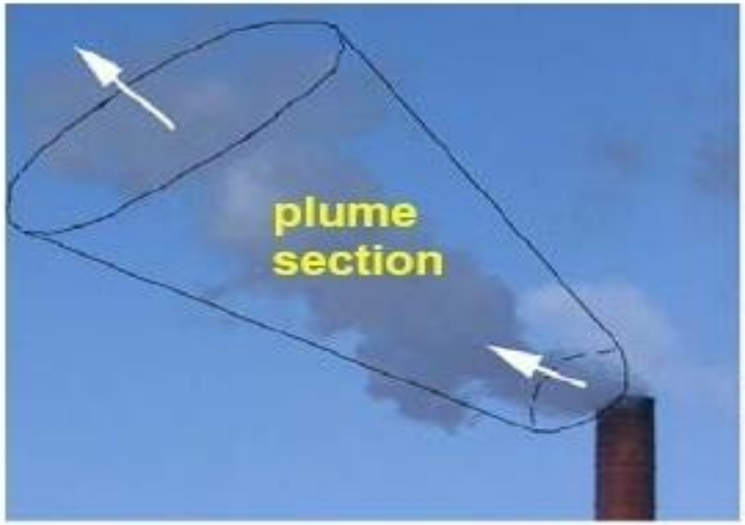
1. Flow chart
2. Compatible mass units
3. Identify system boundary (Control Volume)

- The simplest form of a materials balance or mass balance

Accumulation = input – output

Environmental System (Natural or Device)

Examples of Control Volume



Time as a factor

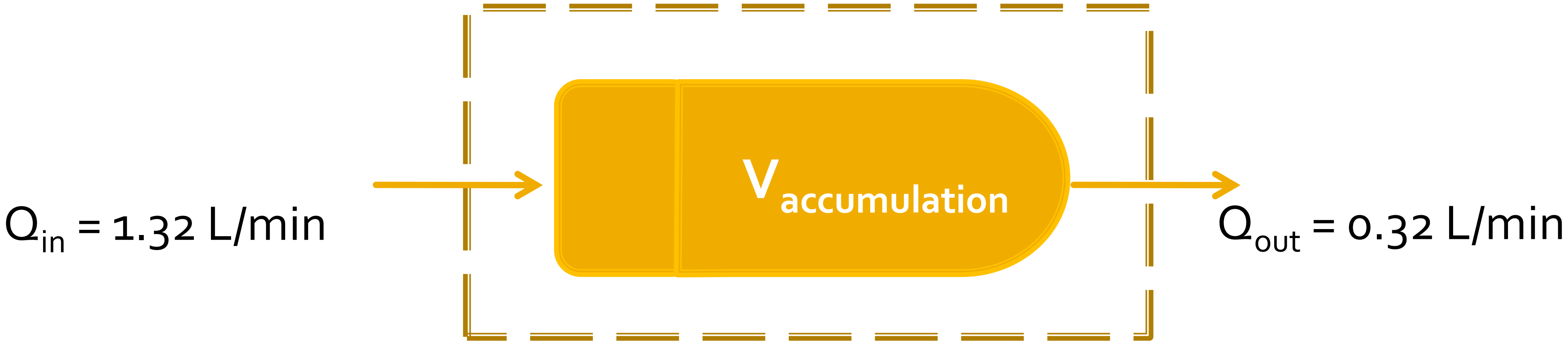
Mass rate of accumulation =
Mass rate of input – Mass rate of output

$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt}$$

Example

- Selam is filling her bathtub but she forgot to put the plug in. if the volume of water for a bath is 0.350 m^3 and the tap is flowing at 1.32 L/min and the drain is running at 0.32 L/min, how long will it take to fill the tub to bath level? Assuming Selam shuts off the water when the tub is full and does not flood the house, how much water will be wasted? Assume the density of water is 1,000 kg/m^3

Solution



Q_{in} = 1.32 L/min

$V_{accumulation}$

Q_{out} = 0.32 L/min

We must convert volumes to masses.

→Mass = (volume)(density)

→Volume = (flow rate)(time) = (Q)(t)

Solution

- From mass balance we have
- Accumulation = mass in –mass out
- $(V_{acc})(\rho) = (Q_{in})(\rho)(t) - (Q_{in})(\rho)(t)$
- $V_{acc} = (Q_{in})(t) – (Q_{in})(t)$
- $V_{acc} = 1.32t – 0.32t$
- 350L = (1.00 L/min)(t)
- t= 350 min
- The amount of wasted water is
  - Waste water = (0.32)(350) = 112 L

A hint on more complex problems

Efficiency

The **effectiveness of an environmental process** in removing a contaminant can be determined using the mass balance technique.

Mass flow rate = $\frac{Mass}{time} = (concentration)(flow\,rate)$

Mass balance → $$\frac{dM}{dt} = c_{in}Q_{in} - c_{out}Q_{out}$$

The ratio of the mass that is **accumulated in the process** to the **incoming mass** is a measure of how effective the process is in removing the contaminant

Efficiency of a system $$\frac{dM/dt}{C_{in}Q_{out}}$$

Efficiency

The **effectiveness of an environmental process** in removing a contaminant can be determined using the mass balance technique.

Efficiency of a system

$$\frac{dM/dt}{C_{in}Q_{out}} = \frac{C_{in}Q_{in} - C_{out}Q_{out}}{C_{in}Q_{out}}$$

OR →

$$\eta = \frac{mass\,in - mass\,out}{mass\,in}$$

Example

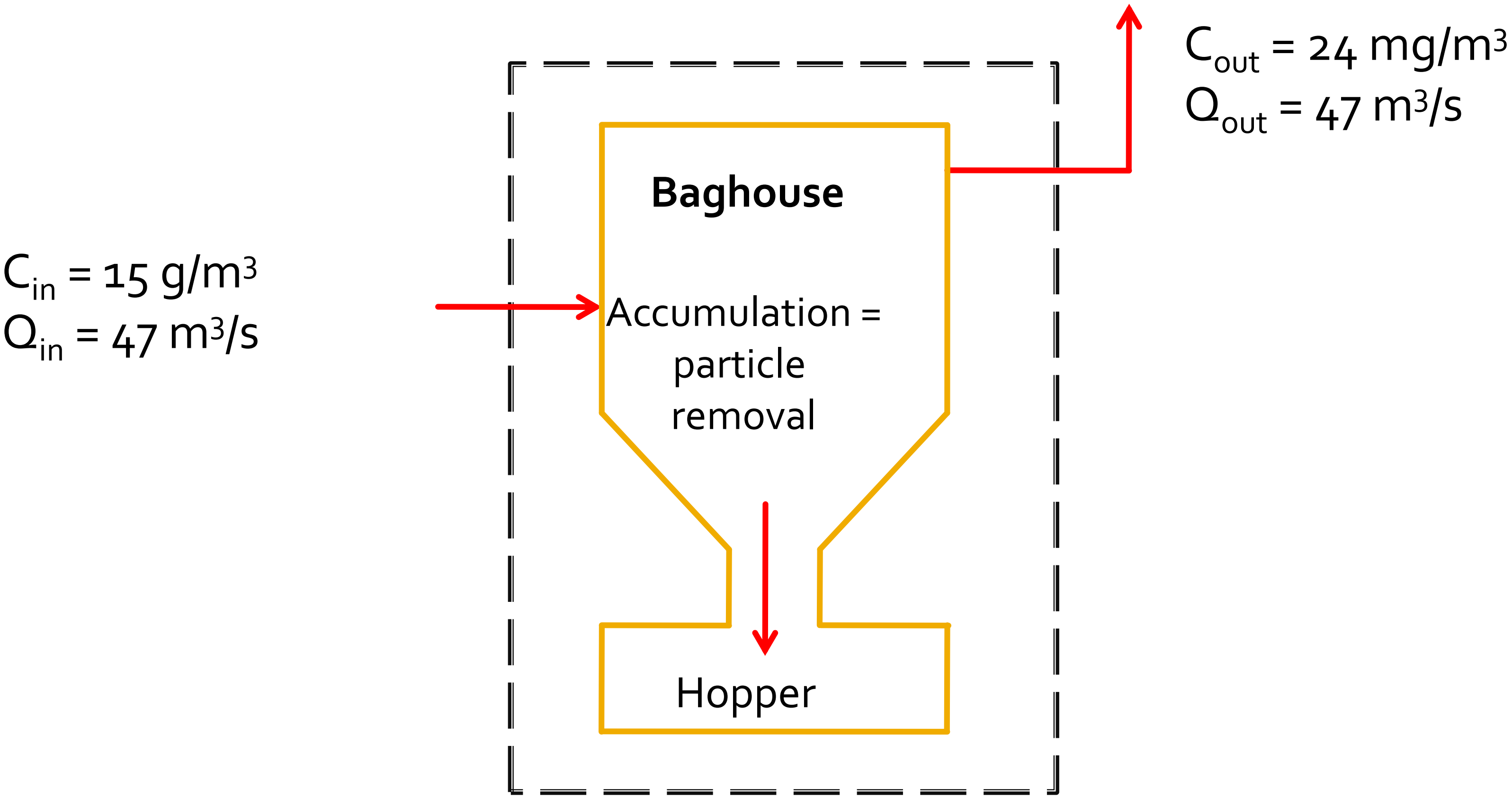
The air pollution control equipment on a municipal waste incinerator includes a fabric filter particle collector (known as a baghouse).The baghouse contains 424 cloth bags arranged in parallel, that is 1/424 of the flow goes through each bag. The gas flow rate into and out of the baghouse is 47 m^3/s, and the concentration of particles entering the baghouse is 15 g/m^3. In normal operation the baghouse particulate discharge meets the regulatory limit of 24 mg/m^3.

Example

Calculate the fraction of particulate matter removed and the efficiency of particulate removal when all 424 bags are in place and the emissions comply with the regulatory requirements.

Estimate the mass emission rate when one of the bags is missing and recalculate the efficiency of the baghouse. Assume the efficiency for each individual bag is the same as the overall efficiency for the baghouse.

Solution



C_{out} = 24 mg/m^3
Q_{out} = 47 m^3/s

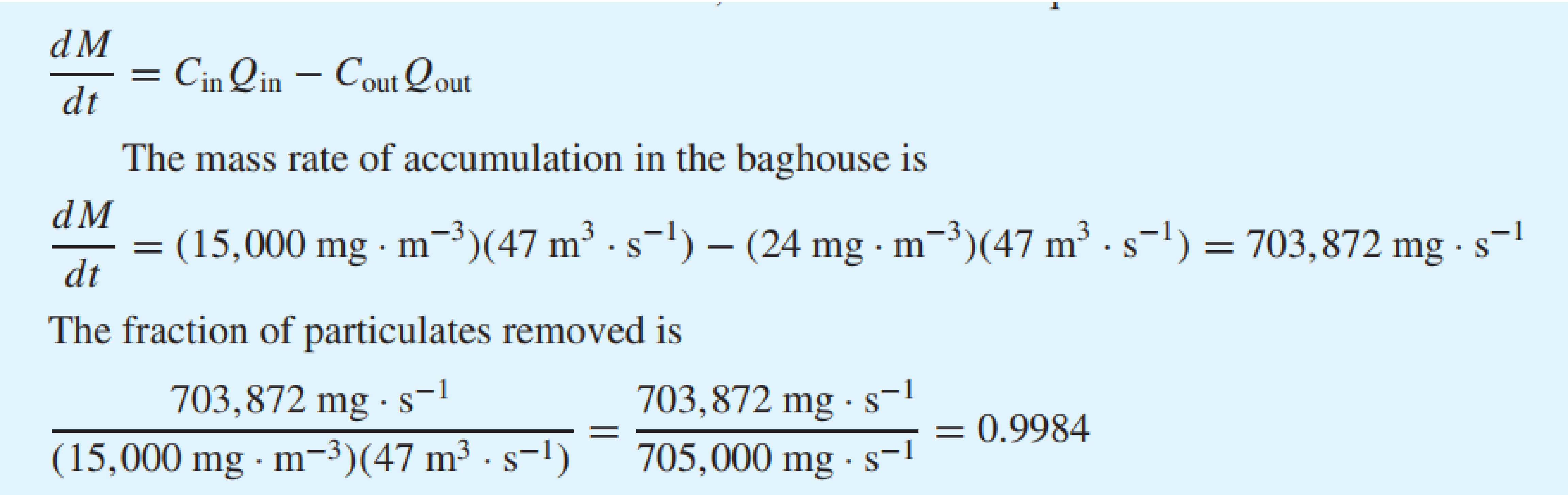
Baghouse

C_{in} = 15 g/m^3
Q_{in} = 47 m^3/s

Accumulation = particle removal

Hopper

Solution



$$\frac{dM}{dt} = C_{\text{in}} Q_{\text{in}} - C_{\text{out}} Q_{\text{out}}$$

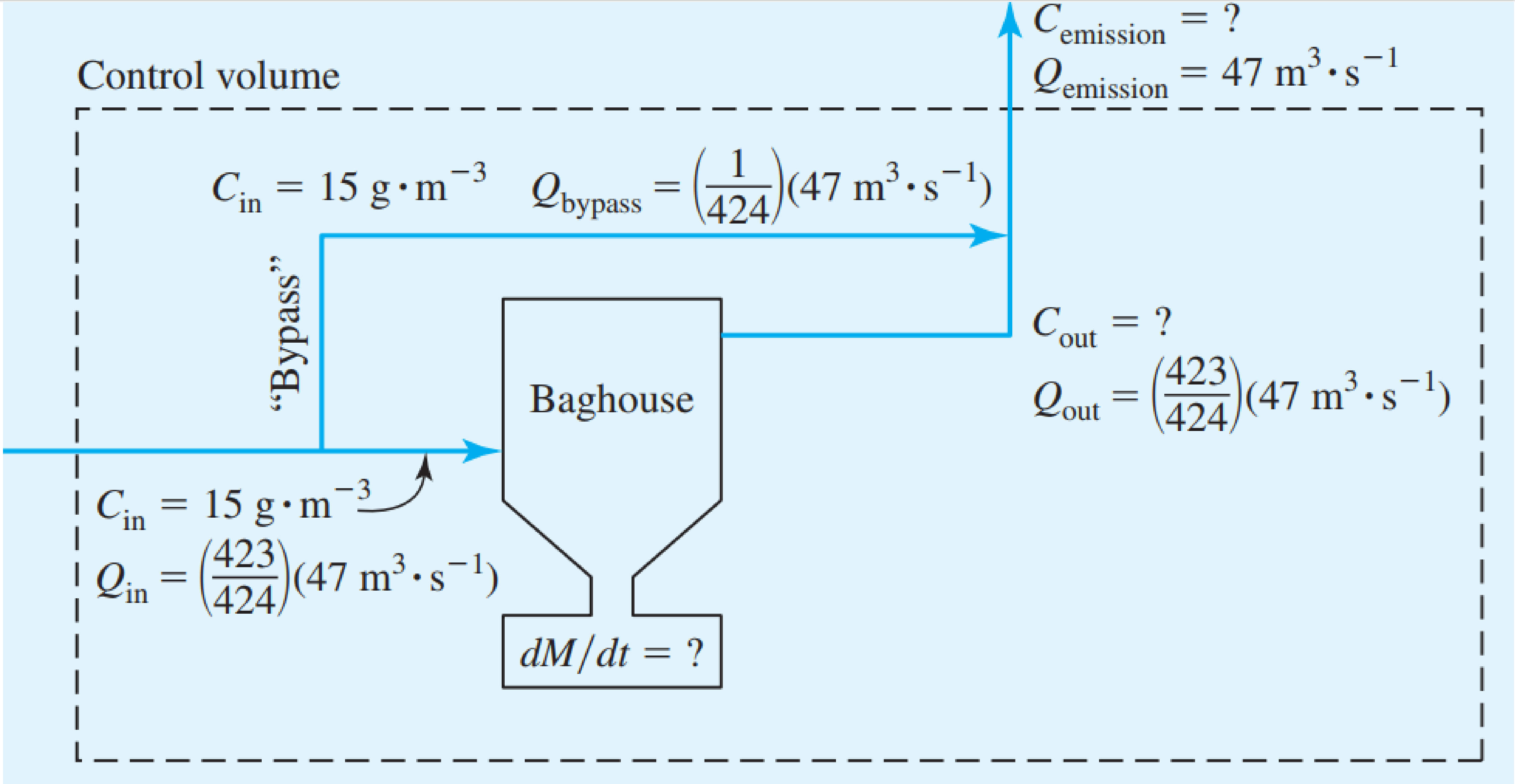
The mass rate of accumulation in the baghouse is

$$\frac{dM}{dt} = (15{,}000 \text{ mg} \cdot \text{m}^{-3})(47 \text{ m}^3 \cdot \text{s}^{-1}) - (24 \text{ mg} \cdot \text{m}^{-3})(47 \text{ m}^3 \cdot \text{s}^{-1}) = 703{,}872 \text{ mg} \cdot \text{s}^{-1}$$

The fraction of particulates removed is

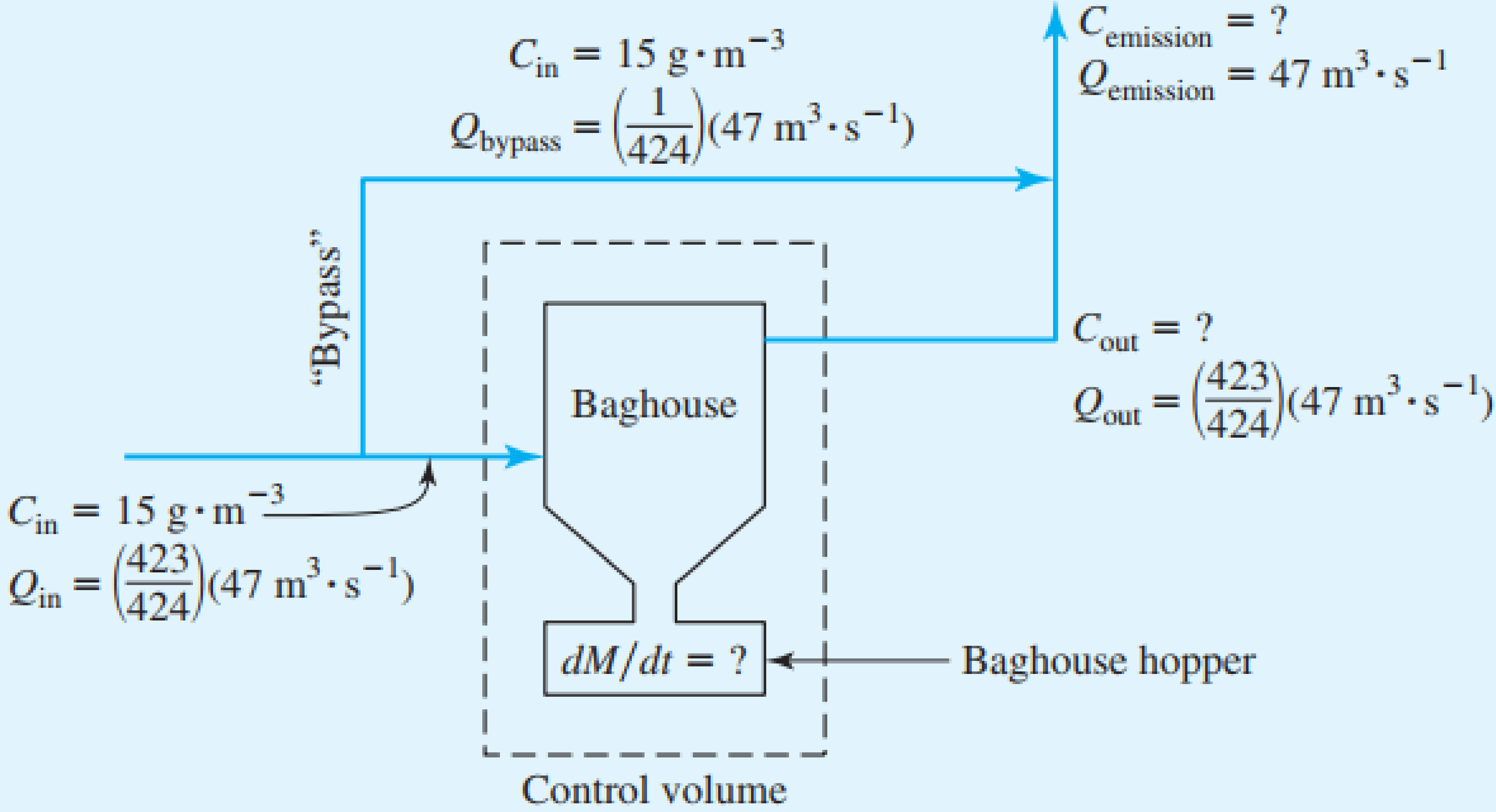
$$\frac{703{,}872 \text{ mg} \cdot \text{s}^{-1}}{(15{,}000 \text{ mg} \cdot \text{m}^{-3})(47 \text{ m}^3 \cdot \text{s}^{-1})} = \frac{703{,}872 \text{ mg} \cdot \text{s}^{-1}}{705{,}000 \text{ mg} \cdot \text{s}^{-1}} = 0.9984$$

Solution



$C_{\text{emission}} = ?$
$Q_{\text{emission}} = 47\ \text{m}^3 \cdot \text{s}^{-1}$
Control volume
$C_{\text{in}} = 15\ \text{g} \cdot \text{m}^{-3}$
$Q_{\text{bypass}} = \left(\frac{1}{424}\right)(47\ \text{m}^3 \cdot \text{s}^{-1})$
"Bypass"
Baghouse
$C_{\text{out}} = ?$
$Q_{\text{out}} = \left(\frac{423}{424}\right)(47\ \text{m}^3 \cdot \text{s}^{-1})$
$C_{\text{in}} = 15\ \text{g} \cdot \text{m}^{-3}$
$Q_{\text{in}} = \left(\frac{423}{424}\right)(47\ \text{m}^3 \cdot \text{s}^{-1})$
$dM/dt = ?$

Solution



$C_{\text{emission}} = ?$

$Q_{\text{emission}} = 47\ \text{m}^3 \cdot \text{s}^{-1}$

$C_{\text{in}} = 15\ \text{g} \cdot \text{m}^{-3}$

$Q_{\text{bypass}} = \left(\frac{1}{424}\right)(47\ \text{m}^3 \cdot \text{s}^{-1})$

"Bypass"

Baghouse

$C_{\text{out}} = ?$

$Q_{\text{out}} = \left(\frac{423}{424}\right)(47\ \text{m}^3 \cdot \text{s}^{-1})$

$C_{\text{in}} = 15\ \text{g} \cdot \text{m}^{-3}$

$Q_{\text{in}} = \left(\frac{423}{424}\right)(47\ \text{m}^3 \cdot \text{s}^{-1})$

$dM/dt = ?$

Baghouse hopper

Control volume

Solution

Because we know the efficiency and the influent mass flow rate, we can solve the mass balance equation for the mass flow rate out of the filter.

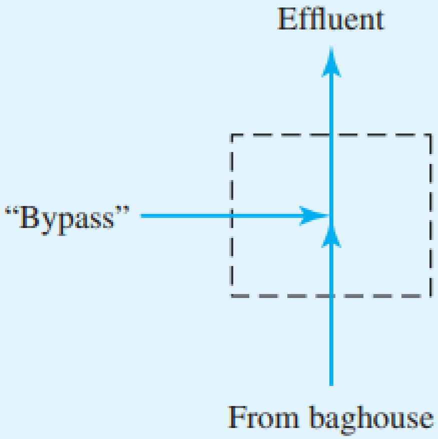
$$\eta = \frac{C_{\text{in}} Q_{\text{in}} - C_{\text{out}} Q_{\text{out}}}{C_{\text{in}} Q_{\text{in}}}$$

Solving for $C_{\text{out}} Q_{\text{out}}$

$$C_{\text{out}} Q_{\text{out}} = (1 - \eta) C_{\text{in}} Q_{\text{in}}$$

$$= (1 - 0.9984)(15{,}000 \text{ mg} \cdot \text{m}^{-3})(47 \text{ m}^3 \cdot \text{s}^{-1})(423/424) = 1125 \text{ mg} \cdot \text{s}^{-1}$$

This value can be used as an input for a control volume around the junction of the bypass, the effluent from the baghouse and the final effluent.



Solution

A mass balance for the control volume around the junction may be written as

$$\frac{dM}{dt} = C_{\text{in}}Q_{\text{in from bypass}} + C_{\text{in}}Q_{\text{in from baghouse}} - C_{\text{emission}}Q_{\text{emission}}$$

Because there is no accumulation in the junction

$$\frac{dM}{dt} = 0$$

and the mass balance equation is

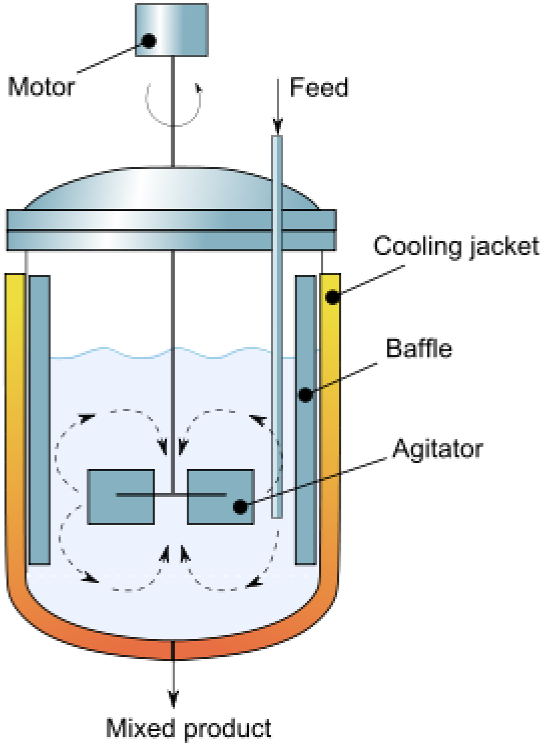
$$\begin{aligned} C_{\text{emission}}Q_{\text{emission}} &= C_{\text{in}}Q_{\text{in from bypass}} + C_{\text{in}}Q_{\text{in from baghouse}} \\ &= (15{,}000\ \text{mg}\cdot\text{m}^{-3})(47\ \text{m}^3\cdot\text{s}^{-1})(1/424) + 1125 = 2788\ \text{mg}\cdot\text{s}^{-1} \end{aligned}$$

The concentration in the effluent is

$$\frac{C_{\text{emission}}Q_{\text{emission}}}{Q_{\text{out}}} = \frac{2788\ \text{mg}\cdot\text{s}^{-1}}{47\ \text{m}^3\cdot\text{s}^{-1}} = 59\ \text{mg}\cdot\text{m}^{-3}$$

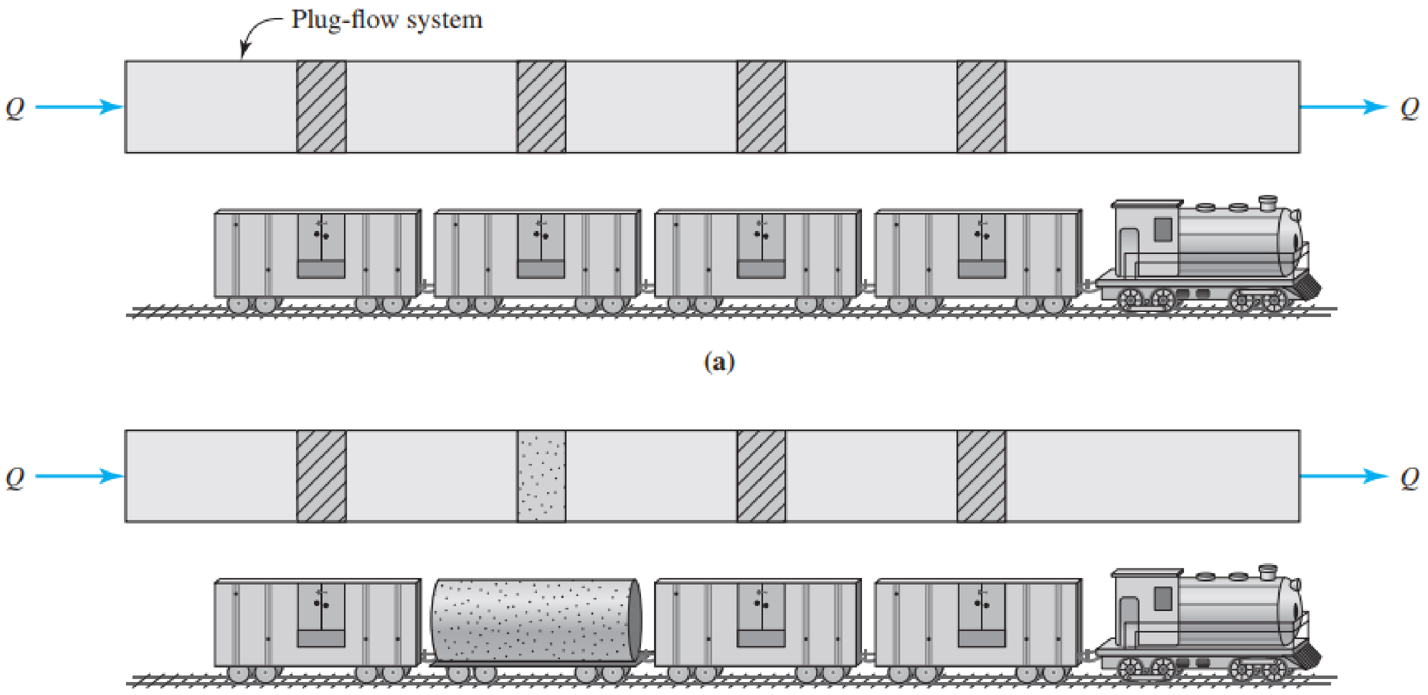
The state of mixing

- **Completely mixed system**: those in which every drop of fluid is homogeneous with every other drop (every other fluid contains the same concentration of material)



The state of mixing

- **Plug flow systems** : each drop of fluid along the direction of flow remains unique, and if no reaction takes place, contain the same concentration of material of physical property that it had when it entered the system.



State of mixing

- Steady-state: rate of input and rate of output are constant and equal
  Input rate = Output rate ➔ Accumulation =0

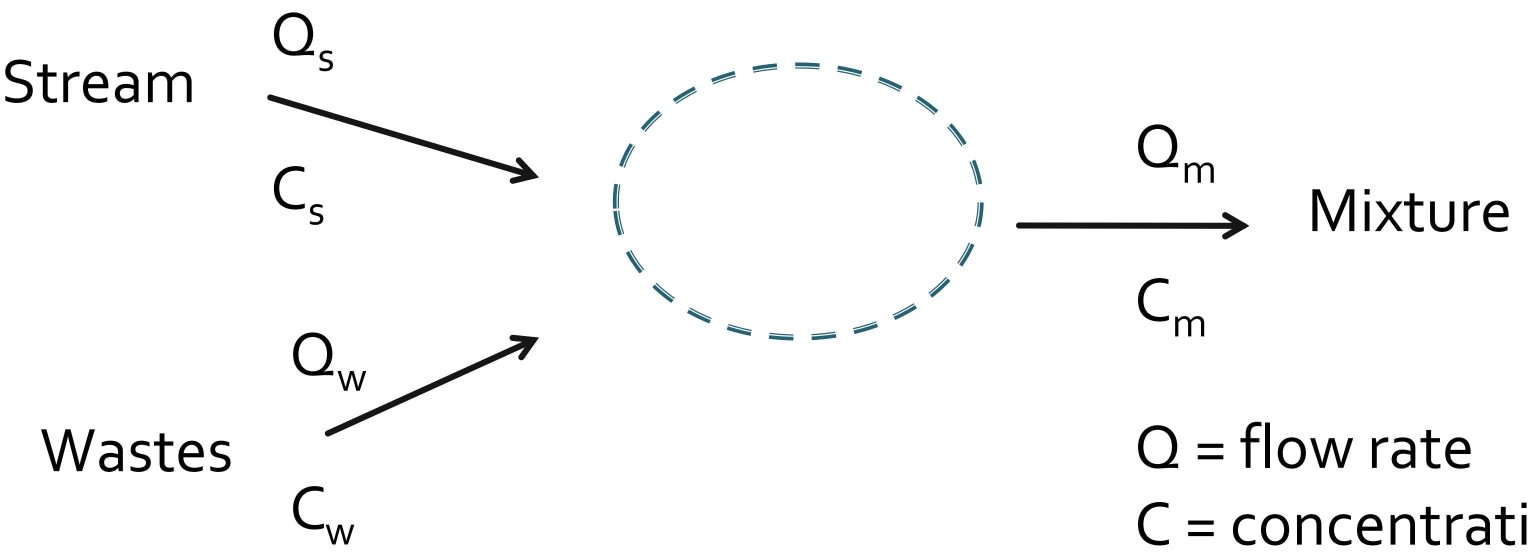
Including reactions

- *Conserved system:* where no chemical or biological reaction takes place and no radioactive decay occurs for the substance in the mass balance.
  - Examples of conservative substances include salt in water and argon in air

Including reactions

- *Non-conserved system*: Pollutants that are transformed to non-toxic substances through physical, chemical, or biological processes in the receiving water.
  - An example of a nonconservative substance (i.e., those that react or settle out) is decomposing organic matter.

Steady-state conservative system



Decay rate = 0
Accumulation rate = 0

Stream Q_s C_s

Wastes Q_w C_w

Q_m C_m Mixture

Q = flow rate
C = concentration

$$C_sQ_s + Q_wC_w = Q_mC_m$$

Example on mixed state

- A storm sewer is carrying snow melt contacting 1.2 g/L of sodium chloride into a small stream. The system has a naturally occurring sodium chloride concentration of 20 mg/L. If the storm sewer flow rate is 2,000L/min and the stream flow rate is 2.0 Cumec, what is the concentration of salt in the system after the discharge point?

Assume the sewer flow and the stream flow are completely mixed, that salt is a **conservative substance** (does not react of decay), and the system is at steady state.

Including reactions

- For *non-conservative* substances

kinetic reactions : reactions that are time dependent.

Accumulation rate = input rate – output rare ± transformation rate

$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + r$$

- The **rate of reaction**, r_i, the rate of formation or disappearance of a substance.
- ***Homogenous reactions***. single phase reactions
- ***Heterogeneous reactions*** : *multiphase reactions* (between phases surface)

Rate of reaction

- **Reaction kinetics:** the study of the effects of temperature, pressure, and concentration on the rate of a chemical reaction.
  - $r_i = k f_1(T,P); f_2([A],[B], \ldots)$

Rate constant

Concentration of reactant

Assuming that the pressure and temperature are constant

$$aA + bB \longrightarrow cC$$

Rate of reaction ➔ $r_A = -k[A]^{\alpha}[b]^{\beta} = k[C]^{\gamma}$

Order of reaction

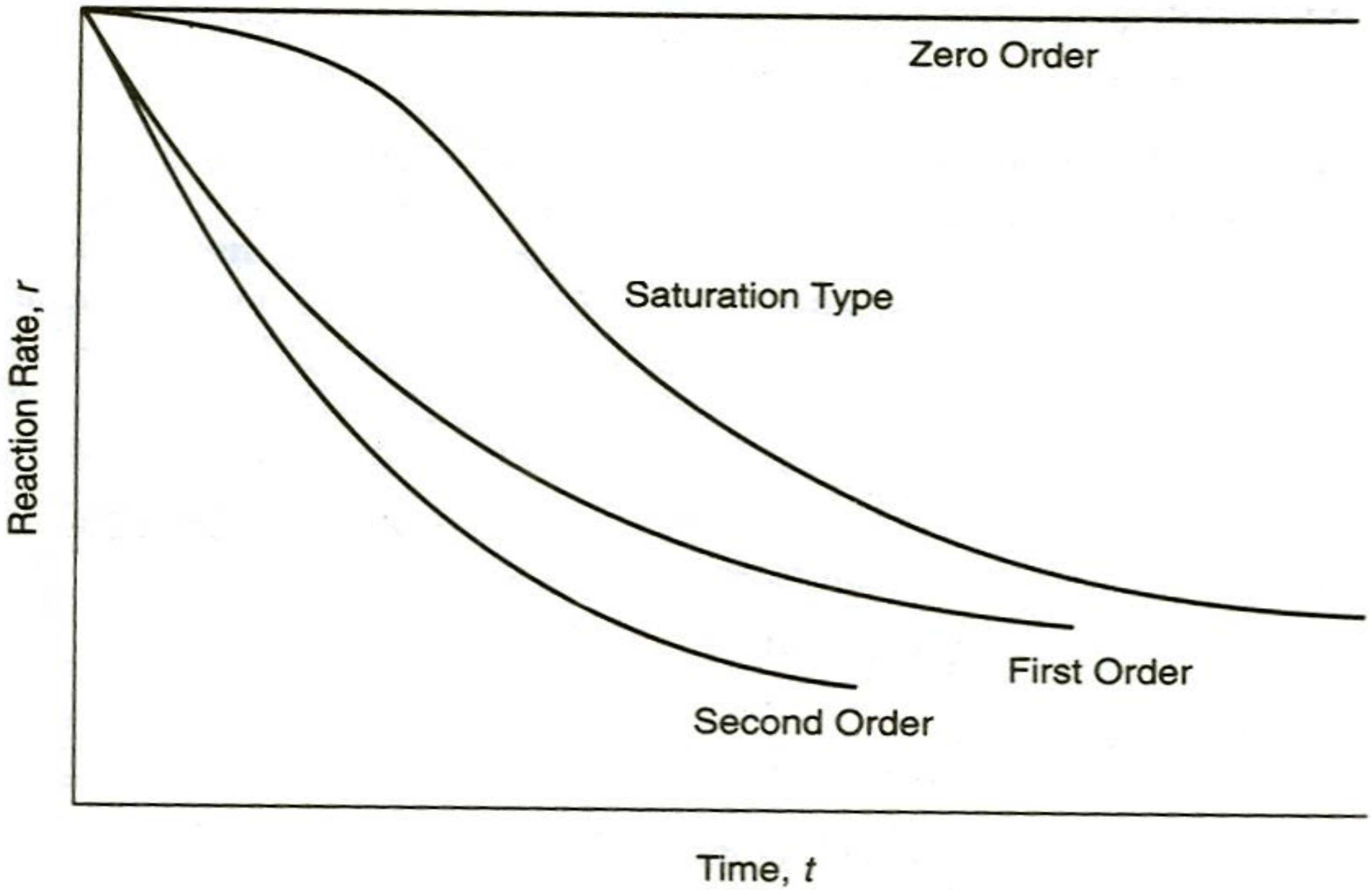
Rate of reaction → $r_A = - k[A]^{\alpha}[b]^{\beta} = k[C]^{\gamma}$

order of reaction = $\alpha + \beta$,

the order with respect to reactant A is α, to B is β, and to product C is γ.

$r_A = -k$	zero-order reaction
$r_A = -k[A]$	first-order reaction
$r_A = -k[A^2]$	second-order reaction
$r_A = -k[A][B]$	second-order reaction

Order of reaction



Order of reaction

- Reaction rate, R is assumed to be directly proportional to the amount of material remaining.

Eg. Oxidation of organic matter in a stream

- The rate of loss of a substance is proportional to the amount of substance present at any given time, t.

Simple completely mixed systems

- **With first-order reactions**

Total mass of substance = concentration x volume

  - when V is a constant, the mass rate of decay of the substance is

$$\frac{dM}{dt} = \frac{d(CV)}{dt} = V\frac{d(C)}{dt}$$

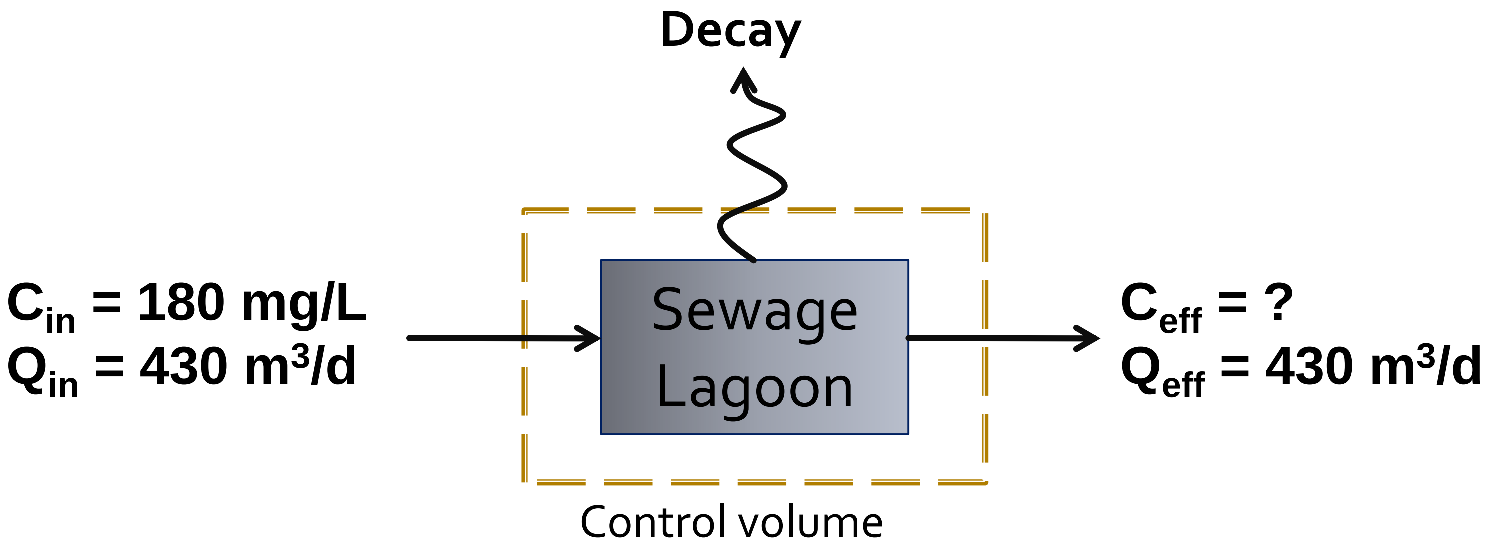
first-order reactions can be described by $r = -kC = dC/dt$,

$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} + r \rightarrow \frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} - kCV$$

Example

A well-mixed sewage lagoon is receiving 430 m^3/d of sewage out of a sewer pipe. The lagoon has a surface area of 10 ha and a depth of 1.0m. The pollutant concentration in the raw sewage discharging into the lagoon is 180 mg/L. The organic matter in the sewage degrades biologically in the lagoon according to first-order kinetics. The reaction rate constant is 0.70 d^{-1}. Assuming no other water losses or gains and that the lagoon is completely mixed, find the steady-state concentration of the pollutant in the lagoon effluent.

Solution



Accumulation=input rate – output rate – decay rate

Assuming steady-state condition, accumulation = 0

input rate = output rate + decay rate

$$C_{in}Q_{in} = C_{eff}Q_{eff} + (K)(C_{lagoon})(V)$$

Plug-flow with reaction

Mass balance for each plug element (differential element of fluid)

$$\frac{dM}{dt} = \cancel{\frac{d(in)}{dt}} - \cancel{\frac{d(out)}{dt}} + V\frac{d(C)}{dt}$$

No mass exchange occurs across the plug boundaries, ➔d(in) and d(out) = 0

$$\frac{dM}{dt} = \frac{dC}{dt}V \Rightarrow \frac{dC}{dt} = -kC \Rightarrow C_{out} = C_{in}e^{-kt}$$

Plug-flow with reaction

$$C_{in} = C_{out} e^{-k\theta}$$

Residence time

The residence time for each plug:

$$\theta = \frac{(L)(A)}{(u)(A)} = \frac{V}{Q}$$

$$\ln \frac{C_{out}}{C_{in}} = -k\theta = -k\frac{(L)}{(u)} = -k\frac{V}{Q}$$

Although the concentration within a given plug changes over time as the plug moves downstream, the concentration at a fixed point in the plug-flow system remains constant with respect to time.

Example-Familiarize yourselves

A wastewater treatment plant must disinfect its effluent before discharging the wastewater to a near-by stream. The wastewater contains 4.5 x 10^5 fecal coliform colony-forming units (CFU) per liter. The maximum permissible fecal coliform concentration that may be discharged is 2,000 fecal coliform CFU/L. It is proposed that a pipe carrying the wastewater be used for disinfection process. Determine the length of the pipe required if the linear velocity of the wastewater in the pipe is 0.75 m/s. Assume that the pipe behaves as a steady-state plug-flow system and that the reaction rate constant for destruction of the fecal coliforms is 0.23 min^{-1}.

Reactors

- Model natural systems

Classified based on

1. Flow characteristics
2. Mixing conditions

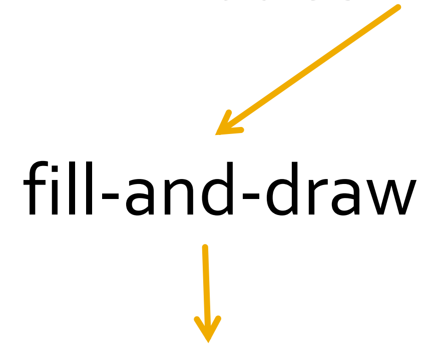
batch reactors and **flow reactors**.

fill-and-draw

Unsteady state

material flows into, through, and out of the reactor

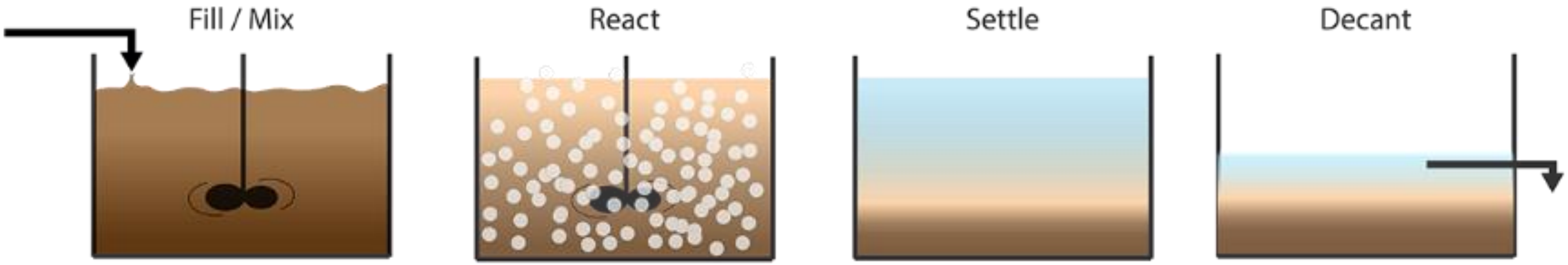
Types of Reactors



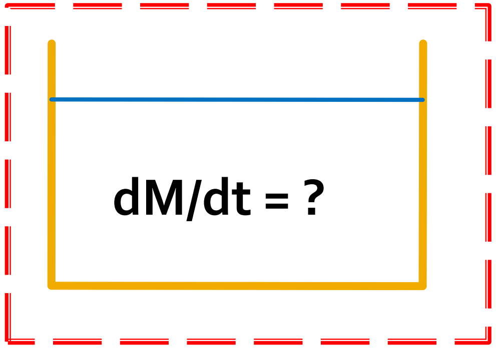
batch reactors

fill-and-draw

Unsteady state



Batch reactors



$$\frac{d(in)}{dt} = \frac{d(out)}{dt} = 0$$

$$\frac{dM}{dt} = \frac{d(in)}{dt} - \frac{d(out)}{dt} - kCV \Rightarrow \frac{dM}{dt} = -kCV$$

$$\frac{dM}{dt} = \frac{dC}{dt}V \Rightarrow \frac{dC}{dt} = -kC \Rightarrow C = C_o e^{-kt}$$

Example

A contaminated soil is to be excavated and treated in a completely mixed aerated lagoon. To determine the time it will take to treat the contaminated soil, a laboratory completely mixed batch reactor is used to gather the following data. Assuming a first-order reaction, estimate the rate constant, k, and determine the time to achieve 99 % reduction in the original concentration.

Time (d)	Waste Concentration (mg/L)
1	280
16	132

Solution

- Using the 1st and 16th day, the time interval t= 16-1 = 15 d

$$\frac{C_t}{C_o} = e^{-kt} \Longrightarrow \frac{132mg/L}{280mg/L} = e^{-k(15d)}$$

Solving for k, we have $k = 0.0501\ \mathrm{d}^{-1}$

To achieve 99 % reduction the concentration at
Time t must be 1 – 0.99 of the original concentration

$$\frac{C_t}{C_o} = 0.01 = e^{-0.05(t)} \Longrightarrow$$

t = 92 days

Types of Reactors

Flow reactors

↓

Continuous type(matter flows into, through and out of the reactor at all time

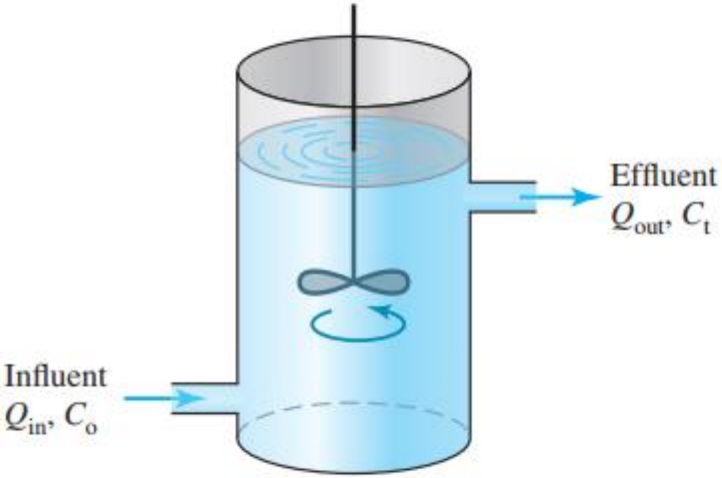
↓

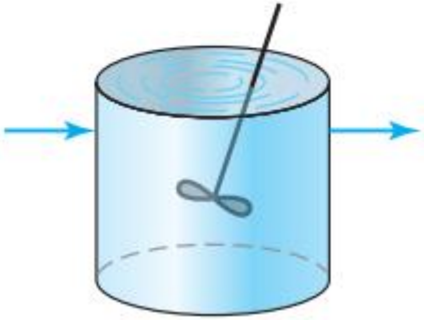
Completely mixed flow reactor & Plug flow reactor

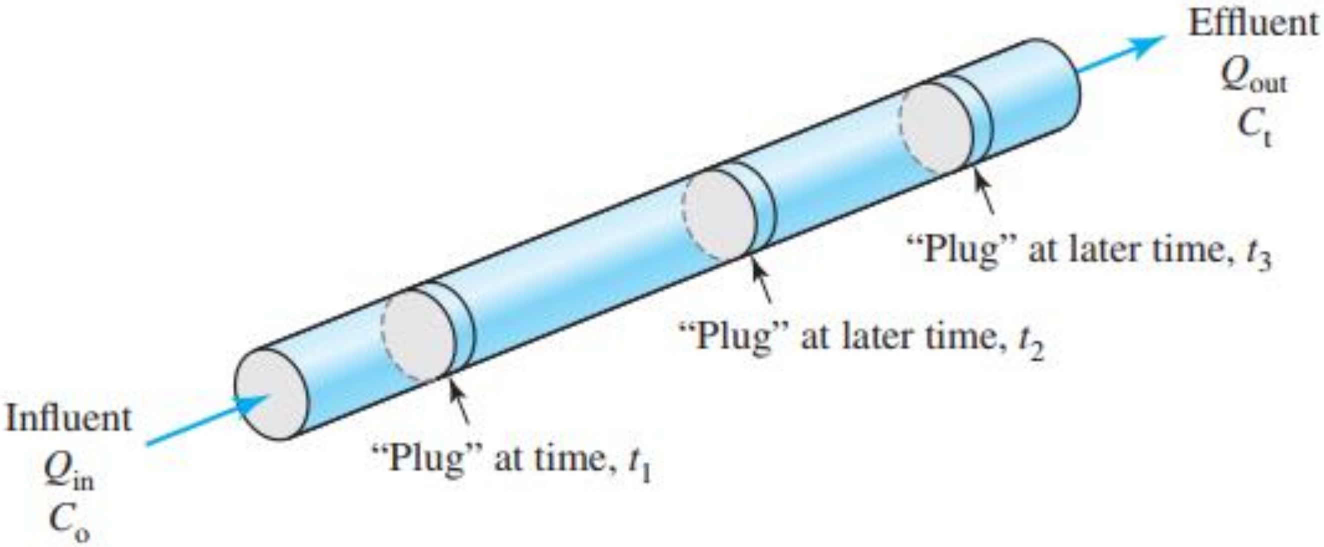
CMFR: The composition of the effluent is the same as the composition in the tank

PFR: no mixing occurs in the lateral direction. Fluid particles pass through the tank in sequence(those that enter first leave first)

Types of Reactors

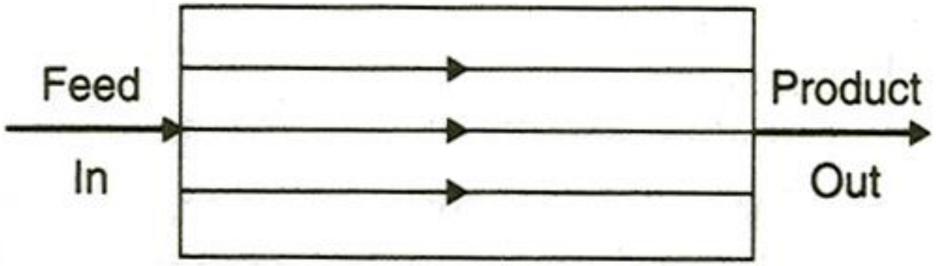




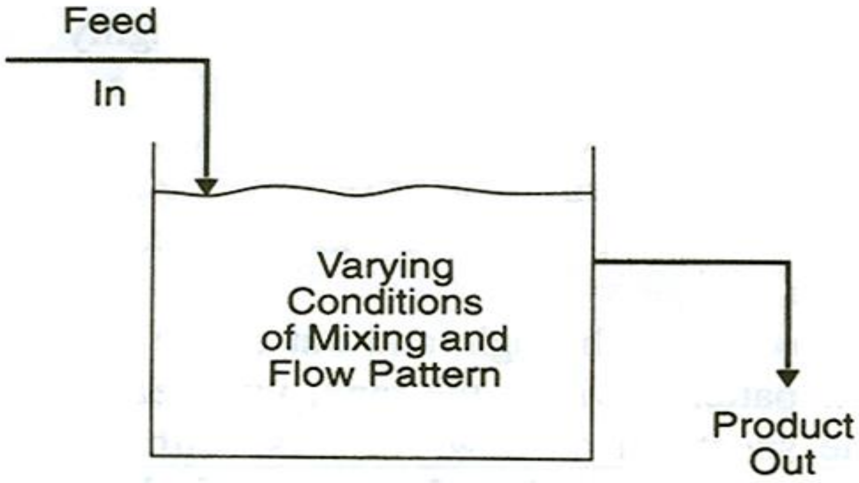


Types of Reactors

IDEAL REACTORS

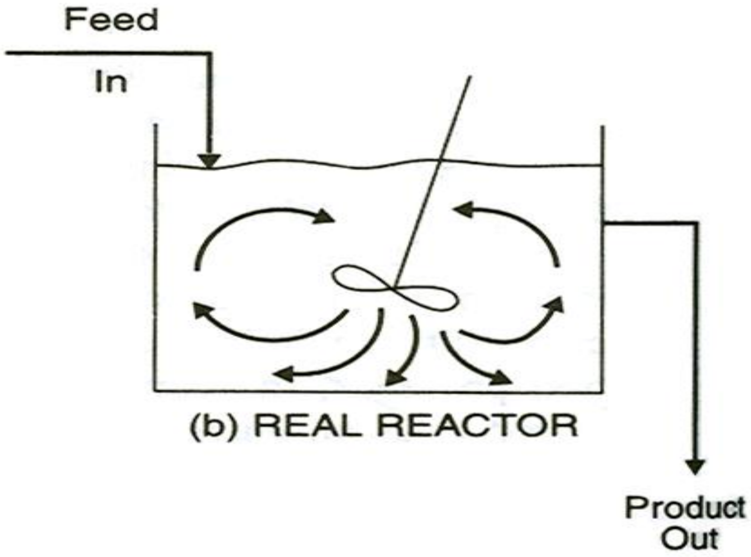


(a) PFTR



(c) CSTR

REAL REACTOR



(b) REAL REACTOR

Types of Reactors

For time-dependent reactions, the time that a fluid particle remains in the reactor obviously affects the degree to which the reaction goes to completion.

In ideal reactors the average time in the reactor (**detention time** or **retention time** or, for liquid systems, **hydraulic detention time** or **hydraulic retention time**) is defined as

$$t_o = \frac{\forall}{Q}$$

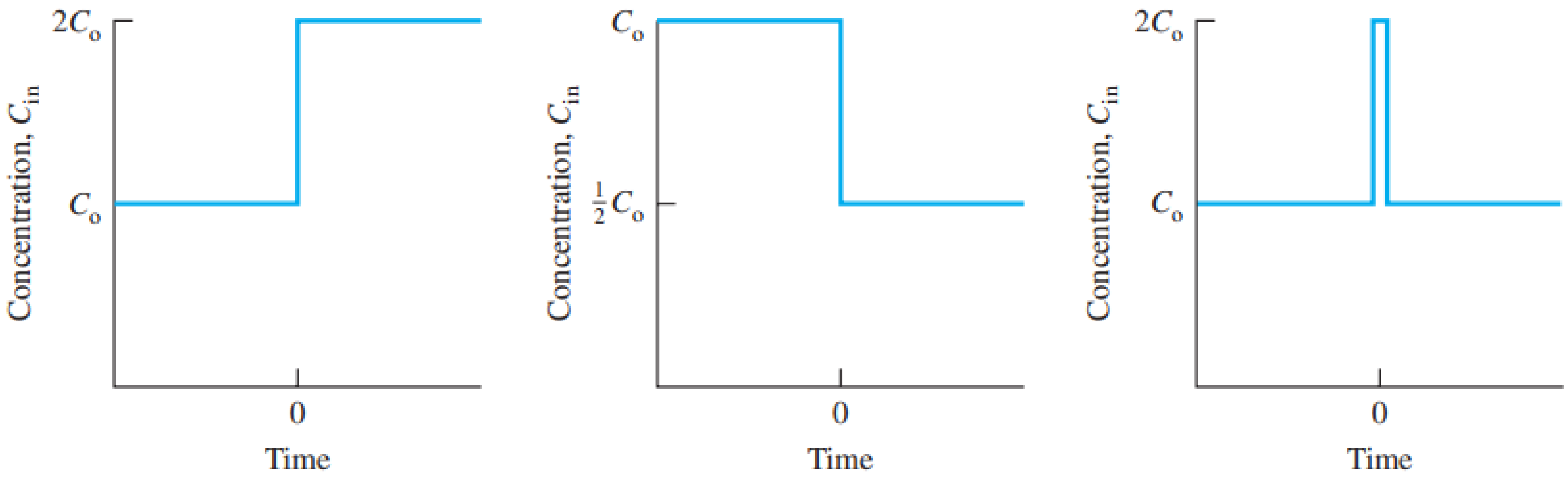
where t_o = theoretical detention time (in s)

$\forall$ = volume of fluid in reactor (in m^3)

Q = flow rate into reactor (in $m^3 \cdot s^{-1}$)

Reactor analysis

- **selection of a reactor**(either as a treatment method or as a model for a natural process)
  - Depends on the behavior desired
- We will examine the behavior of batch, CMFR, and PFR reactors in several situations (response of reacotrs)

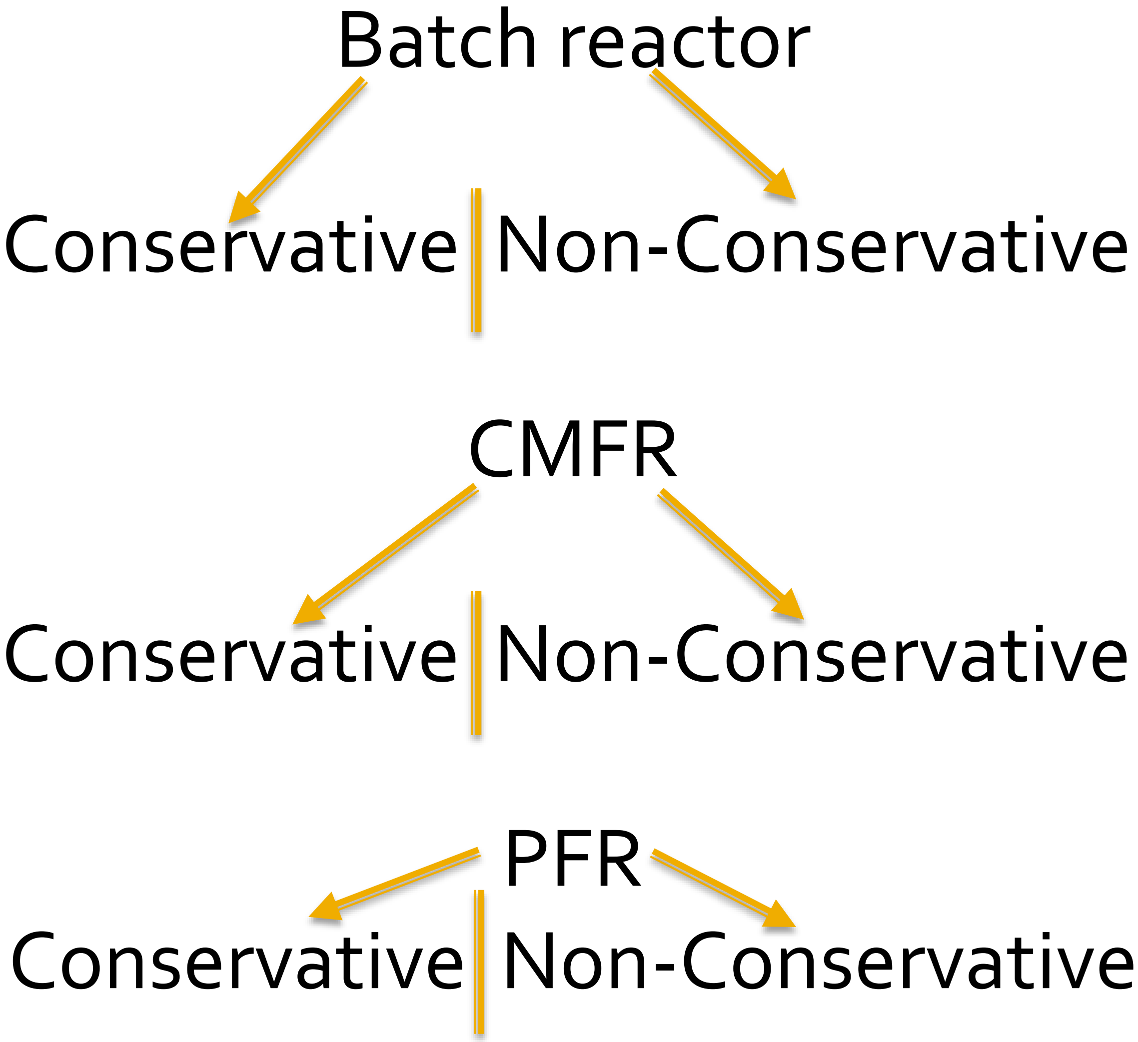


Reactor Analysis

- **Step function response**

- Response of the reactor to a sudden increase or decrease in the steady-state influent concentration for conservative and non conservative species.

- Conditions that are not of steady state



Batch reactor

Conservative | Non-Conservative

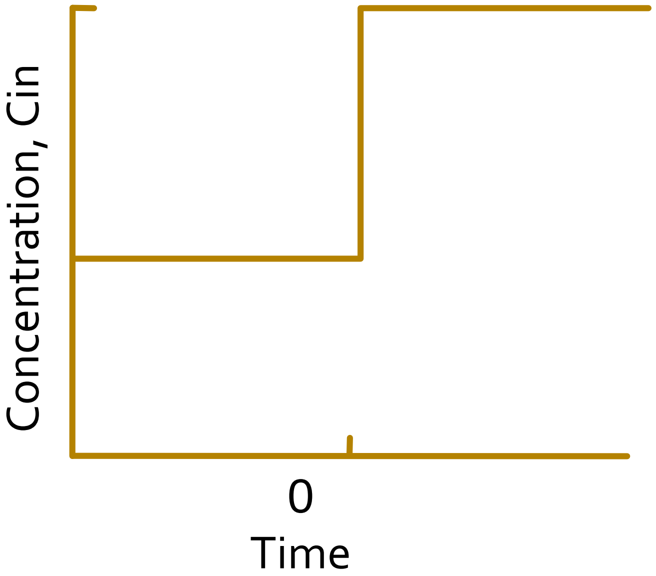
CMFR

Conservative | Non-Conservative

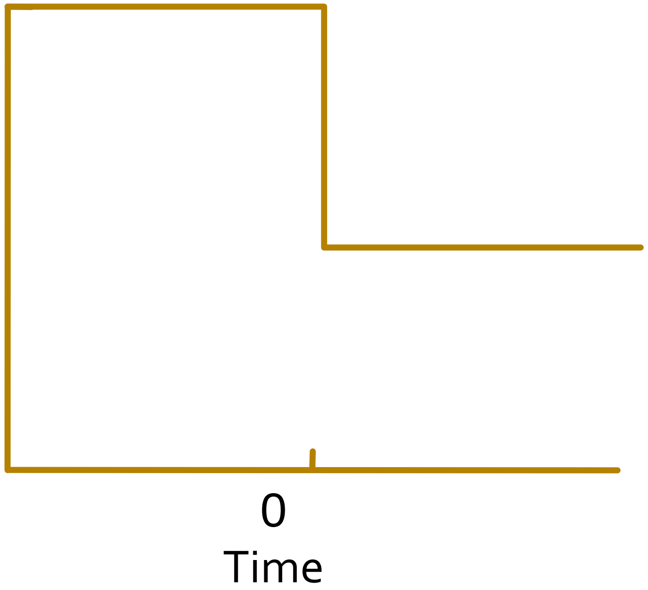
PFR

Conservative | Non-Conservative

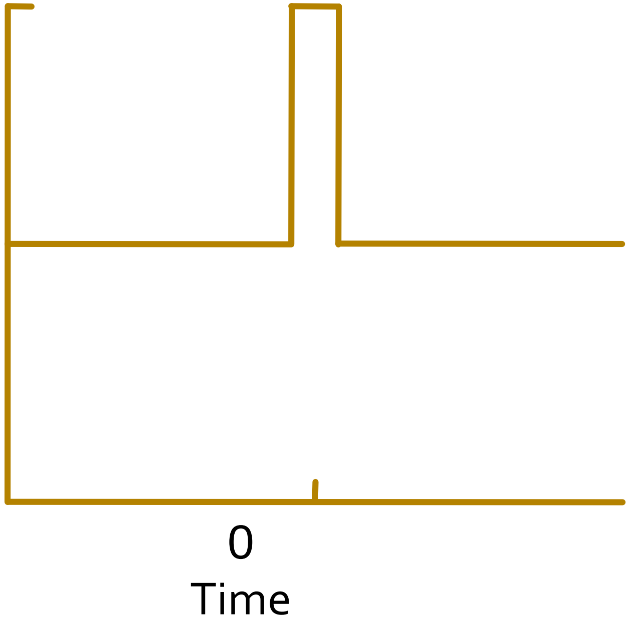
Step function response



Step increase

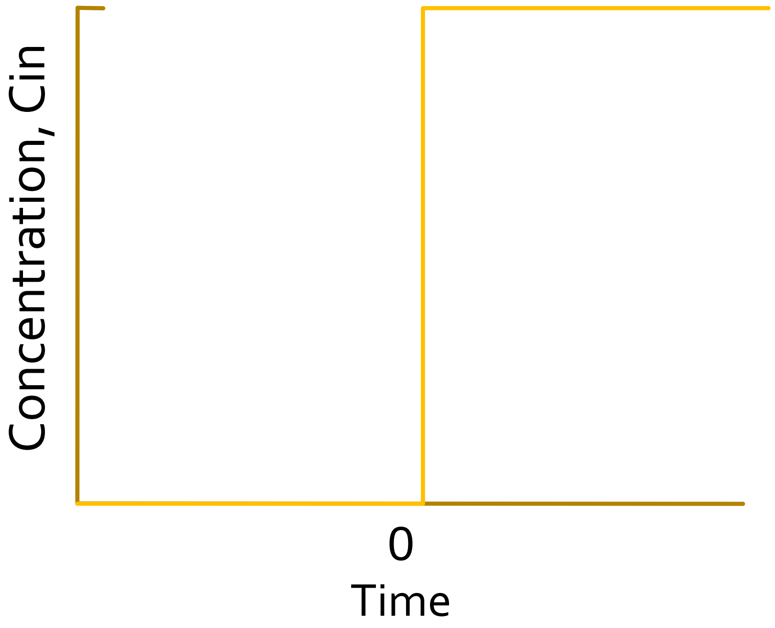


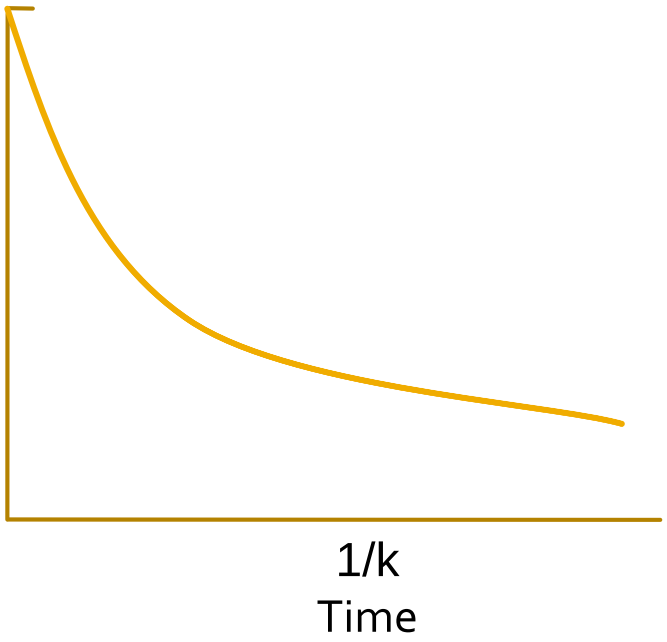
Step decrease



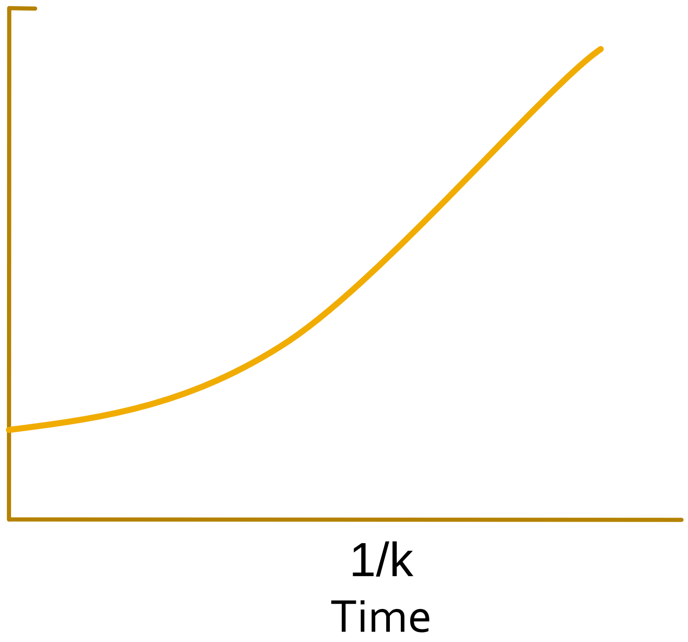
pulse/spike increase

Batch reactor



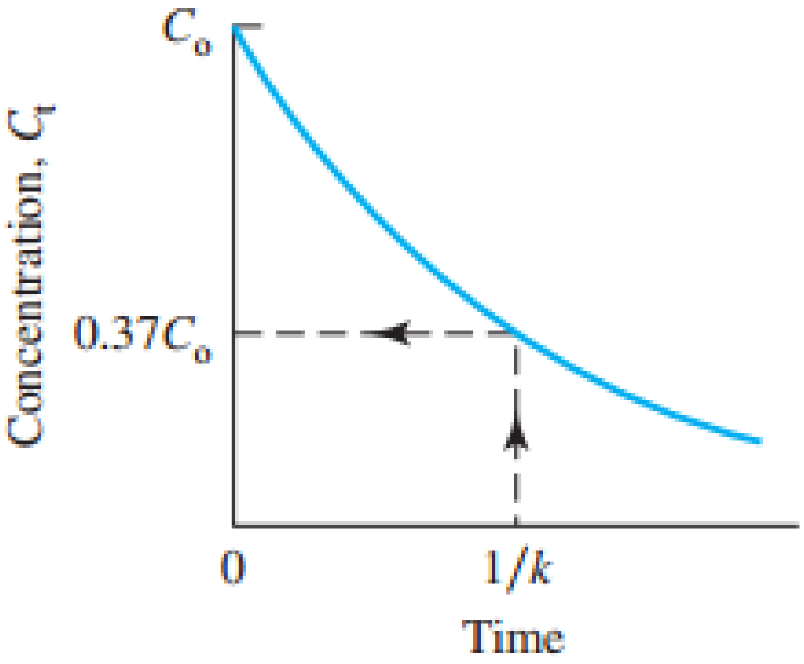


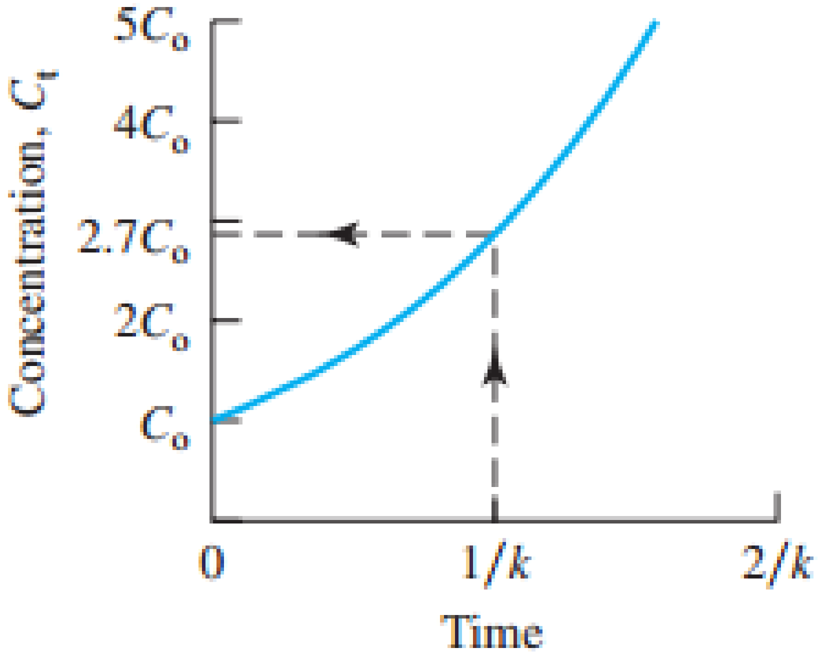
Decay



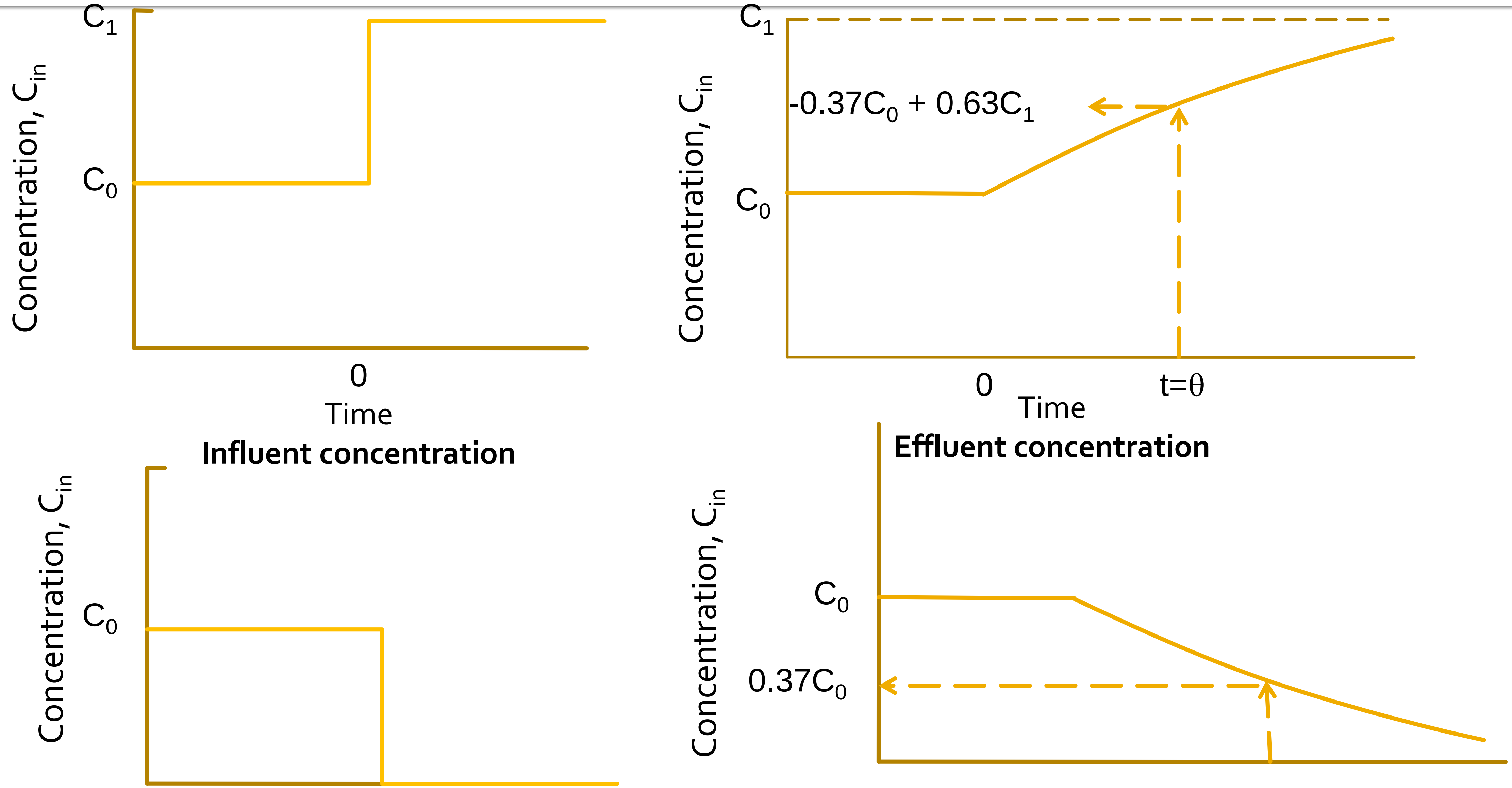
Formation

$$\frac{C_t}{C_o} = e^{-kt}$$


C_o
$0.37C_o$
Concentration, C_t
0
$1/k$
Time


$5C_o$
$4C_o$
$2.7C_o$
$2C_o$
C_o
Concentration, C_t
0
$1/k$
$2/k$
Time

CMFR Conservative



CMFR Conservative

- For balanced flow ($Q_{in} = Q_{out}$) and no reaction, the mass balance becomes

$$\frac{dM}{dt} = c_{in} Q_{in} - c_{out} Q_{out}$$

Where M = CV. The solution is

$$C_t = C_0 \left[\exp\left(-\frac{t}{\theta} \right) \right] + C_1 \left[1 - \exp\left(-\frac{t}{\theta} \right) \right]$$

Where $\theta = V/Q$

CMFR Conservative

- Flushing of nonreactive contaminant from a CMFR by a contaminant-free fluid
- Which means $C_{in} = 0$ and the mass balance becomes

$$\frac{dM}{dt} = -c_{out} Q_{out}$$

Where M = C¥. The initial concentration is C_o=M/V

For time $t \geq 0$ we obtain

$$C_t = C_0 \left[\exp\left(-\frac{t}{\theta} \right) \right]$$

Example

Before entering an underground utility vault to do repairs, a work crew analyzed the gas in the vault and found that it contained 29mg/m^3 of H_2S. Because the allowable exposure level is 14 mg/m^3 the work crew began ventilating the vault with a blower. If the volume of the vault is 160 m^3 and the flow rate of contaminant-free air is 10 m^3/min, how long will it take to lower the H_2S level to that will allow the work crew to enter? Assume the manhole behaves as CMFR and that H_2S is nonreactive in the time period considered.

CMFR NonConservative

- For balanced flow ($Q_{in} = Q_{out}$) and first-order reaction the mass balance becomes

$$\frac{dM}{dt} = C_{in}Q_{in} - C_{out}Q_{out} - kC_{out}V$$

Where M = CV. By dividing with Q and V we have

$$\frac{dC}{dt} = \frac{1}{\theta}\left(C_{in} - C_{out}\right) - kC_{out}$$

CMFR Non Conservative

For **steady-state** conditions $dC/dt=0$

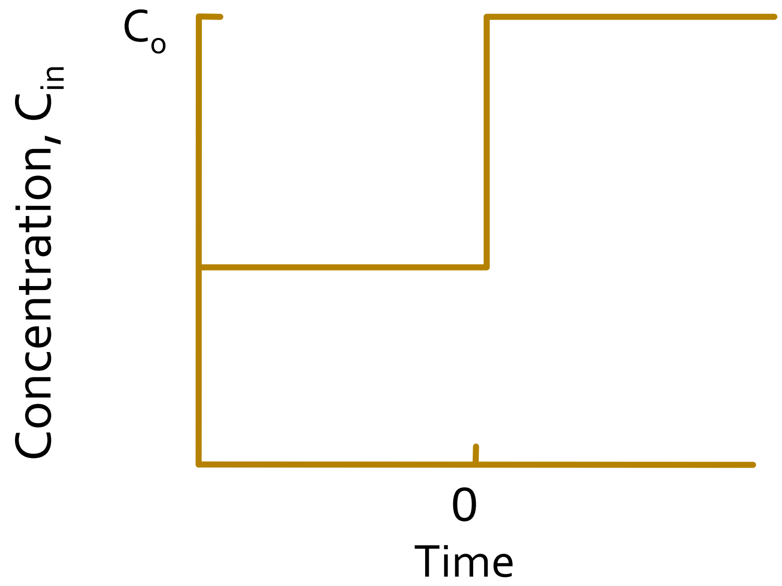
$$C_{out} = \frac{C_o}{1+k\theta}$$

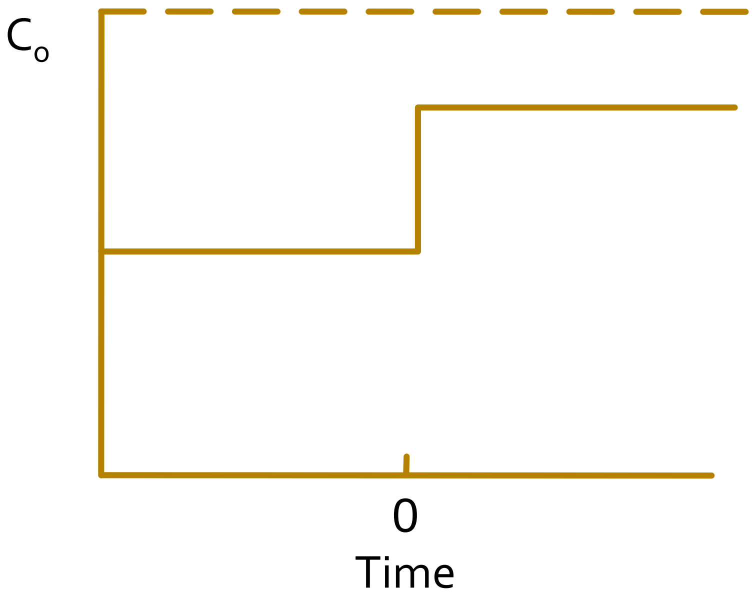
Decay

OR

$$C_{out} = \frac{C_o}{1-k\theta}$$

Formation





CMFR Non Conservative

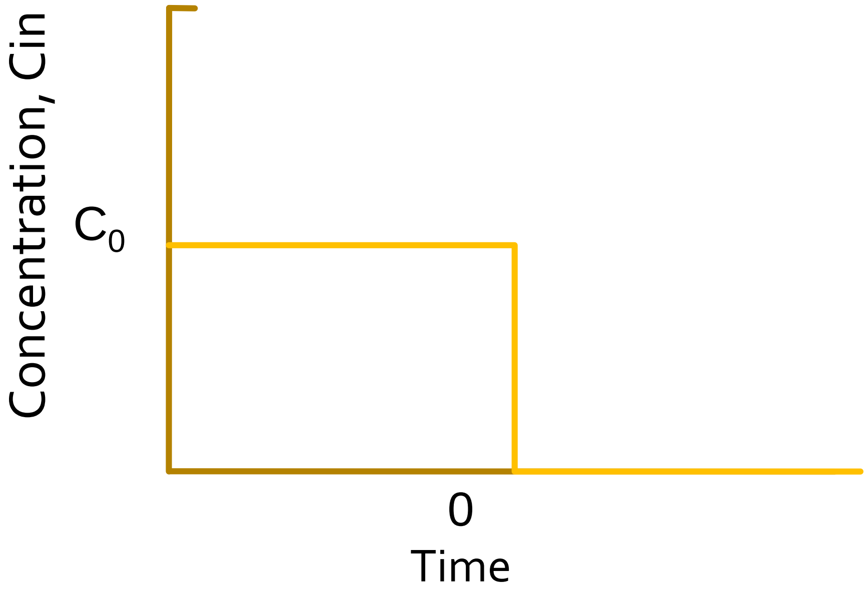
A **step decrease** in influent concentration (C_{in}=O) for **non-steady-state** conditions with first-order decay

$$\frac{dM}{dt} = 0 - C_{out}Q_{out} - kC_{out}V$$

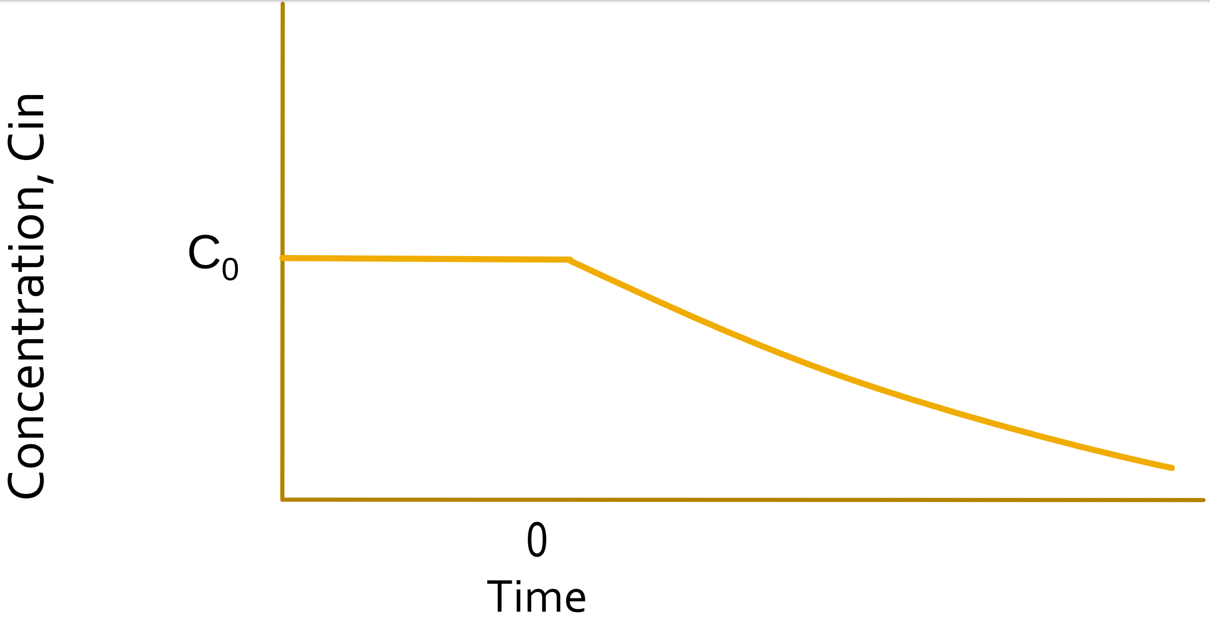
Where M = CV. By dividing with Q and V we have

$$\frac{dC}{dt} = \left(\frac{1}{\theta} + k\right)C_{out} \quad \longrightarrow \quad C_{out} = C_o \exp\left[-\left(\frac{C_o}{1+k\theta}\right)t\right]$$

CMFR Non Conservative



Influent concentration



Effluent concentration

Example

A chemical degrades in a **flow-balanced**, **steady-state** CMFR according to **first-order** reaction kinetics. The upstream concentration of the chemical is 10 mg/L and the downstream concentration is 2 mg/L. Water is being treated at a rate of 29 m^3/min. The volume of the tank is 590 m^3. What is the rate of decay? What is the rate constant?

Solution

- For a first-order reaction, the rate of decay, r =-kC, thus we have to solve for kC from

$$\frac{dM}{dt} = C_{in}Q_{in} - C_{out}Q_{out} - kC_{out}V$$

- For steady-state, dM/dt = 0 and for balanced flow, Q_{in} = Q_{out}

$$r = kC = \frac{C_{in}Q_{in} - C_{out}Q_{out}}{V} = \frac{(10mg/L - 2mg/L)(29m^3/\min)}{580m^3}$$

- r=kC=0.4

Solution...

- For a first-order reaction in a CMFR

$$C_{out} = \frac{C_o}{1+k\theta}$$

- The mean hydraulic detention time is

$$\theta = \frac{V}{Q} = \frac{580m^3}{29m^3/\min} = 20\min$$

- Solving for the rate constant we get

$$k = \frac{(C_o/C_{out})-1}{\theta} = \frac{(10mg/L/2mg/L)-1}{20\min} = 0.20\min^{-1}$$

Environmental Engineering Mini-Project Topics

1. Risk assessment and decision analysis
2. Recycling, reuse and resource recovery
3. Investigate a renewable source of energy (such as wind, solar, geothermal), or a new technology that will reduce greenhouse gas emissions due to energy consumption (such as fuel cells or hydrogen fuel).
4. Indoor air quality and models
5. Ecological Foot prints
6. Air quality measurement and analysis
7. Environmental Impact of Mining
8. Water Quality and Treatment technologies
9. Wastewater treatment and reuse
10. Land use and ecological impacts

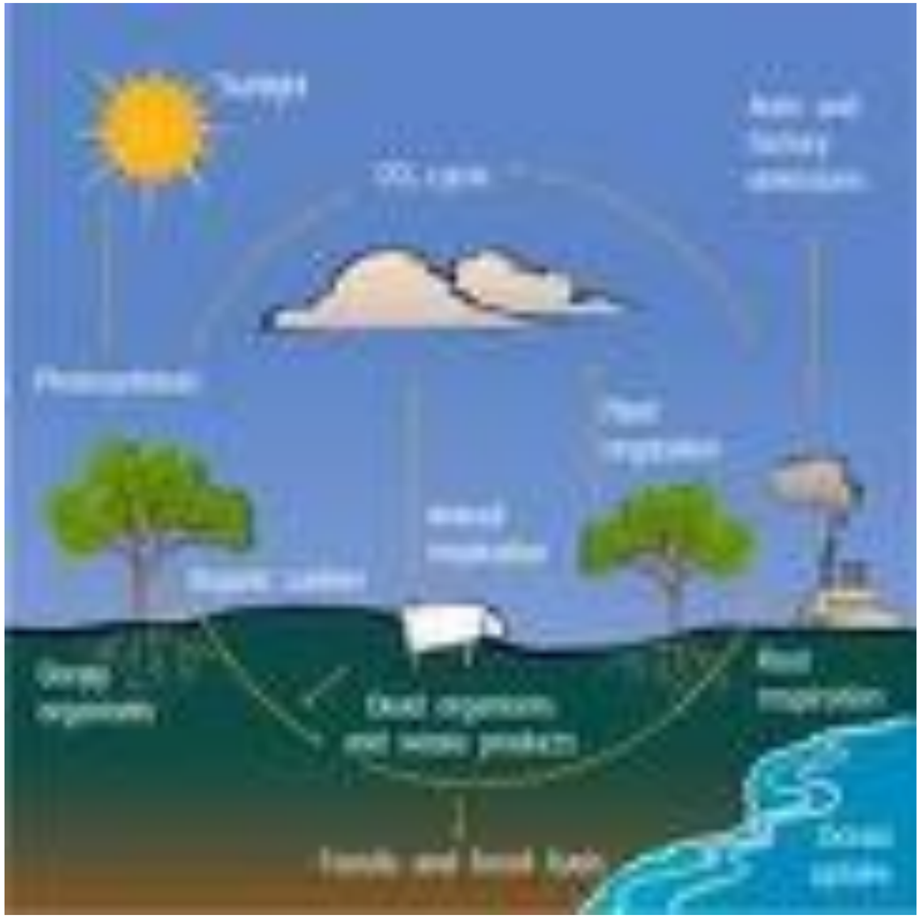
Energy Balances

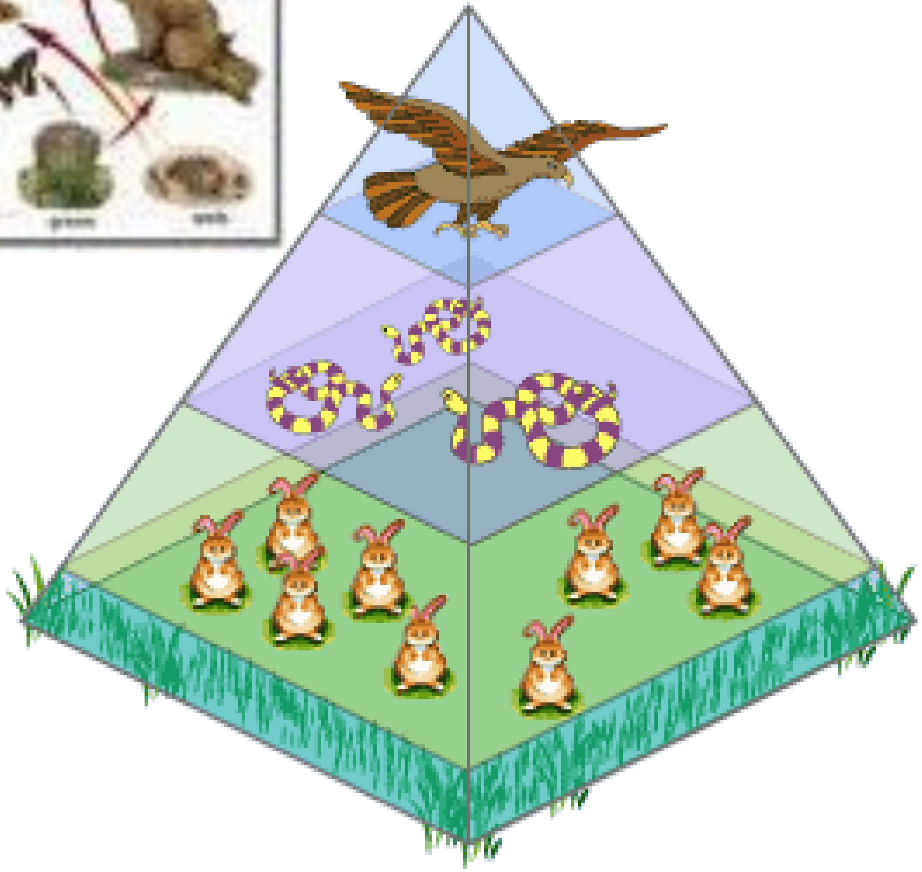
- Energy can access in different forms at given Ecosystem , such as heat, kinetic energy, chemical energy, potential energy
- The primary sources of Energy is sun as a form of Heat Energy
- because of inter-conversions it is not always easy to isolate separate constituents of energy balances.
- Whenever there is a conversion of Energy from one form to another there is loss in other form

No transformation of Energy is possible without its degradation from one form to another form

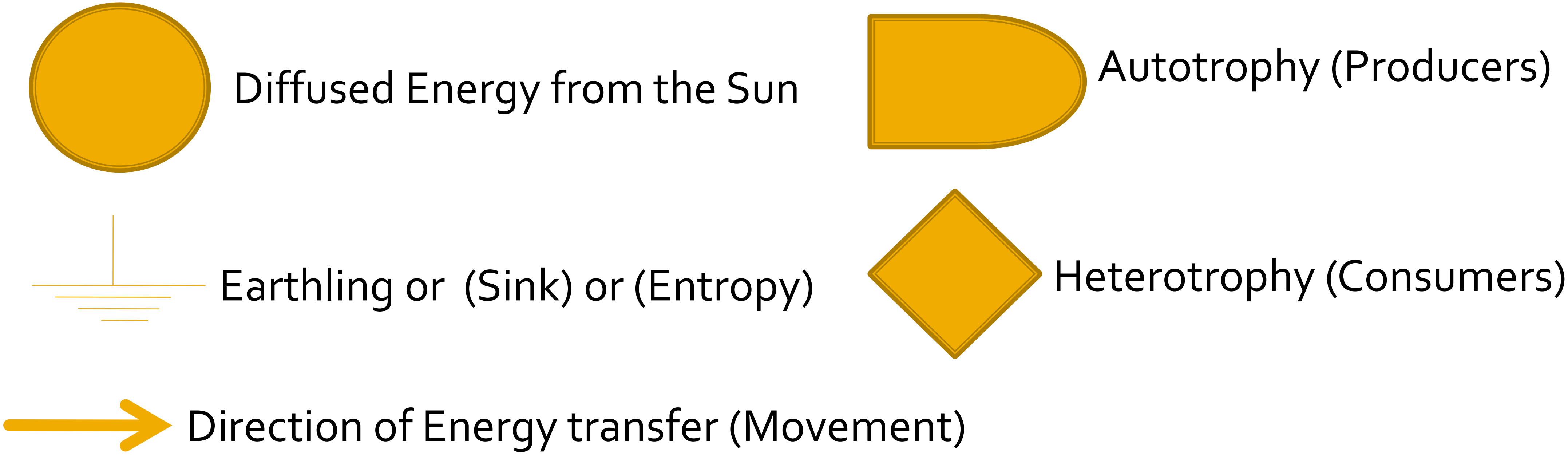
2nd law of Thermodynamics

ENERGY FLOW in ECosystem





Pictorial Presentation of Energy flow



Diffused Energy from the Sun

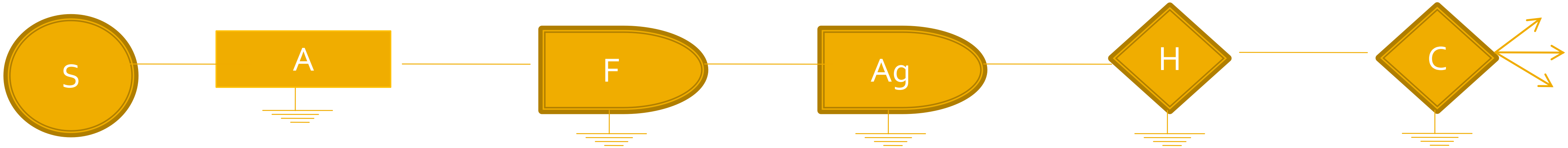
Autotrophy (Producers)

Earthling or (Sink) or (Entropy)

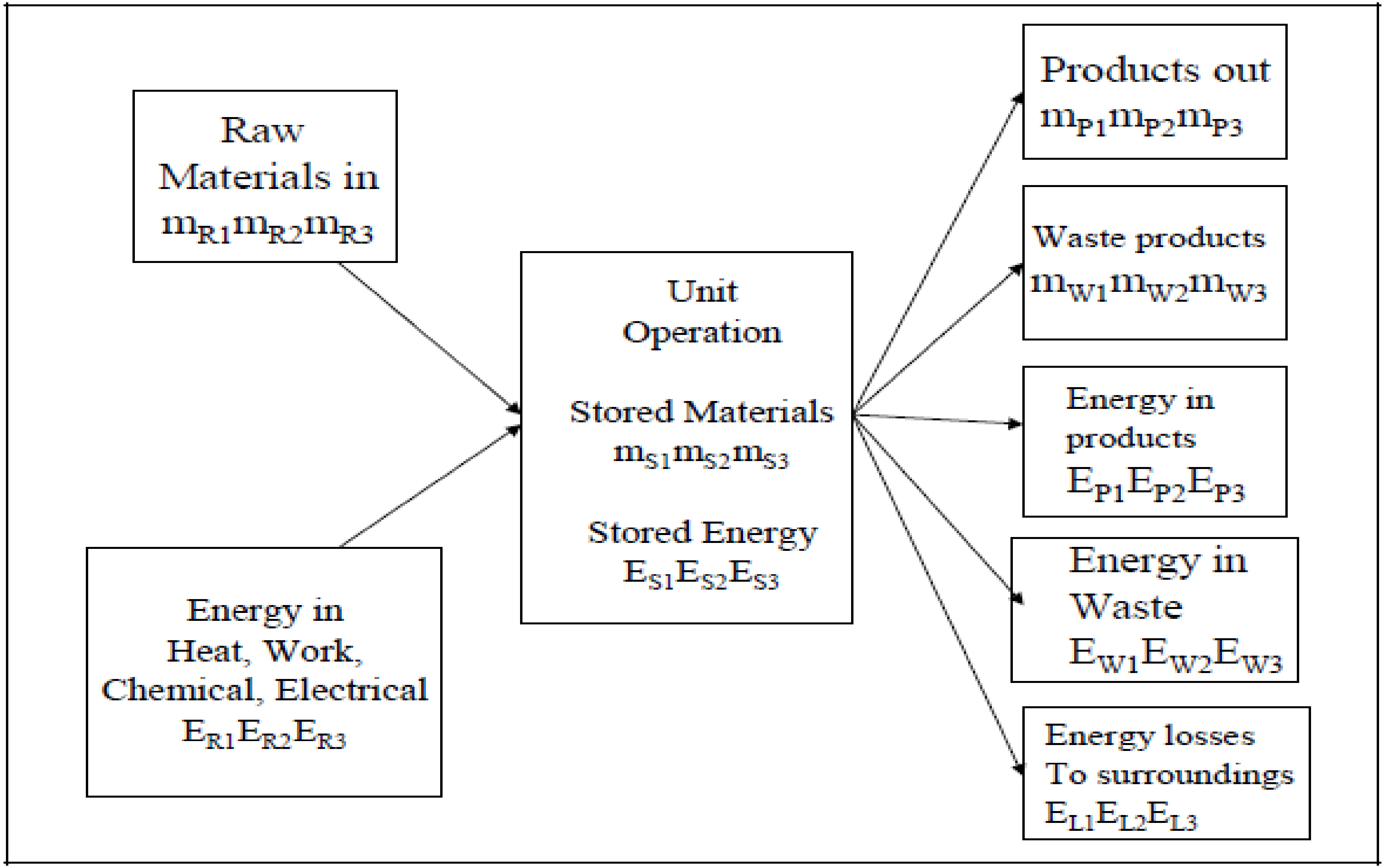
Heterotrophy (Consumers)

Direction of Energy transfer (Movement)

Sun $\xrightarrow[\text{5.0m}]{}$ Atmosphere $\xrightarrow[\text{1.0m}]{}$ Forest $\xrightarrow[\text{0.5m}]{}$ Agricultural $\xrightarrow[\text{2000}]{}$ Herbivores $\xrightarrow[\text{200}]{}$ Carnivorous



General Representation of Material and Energy Balances



Raw Materials in
$m_{R1} m_{R2} m_{R3}$

Energy in
Heat, Work, Chemical, Electrical
$E_{R1} E_{R2} E_{R3}$

Unit Operation

Stored Materials
$m_{S1} m_{S2} m_{S3}$

Stored Energy
$E_{S1} E_{S2} E_{S3}$

Products out
$m_{P1} m_{P2} m_{P3}$

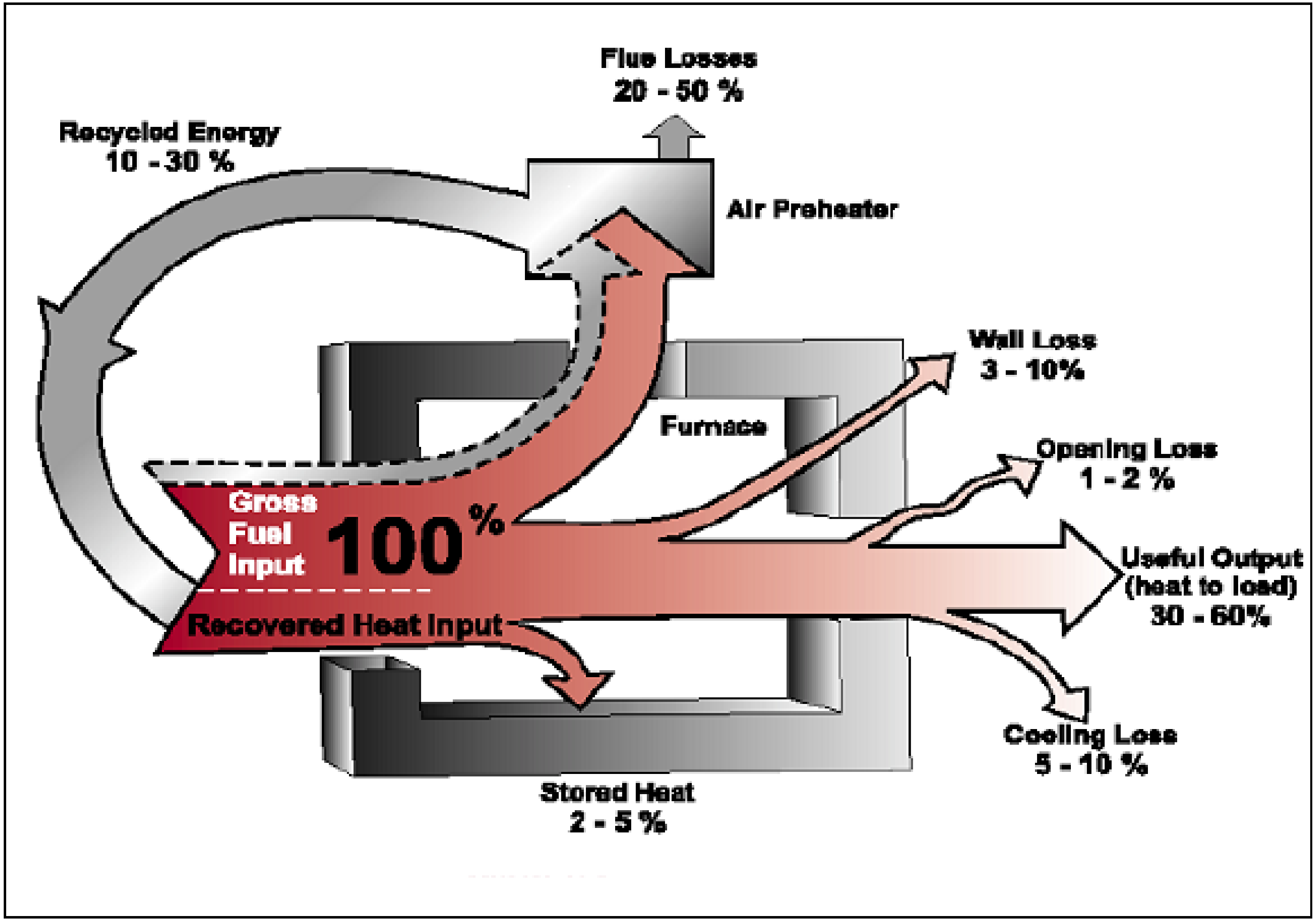
Waste products
$m_{W1} m_{W2} m_{W3}$

Energy in products
$E_{P1} E_{P2} E_{P3}$

Energy in Waste
$E_{W1} E_{W2} E_{W3}$

Energy losses To surroundings
$E_{L1} E_{L2} E_{L3}$

The Sankey Diagram



General Energy Balance Equation

Energy In = Energy Out + Energy Stored

$\Sigma E_R = \Sigma E_P + \Sigma E_W + \Sigma E_L + \Sigma E_S$

where

$\Sigma E_R = E_{R1} + E_{R2} + E_{R3} + \ldots\ldots. =$ Total Energy Entering

$\Sigma E_p = E_{P1} + E_{P2} + E_{P3} + \ldots\ldots. =$ Total Energy Leaving with Products

$\Sigma E_W = E_{W1} + E_{W2} + E_{W3} + \ldots =$ Total Energy Leaving with Waste Materials

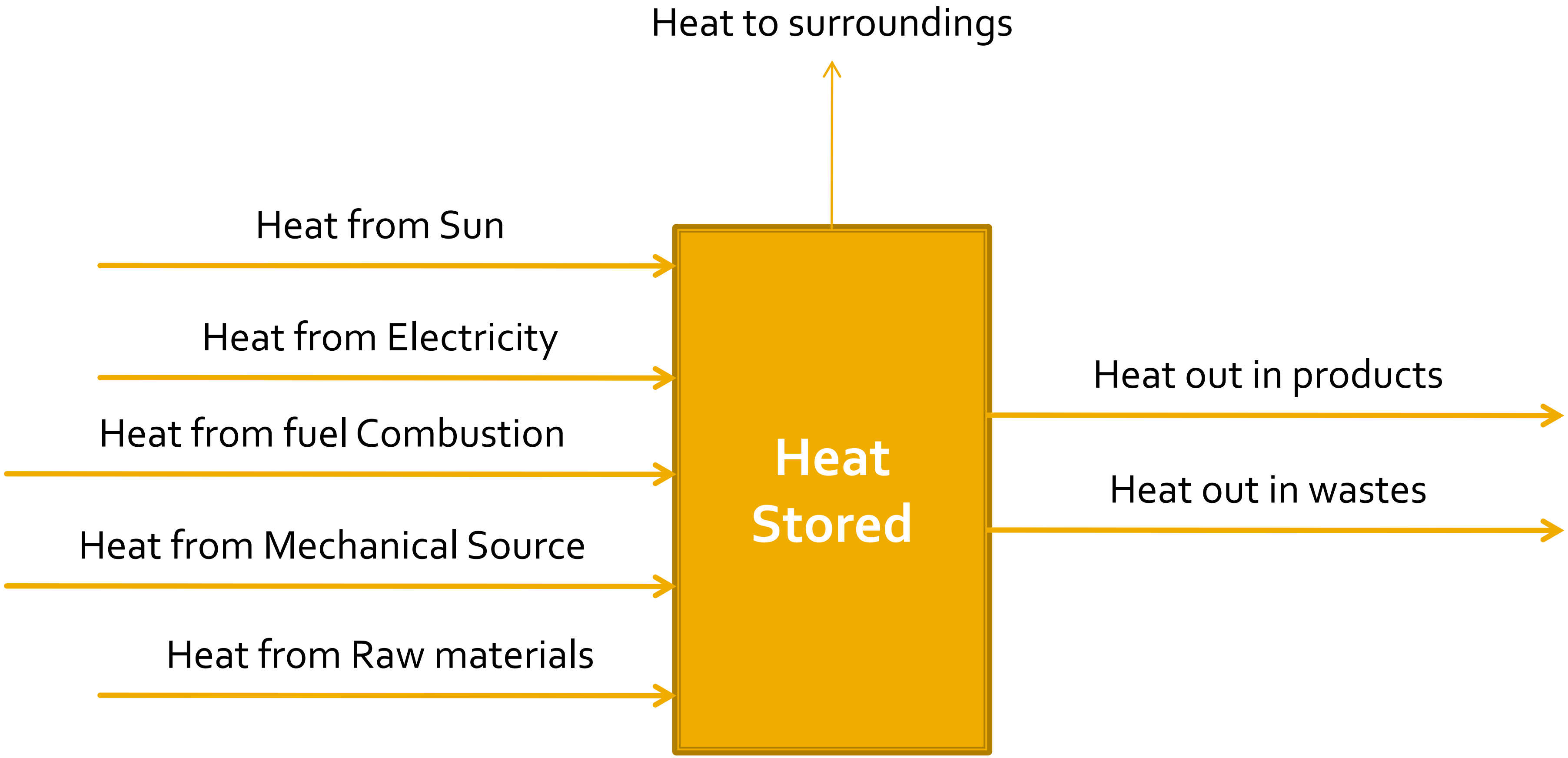
$\Sigma E_L = E_{L1} + E_{L2} + E_{L3} + \ldots\ldots. =$ Total Energy Lost to Surroundings

$\Sigma E_S = E_{S1} + E_{S2} + E_{S3} + \ldots\ldots. =$ Total Energy Stored

Heat Balance

- The most common and important energy form of energy
- conservation of it can be process through different operations such as **heating** and **drying**.
- In these, **enthalpy (total heat)** is conserved and as with the mass balances
- Enthalpy balances can be written round the various items of equipment or process stages, or round the whole plant
- It is assumed that no appreciable heat is converted to other forms of energy such as work.

Heat Balance



Heat to surroundings

Heat from Sun

Heat from Electricity

Heat from fuel Combustion

Heat from Mechanical Source

Heat from Raw materials

Heat Stored

Heat out in products

Heat out in wastes

Terms of Heat Balance

- **Latent heat** is the heat required to change, at constant temperature, the physical state of materials from solid to liquid, liquid to gas, or solid to gas.
- *The units of latent heat are J/kg and total latent heat change is calculated by multiplying the mass of the material, which changes its phase by the latent heat.*
- **Sensible heat** is that heat which when added or subtracted from materials changes their temperature and thus can be sensed.
- The units of specific heat are J/kg K
- sensible heat change is calculated by multiplying the mass by the specific heat by the change in temperature, ($m \times c \times \Delta T$).

Terms of Heat Balance

- Having determined those factors that are significant in the overall energy balance, the simplified heat balance can then be used with confidence in industrial energy studies. Such calculations can be quite simple and straightforward but they give a quantitative feeling for the situation and can be of great use in design of equipment and process.

Example: A water heater

How long would it take to heat the water in a 40 gallon electric water heater from 50F to 140F if the heater element delivers 5KW?

- Assume all of the electrical energy is converted to heat in the water,
- Neglect the energy required to raise the temperature of the tank itself, and
- Neglect any heat losses from the tank to the environment.

Energy input = 5kW x t = 5tkWhr

Energy output = 0

Change in energy = water warming from 50 to 140, (weight of water = 8.34lb/gal)

Change in energy stored = mct

= 40gallons x 8.34lb/gal x (140-50)F

= 30 X 10^3

Energy input = change to stored energy

Example: Power for the hydrological cycle

Global rainfall has been estimated to average about 1m of water per year across the entire 5.10×10^{14} m^2 . Find the **energy** required to cause that much water to evaporate each year.

The energy required to vaporize 1kg of 15C water is 2465kJ

- Compare this to the estimated 1987 world energy consumption of 3.3×10^{17} KJ -

- Compare it to the average rate at which sunlight is absorbed at the surface of the earth, which is about 168 W/m^2

Total energy required to vaporize all the water

Energy needed = $1\text{m/yr} \times 5.10 \times 10^{14} \times 2465\text{kJ/kg}$

$= 1.25 \times 10^{21}$ kJ/yr

$$\text{Average rate} = \frac{1.25 \times 10^{21}\ \text{kJ/yr}}{365 \times 24 \times 3600 \times 5.10 \times 10^{14}}$$

$= 78\ \text{W/m}^2$

Example: Thermal pollution of a river

A coal-fired power plant converts one-third of the coal's energy into electrical energy. The electrical power output of the plant in 1000 MW. The other two-thirds of the energy content of the fuel is rejected to the environment as waste heat. About 15 percent of the waste heat goes up the smokestack and the other 85 percent is taken away by the cooling water that is drawn from a nearby river. The river has an upstream flow of 100 m3/s and temperature of 20C·

a. If the cooling water is only allowed to rise in temperature by 10C, what flow rate from the stream would be required

b. What would be the river temperature just after it receives the heated cooling water?

Summary

- Material and energy balances can be worked out quantitatively knowing the amounts of materials entering into a process, and the nature of the process.
- Material and energy balances take the basic form
- Content of inputs = content of products + wastes/losses + changes in stored materials.
- In continuous processes, a time balance must be established.
- Energy includes heat energy (enthalpy), potential energy (energy of pressure or position), kinetic energy, work energy, chemical energy. It is the sum over all of these that is conserved.
- Enthalpy balances, considering only heat are useful in many processing situations.

The objective of M&E balance is to assess the input, conversion efficiency, output and losses. A M&E balance, used in conjunction with diagnosis, is a powerful tool for establishing the basis for improvements and potential savings.

Exercise

- For the following conditions, determine whether a CMFR or a PFR is more efficient in removing a reactive compound from the waste stream under steady-state conditions with a first-order reaction:

Reaction volume = 280 m^3, flow rate = 14m^3/d and reaction rate coefficient = 0.05d^{-1} .