### 4.3.3.3. Losses caused by recoil of the steel (post-tensioned tendons)

Account must be taken of the loss which occurs when there is a drive-in of the tendons at the anchorage, during the operation of anchoring after tensioning, and of the deformation of the anchorage.

The values to be taken into consideration are defined in the approval documents for the prestressing system concerned.

Due to this drive-in, the highest stress within the tendon is no longer at the anchorage.

### 4.3.3.4. Effect of heat-curing (pretensioning)

Two types of losses have to be taken into account

- reduction of steel stress due to an acceleration of relaxation during heat treatment
- direct thermal effect.

The loss of prestress due to relaxation during the heat treatment can be equated to 75% of the total value of relaxation losses.

*(a) Relaxation losses*

Relaxation losses can be calculated by adding to the value of time a duration defined by the formula

$$t_{ep} = t_{p1}\, 1.04^{(T_{max}-20)} \qquad (4.3\text{–}2)$$

where

$T_{max}$ is the maximum temperature of the concrete during heat treatment in °C,

$t_{p1}$ is the mean duration of the heating cycle, calculated by eq. (4.3–3):

$$t_{p1} = \frac{1}{T_{max} - 20} \int_o^{t_1} [T(t) - 20]\, dt \qquad (4.3\text{–}3)$$

where

$t_1$ is the age of the concrete when its temperature returns to ambient temperature

$T(t)$ is the temperature of concrete, in °C, at time $t$.

*(b) Losses of direct thermal origin*

Direct thermal effect is caused by

- the dilatation of concrete, when it is not bonded to the prestressing steel
- the restraint to the dilatation of concrete presented by the prestressing steel when it is bonded.

This type of loss does not exist with moulds supporting the tension of tendons and heated together with concrete.

The losses of direct thermal origin can be calculated by eq. (4.3–4)

$$\Delta\sigma = 0.9 E_p \alpha_p (T_{max} - T_0) \qquad (4.3\text{–}4)$$

where

$E_p$ is the elastic modulus of steel
$\alpha_p$ is the coefficient of thermal expansion of steel
$T_0$ is the temperature of steel at tensioning
$T_{max}$ is the maximum temperature of steel during heat curing.

### 4.3.3.5. Other immediate losses

Account should be taken of all possible causes of immediate loss of tension related to the tensioning process or the equipment used for prestressing.

## 4.4. VALUE OF PRESTRESSING FORCE

The initial prestressing force in a tendon is the force existing in this tendon at the end of the prestressing operation. The initial prestressing force on a prestressed element is obtained by considering all the forces existing in the tendons, at the end of the last prestressing operation.

The prestressing force at a given time $t$ is obtained by subtracting from the initial prestressing force the value of the time dependent losses at this time $t$.

These losses are due to creep and shrinkage of concrete and relaxation of steel.

The final value of the prestressing force is obtained by subtracting from the initial prestressing force the maximum expected value of the time-dependent losses.

### 4.4.1. Calculation of time-dependent losses

Data for calculation of the deformations of concrete under creep and shrinkage are given in section 2.1.

Ordinary reinforcement has an influence on the value of time-dependent shortening of concrete.

The interaction can be estimated as described in CEB Bulletin 199.

The time-dependent losses are calculated by considering the following two reductions of stress:

(*a*) the reduction of stress, due to the reduction of strain, caused by the deformation of concrete due to creep and shrinkage, under quasi-permanent actions:

(i) for bonded tendons, the local deformation at the level of the tendon has to be considered;
(ii) for unbonded tendons, the deformation of the whole structure

between the constraints of the tendons has to be taken into account;

(*b*) the reduction of stress within the steel due to the relaxation of this material under tension.

Data on the relaxation of steel are given in section 2.3.

The relaxation of steel depends on the reduction of strain due to creep and shrinkage of concrete. This interaction can generally be neglected.

The reduction of strain in steel due to time-dependent losses may be calculated by dividing the stress loss by the modulus of elasticity of steel.

## 4.5. BOND PROPERTIES OF POST-TENSIONED TENDONS

### 4.5.1. General

The main representative elements of the bond of a tendon are the ultimate value of bond stress, and the relationship between the bond stress and the value of the relative slip between concrete and tendon.

Values of design bond stress are given in subsection 6.9.10.

Bond stress and slip are measured at the external surface of the sheathing, but the bond values depend on

Sheathing can be made of steel or plastic; it can be corrugated or smooth.

- the nature of the prestressing steel: wires, strands, bars
- the structure of the surface of these elements.

The origin of variations of deformation is the reference state of decompression at the level of the tendons.

### 4.5.2. Numerical values

The bond properties of post-tensioned tendons can be given by comparison with bond properties of ribbed bars of same diameter, used for ordinary reinforcement. For a given value of bond slip, the value of bond stress for a post-tensioned tendon can be deduced from the bond value of an ordinary bar by a reduction factor $\eta_p$.

Calculation of the distribution of internal stresses is necessary for verification under fatigue effects.

The following values of $\eta_p$ can be adopted for the calculation of crack widths and of the distribution of internal stresses

$\eta_p$ = 0.2 for smooth prestressing steels
$\eta_p$ = 0.4 for strands
$\eta_p$ = 0.6 for ribbed prestressing steels.

These values of $\eta_p$ are established on the basis of tests made on tendons with a corrugated steel sheathing, and a maximum diameter of 56 mm.

These values for $\eta_p$ for grouted tendons, are different from the values given for direct bond strength, see clause 6.9.11.2.

In the general case when ordinary reinforcement is present together with post-tensioned tendons in the same section, the calculation of probable crack widths should be made taking into account the different diameters and bond properties of the reinforcing elements.

Tensile stresses are accepted in partially prestressed structures, where ordinary and prestressing reinforcement are used simultaneously.

# 4.6. DESIGN VALUES OF FORCES IN PRESTRESSING TENDONS

## 4.6.1. General

Prestress is usually exerted by one set of tendons. The total permanent force exerted at a given section (abscissa $x$), and at a time $t$, by the whole set is considered as the prestressing force; in exceptional cases, several prestressing forces (practically never more than two), should be considered separately. These cases should be identified by judgement. The criteria, to be simultaneously satisfied, are that

- the effect of the two sets are of contrary senses
- these effects have the same order of magnitude
- the dispersions are relatively high and there are qualitative reasons why they should not be considered as correlated.

## 4.6.2. Definition of prestress

Depending on the calculation, prestress is represented by various physical quantities, called indicators, each of them corresponding to a reference state. These reference states are specified in the relevant clauses.

If the prestressing steel remains elastic, the prestressing force $P(x, t)$ and the prestressing strain $\varepsilon(x, t)$ are connected by the relation $\varepsilon(x, t) = P(x, t)/E_p A_p$. In general this relation should be supplemented by a term due to the relaxation of the steel.

For example

- For the calculation of losses and the structural analyses the indicator is the prestressing force or stress at a time for a given section ($x$) when the structure is subjected to permanent actions (without $\gamma$ factors) or, more precisely, to the quasi-permanent combination of actions
- The hyperstatic prestress is a complementary indicator deduced from the indicator defined above (see clause 1.4.3.1.)
- For some verifications of resistance (when the yield strength of prestressing steels is exceeded) the indicator is the difference of strain between tendons and the adjacent concrete (see OB on Fig. 1.4.1).

Other indicators, e.g. the strain corresponding to a zero curvature, have been used in some background studies. An analysis of various indicators is given in CEB Bulletin 202.

Whatever the selected indicator is, a mean value of prestress is defined. Two characteristic values (an upper and a lower) are also defined, taking into account the possible variations of the losses and of the tensioning force.

The characteristic values are normally calculated by means of the approximate formulae given in clause 1.4.3.2. If the field of application of these formulae is infringed, for instance in the case of very large immediate losses, more precise formulae have to be used, such as

$$P_{k,\text{sup}}(k, t) = P(0,0) - 0.7[\Delta P(x,0) + \Delta P(x,t)]$$

$$P_{k,\text{inf}}(k, t) = P(0,0) - 1.30[\Delta P(x,0) + \Delta P(x,t)]$$

### 4.6.3. Design values for SLS verifications

For all verifications relating to cracking (decompression included), for the analysis of the fatigue effect and in specific cases where the size of the prestress influences the result in a large overproportional way, the less favourable characteristic value should be taken into account.

It is only acceptable to introduce a mean value $P_m$ for deformation calculations, and for the verification of limiting compressive stresses. See chapter 7.

### 4.6.4. Design values for ULS verifications

For the verifications with regard to fatigue and, if relevant, to static equilibrium, the less favourable characteristic value should be taken into account.

For the other ultimate limit states a factor $\gamma_{P\,sup}$ or $\gamma_{P\,inf}$ shall be applied in accordance with clauses 1.4.3.2. and 1.6.2.4.

Depending on the type of structural analysis the $\gamma_P$ factors may have to be introduced at the level of the input of the analysis or at the level of the output (i.e. applied to the effects of $P$), or even partially at each level (see clause 1.6.2.4(c)). In many cases, $\gamma_{P,inf}$ being equal to 1.0, no practical problem is met. In other cases, e.g. for the assessment of hyperstatic effects or where the yield strength of the prestressing steel is exceeded, $\gamma_P$ may be taken equal to 1.0 and $P$ may be taken equal to $P_m$, as acceptable approximations.

## 4.7. ANCHORAGE AND COUPLING OF PRESTRESSING FORCES (POST-TENSIONING)

### 4.7.1. General

After hardening of the concrete, the tendons are tensioned and their extremities are fixed within anchorages, which transfer the prestressing forces to the concrete.

When cement grouting is applied, the transmission of the prestressing force is not dependent on the anchorages only.

Coupling devices, or couplers, may be used to connect the end of a tendon, which is tensioned first, to the end of a second tendon, placed as a prolongation of the first, and which will be prestressed in a second stage.

With unbonded tendons, special attention should be given to the fact that the anchorages are the only means of transmission of the prestressing force to the structure. It may be necessary to place intermediate anchorages, functioning in both directions, to prevent the risk of progressive collapse, when the strength of the structure is achieved by one set of tendons extending over many spans.

Information relating to anchorage arrangements are given in the approval documents. When the assumptions or service conditions differ from those envisaged by these, additional experimental checks may be necessary.

The deviators have to be designed under the unfavourable assumption that a relative displacement of the tendon takes place, resulting in friction on the deviation device. Generally the calculation of the structure itself can be made assuming the tendons are fixed at the deviating points.



With external prestressing, deviating devices are placed between the tendons and the structure. These devices, and their fixing zones, have to be designed to transfer the corresponding design action, taking the permissible tolerances into account.

With external prestressing, it is recommended to provide for the replacement of the prestressing tendons.

### 4.7.2. Transfer of load from the tendon-anchorage-assembly to the concrete

The strength of the anchorage zones should exceed the characteristic strength of the tendon, both under static load and under slow-cycle load.

Possible formation of small cracks in the anchorage zone may not impair the permanent efficiency of the anchorage if sufficient transverse reinforcement is provided.

This condition is considered to be satisfied if stabilization of strains and cracks widths is obtained during testing.

## 4.8. CORROSION PROTECTION OF TENDONS

### 4.8.1. General

The principles of corrosion protection of tendons in the temporary and permanent state are given in chapters 8 and 11.

# PART II. DESIGN PROCEDURES

## 5. STRUCTURAL ANALYSIS

### 5.1. GENERAL

Structural analysis is defined as the determination of the action effects over the whole or part of a structure, with the purpose of carrying out a verification at the ultimate and serviceability limit states.

This chapter is established in accordance with the concept and rules defined in sections 1.3 and 1.6. Unless stated otherwise, the numerical data for applying it are the design values of the basic variables as defined in sections 1.4, 1.6, 4.6 and 6.2. In some cases, e.g. when using FEM (or also in the cases of clauses 1.6.2.2 and 1.6.3.2), the fulfilment of some verification conditions may be obtained in the course of the analysis itself.

Structural analysis means an overall analysis as defined in section 1.3.

The considered action effects may be

- stresses (see eq. (1.3–3)).
- sectional forces (axial and shear) or moments (bending and torsional) (see eq. (1.3–2))
- geometrical quantities (deflections, rotations, crack widths) (see eq. (1.3–4))
- vibrations, however these are generally indirectly treated by specific methods (see section 7.6).

Except for the determination of stress resultants in isostatic structures, no structural analysis can be carried out without assumptions about material behaviour.

### 5.2. IDEALIZATION OF THE STRUCTURE

#### 5.2.1. Dimensional classification of structural elements

For the analysis the structural elements are classified as

- one-dimensional when one dimension is much larger than the other two
- two-dimensional when one dimension is relatively small compared with the other two
- three-dimensional when no dimension is largely prevailing.

Columns, beams, arches are considered one-dimensional elements, if the length is larger than three times the overall sectional depth.

Slabs, plates, deep beams, walls, shells are considered two-dimensional elements. Slabs and deep beams are defined as

- slabs, if in presence of transverse forces the minimum distance between adjacent regions of zero moments is not less than four times the overall thickness
- deep beams, if the span is less than twice the overall depth.

The definition of structural schemes could be made with reference to the state of stress rather than to geometrical dimensions. According to this approach the definitions are

- one-dimensional schemes when normal stresses in one direction predominate over those in the orthogonal directions
- two-dimensional schemes when normal stresses in two orthogonal directions predominate over those in the third
- three-dimensional schemes when normal stresses do not predominate in any of the three orthogonal directions.

### 5.2.2. Classification in terms of level of discretization

The methods of analysis may refer to

- cross-sections of structural members
- a fibre along a structural member
- finite elements.

### 5.2.3. Geometrical data

#### 5.2.3.1. Effective width of flanges

In T-beams a uniform distribution of longitudinal stresses over a reduced width of the flange, called effective width, may be assumed for all limit states, both for the overall analysis and for section verification.

The effective width may be determined on the basis of calculations by elastic or plastic theory, and may be varied along the axis of the beam.

In the absence of a more accurate determination, the effective width to be used in the overall analysis should be taken equal to the thickness of the web, plus 1/5 of the approximate distance between the points of zero moment, but not exceeding the actual width of the top slab. In this approximation the effective width can be taken as constant over the entire span, including the parts near intermediate supports for continuous beams.

For edge beams the effective width is taken equal to the thickness of the web plus 1/10 of the distance between the points of zero moment.

The points of zero moment considered above may practically be considered as fixed for all calculations and associated with the distribution of moments due to the permanent weight.

#### 5.2.3.2. Effective span

Usually, the span $l$ has to be introduced as the distance between adjacent support axes. When reactions are located significantly away from the axis of support, the effective span has to be calculated taking into account the real position of the support section.

In the overall analysis of frames, when $l_{ef}$ is less than the distance between axes of columns the dimensions of the joints should be taken into account by introducing rigid elements between the centroidal axis of the column and the end section of the beam.

#### 5.2.3.3. Cross-sections

The gross concrete section is defined as the total concrete section, where the contribution to structural stiffness of the reinforcement and the area of ducts for prestressing tendons is disregarded. If these are taken into account, by adopting a suitable modular ratio for the reinforcement, the section is designated as the transformed concrete section.

## 5.3. CALCULATION METHODS

### 5.3.1. Basic principles

Any structural analysis shall satisfy the equilibrium conditions.

If not otherwise stated, the compatibility conditions shall always be satisfied in the limit states considered.

In the cases where verification of compatibility is not directly required, suitable ductility conditions should be satisfied and adequate performance under service conditions be ensured. In general the equilibrium conditions are formulated for the undeformed system (first order theory).

In case of slender structures as defined in clause 1.6.3.1 the equilibrium shall be verified for the deformed system (second order theory). In these cases the influence of deformations on action effects shall be considered.

### 5.3.2. Types of structural analysis

The overall analysis of a structure can be performed according to the following methods

- non-linear analysis
- linear analysis
- linear analysis with redistribution
- plastic analysis.

The choice of method depends on the assumed behaviour of the materials and the possible consideration of the effects of deformation on the action effects (second order effects).

In this context plastic analysis is meant as both upper bound and lower bound solutions.

#### 5.3.2.1. Non-linear analysis

The structural analysis is defined as non-linear if a non-linear behaviour is assumed for the materials. Second order effects may or may not be taken into account.

Equilibrium and compatibility conditions should be fulfilled.

This approach implies that non-linearity arising from a realistic consideration of the structural behaviour should be taken into account in the 'response relationship' (non-linear constitutive laws for materials, non-linear force deformation relationship for cross-section members or sub-assemblages, due to material properties, cracking and second order effects).

The method generally requires an initial definition of the geometry of the structure and of the reinforcement.

The non-linear behaviour of materials and the second order effects are considered by suitable incremental and/or iterative numerical methods.

Non-linear analysis is a realistic description of the physical behaviour and therefore a method completely consistent with the assumptions used for the local verification and member design; it should be used as a reference for other more simplified approaches.

#### 5.3.2.2. Linear analysis

The structural analysis is defined as linear if a linear elastic behaviour is assumed for the materials.

Concerning the cross-section to be taken into account, see clause 5.2.3.3.

This approach implies that the 'response relationship' is linear, and the assumption of reversible deformations is retained. The results are realistic only under the condition that actions are low and members are uncracked.

For ULS verifications existing practice allows the use of linear analysis without direct verification of sufficient ductility. This is based on the assumption that there is ductility enough to balance the lack of compatibility, under the conditions stated in subsection 5.4.2 for beams and frames.

The method is normally used with the gross-section of concrete members; therefore it requires definition of geometry of the structure, but not necessarily of the reinforcement.

Cracked cross-sections may, however, be used if in the limit state under consideration a fully developed crack pattern can be expected.

The results of a linear analysis are also used in the verification for the serviceability limit state.

#### 5.3.2.3. Linear analysis with redistribution

The structural analysis is defined as linear with redistribution, if the action effects derived from a linear analysis are redistributed in the structure. Equilibrium and suitable ductility conditions should be satisfied.

Although the highest redistribution is obtained in the ULS, even some redistribution takes place in SLS due to cracking.

#### 5.3.2.4. Plastic analysis

The structural analysis is defined as plastic if one of the two basic theorems of plasticity upper bound or lower bound is met. Rigid-plastic or elastic-plastic behaviour for the materials may be assumed.

Plastic analysis is allowed only if sufficient ductility for the attainment of the assumed configurations is ensured.

The method is not allowed when consideration of second order effects is required.

## 5.4. BEAMS AND FRAMES

### 5.4.1. Non-linear analysis

#### 5.4.1.1. General

Non-linear analysis may be used both for SLS and ULS verifications under the conditions described in clause 5.4.1.4.

Non-linear analysis can be performed at different levels of complexity according to

- the models of ULS to be considered in the verification
- the model of one-dimensional element to be used in analysis
- the type of constitutive law to be adopted in the analysis and verification of sections
- the type of load history to be adopted in analysis.

In case a) no evaluation of plastic behaviour of materials is needed in performing the analysis, which takes into account cracking and second order effects only. This approach, which may be called 'non-linear elastic' analysis, should be adopted whenever evaluation of plastic behaviour is uncertain, due to the brittle nature of the failure mechanism.

For models b) and c) a trilinear representation of the moment–rotation relationship can be adopted which also describes state 3 (plastic conditions), characterized by the plastic rotation $\theta_{pl}$.

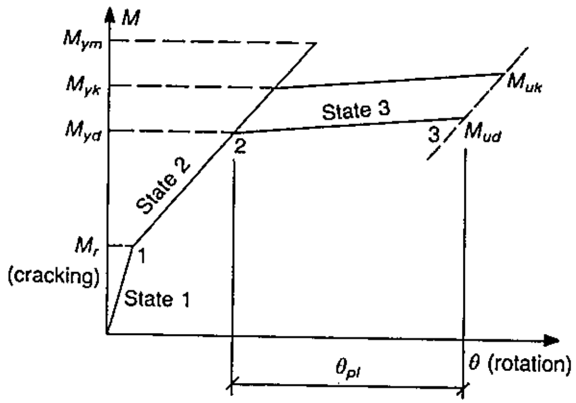



*Fig. 5.4.1. Idealized trilinear diagram*

Notation for Fig. 5.4.1

$M_r$ is the cracking moment
$M_{ym}$ is the moment corresponding to the yielding of the steel, based on mean values of material properties
$M_{yk}$ is the moment corresponding to the yielding of the steel, based on characteristic values of material properties
$M_{yd}$ is the moment corresponding to the yielding of the steel, based on design values of material properties
$M_{uk}$ is the characteristic value of resisting moment
$M_{ud}$ is the design value of resisting moment.

The third branch is derived assuming an affinity with respect to the characteristic values.

## 5.4.1.2. Models for ultimate limit states

Three basic models can be considered.

- *Model* (*a*) represents 'First Yield', corresponding to the load level at which plastic behaviour takes place for the first time at a critical section.
  For the moment–curvature relationship it is often sufficient to adopt a bilinear representation to describe
  — state 1: linear elastic, uncracked
  — state 2: cracked.

  The conditions of compatibility may be obtained by deducing the curvature of each section from an appropriate moment–curvature relationship and by integrating this curvature to obtain the deformed shape.
  For most construction works and design situations a holonomic structural behaviour may be assumed (see clause 5.4.1.4) and therefore normally a law of deformability (e.g. a moment–curvature law) can be applied assuming monotonic conditions.
- *Model* (*b*) represents the formation of a mechanism which can take place whenever critical sections are ductile enough to permit the formation of this mechanism.
- *Model* (*c*) represents local rupture due to exceedance of the limit plastic rotation $\theta_{pl}$ at a critical section.

Models (b) and (c) may be used only for verifications with regard to the ultimate limit state of resistance.

The plastic rotation may be assumed to be concentrated at the critical section; the permissible local plastic rotation $\theta_{pl}$ can be obtained from Fig. 5.4.2. Additional rotation capacity can be obtained considering the favourable influence of possible confining reinforcement (see section 3.5).

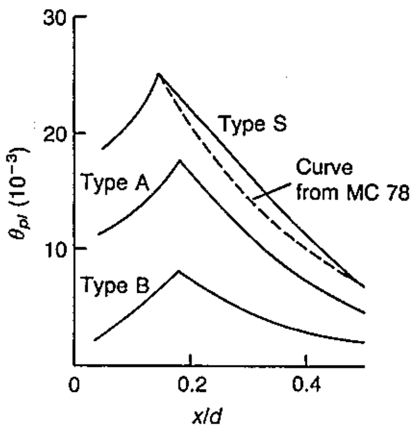



*Fig. 5.4.2. Rotation capacity $\theta_{pl}$ versus* x/d *relative depth of the neutral axis of the critical cross-section at ULS for various steel classes (the* x*-values are conventionally calculated considering design values of material properties, and nominal limit values of $\varepsilon_c = 0.35\%$ and $\varepsilon_s = 1.0\%$)*

### 5.4.1.3. Models of one-dimensional elements used in analysis

With some exceptions, the hypothesis that plane sections remain plane is assumed to be valid.

This approach should be adopted whenever secondary effects such as bond slip, aggregate interlock, and dowel effect, are considered for an adequate modelling of the structure.

The following models may be used in non-linear analysis of one-dimensional structures

- one-dimensional models, where cracking is considered as diffused along the elements while plasticity is considered as concentrated at critical sections by the introduction of 'plastic hinges', at an appropriate load level.
- one-dimensional 'layered' models, where the state of stress in each layer is derived from the state of strain using the constitutive laws of materials and the element stiffness is defined accordingly
- assemblages of one- and two-dimensional finite models, to model the behaviour of the one-dimensional member at 'micro' level.

### 5.4.1.4. Constitutive laws for overall analysis and verification of sections

A non-linear analysis for the ULS may be carried out assuming mean values of the material properties up to the level of design-yield stress in the reinforcing steel; after yield is attained in critical regions, design values of the material properties should be used both for analysis and for resistance determination.

Where no major unloading and reloading of the structure is considered, or the plastic behaviour of the structure is not studied as in non-linear elastic analysis, holonomic constitutive laws may be used.

In the other case the non-holonomic nature of the constitutive laws shall be taken into account.

As an approximation, material properties associated with the specified characteristic strength of the materials may be used.

A constitutive law is defined as holonomic when loading and unloading paths are coincident.

### 5.4.1.5. Type of load history to be adopted in analysis

Two basic types of load history may be adopted whenever an incremental approach is used for non-linear analysis

- proportional, where all loads are incremented using the same multiplying factor
- non-proportional, where the various types of loads are incremented according to a given load history which follows, as far as possible, the order according to which real loads are likely to be applied to the structure.

Whenever an iterative method of analysis is used it should be assumed that the load history is proportional.

Following this approach, it is generally recommended that the calculations be performed in the order

- Apply permanent and quasi-permanent loads ($G + \Psi_2 Q_k$).
- Consider creep and apply the indirect actions with their design values.
- Finally apply progressively the remaining part of the design combination of loads (i.e. $(\gamma_G - 1)G + (\gamma_Q - \Psi_2)Q_k + \ldots$) increasing it proportionally up to the simulated collapse of the structure.

If relevant, due attention should be paid to different stages of construction of structural parts for the same building. When investigating load histories of this type it is relevant to consider the state of internal damage which may have occurred before the load path under consideration. Previous damage may lead to anisotropic constitutive laws.

## 5.4.2. Linear analysis

Linear analysis is to be applied mainly for serviceability limit states and may also be used for verifying the ultimate limit state for continuous beams and non-sway frames.

At the ULS, a linear analysis cannot always satisfy the conditions of compatibility in view of the invalidity of the assumptions relating to the corresponding deformations. The beams shall be capable of sufficient plastic rotation to prevent local rupture before the calculated moment distribution has been attained.

In the absence of more detailed information, this criterion may be assumed to be met if, for regular orthogonal frames with approximately equal geometry in all spans and storeys, the equivalent slenderness ratio $\lambda^*$, according to eq. (5.4–1), does not exceed 30.

For a single isolated column having uniform cross-section, the equivalent slenderness ratio is defined as

$$\lambda^* = \frac{\sqrt{\nu_{sd}}}{1 + 15\rho}\,\lambda \qquad (5.4\text{–}1)$$

where

$\lambda = l/i$ is the Eulerian slenderness (see also clause 6.6.1.3)
$\rho$ is the geometrical percentage of total longitudinal reinforcement, $= A_s/A_c$
$\nu_{sd}$ is the relative axial force $= N_{Sd}/(A_c f_{cd})$.

For a frame, the most unfavourable column with regard to loading and slenderness ratio $\lambda$ should be checked. Where $\lambda^* > 30$, a more rigorous structural analysis should be carried out according to section 6.6.

The consideration of the equivalent slenderness is an application of the concept that the reinforcement ratio considerably affects second order effects (clause 6.6.3.1.1). When the more stringent conditions of clause 6.6.3.1.1 are not met, the above expression for $\lambda^*$ may help to avoid a rigorous analysis.

For continuous beams and beams belonging to non-sway frames, sufficient ductility can be assumed to be present, if the relative depth of the neutral axis $x/d$ in the critical cross-section in the ULS is in accordance with the following:

| *Steel type* | *Concrete grades* | $x/d$ |
|---|---|---|
| S and A | C12 to C35 | $\leqslant 0.45$ |
| S and A | C40 to C80 | $\leqslant 0.35$ |
| B | C12 to C80 | $\leqslant 0.25$ |

The ductility is increased by transverse reinforcement. $x/d$ can be reduced by means of suitable compression reinforcement.

In the absence of more precise information on the influence of the type, quality and bond properties of prestressing steels, post-tensioned steel may be assimilated to type A steel, and pretensioned steel to type B steel.

It can also be used for determining the first order loading effects for sway frames, provided that the reduction in bearing capacity due to second order effects does not exceed the limit defined in clause 6.6.1.3.

## 5.4.3. Linear analysis followed by limited redistribution

### 5.4.3.1. General

For the verification of ultimate limit state, it is allowed to reduce the moments in the sections subjected to the highest action effects, resulting from a linear analysis, provided that in the other sections the moments are increased to maintain equilibrium.

For a structure subject to various load cases, only one redistribution can generally be assumed.

These rules are mainly meant to be applied when type S and A steels are used.

In principle, all the consequences of the assumed redistribution and of the possible scattering of its value, should also be taken into account in the verification procedure, concerning shear, anchorage of the bars and cracking. In particular, the length of the reinforcing bars shall be sufficient to prevent any other section becoming critical.

In curved beams, flexural yielding may produce a sudden increase of torsion, which can lead to a brittle failure before the redistribution of flexural moments is fully exploited.

### 5.4.3.2. Ductility conditions

The reduction coefficient $\delta$ to be used for multiplying the moments in the sections subjected to the highest moments should satisfy the following conditions to be applied to beams with straight axes in horizontal plane only.

| *Steel type* | *Concrete/structure* | *Reduction coefficient* |
|---|---|---|
| S and A | Concrete grades C12 to C35 | $\delta \geqslant 0.44 + 1.25x/d$ |
| S and A | Concrete grades C40 to C60 | $\delta \geqslant 0.56 + 1.25x/d$ |
| S and A | Continuous beams and non-sway frames | $0.75 \leqslant \delta \leqslant 1.00$ |
| S and A | Sway frames | $0.90 \leqslant \delta \leqslant 1.00$ |
| B | Concrete grades C12 to C60 | $\delta \geqslant 0.75 + 1.25x/d$ <br> $0.90 \leqslant \delta \leqslant 1.00$ |

A redistribution of 25% may result after cracking, owing to the reduction in stiffness due to cracking between zones in the span and over the supports. Such a redistribution need not always be linked to a ductility condition and it does not always take place in the desired direction. The given ductility conditions cover the most unfavourable cases.

This is allowed only if the equivalent slenderness ratio $\lambda^*$ does not exceed 15.

Post-tensioned steel may be assimilated to type A steel and pretensioned steel to type B steel.

## 5.4.4. Plastic analysis

It should be verified that the required plastic rotations in the plastic hinges, for the assumed mechanism, are less than the limiting plastic rotations $\theta_{pl}$ given in Fig. 5.4.2.

The method is not allowed for sway frames, nor if type B steel is used.

See also comments on clause 5.3.2.4.

## 5.4.5. Second order effects

In general, second order effects are to be taken into account only for the ultimate limit state.

When second order effects considerably influence the action effects (e.g. sway frames) they should be considered also for the serviceability limit states.

# 5.5. SLABS

## 5.5.1. Scope

Section 5.5 applies to solid slabs subjected to two-dimensional bending with or without prestressing. Non-solid slabs such as ribbed slabs, hollow-core slabs etc. are included provided that their behaviour, especially their stiffness, may be simulated by a corresponding solid slab.

## 5.5.2. Types of analysis

Slabs may be analysed by each of the following methods

- linear analysis
- linear analysis with redistribution
- plastic analysis
  - plastic analysis considering only equilibrium conditions at the ultimate state of the load carrying capacity of slabs (lower bound solutions), using the static method
  - plastic analysis assuming a yield line collapse mechanism for the ultimate state (upper bound solutions), using the kinematic method
- non-linear analysis.

The actual behaviour of slabs under loads and imposed deformations is covered best by a non-linear analysis which may be regarded as the reference solution, whereas linear analysis is covering primarily the serviceability and plastic analysis primarily the ultimate limit state.

This may be carried out by adopting non-linear stress–strain relations or non-linear moment–curvature relations satisfying conditions of equilibrium as well as compatibility.

## 5.5.3. Linear analysis

The method is based on the theory of elasticity adopting a linear moment–curvature relationship and a value for Poisson's ratio between 0.0 and 0.2.

If the reinforcement is determined on the basis of this method, it is not necessary to verify rotation capacities. Moments with local steep gradients (concentrated loads etc.) may be distributed in a convenient wider area provided equilibrium is preserved.

With regard to limit analysis the solution of the linear theory of elasticity is a lower bound solution.

In practice, for the usual cases, the results obtained by the linear analysis adopting a reduced stiffness may give sufficient information for the verification of the limit state of deformation.

## 5.5.4. Linear analysis followed by limited redistribution of bending moments

For built-in continuous slabs, the numerically largest moments resulting from a linear analysis may be reduced without verifying the rotation capacity, provided the modification factor $\delta$ of the moments satisfies the same conditions given in clause 5.4.3.2 for beams and frames.

The reduction of the moment shall be performed assuming the average value for an appropriate width, providing that the average moments for the same width at the corresponding section (supports or midspan) are adjusted to satisfy equilibrium.

The redistribution may modify the moments at the same places in the other direction.

## 5.5.5. Plastic analysis

As this analysis is aimed for the ultimate behaviour, additional boundary conditions regarding rotation capacity and serviceability have to be satisfied. They are

The method should not be employed if type B steel is used or when considerations of second order effects or fatigue are required.

(a) The tensile reinforcement at any point and in any direction should not exceed one half of that which corresponds to a section for which the ULS in bending is characterized by the following strains

$$\varepsilon_s = \varepsilon_y \text{ and } \varepsilon_c = -0.0035 \ (\varepsilon_y = f_y/E_s)$$

In any case, the minimum amount may be disposed as for beams.

For built-in or continuous slabs the ratio of the support moments to the mid-span moments should normally not be less than 0.5 or more than 2.

This condition is required to avoid excessive cracking.

(b) For ribbed slabs it may be necessary also to verify the shearing force capacity of the reinforcement and the compression zone considering the crack depth.

### 5.5.5.1. Lower bound solution of plastic analysis

Statically admissible moment fields which satisfy the equilibrium condition may be found directly (e.g. by applying the strip method) or by starting from a linear analysis.

With this approach, even with the above mentioned limitations, the satisfaction of the serviceability limit states is rather uncertain. Therefore, it is convenient to adopt a field of moments which does not differ substantially from the elastic solution.

### 5.5.5.2. Upper bound solutions of plastic anlaysis

Kinematically admissible plastic deformation modes (e.g. the yield line theory) may be applied covering the corresponding moment field by securing the design ultimate moments at the cross-sections along the necessary yield lines.

## 5.5.6. Non-linear analysis

Application of the physically non-linear theory includes the interaction of all three moments in a slab, $m_x$, $m_y$, $m_{xy}$, and analyses the elastic non-cracked, cracked, and plastic phases of a slab subjected to increasing loads. The non-linear anlaysis covers the serviceability as well as the ultimate limit state.

For the numerical evaluation layered models, finite elements, or finite differences may be chosen.

# 5.6. DEEP BEAMS AND WALLS

## 5.6.1. Methods of analysis

The forces acting in the middle plane of a deep beam may be determined by applying either

(a) linear analysis based on the theory of elasticity
(b) statically admissible stress fields, in accordance with the lower bound theorem of limit analysis (as by analysing an equivalent truss consisting of struts and ties, preferably following the elastic field)
(c) non-linear analysis.

## 5.6.2. Linear analysis

The theory of elasticity may be applied assuming values of Poisson's ratio of 0.0 to 0.2. In most cases only numerical solutions (e.g. finite differences, finite element methods, or boundary element methods) are suitable. The analysis gives the fields of principal stresses and deformations. High stress concentrations, like those at the corner of openings, may be reduced considering cracking effects.

The linear analysis is valid both for serviceability and ultimate limit states.

The analysis for the ULS requires a correct detailing of the reinforcement to withstand the resultant tensile zones in the concrete, satisfying equilibrium conditions.

## 5.6.3. Analysis by statically admissible stress fields

If a stress field is chosen which satisfies the equilibrium conditions, a lower bound solution of limit analysis is considered. For the structure and its loads an equivalent truss may be investigated, consisting of concrete struts and arches as compressive members, and of steel ties, formed by the reinforcement as tensile elements, and their connections (nodes). The equilibrium model may be applied for verifying the ULS and also for the SLS, provided that the evaluated stress distribution is close to the results of the linear analysis.

This method is not allowed if type B steel is used.

## 5.6.4. Non-linear analysis

For a more refined analysis, non-linear stress–strain relations may be taken into account by applying the numerical method for two-dimensional plane structures. The analysis then gives results for the serviceability as well as for the ultimate limit states.

## 5.7. SHELLS AND FOLDED PLATES

For two-dimensional structural elements subjected to combined bending and membrane action, usually a linear analysis of SLS, on the basis of the linear theory of elasticity, may suffice to evaluate the stress fields and deformations of the structure. The analysis should give principal forces and moments.

Shells subjected to compression have to be investigated with regard to buckling failure.

A solution satisfying equilibrium conditions may be found by considering only the membrane solution and neglecting the bending moments. The structure, however, may crack considerably especially at boundary regions (e.g. clamped edges). Hence, in general, bending should be included in the analysis.

At least the following effects should be included in such an analysis: geometrical imperfections of the shell form, the decrease of the bending stiffness caused by bending cracks (state II), displacements due to load actions and creep and settlement of supporting members at the boundaries.

## 5.8. STRUCTURAL EFFECTS OF TIME-DEPENDENT PROPERTIES OF CONCRETE

### 5.8.1. General

The inelastic strains due to creep and shrinkage of concrete may cause non-negligible changes in the long-term state of deformation and/or stress of structures and structural elements.

The characteristics and time sequence of loading and restraint conditions in the construction and service stages are of importance.

By analogy with the effects of other inelastic strains, the performance with respect to serviceability is primarily concerned.

Influence on the safety margins against collapse depends on the ductile behaviour of the structure or of the element and may become significant in cases where collapse is governed by non-plastic failure of concrete.

In slender or thin structures and whenever second order effects are of importance, the increase of deflections due to creep reduces the long-term safety margin with respect to buckling instability and may lead to creep buckling.

The influence of creep on second order effects is dealt with in a simplified way in clause 6.6.3.3.2. For complex problems and non-linear creep buckling reference should be made to specialized literature.

When choosing the level of refinement for the analysis of creep and shrinkage effects the following aspects should be considered

- reliability of the information on material properties (e.g. simple prediction on the basis of prediction laws of the type given in subsection 2.1.6, or prediction accompanied by test control at early ages, or test extrapolation; mean cross-section behaviour or local material properties etc.)

As evidenced in subsection 2.1.6, the available information on the time-dependent behaviour of concrete may suffer from important sources of error due to both lacunae in the model and to randomness in material parameters. Ambient conditions may be subjected to important variations whose influence on the real structure may be difficult to model in the analysis. The attention given to the analysis should therefore be related in particular to the accuracy in establishment of the material parameters, avoiding an excessive refinement in the analysis if the prediction of material parameters is poor.

- importance of the effects of creep and shrinkage on the behaviour of the structure
- importance of the limit state under consideration.

For a complete review of methods of structural analysis for creep and shrinkage effects refer to the CEB Design Manual 'Structural Effects of Time-dependent Behaviour of Concrete', CEB Bulletin No. 142/142 *bis*, Lausanne, 1984, Georgi Publ. Co., St. Saphorin (CH), 1984.

This section applies essentially to the verifications with respect to serviceability limit states.

Creep and shrinkage properties are described in clause 2.1.6.4 as mean cross-section properties; therefore, their use for the determination of internal stresses and strains in time within cross-sections introduces larger errors. A realistic analysis of these effects by appropriate discretization techniques, should in principle be based on the description of local rheological properties, taking into account their intrinsic non-linearities, coupling with moisture and temperature distributions and non-linear effects of cracking.

## 5.8.2. Structural models

The overall analysis of structures for creep effects in terms of sectional forces and displacements may be conducted under the assumption of linearity, and of the consequent validity of the principle of superposition, as indicated in clause 2.1.6.4 for service stress levels $|\sigma_c| < 0.4 f_{cm}(t_o)$ (linear ageing viscoelastic model).

Non-linearities due to higher stress levels may be taken into account on the basis of clause 2.1.6.4.3d.

## 5.8.3. Application of linear model

### 5.8.3.1. General

In the linear ageing viscoelastic model the time-dependent behaviour of concrete is fully characterized by the creep function $J(t, t_0)$ as indicated in clause 2.1.6.4 for both constant and variable stress histories (eqs (2.1–62, 63)).

Alternatively with the same assumptions and range of validity the stress response to a variable imposed strain history may be written as

$$\sigma_c(t) = [\varepsilon_c(t_0) - \varepsilon_{cn}(t_0)]R(t, t_0) + \int_{t_0}^{t} R(t, \tau)d[\varepsilon_c'(\tau) - \varepsilon_{cn}(\tau)] \quad (5.8\text{–}1)$$

where $R(t, t_0)$ is the relaxation function, representing the stress response to a constant unit imposed stress-dependent strain

$$[\varepsilon_c - \varepsilon_{cn}] = \varepsilon_{c\sigma} = 1$$

In the analysis of the global time-dependent behaviour of reinforced or prestressed concrete structures with rigid restraints, in terms of internal forces and displacements, the heterogeneities due to the presence of the

For practical application of linear creep analysis, a distinction between homogeneous structures with rigid restraints and heterogeneous structures is convenient.

steel, as well as to limited variations in the concrete properties between different parts, may be neglected in many cases and the structures be considered as homogeneous.

One should distinguish also between structures under constant restraint conditions and structures subjected to modifications of restraint conditions (variations of statical system) at some stage of construction and/or lifetime of the structure.

Application of the linear model to typical structural problems, in particular for homogeneous structures with rigid restraints, is facilitated, if both the creep function $J$ and the corresponding relaxation function $R$ are available (see clause 5.8.3.2).

The relaxation function $R$ is obtained from the specified creep function $J$ through the integral eq. (2.1-63) for $(\varepsilon_c - \varepsilon_{cn}) = \varepsilon_{c\sigma} = 1$.

Structural problems referring to heterogeneous structures are governed by one or more integral equations (see clause 5.8.3.3).

One of the alternative approaches of subsection 5.8.4 shall be used for the solution of integral equations in practical structural problems.

### 5.8.3.2. Homogeneous concrete structures with rigid restraints

Constancy of creep. Poisson ratio $\nu$ shall be introduced as an additional assumption.

The analysis may be performed on the basis of the theory of linear visco-elasticity.

The stresses $\sigma_{ij}(t)$ and displacements $u_i(t)$ are then obtained on the basis of the stresses $\sigma_{ij,el}(t)$ and displacements $u_{i,el}(t)$ for an elastic structure of constant modulus.

For a reference modulus $E = E_c$ (clause 2.1.4.2) the following criteria shall be applied for the specified imposed actions or restraint conditions.

An addendum to CEB Bulletin 142/142 *bis* (to be published as CEB Bulletin 215) contains the graphs of the adimensional creep and relaxation functions $J(t, t_0)E_c$ and $R(t, t_0)/E_c$ corresponding to the creep function $J$ specified in clause 2.1.6.4.3.

*(a) Imposed loads*

The elastic stresses are not modified by creep (i.e. $\sigma_{ij}(t) = \sigma_{ij,el}(t)$), while the displacements $u_i(t)$ may be obtained by integrating in time the increments of the elastic displacements $du_{i,el}(\tau)$, multiplied by the creep factor $J(t, \tau)E_c$ (first theorem of linear viscoelasticity).

*(b) Imposed deformations*

The elastic displacements are not modified by creep (i.e. $u_i(t) = u_{i,el}(t)$), while the stresses $\sigma_{ij}(t)$ may be obtained by integrating in time the increments of the elastic stresses $d\sigma_{ij,el}(\tau)$, multiplied by the relaxation factor $R(t, \tau)/E_c$ (second theorem of linear viscoelasticity).

(*c*) *Modification of the restraint conditions after the application of loads*
A modification of the restraint conditions, by the introduction of additional restraints at time $t = t_1$, following the application at $t = t_0$ of a system of constant sustained loads, produces a time-dependent variation of the initial elastic stresses and restraint reactions, which cannot be neglected generally.

The long-term distribution of stresses and reactions in a structure whose restraint conditions are modified shortly after loading (when the concrete is still young), may be considered to approach the elastic distribution corresponding to an application of loads to the final statical system of the structure.

Modification of restraint conditions after loading is frequent in practice; if the material behaviour is time-dependent, as for concrete structures, the long-term stress distribution may be widely affected, depending on age and creep deformability of the concrete.

### 5.8.3.3. Heterogeneous structures

The use of the linear ageing viscoelastic model for the concrete portion and of the elastic model for the steel elements leads to compatibility equations in terms of integral equations.

The procedures presented in subsection 5.8.4 can be applied. Attention should be paid, however, to the selection of the proper approach to obtain adequate accuracy for the different types of problems. Some specific procedures have been developed in the literature when the heterogeneity is due only to the presence of steel.

Heterogeneities may be due either to the properties of concrete (e.g. differences in casting ages, mix, size of structural elements, etc.), or to the presence of steel. They may characterize the cross-sections, and/or be diffused throughout the structure, and/or be concentrated in some associated structural element or external restraint.

When the analysis concerns stresses and strains within cross-sections, reference to the comments of subsection 5.8.2 is appropriate in selecting procedures and evaluating results.

(*a*) *Concrete structures made with different concrete portions*
Caution is needed in the application of the algebraic method of clause 5.8.4.3, as the different ages and rheological properties of concrete portions have to be considered; numerical solutions are therefore generally to be preferred.

If the analysis concerns heterogeneous sections, compatibility equations may be written under the usual assumption of sections remaining plane.

(*b*) *Heterogeneous steel–concrete sections*
Compatibility equations may be written under the assumption of sections remaining plane. A system of two integral equations is obtained by the displacement method if the steel cross-section has a non-negligible inertia, and a single equation if it may be reduced to a single fibre.

The use of the algebraic method of clause 5.8.4.3 normally leads to adequate accuracy.

The error introduced by adopting for the concrete a creep model representing the mean cross-section behaviour is diminished if the importance of the concrete portion in comparison with the steel is reduced (e.g. usual type of composite steel–concrete beam sections).

(*c*) *Concrete structures with external elastic restraints*
For structures with $n$ elastic restraints the solution by the force method leads to a system of $n$ integral equations.
The use of the algebraic method of clause 5.8.4.3 normally leads to adequate accuracy.

Forcing the elastic restraints (e.g. prestressing of steel cables in cable-stayed concrete bridges) up to the values of the reactions corresponding to rigid restraint conditions, eliminates the influence of creep deformability on the long-term stress regime, according to the first theorem of linear viscoelasticity (see clause 5.8.3.2a).

## 5.8.4. Practical approaches

The following practical approaches, with different degrees of refinement, may be adopted in structural calculations to deal with the integral equations due to the use of the linear ageing viscoelastic model

- numerical solutions
- conversion to differential forms
- conversion to algebraic expressions
- approximate determination of the relaxation function.

Because of the errors in creep prediction and in the linear model itself, the gain in accuracy achieved by the most refined linear methods, e.g. numerical solutions, is often fictitious compared to the use of the less refined approaches.

Numerical solutions are frequently used as reference procedures to evaluate the accuracy of approximate solutions.

### 5.8.4.1. Numerical solutions

Numerical step-by-step solutions may be obtained applying the trapezoidal rule to replace the superposition integral in the equations.
The relaxation function $R$ corresponding to the specified creep function $J$ is obtained according to clause 5.8.3.1 by applying the same procedure to the solution of eq. (2.1–63).

Proper subdivision of time in steps allows any desired accuracy; for usual time histories of the specified variable, it is convenient to use increasing steps.
For computer programs refer to CEB Bulletin 142/142 *bis*.

### 5.8.4.2. Conversion to differential forms

Rate-type stress–strain differential relations instead of integral-type relations may be obtained approximating, with any desired accuracy, the creep function by rate-type laws based on chains of rheological models with age-dependent parameters.

This procedure is recommended for complex problems involving a large number of time-dependent unknown variables, e.g. in finite-element visco-elastic analysis.

### 5.8.4.3. Approximate algebraic expressions

Adequate accuracy is obtained in most cases by converting the superposition integral in the equations into simplified approximate algebraic expressions and performing calculations in one time-step.

Eq. (5.8–2), with the expression (5.8–3) for the ageing coefficient $\chi$, corresponds exactly to eq. (2.1–53) for all problems resulting from linear combinations of a creep and a relaxation problem. With sufficient accuracy its use may be extended to cover a large number of other asymptotic stress and strain time histories representing a variety of typical problems in creep analysis of structures.

An addendum to CEB Bulletin 142/142 *bis* (to be published as CEB Bulletin 215) contains the graphs of the ageing coefficient $\chi(t, t_0)$ corresponding to the creep function $J$ specified in clause 2.1.6.4.3. Its values vary mainly between 0.5 and 1.0. In a large number of practical cases, particularly for average creep values and long-term creep effects, sufficiently accurate results may be obtained taking $\chi = t_0^{0.5}/(1 + t_0^{0.5})$; a constant value $\chi = 0.8$ can be adopted for typical loading ages between 10 and 30 days.

Adopting a constant value of $\chi = 1$, the following less accurate simple expression may be used (effective modulus method)

$$\varepsilon_c(t) = \sigma_c(t)J(t, t_0) + \varepsilon_{cn}(t) = \frac{\sigma_c(t)}{E_{c,ef}(t, t_0)} + \varepsilon_{cn}(t) \qquad (5.8\text{–}6)$$

Eq. (5.8–6) reduces to the calculation (with the effective modulus $E_{c,ef}$) of an equivalent elastic strain for the final value of the stress $\sigma_c(t)$ at the end of the time interval $(t - t_0)$ considered. Application of this method gives acceptable accuracy only for those cases where stresses are almost constant in time (varying less than 10% to 20%).

Eq. (5.8–7) may be used for any given creep function $J$; maximum error normally does not exceed 10% of the initial value of $R$.

Alternatively, the following expression obtained from eq. (5.8–3) offers similar accuracy, when the approximate values of $\chi$ indicated in clause 5.8.4.3 are introduced

$$R(t, t_0) = E_c(t_0)\left\{1 - \frac{\phi(t, t_0)}{[E_c/E_c(t_0)] + \chi\phi(t, t_0)}\right\} \qquad (5.8\text{–}8)$$

Negative values of $R$ obtained from eqs (5.8–7) and (5.8–8) in extreme cases (e.g. low values of $t_0$ and of the relative humidity RH) should be discarded taking for $R$ a positive value approaching zero.

The following expression (age-adjusted-effective-modulus method), may be used instead of eq. (2.1–63) for a creep function $J$ of the type of eq. (2.1–62)

$$\varepsilon_c(t) = \sigma_c(t_0)J(t, t_0) + [\sigma_c(t) - \sigma_c(t_0)]\left[\frac{1}{E_c(t_0)} + \chi(t, t_0)\frac{\phi(t, t_0)}{E_c}\right] + \varepsilon_{cn}(t)$$

$$= \frac{\sigma_c(t_0)}{E_{c,ef}(t, t_0)} + \frac{\sigma_c(t) - \sigma_c(t_0)}{E_{c,adj}(t, t_0)} + \varepsilon_{cn}(t) \qquad (5.8\text{–}2)$$

having introduced the ageing coefficient

$$\chi(t, t_0) = \frac{1}{1 - R(t, t_0)/E_c(t_0)} - \frac{1}{E_c(t_0)J(t, t_0) - 1}$$

$$= \frac{E_c(t_0)}{E_c(t_0) - R(t, t_0)} - \frac{E_c}{E_c(t_0)\phi(t, t_0)} \qquad (5.8\text{–}3)$$

the effective modulus

$$E_{c,ef}(t, t_0) = \frac{1}{J(t, t_0)} = \frac{E_c(t_0)}{1 + (E_c(t_0)/E_c)\phi(t, t_0)} \qquad (5.8\text{–}4)$$

and the age-adjusted effective modulus

$$E_{c,adj}(t, t_0) = \frac{E_c(t_0)}{1 + \chi(t, t_0)(E_c(t_0)/E_c)\phi(t, t_0)} \qquad (5.8\text{–}5)$$

### 5.8.4.4. Approximate determination of the relaxation function

Accurate values for $R$ may be obtained by the semi-empirical expression

$$R(t, t_0) = \frac{1 - 0.008}{J(t, t_0)} - \frac{0.115}{J(t, t - 1)}\left[\frac{J(t - \Delta, t_0)}{J(t, t_0 + \Delta)} - 1\right] \qquad (5.8\text{–}7)$$

with $\Delta = (t - t_0)/2$.

# 6. VERIFICATION OF THE ULTIMATE LIMIT STATES

## 6.1. GENERAL APPROACH

### 6.1.1. Introduction

This chapter gives methods of verifying that, for a structure as a whole and for its component parts, the probability of an ultimate limit state being reached by the exceedance of the resistance of critical regions is acceptably small.

The determination of the action effects should be carried out in accordance with subsection 5.3.2.

Prestressing may be taken into account by treating the prestress as a loading system. The prestress loading system may be taken to incorporate a part of the other external loading balanced by the effects of the prestress. The magnitudes of the forces in this system should usually be calculated for prestressing forces $P_{do}(x, t)$, i.e. design values of the prestress corresponding to zero stress in the surrounding concrete.

The parts of the resistance of the tendons not used in realizing these forces may be taken into account in the functions determining the resistance of the member to loads applied after the prestressing and bonding of the tendons.

The determination of resistance should be based on physical models of the internal forces and external reactions of the structure.

The models should represent continuous systems of internal forces in equilibrium under the design ultimate conditions and should at least approximately consider compatibility of deformations.

### 6.1.2. Members with only bonded reinforcement

In the case of linear members, where the output of the analysis is values of M, N, V, T the verification should usually be made according to section 6.3. However, for hollow and flanged sections where transverse bending of the section and/or the combined effects of torsion and longitudinal shear are considered reference should be made to section 6.5.

For slabs with the output of analysis in terms of $m_x$, $m_y$, $m_{xy}$ and transverse shear the verification should follow section 6.4.

For deep beams and other plates subjected to in-plane loading, if the output of the analysis is in terms of $n_x$, $n_y$ and $n_{xy}$, the verification should be in accordance with subsection 6.5.3. If the analysis is a lower bound

The ultimate limit state of static equilibrium is treated in chapter 1.

Other treatments of prestressing are not excluded.

This definition of $P_{do}(x, t)$ assumes recovery of elastic losses of prestress. An exception to its use is noted in clause 6.4.2.2.

In circumstances where the results of a verification are especially dependent on the magnitudes of prestressing forces a more refined treatment of them involving consideration of characteristic forces may be justified and reference should be made to clauses 1.4.3.2 and 1.4.3.3.

solution (limit analysis) as in subsection 5.6.3 the models of section 6.8 or analogous models can be adopted.

For plates or slabs subjected to both in-plane and out-of-plane loading the verification should follow subsection 6.5.4.

Verification of local conditions at statical or geometric discontinuities should be in accordance with section 6.8.

### 6.1.3. Members with unbonded reinforcement

Unless an appropriate analysis of the whole system is carried out, the unbonded reinforcement (generally prestressing tendons) may be considered as separate elements acting on the reinforced concrete member. The forces exerted on the member should be given their initial design values reduced by the effects of creep and shrinkage, unless their variations are assessed by means of an appropriate analysis, which should take into account the displacements of the anchorages and of the deviators, as well as relevant second order effects (if any).

### 6.1.4. Combination of stress fields

Stress fields may be superimposed upon one another but the superimposition should avoid the addition of tensions and compressions at small angles to one another.

In general it is preferable that a single model should be derived for each ULS verification but the use of superimposition may be convenient in some cases.

As a guide an angle of 15° can be treated as a minimum for the separation of tensions and compressions in superimposed models.

Where two uniaxial compression stress fields are superimposed the design should respect the upper limit on the maximum compression resulting from the two.

$$\sigma_{max} = \sigma_1 \cos^2 \theta + \sigma_2 \cos^2 (\alpha - \theta) \quad (6.1\text{–}1)$$

where

$\sigma_1$, $\sigma_2$ are greater and lesser uniaxial stresses
$\alpha$ is the angle between $\sigma_1$ and $\sigma_2$ ($\alpha < 90°$)
$\theta$ is the angle between $\sigma_{max}$ and $\sigma_1$
$\cot 2\theta = (\sigma_1 + \sigma_2 \cos 2\alpha)/(\sigma_2 \sin 2\alpha)$,

and compressive stresses are positive.

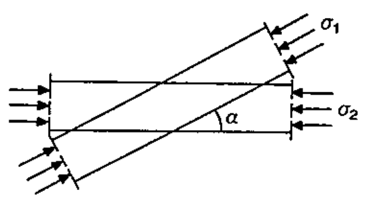



*Fig. 6.1.1. Superimposed compression fields*

### 6.1.5. Reinforcement and anchorage

The development of the forces required in the reinforcement and evaluated according to one of the available models of internal behaviour has to be ensured by bond and/or end anchorages. Consequently adequately anchored reinforcement shall be provided over the entire area in tension, across the trajectories of the compressive forces, at the locations where the directions of the aforementioned trajectories are deviated.

The axes of the ties in the model should coincide with the axes passing through the centroids of the reinforcement.

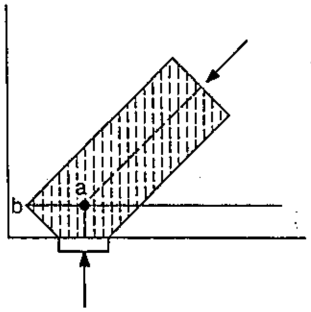



*Fig. 6.1.2. Reinforcement and anchorage*

The point 'a' is the node at which the centre-lines of compression and tension meet. The shaded areas denote zones of concrete in compression. Reinforcement is required out at least to the point 'b'.

Furthermore, the bar must be anchored in accordance with subsection 6.9.2.

Criteria for verification of resistance to fatigue are given in section 6.7.

## 6.2. MATERIAL RESISTANCES

### 6.2.1. General

The resistances given in the following subsections correspond to an increase of load from the service value to the design ultimate value over a period of hours or days. If the rate of loading is significantly greater (impact loading), these values may be replaced by ones appropriate to the rate in question.

Information on the effect of loading rates is given in section 2.1.

Note that in this chapter the sign-convention for stresses and strains differs from the one used in section 2.1.

### 6.2.2. Concrete in compression

#### 6.2.2.1. General

It should be verified that, under the relevant ULS conditions, the maximum compressive force acting on an area of concrete does not exceed a limit value, corresponding to the resultant of the resisting stresses, as given by the constitutive laws of section 2.1 and the appropriate safety factors.

However, appropriate simplifications of these constitutive laws (see clause 2.1.4.4) are allowed especially for cases where the zones checked are subjected to essentially uniaxial compression.

The expression 'essentially uniaxial compression' is used here to describe the stress state in the struts of models, e.g. in the compression chords of beams subjected to shear and bending, which are modelled as simple struts even though a more detailed analysis would indicate the presence of biaxial tension-compression.

## 6.2.2.2. Essentially uniaxial compression

Two alternative simplifications of the basic constitutive laws (see clause 2.1.4.4) which are appropriate are

— a parabola-rectangle stress–strain diagram,
— a uniform stress diagram.

*Parabola-rectangle diagram*
The design resistance of an uncracked zone under essentially uniaxial compression may be determined by means of a parabola-rectangle diagram as follows

$$\left.\begin{aligned} \sigma_{cd} &= 0.85 f_{cd}\left[2\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2\right] && \text{for} \quad \varepsilon_c < \varepsilon_{c1} \\ \sigma_{cd} &= 0.85 f_{cd} && \text{for} \quad \varepsilon_{c1} \leqslant \varepsilon_c \leqslant \varepsilon_{cu} \\ \sigma_{cd} &= 0.00 && \text{for} \quad \varepsilon_{cu} < \varepsilon_c \end{aligned}\right\} \qquad (6.2\text{–}1)$$

where $\varepsilon_{c1} = 0.002$.
For flexure

$$\left.\begin{aligned} \varepsilon_{cu} &= 0.0035 && \text{for} \quad f_{ck} \leqslant 50\,\text{MPa} \\ \varepsilon_{cu} &= 0.0035\left(\frac{50}{f_{ck}}\right) && \text{for} \quad 50\,\text{MPa} < f_{ck} \leqslant 80\,\text{MPa} \end{aligned}\right\} \qquad (6.2\text{–}2)$$

For axial compression

$$\varepsilon_{cu} = 0.002 \qquad (6.2\text{–}3)$$

The strains $\varepsilon_c$ are absolute values and are positive for compression.

The coefficient 0.85 and the use of a constant stress for strains from $\varepsilon_{c1}$ to $\varepsilon_{c2}$ allow for the influence of long-term loading.

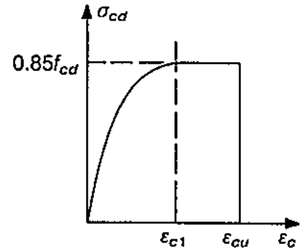



*Fig. 6.2.1. Parabola-rectangle stress–strain diagram*

*Uniform stress diagram*
The design resistance of a zone under essentially uniaxial compression may also be determined by means of a further simplified uniform stress diagram over the full area of this zone if appropriately selected.
The average stress may be taken as

$$f_{cd1} = 0.85\left[1 - \frac{f_{ck}}{250}\right] f_{cd} \qquad (6.2\text{–}4)$$

for uncracked zones, or as

$$f_{cd2} = 0.60\left[1 - \frac{f_{ck}}{250}\right] f_{cd} \qquad (6.2\text{–}5)$$

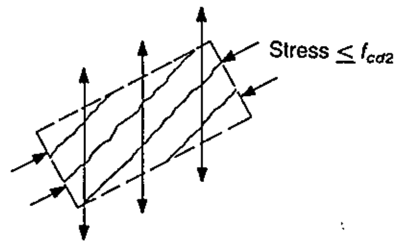



*Fig. 6.2.2. Example of reduced resistance* $f_{cd2}$

for cracked zones where the compressive resistance may be reduced by the effect of transverse tension from reinforcement and by the need to transmit force across the cracks.

These values are valid under the condition that the maximum extreme fibre strain is taken as

$$\varepsilon_{cu}^{*} = 0.004 - 0.002 \frac{f_{ck}}{100} \qquad (6.2\text{–}6)$$

$f_{ck}$ is in MPa.

### 6.2.2.3. Biaxial and triaxial stress states

For biaxial and triaxial stress states reference is made to sections 2.1 and 3.4.

### 6.2.2.4. Compression fields affected by bars or ducts

If a compression zone contains bars or ducts, such that the sum of the diameters of the bars or ducts at one level is greater than 1/6th of the width of the zone, or the sum of their areas is greater than 4% of the area of the zone, account shall be taken of their effect on the compressive stresses.

Compressive stresses inclined to bars or ducts shall be calculated on the basis of a reduced breadth

$$b_{red} = b - \eta \Sigma \phi \qquad (6.2\text{–}7)$$

where

$b$ is the breadth of the compression zone
$\Sigma\phi$ is the sum of the diameters of the bars or ducts determined at the most unfavourable level
$\eta$ is a coefficient depending on the nature of the bars or ducts.

No reduction of breadth is required for ordinary bonded bars at the boundary of the compressed zone, e.g. for ordinary main steel at the boundary of the web of a beam.

This section applies particularly to the webs of prestressed concrete beams, in which case $b = b_w$.

Indicative values for $\eta$ are

$\eta = 0.5$ for bonded bars or grouted tendon ducts,
$\eta = 1.2$ for unbonded tendons and ungrouted ducts.

The value $\eta = 1.2$ is intended for simple cases of individual tendons or ducts if no extra reinforcement is provided for the transverse tension around the embedment. If suitable transverse reinforcement is provided $\eta = 1.0$.

Due consideration shall be given to the transverse tension produced by the local deviation of the compression field, and to the provision of transverse reinforcement.

Compressive stresses parallel or nearly parallel to ducts should be calculated assuming the area in compression to be reduced by the area of the ducts within it.

### 6.2.2.5. Compression zones with confinement

Account may be taken of the effect of appropriately closed hoops or helical reinforcement giving a triaxial behaviour to concrete under longitudinal compression.

This may be done using the stress–strain diagrams for confined concrete given in clause 3.5.2.1.

### 6.2.3. Concrete in tension

The tensile resistance of concrete should not normally be relied upon in any major tie and therefore no general criteria are given for the use of concrete tensile strength. Concrete is, however, relied upon to provide tensile resistance in association with bond/anchorage, shear in members without shear reinforcement, etc. Where such reliance is envisaged, explicit criteria are given.

### 6.2.4. Steel in tension

The design strength of steel in tension is as follows.
For ordinary reinforcement

$$f_{ytd} = f_{ytk}/\gamma_s \qquad (6.2\text{–}8)$$

For bonded prestressed reinforcement, if the prestress is treated as an external load

$$f_{pyd,net} = 0.9 f_{ptk}/\gamma_s - \sigma_{do} \leqslant 600\,\text{MPa} \qquad (6.2\text{–}9a)$$

where $\sigma_{do}$ is the design tendon stress taken into account in the prestress loading system.

The attainment of the stress $f_{ytd}$ or $f_{pyd}$ requires the tensile strain to reach the design yield value. If this is not the case, the stress in the steel in the ULS may be derived from the strain and the stress–strain relationship for the reinforcement.

For the prestressed reinforcement, as an alternative to the use of eq. (6.2–9a) a factored stress–strain diagram derived from the actual characteristic relationship may be employed provided that the relevant ULS strains are calculated. In practice this is only generally possible for the sections of maximum moment in members free from torsion.
If prestress is not treated as an external load and the full tendon strength is to be used in a resistance function

$$f_{pyd} = 0.9 f_{ptk}/\gamma_s \leqslant \sigma_{do} + 600\,\text{MPa} \qquad (6.2\text{–}9b)$$

### 6.2.5. Steel in compression

Bonded longitudinal reinforcement should be assumed to undergo the same changes of strain as the surrounding concrete. If the detailing requirements regarding lateral restraint to the bars (see clause 9.2.3.2) are satisfied the design strength of the steel in compression is

$$f_{ycd} = f_{yck}/\gamma_s \qquad (6.2\text{–}10)$$

Only bars in the direction parallel to a compression strut should be considered effective.

The attainment of the stress $f_{ycd}$ requires the compressive strain to reach the design yield value. If this is not the case, the stress in the steel at the ULS may be derived from the strain and the stress–strain diagram for the reinforcement.

Where bonded post-tensioned tendons are located in a zone which is in compression under the action of the applied loads for the ultimate limit state, the reduction of the prestressing force should not be taken to exceed that corresponding to a change of strain equal to 0.0015.

This limitation is intended to account for possible bond slip.

## 6.3. LINEAR MEMBERS

### 6.3.1. Basic assumptions

(*a*) For simple rectangular members subjected to axial load, flexure and shear, the design models consist of longitudinal chords connected by web lattices comprising concrete struts under diagonal compression and distributed reinforcement in one or more directions.

In the following it is assumed that the minimum thickness of concrete in any part of a member is 80 mm. For members with thinner parts special consideration should be given to dimensional tolerances and possible consequences for design.

For the case of moving loads or multiple load cases, it is necessary to consider the envelopes of the longitudinal forces using different models if relevant. Regarding stirrup forces and diagonal concrete compressive forces in reinforced concrete beams, shear force envelopes can directly be used.

(*b*) For more complex sections and loading conditions, the members can be subdivided into several wall elements (webs, flanges) which can then be designed for their individual action effects. Longitudinal shear in flanges and torsion of beams are treated in this way in subsections 6.3.4 and 6.3.5.

For a more generalized treatment of complex loading conditions, including for example the simultaneous effects of torsion, longitudinal shear and transverse bending reference should be made to section 6.5.

### 6.3.2. Axial action effects

The resistant sectional forces should be derived from the internal resistant stresses and forces in the concrete and reinforcement on the basis of the following assumptions:

(*a*) the distribution of longitudinal strain is linear over the depth of the section;
(*b*) tensile stresses of concrete are neglected;
(*c*) bonded reinforcement is subjected to the same variations in strain as the adjacent concrete;
(*d*) the total deformations of bonded prestressing tendons are calculated taking account of the preliminary elongation corresponding to the design value of the prestressing force in the reference state after losses.

Section 6.3.2 is intended to be applied in cases where loading is only by axial action effects and at sections of maximum bending moment in other cases.

An exception to (c) the reduction of strain in prestressed tendons in a compressed zone should not be taken to exceed 0.0015 (see subsection 6.2.5).

On the basis of these assumptions, if the parabola-rectangle stress–strain diagram for concrete (6.2.2.2) is used, the sectional forces may be derived on the basis illustrated by Fig. 6.3.1 in which it is to be assumed that the strain diagram will pass through either point A or point B.

A diagram passing through A corresponds to simple or compound bending, while one passing through B corresponds to simple compression or compound bending with the section entirely in compression.

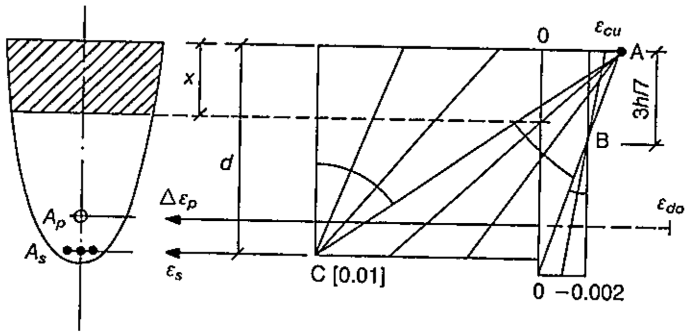



*Fig. 6.3.1. Strain diagram (parabola-rectangle diagram): $\varepsilon_{do}$ is the tendon strain corresponding to* $P_{do}(x, t)$

In some cases, e.g. where different steel types are used or when several steel bars are distributed over the height of the section, it is suggested that $\varepsilon_{s,max}$ or $\Delta\varepsilon_p$ is limited to 0.01. In this case the strain diagram in Fig. 6.3.1 will pass through point C instead of A or B.

In all cases

$$\varepsilon_{s,max} \leqslant \varepsilon_{uk} \quad \text{or} \quad \varepsilon_{do} + \Delta\varepsilon_p < \varepsilon_{uk}$$

For $\varepsilon_{uk}$ see clause 2.2.4.4 or 2.3.4.3.

If the uniform stress diagram is used for the concrete (clause 6.2.2.2), the maximum extreme fibre compressive strain is always limited to $\varepsilon^*_{cu}$ and $\varepsilon_{s,max}$ or $(\varepsilon_{pp} + \Delta\varepsilon_p)$ is limited as above.

For any combination of action effects the applied moment and axial force shall not exceed the corresponding resistances calculated on the basis of the assumptions given above and the criteria of section 6.2.

If prestress is treated as a loading system its effects should be included in the applied moment and axial force (see subsection 6.1.1).

An increase of compressive strength of concrete due to confinement may be taken into account according to clauses 6.2.2.5 and 3.5.2.1 provided that the area of the compression zone is taken as that of the confined concrete.

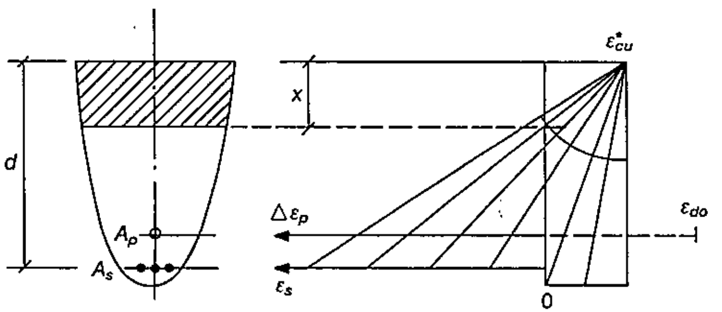



*Fig. 6.3.2. Strain diagram (uniform stress diagram): $\varepsilon_{do}$ is the tendon strain corresponding to* $P_{do}(x, t)$

Beams are linear members mainly subjected to bending and shear without significant compression due to external actions.

Columns are linear members subjected to significant compression due to external actions. They are commonly also subject to significant bending and shear.

The range of models which might represent the detailed behaviour of linear members is very large on account of the range of possible cross-sections (which may be symmetrical or not, solid or not and have various proportions), of possible loading conditions (which may include also lateral bending and shear) and of the proportions between the components of the action effects.

It is assumed that the minimum measures related to longitudinal and transverse reinforcement are satisfied. However, linear members of minor importance such as lintels and floor joists interconnected so as to allow for redistribution of forces between joists may be designed without web reinforcement and should then comply with the requirements of section 6.4.

## 6.3.3. Shear and axial action effects

For the verification with regard to the ultimate limit state of resistance of critical regions three main models are presented below, respectively for

- reinforced concrete beams (clause 6.3.3.2)
- prestressed concrete beams (clause 6.3.3.3)
- reinforced concrete columns (clause 6.3.3.4).

### 6.3.3.1 Conditions for application of models

The application of the models given below is subject to the following conditions.

(*a*) The ratio of the tensile reinforcement should be limited (avoidance of over-reinforced sections) so that

$$0.0035 \frac{d - x}{d} > f_{yd}/E_s \tag{6.3-1a}$$

or

$$0.0035\,\frac{d-x}{d} > f_{yk}/E_s\gamma_s \qquad (6.3\text{–}1b)$$

which leads to a $x/d$-value approximately equal to 0.6.

(*b*) The mechanical ratio of stirrup reinforcement should be not less than 0.2, i.e.

$$\omega_{sw} \;=\; A_{sw}f_{yk}/(b_w s f_{ctm}\sin\alpha) \geqslant 0.2 \qquad (6.3\text{–}2)$$

where $s$ is the spacing of the stirrups ($A_{sw}$) measured along the axis of the member. For $f_{ctm}$ see clause 2.1.3.3.

(*c*) The inclination of stirrups to the axis of the member should be at least 45° and that of bent-up bars at least 30°.

(*d*) The spacing of stirrup legs (in both the longitudinal and transverse directions) should not normally exceed the lesser of $0.75d$ and 800 mm.

(*e*) The shear reinforcement should be adequately anchored to the chords (see clause 9.2.2.2).

The fundamental 'unit-length' model of a typical part of the web of a beam resisting shear and axial action effects is shown in Fig. 6.3.3.

The angle $\theta$ between the web compression and the chords may be chosen freely in the range from 45° (arccot 1) to 18.4° (arccot 3).

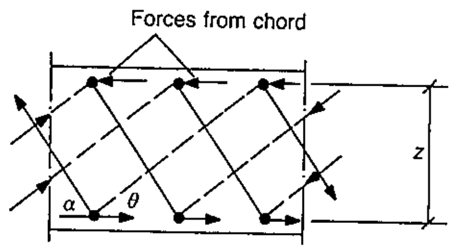



*Fig. 6.3.3. Web model*

The use of a high value for cot $\theta$ increases the stresses in the shear reinforcement at stages between shear cracking and the ULS and also increases demands on the extension (anchorage) of the main steel. The crack control requirements of section 7.4 may then govern the design of the shear reinforcement especially in large members and may not permit cot $\theta$ values as high as 3. The use of high values for cot $\theta$ is not advised where a member is subjected to axial tension.

Longitudinal tension reinforcement should normally be contained within the stirrup cage.

The absolute maximum shear resistance for a given section and concrete strength is obtained with $\theta = 45°$.

$$V_{Rd,\max} \;=\; \frac{f_{cd2}}{2}\,b_w z(1 + \cot\alpha) \qquad (6.3\text{–}3)$$

where $f_{cd2}$ is given by eq. (6.2–5).

It may be noted that where inclined shear reinforcement is used greater values of $V_{Rd,\max}$ would theoretically result from the adoption of $\theta > 45°$. The realization of the greater resistance has, however, not yet been verified experimentally.

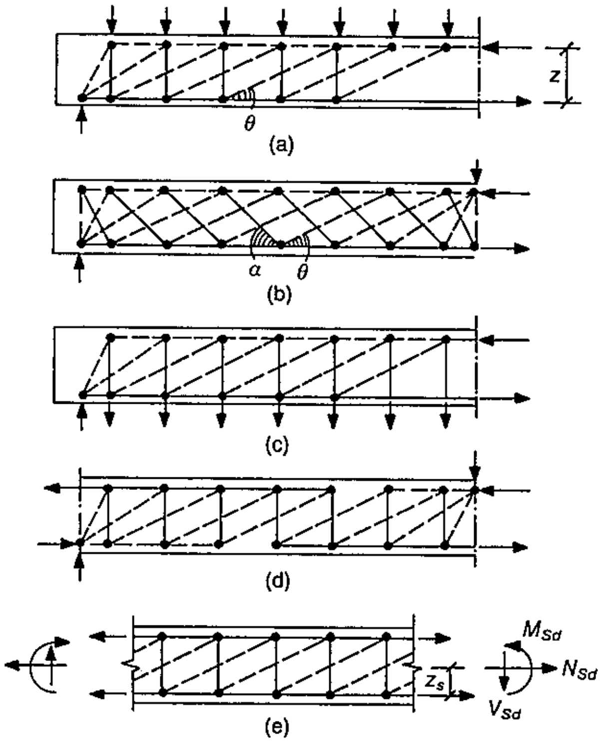



*Fig. 6.3.4. Models for reinforced concrete beams with parallel chords:* (a) continuous top load; (b) concentrated load; (c) continuous hanging load; (d) concentrated load, continuous beam; (e) region under bending, shear and axial tension*

### 6.3.3.2. Reinforced concrete beams

*Chords parallel*

Models for internal action in beams with parallel chords are given in Fig. 6.3.4. The values of the lever arm $z$ and the depth $x$ of the compression zone throughout a region in which bending moments retain the same sign may be taken equal to the values at the section of maximum $M_{Sd}$, evaluated according to chapter 5 on the basis of the assumptions introduced in subsection 6.3.2.

The forces derived from the models and the verifications required are as follows:

*(a) Tension chord*

Acting force

$$F_{St} = \frac{|M_{Sd}|}{z} + N_{Sd}\frac{(z - z_s)}{z} + \frac{V_{Sd}}{2}(\cot\theta - \cot\alpha) \qquad (6.3\text{–}4)$$

where $N_{Sd}$ is the axial load, taken positive for tension and negative for compression.

In the case of support reactions/loads applied so as to create transverse compression over the depth of the member

$$F_{St} \leqslant \frac{|M_{Sd,\max}|}{z} + N_{Sd}\frac{(z - z_s)}{z} \qquad (6.3\text{–}5)$$

where $z_s$ is the distance from the line of action $N_{Sd}$ to the centroid of the tension reinforcement.

In cases where all the tension reinforcement is within the breadth of the web

$$F_{Rt} = A_s f_{yd} \qquad (6.3\text{–}6)$$

In cases where some of the tension reinforcement is outside the breadth of the web the force to be resisted by the reinforcement is generally greater than the chord force (see subsection 6.3.4). It is however limited by eq. (6.3–5) in cases of direct support/loading.

*Concerning beams with inclined chords, see the last part of this section.

Generally no specific verifications are required for $F_{Sc}$ other than at the section of $M_{Sd,max}$ but see subsection 6.3.4 for the design of compression flanges.

The resistance $f_{ycd}$ is reliable only if the strain of the compression reinforcement, calculated according to subsection 6.3.2, is sufficiently high.

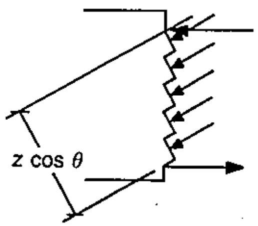



*Fig. 6.3.5. Compression of web concrete*

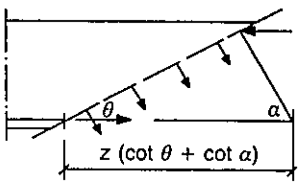



*Fig. 6.3.6. Tension of web steel*

In case of moving loads or multiple load cases, shear force envelopes may be directly used for dimensioning of stirrups provided that the angles $\alpha$ and $\theta$ are constant throughout.

For a region of a member in which the bending moment retains the same sign and the cross-section remains constant, the verification procedure may be as follows.

(*b*) *Compression chord*

$$F_{Sc} = \frac{|M_{Sd}|}{z} - N_{Sd}\frac{z_s}{z} - \frac{V_{Sd}}{2}(\cot\theta - \cot\alpha) \qquad (6.3\text{–}7)$$

except at the section of maximum moment where for direct loading

$$F_{Sc} = \frac{|M_{Sd,max}|}{z} - N_{Sd}\frac{z_s}{z} \qquad (6.3\text{–}8)$$

$$F_{Rc} = f_{cd1}A_c + f_{ycd}A_{sc} \qquad (6.3\text{–}9)$$

where $A_c$ denotes the cross-sectional area of the compression chord.

(*c*) *Compression of web concrete*

$$F_{Scw} = \frac{V_{Sd}}{\sin\theta}\left(\frac{\cot\theta}{\cot\theta + \cot\alpha}\right) \qquad (6.3\text{–}10)$$

$$F_{Rcw} = f_{cd2}b_w z\cos\theta \qquad (6.3\text{–}11)$$

(*d*) *Tension of web steel*

$$F_{Stw} = \frac{V_{Sd}}{\sin\alpha} \qquad (6.3\text{–}12)$$

$$F_{Rtw} = \left[\frac{A_{sw}f_{yd}}{s}\right] z\,(\cot\theta + \cot\alpha) \qquad (6.3\text{–}13)$$

- *Stage I.* The section of maximum moment is designed according to subsection 6.3.2, unless conditions of indirect loading or support require the model shown in Fig. 6.3.7 in which case the resistance of the tension chord must be increased appropriately using eq. (6.3–4).
- *Stage II.* The resistance of the web concrete is verified at the section of maximum shear force.
- *Stage III.* The resistance of the web reinforcement is verified using eqs. (6.3–12) and (6.3–13)
  — for the region further from the support than $z \cot\theta$ in cases of direct loading (Fig. 6.3.8(a)) where the reinforcement determined $z \cot\theta$ from the support is continued to the support (but see stage V below for the cases where major loads act near supports),
  — for the entire region where load is not applied at the top of the beam (Fig. 6.3.7).

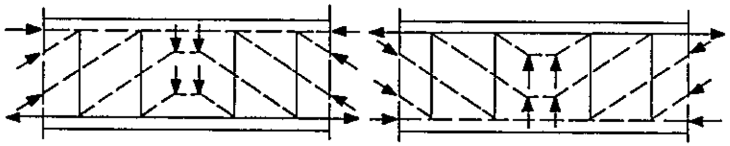


*Fig. 6.3.7. Conditions at loads and supports distributed over the depth*

- *Stage IV.* The admissibility of curtailing main reinforcement is determined by eqs (6.3–4) and (6.3–6).

Higher values of $\cot\theta$ lead to lower amounts of stirrups but increase the forces in the main steel in regions of low moments.

For the vertical stirrups alone, the minimum amount is obtained for

$$\cot\theta = \sqrt{\left[\frac{b_w s f_{cd2}}{A_{sw} f_{ywd}} - 1\right]} \leqslant 3 \qquad (6.3\text{–}14)$$

for which

$$\frac{V_{Sd}}{z b_w f_{cd2}} = \sqrt{\left[\frac{A_{sw} f_{ywd}}{b_w s f_{cd2}}\right]} \sqrt{\left[1 - \frac{A_{sw} f_{ywd}}{b_w s f_{cd2}}\right]} \leqslant 3 \frac{A_{sw} f_{ywd}}{b_w z f_{cd2}} \qquad (6.3\text{–}15)$$

- *Stage V*. In the case of a shear span in which a large part of the transverse loading is applied within a distance $z \cot\theta$ ($\leqslant 3z$) of a support, stage III above permits the shear reinforcement to be designed for only a small force. This has a number of consequences.
  - — The inclined compressive force at the support may be considerably increased and it should be verified that the compressive stresses at the nodes are not excessive — see subsection 6.9.2.
  - — The force in the main steel requiring anchorage at a simple support is increased and the adequacy of the anchorage should be verified — see subsection 6.9.3.
  - — If shear cracking would occur in the serviceability limit state the amount of shear reinforcement controlling the opening of the diagonal crack may be very small and the criteria of serviceability may be violated. In the absence of a more precise calculation the shear force causing shear cracking may be estimated as

$$V_{cr} = 0.15(3d/a_v)^{1/3}\xi(100\rho f_{ck})^{1/3} b_{red} d$$

where

$a_v$ is the distance from major load to support
$\xi = 1 + \sqrt{(200/d)}$ with $d$ in mm
$\rho$ is the ratio of flexural tensile reinforcement $(A_s/b_w d)$ anchored at the support
$b_{red}$ is the reduced web breadth
$(3d/a_v)^{1/3}$ is an empirical expression allowing for the influence of the transverse compression from the loads and support reaction.

If the above present difficulties for the design, conditions can be improved by increasing the angle $\theta$ of the compression struts (Fig. 6.3.8(b)) or by sharing the load between a single direct thrust and a lattice system (Fig. 6.3.8(c)). In the latter case conditions in the strut at the support can be verified for the resultant of the two inclined thrusts.

Further guidance on the design of regions where major loads are applied close to supports is given in clause 6.8.2.2.

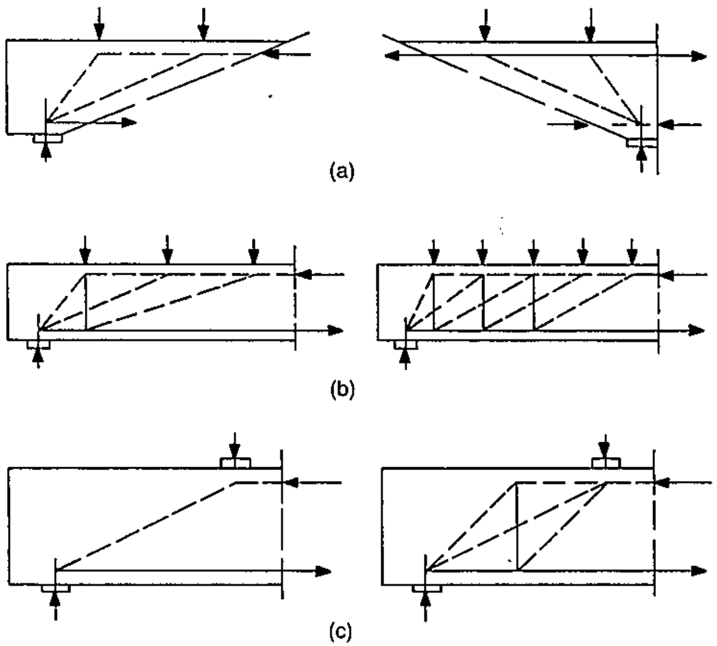



*Fig. 6.3.8. Direct loading near supports: (a) support of loads close to supports by direct thrusts; (b) improvement of conditions at a support by increasing the angle $\theta$; (c) improvement by sharing load between a direct thrust and a lattice system*

An indicative model for a beam with both chords' inclined is given in Fig. 6.3.9. The forces in the chords and web should be determined.

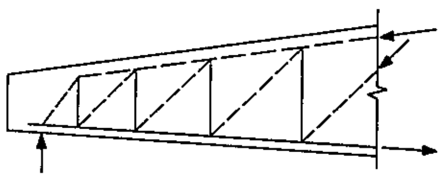


*Fig. 6.3.9. Model for a beam with inclined chords*

## 6.3.3.3. Prestressed concrete beams

### 6.3.3.3.1. General

Prestress is here treated as an external loading system in accordance with subsection 6.1.1.

Three modelling approaches are presented in the following three clauses and clause 6.3.3.3.5 treats special conditions, e.g. those at the ends of pretensioned members.

In all cases the verification is made in individual regions as defined in clause 6.3.3.2 or in zones as defined below. The verification criteria directly applicable are those relating to the 'first yield' defined in clauses 1.6.2.2 and 5.4.1.2.

In all three approaches, two models are superimposed.

The first model represents actions maintaining equilibrium with the prestress (end forces and forces due to tendon curvature) and a part of the other loading. This model shall be such that no reinforcement is necessary for the equilibrium. The model may include a compression arch with a force having a longitudinal component in equilibrium with the longitudinal component of the prestress (Fig. 6.3.10(a)). In this case the other loading considered is such as to maintain equilibrium with the transverse forces due to the curvature of the tendons and of the arch. This version of the first model, including the arching action can reduce the total amount of shear reinforcement required, especially in members with relatively thick webs.

An alternative version of the first model (Fig. 6.3.10(b)) does not include an arch and uses longitudinal forces in the chords to maintain equilibrium with the longitudinal effects of the prestress.

The second model is the truss analogy of an ordinary reinforced concrete beam (Fig. 6.3.10(c)). The forces on it complement those of the first model and are equivalent to the remainder of the actions for which reinforcement is required.

*The first approach* (*clause 6.3.3.3.2*) using a model of the type shown in Fig. 6.3.10(a) together with that of Fig. 6.3.10(c) is of general applicability.

*The second approach* (*clause 6.3.3.3.3*) is a special case of the first, in which the part of the loading applied to the first model is a constant (along the length) proportion of the total load. This approach simplifies the calculations required but is of restricted applicability. In the main it is applicable where the tendons are continuous and their profiles are such that

The models presented below are intended to be used for relatively simple cases of tendon layout, load conditions and section shapes. In other cases (box girder sections, high number of tendons, etc.), the decomposition of the section in plate elements (see section 6.5) may be more convenient in particular to operate on a finite element analysis output.

The choice of approach should depend on various considerations, e.g. the shape of the cross-section, the spatial arrangement of the external loads and of the prestressing tendons and the degree of refinement required.

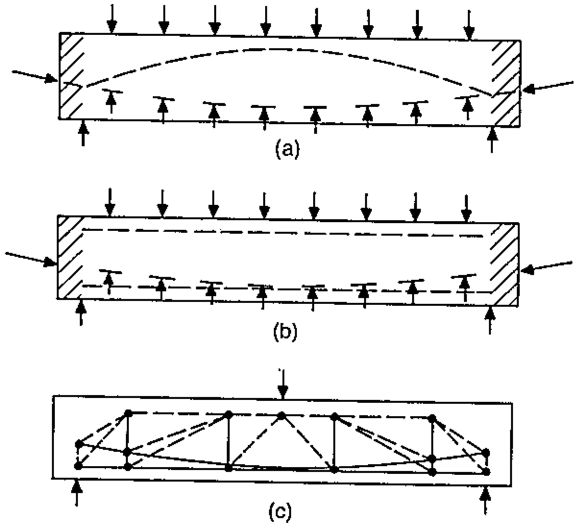



*Fig. 6.3.10. Models 1 and 2 for prestressed beams: (a) model 1 with arch action; (b) model 1 without arch action; (c) model 2*

The first model may in principle be a combination of the types illustrated in Figs 6.3.10(a) and (b).

the forces due to their curvature have a distribution similar to that of the loads, e.g. a simply supported beam with a parabolic tendon profile carrying uniformly distributed loads.

*The third approach* (*clause 6.3.3.3.4*) is the simplest and is of general applicability. The models used are those of Figs. 6.3.10(b) and (c). This approach is realistic for thin-webbed members and somewhat conservative in other cases.

**6.3.3.3.2. General differentiated approach**

In the case of internal prestress by post-tension, the beam is divided into zones (one zone only in the simplest case) generally defined by the points of contraflexure.

Model 1 consists of a compression arch (curved strut) totally internal to the beam. The centre-line of this arch should pass

- through the centre of the compression chord at the section of maximum moment in the zone considered, and
- through points at the extremities of the zone usually at the level of the tendons.

Within a given zone the prestressing force may be considered as constant and equal to the value $P_{do}(x, t)$ at the section of maximum moment defined in subsection 6.2.4.

The horizontal component of the force in the arch is taken as equal to the horizontal component of the prestress.

The profile of the arch between the points defined above may be chosen rather freely to suit the loading.

The equilibrium of the arch must be ensured by the forces at its ends and forces distributed along it (due to the effects of tendon curvature and transverse loading) in such a way that the arch profile is a funicular of these forces.

The simplest case is that of a simple span having tendons without intermediate anchorages.

If several tendons are spread over the height of the section, the resultant force is considered to determine the points of contraflexure.

In cases where intermediate anchorages would exist within a zone, the model should be more differentiated (see clause 6.3.3.3.5).

In the case of pretension the tendons are generally straight and the limits between zones, if relevant, correspond approximately to zero moment points under the loading for the load case considered.

The position of the centre of the compression chord of this section is defined by the design for axial action effects. In the normal case where the moment due to the other actions (loads) exceeds the moment due to prestress, the compression chord is that opposite the chord containing the tendons.

Other points may be adopted at the extremities provided an external moment transferred from model 2 completes the equilibrium of the end sections.

In the absence of concentrated loads at determined abscissae a parabolic or circular profile is recommended. If concentrated loads are applied at determined abscissae, the profile may advantageously include angles at these abscissae and be closer to straight lines between them.

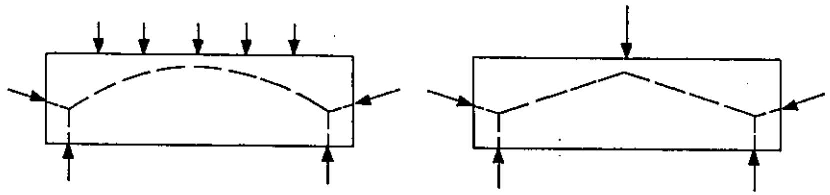


*Fig. 6.3.11. Arch profiles adapted to different patterns of loading*

Model 2 is the truss model of an ordinary reinforced concrete beam in which the remainder of the strength of the tendons not used in $P_{do}(x, t)$ is included, i.e. the design strength corresponding to $f_{pyd,net}$ as defined in subsection 6.2.4.

Model 2 is considered as subject to the following (design values)

- the part of the permanent actions (other than prestress) and the variable actions not applied to model 1
- end shear forces, statically determined, equilibrating the forces above
- hyperstatic end moments and shears of prestress
- end moments due to actions other than prestress, calculated in the overall analysis and equilibrating shears.

In the case where the extremities of the centre-line of the arch do not coincide with the level of the tendons a corrective moment shall be applied there.

The stress fields of the two models are superimposed for the final verifications.

The verifications required are of

- the force in the compression chord (sum of forces from models 1 and 2)
- the force in the tension chord (sum of forces from models 1 and 2, but note that the tension chord force is zero in Fig. 6.3.10(a))
- the web tension from model 2 (model 1 web tension is equal to zero)
- the web compression (sum of effects from models 1 and 2).

The simplified superposition approach of clause 6.3.3.3.3 may be adopted for the web compression.

**6.3.3.3.3. Simplified differentiated approach**

This approach is as above in clause 6.3.3.3.2 but the profiles of the arch and tendons are such that their combined curvature effects equilibrate a constant (along the length) fraction $\lambda$ of the load.

The constant fraction refers to the external loading.

In this case the forces to which model 2 is subjected are simply the fraction $(1 - \lambda)$ of the permanent (prestress excluded) and variable loads together with eventual hyperstatic effects.

The forces derived from the models and the corresponding resistances are as follows

For sections other than that of maximum moment, reference should be made to the truss model.

(*a*) *Tension chord at the section of maximum moment*

$$F_{St} = (1 - \lambda)\frac{M_{Sd}}{z} - F_{pb} \qquad (6.3\text{–}16)$$

For $f_{pyd,net}$ see subsection 6.2.4.

$$F_{Rt} = A_s f_{ytd} + A_p f_{pyd,net} \qquad (6.3\text{–}17)$$

where $F_{pb}$ is the compressive force applied to the tension chord in the model of Fig. 6.3.10(b).

*(b) Compression chord at the section of maximum moment*

$$F_{Sc} = (1 - \lambda)\frac{M_{Sd}}{z} + F_{c\lambda} \qquad (6.3\text{–}18)$$

$$F_{Rc} = f_{cd1} A_c + f_{ycd} A_{sc} \qquad (6.3\text{–}19)$$

where $F_{c\lambda}$ is the compressive force in model 1.

If prestressing tendons are present within the compression chord their influence should be treated in accordance with subsection 6.2.5.

*(c) Compression of web concrete*
Acting stress

$$\sigma_{Scw} = \sigma_l + \sigma_2 \qquad (6.3\text{–}20)$$

$$\sigma_l \cong \frac{F_{c\lambda}}{b_w z}\sqrt{\left[\frac{c_v^2 + z^2}{c_v^2}\right]} \qquad (6.3\text{–}21)$$

where $c_v$ is the horizontal length between the section of maximum moment and the relevant end of the zone, and $b_w$ complies with clause 6.2.2.4

$$\sigma_2 = \frac{(1 - \lambda)V_{Sd}}{b_w z \sin^2\theta(\cot\theta + \cot\alpha)} \qquad (6.3\text{–}22)$$

Resistant stress

$$\sigma_{Rd} = f_{cd2} \qquad (6.3\text{–}23)$$

The direct addition of $\sigma_l$ and $\sigma_2$ is in principle conservative and is adopted here partly for simplicity and partly because of the approximate nature of the expression for $\sigma_l$.

In practice with diffusion of compressive forces, this is a gross simplification to give the average compressive stress. $z$ may be taken as 0.9 h.

For vertical stirrups

$$\sigma_2 = \frac{(1 - \lambda)V_{Sd}}{b_w z \sin\theta\cos\theta}$$

*(d) Tension of web steel*

$$F_{Stw} = (1 - \lambda)\frac{V_{Sd}}{\sin\alpha} \qquad (6.3\text{–}24)$$

$$R_{Rtw} = \left[\frac{A_{sw} f_{yd}}{s}\right] z(\cot\theta + \cot\alpha) \qquad (6.3\text{–}25)$$

To optimize the shear design, in the sense of reducing the amount of stirrups required, having calculated $\sigma_l$, the available $\sigma_2$ can be evaluated by means of eqs (6.3–20) and (6.3–23). Then the corresponding $\theta$ can be obtained from eq. (6.3–22) and can be used in eq. (6.3–25) to determine the stirrups required for $F_{Rtw} \geqslant F_{Stw}$.

**6.3.3.3.4. Simple approach**
This approach is of general applicability. Model 1 is that of Fig. 6.3.10(b) which contains no arch. Thus in effect the design values of all the forces due to prestress and all other actions are applied to the truss model of Fig. 6.3.10(c).

The forces due to prestress are

- the end moments and shears due to hyperstatic prestress
- the end forces (M, N, V) due to isostatic prestress
- the curvature effects of the tendons throughout the zone under consideration.

### 6.3.3.3.5. Special conditions

*Tension chord near beam ends/end zones of pretensioned members*

The distribution of force along the tension chord can be calculated from the models adopted.

In the case of a pretensioned member, in which the transmission length extends into the span, the safety of the end zone with respect to the forces corresponding to model 1 of clauses 6.3.3.3.2 and 6.3.3.3.3 can be assured only if the concrete there remains uncracked under the design action effects, i.e. if the principal tensions at both the centroid and the extreme fibre (calculated for an uncracked section) remain below $f_{ctk,min}/1.5$. If this condition is complied with, the end zone's deviation from simple arching can be accepted.

If this condition is not complied with the approach of clause 6.3.3.3.4 should be adopted.

*Intermediate anchorages*

In the case of several tendons having different profiles/locations of anchorages, separate funicular systems may be considered, generating a number of models type 1, the stress fields of which are to be superimposed upon one another.

If intermediate anchorages are made at or near the compressed face all the effects from the tendons in question should be applied to model 2.

*Inversion of tendon curvature*

In the case of inversion of tendon curvature, the direct effects of downward curvature are to produce additional transverse loading and are thus unfavourable. Model 1 can none-the-less be such that it still resists a constant fraction of the loading and the approach of clause 6.3.3.3.3 remains applicable.

Within the transmission zone, the bond stresses of the tendons may increase beyond their values at transfer, by virtue of the effect of transverse pressure from a reaction, but in many cases supplementary passive reinforcement is required to provide for the change of longitudinal force within the transmission zone.

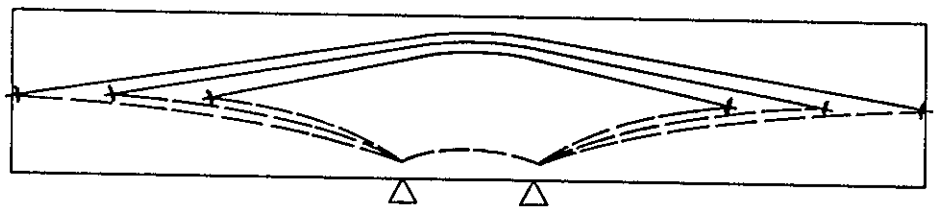


*Fig. 6.3.12. Intermediate anchorages away from the compressed face*

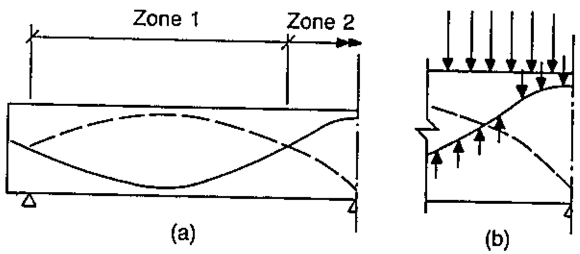



*Fig. 6.3.13. Inversion of tendon curvature (possibility of increasing arch slope shown in (b))*

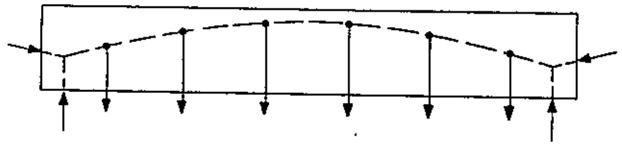


*Fig. 6.3.14. Additional stirrups for suspended loads*

*Suspended loads*

In the descriptions (clauses 6.3.3.3.1 to 6.3.3.3.3) of model 1 it is assumed that the loading is applied in such a manner as to create transverse compression within the member. If this is not the case additional transverse reinforcement is required to 'suspend' the loads.

## 6.3.3.4. Columns subjected to axial load, bending and shear

### 6.3.3.4.1. General

For beam-column joints, see clause 6.8.2.2.

The range of models which might represent the detailed behaviour of columns is very large on account of the range of possible loading conditions and the range of proportions between the longitudinal forces resisted by concrete in compression, steel in compression and steel in tension.

Appropriately precise and consistent models should be used but simplifications are possible and two simple approaches (for different conditions) are given in clauses 6.3.3.4.2 and 6.3.3.4.3.

In all cases the following conditions should be respected.

It should be noted that the design stresses for the reinforcement reach design yield values only if the strains according to subsection 6.3.2 attain their yield values.

- The sections at the ends of the column length considered should be verified vs. axial action effects using the methods of subsection 6.3.2.
- In the case of slender columns (see section 6.6), where sections affected by second order moments are critical these sections should also be verified using the methods of subsection 6.3.2.
- The column should be provided with at least minimum links according to subsection 9.2.3.
- Minimum longitudinal reinforcement should be provided in accordance with subsection 9.2.3.

The two methods given below are intended for use in different situations:

- clause 6.3.3.4.2 where the compression loading is dominant and longitudinal tension reinforcement is not required
- clause 6.3.3.4.3 where the compression is less dominant.

### 6.3.3.4.2. Design of columns in which compression is dominant

Where the compression loading is so dominant that no longitudinal tension reinforcement is required in any part of the length in question, then it should be checked whether or not inclined cracking is to be expected under the design action effects.

For this purpose the longitudinal normal stress and transverse shear stress in the concrete may be calculated approximately as

$$\sigma_n = N_{Sd}/A_c \qquad (6.3\text{–}26)$$

$N_{Sd}$ here is taken positive for compression.

where $A_c$ is the cross-sectional area.

$$\tau = \frac{3V_{Sd}}{2b_w h} \text{ for rectangular cross-sections} \qquad (6.3\text{–}27)$$

where

$b_w$ is the web breadth
$h$ is the overall dimension in the direction of $V_{Sd}$.

Approximate values of the principal stresses may be calculated as

tension: $\sigma_{c1} = \sqrt{(\tau^2 + \sigma_n^2/4)} - \sigma_n/2$

compression: $\sigma_{c2} = \sqrt{(\tau^2 + \sigma_n^2/4)} + \sigma_n/2 \leqslant f_{cd1}$.

Inclined cracking may be assumed to be avoided if

(*a*) for $\sigma_{c2} < f_{cd}/3$:

$$\sigma_{c1} \leqslant f_{ctk,\min}/1.5$$

(*b*) for $\sigma_{c2} \geqslant f_{cd}/3$:

$$\sigma_{c1} \leqslant f_{ctk,\min}\left(1 - \frac{\sigma_{c2}}{f_{cd}}\right)$$

If $\sigma_{c1}$ is larger than the limiting value of (a) or (b), then clause 6.3.3.4.3 should apply.

In this case only the nominal transverse reinforcement defined in subsection 9.2.3 is required.

**6.3.3.4.3. Design of columns in which compression is less dominant**
If longitudinal tension reinforcement is required in any part of the length of the column, the differentiated modelling approach of clause 6.3.3.3.2 is followed, appropriately modified to meet the boundary conditions of the column.

In the case of a building column as in Fig. 6.3.15, the following models may be superimposed

- model 1, in which the only forces acting on the column are end compressions $N_c$, numerically equal to the lesser of the two compressive forces in the concrete obtained from the designs of the end sections for

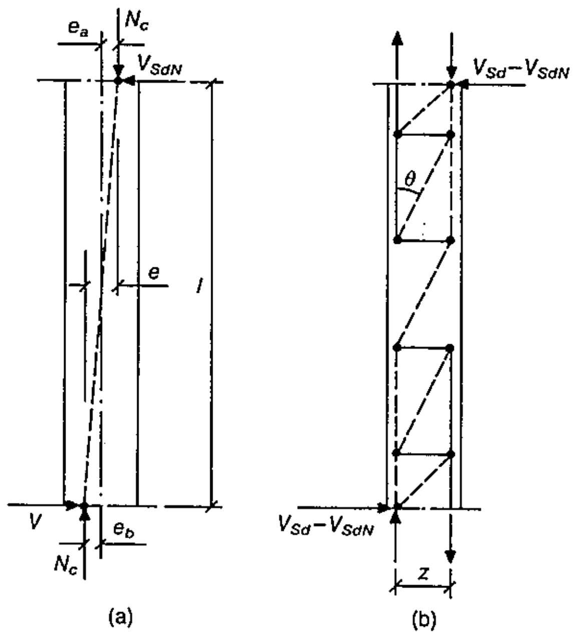



*Fig. 6.3.15. Models for columns with less dominant compression: (a) model 1; (b) model 2*

axial action effects, and shears $V_{SdN}$ needed to deviate the direction of $N_c$ in order to form a diagonal strut across the whole column

- model 2, in which the remaining axial action effects act on the ends, together with the shear forces $V_{Sd} - V_{SdN}$. A truss model represents the internal actions.

The stress fields of the two models are superimposed for the final verifications of the web (and of the curtailment of longitudinal steel if relevant).

The compression of the web concrete is verified from the following

Acting stress

$$\sigma_{Scw} = \sigma_1 + \sigma_2$$

$$\sigma_1 = \frac{N_c}{b_w z} \sqrt{\left(\frac{l^2 + e^2}{l^2}\right)} \qquad (6.3\text{–}28)$$

where $e$ is the sum of the eccentricities of $N_c$ at the two ends

$$\sigma_2 = \frac{V_{Sd} - V_{SdN}}{b_w z \sin\theta \cos\theta} \qquad (6.3\text{–}29)$$

Resistant stress

$$\sigma_{Rd} = f_{cd2}$$

The tension of the stirrups (assumed to be perpendicular to the axis of the column):

$$F_{Stw} = V_{Sd} - V_{SdN}$$

$$F_{Rtw} = \frac{A_{sw} f_{yd}}{s} z \cot\theta$$

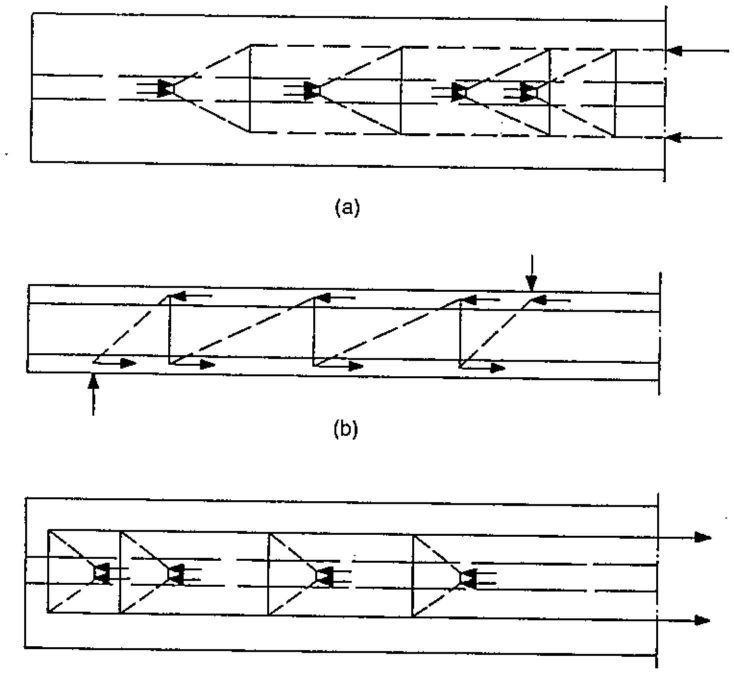



*Fig. 6.3.16. Truss models for an I-beam: (a) compression flange; (b) web; (c) tension flange*

As a simplification the following values of $\tan \theta_f$ may be used

- $\tan \theta_f = 0.5$ for compression flanges
- $\tan \theta_f = 0.8$ for tension flanges.

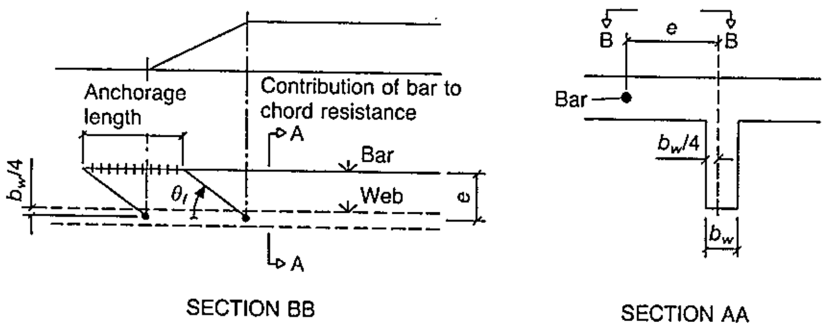



*Fig. 6.3.17. Anchorage of bars in the tension flange*

## 6.3.4. Longitudinal shear in flanged sections

The reinforcement needed to transmit the longitudinal shear in flanged sections may be determined by means of truss models such as those in Fig. 6.3.16.

Along the axis of the beam, the distribution of the longitudinal shear between the web and the flange should be determined from the truss model used for the web.

The longitudinal shear $V$ per unit length of the axis of the beam is determined by the change in the normal (longitudinal) forces in the actual part of the flange:

$$V = \Delta F / \Delta x \qquad (6.3\text{–}30)$$

where

$\Delta x$ is the length under consideration
$\Delta F$ is the change in the normal force in the actual part of the flange in the length $\Delta x$.

The transverse reinforcement per unit length $A_{sf}$ may be determined by the equation

$$A_{sf} f_{yd} \geqslant V \tan \theta_f \qquad (6.3\text{–}31)$$

At any given section of a beam, the curtailment of a bar placed in the tension flange should be appropriately determined. To this purpose, the anchorage length of the bar should be sufficiently ahead of the section where its resistance is required. If $e$ is the distance between the axis of the bar and a line situated at the quarter point of the web breadth, the anchorage length of the bar should end at least $e \cot \theta_f$ ahead of the given section (Fig. 6.3.17).

## 6.3.5. Torsion

### 6.3.5.1. Scope and basic assumptions

The following sections apply to linear members subjected to torsion combined with axial action effects and shear.

They apply to solid cross-sections and to hollow or open sections in which the effects of longitudinal shear and transverse bending are negligible. These effects may be negligible due to the thickness of the walls of the section or to the loading conditions.

For methods of verification for members such as hollow box girders where the effects of longitudinal shear are not negligible reference should be made to section 6.5.

In the case of T-beams, if the torsion is assumed to be resisted only by the web, the web may be designed according to this section and the flange according to subsection 6.3.4.

(*a*) A distinction may be drawn between

(i) equilibrium torsion, in which the torque is necessary for equilibrium

(ii) compatibility torsion, in which the torque is due solely to the restraint of rotation induced by adjoining members.

In the case of compatibility torsion, the torsional moments may be neglected in the calculation for the ULS provided the member is reinforced with closed stirrups perpendicular to the axis, and

(i) the stirrups have their legs close to the boundaries of the section

(ii) the stirrups provide a value of $\rho_{sw} = A_{sw} f_{yk} / (b_w s f_{ctm}) \geqslant 0.2$ where $A_{sw}$ is the area of two legs of a stirrup

(iii) the spacing *s* of the stirrup legs does not exceed the lesser of 0.75*b* and 0.75*d* in the longitudinal direction or 0.75*d* in the transverse direction.

If compatibility torsion is neglected, the neglect should be consistent in the analysis and member design. Thus torsional stiffness should be taken equal to zero in the analysis.

If the analysis is not based on the neglect of torsion a realistic stiffness should be adopted (see section 3.8) and the ULS verification should take account of the torque obtained from the analysis.

(*b*) A distinction may also be drawn between

(i) circulatory torsion, in which equilibrium is maintained by a closed flow of tangential shear

(ii) warping torsion, due to a restraint of longitudinal deformations.

Resistance to torsion is provided by shear forces, which may or may not require longitudinal bending actions. Torsional resistance without longitudinal bending corresponds, in terms of elastic theory, to St. Venant torsion. It is here described by the more general term 'Circulatory torsion' (Fig. 6.3.18).

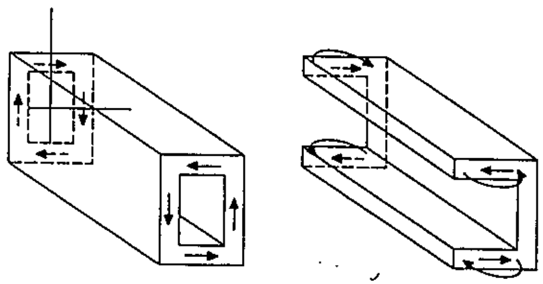


*Fig. 6.3.18. Circulatory- and warping torsion*

(*c*) Away from local disturbances at supports or sections subjected to concentrated loads etc., the action effects due to torsion at any section may be treated as longitudinal moments and shear forces acting on actual or equivalent walls representing the member. Each wall may then be designed for the summed effects of torsion and other load effects.

(*d*) In the verification methods given in clauses 6.3.5.2 and 6.3.5.3 it is assumed that the member is treated as containing inclined cracks and that its walls are designed on the basis of truss models.

There is no absolute division between circulatory and warping actions. The main part of a hollow box girder resists torsion by a circulatory action, but warping effects are produced near diaphragms. An open C-shape may resist torsion primarily by warping but its component rectangles do develop some circulatory torsion. For many members it is sufficient to design for the primary mode of resistance assuming it to carry the total torsion. For major members it is, however, possible to assess a division between circulatory and warping effects.

## 6.3.5.2. Circulatory torsion

*(a) Hollow polygonal convex cross-sections*

The effective thickness $t_{ef}$ taken into account in the calculations shall not

- exceed the actual wall thickness
- be less than twice the distance between the external face of the wall and the line joining the axes of the longitudinal reinforcement.

Subject to the above limitation, the minimum effective thickness required may be determined from consideration of the combined effects of M, N, V and T.

As a first approximation, the effective thickness may be assumed as

- the actual wall thickness if this thickness is less than $(A/u)$, or

  $t_{ef} = A/u$ in the opposite case

where

$u$ is the external perimeter of the cross-section
$A$ is the area limited by this perimeter.

In the flexural tensile zone advantage may be taken of the increased lever arm (for flexure) if the longitudinal reinforcement is placed outside the centre of the wall thickness adopted for tension.

In a solid rectangular cross-section the effects of M, V and N may be considered to act over the full breadth of the section, but may be redistributed to optimize the design. For example either of the two solutions shown in Fig. 6.3.19 for the summation of the effects of V and T is acceptable.

In a similar manner the effects of N may be distributed over the section in proportion to the areas of its parts, may be distributed to all of the walls or may be distributed to only some of the walls.

*(b) Solid sections*

A polygonal convex solid section may be verified as an equivalent hollow section in which the thickness of the 'walls' shall not be less than twice the distance between the external face and a line joining the axes of the longitudinal reinforcement. As a first approximation the effective thickness may be assumed as $t_{ef} = A/u$.

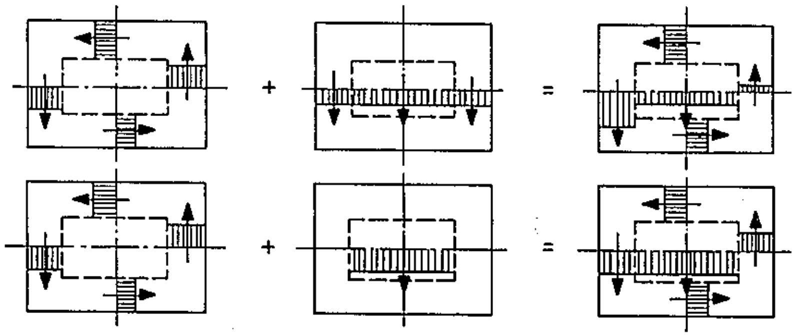


*Fig. 6.3.19. Alternative summations of torsion and shear*

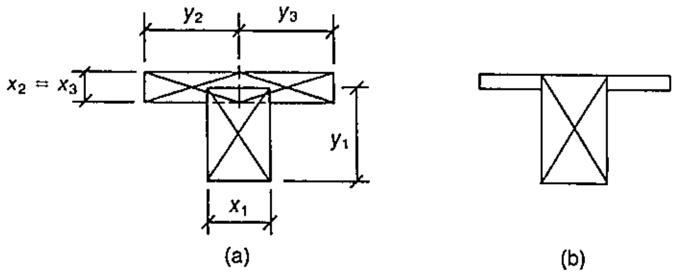



*Fig. 6.3.20. Treatment of sections composed of rectangles: (a) division into component rectangles; (b) use of a single main rectangle*

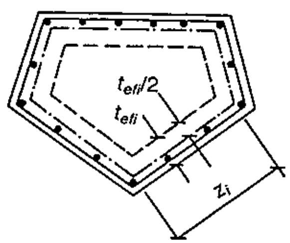



*Fig. 6.3.21. Notation for eqs (6.3–32) and (6.3–33)*

(c) *Sections composed of rectangles*

For sections composed of rectangles the applied torque may be assumed to be distributed between the rectangles in proportion to the values of $x_i^3 y_i$ for each of them

$$T_{Sdi} = T_{Sd} \frac{x_i^3 y_i}{\Sigma x_i^3 y_i}$$

where

$T_{Sdi}$ is the torque assumed to be resisted by the *i*th rectangle
$x_i$ is the smaller dimension of the *i*th rectangle
$y_i$ is the larger dimension of the *i*th rectangle.

Each rectangle *i* is then treated as a solid section subjected to the torque $T_{Sdi}$.

If one of the rectangles has a value of $x_i^3 y_i$ markedly geater than those for the other rectangles then this rectangle should be assigned the entire torque.

## SHEAR FLOW AND SHEAR FORCE

In a hollow polygonal convex cross-section (actual or equivalent) the shear flow due to a torque $T_{Sd}$ may be assumed as constant and acting at the centre of each wall.

With

$t_{efi}$ the effective thickness of the *i*th wall
$z_i$ the distance, for the *i*th wall between the intersections of its centre-line with those of adjacent walls (see Fig. 6.3.21)

the shear flow is

$$\tau_{ti} t_{efi} = T_{Sd}/2A_{ef}\delta \qquad (6.3\text{–}32)$$

and the shear force in the $i$th wall is

$$V_{Sdi,t} = T_{Sd} z_i / 2A_{ef}\delta \qquad (6.3\text{–}33)$$

where

$\tau_{ti}$ is the shear stress due to torsion
$A_{ef}$ is the area encosed by the centre-lines of walls
$\delta$ is a numerical coefficient.

## ACTION EFFECTS TO BE CONSIDERED

Each wall $i$ should be designed for the shear and normal forces due to $M_{Sd}$, $N_{Sd}$, $V_{Sd}$ and $T_{Sd}$ (see Fig. 6.3.22)

$$V_{Sdi} = V_{Sdi,t} + V_{Sdi,v} \qquad (6.3\text{–}35)$$

where

$V_{Sdi,t}$ is the shear force due to torsion, see eq. (6.3–33)
$V_{Sdi,v}$ is the shear force due to transverse shear

$$N_{Sdi} = N_{Sdi,m} + N_{Sdi,n} \qquad (6.3\text{–}36)$$

where

$N_{Sdi,m}$ is the longitudinal force due to flexure
$N_{Sdi,n}$ is the longitudinal force due to axial load.

The effects of prestress should be taken into account in the calculation of $V_{Sdi}$ and $N_{Sdi}$. The prestressing forces should be treated as an external loading system with safety factors as described in clause 1.6.2.4 a1.

## VERIFICATIONS

The verifications differ depending on whether or not the wall is assumed to have inclined cracks.

For a circular section $\delta = 1.0$.

For a rectangular section of side dimensions $b_x$ and $b_y$, where $b_y > b_x$

$$\delta = 1.0 - 0.25 b_x / b_y \qquad (6.3\text{–}34)$$

The coefficient $\delta$ allows for model imperfections/uncertainties probably related to the flow of stress around corners.

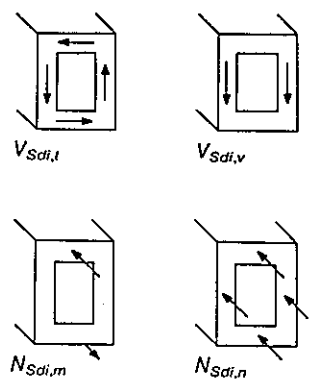



*Fig. 6.3.22. Shear and normal forces on walls*

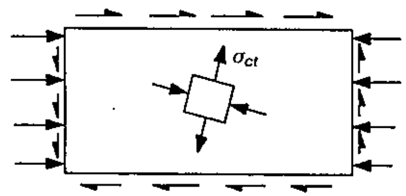



*Fig. 6.3.23. Uncracked wall*

**(*a*) Uncracked wall**
A wall may be treated as uncracked if

- it is parallel to the axis about which the member is bent by flexure, and
- the maximum principal stress $\sigma_{ct,max}$ resulting from $\sigma_{Sdi}$ and $\tau_{Sdi}$ is less than or equal to $f_{ctd}$, where $f_{ctd} = f_{ctk,min}/1.5$ and $f_{ctk,min}$ is given in Table 2.1.2.

Where a wall contains ducts or bars (see clause 6.2.2.4).

It is not prohibited to use different depths of main compression zone for the effects of torsion and flexural/axial load. In this case the relevant areas should be used separately to obtain stresses from forces.

The verification ($\sigma_{ct,max} \leqslant f_{ctd}$) should be made at the centre of the wall and the shear stresses due to torsion should be estimated from eq. (6.3–32), while the stresses due to the remaining load effects should be calculated on the assumption of linear elastic behaviour.

If the condition is satisfied, in addition the principal compressive stress resulting from $\sigma_{Sdi}$ and $\tau_{Sdi}$ should be less than $f_{cd1}$.

**(*b*) Wall with inclined cracks**
The following verifications should be made if either of the conditions in (*a*) is not satisfied.

The model of Fig. 6.3.24 may be used so long as the following provisions are respected.

- The spacing of stirrups should not exceed $u_s/8$ where $u_s$ is the perimeter of the stirrups.
- The stirrups should provide effective continuity from wall to wall.
- At each junction of walls, there should be a longitudinal bar with a diameter at least equal to $s/16$ where $s$ is the spacing of the stirrups.

The angle $\theta_i$ between the wall compression and the longitudinal direction may be chosen freely in the range 18° to 45°.

The forces derived from the model of Fig. 6.3.24 and the verifications required are as follows

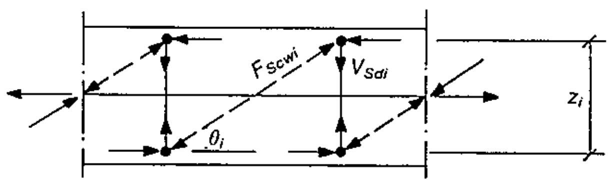



*Fig. 6.3.24. Truss model of a wall*

*1. Longitudinal force*

$$F_{Sli} = N_{Sdi} + V_{Sdi} \cot \theta_i \qquad (6.3\text{–}37)$$

with tension taken positive.

For $f_{pyd,net}$ see subsection 6.2.4.

$$F_{Rli} = A_{Si} f_{yd} + A_{pi} f_{pyd,net} \quad (6.3\text{–}38)$$

The reinforcement required for $F_{Rli}$ should generally be distributed over the length of $z_i$, but for smaller sections it may be concentrated at the ends of this length.

If $F_{Sli} < 0$ no separate verification is required provided that the wall is treated as cracked and its inclined compression is verified as in 2 below.

*2. Inclined compression of wall concrete*

$$F_{Scwi} = V_{Sdi}/\sin\theta_i \quad (6.3\text{–}39)$$

$$F_{Rcwi} = f_{cd2}\, t_i z_i \cos\theta_i \quad (6.3\text{–}40)$$

*3. Tension of transverse reinforcement*

$$F_{Stwi} = V_{Sdi} \quad (6.3\text{–}41)$$

$$F_{Rtwi} = A_{Swi} f_{yd} \cot\theta_i \frac{z_i}{s} \quad (6.3\text{–}42)$$

where $A_{Swi}$ is the area of one unit of shear reinforcement in the *i*th wall, e.g. one leg of a stirrup in a solid section or two units of reinforcement (one at the inner and one at the outer face) in a wall of a box girder.

Warping torsion is found predominantly in linear members with thin open cross-sections formed of at least three walls.

The reduction of warping torsion stiffness due to cracking is similar to that of bending stiffness and smaller than the reduction for circulatory torsion.

### 6.3.5.3. Warping torsion

For open sections having at least three walls in separate planes, the shear forces and bending moments due to torsion in each of the walls should be determined from the requirements of static equilibrium. Each wall may then be designed for the summed effects of torsion and other load effects.

## 6.4. SLABS

### 6.4.1. Bending and torsion

Subsection 6.4.2 is applicable also to prestressed concrete slabs.

In a region of a slab primarily subjected to moments $m_x$ and $m_y$ parallel to the directions of the reinforcement the design for bending should follow subsection 6.3.2 and design for shear should follow subsection 6.4.2.

In more general cases slabs are subjected to moments $m_x$, $m_y$ and $m_{xy}$ per unit width. The design should then be based upon a model in which the outer layers resist the in-plane effects of the moments and the inner layer transmits shear forces between them.

Prestressed slabs may be verified as plates subjected to in-plane compression and transverse loading (see subsection 6.5.4).

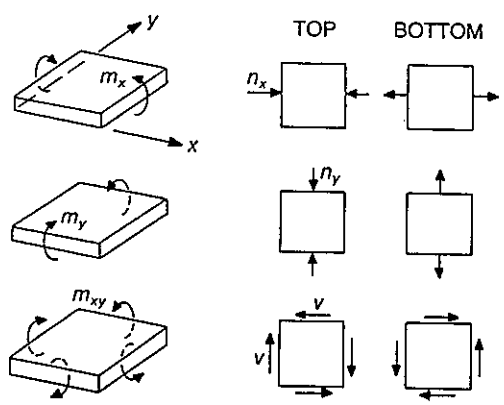



*Fig. 6.4.1. Forces in the outer layers of a slab due to moments* $m_x$, $m_y$, $m_{xy}$*: if the moments are in reversed directions, the in-plane forces have reversed signs*

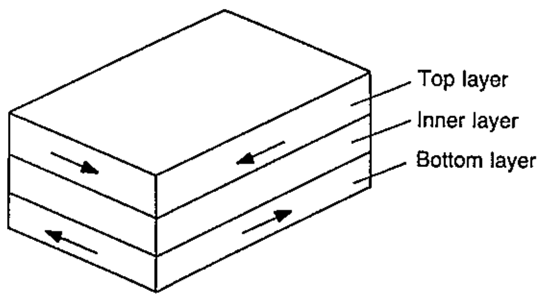



*Fig. 6.4.2. Example of layer forces in a slab resisting torsion*

At free edges of slabs the set of reinforcement perpendicular to the edge, having the greater area per unit width, should be bent into the plane of the edge and returned into the other layer to provide torsional resistance. Alternatively U-bars may be inserted at the edge and lapped with the top and bottom steel.

As illustrated by Fig. 6.4.1 the forces per unit width acting on the outer layers are

$$n_{Sdx} = m_{Sdx}/z_x \qquad (6.4\text{–}1)$$

$$n_{Sdy} = m_{Sdy}/z_y \qquad (6.4\text{–}2)$$

$$v_{Sd} = m_{Sdxy}/z_{xy} \qquad (6.4\text{–}3)$$

with signs as determined by the signs of the moments.

$z_x$ is the internal lever arm between the tensile and compressive normal forces in the $x$-direction

$z_y$ is the internal lever arm between the tensile and compressive normal forces in the $y$-direction

$z_{xy}$ is the internal level arm between the shear forces of the upper and lower layers.

As an approximation $z_{xy}$ may usually be taken as $2h/3$, where $h$ is the overall thickness of the slab.

It should be noted that in the general case the dimension $z_x$ or $z_y$ is the distance between the centres of the resultants of compression in the concrete and tension in the reinforcement. The dimension $z_{xy}$ is always between the centres of concrete forces.

The criteria for ULS are given in subsection 6.5.3.

It results from sections 1.3 and 1.4.5, that this possible deviation should not be taken less than the greater of

- 1.2 times the tolerance
- 2.4 times the expected standard deviation (possibly subjectively assessed).

If the thickness of a slab is smaller than 100 mm, the possible deviation of the depth $d$ of the reinforcement, exceeding 5% of the nominal value of $d$ should be taken into account in the calculations.

## 6.4.2. Transverse shear distributed over the width of a slab

### 6.4.2.1. Scope

The methods of verification given in this section are concerned with failures occurring more or less immediately upon the formation of major shear cracks and the resulting breakdown of normal beam action. In short shear spans where loads and reactions act in such a way as to produce transverse compression an enhanced resistance may be possible due to an arch action — see the final part of clause 6.3.3.2.

The following paragraphs apply to solid slabs in regions subjected to distributed loads. They also apply to slab-like members such as hollow-core units, the T-beams of ribbed or coffered slabs and precast units so connected as to form a slab-like structure.

They may further be applied to linear members of minor importance such as lintels.

Where a slab spans in one direction parallel to reinforcement the verification of its shear resistance is made in terms of the applied shear on sections normal to the span and of the shear resistance associated with the reinforcement in the direction of the span. This sort of verification is treated in clauses 6.4.2.2 to 6.4.2.4 dealing respectively with uncracked zones, cracked zones without shear reinforcement and slabs with shear reinforcement.

### 6.4.2.2. Shear in uncracked zones

See the end of this paragraph for special recommendations for the end regions of prestressed elements.

If the zone in question should not be cracked under the ULS action effects, it should be verified that the maximum principal tensile stress satisfies the inequality

$$\sigma_{ct,\max} < f_{ctd} \qquad (6.4\text{–}4)$$

where

$f_{ctd} = f_{ctk,\min}/1.5$

$f_{ctk,\min}$ is given in Table 2.1.2.

In principle the verification should be carried out at all sections and at all levels. However as a simplification, for rectangular I, or T-sectioned members spanning in one direction and having their centroids within the web, it is sufficient to perform the calculation in terms of a nominal shear stress $\tau = 3V_{Sdc}/(2b_w h)$ and the design longitudinal stress at the centroid.

The applied shear on the concrete ($V_{Sdc}$) should be calculated taking account of the forces in any inclined tendon using the lower design values where the effects are favourable and the higher ones where they are unfavourable.

The presence of prestressing ducts should be taken into account by subtracting from the web breadth the sum of the diameters of the ducts in one layer.

If the centroid of the cross-section does not lie within the web the principal tensile stress verification should be made at the intersection of the web and flange within which the centroid lies.

In pretensioned members account should be taken of the reduction of the effective prestress within the transmission lengths.

Where the centroid of the section is at mid-depth the reduction may be allowed for by making the calculations for a section distance $h/2$ from the inner edge of the support and taking the prestress at the centroid as

$$\sigma_c = \frac{P_{gd}}{A}\left(1 - \frac{s + 0.5h}{l_{bpt}}\right) \quad (6.4\text{–}5)$$

where

$P_{gd}$ is the design prestressing force under permanent loads
$A$ is the area of the section
$s$ is the distance from the end of the member to the inner edge of the supports
$l_{bpt}$ is the transmission length.

$l_{bpt}$ is determined according to clause 6.9.11.4 with $\alpha_9 = 1.0$.

### 6.4.2.3. Shear in cracked zones without shear reinforcement

The net shear forces acting on the concrete should be taken as

(*a*) for ordinary reinforced concrete members, the total shear $V_{Sdc} = V_{Sd}$
(*b*) for prestressed members, the shear corresponding to the part of the external loading system not balanced by the prestress system

$$V_{Sdc} = V_{Sd} - V_\lambda \quad (6.4\text{–}6)$$

where $V_\lambda$ is the shear corresponding to $\lambda q_{sd}$ determined as in clause 6.3.3.3.3.

It is to be verified that

$$V_{Sdc} \leqslant V_{Rd1} \quad (6.4\text{–}7)$$

The transverse shear resistance $V_{Rd1}$ may be described by means of an appropriate model.

Wherever a more precise analysis is not made, the following empirical expression may be used for members with parallel chords:

$$V_{Rd1} = 0.12\xi(100\rho f_{ck})^{1/3} b_{red} d \qquad (6.4\text{–}8)$$

where

$\xi = 1 + \sqrt{(200/d)}$ with $d$ in mm

$\rho$ is the ratio of bonded flexural tensile reinforcement $A_s/b_w d$ or $(A_s + A_p)b_w d$ extending for a distance at least equal to $d$ beyond the section considered, except at end supports where the extension may be considered adequate if the length of bar beyond the centre-line of support is equal to at least 12 times the diameter

$b_{red}$ is the reduced web breadth equal to the full breadth minus the sum of the widths of tendon ducts situated within the web (note no deduction is necessary for ducts at the boundary of the web, i.e. at the level of the main tension reinforcement).

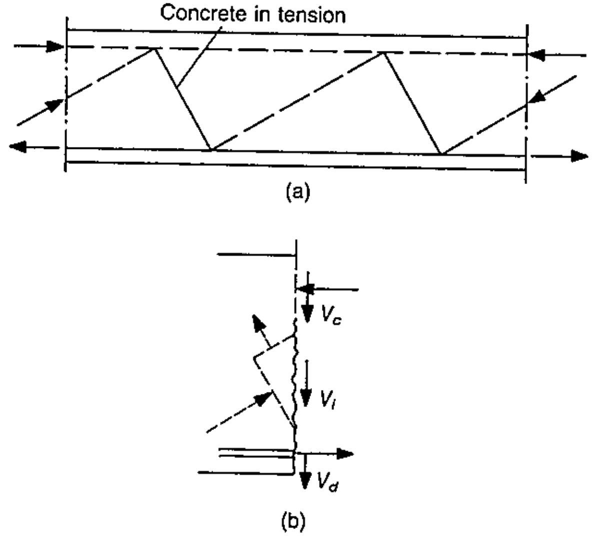



*Fig. 6.4.3. Shear resistance of members without shear reinforcement: total shear = $V_c + V_i + V_d$, where $V_c$ is shear in compression zone, $V_i$ is shear carried across cracks by concrete-to-concrete contact, and $V_d$ is shear carried across cracks by dowel action of reinforcement*

Fig. 6.4.3 illustrates indicative models of the actions resisting shear.

For high concrete strengths the relationship between the shear resistance of a member and the strength of the concrete depends upon the characteristics of the aggregate. If the aggregate fractures at flexural cracks, leaving smooth crack surfaces, the shear resistance may be below that given by eq. (6.4–8) unless $f_{ck}$ is restricted. Relevant experimental evidence may be provided by tests of beams of concrete with the intended value of $f_{ck}$ and the aggregate of the type to be used.

Unless relevant experimental evidence is available for the concrete in question, $f_{ck}$ should be limited to 50 MPa for the purpose of calculation according to eq. (6.4–8).

Except at simple supports, flexural tensile reinforcement should extend at least 0.6$d$ beyond the section at which it is no longer required according to flexural calculations.

In the case of a member subjected to axial tensile loading the reinforcement required to resist this load should be discounted in the calculations of $\rho$.

Bonded prestressing tendons can be included in the calculation of $\rho$ but unbonded tendons should be excluded.

#### 6.4.2.4. Shear in slabs with shear reinforcement

The verification of the resistance of a slab with shear reinforcement should be made in accordance with subsection 6.3.3, but the general requirements of clause 6.3.3.1 may be relaxed as follows

(*a*) The minimum shear reinforcement, giving a value of $\omega_{sw} = A_{sw}f_{yk}/(b_w s f_{ctm})$ of at least 0.2, need be provided only where the applied shear exceeds the shear resistance of the slab without shear reinforcement, and for a distance equal to $d$ in the direction of decreasing shear. The minimum shear reinforcement need not be in the form of stirrups.

(*b*) The inclination of stirrups and bent-up-bars to the axis should be at least 45° and 30° respectively as for linear members.

(*c*) The spacing of members of shear reinforcement in the longitudinal direction should not exceed $0.75d(1 + \cot\alpha)$, where $\alpha$ is the inclination of the shear reinforcement.

(*d*) Longitudinal reinforcement is not required to be contained within a stirrup cage but the shear reinforcement should be anchored at the level of the flexural tensile reinforcement and at the level of the centre of the flexural compression force in the ULS.

It should be noted here, that even in the ULS the depth of the compression zone is the elastic one except at the section of maximum moment.

### 6.4.2.5. General cases of two-way spanning slabs

Where a slab spans in two directions, the results of the analysis include shears $v_x$ and $v_y$ per unit width. The principal shear is then

$$v_1 = \sqrt{(v_x^2 + v_y^2)} \qquad (6.4\text{–}9)$$

and acts on a surface at an angle $\phi$ to the $y$-axis, where

$$\phi = \arctan(v_y/v_x) \qquad (6.4\text{–}10)$$

The shear on a perpendicular surface is zero.

The verification should be made with respect to the principal shear and, for an ordinary reinforced concrete slab, it is required that

$$v_1 b \leqslant V_{Rd1} \qquad (6.4\text{–}11)$$

where

$V_{Rd1}$ is given by eq. (6.4–8) with $d$ taken as the mean effective depth

$$\rho = \rho_x \cos^4\phi + \rho_y \sin^4\phi \qquad (6.4\text{–}12)$$

$\rho_x$ and $\rho_y$ are the ratios of reinforcement near the face in tension in the direction perpendicular to the surface on which $v_1$ acts.

Clause 6.4.2.5 is intended to be applied in cases where neither $v_x$ nor $v_y$ is clearly the major shear.

Eq. (6.4–12) is related to the stiffness of the reinforcement in relation to tension in the direction perpendicular to the surface on which $v_1$ acts.

## 6.4.3. Concentrated loads on slabs/slab-column connections

### 6.4.3.1. General

The resistance to the transverse effects of concentrated forces (loads or reactions) acting on slabs without shear reinforcement may be verified in terms of nominal shear stresses at control perimeters.

Provided that the concentrated force is not opposed by a high distributed pressure, e.g. soil pressure on a base, or by the effects of a load or reaction within a distance equal to 2.0*d* from the periphery of area of application of the force, the control perimeter ($u_1$) may be taken to be at a distance 2.0*d* from the above periphery and should be constructed so as to minimize its length.

The effective depth of the slab is assumed constant and may normally be taken as

$$d_{ef} = (d_x + d_y)/2 \qquad (6.4\text{–}13)$$

where $d_x$ and $d_y$ are the effective depths of the reinforcement in two orthogonal directions.

If the slab contains a drop panel around a column one verification should be made for a perimeter 2.0*d* from the column with *d* taken as the effective depth within the area of the drop.

A second verification should be made for the area outside the drop panel and for this the smaller slab thickness should be used. If the drop panel is large in area it may be more appropriate to make this second verification in accordance with clause 6.4.2.3.

The verification in terms of the perimeter $u_1$ is subject to an upper limit in terms of shear stresses at the periphery of the area to which the force is applied.

In the following subsection a few typical cases are presented and simplified approaches to the analysis of stress distributions are given. For more complex cases, e.g. elongated supports, an appropriate structural analysis should be performed.

The normal shear stress on a section defined by a control perimeter does not have any physical meaning, but this empirical approach gives a close approximation to the resistance obtained by mechanical models of punching behaviour.

In reality, for the axisymmetric case, the failure surface is trumpet-shaped/conical. It runs from the edge of the loaded area through the slab to the opposite face at a mean inclination of about 25° to 30° (see Fig. 6.4.4). The inclined crack is already formed at 1/2 to 2/3 of the failure load without leaving the slab in an unstable condition. The resistance thus depends primarily on the conditions of stress and strain in the concrete in the region close to the column.

The analytical methods, the results of which show good approximation to the empirical treatment, include the Kinnunen/Nylander model and the upper bound solution according to plastic theory.

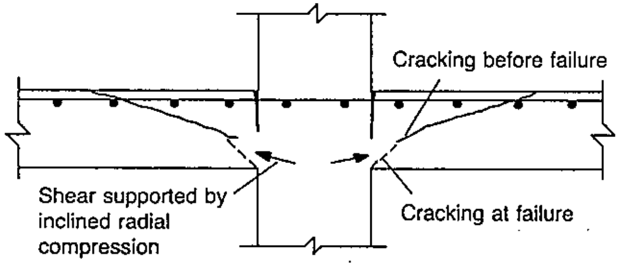



*Fig. 6.4.4. Section through a punching failure*

The upper limit, which is rarely the governing criterion, is given in clause 6.4.3.4.

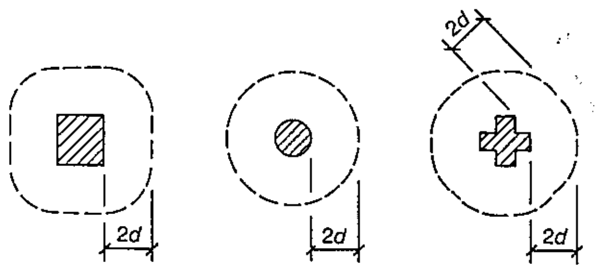



*Fig. 6.4.5. Control perimeters at interior columns*

### 6.4.3.2. Stresses due to applied loads

(*a*) *Symmetric loading*

If the dispersion of the concentrated force is approximately polar-symmetric the applied shear stress at the control perimeter may be taken as

$$\tau_{Sd} = F_{Sd}/u_1 d \qquad (6.4\text{–}14)$$

where

$F_{Sd}$ is the concentrated force
$u_1$ is the length of the control perimeter.

(*b*) *Slab-internal column connections transferring moments*

If the dispersion of the force is non-symmetrical due to the transfer of an unbalanced moment $M_{Sd}$ from the slab to a column the maximum shear at the control perimeter may be calculated as

$$\tau_{Sd} = \frac{F_{Sd}}{u_1 d} + \frac{K M_{Sd}}{W_1 d} \qquad (6.4\text{–}15)$$

where

$W_1$ is a parameter of the control perimeter $u_1$ ($W_1 = \int_0^{u_1} |e| dl$)

$dl$ is an elementary length of the perimeter
$e$ is the distance of $dl$ from the axis about which the moment $M_{Sd}$ acts

$K$ is a coefficient dependent on the ratio between the column dimensions $c_1$ parallel to the eccentricity $M_{Sd}/F_{Sd}$ and $c_2$ perpendicular to the eccentricity; its value is a function of the proportions of the unbalanced moment transmitted by uneven shear on the one hand and by bending and torsion on the other.

The property $W_1$ corresponds to a distribution of shear as illustrated in Fig. 6.4.6.

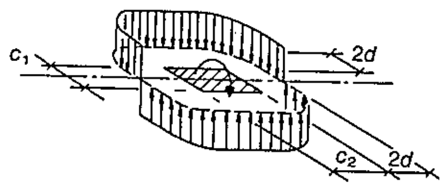



*Fig. 6.4.6. Shear distribution due to an unbalanced moment*

For a rectangular column

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4 c_2 d + 16 d^2 + 2\pi d c_1 \qquad (6.4\text{–}16)$$

where

$c_1$ is the column dimension parallel to the eccentricity of the load
$c_2$ is the column dimension perpendicular to the eccentricity of the load.

Values of $K$ may be obtained from

| $c_1/c_2$ | 0.5 | 1.0 | 2.0 | 3.0 |
|---|---|---|---|---|
| $K$ | 0.45 | 0.60 | 0.70 | 0.80 |

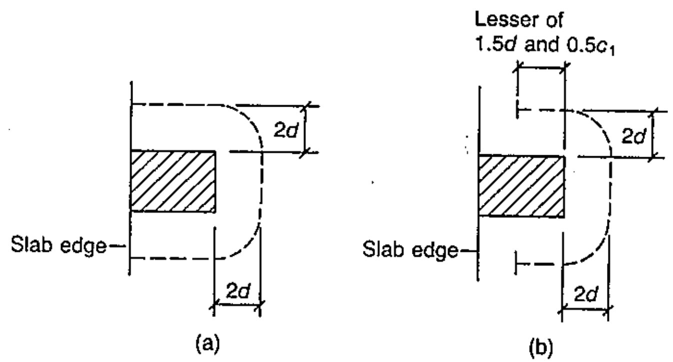



*Fig. 6.4.7. Control perimeters at edge columns: (a) perimeter* $u_1$; *(b) perimeter* $u_1^*$

In such cases the maximum shear is

$$\tau_{Sd} = \frac{P_{Sd}}{u_1^* d} + \frac{K M_{Sd}}{W_1 d} \qquad (6.4\text{–}17)$$

where $K$ may be determined from the table above but with the ratio $c_1/c_2$ replaced by $c_1/2c_2$. $W_1$ is calculated for the full perimeter $u_1$.

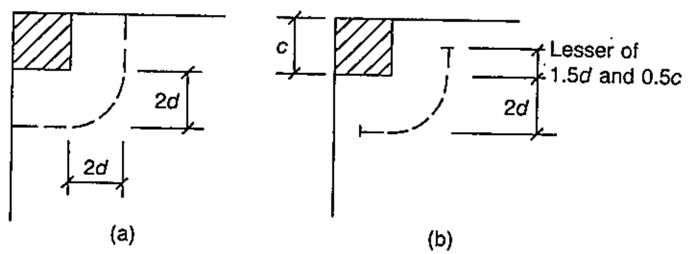



*Fig. 6.4.8. Control perimeters at corner columns: (a) perimeter* $u_1$; *(b) perimeter* $u_1^*$

Unless relevant experimental evidence is available for the concrete in queston, $f_{ck}$ should be limited to 50 MPa for the purpose of calculation according to eq. (6.4–18) (see commentary on clause 6.4.2.3).

*(c) Slab-edge column connections*
In principle the distribution of shear around the perimeter in Fig. 6.4.7(a) should be determined to calculate $\tau_{Sd}$.

However, provided the eccentricity of loading in the direction perpendicular to the slab edge is in the direction of the interior of the slab and there is no eccentricity parallel to the edge, $\tau_{Sd}$ may be calculated on the assumption of uniform shear on the perimeter $u_1^*$ shown on Fig. 6.4.7(b).

Where a moment $M_{Sd}$ acting in the direction parallel to the slab-edge is transferred to the column it should be taken to produce an additional shear stress equal to $KM_{Sd}/W_1 d$ where $W_1$ is calculated for the perimeter $u_1$ of Fig. 6.4.7(a).

*(d) Slab-corner column connections*
In principle the distribution of shear around the perimeter in Fig. 6.4.8(a) should be determined to calculate $\tau_{Sd}$.

However, provided the eccentricity of loading is toward the interior of the slab, $\tau_{Sd}$ may be calculated on the assumption of uniform shear on the perimeter $u_1^*$ shown on Fig. 6.4.8(b).

### 6.4.3.3. Resistances of reinforced slabs

The shear resistance of a reinforced concrete slab, expressed as a shear stress on a control perimeter may be taken as

$$\tau_{Rd} = 0.12\xi(100\rho f_{ck})^{1/3} \qquad (6.4\text{–}18)$$

where

$$\xi = 1 + \sqrt{(200/d)} \quad \text{with } d \text{ in mm.}$$

The ratio $\rho$ of flexural reinforcement may be calculated as $\sqrt{(\rho_x \rho_y)}$ where $\rho_x$ and $\rho_y$ are the ratios in orthogonal directions. In each direction the ratio should be calculated for a width equal to the side dimension of the column (or loaded area) plus $3d$ to either side of it (or to the slab edge if this is closer).

### 6.4.3.4. Maximum resistance

The maximum loading for which any connection (including connections with shear reinforcement and connections involving prestressed slabs) may be designed is defined by

$$F_{Sd,ef}/u_0 d \leqslant 0.5 f_{cd2} \qquad (6.4\text{–}19)$$

where

$F_{Sd,ef}$ is the punching load enhanced to allow for the effects of an eventual moment transferred to a column
for an interior load or column, $u_0$ is the length of the periphery of the load or column
for an edge column $u_0 = c_z + 3d \leqslant c_x + 2c_y$
for a corner column $u_0 = 3d \leqslant c_x + c_y$

where for an edge column

$c_x$ is the column dimension parallel to the slab edge
$c_y$ is the column dimension perpendicular to the slab edge.

At an interior column

$$F_{Sd,ef} = F_{Sd}\left[1 + K \frac{M_{Sd}}{F_{Sd}} \frac{u_1}{W_1}\right] \qquad (6.4\text{–}20)$$

For values of $K$ see clause 6.4.3.2.

At an edge column

$$F_{Sd,ef} = F_{Sd}\left[1 + K \frac{M_{Sd}}{F_{Sd}} \frac{u_1^*}{W_1}\right] \qquad (6.4\text{–}21)$$

where $M_{Sd}$ is the moment parallel to the slab edge.

At a corner column

$$F_{Sd,ef} = F_{Sd} \qquad (6.4\text{–}22)$$

### 6.4.3.5. Verification of column bases

The punching resistances of column bases should be verified at control perimeters at distances up to $2.0d$ from the periphery of the column. The situation at the perimeter giving the lowest column load should be taken to be decisive.

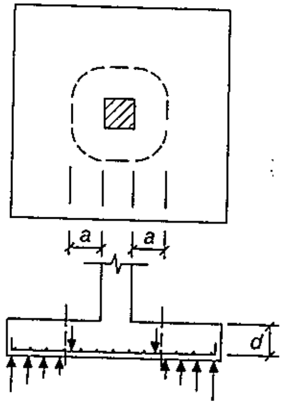



*Fig. 6.4.9. Column base*

For concentric loading the net applied force is

$$F_{Sd,red} = F_{Sd} - \Delta F_{Sd} \quad (6.4\text{–}23)$$

where

$F_{Sd}$ is the column load
$\Delta F_{Sd}$ is the net upward force within the control perimeter considered i.e. upward pressure from soil minus self-weight of base

$$\tau_{Sd} = F_{Sd,red}/ud \quad (6.4\text{–}24)$$

$$\tau_{Rd} = 0.12\xi(100\rho f_{ck})^{1/3} \times 2d/a \leqslant 0.5 f_{cd2} \quad (6.4\text{–}25)$$

where $a$ is the distance from the periphery of the column to the control perimeter in question.

$\Delta F_{Sd}$ should be determined taking account of the design (unfavourable) distribution of soil pressure on the base. $u$ is the control perimeter in question ($u \leqslant u_1$).

For eccentric loading

$$\tau_{Sd} = F_{Sd,red}\left[1 + K \frac{M_{Sd}}{F_{Sd}} \frac{u_1}{W_1}\right] \quad (6.4\text{–}26)$$

For $K$ see clause 6.4.3.2.

### 6.4.3.6. Slabs with shear reinforcement

The punching resistances of slabs with shear reinforcement are to be verified in three zones:

- the zone immediately adjacent to the column or loaded area
- the zone in which the shear reinforcement is placed
- the zone outside the shear reinforcement.

At interior slab-column connections subjected to symmetric loading, at edge column connections where there is no eccentricity parallel to the slab edge and the eccentricity perpendicular to it is toward the interior of the slab, and at corner columns where the eccentricity of the reaction is toward the interior of the slab, the verification may be made as follows, subject to the detailing requirements below.

For an interior column $u_0$ = length of column periphery
for an edge column $u_0 = c_x + 3d \leqslant c_x + 2c_y$
for a corner column $u_0 = 3d \leqslant c_x + c_y$

where for an edge column $c_x$ is the column dimension parallel to the slab edge.

For edge and corner columns, unless an analysis of the shear distribution is made, the perimeters $u_1^*$ should be used for $u_1$ (see Figs 6.4.7 and 6.4.8).

The ratio $\rho$ of flexural reinforcement may be calculated as $\sqrt{(\rho_x \rho_y)}$ where $\rho_x$ and $\rho_y$ are the ratios in orthogonal directions. In each direction of the column (or loaded area) plus $3d$ to either side of it (or to the slab edge if this is closer).

Anchorage of shear reinforcement in slabs thinner than 250 mm requires special attention to detailing.

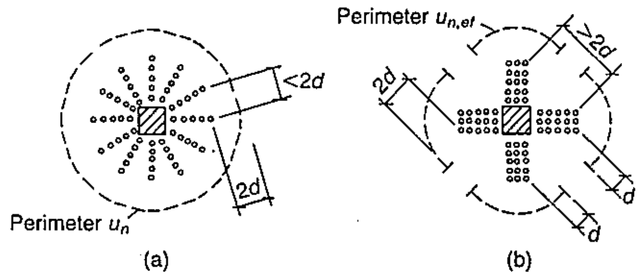



*Fig. 6.4.10. Control perimeters* ($u_n$) *at interior columns*

(*a*) *Adjacent to the column*

$$F_{Sd} \leqslant u_0 d(0.5 f_{cd2}) \qquad (6.4\text{–}27)$$

(*b*) *In the zone with shear reinforcement*

$$F_{Sd} \leqslant 0.09\xi(100\rho f_{ck})^{1/3} u_1 d + 1.5 \frac{d}{s_r} A_{sw} f_{ywd} \sin\alpha \qquad (6.4\text{–}28)$$

where

$A_{sw}$ is the area of shear reinforcement in a layer around the column
$s_r$ is the radial spacing of the layers of shear reinforcement
$\alpha$ is the angle between the shear reinforcement and the plane of the slab

$$1.5 \frac{d}{s_r} A_{sw} f_{ywd} \sin\alpha \geqslant 0.03(100\rho f_{ck})^{1/3} u_1 d$$

The design strength of the shear reinforcement ($f_{ywd}$) should not be taken greater than 300 MPa.

(*c*) *Outside the shear reinforcement*

$$F_{Sd} \leqslant 0.12\xi(100\rho f_{ck})^{1/3} u_{n,ef} d \qquad (6.4\text{–}29)$$

where

$u_{n,ef}$ is the effective length of a perimeter constructed at a distance 2.0$d$ outside the outermost shear reinforcement,
$\rho$ is calculated for the reinforcement crossing $u_{n,ef}$.

If the circumferential spacing of the outermost shear reinforcement exceeds $2d$, $u_{n,ef}$ is the sum of the lengths of perimeters corresponding to parts of the periphery of the shear reinforcement within distances $d$ of the elements of the shear reinforcement.

The detailing requirements referred to above are as follows.

(*a*) The distance from the innermost shear reinforcement to the periphery of the column should not exceed $\beta d$, where

$$\beta = \frac{\text{capacity of slab without shear reinforcement}}{\text{required capacity}} \leqslant 0.5$$

(*b*) The shear reinforcement should be anchored at or beyond the planes of the tensile reinforcement and centre of flexural compression of the slab.

(*c*) The radial spacing of the shear reinforcement should not exceed $0.75d$.

(*d*) At edge and corner columns, the shear reinforcement required by calculation should be placed within the segments indicated in Fig. 6.4.11. Similar reinforcement at the same spacings should be provided in the areas between these segments and the slab edge or edges, but should not be taken into account in calculations.

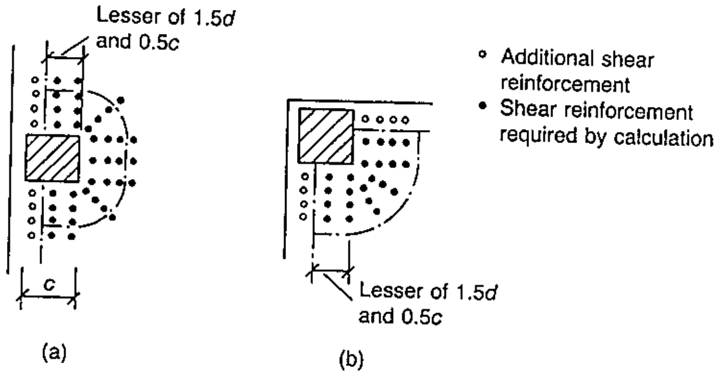



*Fig. 6.4.11. Shear reinforcement at edge and corner columns*

At interior columns to which moments are transferred and at edge columns where there is eccentricity of loading parallel to the slab edge, the force $F_{Sd}$ should be magnified to $F_{Sd,ef}$ to allow for the influence of the transferred moment.

(*a*) Adjacent to the column and in the zone with shear reinforcement

$$F_{Sd,ef} = F_{Sd}\left[1 + K\frac{M_{Sd}}{F_{Sd}}\frac{u_1}{W_1}\right] \qquad (6.4\text{–}30)$$

(*b*) Outside the shear reinforcement

$$F_{Sd,ef} = F_{Sd}\left[1 + K\frac{M_{Sd}}{F_{Sd}}\frac{u_{n,ef}}{W_{n,ef}}\right] \qquad (6.4\text{–}31)$$

$W_{n,ef}$ is the parameter of the perimeter $u_{n,ef}$ analogous to $W_1$ for $u_1$ (see Fig. 6.4.6).
For values of $K$ see clause 6.4.3.2.

The verification may then be made as for a connection without eccentricity of loading and the shear reinforcement should be placed uniformly around the column.

#### 6.4.3.7. Prestressed slabs

For prestressed slabs the sum of the vertical components of the forces in prestressing tendons passing through a column or within a distance $h/2$ of it may be deducted from the load $F_{Sd}$.

Other influences of prestress may be taken into account by methods given in relevant specialist literature, e.g. the FIP Recommendations for the design of flat slabs in post-tensioned concrete.

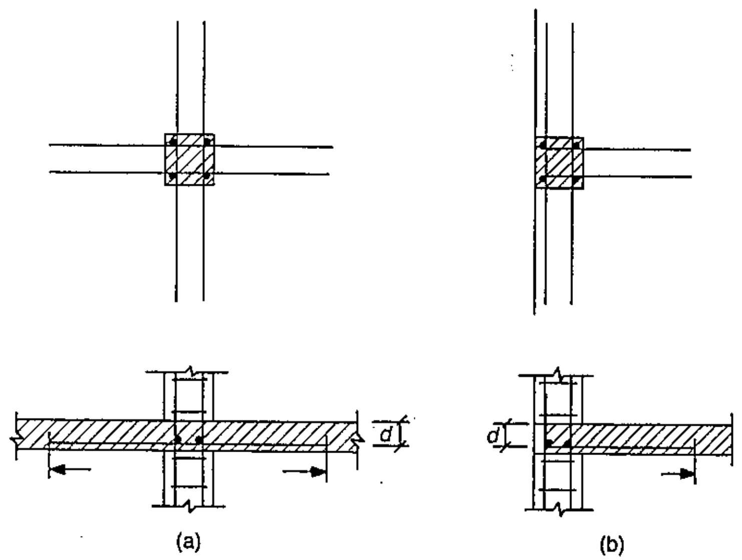



*Fig. 6.4.12. Bottom reinforcement at slab-column interfaces: (a) internal column; (b) edge column*

### 6.4.3.8. Local ductility

To reduce brittleness in the event of a local failure at a slab-column connection the following requirements should be met with respect to bottom steel in the slab crossing the slab-column interfaces.

The total area $A_s$ of reinforcement crossing the interfaces should be such that

$$A_s f_{yd} \geqslant F_{Sd} \qquad (6.4\text{–}32)$$

On either side of each interface, the steel contributing to $A_s$ should be anchored

(*a*) on the slab side, by a full anchorage length plus a length equal to $d$
(*b*) on the column side, either by its anchorage in the slab at the other side of the column or by a full anchorage length within the column.

The bars used in $A_s$ should pass inside the main reinforcement of the column and should be of steel type $S$.

In the example $A_s$ is the area of 8 bars for the internal column and 6 bars for the edge column.

## 6.5. PLATE ELEMENTS

### 6.5.1. Scope

This section gives some methods of generalizing the modelling approaches given for specific cases in sections 6.3 and 6.4.

Subsection 6.5.2 treats the design of thin-walled members subjected to action effects M, N, V and T in circumstances where longitudinal shear is not negligible. The method gives a means of determining the in-plane forces per unit width throughout the walls of the section in regions free from discontinuities.

A separate modelling following the principles of section 6.8 is required in regions of discontinuity, e.g. to connect the forces obtained from subsection 6.5.2 to end reactions.

Subsection 6.5.3 gives means of verifying the resistances of plate elements subjected to in-plane loading, while subsection 6.5.4 extends the approach to treat plates subjected to moments as well as in-plane loads.

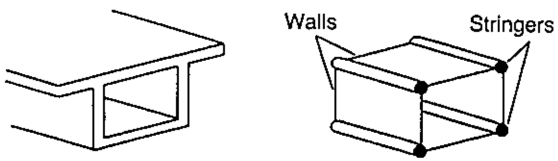



*Fig. 6.5.1. Stringer-and-wall model of a box girder*

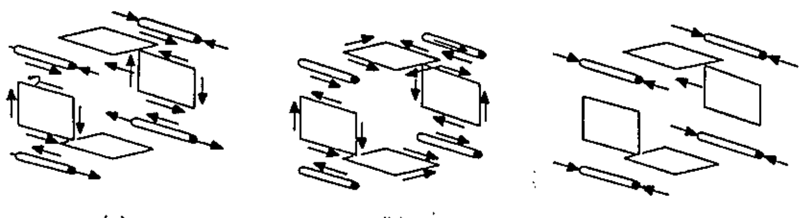


Fig. 6.5.2. *Forces on stringer wall + wall model: (a) due to M and V; (b) due to T; (c) due to N*

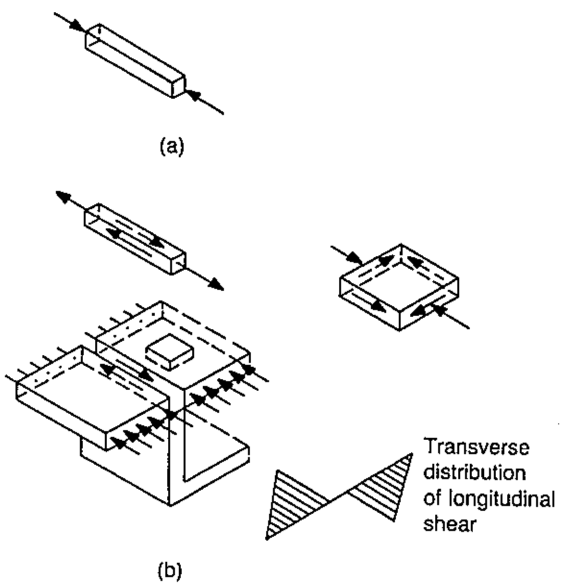



Fig. 6.5.3. *Distribution of a stringer force: (a) stage 1; (b) stage 2*



## 6.5.2. Internal forces in thin-walled sections

In the absence of a more accurate analysis the action effects due to M, N, V and T on a part of a member remote from discontinuities may be determined by a two-stage process.

In the first stage the member is modelled by longitudinal 'stringers' situated at the intersections of the walls of the actual member and by webs connecting the stringers.

The stringers are assumed to resist only axial loads and webs are assumed to resist only shear.

The forces in the stringers and webs are determined from the external action effects M, N, V and T. Prestress is treated as an external action with forces $\gamma_p P_{d,o}(x, t)$.

In the second stage forces from the stringers are distributed over the section to produce longitudinal normal forces per unit width and shear forces per unit width. This distribution should not produce any overall effects (M, N, V or T).

The total forces throughout the section can then be obtained as the sums of the forces from the first and second stages.

In the absence of transverse bending effects the member design may then be verified according to subsection 6.5.3.

Where transverse bending is present the verification should be made according to subsection 6.5.4.

## 6.5.3. Plates subjected to in-plane loading

In-plane loading of a plate may be described in terms of forces per unit width

$$n_{Sdx},\ n_{Sdy} \text{ and } v_{Sd}$$

with the axes $x$ and $y$ chosen to coincide with the directions of orthogonal reinforcement.

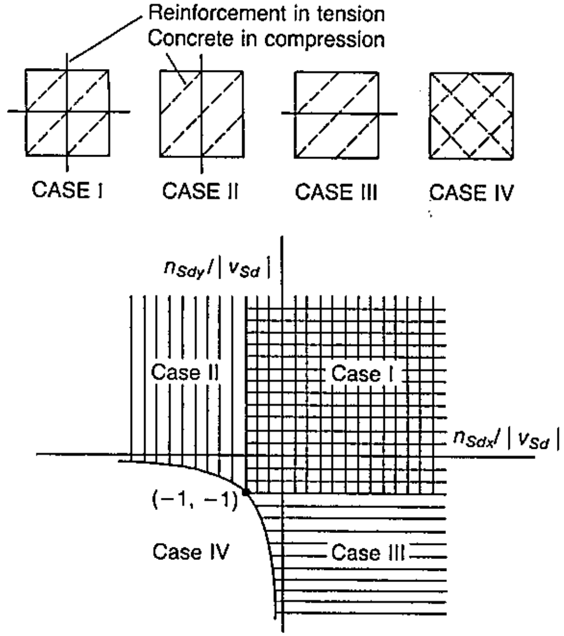



*Fig. 6.5.4. Systems of resistance*

$\theta$ is the angle between the $x$-axis and the direction of compression. In cases I–III, the angle $\theta$ can be chosen freely so long as the compression is inclined at least 15° to both sets of reinforcement. The minimum local reinforcement is obtained with $\theta = 45°$.

The internal system providing resistance to in-plane loading may be of four types

- case I tension in reinforcement in two directions and oblique compression in the concrete
- case II tension in reinforcement in the $y$-direction and oblique compression in the concrete
- case III tension in reinforcement in the $x$-direction and oblique compression in the concrete
- case IV biaxial compression in the concrete.

The relationship between the applied loading and the systems of resistance is illustrated in Fig. 6.5.4.

The resistances for the ULS are

(*a*) for reinforcement $f_{yd}$ or $f_{pyd,net}$
(*b*) for concrete in cases I–III $f_{cd2}$
(*c*) for concrete in case IV $f_{cd1}$ or $hf_{cd1}$ where the coefficient $h$ is the ratio between the biaxial and uniaxial strengths and can be determined from eq. (2.1–11).

## 6.5.4. Plates subjected to moments and in-plane loading

The plate may be modelled as comprising three layers. The outer layers provide resistance to the in-plane effects of both the bending and the in-plane loading, while the inner layer provides a shear transfer between the outer layers (see Fig. 6.5.5).

The action effects of the applied loads are expressed as moments and forces per unit width in directions parallel to the orthogonal reinforcement

$$m_{Sdx},\ m_{Sdy},\ m_{Sdxy},\ n_{Sdx},\ n_{Sdy},\ v_{Sd}$$

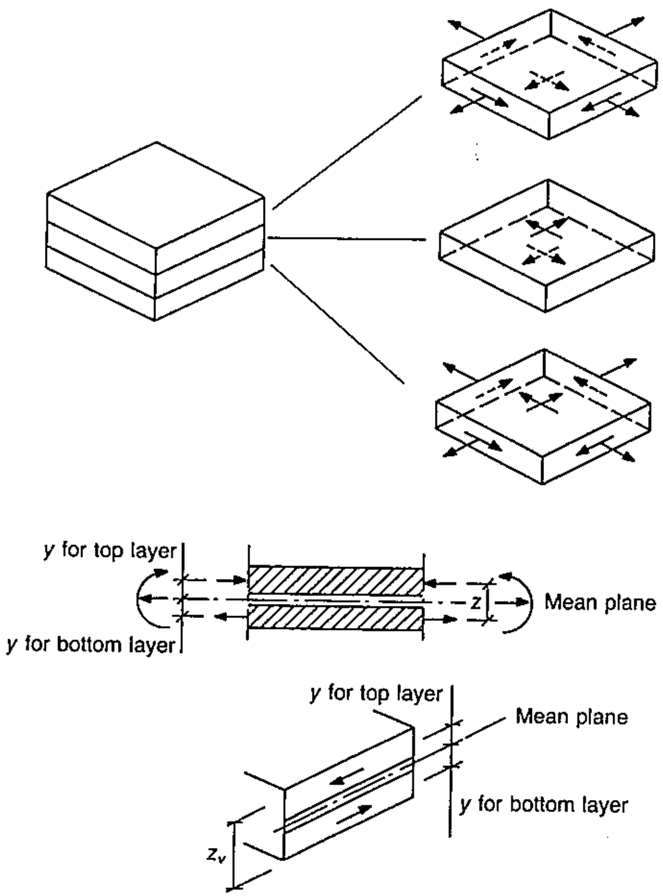



*Fig. 6.5.5. Three-layer plate model*

An exact determination of $z$ and $y$ values is complex and may require iteration since they depend on the levels of the reinforcement and on the thickness of the concrete layers. A reasonable starting point is to take the $z$ values as $2h/3$ where $h$ is the overall thickness of the plate and to take all values of $(z - y)/z$ as 1/2.

These produce the following forces per unit width on the plates:

$$n_{pSdx} = n_{Sdx}\frac{(z_x - y)}{z_x} \pm \frac{m_{Sdx}}{z_x}$$

$$n_{pSdy} = \frac{n_{Sdy}(z_y - y)}{z_y} \pm \frac{m_{Sdy}}{z_y}$$

$$v_{pSd} = v_{Sd}\frac{(z_v - y)}{z_v} \pm \frac{m_{Sdxy}}{z_v}$$

where $z_x$, $z_y$ and $z_v$ are the lever arms between the direct forces in the $x$ and $y$ directions respectively and the shear forces, and $y$ is the distance from the mean plane of the slab to the force in question.

No internal lever arm should be taken greater than the distance between the mean planes of the reinforcement at opposite faces.

The verification of the outer layers can then be made as for plates subjected to in-plane loads (see subsection 6.5.3).

The verification of the inner layer can be made according to subsection 6.4.2.

# 6.6. ULTIMATE LIMIT STATE OF BUCKLING

## 6.6.1. Definitions

The ultimate limit state of buckling is defined in clauses 1.6.3.1 and 1.6.3.2.

The ultimate limit state of buckling is in many cases identical with the so-called ultimate limit state of 'first yield' (see clause 5.4.1.2). The design resistance of a member in the ultimate limit state of buckling is therefore in general smaller than the design resistance in the ultimate limit state of an individual cross-section.

A non-linear analysis should in general be used (see clause 5.3.2.1).

### 6.6.1.1. Range of application

This section applies mainly to uniaxial or mono-dimensional elements made of reinforced or prestressed concrete and subjected to axial compression, with or without bending, for which the effect of combined action of bending and shear as well as bending and torsion may be neglected.

The principles of this section may also be applied to other types of structural elements, such as walls and shells, slender beams subjected to considerable compressive stresses in the compression zone (in which lateral buckling may occur), deep beams and other large or exceptional structures and elements in which significant local deformations may occur or elements made with lightweight aggregate or high strength concrete with normal aggregates or unreinforced concrete.

For the evaluation of shear or torsional deformations, complementary design rules need to be defined (see for example clause 6.6.3.3.4 for lateral buckling of beams).

### 6.6.1.2. Classification of structures and structural elements

For the purpose of design calculations related to buckling, structures or structural elements may be classified as braced or unbraced depending on the provision of bracing elements or not, and as non-sway or sway, depending on the sensitivity to second order effects due to lateral displacements with respect to the direction of compressive forces.

For the classification sway or non-sway, numerical criteria are given in clause 6.6.3.1.2 for bracing elements and in 6.6.3.1.3 for framed building structures.

The bracing elements considered in this section are elements of a sufficiently high stiffness, with respect to the stiffness of the braced sub-assemblies connected to them, which are able to ensure the stability of the whole structure.

A structure may be considered braced if lateral stability of the structure as a whole is provided by bracing elements designed to resist all lateral forces. It should otherwise be considered as unbraced.

For the classification braced or unbraced, a numerical criterion is given in clause 6.6.3.1.2 for framed building structures.

A framed structure is defined as non-sway if the influence of the displacements of the connections upon the design load effects may be neglected.

Isolated elements are either single elements in compression or integral parts of a structure which may be considered as isolated for design purposes. In buildings, the cross-section of an isolated element is commonly constant along its length between restraints.

Special structures are large uniaxial structures such as towers, piers, or structures with variable cross-sections in which the longitudinal forces can vary along the axis, or unbraced frames with irregular dimensions and irregular relative stiffness of their elements.

Plane members in compression, such as walls and shells, which may fail by lateral buckling can be verified by the methods presented in this section.

### 6.6.1.3. Effective length and slenderness

The sensitivity of structures or structural elements to second order effects can be described by the slenderness ratio $\lambda = l_o/i$. The effective length $l_o$ depends on the parameters which influence the deformations and is defined as that length of a pin-ended column which will lead to the same buckling behaviour and load carrying capacity as the real structural element being considered. The radius of gyration $i$ is calculated for the concrete cross-section (gross-section) only and the plane being considered. The actual length of columns, $l_{col}$, should be determined according to the rules for the effective span of members (see clause 5.2.3.2).

Slenderness bounds may be defined, especially those below which second order effects may be neglected for the purpose of stability verifications.

Slenderness bounds below which second order effects may generally be neglected should be related to the reduction in bearing capacity, with respect to the ultimate limit state for bending and longitudinal force according to first order theory, of not more than 10%.

In complex structures, where a reliable definition of the slenderness ratio is difficult to obtain, the sensitivity of the structure to second order effects may be expressed in terms of stiffness.

Normally the effective length $l_0$ is assessed by referring to simplified rules or experience.

For members with variable cross-section, a representative equivalent cross-section may be used in certain cases.

Bearing in mind the uncertainties related to the actual conditions at the end connections of elements, and that all relevant parameters cannot be taken into account, due prudence should be exercised in the determination of the value of $l_0$, especially for unbraced sway structures.

Attention is drawn to the fact that $l_0$ depends on the stiffness in the ultimate limit state of the structural elements, which are connected to the element being considered.

The effective length $l_0$ can in certain cases directly be used for the ultimate limit state design of the structure or structural elements considered as isolated elements.

## 6.6.2. Requirements

### 6.6.2.1. General

It has to be verified that

- under the most unfavourable combination of actions
- giving the materials their design strengths and the associated deformation behaviour
- taking into account geometrical imperfections,

the action effects are smaller than or equal to the action effects in the ultimate limit state of buckling.

For further details see clause 1.6.3.2.

### 6.6.2.2. Differences allowed between rigorous and simplified methods

Simplified or approximate methods should not lead to differences in the design bearing capacity, of the structure or the structural element considered, exceeding 10% in the unsafe direction of that obtained with a rigorous second order analysis.

### 6.6.2.3. Calculation of deformations

Deformations have to be determined using stress–strain diagrams for concrete which are characterized by at least three parameters which are mutually independent

- the strength $f_{cd}$
- the strain belonging to the apex $\varepsilon_{c1}$
- the inclination at the origin, which is the tangent modulus of elasticity $E_{ci}$ (refer to clause 2.1.4.2).

The design values $f_{cd}$ and $E_{cd}$ may be determined by dividing the characteristic values $f_{ck}$ and $E_{ci}$ by a safety coefficient $\gamma_c = 1.2$.

Stress–strain diagrams for reinforcing or prestressing steel, as used for cross-section design (ref. clause 2.2.4.3 or 2.3.4.3) should be applied.

The design strength for evaluating the ultimate resistance of critical sections in the 'local' verification is obtained according to subsection 1.4.1 by dividing the characteristic value of the concrete strength by a safety coefficient $\gamma_c = 1.5$, whereas the corresponding values for the overall deformation behaviour are obtained by using a reduced safety coefficient of $\gamma_c = 1.2$, see clause 1.6.3.4.

Tension stiffening effects may be taken into account.

Tension stiffening effects are only significant for small reinforcement ratios and where the ultimate limit state of buckling is reached before yielding of the reinforcement occurs.

Different models are available for taking tension stiffening effects approximately into account. Reduced steel strains $\varepsilon_s$ (see subsection 3.2.3), or, alternatively, a constant mean concrete tensile stress $\sigma_{c,t}$ in the effective tension zone around the bars in tension, or variable mean concrete tensile stresses $\sigma_{c,t}(\varepsilon_c)$ in the tension zone of the cross-section may be used.

Creep effects are considered to be significant if the increases of the moments due to second order effects caused by creep and longitudinal action effects exceed 10% of the first order bending moments.

The moment increase instead of the bearing capacity reduction has been chosen because the first criterion allows the analytical determination of corresponding slenderness bounds beyond which creep effects become significant (see eq. (6.6–25)).

Creep effects which increase the deformations in the ultimate limit state and which are likely to reduce the structural stability significantly (refer to clause 6.6.2.1) have to be taken into account. They should be analysed for load combinations according to clauses 1.4.2.2 and 1.6.3.3 defining quasi-

The creep coefficient should be considered as a fundamental basic variable (see section 1.3), where the effects of quasi-permanent combinations are dominant of the total action effects. However, in most practical cases of building structures, the favourable effects of some other parameters are permanent combinations of loads and applying an appropriate creep coefficient $\phi$ according to clause 2.1.6.4.3b.

Shrinkage effects can generally be neglected, unless the basic shrinkage of the concrete exceeds the normal range.

neglected and, unless better information on the physical value of the creep coefficient $\phi$ is given, the notation value $\phi_0$ may be used instead of a more unfavourable fractile.

### 6.6.2.4. Minimum reinforcement ratio

A minimum reinforcement ratio $\omega = (A_s f_{yd})/(A_c f_{cd})$ has to be provided.

The minimumm amount of reinforcement min$A_s$ may be determined from the design axial force $N_{Sd}$; min$A_s = 0.15 N_{Sd}/f_{yd}$.

## 6.6.3. Design criteria

### 6.6.3.1. Classification of structures and structural elements

#### 6.6.3.1.1. Isolated elements

Second order effects may be neglected, if the slenderness ratio $\lambda$ of the column satisfies the criterion

$$\lambda \leqslant \lambda_l \qquad (6.6\text{–}1)$$

where $\lambda_l$ is an appropriate slenderness bound, which takes the decrease of load bearing capacity due to second order effects into account.

First order effects of imperfections need always to be considered for columns or other members and structures for which these imperfections are explicitly defined. Ignoring imperfection effects would result in even greater discontinuity between the column bearing capacities for first order and second order theory.

Slenderness bounds are always considerably affected by the reinforcement ratio. The values given below are lower bounds and are valid for minimum reinforcement. Alternatively, other methods showing the limited influence on the strength reduction may be used.

In the absence of a more rigorous analysis the slenderness bound $\lambda_l$ for sway elements may be taken as

$$\lambda_l = 7.5/\sqrt{\nu_{Sd}} \quad \text{if} \quad \nu_{Sd} \leqslant 0.39 \qquad (6.6\text{–}2)$$

$$\lambda_l = 12 \quad \text{if} \quad \nu_{Sd} > 0.39 \qquad (6.6\text{–}3)$$

and for non-sway elements

$$\lambda_l = 7.5(2 - e_{01}/e_{02})/\sqrt{\nu_{Sd}} \quad \text{if} \quad \nu_{Sd} \leqslant 0.39 \qquad (6.6\text{–}4)$$

$$\lambda_l = 12(2 - e_{01}/e_{02}) \quad \text{if} \quad \nu_{Sd} > 0.39 \qquad (6.6\text{–}5)$$

where

$e_{01}$ denotes the smaller value of the first order eccentricity of the axial action effect at one end of the element considered

$e_{02}$ denotes the greater value

$\nu_{Sd}$ denotes the relative design axial force in the bracing elements, $\nu_{Sd} = N_{Sd}/(A_c f_{cd})$.

The introduction of negative values for the ratio $e_{01}/e_{02}$ into eq. (6.6–4) or (6.6–5) is allowed only if the column is designed at least for the combination of the axial force $N_{Sd}$ and a minimum design moment

$$M_{Sd} = N_{Sd}h/20 \qquad (6.6\text{–}6)$$

and the restraints at the column ends are able to resist this moment, so that a double curvature deflection develops.

For the definition of braced structures see clause 6.6.1.2.

Braced structures comprise two groups of elements with different stiffnesses against horizontal actions.

The arrangement of bracing elements should be such that they can also resist torsional actions.

This clause, which refers to the frequent combination, is intended to limit cracking, not to avoid it totally.

If a more precise model is not available, the following formula can be applied for the calcualtion of $\lambda_b$ in the case of regular braced frames with approximately equal height and approximately equal vertical loads in each storey

$$\lambda_b = \frac{h_{tot}\sqrt{(F_v/E_{cm}I_c)}}{0.01\beta\sqrt{v_{Sd}}} \qquad (6.6\text{–}8)$$

#### 6.6.3.1.2. Braced structure

Braced structures may be analysed by considering first the braced sub-assembly neglecting any horizontal load and assuming horizontal restraints at each storey level. Its individual columns may be designed as non-sway isolated elements. Thereafter the bracing elements will be designed as cantilever columns submitted to all horizontal loads acting on the structure and to the reactions of the sub-assembly.

A building structure can be considered as braced if its bracing elements are reasonably symmetrically distributed within the building and where it can be shown, by means of a first order linear analysis with member stiffnesses corresponding to uncracked cross-sections, that they are able to attract, at the foundations level, a shear force at least equal to 90% of the sum of the horizontal forces acting on the building. In addition, the bracing elements have to remain uncracked in service conditions under the frequent combination of all horizontal loads and the corresponding vertical loads. The latter criterion may be considered to be met if the concrete tensile stress $\sigma_{ct}$ does not exceed $f_{ctm}$ defined in clause 2.1.3.3.1.

Depending on their slenderness ratios the bracing elements may be considered as non-sway or sway and designed accordingly as short columns or slender columns, i.e. according to the first or second order theory. The corresponding criterion for non-sway is

$$\lambda_b \leqslant \lambda_l \qquad (6.6\text{–}7)$$

where

$\lambda_b$ is the equivalent slenderness ratio of the bracing elements taking into account the second order moments due to the loads applied to the braced sub-assembly

$\lambda_l$ is the slenderness bound for isolated elements according to clause 6.6.3.1.1.

If this criterion is fulfilled, the bracing elements may be designed according to a first order analysis taking into account the effects of imperfections.

with

$$\beta = 1.8 - 1.44/(n + 0.8) \qquad (6.6\text{–}9)$$

where

$n$ denotes the number of storeys
$h_{tot}$ denotes the total height of the structure measured from the top surface of the foundation or from a non-deformable sub-stratum
$E_{cm}I_c$ denotes the sum of the nominal flexural stiffnesses of all the vertical bracing elements acting in the direction under consideration
$F_v$ denotes the sum of all vertical loads in service conditions (rare combinations), acting on the bracing elements and on the braced sub-assembly, with $\gamma_F = 1.0$ (the effect of $\gamma_F > 1.0$ is covered by the factor 0.01 in eq. (6.6–8)).
$\upsilon_{Sd}$ denotes the relative design axial force in the bracing elements, $\upsilon_{Sd} = N_{Sd}/(A_c f_{cd})$.

If the stiffness of the bracing elements is variable along the height, an equivalent stiffness should be used. As an approximation, the equivalent stiffness may be derived from the assumption that it leads to the same horizontal deflection under a unit load as the actual stiffnesses of the structure.

The limitation of the equivalent slenderness for bracing elements is related to the 10% decrease in bearing capacity under which structures can be considered non-sway (see clause 6.6.3.1.3).

Individual non-sway compression members should be considered as isolated elements and be designed accordingly.

In general, for sway structures a second order analysis will be necessary.

Simplified methods, introducing, for example, instead of the effects of geometrical imperfections increased horizontal design loads or nominal bending moments which take account of second order effects, may be used.

For economic reasons, the second order effects should always be limited. In this case, a rough assessment of their effects may be adequate. A simplified procedure is described here below.

The deflected shape of the frame is assumed to be straight with an angle of $\alpha''$ from the vertical so that the second order effects may be expressed by

As the actual load effects in the braced sub-assembly are likely to be different from the design values, it is recommended that the members of the sub-assembly have adequate ductility.

**6.6.3.1.3. Non-sway framed structures**

Non-sway structures may be analysed according to the first order theory. This analysis can be done with nominal or with reduced member stiffnesses.

A building frame can be considered as non-sway if the displacements of the connections would only result in a 10% increase of the relevant first order bending moments. This analysis should be made taking into account either the non-linear behaviour of the material or, as a simplification, adequately reduced member stiffnesses.

the action of additional horizontal forces $\Delta H_{Sd} = \alpha'' V_{Sd}$. Assuming uniformly distributed horizontal and vertical loads $H_{Sd}$ and $V_{Sd}$, the deformation $\Delta a'$ in the plane of the frame due to these additional forces $\Delta H_{sd}$ can be determined by multiplying the first order deflection $a'(H_{Sd})$ due to forces $H_{Sd}$ by the ratio $\alpha''\Sigma(V_{Sd}x)/\Sigma(H_{Sd}x)$. Finally, the total slope $\alpha''$ can then be calculated from the first order top deflection of the frame $a' = a'(H_{Sd}) + a'(V_{Sd})$ and taking also the deviation $\alpha_a$ from the vertical according to eq. (6.6–13) into account.

In the following $x$ denotes the vertical co-ordinate of the application point of the relevant horizontal load $H_{Sd}$ and the vertical load $V_{Sd}$ (see Fig. 6.6.1).

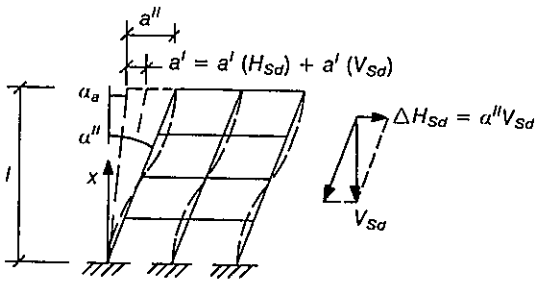



*Fig. 6.6.1. Simplified calculation of the maximum displacement a'' due to second order effects*

$$\alpha'' = \alpha_a + a'/l + \Delta a'(\Delta H_{Sd})/l \qquad (6.6\text{–}10)$$

$$= \alpha_a + a'/l + \frac{\Sigma(\Delta H_{Sd} x)}{\Sigma(H_{Sd} x)} \frac{a'(H_{Sd})}{l}$$

$$= \alpha_a + a'/l + \frac{\alpha''\Sigma(V_{Sd} x)}{\Sigma(H_{Sd} x)} \frac{a'(H_{Sd})}{l}$$

$$\alpha'' = \frac{\alpha_a + a'/l}{1 - (\Sigma V_{Sd} x/\Sigma H_{Sd} x)(a'(H_{Sd})/l)} \qquad (6.6\text{–}11)$$

The first order deflections $a'$ for design actions $H_{Sd}$ and $V_{Sd}$ should be calculated by considering the stiffness reduction due to cracking.

In many cases, a rough estimate is adequate. In the case where the resulting increase of the horizontal loads is not greater than 25%, the deflection may be assumed to be twice the deflection obtained by a first order linear frame analysis with the stiffness $E_{cm} I_c$. This is equivalent to the assumption of a 50% reduction in stiffness. Otherwise, it has to be shown that the assumed stiffness reduction, which may be different for the different members, corresponds reasonably well with the resulting state of stress. The whole structure should be designed for increased horizontal loads $H_{Sd,ef}$

$$H_{Sd,ef} = H_{Sd} + \alpha'' V_{Sd} = (1 + \alpha'' V_{Sd}/H_{Sd}) H_{Sd} \qquad (6.6\text{–}12)$$

The magnification factor $(1 + \alpha'' V_{Sd}/H_{Sd})$ for the horizontal loads constitutes a good indication of the sensitivity of the structure to sway. If the factor is large, it may be advisable to modify the design.

The simple procedure is a special form of the $P/\Delta$-method.

The design value of geometrical imperfections should not include effects of other structural imperfections, such as residual stresses by shrinkage and creep, inhomogeneities or locally poor concrete. These structural imperfections should be covered by adequate design values for the stress–strain diagram of concrete and/or by using appropriately chosen safety coefficients $\gamma_c$. However, the design value should cover the secondary effects of actions (e.g. thermal gradients) which are not considered separately; such effects may be important for high rise buildings and very slender structures.

This rule is mainly intended for common framed building structures. $m$ is the total number of elements, whether they are bracing or not, that support vertical loads and are horizontally tied together. In the case shown in Fig. 6.6.2 $m = 2$ (one bracing element in A and one non-bracing element in B).

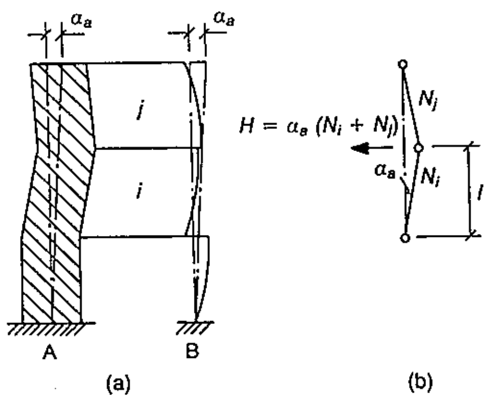



*Fig. 6.6.2. Representation of geometrical imperfections by a deviation $\alpha_a$ from the vertical*

## 6.6.3.2. Imperfections

Geometrical imperfections should be taken into account.

The effects of geometrical imperfections may be taken into account in different ways depending on the type of element considered.

(*a*) For multi-storey structures, a deviation $\alpha_a$ of the structure from the vertical may be assumed as shown, for example, in Fig. 6.6.2(a) for a braced frame

$$\alpha_a = 1/(100\sqrt{l}) \leqslant 1/200 \qquad (6.6\text{–}13)$$

where $l$ denotes the member length (m).

In the case where $m$ vertical multistorey continuous elements are horizontally tied together, $\alpha_a$ as defined by eq. (6.6–13) may be reduced by a factor $\alpha_m$ obtained from eq. (6.6–14)

$$\alpha_m = \sqrt{[0.5(1 + 1/m)]} \qquad (6.6\text{–}14)$$

Figure 6.6.2(b) gives guidance on how to calculate the local horizontal forces $H$ in the floors transferring the stabilizing forces from the braced members to the bracing elements. These local forces $H$, determined with the local length of the braced members, need not be summed for verifying the bracing element. The bracing element has to be verified for horizontal forces corresponding to the inclination as derived from the length of the bracing element.

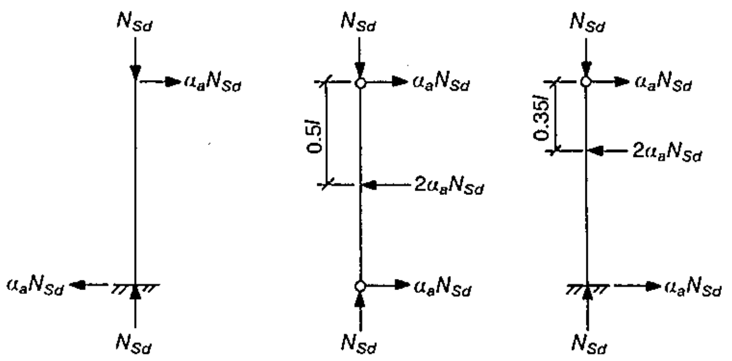



*Fig. 6.6.3. Transverse forces for considering effects of geometrical imperfections*

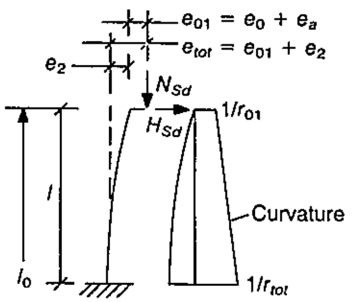



*Fig. 6.6.4. Model column*

(*b*) For isolated elements, the effects of geometrical imperfections may be taken into account by increasing the eccentricity of the design normal forces by an additional eccentricity $e_a$ acting in the most unfavourable direction and defined by eq. (6.6–15):

$$e_a = \alpha_a l/2 \qquad (6.6\text{–}15)$$

Alternatively the effects of geometrical imperfections can be taken into account by additional forces equal to $\alpha_a$ times the normal forces, $N_{Sd}$, and acting transversely to them in the most unfavourable direction. In consequence, pin-ended columns may be loaded according to Fig. 6.6.3 by a transverse load at mid-span which is $2\alpha_a$ times the normal force.

For columns in sway frames, a displacement from the vertical may be considered in order to account for geometrical imperfections or, alternatively, modified horizontal loads at the ends equivalent to this displacement may be used.

## 6.6.3.3. Simplified design methods for isolated elements

### 6.6.3.3.1. Second order deformation

A design method may be used which adopts a simplified shape for the deformed axis of the member. The second order deflection $e_2$ is then calculated as a function of the member length $l$, the eccentricities $e_{01}$ and $e_{02}$ of the axial force at the ends of the member and the curvature $1/r_{tot}$ in the critical section with the total eccentricity $e_{tot}$ according to eq. (6.6–16):

$$e_{tot} = e_0 + e_a + e_2 \qquad (6.6\text{–}16)$$

where

$e_0$ denotes the first order eccentricity $e_0 = M_{Sd,1}/N_{Sd}$

$M_{Sd,1}$ denotes the maximum design bending moment
$N_{Sd}$ denotes the applied design axial force

The 'model column' as shown in Fig. 6.6.4 is a cantilever column with constant cross-section, fixed at the base and free at the top. It is being bent in single curvature under loads and moments which give the maximum moment at the base.

In the case of constant reinforcement the maximum deflection may be assumed to be

$$e_2 = 0.1 K_1 l^2 (4/r_{tot} + 1/r_{01}) \qquad (6.6\text{–}17)$$

$$= 0.1 K_1 l^2 (4 + e_{01}/e_{tot}) 1/r_{tot} \qquad (6.6\text{–}18)$$

and when the reinforcement is curtailed in accordance with the bending moment diagram, it may be assumed to be

$$e_2 = 0.5 K_1 l^2 / r_{tot} \qquad (6.6\text{–}19)$$

where

$1/r_{tot}$ denotes the curvature associated with the eccentricity $e_{tot}$

$1/r_{01}$ denotes the curvature associated with the eccentricity $e_{01}$ and which may be assumed to be $(e_{01}/e_{tot})1/r_{tot}$

$K_1$ denotes a coefficient which is introduced in order to avoid discontinuity of the function describing the design bearing capacity when the slenderness bound $\lambda_1$ is exceeded; it is obtained from eq. (6.6–20)

$$K_1 = 2(\lambda/\lambda_1 - 1) \quad \text{for} \quad \lambda_1 \leqslant \lambda \leqslant 1.5\lambda_1 \qquad (6.6\text{–}20)$$

$$K_1 = 1 \quad \text{for} \quad \lambda > 1.5\lambda_1 \qquad (6.6\text{–}21)$$

The curvature $1/r_{tot}$ is derived from the equilibrium of the internal and external forces.

For pin-ended columns with constant cross-section and reinforcement and subjected to first order moments varying linearly along their length and where $|e_{02}| > |e_{01}|$, an equivalent eccentricity $e_e$ may be taken as

$$e_e = 0.6 e_{02} + 0.4 e_{01} \qquad (6.6\text{–}22)$$

The stability verification may then be done as for a 'model column' but having only half the length $l$ of the real column. The cross-section design with $e_{02}$ is also necessary.

A fictitious curvature $1/r_{tot}$ in eqs. (6.6–17, –18 and –19) may be derived for rectangular cross-sections with symmetrically arranged reinforcement in a top and bottom layer from

$e_a$ denotes the additional eccentricity according to eq. (6.6–15)

$e_2$ denotes the eccentricity due to second order deflection.

In cases where great accuracy is not required, the verification of stability may be done by checking the critical section for the ultimate limit state of bending and axial force under the axial load $N_{Sd}$ and a fictitious design value of the second order deflection $e_2$.

$$1/r_{tot} = 2K_2\varepsilon_{yd}/z_s \qquad (6.6\text{–}23)$$

where

$\varepsilon_{yd} = f_{yd}/E_s$ is the design yield strain of steel reinforcement

$z_s$ is the distance between compression and tension reinforcement, approximately $z_s = 0.9d$

$K_2$ is a coefficient, taking into account the decrease of the curvature with increasing axial force as defined by eq. (6.6–24)

$$K_2 = (N_{ud} - N_{Sd})/(N_{ud} - N_{bal}) \leqslant 1 \qquad (6.6\text{–}24)$$

where

$N_{ud}$ is the design ultimate capacity of the section subjected to axial load only, it may be taken as $0.85f_{cd}A_c + f_{yd}A_s$

$N_{Sd}$ is the actual design axial force

$N_{bal}$ is the design axial load which, when applied to the concrete section, maximizes its ultimate moment capacity; for symmetrically reinforced rectangular sections it may be taken as $0.4f_{cd}A_c$.

It will always be conservative to assume $K_2 = 1$.

For columns with cross-sections other than rectangular or with distributed reinforcement equivalent values may be used for $z_s$.

One of the two following simplified procedures may be used to treat the creep effects approximately.

The element is directly calculated for the ultimate limit state applying the most unfavourable combination of factored actions and including the effects of geometrical imperfections, by using a modified stress–strain relationship, which is obtained by magnifying all strains with the factor $K_3 = 1 + \alpha\beta$. When using eq. (6.6–23) for determining the fictitious curvature $1/r_{tot}$, the curvature may be multiplied by $0.5K_3$ to take creep effects into account.

The creep effects may also be introduced as an additional eccentricity $e_c$ according to eq. (6.6–28)

$$e_c = (e_0 + e_a)\left[\exp\left(\frac{\phi}{N_E/N_{Sg} - 1}\right) - 1\right] \qquad (6.6\text{–}28)$$

where

$N_{Sg}$ denotes the axial force in the element, under the quasi-permanent combination of actions

#### 6.6.3.3.2. Creep effects

Creep effects may be neglected when at least two of the following conditions are fulfilled simultaneously

$$\lambda \leqslant 53/(\sqrt{(\nu_c)}f_{ck}^{1/3}) \qquad (f_{ck} \text{ in (MPa)}) \qquad (6.6\text{–}25)$$

$$e_0 \geqslant 2h \qquad (6.6\text{–}26)$$

$$\alpha\beta \leqslant 0.2 \qquad (6.6\text{–}27)$$

where

$\nu_c = N_{Sg}/(f_{ck}A_c)$ and where $N_{Sg}$ denotes the axial force under quasi-permanent actions

$e_0$ denotes the first order eccentricity of $N_{Sg}$

$h$ denotes the height of the cross-section

$\alpha$ denotes the ratio of the design bending moment $N_{Sg}$ under the quasi-permanent actions and the total factored design normal force $N_{Sd}$ considered for the ultimate limit state

$\beta$ denotes the ratio of the quasi-permanent design bending moment $M_{Sg}$ standard and the total factored design bending moment $M_{Sd}$ considered for the ultimate limit state.

$N_E = E_{cm} I_c (\pi / l_0)^2$ denotes the critical Euler-load of the element, $I_c$ is the moment of inertia of the uncracked concrete section; if cracking is likely to occur under permanent actions an appropriate reduction of $I_c$ should be used

$e_c$ is the remaining additional eccentricity due to creep effects when the column is fictitiously considered as unloaded.

Out-of-plane buckling means buckling transverse to the loaded plane in the case of mono-axial eccentricity.

Geometrical imperfections according to eq. (6.6–15) have to be taken into account by additional eccentricities $e_{ay}$ and $e_{az}$ for both principal planes and by applying them simultaneously.

$$e_{y1} = e_{0y} + e_{ay}$$

$$e_{z1} = e_{0z} + e_{az}$$

Both additional eccentricites are equal, e.g. $e_{ay} = e_{az}$, if the actual lengths between restraints in both principal planes are equal, e.g. $l_y = l_z$.

**6.6.3.3.3. Biaxial eccentricities and out-of-plane buckling**

For members with rectangular cross-sections, separate verifications in the two principal planes $y$ and $z$ are permissible, if the point of application of $N_{Sd}$ is located close to one principal axis, e.g. within the hatched zones in Fig. 6.6.5. The ratios of the corresponding eccentricities $e_{y1}/b$ and $e_{z1}/h$ have to satisfy one of the following conditions: either

$$(e_{z1}/h)/(e_{y1}/b) \leqslant 1/4 \quad (6.6\text{–}29)$$

or

$$(e_{y1}/b)/(e_{z1}/h) \leqslant 1/4 \quad (6.6\text{–}30)$$

The eccentricities $e_{y1}$ and $e_{z1}$ are those in directions of the section dimensions $b$ and $h$ respectively and including imperfections $e_a$ as defined in eq. (6.6–15). A rigorous analysis is required if the above conditions are not met.

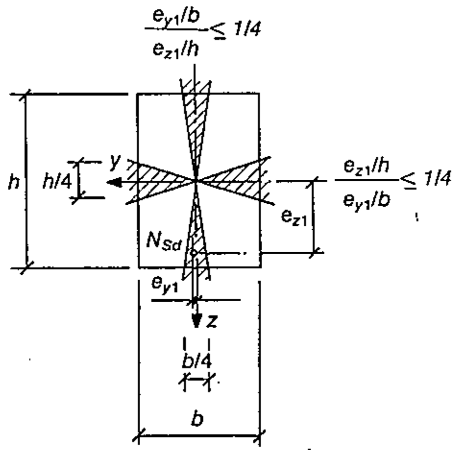



*Fig. 6.6.5. Condition for separate verification in the two principal planes*

For a separate verification with the reduced width $h$ all the reinforcement may be taken into account, also the reinforcement outside the compression zone.

Where $e_{1z}/h > 0.2$, separate verification is permissible only if the check for bending in the $y$-direction is based on the reduced width $h'$, as shown in Fig. 6.6.6. The value $h'$ may be determined as the height of the compression zone and on the assumption of linear stress distribution and uncracked concrete.

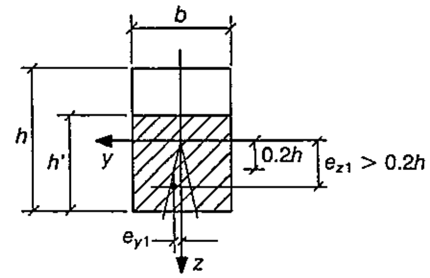



*Fig. 6.6.6. Reduced effective width* h′ *where* $e_{z1} > 0.2h$

The basic idea of this simplified method is to show the existence of equilibrium between actions and member resistances for the deformed system. For this purpose, the structural system, loading and cross-sections as well as the deformation conditions have to be known. The system, loading and cross-section are usually known or determined by the first order design for bending. The actual deformation conditions, which imply the lateral deflection and the rotation related to the shear centre $M$ as indicated in Fig. 6.6.7, are not known. Therefore, the deformation curve is commonly approximated by an assumed function and the magnitude of the deflection is fixed using an ultimate limit state assumption for the admissible rotation $\theta_{adm}$ of the cross-section.

The following developments of the commentary are intended for reinforced concrete beams, as well as for prestressed concrete beams. The symbols $y$ and $z$ denote the horizontal and vertical directions, respectively.

For the limitation of $\theta_{adm}$ and the final verification two conditions have to be considered.

(*a*) *Biaxial bending*

$$M_{z1d} = M_{yd}(\theta + \theta_a)$$

$$\theta_{B,adm} = M_{z1,d}/M_{yd} - \theta_a \qquad (6.6\text{–}32)$$

### 6.6.3.3.4. Lateral buckling of beams

For the verification of adequate safety against lateral buckling of slender beams with slender chords, a second order analysis may be used, which adopts a simplified shape for the lateral deflection and a simplified expression for the rotation of the mid-section caused by torsion. The mid-section of the beam has to be designed for biaxial bending according to the mid-section rotation. The end restraints (in general acting as a fork) shall resist the resulting second order torsion effects caused by the lateral deflection.

Favourable influences such as bracing by horizontal trusses or torsional restraints from purlins may be considered.

The effects of geometrical imperfections may be taken into account by a lateral pre-deflection $e_a$ equal to the eccentricity according to eq. (6.6–15) together with an additional rotation of the mid-section of

$$\theta_a = 1/200 \qquad (6.6\text{–}31)$$

The effects of creep may be taken into account approximately by doubling both values of geometrical imperfections.

The torsional stiffness $GI_T$ may be considered as constant along the beam and corresponding to the depth of the compression zone under first order bending moment $M_{yd}$ with its maximum value neglecting the effects of geometrical imperfections. The shear modulus may be taken as $G = 0.4E_{cd}$.

(*b*) *Onset of cracking due to torsion near the support*

Assuming that no longitudinal stresses result from bending, the principal tensile stresses ae identical to the torsional stresses. The admissible rotation can then be determined from the torsional cracking moment $T_r$. For a sinusoidal shape of the acting second order torsional moments this admissible rotation is defined by

$$\theta_{T,adm} = \frac{T_r l}{\pi G I_T} \quad (6.6\text{–}33)$$

where

$$T_r = 0.25 f_{ck}^{2/3} W_t \quad (6.6\text{–}34)$$

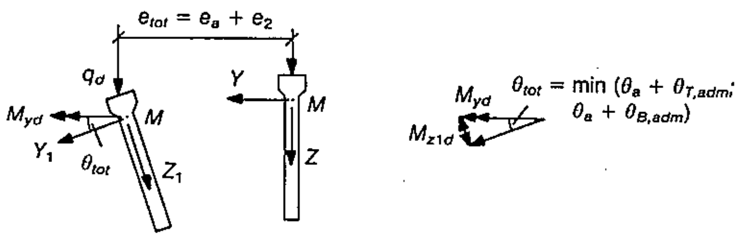



*Fig. 6.6.7. Deformed position of the cross-section in the middle of the beam*

In most cases $\theta_{T,adm}$ is smaller than $\theta_{B,adm}$ and therefore decisive for the design.

The admissible lateral deflection $e_{2,adm}$ has to be limited, so that the second order torsional moment $T_d$ does not exceed $T_r$. For constant vertical loading $q_d$ along the beam, $q_d l = Q_d$, it can approximately be assumed

$$T_d = T_r = Q_d(e_{2,adm} + e_a)/\pi \quad (6.6\text{–}35)$$

$$e_{2,adm} = \pi T_r / Q_d - e_a \quad (6.6\text{–}36)$$

The beam has to be designed such, that the lateral deflection

$$e_2 = \frac{Q_d \theta_{tot} l^3}{E J_z} 0.0112 \quad (6.6\text{–}37)$$

does not exceed the admissible lateral deflection $e_{2,adm}$. The lateral bending stiffness $EJ_z$ can be determined using only the compressive zone due to the first order bending moment $M_{yd}$ and the reinforcement.

The given fatigue strength for concrete is valid for concrete tested under sealed conditions (see subsection 2.1.7). The fatigue strength for steel is given as well for normal environment as for marine environment.

*Notation*

$D$ is fatigue damage
$n_{Si}$ is the number of acting stress cycles associated with the stress range for steel, and the actual stress levels for concrete
$N_{Ri}$ or $N$ is the number of resisting stress cycles
$n$ is the foreseen number of cycles in the desired design lifetime
$\Delta\sigma_{Ss}$ is the steel stress range under the acting loads
$\Delta\sigma_{Rsk}(n)$ is the stress range relevant to $n$ cycles obtained from a characteristic fatigue strength function
$S_{cd,max}$ is the maximum compressive stress levels
$S_{cd,min}$ is the minimum compressive stress levels
$\Delta S_{cd}$ is the stress range
$S_{td,max}$ is the maximum tensile stress level
$\sigma_{c,max}$ is the maximum compressive stress
$\sigma_{c,min}$ is the minimum compressive stress
$\sigma_{ct,max}$ is the maximum tensile stress
$\eta_s$ is the factor which increases the stress in the reinforcing steel due to differences in bond behaviour of prestressing and reinforcing steel
$\eta_c$ is the averaging factor of concrete stresses in the compression zone considering the stress gradient
$f_{cd,fat}$ is the design fatigue reference strength for concrete under compression
$f_{ctd,fat}$ is the design fatigue reference strength for concrete under tension
$\theta_{fat}$ is the angle between the web compression and the chords valid for verification of the reinforcement.

## 6.7. ULTIMATE LIMIT STATE OF FATIGUE

### 6.7.1. Scope

The following design rules apply for the entire lifetime of concrete. The rules for reinforcing and prestressing steel apply for more than $10^4$ repetitions; low-cycle fatigue is not covered.

The verification of the design principle (see clause 1.6.4.2) can be performed according to the three methods given in subsections 6.7.3, 6.7.4 and 6.7.5, with an increasing refinement. The models for the analysis of stresses in reinforced and prestressed concrete members under fatigue loading are treated in subsection 6.7.2 as well as concrete stress gradients.

Subsection 6.7.6 deals with shear design and in 6.7.7 a method for calculating the increased deflections under fatigue loading is given. The relevant combination of loads is treated in clause 1.6.4.5.

## 6.7.2. Analysis of stresses in reinforced and prestressed members under fatigue loading

Linear elastic models may generally be used, and reinforced concrete in tension is considered to be cracked. The ratio of moduli of elasticity for steel and concrete may be taken as $\alpha = 10$.

In the case of prestressed members it should be verified if the relevant section is sensitive to cracking. This holds true if any combination of loads (see clause 1.6.6.5) causes tensile stresses in the surface fibre and then the stress ranges for reinforcing steel and prestressing steel should be calculated as though the member is in the cracked state.

The effect of differences in bond behaviour of prestressing and reinforcing steel has to be taken into account for the stresses in the reinforcing steel. Unless a more refined method is used, this can be done using a linear elastic model for stress calculation which fulfils the compatibility in strains and multiplying the stress in the reinforcing steel by the following factor:

$$\eta_s = \frac{1 + (A_p/A_s)}{1 + (A_p/A_s)\sqrt{[\zeta(\phi_s/\phi_p)]}} \geqslant 1 \qquad (6.7\text{–}1)$$

where

$A_s$ is the area of reinforcing steel
$A_p$ is the area of prestressing steel
$\phi_s$ is the smallest diameter of reinforcing steel in the relevant section
$\phi_p$ is the diameter of prestressing steel (for bundles an equivalent diameter has to be chosen $1.6\sqrt{A_p}$ where $A_p$ is the cross-section area of the bundle)
$\zeta$ is the ratio of bond strength of prestressing steel and high-bond reinforcing steel.

Post-tensioned members

$\zeta = 0.2$ for smooth prestressing steel
$\zeta = 0.4$ for strands
$\zeta = 0.6$ for ribbed prestressing wires
$\zeta = 1.0$ for ribbed prestressing bars

Pretensioned members

$\zeta = 0.6$ for strands
$\zeta = 0.8$ for ribbed prestressing steels

The stress gradient for concrete in the compression zone of a cracked section may be taken into account by multiplying the maximum stress in the compression zone by a factor $\eta_c$

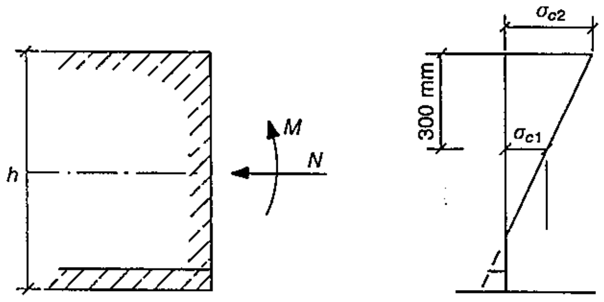



*Fig. 6.7.1. Definition of stresses* $\sigma_{c1}$, $\sigma_{c2}$

$$\eta_c = \frac{1}{1.5 - 0.5|\sigma_{c1}|/|\sigma_{c2}|} \qquad (6.7\text{–}2)$$

where

$|\sigma_{c1}|$ is the lower absolute value of the compressive stress within a distance no more than 300 mm from the surface under the relevant load combination

$|\sigma_{c2}|$ is the larger absolute value of the compressive stress within a distance no more than 300 mm from the surface under the same load combination as for $|\sigma_{c1}|$.

## 6.7.3. Verification by the simplified procedure

This procedure is only applicable to structures subjected to a limited number ($\leqslant 10^8$) of low stress cycles.

*Steel*

The fatigue requirements will be met, if the maximum calculated stress range under the frequent combination of loads, max $\Delta\sigma_{Ss}$, satisfies

$$\gamma_{Sd} \max \Delta\sigma_{Ss} \leqslant \Delta\sigma_{Rsk}/\gamma_{s,fat} \qquad (6.7\text{–}3)$$

where $\Delta\sigma_{Rsk}$ is the characteristic fatigue strength at $10^8$ cycles. Values for $\Delta\sigma_{Rsk}$ are given in Tables 6.7.1 and 6.7.2.

Values for $\gamma_{s,fat}$ and $\gamma_{c,fat}$ are given in clause 1.6.4.4.

*Concrete*

Detailed fatigue design need not be carried out if the maximum calculated stress under the frequent combination of loads, $\sigma_{c,max}$ (compression), $\sigma_{ct,max}$ (tension), respectively, satisfy the following

Compression:

$$\gamma_{Sd}\sigma_{c,max}\eta_c \leqslant 0.45 f_{cd,fat} \qquad (6.7\text{–}4)$$

where $\eta_c$ is the averaging factor considering the stress gradient eq. (6.7–2).

Tension

$$\gamma_{Sd}\sigma_{ct,max} \leqslant 0.33 f_{ctd,fat} \qquad (6.7\text{–}5)$$

The fatigue reference strength is defined as follows (see also clause 2.1.7.1).

Compression

$$f_{cd,fat} = 0.85\beta_{cc}(t)\left[f_{ck}\left(1 - \frac{f_{ck}}{25 f_{cko}}\right)\right]\Big/\gamma_c$$

where

$\beta_{cc}(t)$ is the coefficient which depends on the age of concrete $t$ in days when fatigue loading starts (see clause 2.1.6.1)

$f_{cko}$ = 10 MPa (reference strength).

Tension

$$f_{ctd,fat} = f_{ctk0.05}/\gamma_{c,fat}$$

For value of $\gamma_{c,fat}$ see clause 1.6.4.4.
For $\sigma_{c,\max}$, $\sigma_{ct,\max}$ see clause 1.6.4.2.

When the unique value $Q$ can be chosen satisfactorily, (e.g. as fatigue equivalent) this method is a more precise assessment than the simplified procedure.

When it is considered necessary to carry out fatigue tests to determine the performance of reinforcing steel, the tests should be made according to RILEM-FIP-CEB Recommendations, 1973. These data should normally be expressed as 5% fractiles and 75% confidence levels.

The characteristic fatigue strength function for steel consists of segments (see Fig. 6.7.2) of the form $\Delta\sigma^{m}_{Rsk}N = $ const. Values for the S–N curves are given in Tables 6.7.1 and 6.7.2.

The values given in Tables 6.7.1 and 6.7.2 are characteristic and do not incorporate partial safety factors. These values or higher values shall be validated by appropriate approval documents.
The code does not cover coiled and restraightened bars.

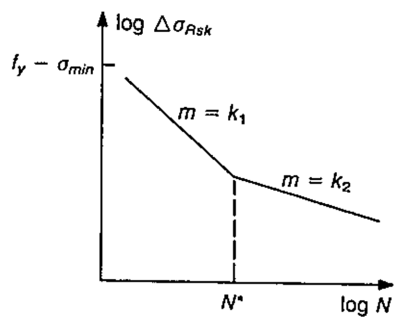



*Fig. 6.7.2. Shape of the characteristic fatigue strength curves (S–N curves) for steel*

## 6.7.4. Verification by means of single load level

This method takes account of the required lifetime with a foreseen number, $n$, of cycles. This number intervenes in the verification with the maximum fatigue effects of the action, $Q$, as defined in clauses 1.6.4.2.(c), 6.7.2 and below.

*Steel*

The fatigue requirement will be met if the calculated maximum acting stress range, max $\Delta\sigma_{Ss}$, satisfies

$$\gamma_{Sd} \max \Delta\sigma_{Ss} \leqslant \Delta\sigma_{Rsk}(n)/\gamma_{s,fat} \qquad (6.8\text{-}6)$$

where

$n$ is the foreseen number of cycles in the required design lifetime,
$\Delta\sigma_{Rsk}(n)$ is the stress range relevant to $n$ cycles obtained from a characteristic fatigue strength function.

*Table 6.7.1. Parameters of S–N curves for reinforcing steel (embedded in concrete)*

| | $N^*$ | Stress exponent | | $\Delta\sigma_{Rsk}$ (MPa)[c] | |
|---|---|---|---|---|---|
| | | $k_1$ | $k_2$ | At $N^*$ cycles | At $10^8$ cycles |
| Straight and bent bars $D \geqslant 25\phi$ | | | | | |
| $\phi \leqslant 16$ mm | $10^6$ | 5 | 9 | 210 | 125 |
| $\phi > 16$ mm[a] | $10^6$ | 5 | 9 | 160 | 95 |
| Bent bars $D < 25\phi$[b] | $10^6$ | 5 | 9 | –[c] | –[c] |
| Welded bars[b] including tack welding and butt joints mechanical connectors | $10^7$ | 3 | 5 | 50 | 30 |
| Marine environment[b],[d] | $10^7$ | 3 | 5 | 65 | 40 |

[a]The values given in this line represent the S–N curve of a 40 mm bar; for diameters between 16 and 40 mm interpolation between the values of this line and those of the line above is permitted.
[b]Most of these S–N curves intersect the curve of the corresponding straight bar. In such cases the fatigue strength of the straight bar is valid for cycle numbers less than that of the intersection point.
[c]Values are those of the according straight bar multiplied by a reduction factor $\xi$ depending on the ratio of the diameter of mandrel $D$ and bar diameter $\phi$: $\xi = 0.35 + 0.026D/\phi$.
[d]Valid for all ratios $D/\phi$ and all diameters $\phi$.
[e]In cases where $\Delta\sigma_{Rsk}$-values calculated from the S–N curve exceed the stress range $f_{yd} - \sigma_{min}$, the value $f_y - \sigma_{min}$ is valid.

In case appropriate information is provided by specific approval documents for the steel to be used, higher fatigue strength values may be used accordingly.

*Table 6.7.2. Parameters of S–N curves for prestressing steel (embedded in concrete)*

| Prestressing steel | $N^*$ | Stress exponent | | $\Delta\sigma_{Rsk}$ (MPa) | |
|---|---|---|---|---|---|
| | | $k_1$ | $k_2$ | At $N^*$ cycles | At $10^8$ cycles |
| *Pretensioning* | | | | | |
| Straight steels | $10^6$ | 5 | 9 | 160 | 95 |
| *Post-tensioning* | | | | | |
| Curved tendons[a] | $10^6$ | 3 | 7 | 120 | 65 |
| Straight tendons | $10^6$ | 5 | 9 | 160 | 95 |
| Mechanical connectors | $10^6$ | 3 | 5 | 80 | 30 |

[a]In cases where the S–N curve intersects that of the straight bar, the fatigue strength of the straight bar is valid.

The values given in Table 6.7.2 are on the safe side compared to the strength values for the basic material given in section 2.3.

The reduction of the $\Delta\sigma_{Rsk}$ values of curved tendons compared with the values of straight tendons is due to fretting corrosion which results from the lateral pressure and slip between prestressing strands and/or ribs of the steel sheaths.

*Concrete*

The fatigue requirements under cyclic loading will be met if the required lifetime (number of cycles) is less than or equal to the number of cycles to failure:

$$n \leqslant N$$

$N$ should be calculated from the fatigue strength functions given below.

Characteristic S–N curves for concrete can be used without any restriction for frequencies higher than 0.1 Hz. For lower frequencies, fatigue life should be reduced, see chapter 3 in CEB Bulletin 188 for guidance.

In the case of compression–tension the criteria for compression as well as the criteria for tension shall be fulfilled.

For $S_{cd,\min} \geqslant 0.8$, the S–N relations for $S_{cd,\min} = 0.8$ are valid (see also clause 2.1.7.1).

The value $\log N_3$ is to be calculated only if $\log N_1 > 6$.

Compression

For $0 < S_{cd,\min} < 0.8$

$$\log N_1 = (12 + 16 S_{cd,\min} + 8 S_{cd,\min}^2)(1 - S_{cd,\max}) \qquad (6.7\text{–}7a)$$

$$\log N_2 = 0.2 \log N_1 (\log N_1 - 1) \qquad (6.7\text{–}7b)$$

$$\log N_3 = \log N_2 (0.3 - \tfrac{3}{8} S_{Sd,\min}) / \Delta S_{cd} \qquad (6.7\text{–}7c)$$

(*a*) If $\log N_1 \leqslant 6$, then $\log N = \log N_1$
(*b*) If $\log N_1 > 6$ and $\Delta S_{cd} \geqslant 0.3 - \tfrac{3}{8} S_{cd,\min}$, then $\log N = \log N_2$
(*c*) If $\log N_1 > 6$ and $\Delta S_{cd} < 0.3 - \tfrac{3}{8} S_{cd,\min}$, then $\log N = \log N_3$

where

$$S_{cd,\max} = \gamma_{Sd} \sigma_{c,\max} \eta_c / f_{cd,fat}$$

$$S_{cd,\min} = \gamma_{Sd} \sigma_{c,\min} \eta_c / f_{cd,fat}$$

$$\Delta S_{cd} = S_{cd,\max} - S_{cd,\min}$$

For $\eta_c$ see eq. (6.7–2).
For $\gamma_{Sd}$ see clause 1.6.4.4.
For the assessment of $\sigma_{c,\max}$, $\sigma_{c,\min}$ and $\sigma_{ct,\max}$ see clauses 1.6.4.2 and 6.7.2 using the fatigue equivalent or frequent value of the variable action $Q$.
$\sigma_{c,\max}$ and $\sigma_{ct,\max}$ are to be calculated under the upper load effect.
$\sigma_{c,\min}$ is determined as the maximum stress in the compression zone at a distance no more than 300 mm from the surface where $\sigma_{c,\max}$ occurs, but under the lower load effect.

Tension

$$\log N = 12(1 - S_{td,\max}) \qquad (6.7\text{–}8)$$

where

$$S_{td,\max} = \gamma_{Sd} \sigma_{ct,\max} / f_{ctd,fat}$$

## 6.7.5. Verification by means of spectrum of load levels

This method takes account of the required lifetime, the load spectrum (which is divided into $j$ blocks) and the characteristic fatigue strength functions.

Fatigue damage $D$ is calculated using the Palmgren-Miner summation

$$D = \sum_{i=1}^{j} \frac{n_{Si}}{N_{Ri}}$$

where

$n_{Si}$ denotes the number of acting stress cycles associated with the stress range for steel and the actual stress levels for concrete
$N_{Ri}$ denotes the number of resisting stress cycles.

The partial coefficients are applied in this procedure as follows.
For steel, values $N_{Ri}$ are calculated from the S–N curves given in Tables 6.7.1 and 6.7.2 using an increased stress range $\gamma_{Sd} \gamma_{s,fat} \Delta\sigma_{Ssi}$.
For concrete, values $N_{Ri}$ are calculated directly from the fatigue strength functions given in subsection 6.7.4.

The fatigue requirement will be satisfied if $D \leqslant D_{\lim}$.
Using an appropriate counting method (e.g. rainflow method) a value of $D_{\lim} = 1$ can normally be used.

## 6.7.6. Shear design

*Members without shear reinforcement*

The fatigue requirements will be met, if under cyclic loading the required life (number of cycles $n$) is less than or equal to the numbers of cycles to failure

$$n \leqslant N$$

$N$ should be calculated from the fatigue strength functions given below.

$$\log N = 10(1 - V_{max}/V_{ref}) \qquad (6.7\text{–}9)$$

where

$V_{max}$ is the maximum shear force under the relevant representative values of permanent loads including prestress and maximum cyclic loading

$V_{ref} = V_{Rd1}$ (see clause 6.4.2.3).

*Members with shear reinforcement*

The stress in the shear reinforcement should be calculated according to chapter 6 assuming the following inclination of the compression struts under fatigue loading:

$$\tan \theta_{fat} = \sqrt{(\tan \theta)}$$

For assessment of the $\theta$ value see subsection 6.3.3.

The resistance of compressive struts can be verified using eq. (6.7–4) or eqs. (6.7–7a), (6.7–7b) and (6.7–7c) reducing the fatigue reference strength given in subsection 6.7.3 by a factor of 0.7. The compression of web concrete subjected to fatigue loading should be calculated using the angle $\theta$ (see subsection 6.3.3).

The fatigue reference strength is to be reduced in the same way as the compressive strength of concrete subjected to simultaneously acting compressive and transverse tensile forces. The factor 0.7 applied to the fatigue reference strength corresponds to the reduction of $f_{cd1}$ value (average design stress for cracked compressive zones). $f_{cd2}/f_{cd1} = 0.6/0.85 = 0.7$ (see clause 6.2.2.2).

## 6.7.7. Increased deflections under fatigue loading

Under cyclic loading progressive deflection can occur in reinforced concrete members in addition to the deflection produced by creep. The cyclic effect can be calculated from

$$a_n = a_1[1.5 - 0.5\exp(-0.03n^{0.25})] \qquad (6.7\text{–}10)$$

where

$a_1$ is the deflection in the first cycle due to the maximum load including effects of shear strains

$n$ is the number of cycles.

## 6.8. DEEP BEAMS AND DISCONTINUITY REGIONS

### 6.8.1. Scope and basic criteria

This section applies to members or parts of members where the assumption of linear strain distribution is not valid.

The verification shall be based on physical models according to the requirements given in section 6.1.

This applies to deep beams, regions of other members with concentrated loads or reactions (statical discontinuities) and regions with geometrical discontinuities, including the connections of different members.

According to section 5.6 the analysis may be carried out by applying

- linear analysis
- analysis by admissible stress fields (truss analogy)
- non-linear analysis.

For truss analogy the model of resistance is an arrangement of compression fields: struts, ties and nodes. The compatibility of deformations should be considered by orientating the models at the force systems given by linear elastic analysis of uncracked members and connections.

The orientation at the linear elastic stress system is more important for the compression struts than for the ties which usually can be arranged parallel to the edges of the member following practical considerations of reinforcement layout. In highly stressed node regions (e.g. near supports or concentrated loads) the main struts and ties of the model should normally meet at angles of about 60° and not less than 45°.

Energy criteria may be used for the selection of the model.

Alternatively, this selection may be based on previous experimental or analytical data available in the literature; however, in such a case, a sensitivity check might be needed for several trial schemes.

It is to be verified that under the action of the design loads the stresses in the compression fields and ties do not exceed the basic strength criteria given in section 6.2, and that the nodes and anchorages comply with section 6.9.

#### 6.8.1.1. Ties

The steel ties have to be dimensioned so that the resistance according to subsection 6.2.4 is not exceeded. The arrangement of the reinforcing bars shall be chosen to avoid disproportionate cracking at the SLS (see section 7.4). The detailing of the bars should follow the rules of chapter 9.

Normally the tensile resistance of concrete should not be relied upon in any major tie (see subsection 6.2.3).

If the arrangement of the reinforcement is made in accordance with a linear elastic stress field a SLS verification can normally be avoided.

Tension in re-entrant corners leads to relatively wide cracks. Additional diagonal bars are recommended there, even if a corresponding tie is not provided in the model or analysis.

#### 6.8.1.2. Concrete compression fields or struts

The concrete dimensions shall be such that the strength criteria given in clause 6.2.2.2 are not exceeded.

In case of linear elastic and non-linear analyses the stress field is directly determined. Further guidance for applying the strength criteria and safety factors is given in chapter 5. Additional to the analysis the anchorages shall be verified.

The stresses in struts of truss models need normally not be verified, if their singular nodes are checked and if reinforcement transverse to the strut axis is provided (see subsection 6.9.1).

The total transverse force between a concentrated node and the bulge of a compression stress field can be assumed not to exceed 25% of the total strut force unless shown otherwise. For struts with concentrated nodes at both ends the transverse reinforcement between the nodes may be designed for 30 to 40% of the strut force, considering stress redistribution after cracking.

For continuous deep beams, special consideration should be given to differential settlements of the supports.

A further departure from such an orientation may be considered, accounting for redistribution of forces due to the cracked state; such more pragmatic solutions are more economical, provided the SLS is more strictly considered.

Transverse tension due to the deviation of the bulging compression stress trajectories within a stress field or strut requires transverse reinforcement unless the concrete tensile strength is sufficient and reliable enough for carrying the tension (see section 3.3).

### 6.8.1.3. Nodes and anchorages

It shall be secured that the forces of the struts and ties are balanced in the node region (see subsection 6.1.5) and that the strength criteria for nodes and anchorages, given in section 6.9, are not exceeded.

## 6.8.2. Examples of application of admissible stress fields

### 6.8.2.1. Plate elements

Based on a linear-elastic solution of stress pattern, dimensioning may be carried out by means of a truss model, composed of struts and ties as follows.

(*a*) For the plate-element under consideration, an appropriate number of typical sections is selected approximately perpendicular to the principal stress-trajectories, both compressive and tensile.

(*b*) Across each of these sections an appropriate number of struts and ties are positioned as follows.

- (i) Struts are roughly oriented in alignment with compressive stress-trajectories, and are placed in the respective centres of gravity of these stresses across each selected cross-section.
- (ii) Ties are oriented and placed in a similar way; however, their orientation may be influenced by the practicality of the layout of reinforcement, whereas their position may be dictated by the importance of the expected cracks; thus, ties parallel or perpendicular to the edges of the plate-element are generally preferred.

(*c*) Axial forces acting along each bar of the truss are subsequently calculated. Verification follows as in clauses 6.8.1.1 and 6.8.1.2.

Single-span deep beams as shown in Fig. 6.8.1 may be designed for chord forces derived from an internal lever arm $z = 0.6–0.7l$, but not more than the lever arm obtained from a standard analysis of linear members according to section 6.3.

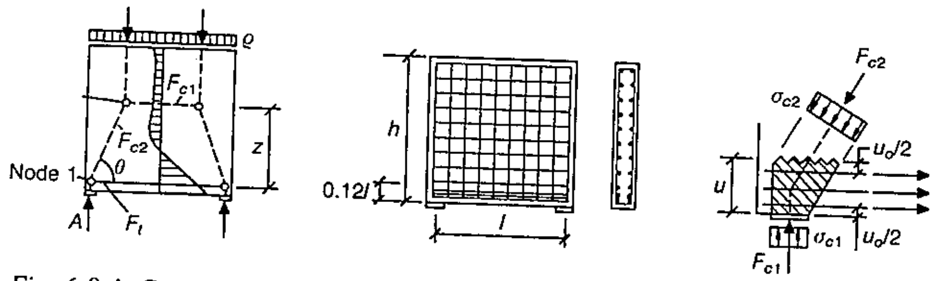



*Fig. 6.8.1. Strut-and-tie model, reinforcement and idealized support node for a deep beam*

For the deep beam shown in Fig. 6.8.1 the following checks are necessary.

(*a*) Tension chord $F_t$. Reinforcement according to subsection 6.2.4 for $F_t \approx 0.4A$.

(*b*) Support node 1

(i) Bearing pressure of support according to clause 6.9.2.3 and subsection 6.9.1. Alternatively the pressure $\sigma_{c2}$ from the strut $F_{c2}$ must be checked, if the height $u$ of the node and the strut angle $\theta$ is relatively low ($u < a_1 \cot \theta$).

(ii) Anchorage length according to clause 6.9.2.3 and subsection 6.9.5.

(*c*) Strut $F_{c2}$. Transverse reinforcement between node 1 and 2 for deviation forces $F_{t2} \approx 0.25F_{c2}$ of the bulging stress field according to clause 6.8.1.2. The nominal mesh reinforcement on both sides may have to be increased in the strut region.

Simplified design procedures are also allowed if based on sufficient evidence and experience.

Special attention shall be paid to dimensioning the anchorage lengths and the nodes at the supports according to section 6.9.

## 6.8.2.2. Discontinuity regions

### 6.8.2.2.1 General

This section applies to regions of frames near concentrated forces or regions with geometrical discontinuities (including the connections of different building elements). The design provisions of clause 6.8.2.1 are equally valid in discontinuity regions.

However, simpler techniques may be applied. A direct application of a truss-model may be used, without previous knowledge of an elastic solution of stress-pattern.

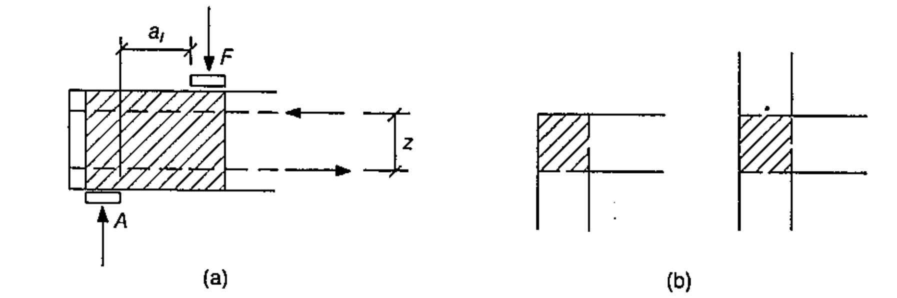



*Fig. 6.8.2. Discontinuity regions: (a) region near concentrated load if* $a_l < z \cot \theta$ *(where θ as in clause 6.3.3.1.); (b) geometrical discontinuity*

The technique of 'standard discontinuity regions' may profitably be used to this end.

A broader area is considered, including the 'discontinuity region' ($D_2$) and nearby 'transition regions' ($D_1$) where the provisions of subsections 6.3.2 and 6.3.3 are applicable. Input forces coming from $D_1$-regions (determined as in subsections 6.3.2 and 6.3.3) are acting on the truss model of a $D_2$-region.

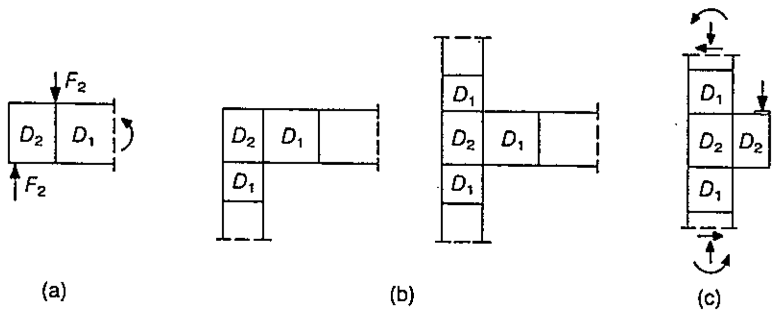



*Fig. 6.8.3. Examples for the application of standard discontinuity regions* $D_1$ *and* $D_2$*: (a) in a beam; (b) in frames; (c) in a corbel*

Standard solutions of truss models in $D_2$-regions may be derived and subsequently used in several cases; an example is given in Fig. 6.8.4.

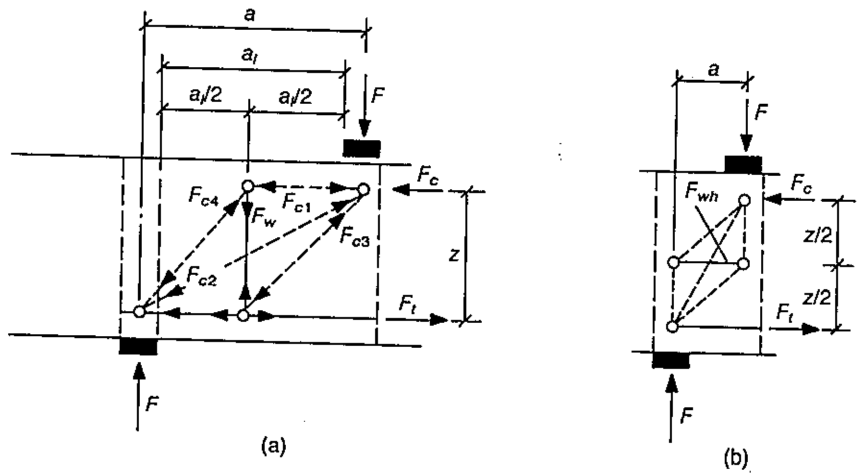



*Fig. 6.8.4. Modelling of a standard discontinuity region:* (*a*) a > ½z, a < z cot θ; (*b*) a < ½z

Appropriate stirrups, accommodated close to $F_w$ will secure the transfer of this force. Conservative approximate expressions for $F_w$ as a function of the ratio $a/z$ and of the resultant axial and shear forces may be used, e.g.

$$F_w = \frac{2a/z - 1}{3 - N_{Sd/F}} F \begin{Bmatrix} > 0 \\ < F \end{Bmatrix}$$

$$F_{wh} = \frac{2z/a - 1}{3 + F/F_c} F_c \begin{Bmatrix} > 0 \\ < F_c \end{Bmatrix}$$

where axial tension $N_{Sd}$ (not shown in Fig. 6.8.3) is defined positive.

Reference is made to subsection 6.9.2.

Appropriate verifications should be carried out within discontinuity regions, regarding

- web reinforcement
- diagonal web forces (if web-width is smaller than the width of the nodes where concentrated forces are acting)
- strut forces acting on nodes
- anchorage of reinforcement in the node regions.

*High beam/narrow column ($h_2/h_1$ > 1.5)*

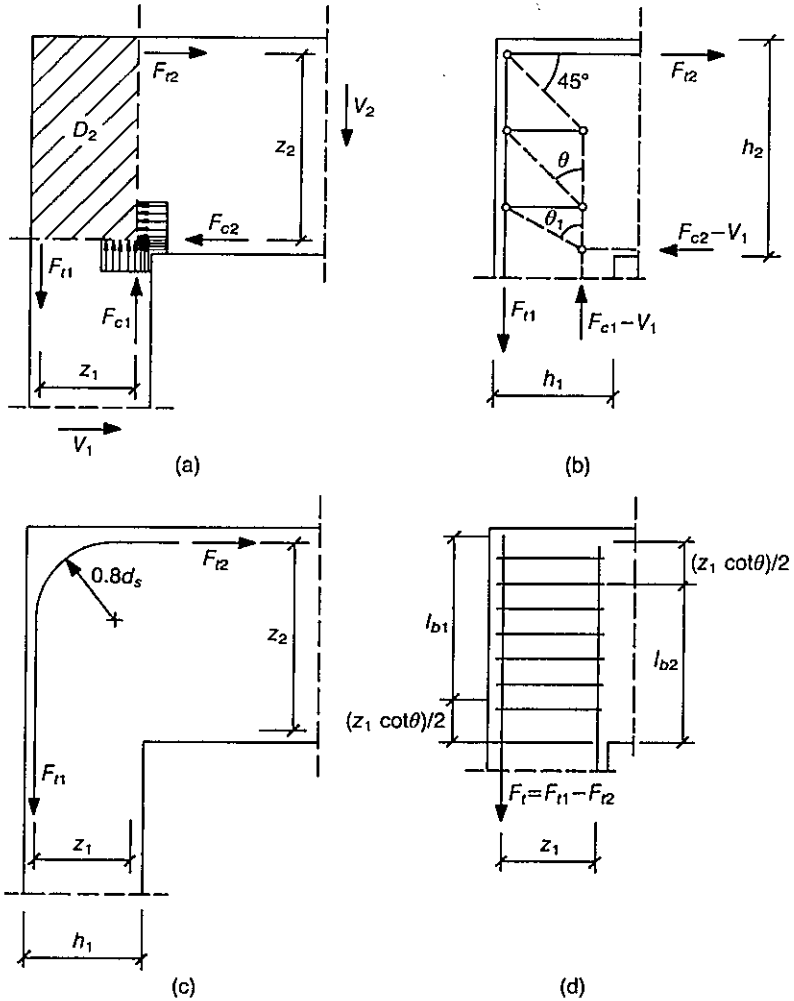



*Fig. 6.8.5. Frame corner with closing moments: (a) input forces; (b) truss model; (c) bent around bars; (d) anchorage lengths of column reinforcement, $l_{b1}$ (tensile) and $l_{b2}$ (compressive)*

**6.8.2.2.2. Frame corners with closing moments**

It should be secured that all input forces acting on the beam-column joint area will be safely transferred through the body of the joint.

Assuming a strut angle $\theta = 45°$, a safe value of the total horizontal force $F_w$ of the stirrup reinforcement is given by $F_w = F_{t1} - F_{t2}$. Appropriate anchorage lengths should be secured: $l_{b1}$ for a tensile force $F_{11} - F_{12}$ (outer reinforcement), $l_{b1}$ for the compressive force of inner column reinforcement.

*Approximately equal depths of column and beam*
If

$$\frac{2}{3} < \frac{h_2}{h_1} < \frac{3}{2}$$

no check of stirrup reinforcement or anchorage lengths within the beam-column joint is needed, provided that all the longitudinal reinforcement of the beam and column is bent around the corner.

The standard solutions given in Fig. 6.8.3 may be applied for the cantilevering part of the structure and its support region.

### 6.8.2.2.3. Corbels

Corbels shall be designed using truss models. If $a > z$ the corbel may be designed as a linear member (section 6.3).

Normally, in addition to the main chord reinforcement, closed horizontal stirrups shall be provided if $a < z/2$ and closed vertical stirrups if $a > z/2$ or a combination of both in all cases $a < z$. Instead, inclined stirrups can be used where suitable (e.g. Fig. 6.8.7).

The internal lever arm $z$ of the models, the main tensile chord force $F_t$ and the compression node 1 (see clause 6.9.2.2) can be checked by application of standard methods for axial action effects (subsection 6.3.2). The critical design section for these verifications (corresponding to the boundary line of the $D_2$-region shown in Fig. 6.8.3) is defined by $x_1$ in Figs 6.8.6 and 6.8.7.

Node 2 in Fig. 6.8.6 is of the type shown in Fig. 6.9.3(b) and can be checked accordingly if the diagonal compression struts $F_{c2}$ and $F_{c3}$ are replaced by their resultant. This resultant force is inclined at the angle $\theta_r$ derived from

$$\cot\theta_r = \frac{a}{z}\left(1 - \frac{F_w}{2F_v}\right)$$

For the anchorage of main reinforcement in node 2, it is recommended to use relatively small diameter bars in form of horizontal U-loops in several layers (see clause 6.9.2.3). Instead of loops, the bars may end with anchor plates or with a welded transverse bar of equal diameter near the outer surface. For bars bent in the vertical plane the anchorage length begins below the inner edge of the loading plate.

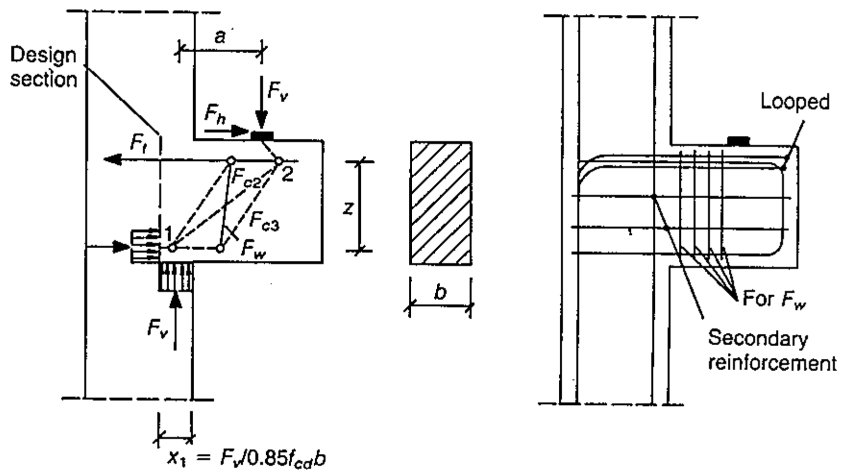



*Fig. 6.8.6. Truss model and reinforcement for a corbel (compare Fig. 6.8.3)*

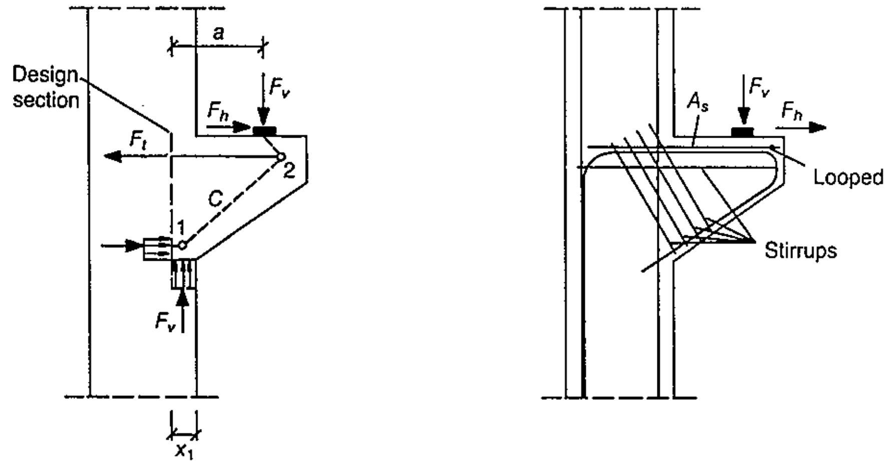



*Fig. 6.8.7. Simple strut-and-tie model and reinforcement for a corbel*

# 6.9. VERIFICATION OF NODES AND ANCHORAGES

## 6.9.1. General

A node is defined as a volume of concrete contained within the intersections between compression fields of struts, in combination with anchorage forces and/or external compressive forces (imposed loads or support reactions).

The nodes should be dimensioned so that all forces are anchored and balanced safely.

Examples of typical nodes are given in subsection 6.9.2.

The geometry of the node region and the arrangement of reinforcement in it should be consistent with the model on which the design of the structure is based and with the applied forces. Thereby the equilibrium conditions should be fulfilled.

Nodes should be verified accordingly

- verification of the stresses from the compressive struts in the node according to subsection 6.9.2
- verification of the anchorages of ties according to subsection 6.9.3 and following.

Normally, the geometry of acting forces and the directions of struts allow for the determination of the dimensions of a node.

In areas of large supports, the distribution of strains should possibly be known or approximately estimated in order to determine the dimensions of nodes (especially in areas of non-rotative supports).

The compressive stresses of the adjoining struts of the node should be within the design limits specified in section 6.2.

Normally the compressive stresses of nodes need to be checked only where concentrated forces are applied to the surface of the structural element, e.g. below bearing plates, anchor plates and over supports. A verification of node pressures within the structure may become necessary at geometrical discontinuities (holes, corners). Often such node pressures near discontinuities can be checked well enough by the application of standard methods for linear members (subsection 6.3.2) on a section through the whole member; in such a case the node pressures replace the compressive stresses due to the respective bending moment. The design strength for the compression zone should in these cases comply with subsection 6.9.2.

If the length of the bar in the node region is less than the anchorage length required in subsection 6.9.5, the anchored bar may be extended beyond the node region and thus introduce part of its force into the node by compression from behind (see Fig. 6.9.3(b)). In this case tension bars and compression stresses behind the node work like a bowden cable with no perceptible resultant force in the model.

These transverse tensile forces are orthogonal to the plane in which Figs 6.9.2–6.9.5 are drawn.

The anchorage of bars shall comply with subsection 6.9.5. The anchorage length will be assumed to begin at the section where the transverse compressive stress trajectories of a strut meet the anchored bar and are deviated. The anchored bar shall extend at least over the whole length of the compression field which is deviated by it.

Minimum radii of bent bars according to clause 9.1.1.2 shall be observed in order to limit the concrete stresses in the supported strut.

Transverse tensile forces from bond action and minor non-uniformities of applied strut stresses should normally be covered by structural reinforcement (e.g. stirrups) which is arranged near the surfaces.

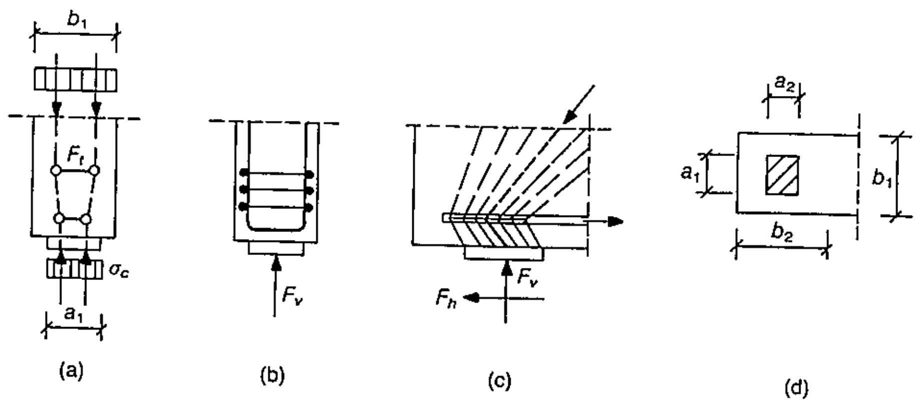



*Fig. 6.9.1. Support area: (a) structural model; (b) reinforcement layout; (c) horizontal force at support; (d) homologous areas*

## 6.9.2. Standard cases of nodes

### 6.9.2.1. Bearing stresses and other node stresses

Average design stresses in any surface or section through a singular node shall normally not exceed the following values of concrete strength

$f_{cd1}$ for nodes where only compression struts meet
$f_{cd2}$ for nodes where main tensile bars are anchored.

$f_{cd1}$ and $f_{cd2}$ are given in clause 6.2.2.2 for essentially uniaxial compression but can be applied also to nodes with multiaxial states of stress.

$f_{cd1}$ may also be applied in other nodes if the angle between ties and major struts is not less than 55° and if the reinforcement layout in the node region is designed with special care (e.g. arranged in several layers, with transverse ties).

For concentrated loads as shown in Fig. 6.9.1 the design bearing pressure can be increased by a factor

$$\beta = \min \begin{cases} b_1/a_1 \\ b_2/a_2 \end{cases} > 4$$

For nodes with secured triaxial compression, e.g. due to local compression or due to lateral confinement by reinforcement, the increased strength values given in section 3.3 for local compression or in clause 2.1.3.4 for multiaxial states of stress may be applied to individual node surfaces (e.g. at bearings), provided that all tensile forces in the node regions and adjacent struts are carried by reinforcement.

where

$a_1$ and $a_2$ are the dimensions of the loaded area (Fig. 6.9.1)
$b_1$ and $b_2$ (homologous to the loaded area) are determined from limitations to the dispersion of the stresses (Fig. 6.9.1).

Transverse tension in the case (Fig. 6.9.1(a)) may be estimated from the formula

$$F_t = \frac{1}{4}\frac{b_1 - a_1}{b_1} F_v \qquad (6.9\text{–}1)$$

If an additional horizontal force $F_h$ is acting at a support (Fig. 6.9.1(c)), the acting stress may be estimated using

$$\sigma_c = \frac{F_v}{a_1 a_2} \sqrt{[1 + (F_h/F_v)^2]} \qquad (6.9\text{–}2)$$

Ducts crossing the node region shall be considered like ducts in compression struts (see clause 6.2.2.4).

### 6.9.2.2. Nodes with only compressive forces

Such nodes occur, for example, under concentrated loads or over inner supports of continuous beams or at supports where prestressing tendons are anchored or in compressed re-entrant corners.

The node region may be assumed to be limited by a polygon not necessarily at right angles to the strut direction. The stresses along the individual node surface can normally be assumed evenly distributed.

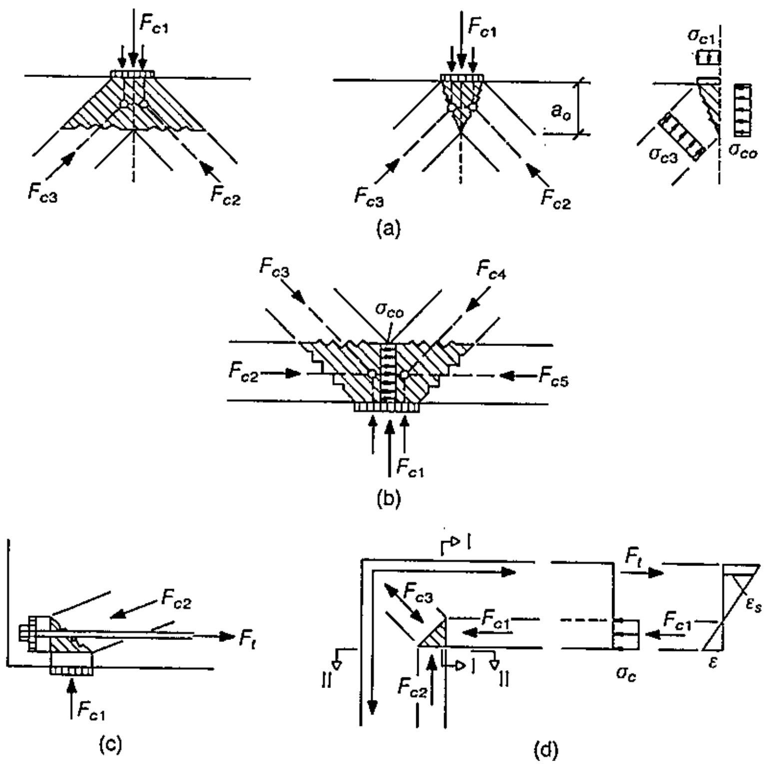



*Fig. 6.9.2. Nodes with only compression forces: (a) node under concentrated load; two alternative layouts of the node region with identical pressures; (b) node over inner support of continuous beam; (c) node over end support of prestressed beam; (d) node in compressed re-entrant corner with simplified check of node pressure in section I–I assuming linear strain distribution*

In the cases of Fig. 6.9.2(a) and (b) it is normally sufficient to check only the bearing pressure with respect to $f_{cd1}$. However, if the height $a_o$ of the nodes is restricted by a crack or by the height of the on-coming struts $F_{c2}$ and $F_{c5}$ as is the case for the compression chords of beams, the pressure $\sigma_{co}$ in the section orthogonal to the bearing should also be verified, e.g. by standard methods for beams (subsection 6.3.2). Accordingly in the cases of Fig. 6.9.2(c) and (d) the pressures in both orthogonal faces of the node should be checked.

### 6.9.2.3. Nodes with anchorage of parallel bars only (compression-tension node)

Such nodes occur where in the strut-and-tie model one tie meets two or more compression struts, e.g. at end-supports and below concentrated loads which are applied to corbels or near the corner of deep beams. They further occur frequently in regions where one-dimensional members are connected to other members and apply concentrated chord forces on them, e.g. in frame corners with opening moments and in beam-column-connections.

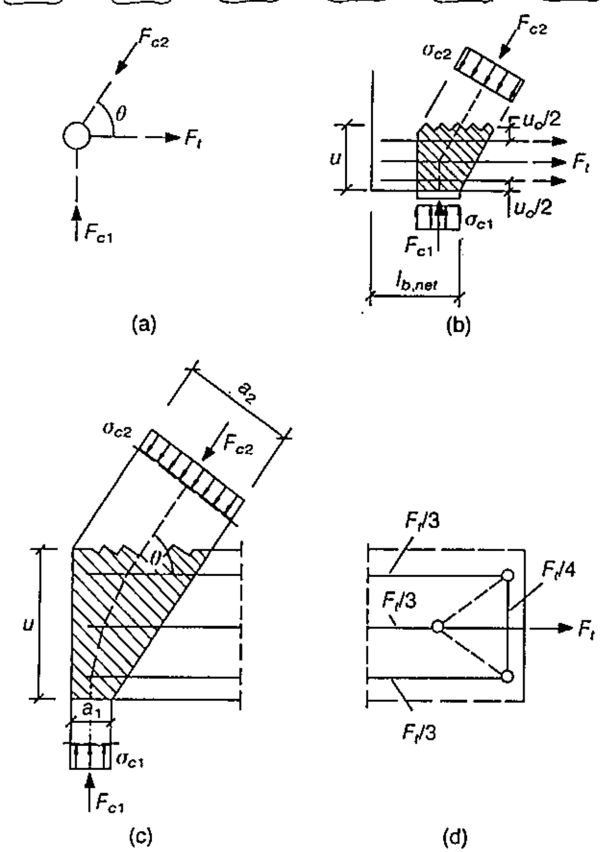



*Fig. 6.9.3. Node with anchorage of parallel bars only: (a) scheme; (b) node with reinforcement extended beyond the bearing; (c) node with bars ending within the node region; (d) distribution of a concentrated tie* $F_t$ *over several layers of reinforcement (stirrups, loops)*

Detailing of the reinforcement is essential in such nodes (Fig. 6.9.3). The reinforcement shall preferably be distributed over the height $u$ in several layers and anchored by loops or hooks which are bent in the plane orthogonal to the compression $F_{c1}$.

Normally the tie force $F_t$ cannot be anchored within the short length of the node if the reinforcement capacity is fully used and if the bars cannot be extended beyond the node, as e.g. in member connections. In such cases it is without further check of anchorage lengths sufficient to transfer 2/3 of the tie force to parallel bars (stirrups or loops) which are arranged in several layers (Fig. 6.9.3(c) and (d)) in order to extend the node region over a greater height $u$ and to reduce the individual anchor forces of the bars. The transverse tie force due to the lap requires additional transverse reinforcement for approximately $F_t/4$, unless this force is compensated by compression.

The transverse pressure reduces the design anchorage length according to subsection 6.9.5.

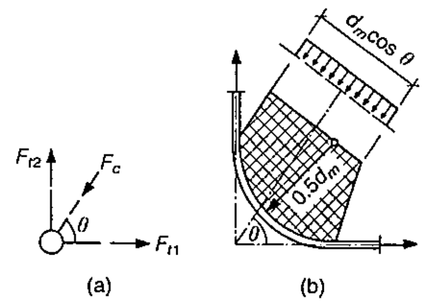



*Fig. 6.9.4. Node with bent bars*



### 6.9.2.4. Nodes with bent bars

Such nodes occur where a strut force is balanced mainly by the deviation forces of bent bars and also by bond forces if the node is unsymmetric with respect to the strut.

The radius of bend shall comply with clause 9.1.1.2.

Bars bent around a rectangular corner tend to anchor the diagonal strut partly ahead of the bend by bond similar to the node described in clause 6.9.2.5. Therefore transverse ties (stirrups, loops) should be arranged immediately before and after the bend.

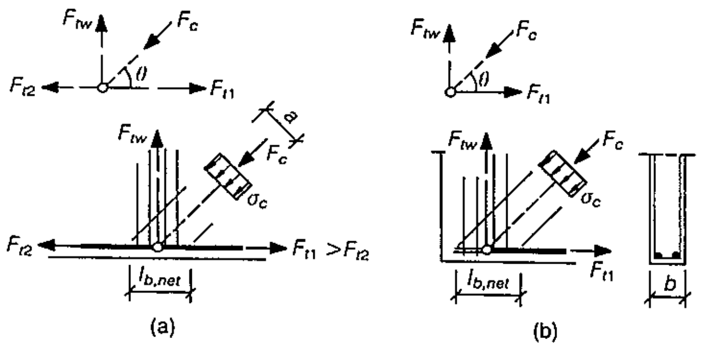



*Fig. 6.9.5. Nodes with ties in orthogonal directions: (a) node in a chord at the edge of a member; (b) node in the corner of a member*

### 6.9.2.5. Nodes with ties in orthogonal directions

Such nodes typically occur distributed over considerable lengths of bars in the edges and corners of members, if bond forces are applied to them, e.g. in the tension chords of beams or deep beams and in the chords of standard discontinuity regions (clause 6.8.2.2).

The diagonal compression stresses in the node need normally not be verified if the anchorage length or bond stresses comply with subsections 6.9.2–6.9.5. However, if a multi-layered reinforcement chord is anchored, the diagonal concrete stresses may become critical. They have to comply with the appropriate design strengths.

The reinforcement $F_{tw}$ orthogonal to the edge of the member can be anchored only by hooks or bends (stirrups, loops) which enclose the longitudinal bars (chord reinforcement). Therefore small diameter bars at small spacings should be chosen for $F_{tw}$. The detailing rules for shear reinforcement in beams (clause 9.2.2.2) and the anchorage of stirrups and shear assemblies should be observed accordingly.

### 6.9.3. Design bond stress for reinforcing bars

The design value of the bond stress $f_{bd}$ is

$$f_{bd} = \eta_1 \eta_2 \eta_3 f_{ctd} \tag{6.9-4}$$

where

$f_{ctd}$ is the design value of concrete tensile strength ($= f_{ctk,min}/1.50$)
$\eta_1$ considers the type of reinforcement: $\eta_1 = 1.0$ for plain bars, $\eta_1 = 1.4$ for indented bars and $\eta_1 = 2.25$ for ribbed bars

$\eta_2$ considers the position of the bar during concreting: $\eta_2 = 1.0$ when good bond conditions are obtained, as for

- all bars with an inclination of 45°–90° to the horizontal during concreting and
- all bars with an inclination less than 45° to the horizontal, which are up to 250 mm from the bottom or at least 300 mm from the top of the concrete layer during concreting;

$\eta_2 = 0.7$ for all other cases and for bars in structural parts built with slip forms

$\eta_3$ considers the bar diameter

$\eta_3 = 1.0$ for $\phi \leqslant 32$ mm

$\eta_3 = \dfrac{132 - \phi}{100}$ for $\phi > 32$ mm

with $\phi$ in mm.

## 6.9.4. Basic anchorage length

The basic length necessary for the transfer of the yield force of a bar or wire of diameter $\phi$ is

$$l_b = \frac{\phi}{4}\frac{f_{yd}}{f_{bd}} \qquad (6.9\text{–}5)$$

$f_{bd}$ having the values defined in subsection 6.9.3.

For a welded mesh made up of plain or indented wires, $l_b$ in eq. (6.9–5) can be calculated with the values for $f_{bd}$ obtained with $\eta_1$ for ribbed bars in subsection 6.9.3, provided there is a sufficient number of welded cross bars in the anchorage zone (see subsection 6.9.5).

Each welded joint should be capable of withstanding the shearing force given in clause 2.2.5.1b.

## 6.9.5. Design anchorage length

The design anchorage length $l_{b,net}$ can be calculated from eq. (6.9–6)

$$l_{b,net} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 l_b A_{s,cal}/A_{s,ef} \geqslant l_{b,\min} \qquad (6.9\text{–}6)$$

where

$A_{s,cal}$ is the calculated area of reinforcement required by the design
$A_{s,ef}$ is the area of reinforcement provided.
$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are coefficients given by Table 6.9.1 and defined as

- $\alpha_1$ coefficient taking into account the form of the bar (straight, bent, loop)
- $\alpha_2$ coefficient taking into account the influence of one or more welded transverse bars ($\phi_t > 0.6\phi$) along the design anchorage length $l_{b,net}$ (see Fig. 6.9.6)
- $\alpha_3$ coefficient taking into account the effect of confinement by the concrete cover
- $\alpha_4$ coefficient taking into account the effect of confinement by transverse reinforcement
- $\alpha_5$ coefficient taking into consideration the effect of the pressure transverse to the plane of splitting along the design anchorage length

$l_b$ is taken from eq. (6.9–5)

The effect of other types of anchoring devices may be taken into account in the same way. The appropriate $\alpha_2$-value should be determined by tests.

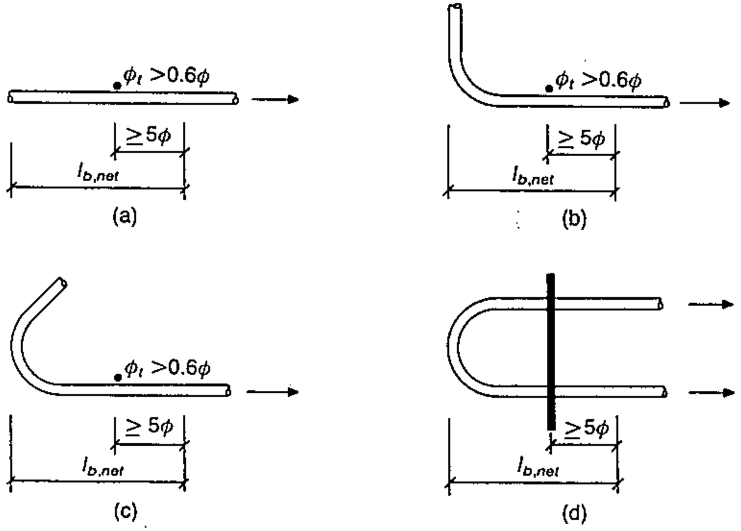



*Fig. 6.9.6. Requirement on position of the welded transverse bar along* $l_{b,net}$

The limitations of $l_{b,min}$ are given

- to ensure minimum active anchorage length
- to take into account tolerances.

$l_{b,min}$ denotes the minimum anchorage length:

- for bars in tension: $l_{b,min} > \max\{0.3l_b; 10\phi; 100\,\text{mm}\}$
- for bars in compression: $l_{b,min} > \max\{0.6l_b; 10\phi; 100\,\text{mm}\}$.

The product $(\alpha_3\alpha_4\alpha_5)$ is limited:

— for high-bond bars: $\alpha_3\alpha_4\alpha_5 > 0.7$,
— for plain or indented bars or wires: $\alpha_3\alpha_4\alpha_5 = 1$.

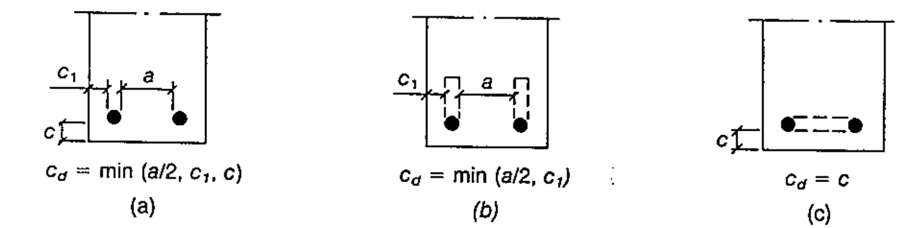



*Fig. 6.9.7. Notation for Table 6.9.1: (a) straight bars; (b) hooks, bends; (c) loops, horizontally placed hooks, bends*

Minimum concrete cover for the bond model to be valid is one bar diameter.

The transverse reinforcement should be evenly distributed along

- $l_{b,net}$ in the case of tensile anchorages
- $l_{b,net} + 4\phi$ in the case of compression anchorages; at least one unit of the transverse reinforcement should be placed along this $4\phi$ region outside the anchored bar.

At least one unit of the transverse reinforcement should be placed in the region of a hook, a bend or a loop.

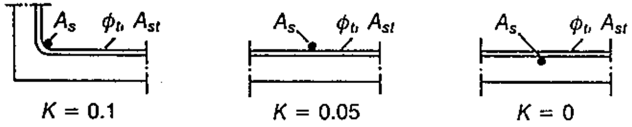
*Table 6.9.1. $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, $\alpha_5$ coefficients*

| Influencing factor | Type of anchorage | Reinforcement bar | | |
|---|---|---|---|---|
| | | In tension | | In compression |
| Form of bars | — | $\alpha_1 = 1.0$ | | $\alpha_1 = 1.0$ |
| | ⊂ ∟ ⊏ | $\alpha_1 = 0.7$[a] | | $\alpha_1 = 1.0$ |
| Welded transverse bars | ⊂ ∟ ⊏ | $\alpha_2 = 0.7$ | | $\alpha_2 = 0.7$ |
| Confinement by concrete | — | $\alpha_3 = 1 - 0.15\dfrac{c_d - \phi}{\phi}$ | $\begin{cases} \geqslant 0.7 \\ \leqslant 1.0 \end{cases}$ | $\alpha_3 = 1.0$ |
| | ⊂ ∟ ⊏ | $\alpha_3 = 1 - 0.15\dfrac{c_d - 3\phi}{\phi}$ | $\begin{cases} \geqslant 0.7 \\ \leqslant 1.0 \end{cases}$ | $\alpha_3 = 1.0$ |
| Confinement by not welded transverse reinforcement | — ⊂ ∟ ⊏ | $\alpha_4 = 1 - K\lambda$[b] | $\begin{cases} \geqslant 0.7 \\ \leqslant 1.0 \end{cases}$ | $\alpha_4 = 1.0$ |
| Confinement by transverse pressure | — ⊂ ∟ ⊏ | $\alpha_5 = 1 - 0.04p$ | $\begin{cases} \geqslant 0.7 \\ \leqslant 1.0 \end{cases}$ | |

[a] If $c_d > 3\phi$, otherwise $\alpha_1 = 1.0$.
[b] $\lambda = (\Sigma A_{st} - \Sigma A_{st,\min})/A_s$.

Notation:

$\Sigma A_{st}$ is the cross-sectional area of the transverse reinforcement along the design anchorage length $l_{b,net}$
$\Sigma A_{st,\min}$ is the cross-sectional area of the minimum transverse reinforcement $= 0.25A_s$ for beams and 0 for slabs
$A_s$ is the area of a single anchored bar with maximum bar diameter
$K$ values are




$p$ is the transverse pressure (MPa) at ultimate limit state along $l_{b,net}$, perpendicular to splitting plane.

Anchorage of stirrups is given in clause 9.1.1.4.

For a welded mesh made of plain or indented wires, the design anchorage length $l_{b,net}$ can be calculated as for mesh made of high-bond wires, provided the number of welded cross wires over the design anchorage length is

$$n = 4A_{s,cal}/A_{s,ef}$$

For the shear strength of welds, see clause 2.2.5.1b.

## 6.9.6. Design lap length of bars in tension

The design lap length is

$$l_0 = \alpha_1\alpha_3\alpha_4\alpha_5\alpha_6 l_b A_{s,cal}/A_{s,ef} \geqslant l_{0,\min} \qquad (6.9\text{–}7)$$

where

$l_b$ is calculated from eq. (6.9–5)
$l_{0,\min} > \max\{0.3\alpha_6 l_b;\ 15\phi;\ 200\,\text{mm}\}$
$\alpha_1$, $\alpha_3$, $\alpha_4$ and $\alpha_5$ can be taken from Table 6.9.1,

however, for the calculation of $\alpha_4$, $\Sigma A_{st,\min}$ should be taken as $1.0A_s$, with $A_s$ = area of one spliced bar

$\alpha_6$ is a coefficient given in Table 6.9.2 as a function of the percentage of the reinforcement lapped within $1.3l_0$ from the centre of the lap length considered.

For transverse distribution reinforcement $\alpha_6$ can be taken equal to 1.0.

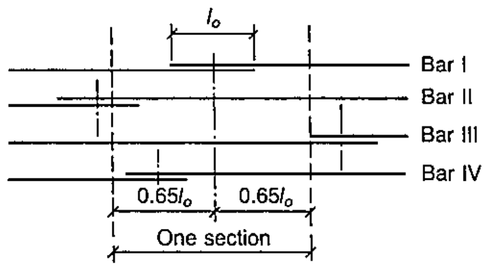



*Fig. 6.9.8. Percentage of lapped bars in one section*

*Table 6.9.2. Values of the coefficient $\alpha_6$*

| Percentage of lapped bars relative to the total cross-section of steel | ⩽20% | 25% | 33% | 50% | >50% |
|---|---|---|---|---|---|
| $\alpha_6$ | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |

## 6.9.7. Design lap length of bars permanently in compression

The design length of lap $l_0$ should comply with the condition

$$l_0 > l_b$$

$l_b$ from eq. (6.9–5).

Concerning the transverse reinforcement, see clause 9.1.2.2.3.

$l_o$ is the lap length of bar I to be designed. II and III situated out of section. Lap IV situated in the section. Percentage = 50% and $\alpha_6 = 1.8$.

## 6.9.8. Design lap length of welded fabric in tension

### 6.9.8.1. Lap of the main reinforcement

The minimum number of welded cross wires over the lap length is

— $n = 1$ for fabric made of ribbed wires
— $n = 5(A_{s,cal}/A_{s,ef})$ for fabric made of plain indented wires ($n$ to be rounded up to the next whole number).

The design lap length is

- with intermeshed fabric (Fig. 9.1.9), according to eq. (6.9–7),
- with layered fabric

$$l_0 \geqslant \alpha_7 l_b A_{s,cal}/A_{s,ef} \geqslant l_{0,\min} \qquad (6.9\text{–}8)$$

where

$$\alpha_7 = 0.5 + \frac{A_s/s}{A_{s,ef}}, \quad \text{and} \quad 1.0 \leqslant \alpha_7 \leqslant 2.0 \qquad (6.9\text{–}9)$$

$l_b$ from eq. (6.9–5)

$A_{s,cal}$ and $A_{s,ef}$ from subsection 6.9.5

$$l_{0,\min} > \max\{0.75 l_b;\ 15\phi;\ s;\ 200\,\text{mm}\} \qquad (6.9\text{–}10)$$

$A_s/s$ corresponds to the specific cross-sectional area of the fabric (mm$^2$/m).

Additional transverse reinforcement is not necessary in the zone of lapping.

Splicing of welded fabric in structures assessed for fatigue loads should be done with intermeshed fabrics (see Fig. 9.1.9 in clause 9.1.2.3).

For welded fabric placed in more than one layer, the values of $l_o$ from eq. (6.9–8) can be reduced by 20% for the fabric furthest from a surface (see clause 9.1.2.3.2).

### 6.9.8.2. Laps in the transverse direction: secondary reinforcement

For intermeshed fabric, clause 6.9.8.1 applies.

For layered fabric, the length of lap is chosen from Table 9.1.3 (see clause 9.1.2.3.2).

## 6.9.9. Design lap length of welded fabric in compression

For the main reinforcement the design lap length should comply with the condition

$$l_0 \geqslant l_b \qquad (6.9\text{–}11)$$

$l_b$ from eq. (6.9–5)

For the secondary reinforcement, clause 6.9.8.2 applies.

## 6.9.10. Anchorage of prestressing tendons

It is necessary to ensure that the anchorage device in the case of post-tensioned tendons or the anchorage length in the case of pretensioned tendons able to transfer the design strength of the tendon to the concrete.

If sleeves or couplers are used, these should be so located that the required strengths can be obtained in all sections and the anchorages specified above can be attained.

## 6.9.11. Anchorage of pretensioned prestressing reinforcement

### 6.9.11.1. General

The bond strength of pretensioned prestressing tendons depends on the loading case. The highest value applies to the transmission length — length to introduce the prestressing force. Beyond that length a lower bond strength as to be taken into account. This results in a bilinear diagram for the embedment length that is required to develop the design steel stress (Fig. 6.9.9).

Two different bond situations should be considered due to the transverse deformations of the tendon. A 'push-in' along the transmission length, where the tendons become thicker at release, and a 'pull-out', which refers to the anchorage length where the opposite occurs when the steel stress is increased due to loading.

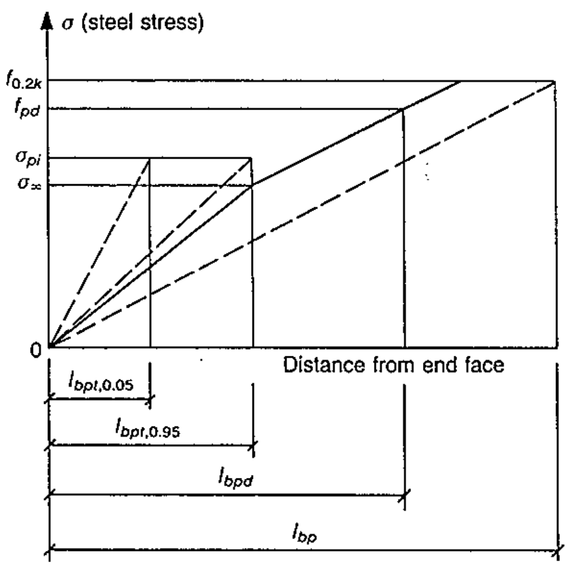



*Fig. 6.9.9. Course of steel stresses along the anchorage zone of a pretensioned member*

### 6.9.11.2. Design bond strength

The design value of the bond strength for prestressing tendons is

$$f_{bpd} = \eta_{p1}\eta_{p2}f_{ctd} \qquad (6.9\text{–}12)$$

where

$f_{ctd} = f_{ctk}(t)/1.50$ is the lower design concrete tensile strength; for the transmission length the strength at the time of release, for the anchorage length the strength at 28 days

$\eta_{p1}$ takes into account the type of prestressing tendon: $\eta_{p1} = 1.4$ for indented and crimped wires, and $\eta_{p1} = 1.2$ for 7-wire strands

$\eta_{p2}$ takes into account the position of the tendon: $\eta_{p2} = 1.0$ for all tendons with an inclination of 45°–90° with respect to the horizontal during concreting, $\eta_{p2} = 1.0$ for all horizontal tendons which are up to 250 mm from the bottom or at least 300 mm below the top of the concrete section during concreting, and $\eta_{p2} = 0.7$ for all other cases.

### 6.9.11.3. Basic anchorage length

The basic anchorage length of an individual pretensioned tendon is

$$l_{bp} = \frac{A_{sp}}{\phi\pi}\frac{f_{ptd}}{f_{bpd}} \qquad (6.9\text{–}13)$$

$f_{ptd} = f_{ptk}/1.15$, where $f_{ptk}$ as defined in clause 2.3.4.3.

The basic anchorage length defines the length that is required to develop the full strength in an untensioned tendon.

The factor $A_{sp}/\phi\pi$ depends on the type of tendon.

For tendons with a circular cross-section

$$\frac{A_{sp}}{\phi\pi} = \frac{\phi}{4}$$

For 7-wire strands

$$\frac{A_{sp}}{\phi\pi} = \frac{7}{36}\phi$$

### 6.9.11.4. Transmission length

The transmission length of a pretensioned tendon is

$$l_{bpt} = \alpha_8\alpha_9\alpha_{10}l_{bp}\frac{\sigma_{pi}}{f_{pd}} \qquad (6.9\text{–}14)$$

where

$\alpha_8$ considers the way of release: $\alpha_8 = 1.0$ for gradual release and $\alpha_8 = 1.25$ for sudden release;

$\alpha_9$ considers the action effect to be verified: $\alpha_9 = 1.0$ for calculation of anchorage length when moment and shear capacity is considered, and $\alpha_9 = 0.5$ for verification of transverse stresses in anchorage zone

$\alpha_{10}$ considers the influence of bond situation: $\alpha_{10} = 0.5$ for strands and $\alpha_{10} = 0.7$ for indented or crimped wires

$\sigma_{pi}$ is the steel stress just after release.

The use of narrow spaced stirrups or helices around the tendons and transverse prestressing may result in a shorter transmission length. This is not considered due to lack of experimental data.

Tendon release that is obtained by sawing through the concrete and the steel should be considered as gradual release.

The transmission length can be estimated from the draw-in value ($\delta_e$) of the tendons at the end face of the concrete member. Assuming a linear steel stress along the transmission length, this draw-in shall be

$$\delta_e < 0.5\frac{\sigma_{pi}}{t_p}l_{bpt}$$

with $\alpha_9 = 1.0$ in the expression for $l_{bpt}$.

When the concrete member is sawn from a longer production unit, the draw-in cannot be estimated properly.

See commentary to clause 6.9.11.1 for different bond situations. The basic anchorage length is related to 'pull-out'. The transmission length is connected to 'push-in'. The ratio between the two is given by $\alpha_{10}$.

### 6.9.11.5. Design anchorage length

The design anchorage length of a pretensioned prestressing tendon is

$$l_{bpd} = l_{bpt} + l_{bp}\frac{\sigma_{pd} - \sigma_{pcs}}{f_{pd}} \qquad (6.9\text{–}15)$$

If necessary, the required anchorage capacity may be obtained by additional end anchorages or non-prestressed reinforcement.

where

$\sigma_{pd}$ tendon stress under design load ($\sigma_{pd} \leqslant f_{pd}$)
$\sigma_{pcs}$ tendon stress due to prestress including all losses.

### 6.9.11.6. Development length

The development length is the distance from the end face to the concrete cross-section beyond which the distribution of the longitudinal stresses is considered linear.

For a rectangular cross-section and straight tendons situated near the bottom edge of the concrete section the development length is

$$l_p = \sqrt{[h^2 + (0.6 l_{bpt})^2]} > l_{bpt} \qquad (6.9\text{–}16)$$

where $h$ is the total depth of the concrete section.

For non-rectangular sections the development length can be found in a similar way as assumed for post-tensioning (clause 9.1.6.1).

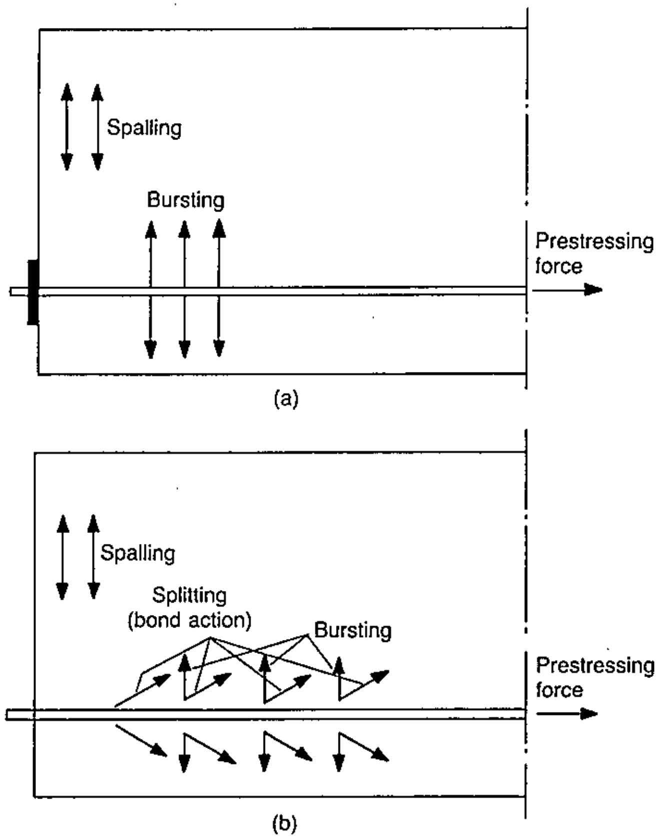



*Fig. 6.9.10. Transverse tensile stresses in the anchorage zone: (a) of a post-tensioned member; (b) of a pretensioned member*

## 6.9.12. Transverse stresses in the anchorage zone of prestressed tendons

### 6.9.12.1. General

The anchorage zone of prestressed tendons is a discontinuity region that should be treated according to section 6.8. Should the use of the strut-and-tie model be too problematic because of the complexity of the stress field, the verification may be performed on the basis of the stresses in a linear, uncracked member. For design purposes, the tensile stresses, due to the development and distribution of the prestressing force, are subdivided into three groups (Fig. 6.9.10).

If the strut-and-tie model is not applicable due to lack of transverse reinforcement, the verification may be performed on the basis of stress and strain analysis.

It should be realized that splitting—in case of pretensioned tendons—and bursting occur in the same region. Hence, they have to be superimposed.

### 6.9.12.2. Bursting

For the calculation of the bursting force the symmetric prism analogy may be used (Fig. 6.9.11). The height and the width of the prism follow from the possible enlargement of the anchor plates (post-tensioning) or the tendon pattern (pretensioning). For multiple tendons the most unfavourable situations shall be considered: a single tendon or a group of tendons. The

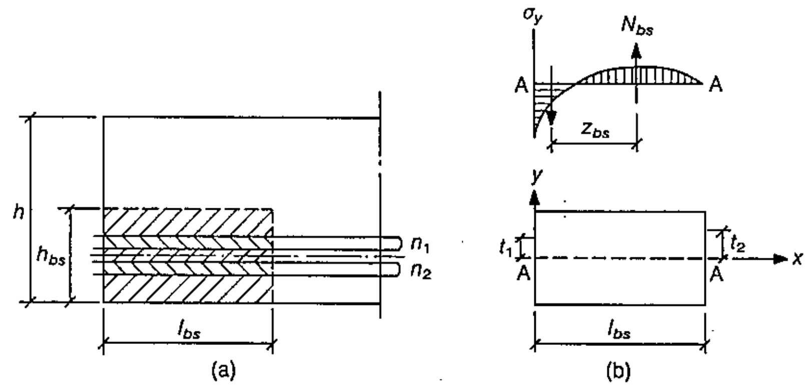



*Fig. 6.9.11. For calculation of the bursting force: (a) dimensions of the symmetrical prism; (b) moment equilibrium along section A–A*

bursting action shall be determined both in the vertical and in the horizontal direction.

The length of the prism is for end anchored tendons

$$l_{bs} = h_{bs}$$

and for tendons anchored by bond

$$l_{bs} = \sqrt{[h_{bs}^2 + (0.6 l_{bpt})^2]} < l_{bpt}$$

The internal lever arm for the bursting force is

$$z_{bs} = 0.5 l_{bs}$$

The bursting force follows from the moment equilibrium along section A-A (Fig. 6.9.11)

$$N_{bs} = \frac{\frac{1}{2}(n_1 + n_2)t_2 - n_1 t_1}{z_{bs}} \gamma_1 F_{sd} \qquad (6.9\text{–}17)$$

where

$t_1$ is the distance between the centroid of tendons above section A-A to the centroid of the prism

$t_2$ is the distance between the centroid of the concrete stress block above section A-A to the centroid of the prism

$n_1$, $n_2$ are the numbers of tendons above and below section A-A, respectively

$F_{sd}$ is the design force per tendon

$\gamma_1 = 1.1$ is the supplementary partial safety factor against overstressing by overpumping.

The maximum bursting stress follows from

$$\sigma_{bs} = 2N_{bs}/b_{bs}l_{bs} \qquad (6.9\text{–}18)$$

where $b_{bs}$ is the width of the prism.

The model is based on uncracked concrete but it is sufficiently accurate to be used also for cracked concrete with tensile forces resisted by reinforcement.

For $\sigma_{bs} > f_{ctd}$ the bursting force shall be resisted by confining or net reinforcement distributed within $l_{bs}/3$ to $l_{bs}$ from the end face, with

$$A_{sbs} = N_{bs}/f_{sy} \qquad (6.9\text{–}19)$$

## 6.9.12.3. Spalling

The spalling force may be calculated with the equivalent prism analogy (Fig. 6.9.12). The length of the prism is defined as, for end anchored tendons

$$l_{sl} = h$$

and for tendons anchored by bond

$$l_{sl} = \sqrt{[h^2 + (0.6 l_{bpt})^2]} < l_{bpt}$$

The internal lever arm for the spalling force is

$$z_{sl} = 0.5 l_{sl}$$

Section B-B shall be chosen so that along this section no shear force results. The spalling force results from the moment equilibrium along section B-B

$$N_{sl} = M / z_{sl} \qquad (6.9\text{–}20)$$

with the moment $M$ given by the concrete stresses above section B-B.

The maximum spalling stress follows from

$$\sigma_{sl} = 8 N_{sl} / b_{sl} l_{sl} \qquad (6.9\text{–}21)$$

with $b_{sl}$ width of the cross-section at section B-B.
For $\sigma_{sl} \geqslant f_{ct,fl} / \gamma_c$, where

$\gamma_c = 1.5$
$f_{ct,fl}$ is the flexural tensile strength (given in clause 2.1.3.3.1)

the spalling force shall be resisted by the reinforcement

$$A_{s,sl} = N_{sl} / f_{sy}$$

The spalling force resisting reinforcement shall be put parallel to the end face in its close vicinity.

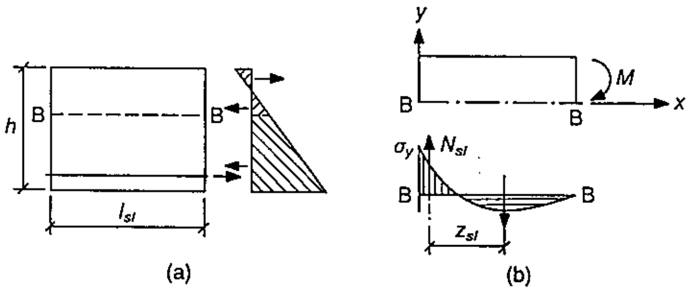



*Fig. 6.9.12. For calculation of the spalling force: (a) definition of the equivalent prism; (b) moment equilibrium along section B–B*

The equivalent prism approach overestimates the spalling stress. For shallow members (i.e. hollow core slabs) a more accurate value may be obtained from Fig. 6.9.13.

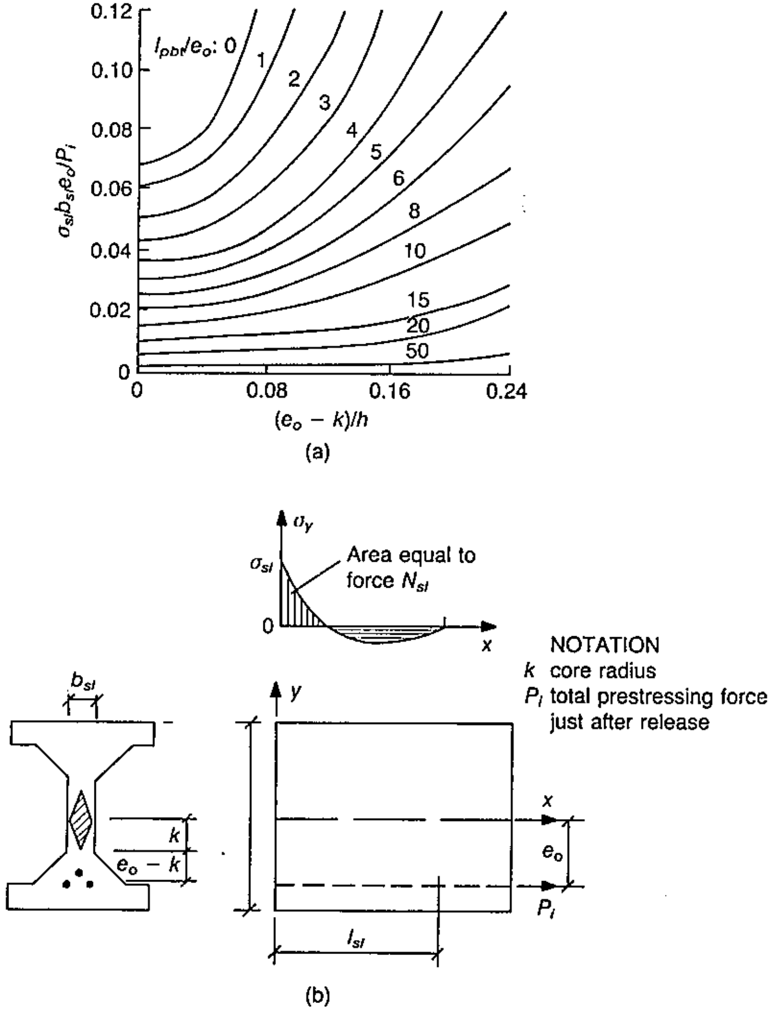



*Fig. 6.9.13. Maximum spalling stress as a function of eccentricity and transmission length (based on linear analysis) for members with* h $<$ *400 mm*

### 6.9.12.4. Splitting

Splitting stresses due to bond of pretensioned tendons are sufficiently accounted for when the transverse reinforcement required for bursting and spalling confines the tendons.

If no such confining reinforcement is applied, the concrete cover should be as given in Table 6.9.3.

*Table 6.9.3. Minimum cover as a function of the clear spacing to resist the splitting stresses around pretensioned tendons*

| Clear spacing | Cover |
|---|---|
| $\geqslant 3\phi$ | $\geqslant 3\phi$ |
| $< 3\phi$ | $\geqslant 4\phi$ |

## 6.10. ULS OF SHEAR JOINTS

### 6.10.1. General

This section covers design of reinforced concrete interfaces along which shear forces have to be transferred (cracked areas, joints between precast elements, etc.).

For a given value of relative displacement, the shear force which can be transferred along a reinforced concrete interface may be calculated on the basis of the basic models given in sections 3.9 (Concrete-to-concrete friction) and 3.10 (Dowel action).

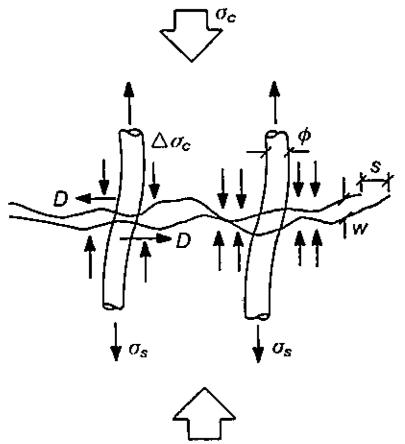



*Fig. 6.10.1. Model of behaviour of shear joints*

Any shear slip imposed on an interface results in a dilatancy, $w$, whose value is a function of the geometry (natural rough, smooth, keyed) of the interface.

This dilatancy induces axial stresses in the reinforcing bars crossing the interface. These axial stresses may be calculated according to section 6.9.

The tensioned bars impose normal compressive stresses, $\Delta\sigma_c$, on the interface (clamping effect). Thus, the interface under the normal stress $\sigma_c$ (due to external actions or prestressing) increased by $\Delta\sigma_c$ resists the imposed shear slip by means of friction. The friction resistance may be evaluated on the basis of section 3.9.

The reinforcing bars themselves contribute to the shear resistance of the joint by means of their dowel action (to be evaluated according to section 3.10).

The shear resistance of the joint, for a given slip value, may be calculated as the sum of the contributions from all resisting mechanisms.

## 6.10.2. Design of shear joints

Where a more general model is not available and SLS aspects are not governing, the following expression may be used for the calculation of the shear resistance of joints

$$\tau_{Rd} = \beta f_{ctd} + \mu(\rho f_{yd} + \sigma_{cd}) < 0.25 f_{cd} \qquad (6.10\text{–}1)$$

where

$\beta f_{ctd}$ is the cohesion between the two concrete parts of the joints,
$\mu$ is the coefficient of friction,

the factors $\beta$ and $\mu$ depend on the roughness-category of the interfaces, according to Table 6.10.1,

If the construction is carried out according to the approximate rules, with particular regard to surface treatment, concrete compaction and plasticity, two categories of surface roughness are considered:

- Category 1 ('smooth')

I a smooth surface, as obtained by casting against a steel or timber shutter
II a surface which lies between trowelled or floated to a degree, which is effectively as smooth as (I)
III a surface which has been trowelled or tamped in such a way that small ridges, indentations or undulations have been left
IV a surface achieved by slip forming or vibro-beam screeding
V a surface achieved by extrusion
VI a surface, which has been deliberately textured by lightly brushing the concrete when wet

- Category 2 ('rough')

VII as for (VI), but with more pronounced texturing, as obtained by brushing, by a transverse screeder, by combining with a steel rake or with an expanded metal

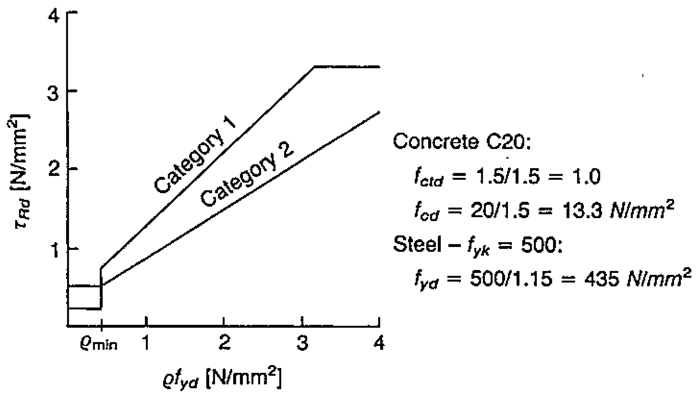



*Fig. 6.10.2. Example of application*

VIII a surface which has been thoroughly compacted, but no attempt has been made to smooth, tamp or texture the surface in any way, having a rough surface with coarse aggregate protruding, but firmly fixed in the matrix

IX where the concrete has been sprayed when wet, to expose the coarse aggregate without disturbing it

X a surface which has been provided with mechanical shear keys.

Eq. (6.10–1) applied for category 2, is an approximate linearization of eq. (3.9–3).

*Table 6.10.1. Factors $\beta$ and $\mu$ used in eq. (6.10–1)*

| | Surface category 1 | Surface category 2 |
|---|---|---|
| $\beta$ | 0.2* | 0.4 |
| $\mu$ | 0.6 | 0.9 |

*For very smooth surfaces I and II (see commentary on left-hand side) the use of $\beta$ = 0.1 is recommended.

$\rho$ is the ratio of reinforcing steel crossing the joint $\geqslant 0.001$
$f_{yd}$ is the design value of the yield stress of the steel
$\sigma_{cd}$ is the external normal stress acting on the joint ($\sigma_{cd} > 0$ for compression)
$f_{ctd}$ is the design tensile strength of concrete with lowest strength ($f_{ctk,\min}/1.50$).

For low shear situations, the design shear strength may be estimated according to

$$\tau_{Rd} = \beta f_{ctd}$$

In this case no reinforcement is necessary.

For the definition of shear joint geometry, and the role of an inclined reinforcement, reference is made to subsection 14.3.3.

# 7. VERIFICATION OF SERVICEABILITY LIMIT STATES

## 7.1. REQUIREMENTS

It should be demonstrated that the structure and the structural elements perform adequately in normal use. To meet this requirement the serviceability limit states should be verified.

Depending on the type and function of a structure or a structural element the verification of different serviceability limit states may be relevant, such as the limitation of

- stresses (see section 7.3)
- crack widths (see section 7.4)
- deformations (see section 7.5)
- vibrations (see section 7.6).

The serviceability limit states are listed in subsection 1.2.3.

The verification of SLS is performed under service load conditions and the operational failure probability to exceed the limit state is about a thousand times higher than that of ULS. Obviously it can be observed sporadically, that the requirements are not met. Such a situation should not justify rejection of the structure.

Exceeding the limit state of stresses or limit state of cracking may lead to limited local structural damage mainly affecting the durability of the structure.

Excessive deformations may produce damage in non-structural elements or load bearing walls and affect the efficient use or appearance of structural or non-structural elements.

Vibrations may cause discomfort, alarm or loss of ability to use.

## 7.2. DESIGN CRITERIA

For the verification of serviceability limit states all direct and indirect actions (loads or imposed or restrained deformations) should be taken into account.

The design criteria depend on the type of SLS and are given in clause 1.6.6.2.

The partial safety coefficients are taken equal to 1.0.

The combination of loads to be considered depends on the type of SLS and on the specific problem. It is suitable to utilize one of the combinations given in clause 1.6.6.5, i.e.

- rare combination
- frequent combination
- quasi-permanent combination.

For special problems other suitable combinations may be agreed by client and designer.

Prestressing forces should be considered as permanent actions.

The relevant values of the prestressing force depend on the type of SLS and the problem considered. Prestressing force values to be considered are suggested in section 4.6.

In general for LS of deformations the mean value is sufficient while for LS of cracking or stresses upper or lower fractiles are suitable.

For structural analysis any appropriate method may be used, which takes account of the material behaviour under service loads.

Linear or non-linear methods may be used. For most SLS problems linear analysis is sufficient. If, however, a non-linear analysis is carried out for ULS the action effects under service loads may be calculated by the same model. Plastic analysis is unsuitable for SLS.

## 7.3. STRESS LIMITATION

Under service load conditions the limitation of stresses may be required for

- tensile stresses in concrete
- compressive stresses in concrete
- tensile stresses in steel.

The limitation of tensile stresses in concrete is an adequate measure to reduce the probability of cracking.

The limitation of compressive stresses in concrete should avoid excessive compression, producing irreversible strains and longitudinal cracks.

Tensile stresses in reinforcement should be limited with an appropriate safety margin below the yielding strength, preventing uncontrolled cracking.

In calculating the stress, account shall be taken of whether the section is expected to crack under service loads and also of the effects of creep, shrinkage and relaxation of prestressing steel. Other indirect actions which could influence the stress, such as temperature, should also be considered.

In selecting appropriate stress limits, the effect of the absolute dimensions of the member should be taken into account. Lower limits will be appropriate for larger members due to size-effects.

For more detailed information see section 7.4.

Stresses are calculated using section properties corresponding to either the uncracked or the fully cracked condition, whichever is appropriate.

In general where the maximum tensile stress in the concrete calculated on the basis of an uncracked section under the rare combination of loads exceeds $f_{ctm}$ (see Table 2.1.2), the cracked state should be assumed.

Where an uncracked section is used, the whole of the concrete section is assumed to be active and both concrete and steel are assumed to be elastic in both tension and compression. Where a cracked section is used, the concrete is assumed to be elastic in compression but to be incapable of sustaining any tension (in verifying stresses in accordance with these results, no allowance should be made for the stiffening effect of the concrete in tension after cracking).

At least the minimum area of reinforcement given by subsection 7.4.5 is required to satisfy the limitation of the stress in ordinary bonded reinforcement.

### 7.3.1. Tensile stresses in concrete

Depending on the stress limit chosen different limit states may be required, but the LS of decompression is considered to be the most significant. Stresses may be calculated on the basis of a homogeneous uncracked concrete section (state I). The contribution of reinforcement to the area and section modulus of the cross-section may be taken into account.

In specific cases, e.g. in incremental launching with precast elements, a minimum compressive stress may be required.

#### 7.3.1.1. Limit state of decompression

The limit state of decompression is defined as the state where all axial concrete stresses are below or equal to zero.

As a rule the limit state of decompression should be required, if cracking or reopening of cracks are to be avoided under a given load combination. The margin between zero stress and tensile strength may also be reserved for self-equilibrating stresses not under consideration.

## 7.3.2. Compressive stresses in concrete

Excessive compressive stress in the concrete under service load may lead to longitudinal cracks and high and hardly predictable creep, with serious consequences to prestress losses. When such effects are likely to occur, measures should be taken to limit the stresses to an appropriate level.

The occurrence of longitudinal cracks may lead to a reduction in durability. In the absence of other measures (such as an increase of concrete cover) it is recommended to limit the compressive stress for exposure classes 3 and 4 (see Table 1.5.1). However, no limitation in serviceability conditions is necessary for stresses under bearings and anchorages.

If the stress does not exceed $0.6f_{ck}(t)$

- under the rare combination, longitudinal cracking is unlikely to occur
- under the quasi-permanent combination, creep and the corresponding prestress losses can be correctly predicted.

If under the quasi-permanent combination the stress exceeds $0.4f_{ck}(t)$, the non-linear model shall be used for the assessment of creep (see clause 2.1.6.4.3(d)).

The limit of $0.6f_{ck}(t)$ is not sharp. Consequently, in the corresponding verification prestress may be represented by its mean value, and in transient situations where the magnitude of variable actions is small (especially at transfer in prestressed beams) the quasi-permanent combination may be substituted by the rare combination. On the other hand prestress and concrete strength should be introduced in the verification by their values at the time at which the maximum stresses are reached.

These measures should be envisaged for deformations if the span/effective depth ratio exceeds 85% of the value given in Table 7.5.2 for the case considered.

If creep is likely to significantly affect the functioning of the member considered (e.g. with regard to loss of prestress, deformation, validity of the structural analysis) an alternative measure would be a limitation of the stress to $0.4f_{ck}(t)$. However, the limitation may be taken as a value between $0.4f_{ck}(t)$ and $0.6f_{ck}(t)$ for verifications relating to a transient situation (e.g. during construction) depending on the duration of the loading.

Creep effects in a cracked cross-section may be taken into account by assuming a modular ratio of 15 for situations where more than 50% of the stress arises from quasi-permanent actions. Otherwise, they may be ignored.

## 7.3.3. Steel stresses

Tensile stresses in the steel under serviceability conditions which could lead to inelastic deformation of the steel shall be avoided as this will lead to large, permanently open, cracks.

This requirement will be met provided that, under the rare combination of loads, the tensile stress in ordinary reinforcement does not exceed $0.8f_{yk}$. Where the stress is due only to imposed deformations, a stress of $1.0f_{yk}$ will be acceptable.

The stress in prestressing tendons should not exceed $0.75f_{pyk}$ after allowance for losses (see clause 2.3.4.1).

### 7.3.4. Cases where stress calculation is not essential

The stress limitations given in subsections 7.3.2 and 7.3.3 above may generally be assumed to be satisfied without further calculations provided

(*a*) the design for ultimate limit state has been carried out in accordance with chapter 6
(*b*) the minimum reinforcement provisions of subsection 7.4.5 are satisfied
(*c*) detailing is carried out in accordance with chapter 9
(*d*) not more than 30% of redistribution has been carried out in the analysis for the ultimate limit state.

Stress verifications should be carried out for partially prestressed members because there may be fatigue problems.

As mentioned in subsection 7.3.2 it may be necessary to verify the compressive stresses in prestressed beams at transfer.

## 7.4. LIMIT STATE OF CRACKING

### 7.4.1. Requirements

It should be ensured that, with an adequate probability, cracks will not impair the serviceability and durability of the structure.

Cracks do not, per se, indicate a lack of serviceability or durability; in reinforced concrete structures, cracking may be inevitable due to tension, bending, shear, torsion (resulting from either direct loading or restraint of imposed deformations), without necessarily impairing serviceability or durability.

However, the following specific requirements should generally be respected.

Cracks may be due to other causes such as plastic shrinkage or chemical reactions accompanied by expansion of the hardened concrete. The avoidance and the control of the width of such cracks are not covered by this chapter; see chapter 8 and Appendix d, sections d.6.3. and d.12.4.

#### 7.4.1.1. Function requirements

The function of the structure should not be harmed by the cracks formed.

This may be the case in reservoirs or external water-insulation structures.

Alternatively, cracks may be allowed to form without any control of their width or the probability of their formation may be reduced by special measures, such as the provision of joints (provided that this does not impair the functioning of the structure).

In relevant cases, nominal crack width limits may be agreed with the client, unless reference is made to more simplified design means.

However, due to the actual state-of-the-art and the highly probabilistic nature of the related phenomena, such nominal crack width values may only serve as means to apply the design criterion of subsection 7.4.2a, and can in no case be compared to actual crack widths measured in situ.

#### 7.4.1.2. Durability

The durability of the structure during its intended lifetime should not be harmed by the cracks formed.

This might be the case in reinforced or prestressed structures, depending on the aggressiveness of the environment and the sensitivity of the steel.

However, under some well defined conditions, crack formation in reinforced concrete does not necessarily increase the corrosion risk of normal reinforcing steel; provided that the characteristic crack width does not exceed an appropriately specified value $w_{lim}$.

A typical exception of this rule should be carefully considered. It is the case of frequent use of de-icing agents on top of tension zones of reinforced concrete elements.

### 7.4.1.3. Appearance of the structure

The appearance of the structure should not be unacceptable because of cracking.

The satisfaction of this requirement should exclusively be made by means of deemed to satisfy rules. If not otherwise specified, the design criteria presented in this chapter to cover durability requirements are considered as satisfying this requirement too.

### 7.4.1.4. Uncertainties

Uncertainties related to the actual local concrete tensile strength, as well as to unforeseen tensile stresses, should be appropriately covered in design and construction.

### 7.4.1.5. Further requirements

Further requirements for an appropriate control of cracking may result from the necessity to limit or to avoid

- vibrations
- damage caused by excessive deformations
- brittle failure.

Brittleness of structural elements due to crack formation is avoided, if the minimum reinforcement ratios required in chapter 9 are observed.

## 7.4.2. Design criteria vs. cracking

(*a*) The specific requirement of clauses 7.4.1.1 to 7.4.1.4 may be met by an appropriate limitation of crack widths. This may be achieved either by means of analytical procedures (clause 7.4.3.1 or 7.4.3.2) or by appropriate practical rules (subsection 7.4.4).

(*b*) In cases where the ULS design leads to low reinforcement ratios, the specific requirements of subsection 7.4.1 may be met by providing an appropriate minimum amount of reinforcement.

Thus, at all sections which are expected to be subjected to significant tension (due to restraint, combined or not with direct loading), a minimum amount of reinforcement should be placed, ensuring that yield of the reinforcement will not occur after cracking in the SLS.

This applies also to prestressed members in regions where tension is expected to develop in the concrete.

## 7.4.3. Verification of crack width

(*a*) Whenever an analytical procedure is needed, the criterion of subsection 7.4.2a is applied as follows for transverse cracks.
The following inequality should be observed

$$w_k \leqslant w_{lim} \qquad (7.4\text{–}1)$$

where

$w_k$ denotes the characteristic crack width calculated as in clause 7.4.3.1 or 7.4.3.2 under the appropriate combination of actions (see clause 1.6.6.5)

$w_{lim}$ denotes the nominal limit value of crack width which is specified for cases of expected functional consequences of cracking, or for some particular cases related to durability problems.

Different combinations of actions may be considered under particular conditions (e.g. partially prestressed structures under special conditions).

(*a*) In the absence of specific requirements (e.g. watertightness), it may be assumed that for exposure classes 2–4 (as specified in section 1.5), a $w_{lim}$-value equal to 0.30 mm under the quasi-permanent combination of actions is satisfactory for reinforced concrete members with respect to both appearance and durability.

For exposure class 1, this limit may be relaxed provided that it is not necessary for reasons other than durability.

When de-icing agents are expected to be used on top of tensioned zones of reinforced concrete elements, appropriate $w_{lim}$-values should be specified in accordance with the client, depending on the thickness and quality of the concrete, and of additional protective layers.

(*b*) For prestressed members, if more detailed data are not available, the crack width limiting values presented in Table 7.4.1 may be used.

*Table 7.4.1. Crack width limits for prestressed members*

| Exposure class | Limiting crack widths (in mm) under the frequent load combination | |
|---|---|---|
| | Post-tensioned | Pretensioned |
| 1 | 0.20 | 0.20 |
| 2 | 0.20 | No tension within the section is allowed |
| 3 and 4 | (*a*) No tension is allowed within the section, or<br>(*b*) if tension is accepted, impermeable ducts or coating of the tendons should be applied; in this case, $w_{lim}$ = 0.20 | |

Longitudinal cracks due to the corrosion of steel bars are not covered by these criteria and shall be avoided by means of measures taken to ensure durability (see section 8.4).

If

$$\rho_{s,ef}\sigma_{s2} > f_{ctm}(t)(1 + \alpha_e \rho_{s,ef})$$

it may be assumed that the stabilized cracking condition has been reached, otherwise the formation of single cracks should be considered; where

$f_{ctm}(t)$ is the mean value of the tensile strength of the concrete at the time $t$ when the crack appeared,
$\alpha_e$ is the ratio $E_s/E_{ci}$ (for $E_{ci}$ see clause 2.1.4.2; in the case of early cracking the modulus of elasticity should be reduced according to clause 2.1.6.3)
$\rho_{s,ef}$ is the effective reinforcement ratio ($= A_s/A_{c,ef}$)
$A_{c,ef}$ is the effective area of concrete in tension; this is generally the area of concrete surrounding the tension reinforcement (see Fig. 7.4.2)
$\sigma_{s2}$ denotes the steel stress at the crack.

For the sake of simplicity $(1 + \alpha_e \rho_{s,ef})$ can be set equal to 1.

$$l_{s,\max} = 2\,\frac{\sigma_{s2} - \sigma_{sE}}{4\tau_{bk}}\,\phi_s \qquad (7.4\text{–}3)$$

$$= \frac{\phi_s}{3.6\rho_{s,ef}} \text{ for stabilized cracking}$$

$$= \frac{\sigma_{s2}}{2\tau_{bk}}\,\phi_s\,\frac{1}{1 + \alpha_e \rho_{s,ef}} \text{ for single crack formation}$$



(*b*) For the control of longitudinal cracks (parallel to main steel bars), the following design criteria apply.

(i) The thickness of concrete cover as well as, where necessary, its secondary reinforcement (skin reinforcement) transverse to the main steel bars should be appropriately selected (as a function of their diameter), in order to secure the full development of bond resistance without any longitudinal cracking.

(ii) For plane elements reinforced in two directions, tensile stresses generated in sections parallel to the direction of a steel bar should be appropriately limited.

## 7.4.3.1. Calculation of crack width in reinforced concrete members

### 7.4.3.1.1. Basic crack width formula

For all stages of cracking, the design crack width may be calculated according to

$$w_k = l_{s,\max}(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) \qquad (7.4\text{–}2)$$

where

$l_{s,\max}$ denotes the length over which slip between steel and concrete occurs; steel and concrete strains, which occur within this length, contribute to the width of the crack; $l_{s,\max}$ is calculated by means of eq. (7.4–3)
$\varepsilon_{sm}$ is the average steel strain within $l_{s,\max}$
$\varepsilon_{cm}$ is the average concrete strain within $l_{s,\max}$
$\varepsilon_{cs}$ denotes the strain of concrete due to shrinkage; it has to be introduced algebraically.

where

$\sigma_{sE}$ is the steel stress at the point of zero slip
$\tau_{bk}$ is the lower fractile value of the average bond stress; it may be taken from Table 7.4.2
$\phi_s$ denotes the diameter of the steel bar, or the equivalent diameter of bundled bars.

From eq. (7.4–2) can be derived with Fig. 7.4.1 for the mean strains

$$\varepsilon_{sm} - \varepsilon_{cm} = (\varepsilon_{s2} - \beta\Delta\varepsilon_{sr}) - \beta\varepsilon_{sr1} = \varepsilon_{s2} - \beta\varepsilon_{sr2} \qquad (7.4\text{–}4)$$

with

$$\varepsilon_{sr2} = \frac{f_{ctm}(t)}{\rho_{s,ef}E_s}(1 + \alpha_e\rho_{s,ef})$$

where

$$\Delta\varepsilon_{sr} = \varepsilon_{sr2} - \varepsilon_{sr1}$$

$\varepsilon_{s2}$ is the steel strain at the crack
$\varepsilon_{sr2}$ is the steel strain at the crack, under forces causing $f_{ctm}(t)$ within $A_{c,ef}$; if the internal forces are lower than or equal to these forces (e.g. in a working joint), then $\varepsilon_{sr2} = \varepsilon_{s2}$
$\varepsilon_{sr1}$ is the steel strain at the point of zero slip under cracking forces reaching $f_{ctm}(t)$
$\beta$ is an empirical factor to assess averaged strain within $l_{s,max}$; it can be taken from Table 7.4.2
$f_{ctm}(t)$ is the mean value of concrete tensile strength at time $t$ at which the crack appeared.

For direct calculation of the reinforcement area $A_s$, required for crack width control with a given bar diameter, the following formula can be used

$$A_s = \sqrt{\left[\frac{\phi_s F_{cr}(F_s - \beta F_{cr})}{2E_s w_k \tau_{bk}(1 + \alpha_e\rho_{s,ef})}\right]} \qquad (7.4\text{–}5)$$

where

$(1 + \alpha_e\rho_{s,ef}) = 1$, is allowed for a simple calculation
$F_s$ is the force in the crack transmitted by the reinforcement
$F_{cr}$ indicates the force, which has to be introduced into concrete by bond (or interaction with other parts of the structure) in order to provoke cracking within $A_{c,ef}$ at the end of the transmission length

$$F_{cr} = F_s$$

for the crack formation phase $[\rho_{s,ef}\sigma_{s2} \leqslant f_{ctm}(t)(1 + \alpha_e\rho_{s,ef})]$

$F_{cr} = A_{c,ef} f_{ctm}(t)(1 + \alpha_e \rho_{s,ef})$ for stabilized cracking $[\rho_{s,ef}\sigma_{s2} > f_{ctm}(t)(1 + \alpha_e \rho_{s,ef})]$.

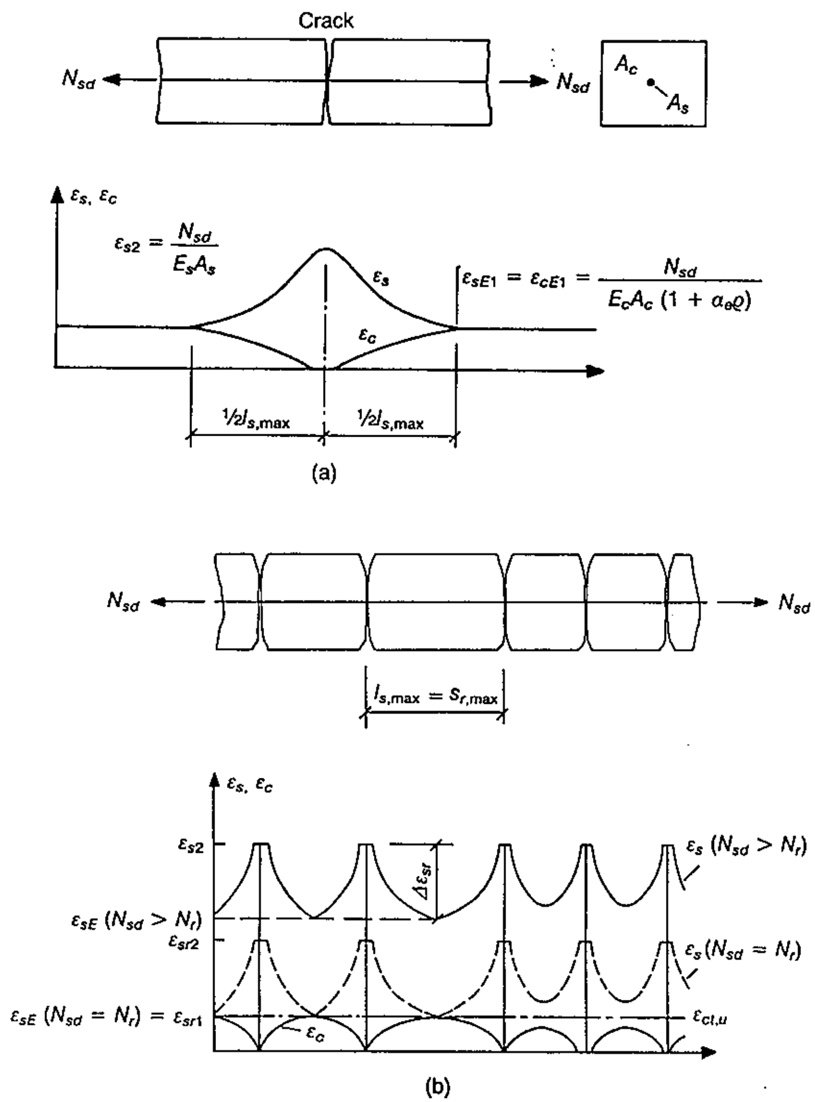



*Fig. 7.4.1. Strains for calculating the crack spacing and the average strains: (a) for single cracks; (b) for stabilized cracking*

For stabilized cracking the average width may be estimated on the basis of an average crack spacing: $s_{rm} \approx \frac{2}{3} l_{s,\max}$.

*Table 7.4.2. Values for $\beta$ and $\tau_{bk}$ (assuming that only deformed bars are used)*

| | Single crack formation | | Stabilized cracking | |
|---|---|---|---|---|
| | $\beta$ | $\tau_{bk}$ | $\beta$ | $\tau_{bk}$ |
| Short term/instantaneous loading | 0.6 | $1.8 f_{ctm}(t)$ | 0.6 | $1.8 f_{ctm}(t)$ |
| Long term/repeated loading | 0.6 | $1.35 f_{ctm}(t)$ | 0.38 | $1.8 f_{ctm}(t)$ |

The effective concrete area in tension ($A_{c,ef}$) accounts for the non-uniform normal stress distribution by bond forces into the concrete cross-section at the end of the transmission length.

In absence of a refined model, Fig. 7.4.2 may be used in order to assess the effective concrete area in tension.

S. 250

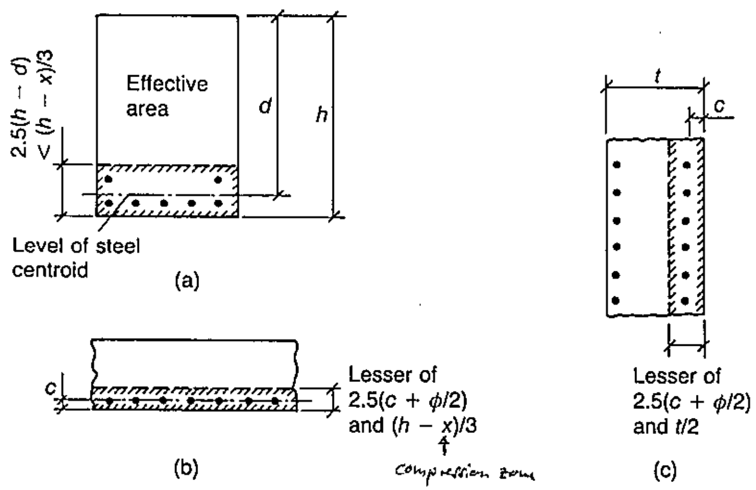



*Fig. 7.4.2. Effective tension area: (a) beam; (b) slab; (c) member in tension*

By means of the method given in this section, the design crack width within the effective tension area may be calculated. It should be noted that outside this region, larger cracks may occur.

#### 7.4.3.1.2. Combined effects of load and imposed deformations

Where cracking is due also to imposed deformations, the steel strain at cracks due to imposed loads ($\varepsilon_{s2}$ in eq. (7.4–4)) should be increased by that caused by imposed deformations.

#### 7.4.3.1.3. Orthogonal reinforcement directions

When in a member reinforced in two orthogonal directions, cracks are expected to form at a significant angle (> 15°) with respect to the direction of reinforcement, approximations may be used in calculating crack spacings.

When a more refined model is not available, the following expression may be used:

$$l_{s,\max} = \left(\frac{\cos\theta}{l_{sx,\max}} + \frac{\sin\theta}{l_{sy,\max}}\right)^{-1} \qquad (7.4\text{–}7)$$

where

$\theta$ denotes the angle between the reinforcement in the $x$-direction and the direction of the principal tensile stress

$l_{sx,\max}$, $l_{sy,\max}$ denote crack spacings in the two orthogonal directions, calculated according to eq. (7.4–3).

### 7.4.3.2. Calculation of crack width for prestressed concrete

The calculation of crack width concerns structural members with bonded prestressing reinforcement.

The calculation of crack width follows generally the procedure and the formulae given in clause 7.4.3.1.

When prestressed and non-prestressed types of steel are simultaneously used, since the bond behaviour of prestressing tendons is different from the bond behaviour of deformed reinforcing bars, different steel stresses will be developed in each type of steel.

Both equilibrium and compatibility should be respected for the calculated stresses in prestressing and reinforcing steel.

For single cracks, different transmission lengths $l_s$ and $l_p$ should be calculated for reinforcing and prestressing steel respectively.

The crack spacing calculated taking into account prestressing tendons is valid for a square region (300 mm × 300 mm) surrounding the prestressing tendon.

*For single cracks* ($\Delta F_{s+p} \leqslant f_{ctm}(t) A_{c,ef}$)
For the sake of simplicity the factor $(1 + \alpha_e \rho)$ is taken as 1 in the whole clause.

For reinforcing steel $l_{s,\max}/2 = \sigma_{s2}\phi_s/(4\tau_{bs,k})$.

For prestressing steel $l_{p,\max}/2 = \Delta\sigma_p\phi_p/(4\tau_{bp,k})$.

Assuming that $w_{sk} = w_{pk}$ according to eq. (7.4–2), the total force after decompression

$$\Delta F_{s+p} = A_s \sigma_{s2} + A_p \Delta\sigma_p$$

leads to different stresses in the reinforcing and prestressing steel

$$\sigma_{s2} = \Delta F_{s+p}/(A_s + \sqrt{(\xi_1)}A_p) \quad \text{in the reinforcing steel} \quad (7.4\text{–}8)$$

$$\Delta\sigma_p = \sqrt{(\xi_1)}\Delta F_{s+p}/(A_s + \sqrt{(\xi_1)}A_p) \text{ in the prestressing steel} \quad (7.4\text{–}9)$$

where

$$\xi_1 = \tau_{bp,k}\phi_s/(\tau_{bs,k}\phi_p)$$

$l_{s,\max}$ and $\varepsilon_{s2} = \sigma_{s2}/E_s$ can be introduced in eqs (7.4–2) and (7.4–4) in order to determine explicitly the crack width.

The force $\Delta F_{s+p}$ can be taken as follows.

— For tension chords of T-beams or box girders

$$\Delta F_{s+p} = 0.9 f_{t1} \quad (7.4\text{–}10a)$$

— For rectangular sections or for the part of the web, where no tension chord is connected:

$$\Delta F_{s+p} = 0.9 F_{t1}\left(1 - \frac{\sigma_{cs}}{\sigma_{cs}^*}\right) \quad (7.4\text{–}10b)$$

where

$F_{t1}$ is the tensile force within the total tensile zone before formation of the first crack

$\sigma_{cs}$ is the compressive stress at the centre of the section, due to external normal force $N$ caused by load or restraint and the characteristic prestressing force including the losses due to creep and shrinkage

$\sigma_{cs}^*$ is the compressive stress at the centre of gravity of the section which is able to control the crack width without any additional reinforcement in the tensile zone. This condition is fulfilled, if the depth of the tension zone, calculated on the basis of a cracked section under the loading conditions leading to formation of the first crack, does not exceed the lesser of $h/4$ or 0.3 m.

*For stabilized cracking* $(\Delta F_{s+p} > f_{ctm}(t) A_{c,ef})$

$$l_{s,\max} = l_{p,\max} = \frac{\phi_s}{3.6(\rho_{s,ef} + \xi_1 \rho_{p,ef})} \qquad (7.4\text{–}11)$$

$$\sigma_{s2} = \sigma_{sm,m} + \tfrac{2}{3}\beta \frac{f_{ctm}(t)}{(\rho_{s,ef} + \xi_1 \rho_{p,ef})} \qquad (7.4\text{–}12)$$

$$\Delta\sigma_p = \Delta\sigma_{pm,m} + \tfrac{2}{3}\beta \frac{\xi_1 f_{ctm}(t)}{(\rho_{s,ef} + \xi_1 \rho_{p,ef})} \qquad (7.4\text{–}13)$$

where the subscript *ef* implies that steel areas should be normalized to appropriate effective concrete area in tension, and

$\sigma_{s2}$ denotes the stress in the reinforcing steel at the crack, calculated on the basis of different bond characteristics for reinforcing bars and prestressing tendons
$\rho_{p,ef} = A_p / A_{c,ef}$
$\rho_{s,ef} = A_s / A_{c,ef}$
$\sigma_{sm,m} = \Delta\sigma_{pm,m}$ are the mean steel stresses; they can be calculated according to state II but taking the tension stiffening effect into account; the higher the percentage of steel the more the mean steel stress reach the steel stress according to the 'naked' state II

$l_{s,\max}$ and $\varepsilon_{s2} = \sigma_{s2}/E_s$ can be introduced in eqs (7.4–2) and (7.4–4) in order to determine explicitly the crack widths.

When the maximum slip between steel and concrete does not exceed the value of 0.25 mm (i.e. for $w < 0.50$ mm), the fractile value of the average bond stress for ribbed bars is given by

$$\tau_{bs,k} = k_1 f_{ctm}(t)$$

where

$k_1 = 2.25$ leading to 50% fractile
$k_1 = 1.80$ leading to 75% fractile of crack width for ribbed bars.

For smooth bars, 50% of the values of the average bond stress for ribbed bars should be taken. For prestressing steel the following apply

$\tau_{bp,k}/\tau_{bs,k} = 0.20$ for post-tensioned tendons, smooth bars
$\tau_{bp,k}/\tau_{bs,k} = 0.40$ for post-tensioned tendons, indented wires or strands
$\tau_{bp,k}/\tau_{bs,k} = 0.60$ for post-tensioned tendons, ribbed bars

In case of stabilized cracking, the maximum crack spacing may be calculated as in clause 7.4.3.1, provided that the different diameters and bond behaviour of reinforcing and prestressing steel are appropriately taken into account.

## 7.4.4. Control of cracking without calculation of crack width

Under well specified conditions, the fulfilment of the requirements of clauses 7.4.1.1, 7.4.1.2 and 7.4.1.3 may also be achieved by means of appropriate practical rules.

(*a*) When small depth elements subjected mainly to bending are considered, no special measures are needed for crack control.

(*b*) Under the condition that the minimum reinforcement specified in subsection 7.4.5 is provided, the design crack width may be kept to acceptable low values, if appropriately chosen bar diameters and bar spacings are used.

$\tau_{bp,k}/\tau_{bs,k}$ = 0.80 for pretensioned tendons, ribbed bars
$\tau_{bp,k}/\tau_{bs,k}$ = 0.60 for pretensioned tendons, indented bars or strands.

For reinforced or prestressed slabs subjected to bending without significant axial tension, no special measures to control cracking are needed, provided that the overall depth of the slab does not exceed 160 mm.

When the values given in the following Tables 7.4.3 and 7.4.4 are respected, crack widths do not generally exceed the value of 0.30 mm for reinforced elements and 0.20 mm for prestressed elements.

(*a*) For cracking caused mainly by restraint, crack widths will not generally be excessive provided that the bar sizes given in Table 7.4.3 are not exceeded; the $\sigma_s$-value of Table 7.4.3 is that calculated at cracking of the element ($\sigma_{sr}$).
(*b*) For cracks caused mainly by imposed loads, crack widths will not generally be excessive provided that either the provisions of Table 7.4.3 or those of Table 7.4.4 are satisfied.

*Table 7.4.3. Maximum bar diameters (deformed bars) for which no calculation of crack width is needed*

| Steel stress (MPa)* | Maximum bar diameter (mm) | |
|---|---|---|
| | Reinforced sections | Prestressed sections |
| 160 | 32 | 25 |
| 200 | 25 | 16 |
| 240 | 20 | 12 |
| 280 | 14 | 8 |
| 320 | 10 | 6 |
| 360 | 8 | 5 |
| 400 | 6 | 4 |
| 450 | 5 | — |

* Steel stresses are calculated under quasi-permanent loads (reinforced concrete) or under frequent loads and the characteristic value of prestress (prestressed concrete).

Further guidance concerning the choice of bar diameters is given in chapter 8.

For prestressed concrete sections, the stresses in the reinforcement should be calculated regarding prestress as an external force. In general the stress increase of the tendons, i.e. the contribution of tendons to the limitation of crack widths, may be disregarded.

For reinforced concrete the maximum bar diameter may be modified as follows.

For restraint cracking

$$\phi = \phi_{s,max} \frac{f_{ctm}}{2.9} \qquad (7.4\text{–}14)$$

For load induced cracking

$$\phi_s = \phi_{s,max} \frac{h_t}{7.5(h - d)} > \phi_{s,max} \qquad (7.4\text{–}15)$$

where

$\phi_s$ is the adjusted maximum bar diameter
$\phi_{s,max}$ is the maximum bar size given in the table
$h$ is the overall depth of the section
$h_t$ is the depth of the tension zone just before cracking
$d$ is the effective depth
$f_{ctm}(t)$ is the mean value of the concrete tensile strength, at the time $t$ when the crack appeared.

*Table 7.4.4. Maximum bar spacing for which no calculation of crack width is needed*

| Steel stress (MPa) | Maximum bar spacing (mm)* | |
|---|---|---|
| | Reinforced sections | Prestressed sections |
| 160 | 300 | 200 |
| 200 | 250 | 150 |
| 240 | 200 | 100 |
| 280 | 150 | 50 |
| 320 | 100 | — |
| 360 | 60 | — |

* For members in pure tension with an overall depth $h \leqslant 200$ mm and in pure bending with $h \leqslant 400$ mm a more accurate calculation could give greater spacings, but they should not exceed 300 mm.

## 7.4.5. Minimum reinforcement areas

### 7.4.5.1. General

A minimum amount of reinforcement should be provided in order to satisfy the requirements of clauses 7.4.1.1, 7.4.1.2 and 7.4.1.3.

To this end, in every area where (under SLS conditions) the tensile strength of concrete may be exceeded, an appropriate amount of reinforcement, for crack control, should be provided.

A combination of external loads and restraint or imposed deformations (intrinsic, like shrinkage or extrinsic, like differential settlements) may lead to this situation.

### 7.4.5.2. Mechanical basis

In calculating the minimum amount of reinforcement, redistribution of internal stresses after cracking, as well as fracture mechanics effects may be taken into account.

### 7.4.5.3. Simplified methods

Simplified calculation methods based on experimental evidence may be used.

For the combination of pure tension and flexure, in the absence of a more rigorous method, the following simplified procedure may be applied for the calculation of the required area of minimum reinforcement within the tensioned concrete zone

$$A_{s,min} = k_c k f_{ct,max} A_{ct} / \sigma_{s2} \qquad (7.4\text{–}16)$$

where

$A_{ct}$ denotes the area of the concrete tension zone just before the formation of cracks, calculated with the technical theory in the uncracked stage

$\sigma_{s2}$ may be taken equal to $f_{yk}$ if adequate anchorage is secured; a lower value may, however, be needed to satisfy the crack width limits (see Tables 7.4.3 and 7.4.4)

$f_{ct,max}$ is the upper fractile of the concrete strength in tension at the moment when the first crack is expected to appear; values of $f_{ct,max}$ may be obtained from Table 2.1.2

$k$ is a factor correcting the value $A_s$ as found by technical theory, against the real $A_{ct}$-values taking into account non-linear stress distributions (self-equilibrating effects); the following rules may be applied

- restraint of extrinsic imposed deformations

  $k = 1.0$

- restraint of intrinsic imposed deformations of rectangular sections

  $k = 0.8$ for $h < 0.3\,m$
  $k = 0.5$ for $h > 0.8\,m$

(linear interpolation is possible)

$k_c$ accounts for the scheme of tensile stress distribution

$k_c = 1.0$ for pure tension
$k_c = 0.4$ under flexural conditions without axial compressive force
$k_c = 0.4$–$1.0$ for a combination of pure tension and flexure.

In prestressed members, the minimum reinforcement for crack control is not necessary in areas where, under the rare combination of loads and the characteristic value of prestress or normal force, the concrete remains in compression.

Otherwise, the required minimum area may be calculated by means of equation (7.4–16), with the following values for $k_c$.

For box sections

$k_c = 0.45$ for the webs
$k_c = 0.9$ for the tension chord.

### 7.4.5.4. Reduced minimum reinforcement

In prestressed members or reinforced concrete members subject to compressive normal force, the minimum reinforcement area may be reduced below that necessary for ordinary reinforced concrete due to the influence of

- the increased flexural stiffness of the compression zone
- the contribution of the prestressing tendons
- the effect of prestress or compressive normal force contributing to crack width limitation of single cracks.

The minimum reinforcement may be reduced or even dispensed with

For rectangular sections

$k_c$ = 0.45 under flexural conditions without axial compressive force

$k_c$ = 0 when concrete remains in compression, or the depth of the tension zone, calculated on the basis of a cracked section under the loading conditions leading to formation of the first crack, does not exceed the lesser of $h/4$ or 0.3 m.

Linear interpolation between both values is possible.

Prestressing tendons may be taken into account as minimum reinforcement within a 300 mm square surrounding the tendon, provided the different bond behaviour of the tendons and reinforcement are taken into account.

altogether if the imposed deformation is so small that it is unlikely to cause cracking. In such cases minimum reinforcement is only needed to resist the forces due to restraint.

# 7.5. LIMIT STATES OF DEFORMATION

## 7.5.1. General

### 7.5.1.1. Requirements

In-service deformations (deflections and rotations) may be harmful to

- the appearance of the structure
- the integrity of non-structural parts
- the proper function of the structure or its equipment.

To avoid harmful effects of deformations appropriate limiting values should be respected.

To establish such limits is not within the scope of this Model Code. However, some practical rules are given in clause 7.5.2.3 for some categories of simple buildings.

Where applicable, acceptable limit values should be established in agreement with the client or his representative.

### 7.5.1.2. Combination of actions

The combinations of actions to be considered depend on the criteria in question and are defined in section 7.2.

In order to ensure a satisfactory behaviour in the serviceability limit state, deformations should be calculated as follows

- the long-term deformations are calculated for the quasi-permanent combinations
- the instantaneous deformations should be calculated for the rare combinations.

For the calculation of camber, only the quasi-permanent combinations are considered.

### 7.5.1.3. Data for the materials

The values of the material properties to be applied depend on the criteria in question.

In order to prevent damage due to deformations, prudent values of the material properties should be used.

In order to calculate camber, the mean values of the material properties may be used.

### 7.5.1.4. Modelling

Depending on the precision needed, appropriate deformation models should be used, as described in the following subsections.

## 7.5.2. Deformations due to bending with or without axial force

### 7.5.2.1. General methods

The deformations are calculated from the curvatures (see section 3.6) by applying appropriate procedures, such as the principle of virtual work or double integration.

In state I the assumption of plane sections remaining plane is accepted. The principle of superposition and, thus, linearity are assumed valid.

In state II-naked the assumption of plane sections is applied.

For quasi-permanent load combinations, the time-dependent behaviour of the materials should be taken into account.

The actual deformations may differ appreciably from the calculated values; in particular if the values of the applied moments are close to the cracking moment. The difference will depend on the dispersion of the material properties, on the ambient conditions, on the loading conditions and the previous loading conditions, on the restraints at the supports, etc.

For prestressed concrete it may be necessary to control deflections assuming unfavourable deviations of the prestressing force and the dead load.

General methods are proposed in CEB Bulletin d'Information No. 158, CEB-Manual Cracking and Deformations, Lausanne, 1985.

### 7.5.2.2. Simplified method

For building members, long-term deflections can be evaluated by the following relations based on a bilinear relationship between load and deflection

$$a = (1 + \phi)a_c \quad \text{for} \quad M_d < M_r \qquad (7.5\text{–}1a)$$

$$a = \left(\frac{h}{d}\right)^3 \eta(1 - 20\rho_{cm})a_c \quad \text{for} \quad M_d \geqslant M_r \qquad (7.5\text{–}1b)$$

Equation (7.5–1b) is only valid for reinforced concrete members.

with cracking moment $M_r = W_c f_{ct}$

where

$a_c$ is the elastic deflection calculated with the rigidity $E_c I_c$ of the cross section (neglecting the reinforcement)

$M_d$ is the bending moment at mid-span of a beam or a slab, or at the fixed end of a cantilever under frequent actions
$\rho_{tm}$ is the geometrical mean percentage of tensile reinforcement, see eq. (7.5–2)
$\rho_{cm}$ is the geometrical mean percentage of compressive reinforcement
$\eta$ is a correction factor (see Table 7.5.1), which includes the effects of cracking and creep
$\phi$ is the creep coefficient (see clause 2.1.6.4.3).

*Table 7.5.1. Correction factor $\eta$ for estimation of deflections*

| $\rho_m$(%) | 0.15 | 0.2 | 0.3 | 0.5 | 0.75 | 1.0 | 1.5 |
|---|---|---|---|---|---|---|---|
| $\eta$ | 10 | 8 | 6 | 4 | 3 | 2.5 | 2 |

$\eta \approx \frac{3}{2} \cdot \frac{1}{\rho_m(\%)}$

The mean percentage $\rho_m$ of tensile reinforcement is determined according to the bending moment diagram (see Fig. 7.5.1).

$$\rho_m = \rho_a \frac{l_a}{l} + \rho \frac{l_o}{l} + \rho_b \frac{l_b}{l} \qquad (7.5\text{–}2)$$

where

$\rho_a$, $\rho_b$ are the percentages of tensile/compressive reinforcement at the left and right supports, respectively,
$\rho$ is the percentage of tensile reinforcement at the $M_{max}$-section.

An estimate of the lengths $l_a$, $l_b$ is generally sufficient.

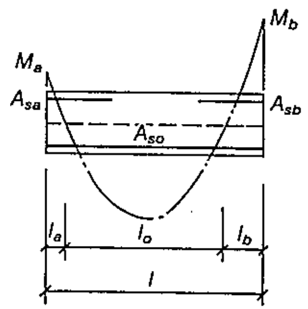



*Fig. 7.5.1. Bending moment diagram defining $l_o$, $l_a$, $l_b$*

### 7.5.2.3. Practical verification of deflections

For simple building elements under specified circumstances it may not be necessary to calculate deflections explicitly if certain limitations of the span-depth ratio are respected.

Making use of the simplified provisions of clause 7.5.2.2, the following rule may be applied

$$l/d \leqslant 1/\sqrt{[\delta\eta(l/a)_{lim}]^{1/3}} \qquad (7.5\text{–}3)$$

where $\delta$ is a coefficient characterizing the system

$$\delta = a_c h^3/l^4 \qquad (7.5\text{–}4)$$

For the derivation of such limiting values, the following criteria should be fulfilled

- the deformation sensitivity of non-structural elements attached to (or in contact with) the flexural member under consideration, should be accounted for
- simple but sound models for deflection estimation should be used
- previous experience within the prescribed fields of application should be available.

For simple verifications $(l/a)_{lim} = 300$.

$(l/a)_{lim}$ is an appropriate span-deflection limiting value.

When applying eqs (7.5–3) and (7.5–4), it is allowable to replace any shape of a cross-section by a rectangular one with the same height and the same moment of inertia. The cracking moment $M_r$ is calculated for the original cross-section.

For two-way spanning slabs, the verification should be carried out for the shorter span. For flat slabs the longer span should be taken.

For reinforced concrete flexural elements without axial force the following rule may be applied

$$\frac{l}{d} \leqslant \lambda = \lambda_o k_T k_l \left(\frac{400}{f_{yk}}\right) \qquad (7.5\text{–}5)$$

where

$\lambda_o$ is taken from Table 7.5.2
$k_T = 1.0$ for flanged section with flange-web width ratio lower than 3 and 0.8 for ratios higher than 3
$k_l = 7/l \leqslant 1$, with $l$ in m
$f_{yk}$ is the yield stress of the reinforcing steel (MPa).

Members where the concrete is lightly stressed are those where $\rho < 0.5\%$ $(\rho = A_s/bd)$. It may normally be assumed that slabs are lightly stressed.

If the reinforcement ratio $\rho$ is known, $(\lambda_o)$-values between the 'highly stressed' and 'lightly stressed' values in Table 7.5.2 may be obtained by interpolation, assuming the 'lightly stressed' values to correspond to $\rho = 0.5\%$ and the 'highly stressed' to $\rho = 1.5\%$.

The limits given for flat slabs correspond to a less severe limitation than a mid-span deflection $l/250$. Experience has shown this to be satisfactory.

*Table 7.5.2. Values of $\lambda_o$ for reinforced concrete members without axial compression*

| Structural system | Concrete highly stressed | Concrete lightly stressed |
|---|---|---|
| Simply supported beam, one- or two-way spanning simply supported slab | 18 | 25 |
| End span of a series of continuous spans, two-way spanning slab continuous over one long side | 23 | 32 |
| Interior span of beam or one-way or two-way spanning slab | 25 | 35 |
| Slab supported on columns without beams (flat slab), verification effected on the longer span | 21* | 30* |
| Cantilever | 7 | 10 |

* These values should be verified.

## 7.5.3. Other deformations

### 7.5.3.1. Deformations due to pure tension

The deformations due to pure tension may be calculated under service conditions when the tension stiffening is assumed constant (see section 3.2).

The mean strain $\varepsilon$ (instantaneous or long-term) in any section of a tie is defined, for the first loading, as follows

$$\varepsilon = \varepsilon_1 \qquad \text{for } N < N_r \qquad (7.5\text{–}6a)$$

$$\varepsilon = \varepsilon_2 - (\varepsilon_{2r} - \varepsilon_{1r})\beta_t \qquad \text{for } N \geqslant N_r \qquad (7.5\text{–}6b)$$

where

$N$ is the applied normal force on the section
$N_r$ is the cracking normal force

$$N_r = f_{ct} A_1 \qquad (7.5\text{–}7)$$

where

$f_{ct}$ is the tensile strength
$A_1$ is the section area in state I
$\varepsilon_1$, $\varepsilon_{1r}$ are the strains in state I corresponding to the actions $N$ and $N_r$ respectively
$\varepsilon_2$, $\varepsilon_{2r}$ are the strains in state II-naked corresponding to the actions $N$ and $N_r$ respectively
$\beta_t$ = 0.40 for instantaneous loads and 0.25 for long-term or repeated loads.

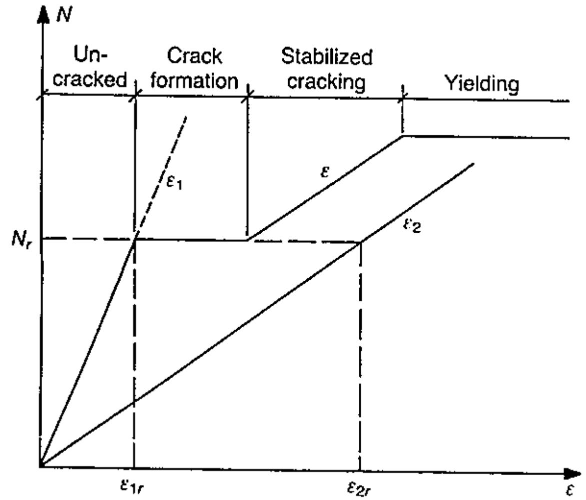



*Fig. 7.5.2. Instantaneous mean strain*

### 7.5.3.2. Deformations due to shear forces

If diagonal cracking does not occur, deformations due to shear forces may be neglected.

Influences on shear stiffness from normal force or prestress may generally be neglected.

### 7.5.3.3. Deformation due to torsion

This clause applies to linear elements of hollow or solid cross-section, subjected to pure or combined torsion.

The torsional stiffness should be determined taking account of cracking under the relevant load effects.

The torsional stiffness can be determined for the compressed part of the cross-section which results from bending and prestressing. A verification of the position of the neutral axis should be carried out.

The shear modulus can be taken as

$$G = 0.4E_{cm}$$

$E_{cm}$ is the mean secant modulus in the compression zone using the stress-strain relation for concrete in compression.

For simplification the shear centre can be determined by using the uncracked cross-section.

## 7.6. VIBRATIONS

### 7.6.1. General

Vibrations of structures may affect the serviceability of a structure as follows

- functional effects (discomfort to occupants, affecting operation of machines, etc.),
- structural effects (mostly on non-structural elements, as cracks in partition, loss of cladding, etc.).

Vibrations can be caused by several variable actions, e.g.

- rhythmic movements made by people such as walking, running, jumping and dancing
- machines
- waves due to wind and water
- rail and road traffic
- construction work such as driving or placing by vibration of sheet piles, compressing soil by means of vibrations as well as blasting work.

Vibrations that endanger the structure, such as very large deflections due to resonance or the loss of resistance due to fatigue, should be included in the verification for ULS of the structure.

### 7.6.2. Vibrational behaviour

To secure satisfactory behaviour of a structure subject to vibrations, the natural frequency of vibration of the relevant structure should be kept sufficiently apart from critical values which depend on the function of the corresponding building, see Table 7.6.1.

$$f > kf_{crit} \quad \text{or} \quad f < f_{crit}/k$$

where $k$ takes integer values.

The vibrational behaviour of structures can be influenced by the following measures

- changing the dynamic actions
- changing the natural frequencies by changing the rigidity of the structure or the vibrating mass
- increasing the damping features, etc.

*Table 7.6.1. Critical frequency in structures subject to vibrations caused by movements of people*

| Structures | Frequency (Hz) $f_{crit}$ |
|---|---|
| Gymnasia and sports halls | 8.0 |
| Dance rooms and concert halls without permanent seating | 7.0 |
| Concert halls with permanent seating | 3.4 |
| Structures for pedestrians and cyclists | See below* |

* Natural frequencies between 1.6 and 2.4 Hz and between 3.5 and 4.5 Hz are to be avoided in structures for pedestrians and cyclists. Joggers can also cause vibrations in structures with natural frequencies between 2.4 and 3.5 Hz.

# 8. DURABILITY

## 8.1. GENERAL

The basic requirement for design versus durability (subsection 1.5.1) is

> Concrete structures shall be designed, constructed and operated in such a way that, under the expected environmental influences, they maintain their safety, serviceability and acceptable appearance during an explicit or implicit period of time without requiring unforeseen high costs for maintenance and repair.

This means that the performance of the structures and structural components in their anticipated working environment should be such that deterioration of material properties or structural components will not lead to an unacceptable probability of failure nor to an unacceptable service performance or appearance.

Measures necessary to ensure the required service life are chosen according to the environmental conditions and the significance of the structure.

Service life depends equally on the behaviour of structural and non-structural elements. Both shall be considered during design, construction and use of the structure.

Non-structural elements such as drainage, joints, bearings, installations etc. may require specialist attention other than that of structural engineering. Particular structural components such as anchorages, couplers and deviators for prestressing tendons and their location in the structure may require particular attention.

The avoidance of durability problems throughout the expected life of a structure requires the co-ordinated efforts of all parties involved in all phases of the planning, construction and use of the structure.

The whole process of creating structures and keeping them in satisfactory use and service requires co-operation between the following four parties

- the owner, by defining his present and foreseen future demands and wishes, if any
- the designers (engineers and architects) by preparing design specifications (including proposed quality control schemes) and conditions
- the contractor who should follow these intentions in his construction works; most commonly also subcontractors are involved
- the user, who will normally be responsible for the maintenance of the structure during the period of use.

Any of these four parties may, by their actions or lack of actions, contribute to an unsatisfactory state of durability of the structure and thus cause a reduction of the service life. Also interactions between two parties may cause faults which can have an adverse effect on durability and service life.

## 8.1.1. Design strategy

Prior to commencement of design, the owner together with the designer shall determine the required service life of the structure and the required exposure classes.

The design options presented in this chapter can accommodate very short as well as very long design lives of the structures for any of the exposure classes presented in subsection 1.5.2.

National or regional regulations may govern these decisions.

The Model Code clauses on design, execution and maintenance will generally lead to a service life of the structure in excess of 50 years according to section 1.5.

However, some structures will require a substantially longer service life, say 100 years or more, and other structures may need a considerably shorter service life, say less than 25 years.

A structure shall be designed and constructed as good and as robust as necessary in order to satisfy the required service life with a minimum amount of foreseen maintenance.

Accessories such as drainage, joints, bearings, railings, connections, installations etc. usually have a shorter service life than the structure itself, and adequate provisions for maintenance and replacement of such elements should be provided in the design.

The design should consider detailing which increases self-protection and robustness of the structure against aggressive environment.

This includes provisions to ensure satisfactory weathering and ageing of exposed surfaces thus allowing buildings to grow old gracefully without expensive maintenance. An appropriate selection of structural form should be ensured at an early, conceptual stage of the project.

The required service life should be obtained without relying on special protections needing frequent maintenance or redoing.

However, in cases of especially aggressive environments special protective measures may be foreseen, see subsection 8.4.6.

The service life of special protective measures is usually shorter—at times very much shorter—than the intended service life of the structure.

Surface treatments shall be chosen very carefully. The effects cannot always be foreseen in full, and very adverse effects may be experienced.

The Service Life Design Concept is based on a simple two-phase modelling of the technical ageing and deterioration of structures (see Fig. 8.1.1).

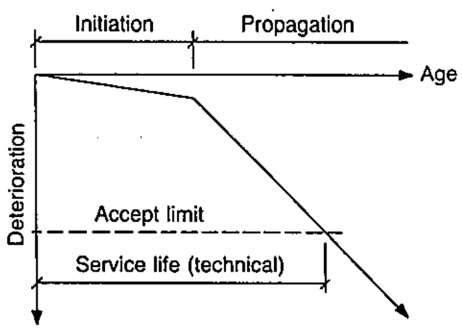



*Fig. 8.1.1. Technical service life*

During the Initiation Phase no noticeable weakening of the material or of the function of the structure occurs, but some protective barrier is broken down or overcome by the aggressive media of the environment. Examples are carbonation, chloride penetration, sulphate accumulation or leaching of lime.

During the Propagation Phase active deterioration normally proceeds rapidly and in a number of cases at accelerating pace.

The protective measures available may influence favourably either the duration of the initiation phase or the rate of propagation of the active deterioration.

The strategy of the service life design is to select intelligently an appropriate number and types of co-operating measures to ensure the required service life, considering the environment in question.

Robustness in the design is achieved by focusing primarily on extending the initiation period as far as possible towards the required service life, and by ensuring a relatively low rate of propagation if active deterioration should develop within the actual period of use.

The design strategy should consider possible measures to protect the structure against premature deterioration. A set of appropriate measures (one or more) can be combined to ensure that the required service life is obtained with a sufficiently high probability.

This design strategy is considered a 'Multi-Stage Protection Strategy' which leaves the selection of individual protective measures to the designer.

The different measures may act simultaneously in contributing to the protection, or one measure may be substituted by the next, once the former has been overcome, eliminated or surpassed by the aggressive substance.

Protective measures may be established by, *inter alia*

- the selected structural form
- the concrete composition, including special additions or admixtures
- the reinforcement detailing including concrete cover
- a special skin concrete quality, including skin reinforcement
- limiting or avoiding crack development and crack widths, e.g. by prestressing
- additional protective measures such as tanking, membranes or coatings, including coating of reinforcement
- specified inspection and maintenance procedures during in-service operation of the structure, including monitoring procedures
- special active protective measures such as cathodic protection or monitoring by way of sensors.

A different level of reliability is associated with the protective effect of each type of measure. This level of reliability depends much on the quality assurance scheme associated with the establishing and possible maintenance of each protective measure.

The choice of protective measures shall be carefully considered in relation to the particular aggressive environment encountered. Possible secondary effects shall be evaluated.

A secondary effect is associated with e.g. the selection of epoxy coated reinforcement with individually coated bars, which rules out later use of cathodic protection.

An unfavourable secondary effect could be the increased rate of carbonation following water repellant impregnations.

A characteristic example of multi-stage protection is post-tensioned structures in a very aggressive environment (high chloride content) where the different protective stages—or measures—could be

(*a*) tanking, coating or a stainless steel lining on the concrete surface
(*b*) a highly impermeable (low water/cement ratio) concrete with pozzolanic additions
(*c*) reasonably large and crack-free concrete cover
(*d*) protective sheathing or ducts, metallic, polyethylene or similar
(*e*) cementitious grout; for unbonded tendons the grouting could be made with corrosion protective grease.

In this example (*a*), (*b*) + (*c*), (*d*) and (e) act consecutively, whereas (*b*) and (*c*) act simultaneously. Also (*b*) as well as (*e*) (with cementitious grout), containing two protective elements each, are acting simultaneously.

For a non-prestressed structure in a similar environment the protective stages could be

— (*a*) and (*b*) unchanged
— (*c*) not considered (as crack-free concrete cannot be ensured)
— (*d*) epoxy coated reinforcement or stainless steel reinforcement or foreseen cathodic protection.

In both cases, an alternative to (*a*) could be larger, high quality concrete cover with low permeability and with a separate small diameter skin reinforcement made of epoxy coated bars or stainless steel, to ensure crack distribution and avoid longitudinal cracking.

The design shall, wherever possible, ensure adequate access to all parts of the structure, including voids and accessories, to allow for inspection and possible maintenance to be performed throughout the intended service life of the structure.

The design shall take into account the execution and maintenance policy foreseen for the structure. In case of doubts, the design strategy should be modified accordingly.

## 8.1.2. Execution

The reduced quality of execution and curing shall be thoroughly specified and subsequently controlled and documented during the construction phase. The quality control procedures should follow a preplanned quality assurance scheme adjusted in detailing and technical level to

- the type, shape, complexity and sensitivity of the structure
- the type and aggressivity of the (local) environment
- the experience and competence of the contractor.

The chosen quality assurance system should provide a high probability of the structure being made right in the first place.

All numerical values given are to be considered as minimum measures and should not be violated.

The quality of execution including curing has a dominant influence on the quality of concrete and on the dimensions, such as cover, obtained in the structure.

The intention is that violation of one minimum measure should not be attempted and compensated for by going beyond the minimal value of another measure, such as increased concrete cover to compensate for reduced concrete quality (e.g. higher water/cement ratio).

## 8.1.3. Use and maintenance strategy

Regular and systematic inspection of the structure and all accessories shall, where possible and relevant, be exerted throughout the intended service life.

A pre-chosen maintenance strategy influence the decisions regarding structural design and layout. The strategy should be decided upon prior to the commencement of the design.

Results of the findings during inspection may determine the subsequent intervals between inspection and their intensity.

In some cases inspection is impossible or very complicated, e.g. for foundations. Robust design and intensified quality control during execution may then be the only available measures.

Accessories usually exhibit a shorter service life than the structure itself and maintenance, repair and replacement must be foreseen in the design and during use.

## 8.2. DETERIORATION MECHANISMS

The combined transportation of heat, moisture and chemical substances, both within the concrete mass and in exchange with the surroundings (micro-climate), and the parameters controlling these transport mechanisms, constitute the principal elements of durability. Transport mechanisms are treated in detail in subsection 2.1.9.

The presence of water or moisture is the one single, most important factor controlling the various types of deterioration mechanisms, excluding mech-

In applying these criteria, the designer has an interest to keep in mind the essential deterioration mechanisms considered in this Model Code. These mechanisms are presented in detail in the CEB Design-Guide to Durable Concrete Structures, Bulletin 182.

anical deterioration. The transport of water within the concrete is determined by the pore type, size and distribution. Thus, controlling the nature and distribution of pores becomes an essential task during the initial process of creating concrete structures.

In turn, the type and rate of degradation processes for concrete (physical, chemical and biological) and for reinforcing or prestressing steel (corrosion) determine the resistance and the rigidity of the materials, the sections and the elements making up a structure. Also the surface conditions are determined in this way, and this is reflected in the safety, the serviceability and the appearance of a structure (i.e. determines the performance of the structure).

## 8.3. ENVIRONMENTAL CONDITIONS

### 8.3.1. Exposure classes

Environmental conditions mean those chemical and physical actions to which the concrete structure is exposed and which result in effects that are not considered loads or action effects in structural design.

In absence of a more specific study, these environmental conditions may be classified into the exposure classes given in Tables 1.5.1 and 1.5.2 of section 1.5.

### 8.3.2. Micro-environment

Classification of environmental conditions (Table 1.5.1 in subsection 1.5.2) should be related to micro-environment and not to macro-environment.

The micro-environment to which a building material or component is exposed reflects the macro-environment acting at the location of the structure, modified by the structure itself.

The micro-environment is the environment in the immediate vicinity of the point considered on the surface of the structure or the structural component.

The micro-environment may differ considerably from the original macro-environment.

Concrete absorbs water quicker than it dries out. The water content in the surface layers will thus usually be higher than corresponding to the average relative humidity of the environment. Due to the hygroscopic effects, this tendency is even more pronounced if the concrete contains some chlorides, either accidentally mixed into the concrete (as polluted aggregates or mixing water, or as an accelerator) or penetrated from the outside, i.e. as de-icing salts or from spray of seawater.

In most cases a cyclic wetting and drying (seasonal changes or on a day-by-day basis) will provide sufficient moisture to allow deterioration to develop.

Water will especially accumulate at intersections between exposed horizontal and vertical surfaces where dust and dirt may accumulate and water is maintained, thus keeping the concrete wet locally over an extended period of time.

The different orientations of structural components will experience different amounts of wind, sunshine and driving rain. In the northern hemisphere southern and western orientations usually have higher maximum temperatures, higher temperature variations, stronger effect from UV-light, and also are more often affected by driving rain. This changing microclimate may promote fast carbonation, quicker corrosion once started and rapid deterioration of a surface protective coating. This may reflect on the quality of concrete selected or on the maintenance strategy adopted.

Northern and eastern orientations may exhibit slower drying-out which may increase risk of frost damage and may increase the tendency of vegetation and bacteriological growth on the concrete surface.

The regular washing and drainage of water from the façades is decisive of the long-term appearance of structures.

The appearance of the structure is much affected by the miscolouring caused by dust, dirt and soot depositing on the concrete surfaces, especially the vertical ones.

On high rise buildings driving rain will only affect the top 2–3 storeys. The remaining stories will not be noticeably affected, i.e. a washing of the façade due to rain cannot be relied upon.

In the design, attempts shall be made to minimize the possibilities of such depositing and to profit from the natural clean-washing which occasionally may be provided by driving rain. This requires a carefully planned channelling of water run-off on the surfaces of structures.

Exposed aggregate surfaces with high quality impermeable coarse aggregate have a relatively high degree of self-protection or self-rinsing ability. This is partly due to the low percentage of the surface being exposed mortar with high capillary effect, partly due to such a surface providing a certain visual camouflage of the deposited dirt.

Window sills, balconies etc. protruding from the walls cause shadow effects on the wall section below, thus preventing possible washing of the wall by falling rain. Very unsightly dirt patterns may result.

Some concrete compositions and surface textures are more sensitive to dirt depositing than others. This shall be considered at the initial choice of materials and texture of exposed surfaces.

The different aggressivity of the micro-environments of a structure may be taken care of in the design by applying a zoning strategy. According to this, the structural elements, or the most adversely affected parts of the elements may be considered as belonging to different zones, depending on the aggressivity of the micro-environment foreseen.

A division in two or three zones can be recommended, i.e.

zone 1: slightly susceptible
zone 2: moderately susceptible
zone 3: highly susceptible.

The substance causing the aggressiveness may vary from structure to structure, or even within different parts of the same structure.

Usually the design and execution should provide a high quality structure to all zones, but special provisions can be added for the more susceptible zones. Such provisions could be the application of a surface protection with the more intensified maintenance associated with such provisions. Alternatively a stainless steel lining, epoxy coated reinforcement or easy replacement of the most exposed structural components may be options to choose between.

Accessories shall be maintained regularly at intervals determined individually depending on the type of structure, its use and its environment.

## 8.4. DURABILITY DESIGN CRITERIA

### 8.4.1. General

The design strategy of this Model Code vs. durability is dissuasive, i.e. the satisfaction of the basic requirement of durability is secured by means of appropriate anticipatory measures, which ensure (with an acceptable probability) that the behaviour of the structure in the ULS and the SLS will not be finally affected during its service life.

In order to satisfy the requirements of section 8.1 the following criteria should be used.

(*a*) An appropriate structural form should be selected at an early stage of the project, in order to avoid disproportionately sensitive structural arrangements and to secure adequate access to all critical parts of the structure for inspection and maintenance.

The relevant rules are described in subsection 8.4.2.

(*b*) An appropriate quality of concrete in the outer layer ('skin') of the structural elements shall be secured. A dense, well compacted and well cured, strong and low permeability concrete is needed, which should not exhibit map cracking. Besides, an adequate thickness of concrete cover should be provided.

The relevant rules are described in subsection 8.4.3.

Low permeability towards gases, moisture and aggressive agents is needed.

(*c*) Adequate detailing of reinforced and prestressed concrete structural elements should ensure the integrity of critical surfaces or corners and edges in order to avoid any unforeseen concentration of aggressive influences.

The relevant rules are described in subsection 8.4.4.

(*d*) Under specified environmental conditions and/or for small diameter reinforcing bars or prestressing single wires, nominal crack widths should be controlled under specified load conditions to avoid depassivation during the specified design life.

The relevant rules are described in subsection 8.4.5.

(*e*) Under strongly aggressive environmental conditions, protective surface coatings may be needed.

The relevant rules are described in subsection 8.4.6.

(*f*) The design for durability should be based on accepted sets of standards for materials and recommendations for execution, and on a given maintenance policy.

Detailed information on concrete technology is presented in Appendix d.

## 8.4.2. Selection of structural form

The selected structural form of exposed concrete structures has a decisive influence on the interaction between the structural material and the environment.

An architectural design well thought of from the point of view of required long service life may lead to considerable improvements in the durability and appearance of concrete structures.

All exposed concrete surfaces should be adequately drained. Only preplanned ponding may take place.

Complexity in structural form, as well as in execution and use, will usually increase the sensitivity of the structure to deterioration, shorten service life or require increased efforts in maintenance. Also the ageing characteristics may deteriorate with increased complexity.

In the selection of structural form adequate care should be taken to provide robustness against deleterious liquid or gaseous substances penetrating into the structure.

Most deterioration mechanisms are governed by some substance which in liquid, dissolved or gaseous form penetrates from the surrounding environment through the concrete surface into the concrete. Exceptions are physical and some mechanical damage.

Water may trigger inherent deleterious reactions and may cause leaching of lime, thus causing dissolution of the cementitious effect in the concrete.

The geometry of exposed structural components and the form, type and placing of joints, including construction joints (see subsection 8.4.4), connections, and supports should be chosen such as to minimize the risks of local concentrations of deleterious substances. These concentrations may develop on the surface of the structure as well as within the concrete when these substances enter the concrete by permeation, diffusion, capillary action or similar.

The robustness of an exposed structure or structural component is partly related to the ratio between the exposed surface area and the volume of concrete. The larger this ratio the greater is the risk of some deleterious substance penetrating into the concrete in sufficient quantity to initiate deterioration of the concrete or the reinforcement. For building façades exposed only on one side, the ratio between the exposed surface area and the projection of the façade on a vertical plane is a relative measure of the vulnerability of the structure.

The higher the risk of deterioration (relative risk factor) the more the need for care in selecting the concrete composition, the concrete cover, the correct execution and curing, and an adequate and appropriate maintenance scheme.