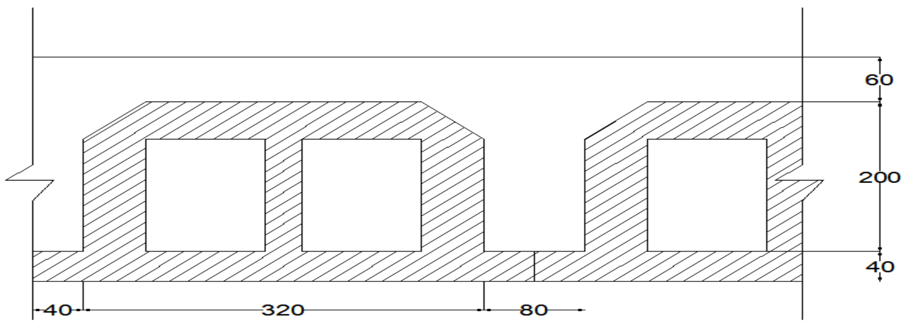
Example 2.2 [Ribbed slab design]

A typical floor system of a lecture hall is to be designed as a ribbed slab. The joists which are spaced at 400mm are supported by girders. The overall depth of the slab without finishing materials is 300 mm. Imposed load of 1.5KN/m^2 for partition and fixture is considered in the design. In addition the floor has a floor finish material of 3cm marble over a 2cm cement screed and it ha 2cm plastering as ceiling. Take the unit weight of ribbed block to be 2KN/m2.

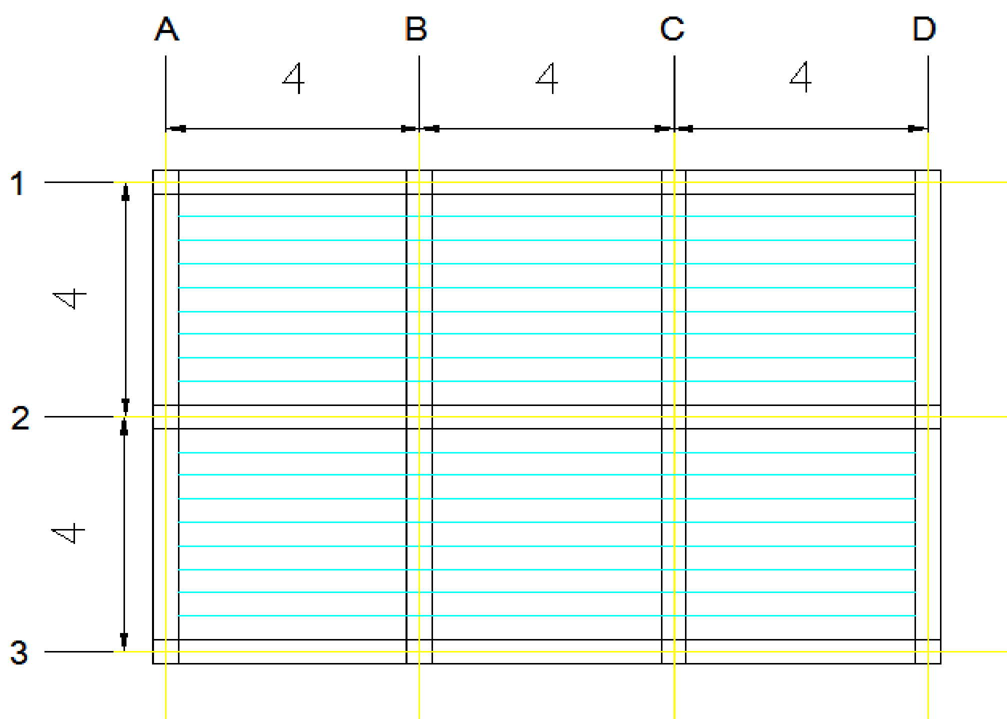


Use: C 20/25

S – 300

Class 1 works

a) Analyze the ribbed slab system, considering the effects of loading pattern
b) Design the ribbed slab system



Solution:

Step 1: Material property

Concrete:

$$fctk, 0.05 = 1.5Mpa$$

$$fctm = 2.2Mpa$$

$$\gamma c = 1.5$$

$$fck = 20Mpa, fcu = 25Mpa$$

$$fcd = \frac{0.85 * 20}{1.5} = 11.33Mpa$$

Rebar

$$fyk = 300Mpa$$

$$fyd = \frac{fyk}{1.15} = 260.87Mpa$$

$$\varepsilon yd = \frac{fyd}{Es} = \frac{260.87}{200} = 1.74\%$$

Step 2: Loading

Dead load:

- Joist→ 0.2 * 0.08 * 25 = 0.4
- Topping→ 0.4 * 0.06 * 25 = 0.6
- Floor finish → 0.4 * 0.03 * 27 = 0.32
- Cement Screed → 0.4 * 0.02 * 23 = 0.184
- Plastering → 0.4 * 0.02 * 23 = 0.184
- Partition and fittings → 0.4*1.5 = 0.6
- Ribbed block → 0.4 * 2 = 0.8

G_k = 3.092 KN/m

Live load:

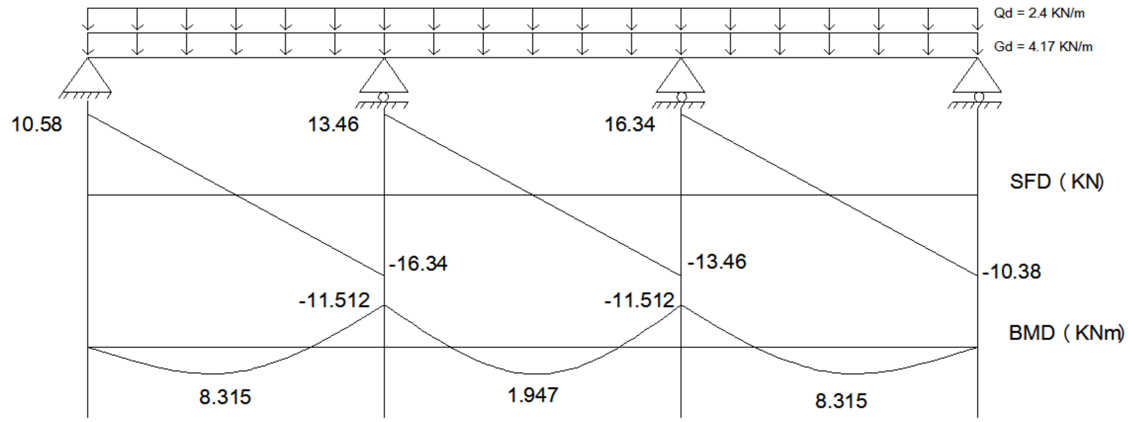
- Q_k = 4KN/m^2 * 0.4 = 1.6 KN/m

Design load:

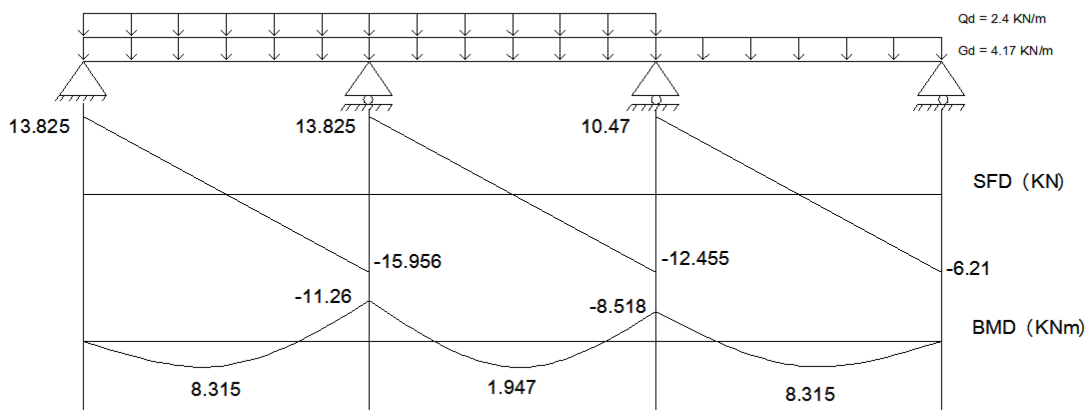
- $Gd = 1.35 * Gk = 1.35 * 3.092 = 4.174KN/m$
- $Qd = 1.5 * Qk = 1.5 * 1.6 = 2.4KN/m$

Step 3: Analysis (for Ribs)

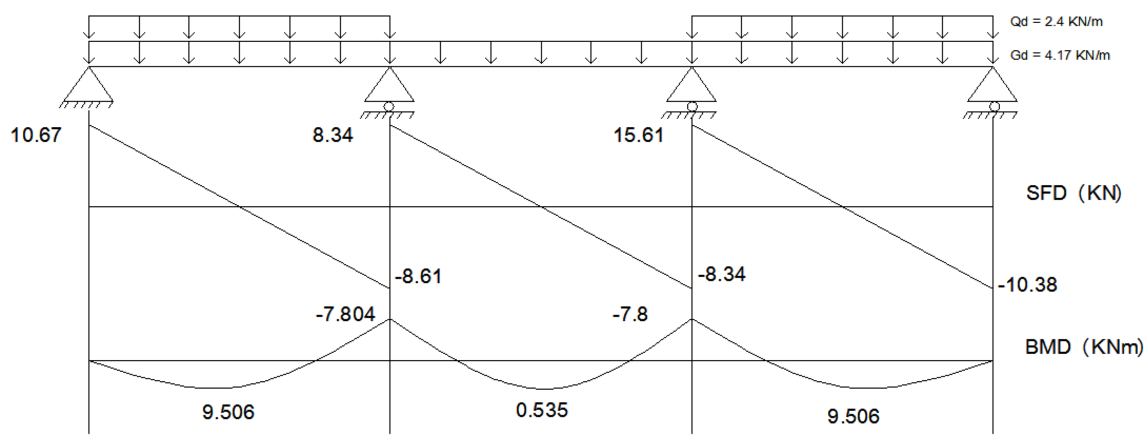
i) Full design load



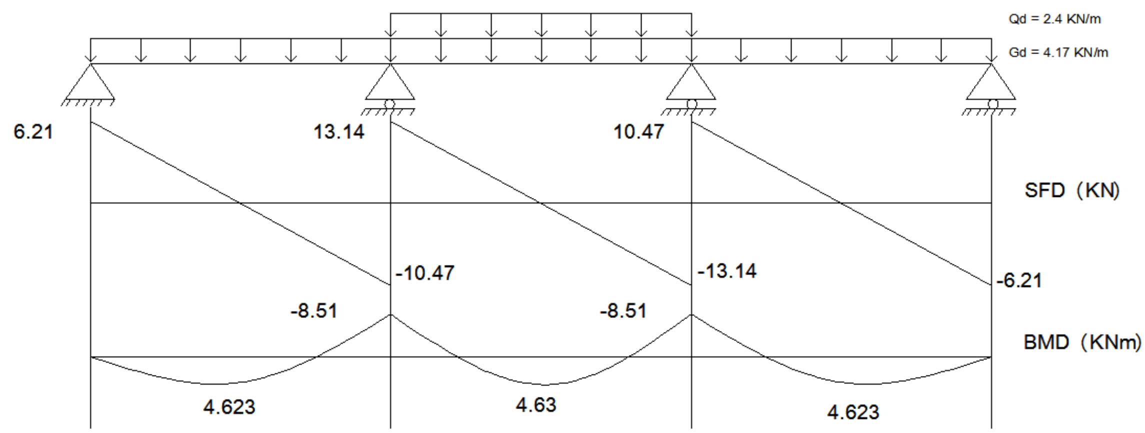
ii) Maximum support moment [at B and C]



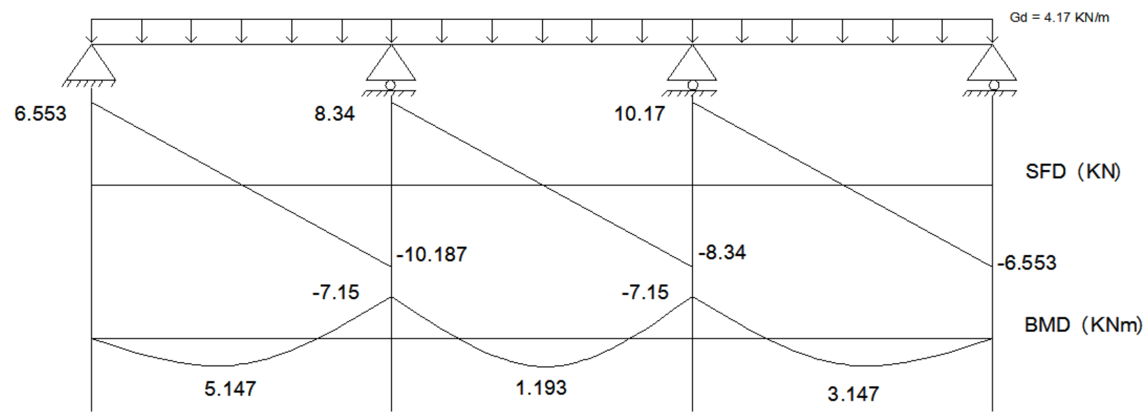
iii) For maximum span moment [at span AB and CD]



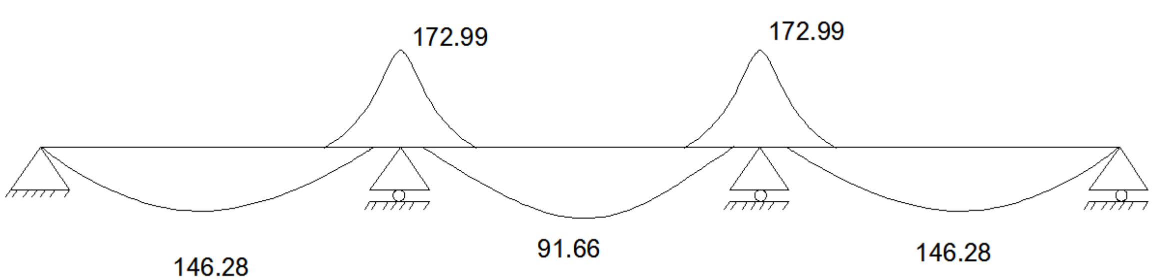
iv) Maximum span moment [at BC]



v) Only dead load acting

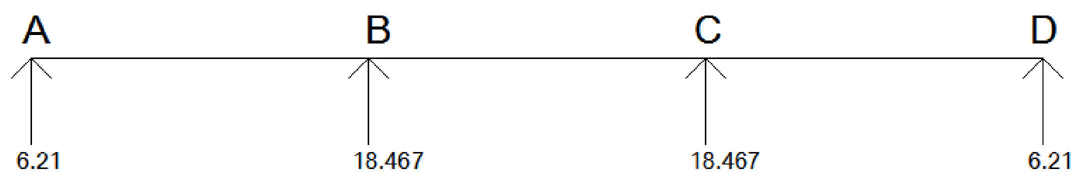


- ***Moment envelope diagram for the rib***



- ***Maximum reaction envelope***

- ***Minimum reaction envelope***



Step 4. Loading on Girders

- Assume Width of girders A , D ....W=300mm
  B, C ....W=600mm

For all girders........... D=300mm

- Note: the section should be checked for serviceability

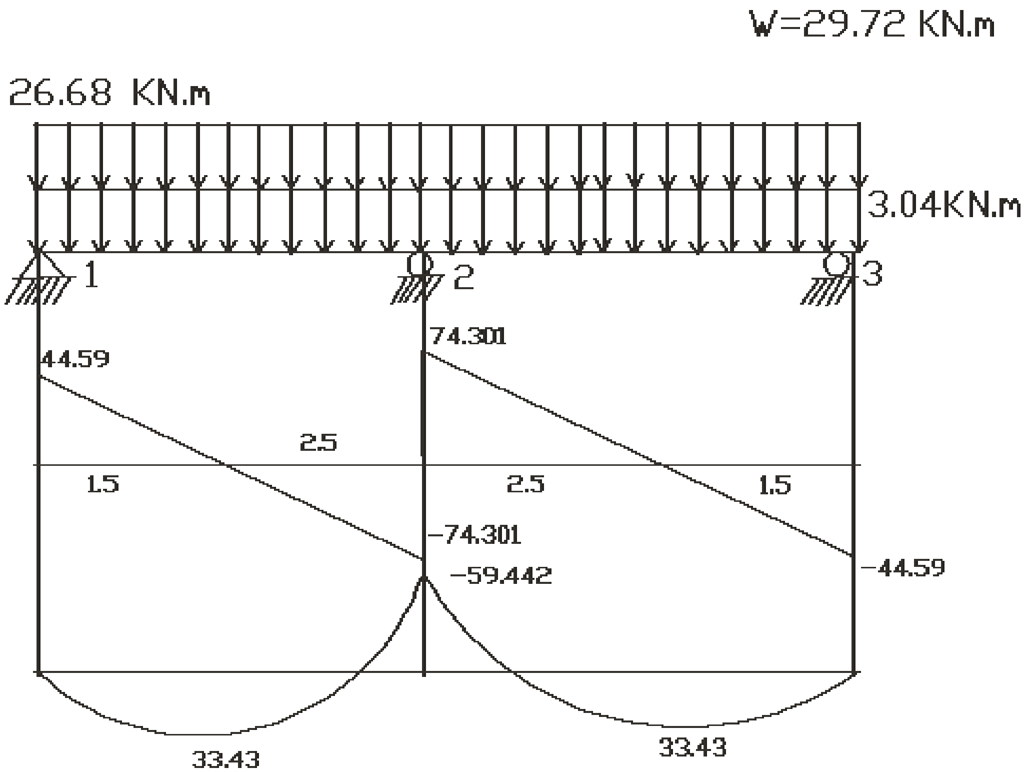
  - Self-weight: A & D ....= 0.3 x 0.3 x 25 = 2.25 KN. m
    B&C ....= 0.6 x 0.3 x 25 = 4.5 KN. m

  - Design loads: A & D ....Gd = 1.35 x2.25 = 3.04 KN. m
    B & C ....Gd= 1.35 x4.5 = 6.08 KN. M

Step 5. Analysis of Girders

i. For Girder on axis "A" and "D"

- *To get to maximum support moment [at 2]*



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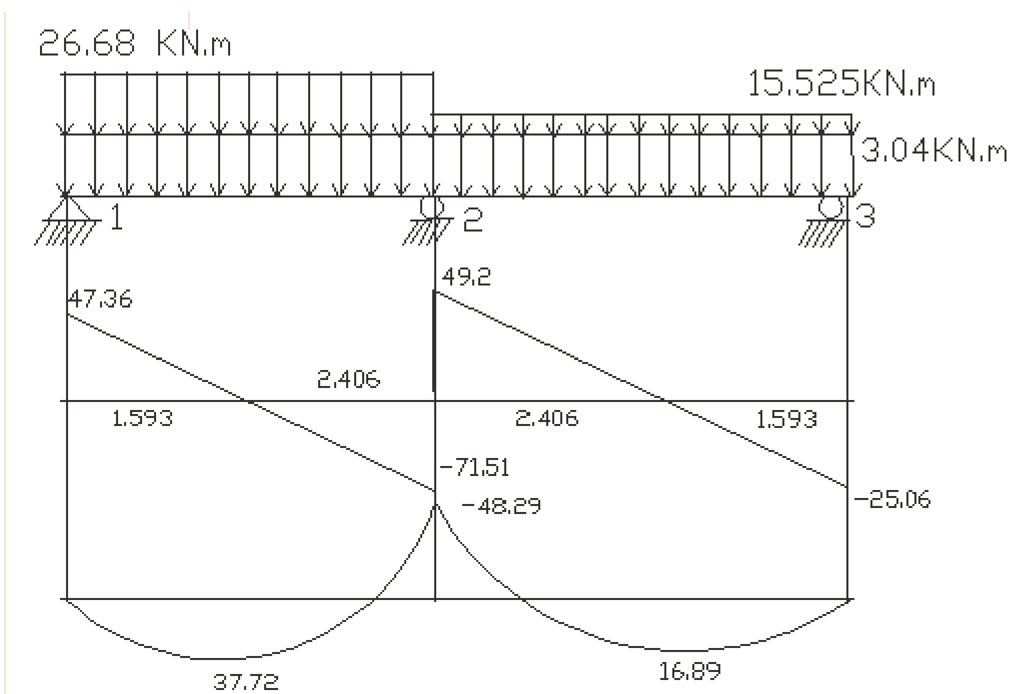
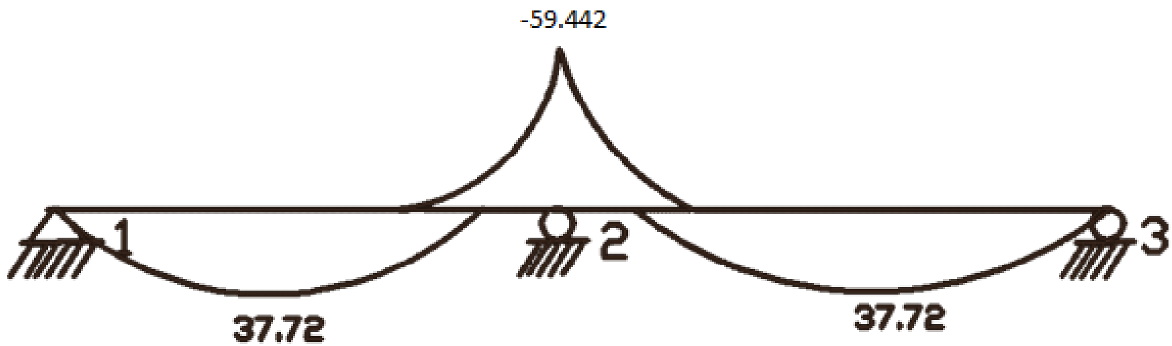
- *To get maximum span moment on girder A & D at 12 & 23*



- ***Moment envelop*** *for girder A and D*



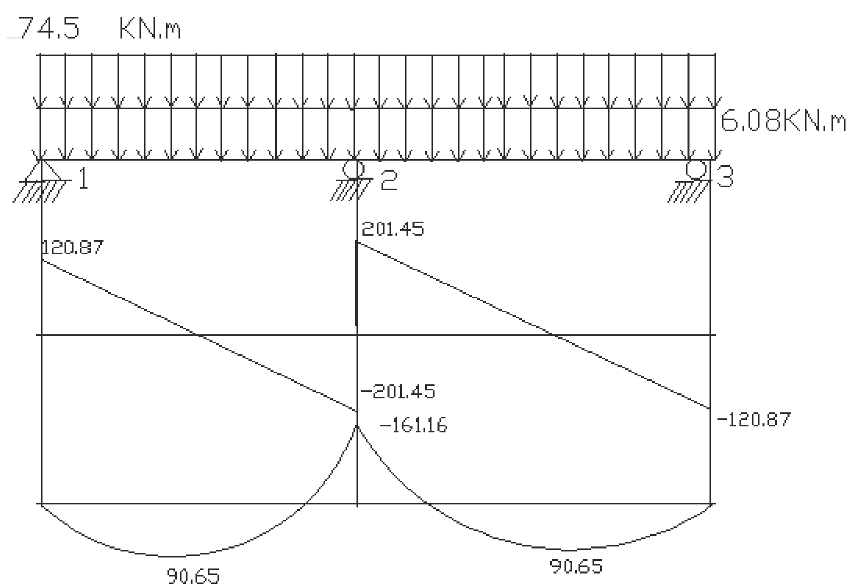
ii. <u>For Girder on axis "B" and "C"</u>

- Loading: Self-weight = 6.08 KN.m

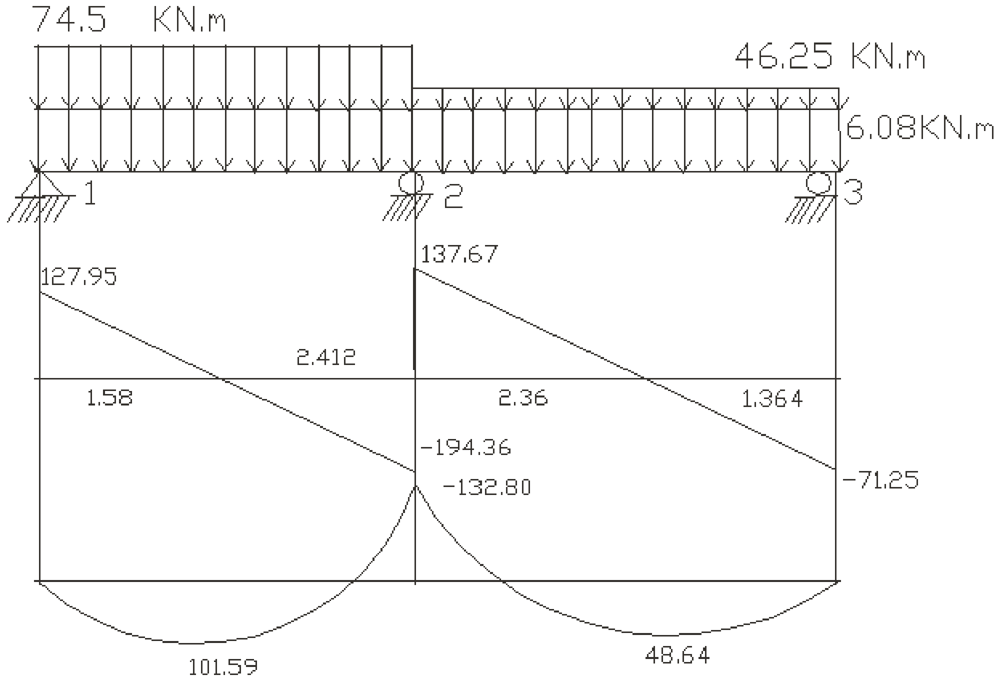
  Reactions from the ribs (divided by the rib spacing)

  $\frac{29.8}{0.4} = 74.5\ KN.m$ and $\frac{18.467}{0.4} = 46.15\ KN.m$

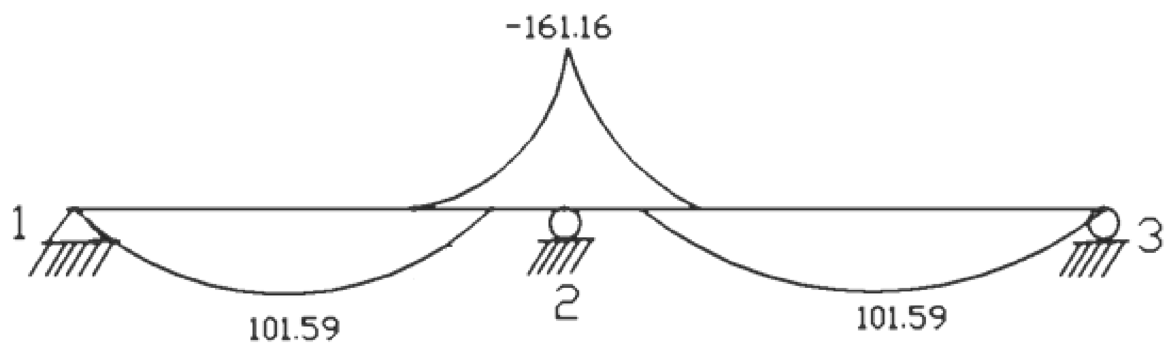
- *To get maximum support moment [at "2"]*



- *To get maximum span moment [at “12” or “23”]*



- ***Moment envelop*** *diagram for girders on axis B and C*



Step 6. Loading on the **Beam** …. Axis 1, 2 and 3

- Self-weight width= 200 mm
  Depth= 300 mm

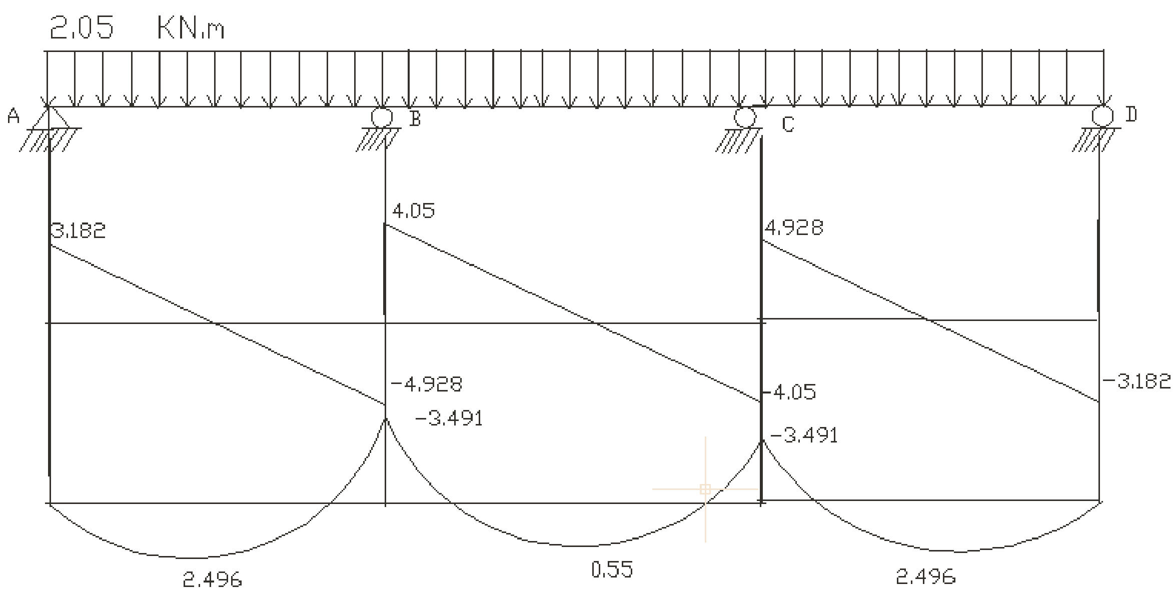
N.B: cross section should be checked for serviceability.

Since there are columns at the intersection of the beams and girders, the beams will only support their own loads.

DL = 0.2 x 0.3 x 25 = 1.5 KN/m

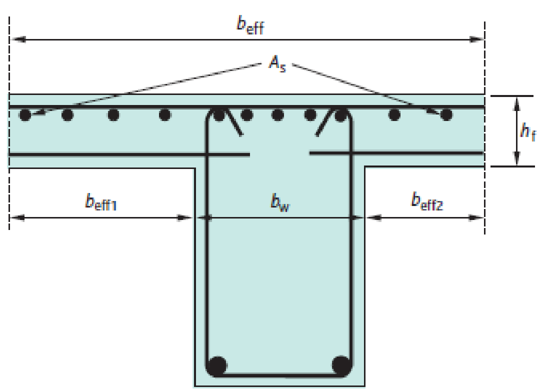
Gd= 1.35 x 1.5 = 2.025 KN/m

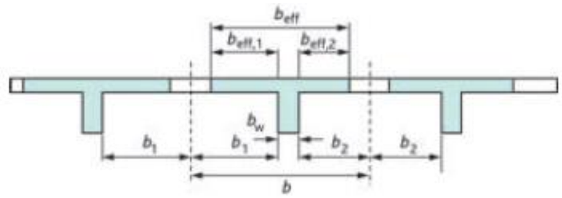
- *Beam analysis*



Step 7. Design

1. **Rib design**





Cross section at span

$$h_f = 60\ mm$$

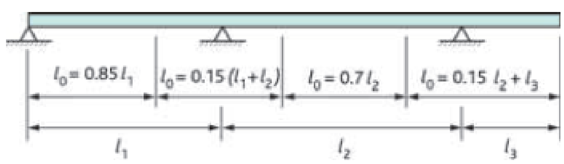
$b_w = 80\ mm$

$h = 260\ mm$

$Take\ cover\ 15\ mm$

$d = 260 - 15 - 6 - \frac{12}{2} = 233\ mm$

- ***Effective width computation***



$b_{eff} = \sum b_{eff,i} + b_w \leq b$

$b_{eff,i} = 0.2b_i + 0.1l_o \leq 0.2l_o$

I. *For end span (sagging moment)*

$l_o = 0.85l_1$
$l_o = 0.85 * 4000 = 3400mm$
$b_1 = b_2 = 160\ mm$

$b_{eff\ 1} = b_{eff\ 2} = 372 < 680 < b_1$

$b_{eff} = \sum b_{eff,i} + b_w \leq b$

$b_{eff} = 824 \leq 400\ \ \boldsymbol{NOT\ OK}$

$$\boldsymbol{b_{eff} = 400\ mm}$$

II. *For interior sagging moment*

$l_o = 0.7l_2$
$l_o = 0.7 * 4000 = 2800mm$
$b_1 = b_2 = 160\ mm$

$b_{eff\ 1} = b_{eff\ 2} = 312 < 560 < b_1$

$b_{eff} = \sum b_{eff,i} + b_w \leq b$

$b_{eff} = 704 \leq 400\ \ \boldsymbol{NOT\ OK}$

$$b_{eff} = 400\ mm$$

III. *For interior sagging moment*

$l_o = 0.15(l_1 + l_2)$
$l_o = 1200mm$
$b_1 = b_2 = 160\ mm$

$b_{eff\,1} = b_{eff\,1} = 152 < 240 < b_1$

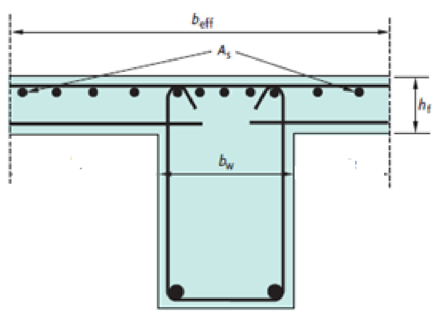
$b_{eff} = \sum b_{eff,i} + b_w \leq b$

$b_{eff} = 384 \leq 400$ ***OK***

$$b_{eff} = 384\ mm$$

- ***Design of the T-section***

A. Positive span moment AB and CD



$M_{sd} = 9.506\ KNm \quad b_{eff} = 384mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{9.506 * 10^6 Nmm}{11.33 * 384 * 233^2} = 0.0386$$

$\mu_{sd} < \mu_{sd,lim} = 0.295$ ***Singly reinforced***

$K_x = 0.055 \quad X = K_x d = 12.815\ mm < h_f$ ***design as a rectangular section***

$K_z = 0.975 \quad Z = K_z d = 227.175mm$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{9.506 * 10^6 Nmm}{260.87 * 227.175} = 160.40\ mm^2$$

$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \quad where\ b_t = b_w \quad d = 233mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$

$A_{smin} = 35.54\ mm^2 < A_s \quad \boldsymbol{OK}!$

$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 1.418 \quad \boldsymbol{use\ \ 2\emptyset 12\ \ bottom\ bars}$

B. Negative moment on the rib support B and C

$M_{sd} = 11.512\ KNm \qquad b_w = 80\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$

$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{11.512 * 10^6 Nmm}{11.33 * 80 * 233^2} = 0.2339$

$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$

$K_z = 0.88 \quad Z = K_z d = 205.04\ mm$

$A_s = \frac{M_{sd}}{f_{yd} Z} = \frac{11.512 * 10^6 Nmm}{260.87 * 205.04} = 215.22mm^2$

$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \quad where\ b_t = b_w \quad d = 233mm \ \ f_{ctm} = 2.2\ mpa$

$f_{yk} = 300\ mpa$

$A_{smin} = 35.54\ mm^2 < A_s \quad \boldsymbol{OK}!$

$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 1.9029 \quad \boldsymbol{use\ \ 2\emptyset 12\ \ bars\ at\ the\ top}$

C. Span moment between B and C

$M_{sd} = 4.63\ KNm \qquad b_{eff} = 384mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$

$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{4.63 * 10^6 Nmm}{11.33 * 384 * 233^2} = 0.0188$

$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad \boldsymbol{Singly\ reinforced}$

$K_x = 0.07 \quad X = K_x d = 16.31\ mm < h_f \qquad \boldsymbol{design\ as\ a\ rectangular\ section.}$

$K_z = 0.985 \quad Z = K_z d = 229.505mm$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{4.63 * 10^6 Nmm}{260.87 * 229.505} = 77.33mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 233mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 35.54\ mm^2 < A_s \quad \boldsymbol{OK}!$$

$$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 0.6837 \quad \boldsymbol{use\ \ 2\emptyset 12 \quad bottom\ bars}$$

2. Rib cross section at end (rectangular)

a. <u>Positive span moment AB,CD</u>

$$M_{sd} = 9.506\ KNm \qquad b_w = 80\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{9.506 * 10^6 Nmm}{11.33 * 384 * 233^2} = 0.193$$

$$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$$

$$K_z = 0.89 \quad Z = K_z d = 207.37mm$$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{9.506 * 10^6 Nmm}{260.87 * 207.37} = 175.61mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 233mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 35.54\ mm^2 < A_s \quad OK!$$

$$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 1.552 \quad \boldsymbol{use\ \ 2\emptyset 12 \quad bottom\ bars}$$

b. <u>Negative support moment at B and C</u>

$$M_{sd} = 11.512\ KNm \qquad b_w = 80\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{11.512 * 10^6 Nmm}{11.33 * 80 * 233^2} = 0.2339$$

$$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad \boldsymbol{Singly\ reinforced}$$

$$K_z = 0.863 \quad Z = K_z d = 201.79\ mm$$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{11.512 * 10^6 Nmm}{260.87 * 201.79} = 219.46mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 233mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 35.54\ mm^2 < A_s \quad \boldsymbol{OK}!$$

$$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 1.9 \quad \boldsymbol{use\ \ 2\emptyset 12 \ \ bars\ at\ the\ top}$$

c. Positive span moment at BC

$$M_{sd} = 4.63\ KNm \qquad b_w = 80\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{4.63 * 10^6 Nmm}{11.33 * 80 * 233^2} = 0.094$$

$$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$$

$$K_z = 0.951 \quad Z = K_z d = 221.583\ mm$$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{4.63 * 10^6 Nmm}{260.87 * 229.505} = 80.09mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 233mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 35.54\ mm^2 < A_s \quad \boldsymbol{OK}!$$

$$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 0.6837 \quad \boldsymbol{use\ \ 2\emptyset 12 \ \ bottom\ bars}$$

2. Girder design

a. Girder at A and D

- Positive span moment

$$M_{sd} = 37.728\ KNm \qquad b_w = 300\ mm \quad D = 300\ mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$d = 300 - 25 - 8 - \frac{16}{2} = 259\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{37.728 * 10^6 Nmm}{11.33 * 300 * 259^2} = 0.165$$

$$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$$

$$K_z = 0.907 \quad Z = K_z d = 234.91\ mm$$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{37.728 * 10^6 Nmm}{260.87 * 234.91} = 615.525mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 259\ mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 148.148mm^2 < A_s \quad \boldsymbol{OK}!$$

$$using\ \emptyset\ 16 a_s = 200.96\ mm^2 \quad n = \frac{A_s}{a_s} = 3.0629 \quad \boldsymbol{use\ \ 4\emptyset 16\ \ bottom\ bars}$$

- Negative support moment

$$M_{sd} = 59.442\ KNm \qquad b_w = 300\ mm \quad D = 300\ mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$d = 300 - 25 - 8 - \frac{16}{2} = 259\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{59.442 * 10^6 Nmm}{11.33 * 300 * 259^2} = 0.260$$

$$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$$

$$K_z = 0.841 \quad Z = K_z d = 217.819\ mm$$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{59.442 * 10^6 Nmm}{260.87 * 217.819} = 1046.1mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 259\ mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 148.148\ mm^2 < A_s \quad \boldsymbol{OK}!$$

$$using\ \emptyset\ 16 \quad a_s = 200.96\ mm^2 \quad n = \frac{A_s}{a_s} = 3.0629 \quad \boldsymbol{use\ \ 6\emptyset 16\ \ bottom\ bars}$$

b. Girder at B and C

Positive span moment

$$M_{sd} = 101.59\ KNm \qquad b_w = 600\ mm \quad D = 300\ mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$d = 300 - 25 - 8 - \frac{20}{2} = 257\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{101.59 * 10^6 Nmm}{11.33 * 600 * 257^2} = 0.226$$

$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$

$K_z = 0.867 \quad Z = K_z d = 222.819\ mm$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{101.59 * 10^6 Nmm}{260.87 * 222.819} = 1747.73mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 259\ mm\ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$A_{smin} = 148.148\ mm^2 < A_s \quad \boldsymbol{OK}!$

$$using\ \emptyset\ 20 a_s = 314mm^2 \quad n = \frac{A_s}{a_s} = 5.566 \quad \boldsymbol{use\ \ 6\emptyset 20\ \ bottom\ bars}$$

Negative support moment

$M_{sd} = 161.16\ KNm \qquad b_w = 600\ mm \quad D = 300\ mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$

$$d = 300 - 25 - 8 - \frac{20}{2} = 257\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd} b d^2} = \frac{161.16 * 10^6 Nmm}{11.33 * 300 * 257^2} = 0.358$$

$\mu_{sd} > \mu_{sd,lim} = 0.295 \quad Doubly\ reinforced\ section$

$K_{z,lim} = 0.814$

$M_{sd,lim} = \mu_{sd,lim} f_{cd} b d^2 = 0.295 * 11.33 * 600 * 257^2 = 132.455\ KNm$

$Z = K_{z,lim} * d = 0.814 * 257 = 209.19\ mm$

$$A_{s1} = \frac{M_{sd,lim}}{Z f_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_{2)}} = \frac{132.455 * 10^6}{260.87 * 209.19} + \frac{(161.16 - 132.455) * 10^6}{260.87 * (257 - 35)} = 2915.617mm^2$$

$\boldsymbol{use\ 10\ \emptyset 20}$

- *Compression reinforcement design*

  - Check if the reinforcement has yielded

$$\frac{d_2}{d} = \frac{35}{257} = 0.14 \varepsilon_{s2} = 2.6‰ \ \ (read\ from\ chart)$$

$\varepsilon_{s2} = 2.6‰ > \varepsilon_{yd}\ use\ f_{yd} = 260.87$

- Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.

  $\varepsilon_{cs2} = 2.6‰ \geq 2‰$, Therefore, we take

  $\varepsilon_c = 3.5‰ \ and\ \sigma_{cd,s2} = 11.33\ mpa$

$$A_{s2=\frac{1}{(\sigma_{s2}-\sigma_{cd,s2})}(\frac{M_{sds}-M_{sd,lim}}{d-d_2})} = \frac{1}{(260.87-11.33)}(\frac{(161.16-132.455)*10^6}{(257-35)} = 518.160mm^2$$

$$\boldsymbol{use\ 2\varnothing 20}$$

3. Beams Design

i) Negative support moment

$$M_{sd} = 3.491KNm \quad b_w = 200\ mm \quad D = 300\ mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$$

$$d = 300 - 25 - 8 - \frac{12}{2} = 261\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{3.491*10^6 Nmm}{11.33*200*261^2} = 0.0226$$

$$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad \boldsymbol{Singly\ reinforced}$$

$$K_z = 0.985 \quad Z = K_z d = 257.08\ mm$$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{3.491*10^6 Nmm}{260.87*257.08} = 52.04mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w = 200 \quad d = 261\ mm \ \ f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$$

$$A_{smin} = 99.52mm^2 > A_s Not\ \ \boldsymbol{OK}!$$

$$using\ \emptyset\ 12 a_s = 113.04mm^2 \quad n = \frac{A_{s,min}}{a_s} = 0.88 \quad \boldsymbol{use\ \ 2\emptyset 12\ \ bottom\ bars}$$

$$\boldsymbol{Use\ 2\emptyset 12\ bottom\ and\ top\ bar\ for\ the\ total\ lenth\ of\ the\ beam}.$$

Step 8. Transverse reinforcement

Secondary reinforcement is required for temperature and shrinkage.

$$A_{s2} = 20\%\ A_{s,min}$$

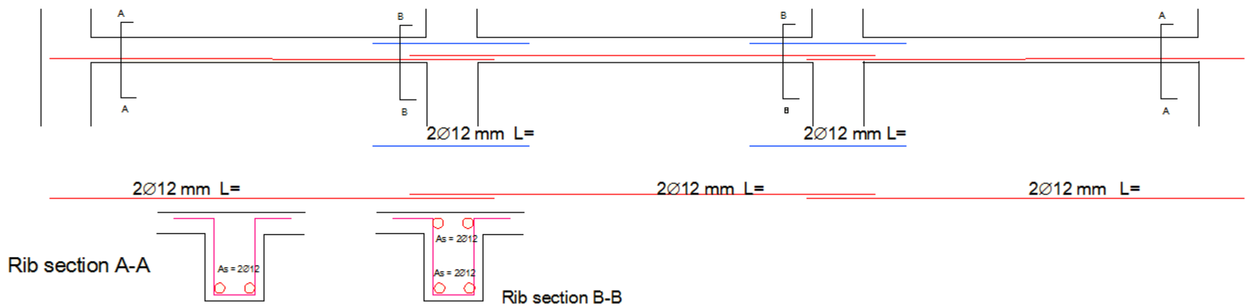
$$A_{s2} = 0.12\%\ A_{topping}$$

$$Spacing\ S = \frac{ba_s}{A_s}$$

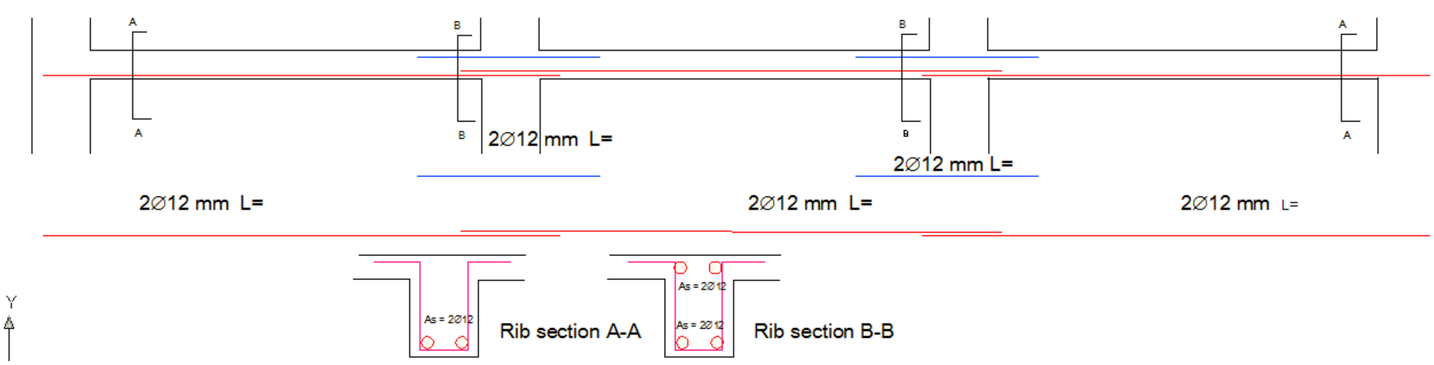
$$\boldsymbol{use\ \emptyset\ 8\ ^{c}/_{c}\ 200\ mm}$$

Step 8. Detailing

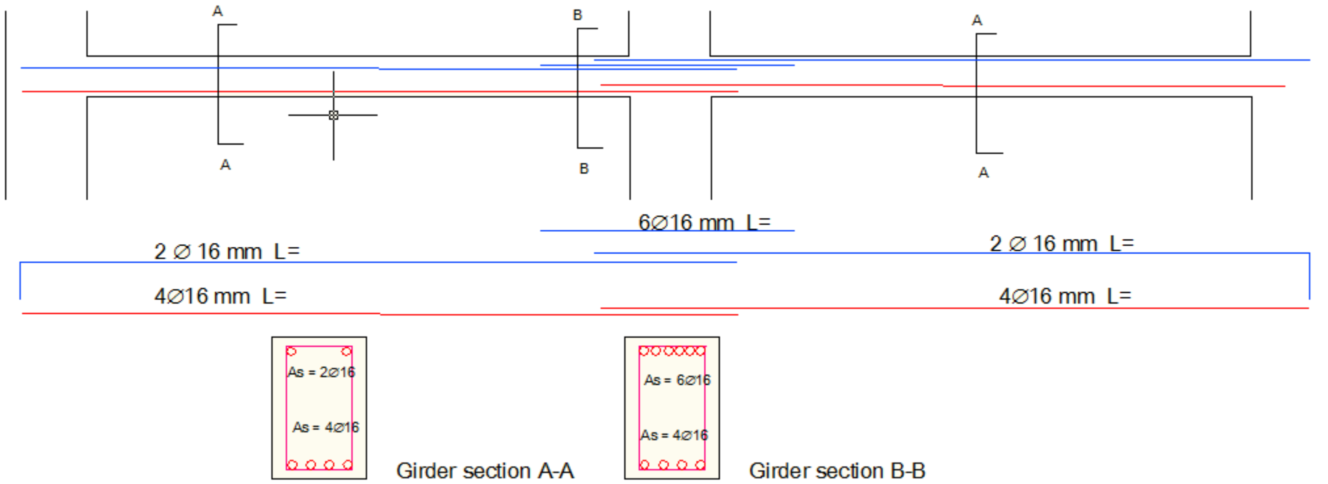
Flexure Reinforcement detailing for Ribs [middle span]



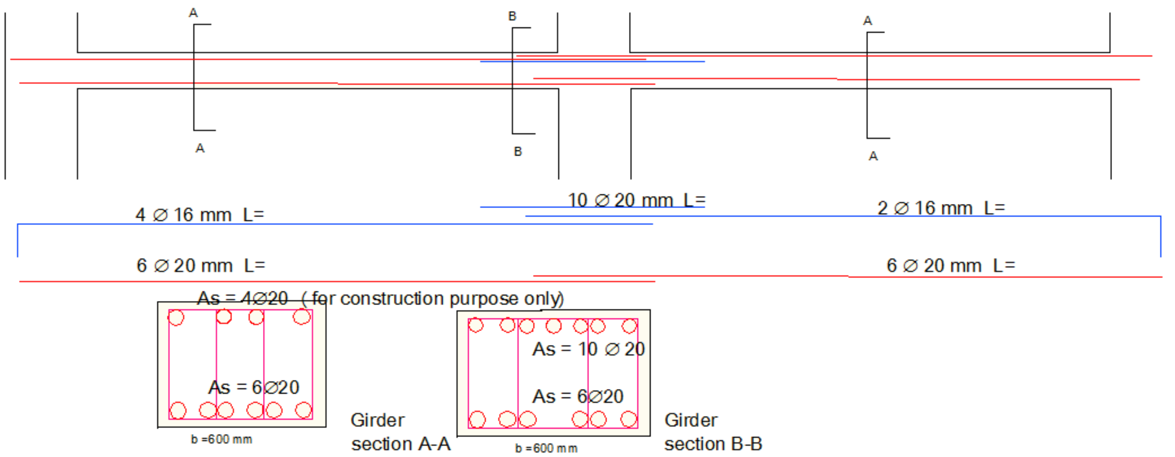
Flexure Reinforcement detailing for Ribs [end span]



Flexure Reinforcement detailing for Girders [A and D]



Flexure Reinforcement detailing for Girders [B and C]



Flexure Reinforcement detailing for Beam