

IS EN 1992

(Eurocode 2) Design of Concrete Structures

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Introduction

EUROCODE 2: DESIGN OF CONCRETE STRUCTURES is published in four parts:

- IS EN 1992-1-1:2005 General Rules and Rules for Buildings
Irish National Annex due 19th October 2009
Replaces BS 8110-1,2,3
- IS EN 1992-1-2:2005 Design of Concrete structures. Structural fire design
Irish National Annex due 19th October 2009
Replaces BS 8110-1,2
- IS EN 1992-2: 2005 Design of Concrete Structures. Bridges. 2005
Irish National Annex due 30th October 2009
Replaces BS 5400
- IS EN 1992-3: 2006 Design of Concrete Structures. Liquid-retaining and
containment structures
Irish National Annex due 19th October 2009
Replaces BS 8007

Useful Resources

- www.eurocode2.info
- www.concretecentre.com
- I.S.E. Manual for Design of Concrete Structures to Eurocode 2
- Companion Document BD 2403 U.K. Dept. of Communities and Local Government
- Designed and Detailed Eurocode 2, Concrete Society
- Concrete Society / I.S.E. Standard Method of Detailing Structural Concrete, third edition to Eurocode 2
- Lecture and notes on www.ieicork.ie under downloads

Eurocode 2 Differences

1. Eurocode 2 is generally laid out to give advice on the basis of phenomena (e.g. bending, shear etc.) rather than by member types as in BS 8110 (e.g. beams, slabs, columns, etc)
2. Design is based on characteristic cylinder strengths not characteristic cube strengths
3. Code does not provide derived formulae (e.g. for bending, only the details of the stress block are expressed).
4. Units for stress are mega Pascals, MPa ($1 \text{ MPa} = 1 \text{ N/mm}^2$)

Differences continued

5. A comma used for a decimal point
6. One thousandth is represented by ‰
7. Axes changed x, y to y, z
8. The partial factor for steel reinforcement is 1.15. However, the characteristic yield strength of steel that meets the requirements will be 500 MPa; so overall the effect is negligible
9. Higher strengths of concrete are covered up to class C90/105. However, because the characteristics of higher strength concrete are different, some expressions in the Eurocode are adjusted for classes above C50/60

Differences continued

9. The 'variable strut inclination' method is used in for the assessment of the shear capacity of a section
10. Serviceability checks can still be carried out using 'deemed to satisfy' span to effective depth rules similar to BS 8110
11. The rules for determining the anchorage and lap lengths are more complex than the simple tables in BS 8110

IS EN 1992-1-1:2005



Twelve sections:

Section 1: General

Section 2: Basis of design

Section 3: Materials

Section 4: Durability and cover to reinforcement

Section 5: Structural analysis

Section 6: Ultimate limit states

Section 7: Serviceability limit states

Section 8: Detailing of reinforcement and prestressing tendons -
General

Section 9: Detailing of members and particular rules

Section 10: Additional rules for precast concrete elements and
structures

Section 11: Lightweight aggregate concrete structures

Section 12: Plain and lightly reinforced concrete structures

Section 1 General Information

- Scope of code
- Principles and application rules
- Symbols

EC 2	M_{Ed}	V_{Ed}	b	d	d_2	A_s, A_{s1}	A_{s2}	x	z	N_{Ed}	I_0
BS 8110	M	V	b	d	d'	A_s	A_s'	x	z	N	I_e

Section 2 Basis of Design

Refers to IS EN 1990 / IS EN 1991 for design life, limit state principles, actions, etc.

2.4.2.2 Partial factors for materials UK values

Table 2.1N: Partial factors for materials for ultimate limit states

Design situations	γ_c for concrete	γ_s for reinforcing steel	γ_s for prestressing steel
Persistent & Transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0

Section 3 Materials

Concrete: Table 3.1

Strength classes for concrete															Analytical relation / Explanation
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90	
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105	
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck} + 8$ (MPa)
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0	$f_{ctm} = 0,30 \times f_{cm}^{0,67} \leq C50/60$ $f_{ctm} = 2,12 \cdot \ln(1 + (f_{cm}/10)) > C50/60$
$f_{ctk,0,05}$ (MPa)	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5	$f_{ctk,0,05} = 0,7 \times f_{ctm}$ 5% fractile
$f_{ctk,0,95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6	$f_{ctk,0,95} = 1,3 \times f_{ctm}$ 95% fractile
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44	$E_{cm} = 22[(f_{cm}/10)]^{0,3}$ (f_{cm} in MPa)
ϵ_{c1} (‰)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8	see Figure 3.2 $\epsilon_{c1}(\text{‰}) = 0,7 \cdot f_{cm}^{0,25} < 2,6$
ϵ_{cut} (‰)	3,5									3,2	3,0	2,8	2,8	2,8	see Figure 3.2 for $f_{ck} \geq 50$ Mpa $\epsilon_{cut}(\text{‰}) = 2,8 + 27[(98 - f_{cm})/100]^4$
ϵ_{c2} (‰)	2,0									2,2	2,3	2,4	2,5	2,6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa $\epsilon_{c2}(\text{‰}) = 2,0 + 0,085(f_{ck} - 50)^{0,25}$
ϵ_{cu2} (‰)	3,5									3,1	2,9	2,7	2,6	2,6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu2}(\text{‰}) = 2,6 + 35[(90 - f_{ck})/100]^4$
n	2,0									1,75	1,6	1,45	1,4	1,4	for $f_{ck} \geq 50$ Mpa $n = 1,4 + 23,4[(90 - f_{ck})/100]^4$
ϵ_{c3} (‰)	1,75									1,8	1,9	2,0	2,2	2,3	see Figure 3.4 for $f_{ck} \geq 50$ Mpa $\epsilon_{c3}(\text{‰}) = 1,75 + 0,55[(f_{ck} - 50)/40]$
ϵ_{cu3} (‰)	3,5									3,1	2,9	2,7	2,6	2,6	see Figure 3.4 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu3}(\text{‰}) = 2,6 + 35[(90 - f_{ck})/100]^4$

Concrete Table 3.1 (extract)

f_{ck} MPa	25	30	35	40	45	50
f_{cu} MPa	30	37	45	50	55	60
f_{ctm} MPa	2.6	2.9	3.2	3.5	3.8	4.1
E_{cm} GPa	31	33	34	35	36	37
ϵ_{c2}	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035

Irish N.A. may provide data for C28/35 & C32/40

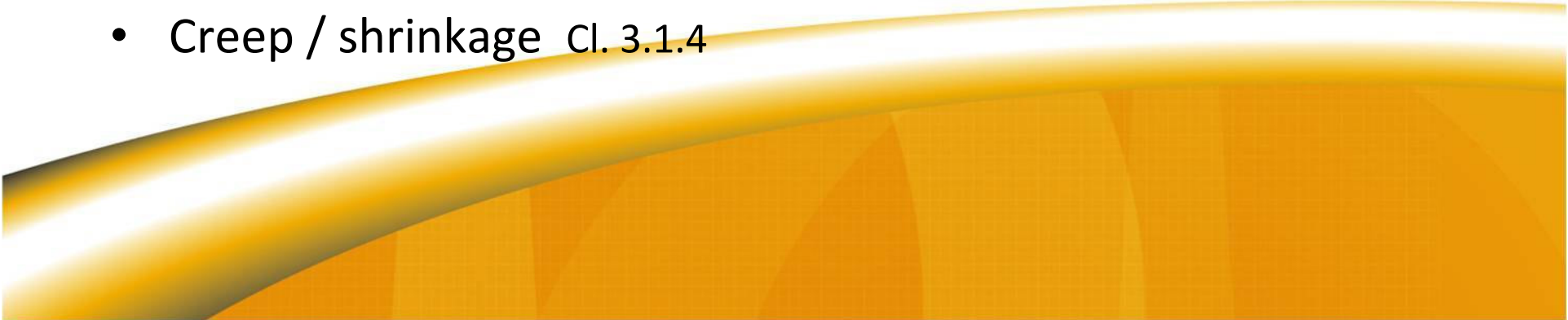
- Concrete design strength Cl. 3.1.6

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c}$$

α_{cc} = coefficient; 0.85 flexure & axial load, 1.0 shear

UK values: Irish N.A. may change α_{cc} in range 0.85 – 1.0

$$\gamma_c = 1.5$$

- Poisson's ratio = 0.2 Cl. 3.1.3 (4)
 - Coefficient of thermal expansion 10E-6/k Cl. 3.1.3 (5)
 - Creep / shrinkage Cl. 3.1.4
- 

Steel Reinforcement Cl. 3.2.2

- Ranges from 400 to 600 MPa, generally 500MPa
- Bar sizes unchanged
- Modulus of elasticity, $E_s = 200\text{GPa}$
- Mild steel reinforcement not covered

$$f_{yd} = \frac{f_{yk}}{\gamma_s}$$

$\gamma_s = \text{partial factor for steel} = 1.15$

Section 4 Durability and Cover

Cl. 4.4.1.1 (2)

$$C_{\text{nom}} = C_{\text{min}} + \Delta C_{\text{dev}}$$

$$C_{\text{min}} = \max\{C_{\text{min,b}}; C_{\text{min,dur}}\}$$

$C_{\text{min,b}}$ from Table 4.2 (generally bar size)

$C_{\text{min,dur}}$ from BS 8500 UK: IRISH N.A. & IS EN 206

$\Delta C_{\text{dev}} = 10\text{mm UK N. A.}$



Cover for Fire Protection

EN 1992-1-2 Typical dimensions / axis distance to satisfy fire resistance

Fire Resistance	Beam		One-way solid slab		Braced column	
	Simply Supported b_{min}/a (mm)	Continuous b_{min}/a (mm)	Simply Supported h_{min}/a (mm)	Continuous h_{min}/a (mm)	Exposed on one side b_{min}/a (mm)	Exposed on more than one side b_{min}/a (mm)
R60	120/40 160/35 200/30 300/25	120/25 200/12	80/20	80/10	155/25	250/46 350/40
R90	150/55 200/45 300/40 400/35	150/35 250/25	100/30	100/15	155/25	350/53 450/40
R120	200/65 240/65 300/55 500/50	200/45 300/35 450/35 500/30	120/40	200/20	175/35	350/57 450/51
R240	280/90 350/80 500/75 700/70	280/75 500/60 650/60 700/50	175/65	280/40	295/70	

Notes

b_{min}, h_{min} = beam or column width

a = axis distance, generally distance to centre of reinforcing bar

Section 5 Structural Analysis

5.1.1 Common idealisations of the behaviour used for analysis are:

- linear elastic behaviour (Cl. 5.4)
- linear elastic behaviour with limited redistribution (Cl. 5.5)
- plastic behaviour at ULS including strut and tie models (Cl. 5.6)
- non-linear behaviour (Cl. 5.7)

5.2 Geometric imperfections

Structure assumed to be out of plumb with inclination of 1/200. Analysis must include an equivalent horizontal load acting **with** the other actions such as wind

5.8 second order effects with axial loads (columns)

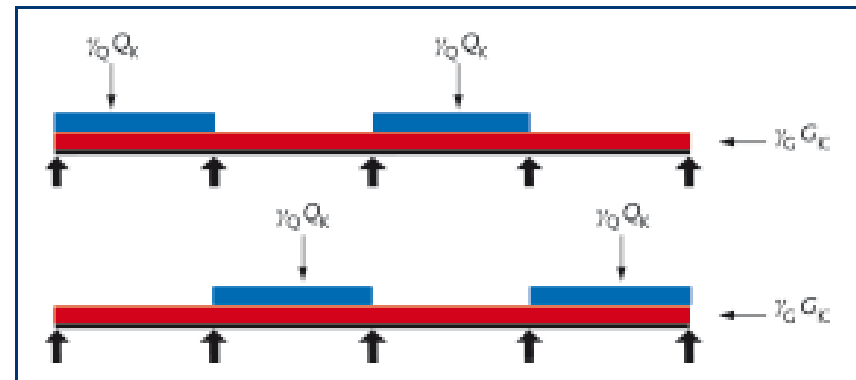
Load Cases and Combinations Continuous Beams

5.1.3(1) permits analysis based on either:

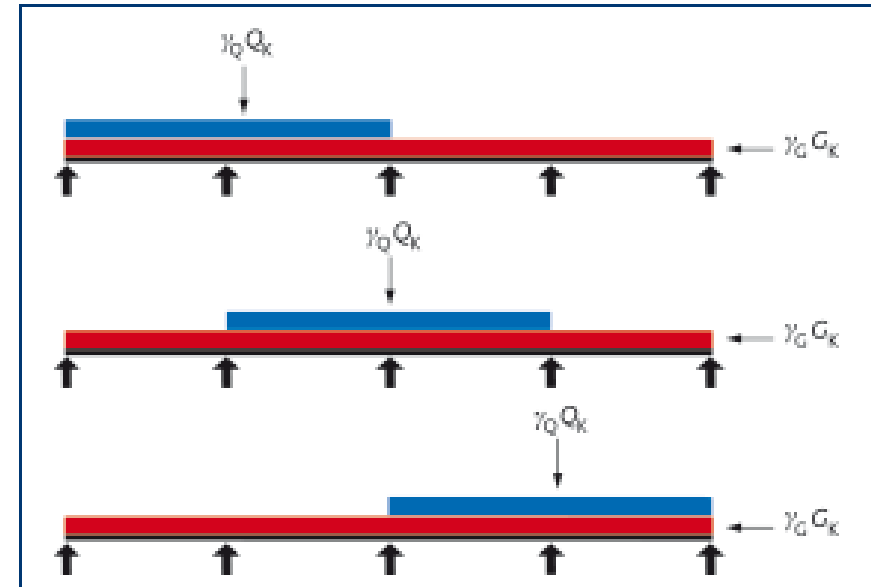
- (a) Alternate spans carrying the design variable and permanent load, and other spans carrying the permanent load
- (b) Any two adjacent spans carrying the variable and permanent load, and all other spans carrying only the design permanent load

UK NA recommends (a), which leads to three load cases considered

Alternate spans loaded



Adjacent spans loaded

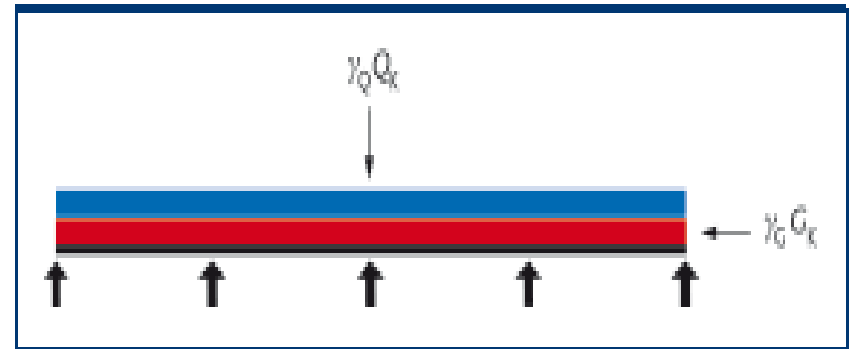


Floor slab simplification

UK N.A. recommends

analysis based on all spans
loaded where:

- (a) For one way spanning
slabs with bay areas $> 30\text{m}^2$
- (b) Ratio of variable to
permanent load ≤ 1.25
- (c) Characteristic variable load
does not exceed 5 kN/m^2
excluding partitions



Bending moment and shear co-efficients for beams

	Moment	Shear
Outer support	25% of span moment	$0.45 (G + Q)$
Near middle of end span	$0.090 Gl + 0.100 Ql$	
At first interior support	$- 0.094 (G + Q)l$	$0.63 (G + Q)^a$
At middle of interior spans	$0.066 Gl + 0.086 Ql$	
At interior supports	$- 0.075 (G + Q)l$	$0.50 (G + Q)$

Key

a $0.55 (G + Q)$ may be used adjacent to the interior span.

Notes

- 1** Redistribution of support moments by 15% has been included.
- 2** Applicable to 3 or more spans only and where $Q_k \leq G_k$.
- 3** Minimum span ≥ 0.85 longest span.
- 4** l is the effective length, G is the total of the ULS permanent actions, Q is the total of the ULS variable actions.

Effective Width of Flanges

5.3.2.1

The effective flange width b_{eff} of T and L beams is determined from:

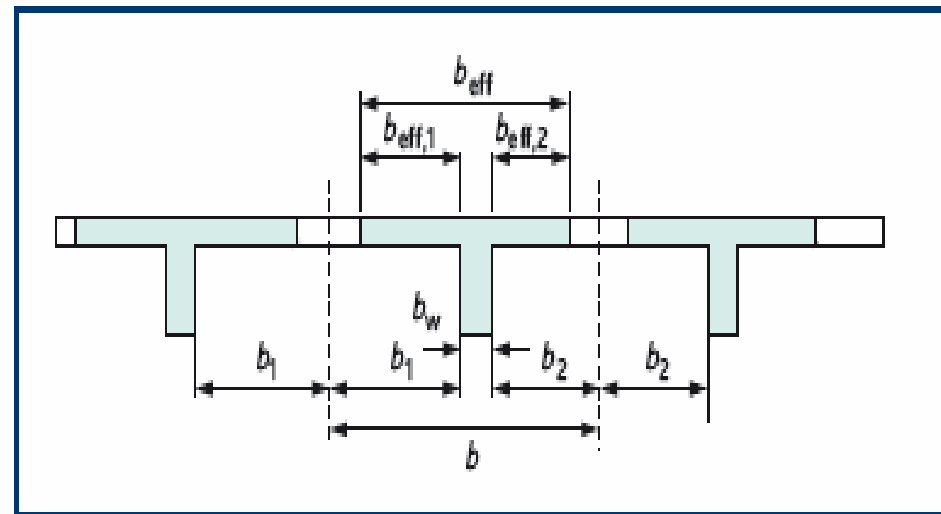
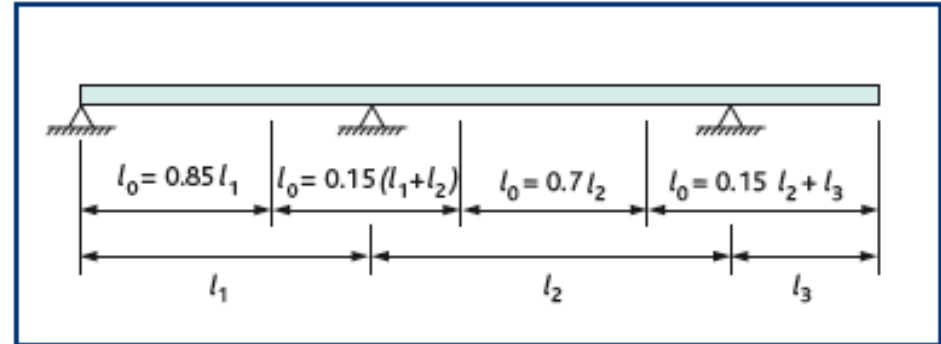
$$b_{eff} = \sum b_{eff,j} + b_w \leq b$$

where

$$b_{eff,j} = 0,2b_1 + 0,1l_0 \leq 0,2l_0$$

and

$$b_{eff,j} \leq b_1$$



Effective Span of Beams and Slabs

5.3.2.2 The effective span l_{eff} of a beam or slab is:

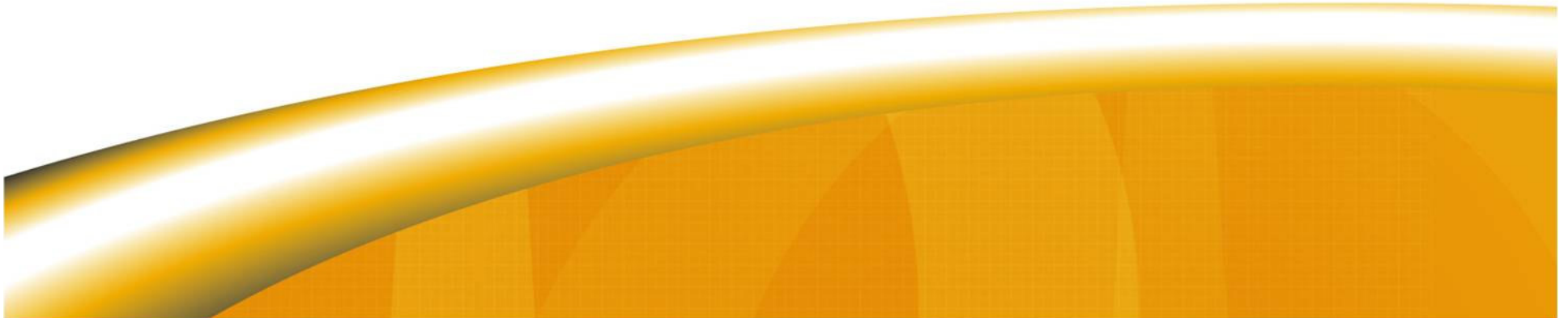
$$l_{eff} = l_n + a_1 + a_2$$

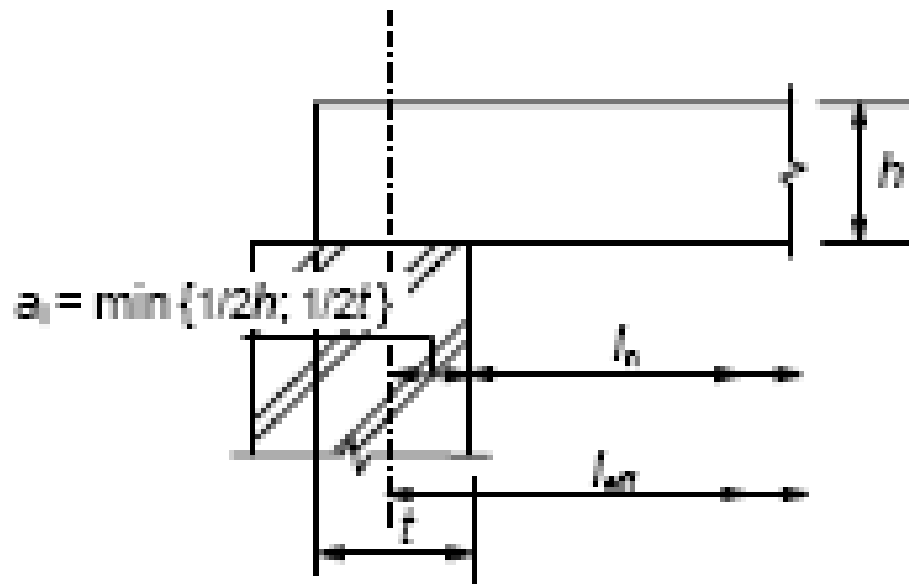
Generally the lesser of:

Clear span + $h/2$ or

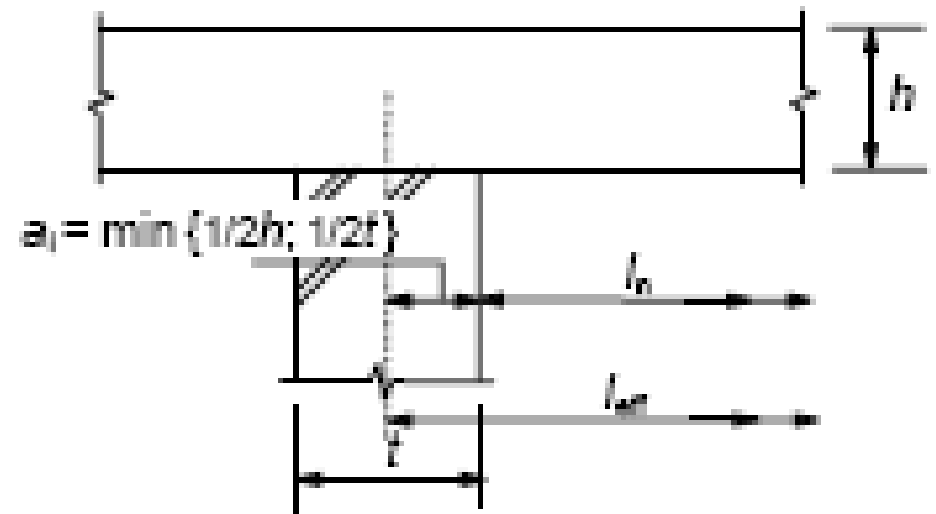
Clear span + $t/2$

Where t is the width of the support





(a) Non-continuous members



(b) Continuous members

Extract Figure 5.4

Section 6 Ultimate Limit State

Bending with or without axial force

6.1 (2)

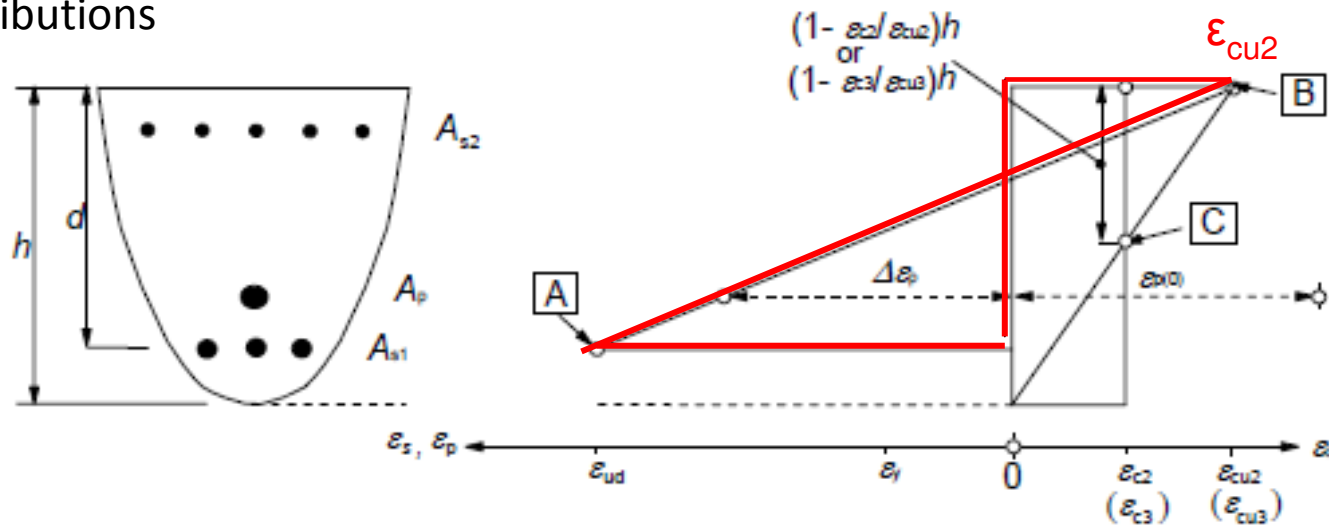
Assumptions

- Plane sections remain plane after bending so that:
 - (a) the strains are linearly proportional to the distance to the neutral axis and
 - (b) the strain in the concrete is equal to the strain in the reinforcement at the same depth in the section
- The tensile strength of the concrete is ignored and no contribution is taken for the concrete below the neutral axis in tension



Strain distribution at ULS

Range of possible strain distributions



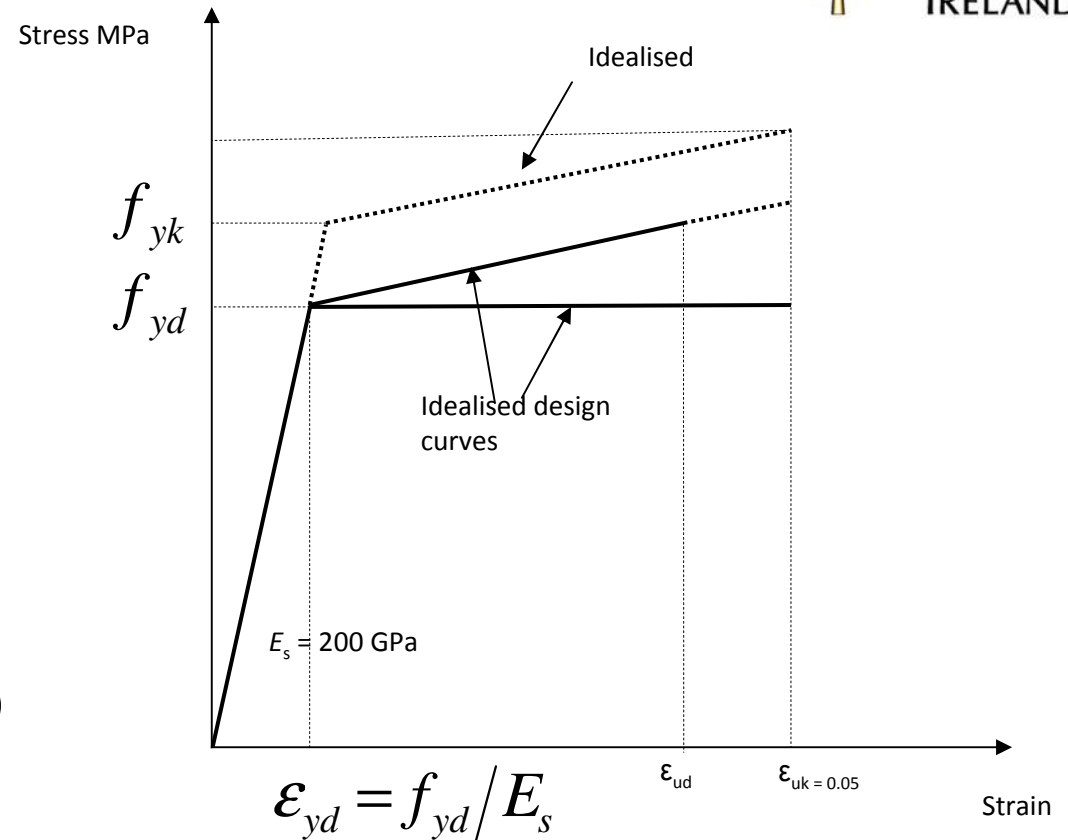
- A** - reinforcing steel tension strain limit
- B** - concrete compression strain limit
- C** - concrete pure compression strain limit

Figure 6.1: Possible strain distributions in the ultimate limit state

6.1(2)P Stress in the reinforcement

- The stress in the reinforcement is derived from its stress-strain curve given in Figure 3.8

$$\begin{aligned}\epsilon_{yd} &= f_{yd} / E_s \\ &= 500 / (200 * E3 * 1.15) \\ &= 0.0022\end{aligned}$$



Where: f_t = tensile strength of the reinforcement

f_{yk} = yield strength of the reinforcement

Figure 3.8

6.1 (3)P Stress in the concrete

- The ultimate strain in the concrete is $\frac{0.85f_{ck}}{\gamma_c}$
 $\epsilon_{cu2} = 0.0035$ from Table 3.1
- The stress in the concrete is obtained from the stress-strain curve Figure 3.3

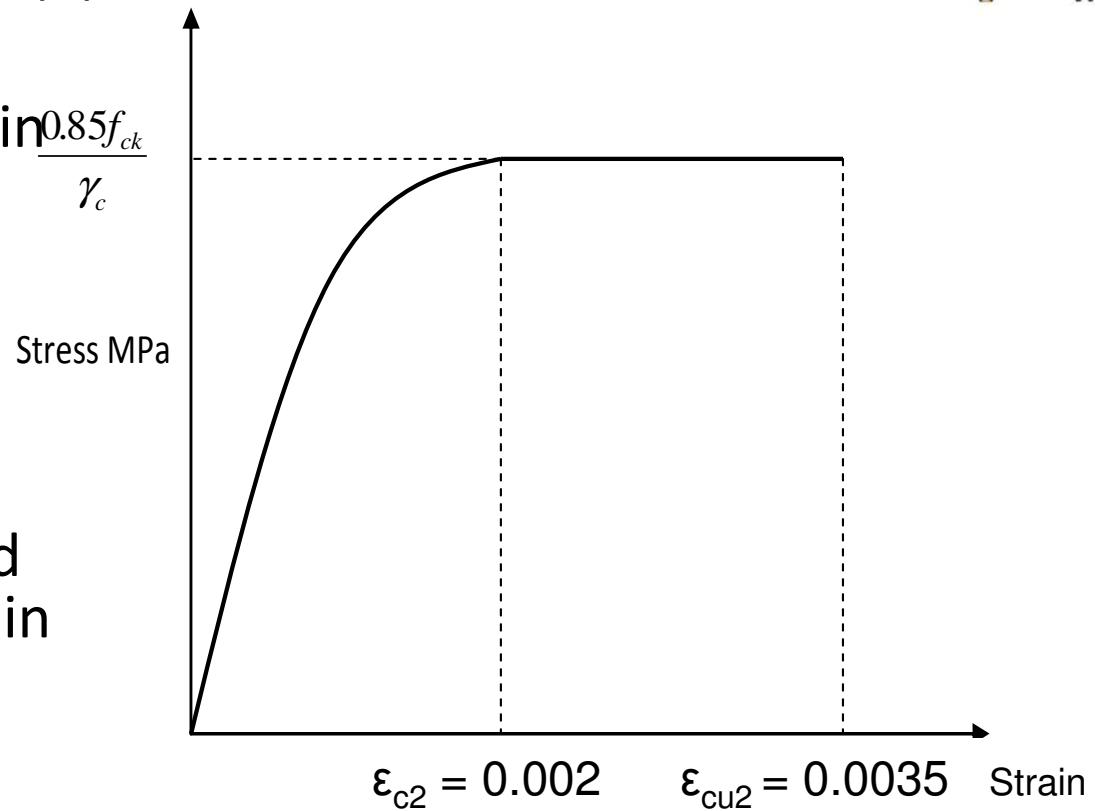
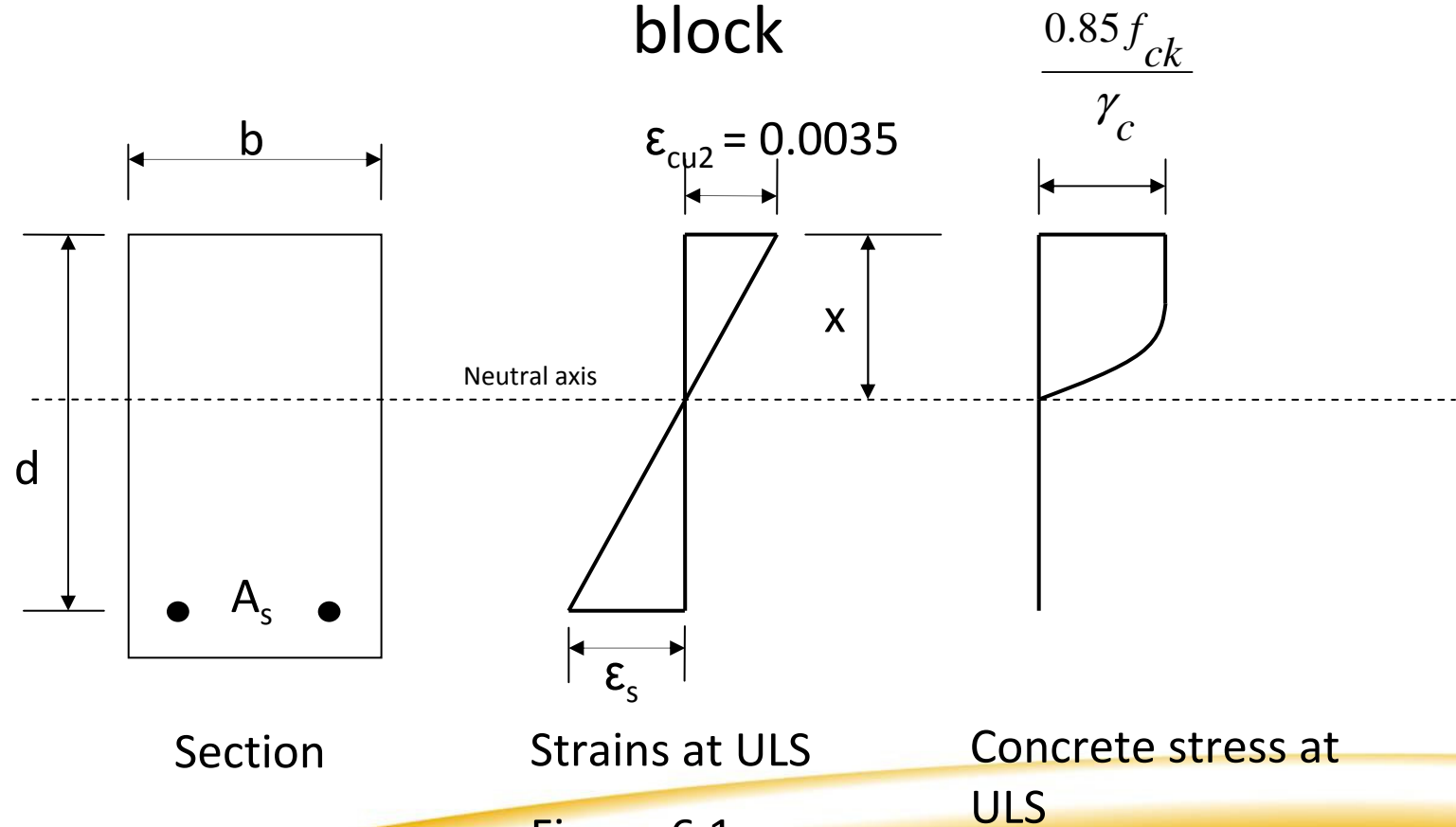


Figure 3.3

Singly reinforced beams rectangular – parabolic stress block



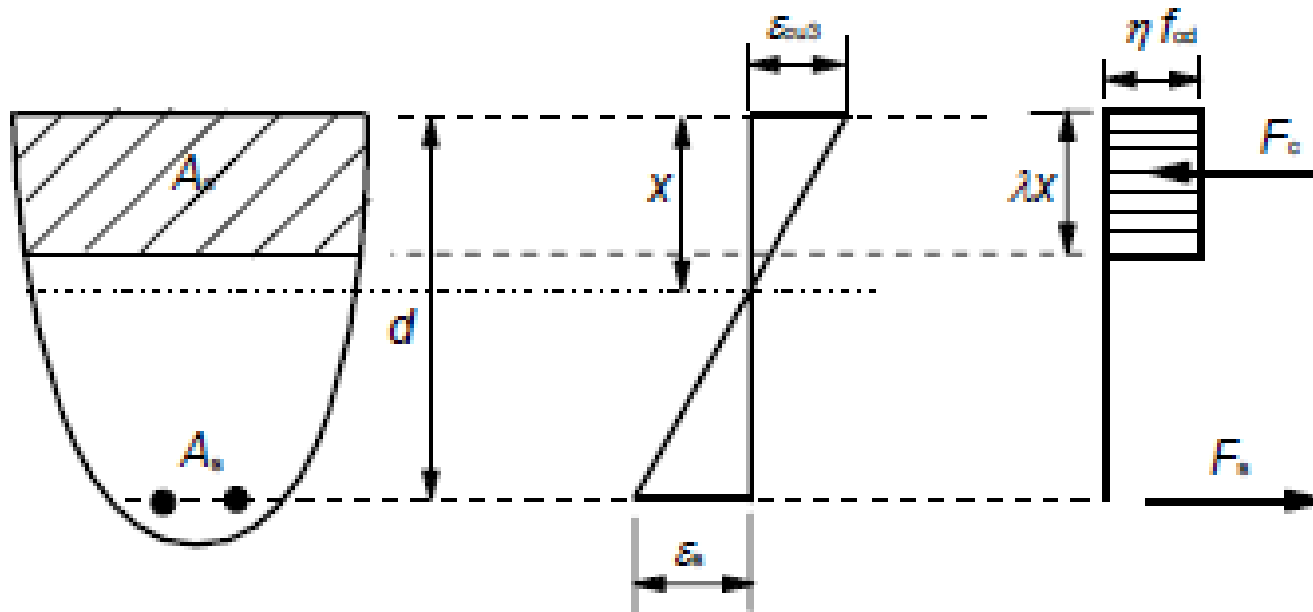
Strains at ULS

Concrete stress at
ULS

Figure 6.1

'for use'

EC2 Rectangular Stress Block Cl. 3.1.7 Figure 3.5



$$\lambda = 0,8 \quad \text{for } f_{ck} \leq 50 \text{ MPa}$$

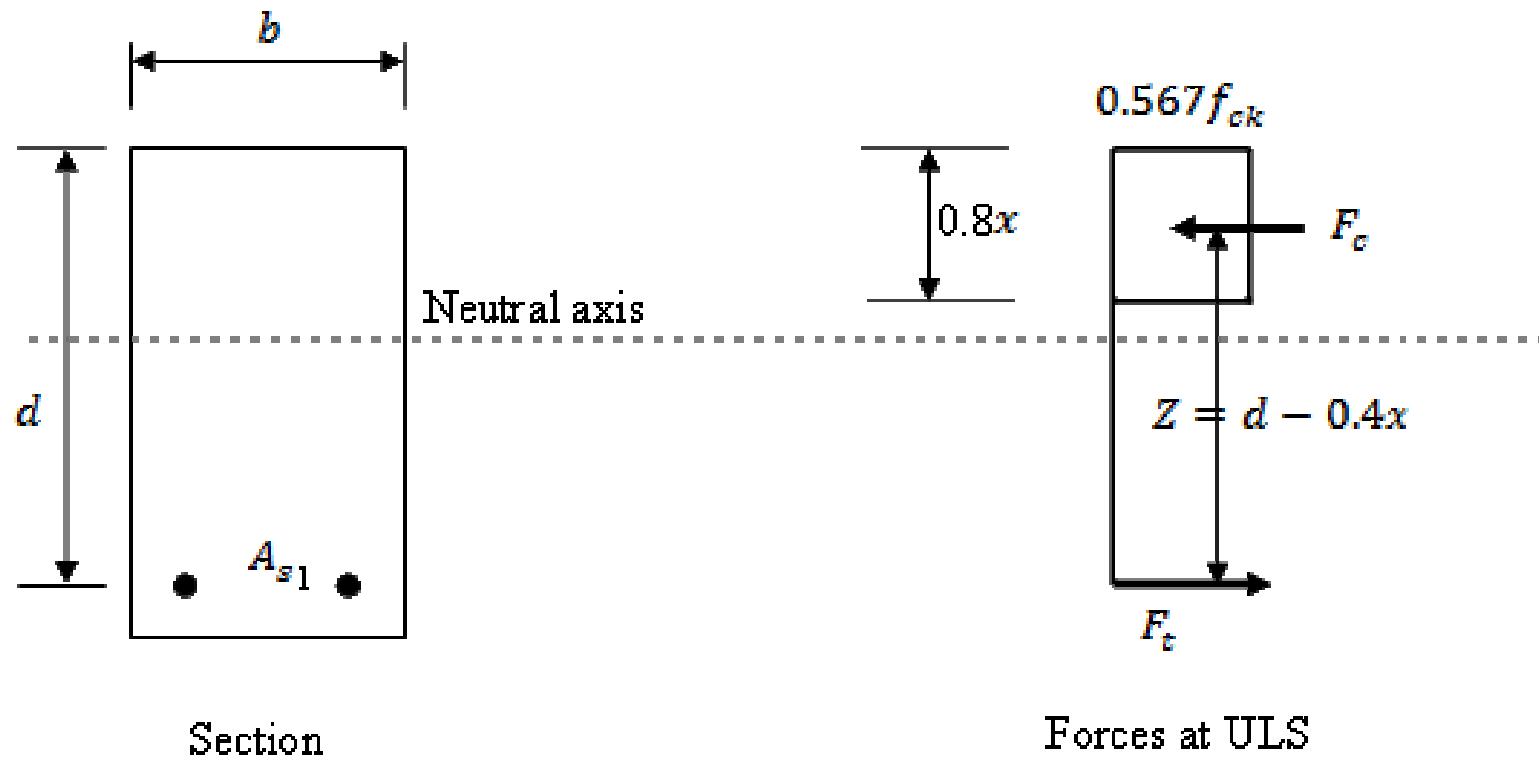
$$\lambda = 0,8 - (f_{ck} - 50)/400 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa}$$

and

$$\eta = 1,0 \quad \text{for } f_{ck} \leq 50 \text{ MPa}$$

$$\eta = 1,0 - (f_{ck} - 50)/200 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa}$$

Singly Reinforced Sections



Design Equations

Moment of resistance based on concrete reaching ULS

$$M_R = F_c Z$$

$$F_c = 0.567 f_{ck} (0.8x) b$$

$$Z = d - 0.4x$$

$$M_R = (0.567 f_{ck} (0.8x) b) (Z)$$

$$M_{Ed} = M_{\underline{R}}$$

for equilibrium at ULS

Let

$$k = \frac{M_{Ed}}{bd^2 f_{ck}}$$

And let

$$x = \frac{d - z}{0.4}$$

Hence

$$k b d^2 f_{ck} = \left(0.45 f_{ck} b \left(\frac{z - d}{0.4} \right) \right) (z)$$

Therefore

$$\frac{z^2}{d^2} - \frac{z}{d} + \frac{k}{1.134} = 0$$

Which is a quadratic equation in terms of $\frac{M}{bd^2}$ With the positive root of:

$$Z = d \left\{ 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right\}$$

Which is an equation for the lever arm in terms of k and d

The area of steel reinforcement required to resist M can be derived from:

$$M_{Ed} = F_t Z = \frac{f_{yk}}{\gamma_m} A_s Z$$

With $\gamma_m = 1.15$

Therefore:

$$A_s = \frac{M_{Ed}}{0.87 f_{yk} Z}$$

Summary: Design Equations

$$k = \frac{M_{Ed}}{bd^2 f_{ck}}$$

$$Z = d \left\{ 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right\}$$

$$A_s = \frac{M_{Ed}}{0.87 f_{yk} Z}$$

Limit on k

These derived equations can be used to design the reinforcement in a singly reinforced beam subject to the limits on the lever arm of:

1. Balanced design $x = 0.636d$, which limits $Z = 0.75d$ as a minimum
2. Maximum $x = 0.45d$ (5.6.3(2)), which limits $Z = 0.82d$ as a minimum

These limits expressed in terms of k are:

1. Balanced design $x = 0.636d$ or $Z = 0.75d$ substitutes to give:

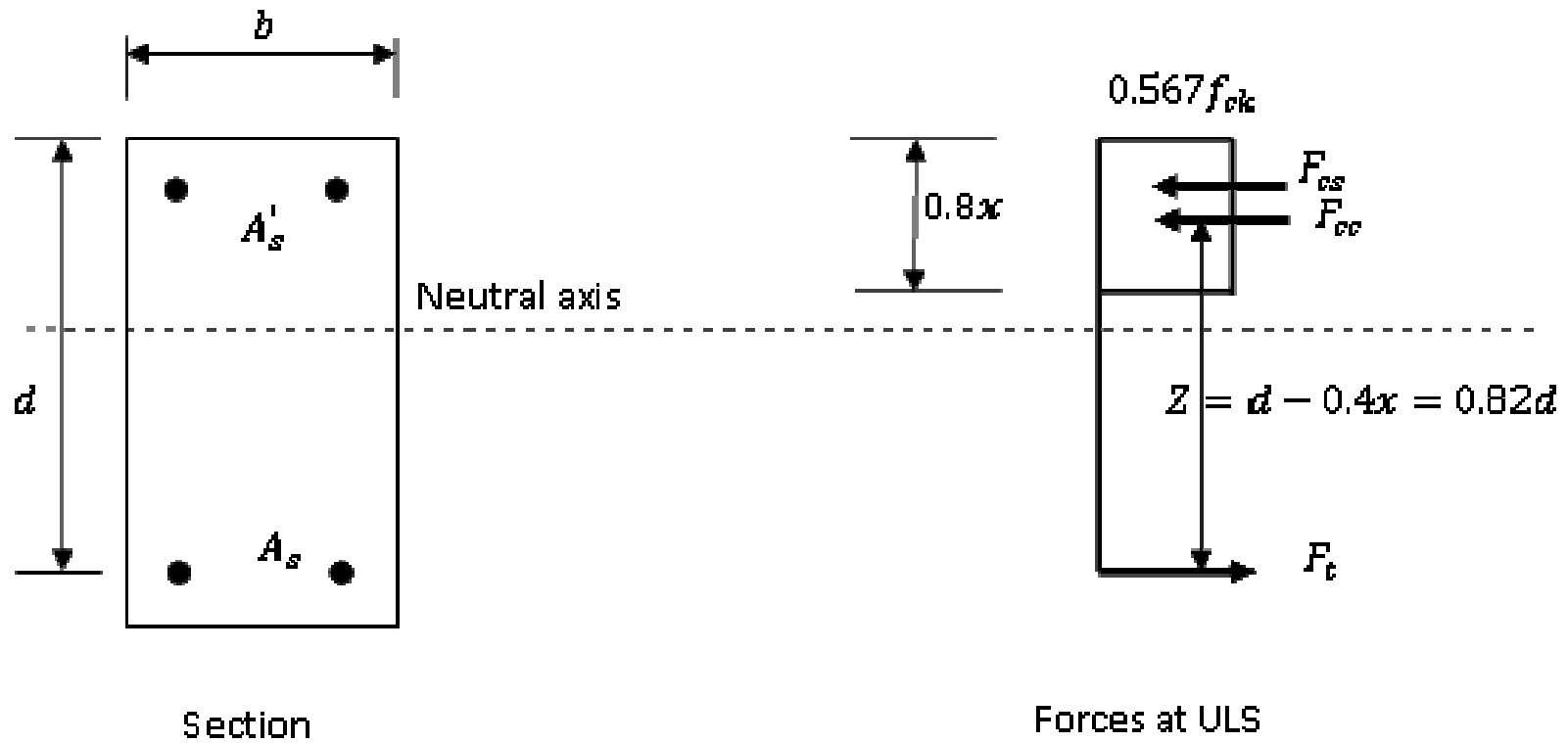
$$0.75d = d \left\{ 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right\}$$

Which solves for $k = 0.207$ maximum

2. Maximum $x = 0.45d$ or $Z = 0.82d$
solves for $k = 0.167$ maximum

It is also good practice to avoid failure by premature crushing of weak concrete near the top of the section $Z = 0.95d$ as a maximum

Doubly Reinforced Beams



For the EC2 limit $x = 0.45d$ the equilibrium of forces is:

$$0.87f_{yk}A_{s1} = 0.567f_{ck}b(0.8)(0.45d) + 0.87f_{yk}A_{s2}$$

Equation A

$$M = F_{cc}(0.82d) + F_{sc}(d - d')$$

$$= 0.167f_{ck}bd^2 + 0.87f_{yk}A_{s2}(d - d')$$

$$A_{s2} = \frac{M - 0.167f_{ck}bd^2}{0.87f_{yk}(d - d')}$$

Equation B



By multiplying both sides of Equation A by $Z = 0.82d$ and rearranging gives

$$A_{s1} = \frac{0.167 f_{ck} b d^2}{0.87 f_{yk} (Z_{bal})} + A_{s2}$$

Equation C

With $Z_{bal} = 0.82d$

Substitution of

$$k_{bal} = 0.167 \text{ and } k = \frac{M}{b d^2 f_{ck}}$$

into Equation B and Equation C converts them to

$$A_{s2} = \frac{(k - k_{bal}) f_{ck} b d^2}{0.87 f_{yk} (d - d')}$$

And

$$A_s = \frac{k_{bal} f_{ck} b d^2}{0.87 f_{yk} Z_{bal}} + A_{s1}$$

Shear

Section 6.2

- The strut inclination method is used for shear capacity checks
- The shear is resisted by concrete struts in compression and shear reinforcement acting in tension
- Shear formulae expressed in terms of force rather than stress
- Designer free to choose a strut angle $22^\circ \leq \theta \leq 45^\circ$

BO1

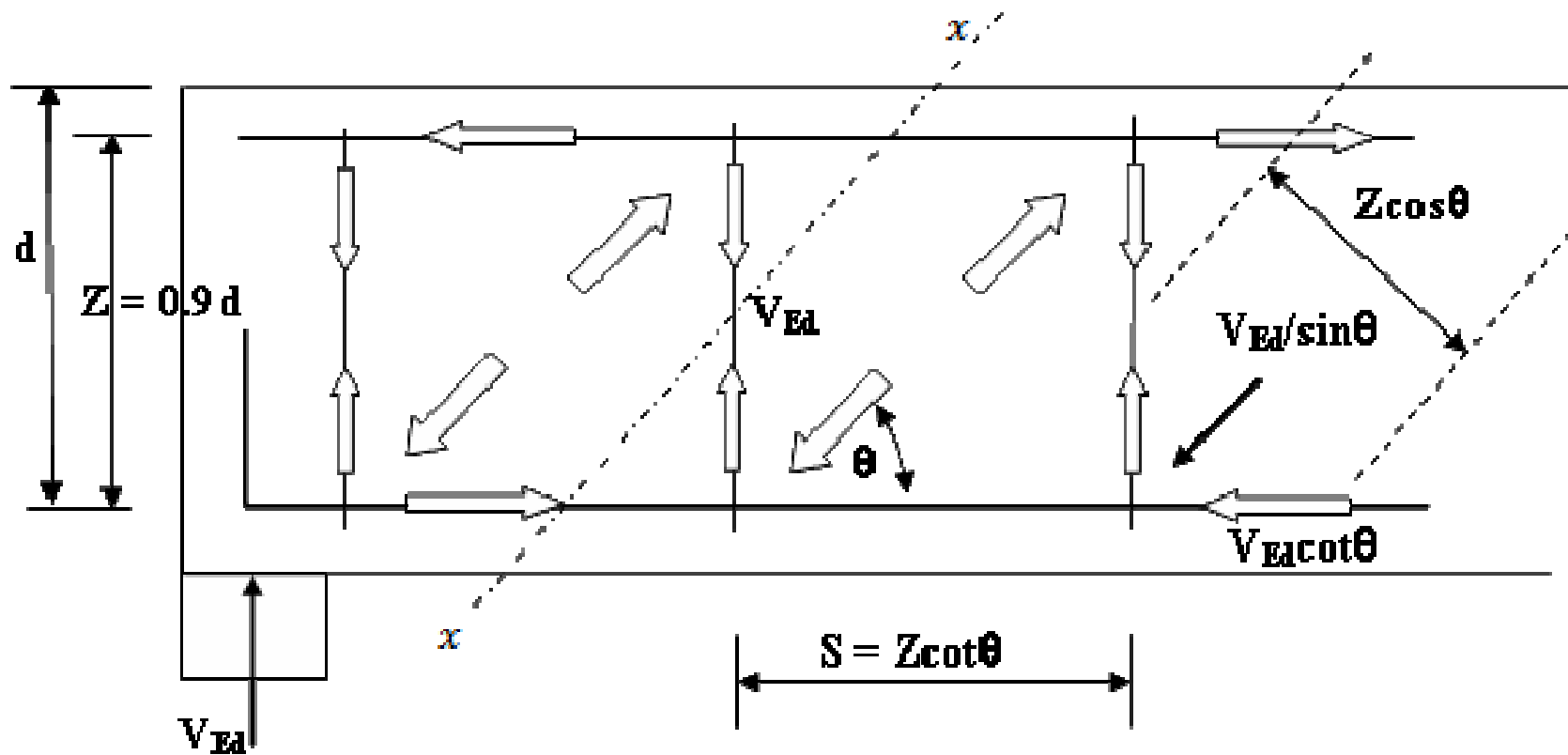


Slide 38

BO1

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Strut Inclination Method



notation

$V_{Rd,c}$ is the design shear resistance of the member without shear reinforcement.

$V_{Rd,s}$ is the design value of the shear force which can be sustained by the yielding shear reinforcement.

$V_{Rd,max}$ is the design value of the maximum shear force which can be sustained by the member, limited by crushing of the compression struts.

Design equations are derived as follows:

The maximum design strength of the concrete strut

= Ultimate design strength x cross-sectional area

$$= (f_{ck}/1.5)(b_w Z \cos\theta)$$

And its vertical component

$$= [(f_{ck}/1.5)(b_w Z \cos\theta)] \sin\theta$$

this is the maximum vertical shear that can be resisted by the concrete strut, $V_{Rd,max}$

Trigonometrical conversion yields:

$$V_{Rd,max} = \frac{f_{ck} b_w Z}{1.5(\cot\theta + \tan\theta)}$$

In EC2 this equation is modified by a strength reduction, v_1 factor for concrete cracked in shear:

$$v_1 = 0.6(1 - f_{ck}/250)$$

$$\text{And } Z = 0.9d$$

Therefore:

$$V_{Rd,max} = \frac{v_1 f_{ck} b_w 0.9d}{1.5(\cot\theta + \tan\theta)}$$

When the design shear force, V_{Ed} exceeds $V_{Rd,c}$ shear links must be provided.
Their area and spacing is obtained by taking a method of sections cut at x-x

The vertical shear force in the link, V_{wd} is:

$$\begin{aligned} V_{wd} &= V_{Ed} = f_{ywd} A_{sw} \\ &= \frac{f_{yk} A_{sw}}{1.15} \\ &= 0.87 f_{yk} A_{sw} \end{aligned}$$

If the links are spaced at s then the force in each link is proportionately:

$$V_{Ed} \frac{s}{z \cot \theta} = 0.87 f_{yk} A_{sw}$$

The shear resistance must equal the shear applied hence and by rearrangement:

$$V_{Rd} = V_{Rd,s} = \frac{A_{sw}}{s} 0.78 d f_{yk} \cot \theta$$

EC2 minimum links are:

$$\frac{A_{sw,min}}{s} = \frac{0.8f_{ck}^{0.5}b_w}{f_{yk}}$$

For minor members that do not require shear reinforcement the shear capacity is given by an empirical equation:

$$V_{Rd,c} = \left[C_{Rd,c} k (100\rho_1 f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \right] b_w d$$

With a minimum value of:

$$V_{Rd,c} = (v_{min} + k_1 \sigma_{cp}) b_w d$$

Where

$$C_{Rd,c} = \frac{0.18}{\gamma_c}$$

$$\rho_1 = \frac{A_{sl}}{b_w d} \leq 0.02$$

A_{sl} is taken from Figure 6.3 in EC2

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$$

$$k_1 = 0.15$$

$$\sigma_{cp} = \text{axial stress}$$

Shear Formulae Summary

$$V_{Rd,max} = \frac{v_1 f_{ck} b_w 0.9d}{1.5(\cot\theta + \tan\theta)} \quad \approx \text{EC2 EQN.6.9}$$

$$V_{Rd} = V_{Rd,s} = \frac{A_{sw}}{s} 0.78 d f_{yk} \cot\theta \quad \approx \text{EC2 EQN.6.8}$$

$$\frac{A_{sw,min}}{s} = \frac{0.8 f_{ck}^{0.5} b_w}{f_{yk}} \quad \approx \text{EC2 EQN.9.4}$$

$$V_{Rd,c} = \left[C_{Rd,c} k (100 \rho_1 f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} \right] b_w d \quad \text{EC2 EQN.6.2a}$$

But not less than

$$V_{Rd,c} = (v_{min} + k_1 \sigma_{cp}) b_w d \quad \text{EC2 EQN.6.2b}$$

Suggested Design Procedure for Shear

1. Determine V_{Ed}
2. Calculate the concrete compressive strut capacity for $\theta = 22^\circ$ from:

$$V_{Rd,max} = \frac{v_1 f_{ck} b_w 0.9d}{1.5(\cot\theta + \tan\theta)}$$

3. If $V_{Rd,max} 22^\circ \geq V_{Ed}$ proceed to step 6
4. If $V_{Rd,max} 22^\circ \leq V_{Ed}$ check that the strut angle lies between 22° and 45° by calculating $V_{Rd,max} 45^\circ$
5. Determine the strut angle from:

$$\theta = 0.5 \sin^{-1} \left\{ \frac{V_{Ed}}{V_{Rd,max} (45^\circ)} \right\} \leq 45^\circ$$

6. Determine the area and spacing of the shear links from:

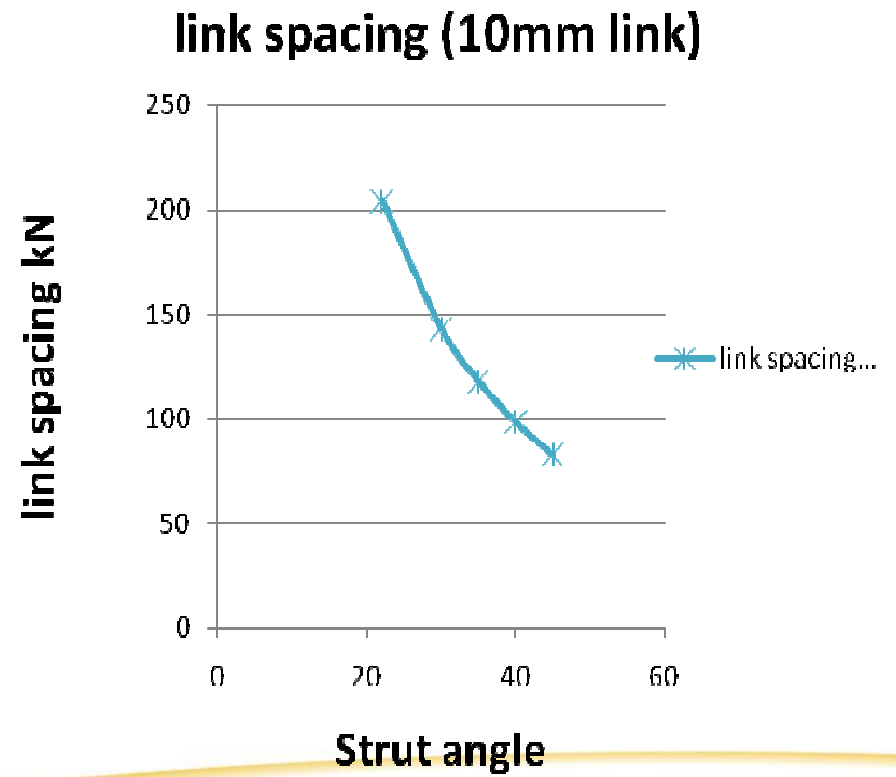
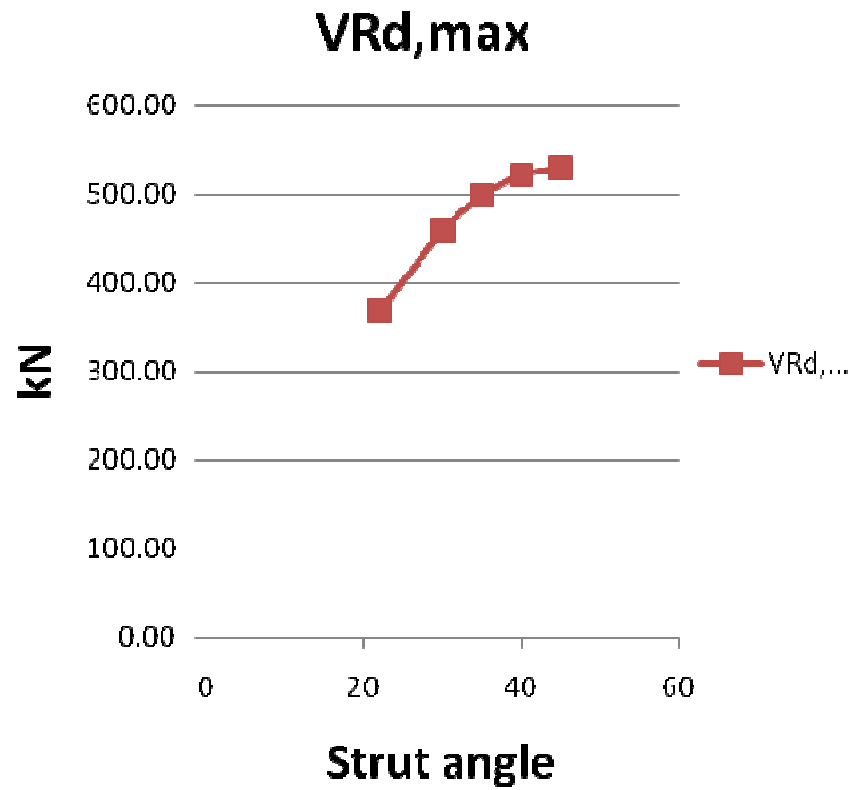
$$V_{Ed} = V_{Rd,s} = \frac{A_{sw}}{s} 0.78d f_{yk} \cot\theta$$

7. Check minimum links from:

$$\frac{A_{sw,min}}{s} = \frac{0.8 f_{ck}^{0.5} b_w}{f_{yk}}$$

8. Check link spacing maximum $0.75d$
9. Calculate additional longitudinal force in tension reinforcement.

Strut angle choice 22° - 45°



Cl. 6.4 Punching Shear

- Basic control perimeter radius at corners
- Located at $2d$ from the face of the loaded area

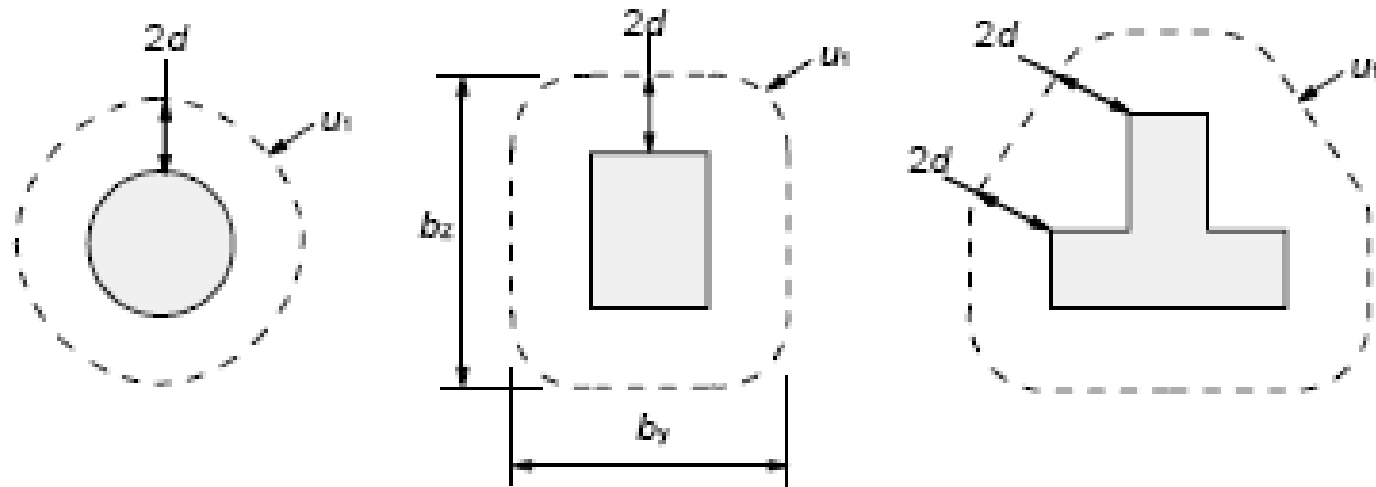


Figure 6.13: Typical basic control perimeters around loaded areas

Section 7 S.L.S.

Cl. 7.4 SLS Deflection

The serviceability limit state of deflection can be checked using span-effective depth ratios. A more rigorous approach is possible but is seldom used in practice. the verification equation is:

$$\text{Allowable } l/d = N \times k \times F1 \times F2 \times F3 \geq \text{actual } l/d$$

Where:

N = Basic span-effective depth factor

K = Element typefactor

F1 = Flange beam factor

F2 = Brittle finishes factor

F3 = reinforcement stress factor

Where:

$$\sigma_s = \frac{f_{yk}}{\gamma_c} [(G_k + \psi_2 Q_k) / (1.25G_k + 1.5Q_k)] [A_{s,reqd} / A_{s,prov}] (1/\delta)$$

Table 7.4N: Basic ratios of span/effective depth for reinforced concrete members without axial compression

Structural System	K	Concrete highly stressed $\rho = 1,5\%$	Concrete lightly stressed $\rho = 0,5\%$
Simply supported beam, one- or two-way spanning simply supported slab	1,0	14	20
End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	1,3	18	26
Interior span of beam or one-way or two-way spanning slab	1,5	20	30
Slab supported on columns without beams (flat slab) (based on longer span)	1,2	17	24
Cantilever	0,4	6	8

Note 1: The values given have been chosen to be generally conservative and calculation may frequently show that thinner members are possible.

Note 2: For 2-way spanning slabs, the check should be carried out on the basis of the shorter span. For flat slabs the longer span should be taken.

Note 3: The limits given for flat slabs correspond to a less severe limitation than a mid-span deflection of span/250 relative to the columns. Experience has shown this to be satisfactory.

Columns

- Sections 5.8 & 6.1
- Design more complex than BS 8110
- Braced / Unbraced
- Geometric imperfections must be included in M_{Ed}

Procedure is to:

- Determine the slenderness ratio, λ
- Determine the limiting slenderness, λ_{lim}
- Design for axial load and first order moments
- Include for second order effects where they occur

CI 5.8.3.2 Column Slenderness, λ

(1) The slenderness ratio is defined as follows:

$$\lambda = l_0 / i$$

where:

l_0 is the effective length, see 5.8.3.2 (2) to (7)

i is the radius of gyration of the uncracked concrete section

Braced members (see Figure 5.7 (f)):

$$l_0 = 0,5l \cdot \sqrt{\left(1 + \frac{k_1}{0,45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0,45 + k_2}\right)} \quad (5.15)$$

Unbraced members (see Figure 5.7 (g)):

$$l_0 = l \cdot \max \left\{ \sqrt{1 + 10 \cdot \frac{k_1 \cdot k_2}{k_1 + k_2}} ; \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right) \right\} \quad (5.16)$$

where:

k_1, k_2 are the relative flexibilities of rotational restraints at ends 1 and 2 respectively:



Table 5.1
Effective length l_0 : conservative factors for braced columns

End condition at top	End condition at bottom		
	1	2	3
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

Key

- Condition 1 Column connected monolithically to beams on each side that are at least as deep as the overall depth of the column in the plane considered
Where the column is connected to a foundation this should be designed to carry moment in order to satisfy this condition
- Condition 2 Column connected monolithically to beams on each side that are shallower than the overall depth of the column in the plane considered by generally not less than half the column depth
- Condition 3 Column connected to members that do not provide more than nominal restraint to rotation

Note

Table taken from *Manual for the design of concrete building structures to Eurocode 2*^[21]. The values are those used in BS 8110: Part 1: 1997^[14] for braced columns. These values are close to those values that would be derived if the contribution from adjacent columns were ignored.

Limiting slenderness λ_{lim}

5.8.3.1 Slenderness criterion for isolated members

(1) As an alternative to 5.8.2 (6), second order effects may be ignored if the slenderness λ (as defined in 5.8.3.2) is below a certain value λ_{lim} .

Note: The value of λ_{lim} for use in a Country may be found in its National Annex. The recommended value follows from:

$$\lambda_{lim} = 20 \cdot A \cdot B \cdot C / n \quad (5.13N)$$

where:

$$A = 1 / (1 + 0,2 \varphi_{ef}) \quad (\text{if } \varphi_{ef} \text{ is not known, } A = 0,7 \text{ may be used})$$

$$B = \sqrt{1 + 2\omega} \quad (\text{if } \omega \text{ is not known, } B = 1,1 \text{ may be used})$$

$$C = 1,7 - r_m \quad (\text{if } r_m \text{ is not known, } C = 0,7 \text{ may be used})$$

φ_{ef} effective creep ratio; see 5.8.4;

$\omega = A_s f_{yd} / (A_c f_{cd})$; mechanical reinforcement ratio;

A_s is the total area of longitudinal reinforcement

$n = N_{Ed} / (A_c f_{cd})$; relative normal force

$r_m = M_{01} / M_{02}$; moment ratio

M_{01}, M_{02} are the first order end moments, $|M_{02}| \geq |M_{01}|$

If the end moments M_{01} and M_{02} give tension on the same side, r_m should be taken positive (i.e. $C \leq 1,7$), otherwise negative (i.e. $C > 1,7$).

Columns where $\lambda \leq \lambda_{lim}$ and Braced

Design for N_{Ed} and M_{Ed}

5.8.8 Nominal curvature method:

$$M_{Ed} = M_{0Ed} + N_{Ed}e_i$$

M_{0Ed} = the larger end moment from analysis

e_i = the eccentricity due to geometric imperfection from
5.2(7)

$$e_i = \left(\frac{\theta_i l_0}{2} \right)$$

$$\theta_i = l / 200$$

With a minimum eccentricity = $h/30$ or 20mm from 6.1(4)

Solve using equilibrium of forces or column design charts

Columns where $\lambda \geq \lambda_{lim}$ and Braced

Design for N_{Ed} and M_{Ed}

5.8.8 Nominal curvature method:

M_{Ed} = maximum of

- (i) M_{02}
- (ii) $M_{0Ed} + M_2$
- (iii) $M_{01} + 0.5M_2$

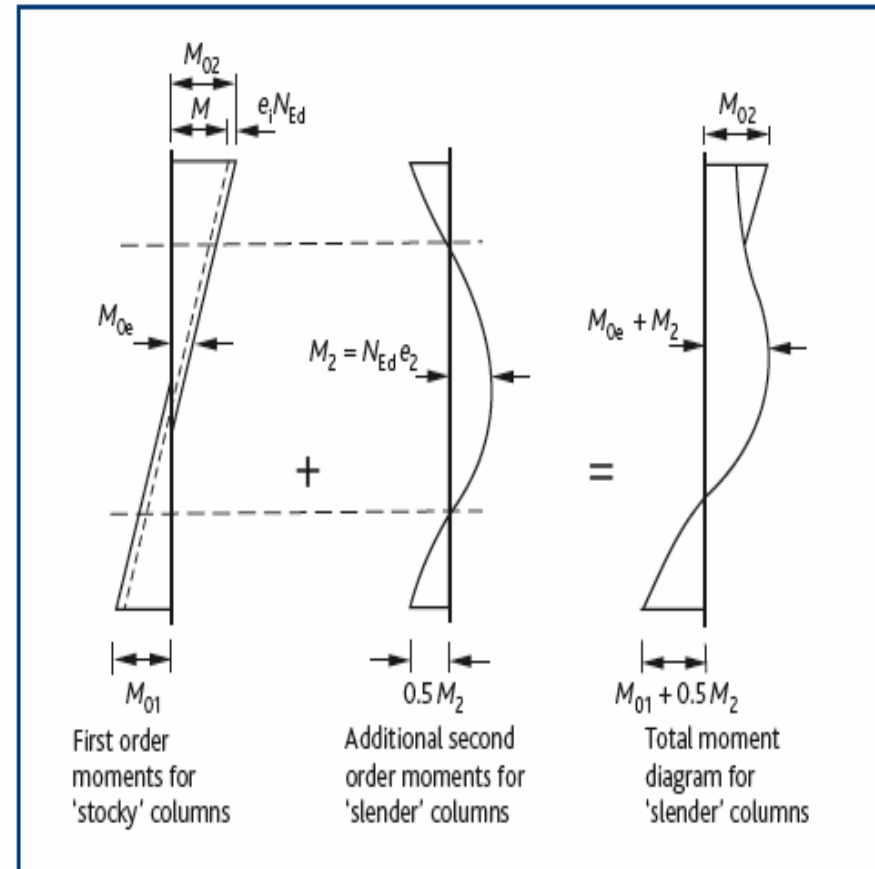
M_{0Ed} is the equivalent first order moment including the effect of imperfections at about mid-span height of the column, given as:

$$M_{0Ed} = (0.6 M_{02} + 0.4M_{01}) \geq 0.4M_{02} \text{ Exp 5.32}$$

M_2 is the nominal 2nd order moment, given as :

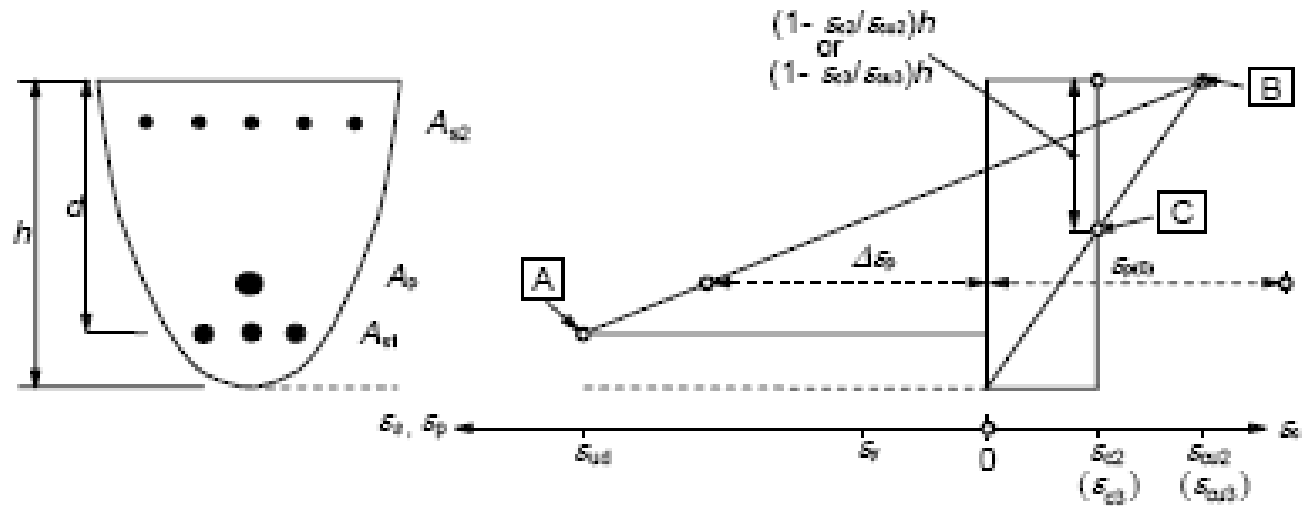
$$M_2 = N_{Ed} e_2$$

e_2 = deflection curvature from Exp 5.33



Again solve using equilibrium of forces or column design charts

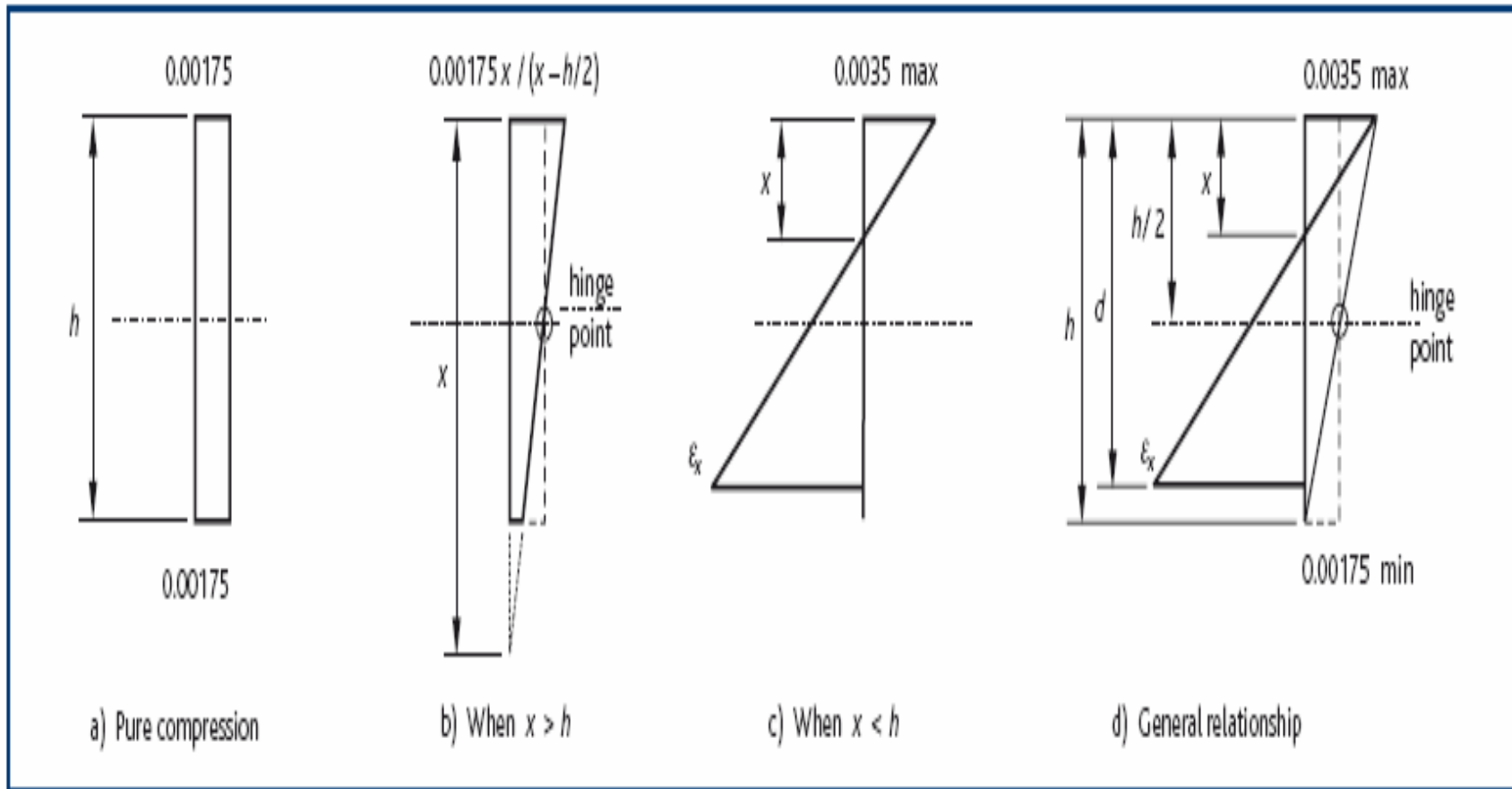
ULS Strain distribution Figure 6.1 for solution by equilibrium



- A** - reinforcing steel tension strain limit
- B** - concrete compression strain limit
- C** - concrete pure compression strain limit

Figure 6.1: Possible strain distributions in the ultimate limit state

Figure 6.1 column strain relationships



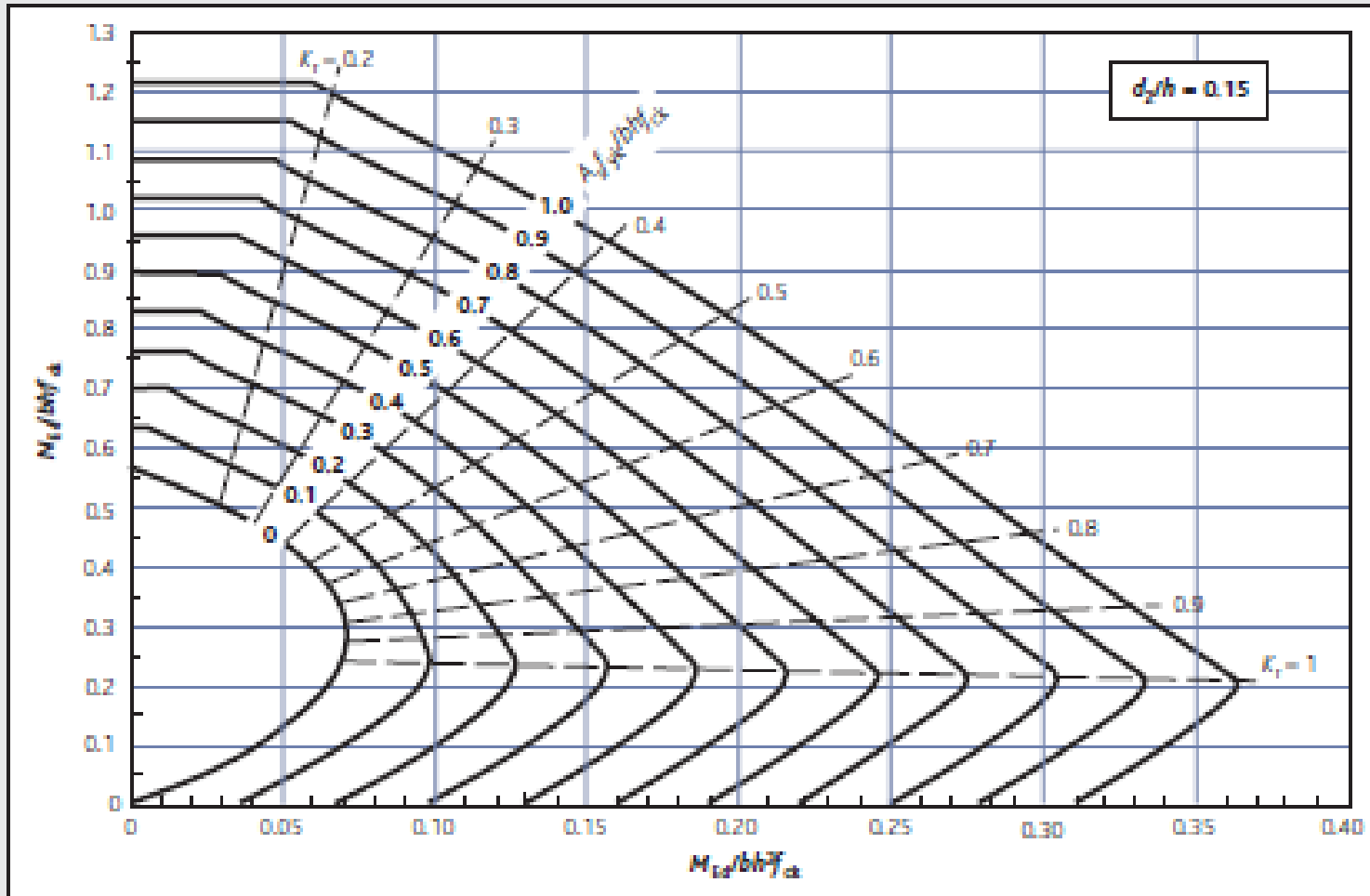
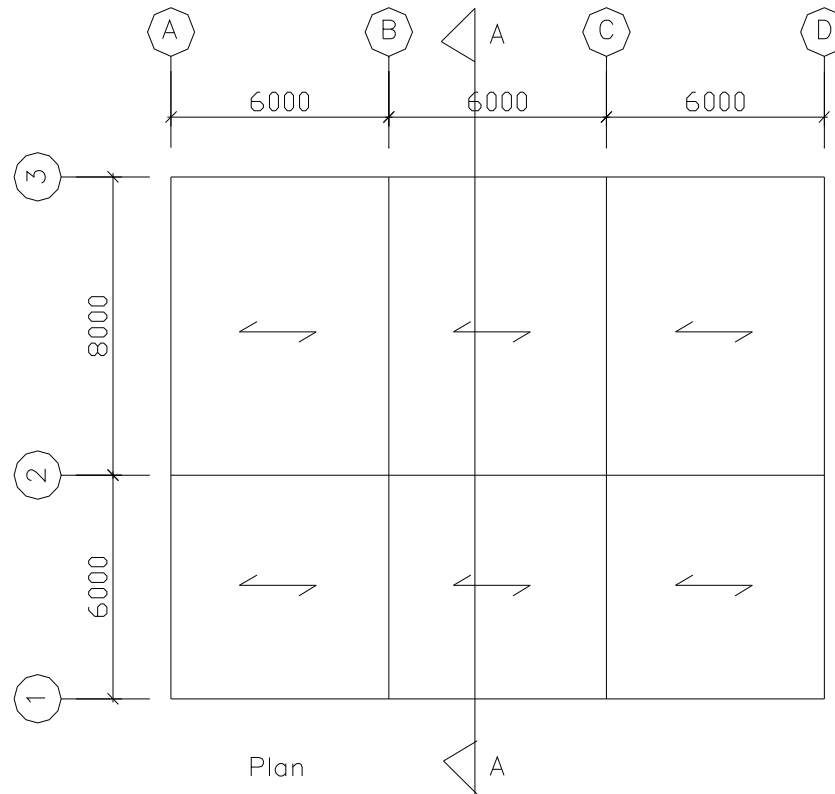


Figure 15.5c)
Rectangular columns $d_2/h = 0.15$

Column Design Example



Loading g_k roof = 3.0 kN/m² g_k floor = 4.5 kN/m²
 q_k roof = 0.6 kN/m² q_k floor = 4.0 kN/m²

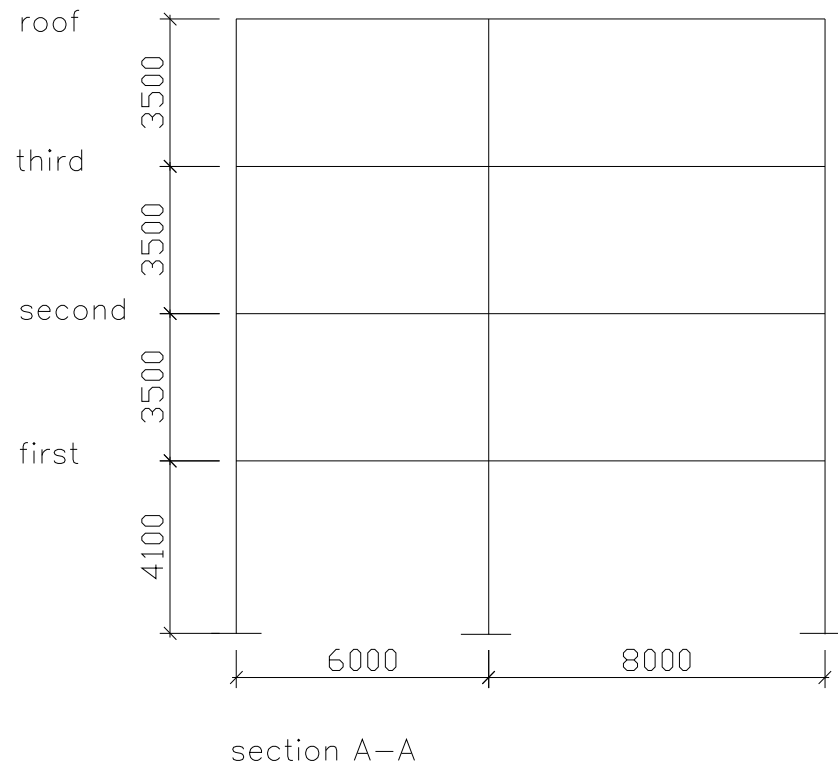
Main beams 600 mm deep x 300 mm wide

Other beams 400 mm deep x 300 mm wide

Columns 450 mm square

f_{ck} = 40 MPa f_{yk} = 500 MPa

Assume pinned foundations and structure braced



Thank you for your attention

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