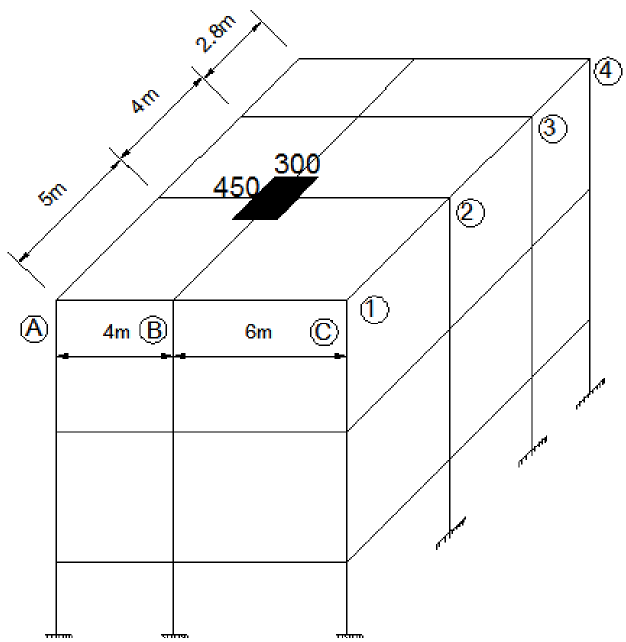
4.5 Column Design

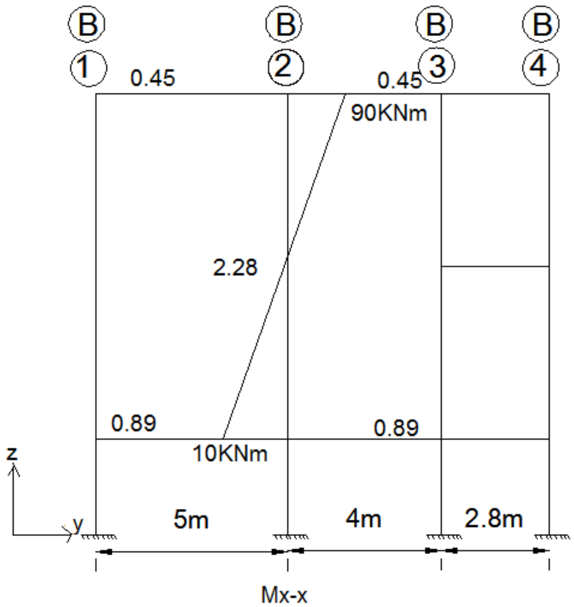
A non-sway column AB of 300*450 cross-section resists at ultimate limit state, an axial load of 1700 KN and end moment of 90 KNM and 10 KNM in the X direction ,60 KNM and 27 KNM in the Y direction causing double curvature about both axes. The column is braced with beams as shown in the figure. The concrete used Is C25/30 and rebar S500.

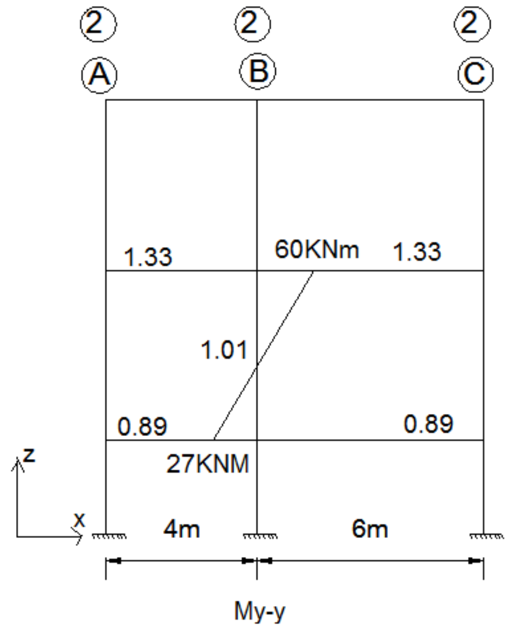
Take $\emptyset_{eff} = 1$

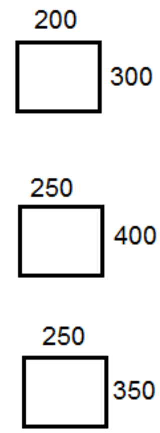


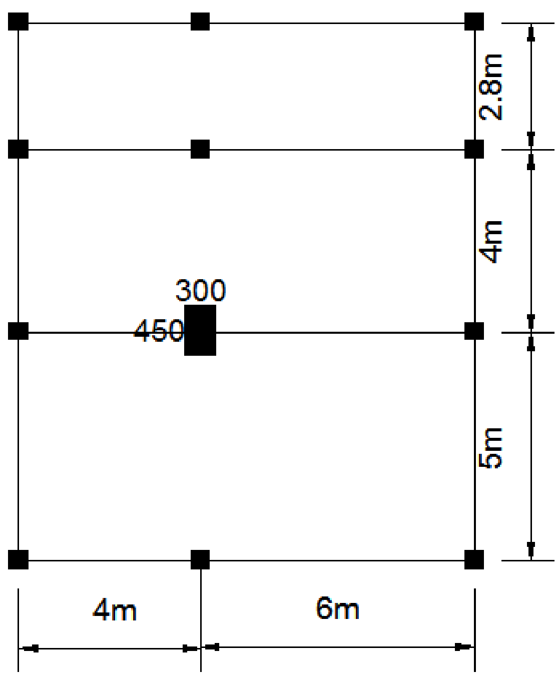
Step 1: Material data

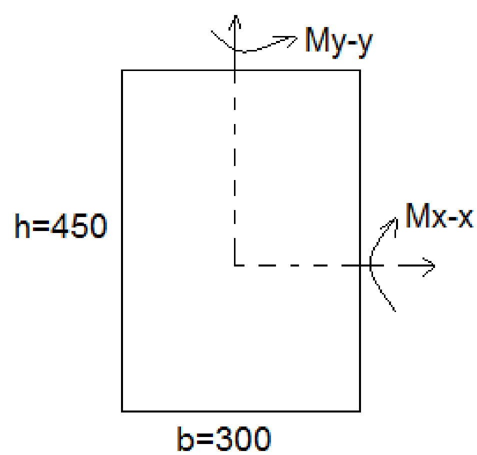
$$f_{cd} = \frac{0.85 * 25}{1.5} = 14.16667\ mpa$$
$$f_{yd} = \frac{500}{1.15} = 434.7826\ mpa$$
$$E_{cm} = 30\ Gpa$$
$$E_s = 200\ Gpa$$
$$\epsilon_{yd} = 2.1739‰$$


B
B
B
B
1
2
3
4
0.45
0.45
90KNm
2.28
0.89
0.89
10KNm
z
y
5m
4m
2.8m
Mx-x


2
2
2
A
B
C
1.33
60KNm
1.33
1.01
0.89
0.89
27KNM
z
x
4m
6m
My-y


200
300
250
400
250
350


2.8m
4m
5m
300
450
4m
6m


My-y
h=450
Mx-x
b=300

Step 2- Check slenderness limit

2.1 In the X direction

$$\lambda_{lim} = {}^{20ABC}/_{\sqrt{n}} \quad take\ A = 0.7 \quad B = 1.1 \quad C = 1.7 - r_m$$

$$where\ r_m = \frac{m_{01}}{m_{02}} = \frac{-27}{60} = -0.45$$

$$C = 1.7 - (-0.45) = 2.15$$

$$n = \frac{N_{ed}}{A_c f_{cd}} = \frac{1700 * 10^3}{14.1667 * 300 * 450} = 0.8888$$

$$\lambda_{lim} = \frac{20 * 0.7 * 1.1 * 2.15}{\sqrt{0.8888}} = 35.11845$$

2.1.1 Effective length

For Braced member

$$l_0 = 0.5l\sqrt{\left[1 + \frac{K_1}{0.5 + K_1}\right]\left[1 + \frac{K_2}{0.5 + K_2}\right]}$$

$$K = \frac{Column\ stiffness}{\sum beam\ stiffness}$$

$$K_i = \frac{\left({}^{EI}/_{l}\right)column}{\sum\left(2\,{}^{EI}/_{l}\right)beam}$$

$$I_{column} = \frac{450 * 300^3}{12} = 1012500000\ mm^4$$

$$I_{beam\ top} = \frac{250 * 400^3}{12} = 1333333333\ mm^4$$

$$K_1 = \frac{\frac{1012500000E}{4500}}{\left(\frac{2 * 1333333333E}{4000} + \frac{2 * 1333333333E}{6000}\right)} = 0.2025$$

$$I_{column} = \frac{450 * 300^3}{12} = 1012500000\ mm^4$$

$$I_{beam\ bottom} = \frac{250 * 350^3}{12} = 893229166.7\ mm^4$$

$$K_2 = \frac{\frac{1012500000E}{4500}}{(\frac{2*893229166.7E}{4000} + \frac{2*893229166.7E}{6000})} = 0.30227$$

$$l_0 = 0.5*4500\sqrt{\left[1+\frac{0.2025}{0.5+0.2025}\right]\left[1+\frac{0.30227}{0.5+0.30227}\right]} = 2996.5021\ mm$$

$$ƛ = \frac{l_o}{i} \quad i = \sqrt{\frac{I}{A}} = \sqrt{\frac{1012500000}{135000}} = 86.6025\ mm$$

$$ƛ = \frac{2996.5021}{86.6025} = 34.6006$$

$$ƛ < ƛ_{lim} \quad Short\ column$$

2.2 In the Y direction

$$ƛ_{lim} = {}^{20ABC}/_{\sqrt{n}} \quad take\ A = 0.7 \quad B = 1.1 \quad C = 1.7 - r_m$$

$$where\ r_m = \frac{m_{01}}{m_{02}} = \frac{-10}{90} = -0.111 \qquad C = 1.7 - (-0.111) = 1.8111$$

$$n = \frac{N_{ed}}{A_c f_{cd}} = \frac{1700*10^3}{14.1667*300*450} = 0.8888$$

$$ƛ_{lim} = \frac{20*0.7*1.1*1.8111}{\sqrt{0.8888}} = 29.58299$$

2.2.1 Effective length

For Braced member

$$l_0 = 0.5l\sqrt{\left[1+\frac{K_1}{0.5+K_1}\right]\left[1+\frac{K_2}{0.5+K_2}\right]}$$

$$K = \frac{Column\ stiffness}{\sum beam\ stiffness}$$

$$K_i = \frac{\left({}^{EI}/_{l}\right)column}{\sum\left(2\,{}^{EI}/_{l}\right)beam}$$

$$I_{column} = \frac{300*450^3}{12} = 2278125000\ mm^4$$

$$I_{beam\ top\ tie\ beam} = \frac{200*300^3}{12} = 450000000\ mm^4$$

$$K_1 = \frac{\frac{2278125000E}{9000}}{(\frac{2 * 450000000E}{5000} + \frac{2 * 450000000E}{4000})} = 0.625$$

$$I_{column} = \frac{300 * 450^3}{12} = 2278125000\ mm^4$$

$$I_{beam\ bottom} = \frac{250 * 350^3}{12} = 893229166.7\ mm^4$$

$$K_2 = \frac{\frac{2278125000E}{9000}}{(\frac{2 * 893229166.7E}{5000} + \frac{2 * 893229166.7E}{4000})} = 0.31486$$

$$l_0 = 0.5 * 9000\sqrt{\left[1 + \frac{0.625}{0.5 + 0.625}\right]\left[1 + \frac{0.31486}{0.5 + 0.31486}\right]} = 6608.4435\ mm$$

$$ƛ = \frac{l_o}{i} \quad i = \sqrt{\frac{I}{A}} = \sqrt{\frac{2278125000}{135000}} = 129.9038\ mm$$

$$ƛ = \frac{6608.4435}{129.9038} = 50.871$$

$$ƛ > ƛ_{lim} \qquad \textbf{Slender column}$$

Step 3. Accidental eccentricity

3.1 In the X direction

$$e_a = \frac{l_o}{400} = \frac{3176.688}{400} = 7.94172\ mm$$

3.2 In the Y direction

$$e_a = \frac{l_o}{400} = \frac{6608.4435}{400} = 16.52111\ mm$$

Step 4. Equivalent first order eccentricity

4.1 In the X direction

$$e_e = max\begin{cases} 0.6e_{02} + 0.4\ e_{01} \\ 0.4e_{02} \end{cases}$$

$$e_{02} = \frac{M_{02}}{N_{sd}} = \frac{60 * 10^6}{1700 * 10^3} = 35.294117\ mm$$

$$e_{01} = \frac{M_{01}}{N_{sd}} = \frac{-27 * 10^6}{1700 * 10^3} = -15.882353\ mm$$

$$e_e = max\begin{cases} 0.6e_{02} + 0.4\ e_{01} \\ 0.4e_{02} \end{cases} = 14.823 \text{ mm}$$

4.2 In the Y direction

$$e_e = max\begin{cases} 0.6e_{02} + 0.4\ e_{01} \\ 0.4e_{02} \end{cases}$$

$$e_{02} = \frac{M_{02}}{N_{sd}} = \frac{90 * 10^6}{1700 * 10^3} = 52.94117\ mm$$

$$e_{01} = \frac{M_{01}}{N_{sd}} = \frac{-10 * 10^6}{1700 * 10^3} = -5.88235\ mm$$

$$e_e = max\begin{cases} 0.6e_{02} + 0.4\ e_{01} \\ 0.4e_{02} \end{cases} = 29.412 \text{ mm}$$

Step 5. Second order moment

5.1 In the X direction

Because the column is short $e_2 = 0$

$$e_{tot} = e_o + e_e + e_2 = 7.94172 + 14.823 + 0 = 22.76475\ mm$$
$$check\ with\ \ e = e_{02} + e_a = 35.294117 + 7.94172 = 43.2358\ mm$$
$$So\ take\ e_{tot} = 43.2358\ mm$$

$$M_{sd,y} = N_{sd} * e_{tot} = 73.501\ KNm$$

$$\mu_{sd,y} = \frac{M_{sd,y}}{f_{cd}bd^2} = \frac{73.501 * 10^6}{14.1667 * 450 * 300^2} = 0.12758$$

5.2 In the Y direction

Calculate the second order moment using either Nominal curvature or Nominal stiffness method

Using Nominal curvature method

$$e_2 = \frac{1}{r}{l_o}^2/_C \qquad \boldsymbol{C = 10 \quad For\ constant\ cross - section}$$

$$\frac{1}{r} = K_r K_\emptyset \frac{1}{r_o} \qquad K_\emptyset = 1 + \beta\emptyset_{eff}$$

$$\beta = 0.35 + {}^{f_{ck}}/_{200} - {}^{\lambda}/_{150} = 0.35 + {}^{25}/_{200} - {}^{50.871}/_{150} = 0.13586$$

$$K_\emptyset = 1 + \beta\emptyset_{eff} = 1 + 0.13586 * 1 = 1.13586$$

$$\frac{1}{r_o} = \frac{\varepsilon_{yd}}{0.45d} = \frac{2.173913 * 10^{-3}}{0.45 * 405} = 1.19282 * 10^{-5} \qquad d = effective\ depth$$

$$K_r = \frac{(n_u - n)}{(n_u - n_{bal})} \quad n_u = 1 + \omega$$

$$n = \frac{N_{ed}}{A_c f_{cd}} = \frac{1700 * 10^3}{14.166 * 450 * 300} = 0.8888 \quad n_{bal} = 0.4$$

For first iteration take $\boldsymbol{e_2 = 0}$

$$e_{tot} = e_o + e_e + e_2 = 16.52111 + 29.412 + 0 = 45.933\ mm$$

$$N_{sd} = 1700\ KN \quad M_{sd,x} = N_{sd} * e_{tot} = 78.0862\ KNm$$

$$\mu_{sd,x} = \frac{M_{sd,x}}{f_{cd} b d^2} = \frac{78.0862 * 10^6}{14.1667 * 300 * 450^2} = 0.0907$$

*Using $\frac{h'}{h} = \frac{b'}{b} = 0.10$ read the mechanical steel ratio from biaxial interaction chart for

$v_{sd} = 0.888 \quad \mu_{sd,x} = 0.0907 \quad \mu_{sd,y} = 0.12758$

Interpolating between v_{sd} 0.8 *and* 1 $\quad \omega = 0.28$

So

$$n_u = 1 + \omega = 1.25 \quad K_r = \frac{(n_u - n)}{(n_u - n_{bal})} = \frac{(1.28 - 0.8888)}{(1.28 - 0.4)} = 0.444$$

$$\frac{1}{r} = 0.444 * 1.13586 * 1.19282 * 10^{-5} = 6.02167 * 10^{-6}$$

$$e_2 = \frac{1}{r} 6608.4435^2 / 10 = 26.29757\ mm$$

For Second iteration take $\boldsymbol{e_2 = 26.29757\ mm}$

$$e_{tot} = e_o + e_e + e_2 = 16.52111 + 29.412 + 26.29757 = 72.2306\ mm$$

$$N_{sd} = 1700\ KN \quad M_{sd,x} = N_{sd} * e_{tot} = 122.7922\ KNm$$

$$\mu_{sd,x} = \frac{M_{sd,x}}{f_{cd} b d^2} = \frac{122.7922 * 10^6}{14.1667 * 300 * 450^2} = 0.142677$$

*Using $\frac{h'}{h} = \frac{b'}{b} = 0.10$ read the mechanical steel ratio from biaxial interaction chart for

$v_{sd} = 0.888 \quad \mu_{sd,x} = 0.142677 \quad \mu_{sd,y} = 0.12758$

Interpolating between v_{sd} 0.8 *and* 1 $\quad \omega = 0.61$

So

$$n_u = 1 + \omega = 1.25 \quad K_r = \frac{(n_u - n)}{(n_u - n_{bal})} = \frac{(1.61 - 0.8888)}{(1.61 - 0.4)} = 0.59596$$

$$\frac{1}{r} = 0.59596 * 1.13586 * 1.19282 * 10^{-5} = 8.074616 * 10^{-6}$$

$$e_2 = \frac{1}{r} 6608.4435^2 / 10 = 35.26308\ mm$$

For 3rd iteration take $\boldsymbol{e_2 = 35.26308\ mm}$

$$e_{tot} = e_o + e_e + e_2 = 16.52111 + 29.412 + 35.26308 = 81.1962\ mm$$
$$N_{sd} = 1700\ KN \quad M_{sd,x} = N_{sd} * e_{tot} = 138.0335\ KNm$$

$$\mu_{sd,x} = \frac{M_{sd,x}}{f_{cd}bd^2} = \frac{138.0335 * 10^6}{14.1667 * 300 * 450^2} = 0.1604$$

*Using $\frac{h'}{h} = \frac{b'}{b} = 0.10$ read the mechanical steel ratio from biaxial interaction chart for

$$v_{sd} = 0.888 \quad \mu_{sd,x} = 0.1604 \quad \mu_{sd,y} = 0.12758$$
$$\text{Interpolating between } v_{sd}\ 0.8\ and\ 1 \quad \omega = 0.656$$

So

$$n_u = 1 + \omega = 1.25 \quad K_r = \frac{(n_u - n)}{(n_u - n_{bal})} = \frac{(1.656 - 0.8888)}{(1.656 - 0.4)} = 0.61075$$
$$\frac{1}{r} = 0.61075 * 1.13586 * 1.19282 * 10^{-5} = 8.275016 * 10^{-6}$$
$$e_2 = \frac{1}{r} 6608.4435^2 / 10 = 36.13825\ mm$$

For 4th iteration take $\boldsymbol{e_2 = 36.13825\ mm}$

$$e_{tot} = e_o + e_e + e_2 = 16.52111 + 29.412 + 36.13825 = 82.07136\ mm$$
$$N_{sd} = 1700\ KN \quad M_{sd,x} = N_{sd} * e_{tot} = 139.521\ KNm$$

$$\mu_{sd,x} = \frac{M_{sd,x}}{f_{cd}bd^2} = \frac{138.0335 * 10^6}{14.1667 * 300 * 450^2} = 0.1621$$

*Using $\frac{h'}{h} = \frac{b'}{b} = 0.10$ read the mechanical steel ratio from biaxial interaction chart for

$$v_{sd} = 0.888 \quad \mu_{sd,x} = 0.1621 \quad \mu_{sd,y} = 0.12758$$
$$\text{Interpolating between } v_{sd}\ 0.8\ and\ 1 \quad \omega = 0.65$$

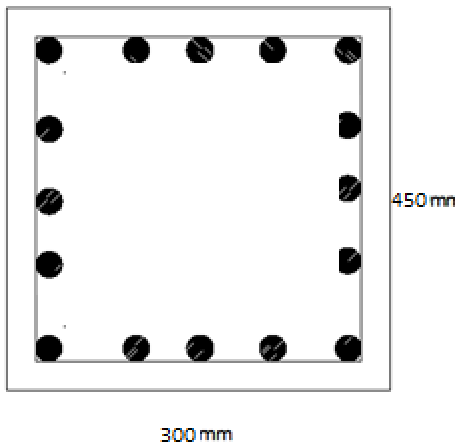
The iteration converges with similar mechanical steel ratio $\boldsymbol{\omega = 0.65}$

$$A_{s,tot} = \frac{\omega f_{cd} bd}{f_{yd}} = \frac{0.65 * 14.166 * 300 * 450}{434.7826} = 2859.187\ mm^2$$
$$A = \frac{A_{s,tot}}{4} = \frac{2859.187}{4} = 714.7968\ mm^2$$

Check with maximum and minimum reinforcement limit

$$A_{s,min} = max \begin{cases} \frac{0.1\ N_{ED}}{f_{yd}} \\ 0.002A_c \end{cases} = 391\ mm^2 \quad \boldsymbol{OK}!$$
$$A_{s,max} = 0.08\ A_c = 0.08 * 300 * 450 = 10800\ mm^2 \quad \boldsymbol{OK}!$$

Using $\emptyset 16$ $\quad$ $provide\ total\ of\ 16\emptyset 16$



- Check using nominal stiffness method and compere the results.