# FOUNDATION ENGINEERING I (CONTD…)

CEng 3204

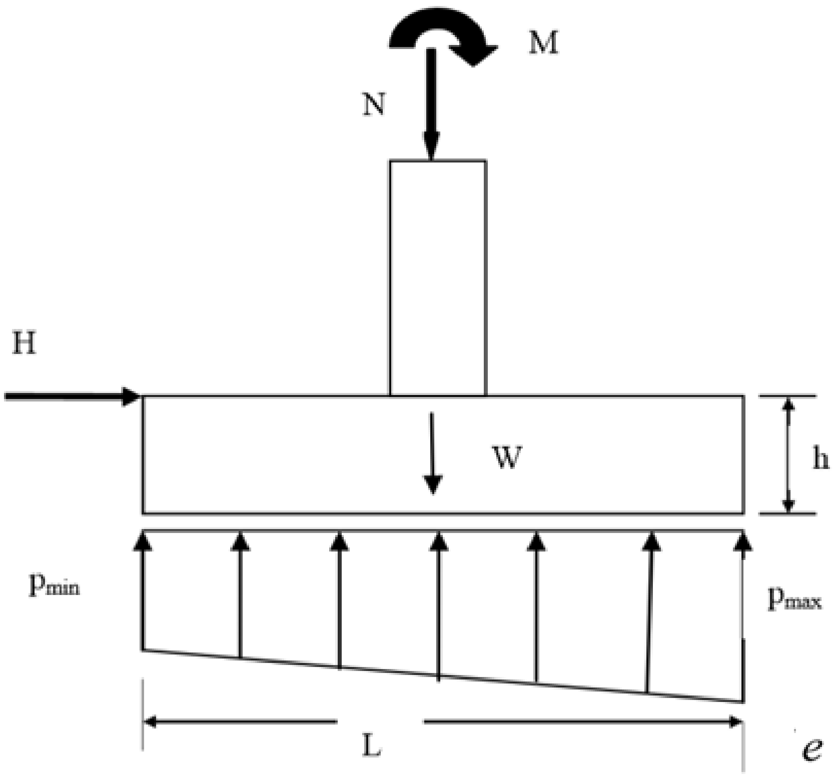
# CHAPTER THREE

Design of Shallow Foundations**: ISOLATED FOOTINGS**

# Isolated Footing Design

## Eccentrically Loaded Pad Bases

- Base pressure is assumed rigid as compared with the soil.
- Therefore, varies linearly across base.




Base area $A = B \times L$

Section modulus $Z = B \times L^2/6$

$$p_{max} = \frac{(N+W)}{A} + \frac{(M+Hh)}{Z}$$

$$p_{min} = \frac{(N+W)}{A} - \frac{(M+Hh)}{Z}$$

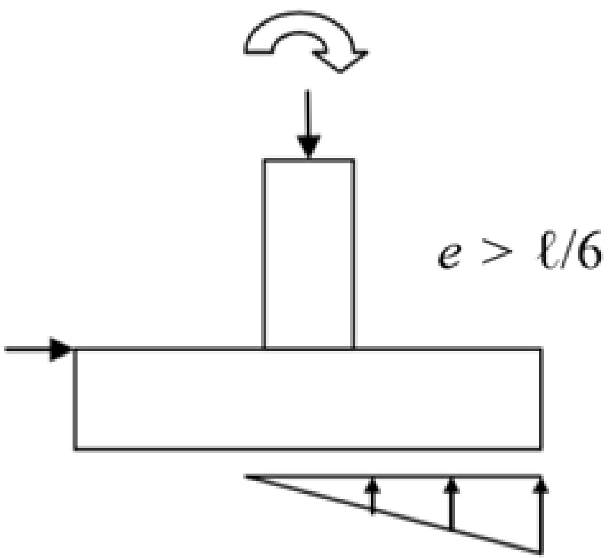
$p_{max}$ should not exceed the safe bearing pressure.

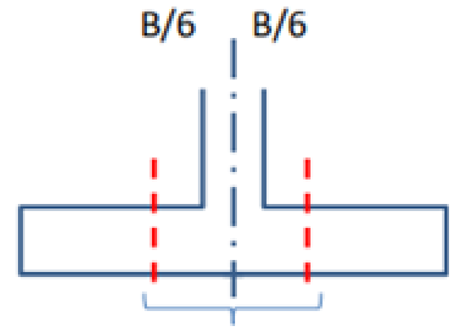
$e \leq L/6.$

# Isolated Footing Design

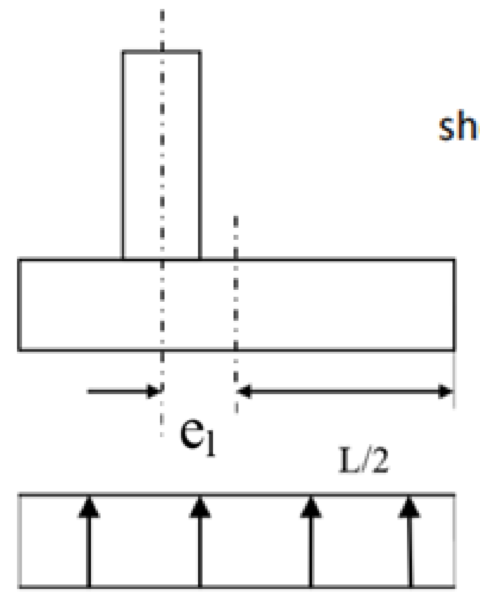
The eccentricity e of the resultant reaction is

$$e = \frac{(M + Hh)}{(N + W)}$$





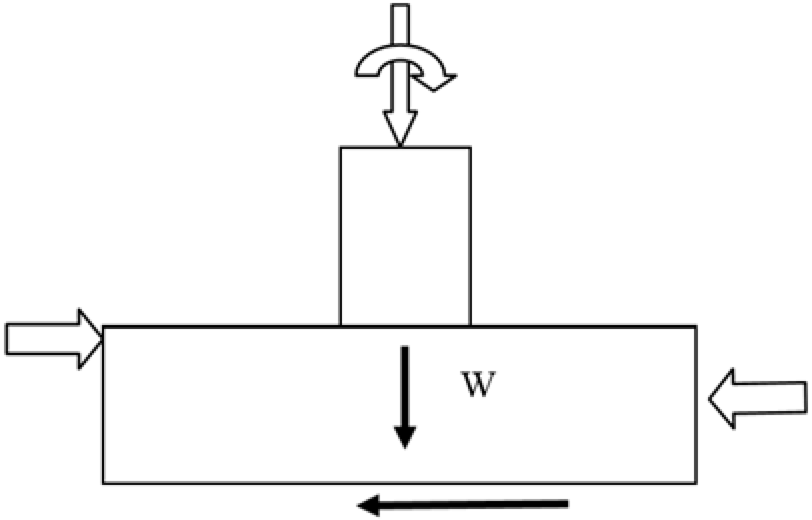

Eccentricity should be within this zone




$e_1 = (M + Hh)/N$

# Isolated Footing Design

- **Resistance to horizontal loads**
  - Resisted by ***passive earth pressure,*** ***friction*** between the base and ground for cohesionless soils, or ***adhesion*** for cohesive soils.




**Cohesive soils**

$$R_{base} = 2cBh + 0.5B\,h\,(p_1 + p_2) + \beta\,L\,B$$

**Cohesionless soils**

Passive Earth pressure

$$p = \gamma\,h\,k_p$$

$$k_p = (1 + \sin\varphi)/\,(1 - \sin\varphi)$$

$$R_{passive} = 0.5\,B\,h\,(p_1 + p_2)$$

$$R_{friction} = \mu\,(N + W) \qquad \mu = \tan\phi$$

# Worked Example

The characteristics load for an internal column footing in a building are given.

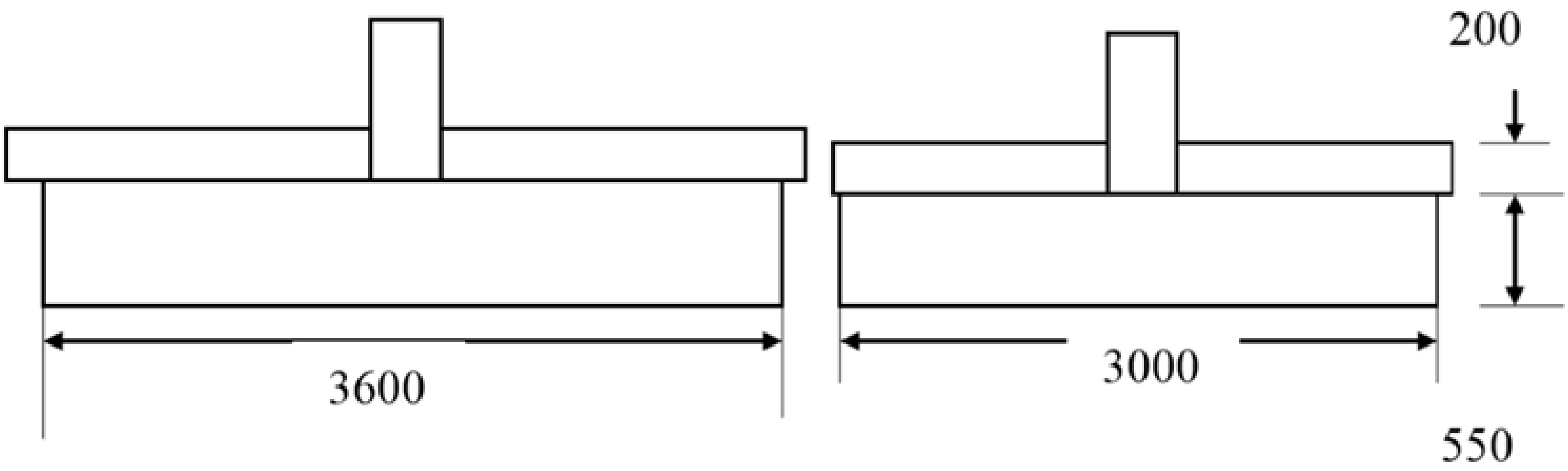
The proposed dimensions for the 450 mm column square and base (3600 x 3000 mm) are shown. The base supports a ground floor slab of 200 mm thick.

The soil is firm well drained clay with the following properties:

Unit weight = 18 $kN/m^3$,
Safe bearing pressure = 150 $kN/m^2$,
Cohesion = 60 $kN/m^2$

The materials to be used in the foundation are $f_{ck}$ = 30 MPa and $f_{yk}$ = 500 MPa.

# Worked Example




| | Vertical load, kN | Horizontal load, kN | Moment, kNm |
|---|---|---|---|
| Dead | 770 | 35 | 78 |
| Imposed | 330 | 15 | 34 |

# Worked Example

a. Maximum base pressure on soil

The maximum base pressure is checked for the service loads.

$$\text{Weight of base + slab} = (550 + 200) \times 10^{-3} \times 3.6 \times 3.0 \times 25 = 202.5 \text{ kN}$$

$$\text{Total axial load} = 770 + 330 + 202.5 = 1302.5 \text{ kN}$$

$$\text{Total moment} = 78 + 34 + 0.550 \times (35 + 15) = 139.5 \text{ kN m}$$

$$\text{Base area } A = 3.0 \times 3.6 = 10.8 \text{ m}^2$$

$$\text{Section modulus } Z = 3.0 \times 3.6^2/6 = 6.48 \text{ m}^3$$

$$\text{Maximum base pressure} = 1302.5/10.8 + 139.5/6.48 = 120.6 + 21.5 = 142.1 \text{ kN/m}^2$$

$$\text{Maximum base pressure} < (\text{safe bearing pressure} = 150 \text{ kN/m}^2)$$

# Worked Example

b. Resistance to horizontal load

Check the passive earth resistance assuming no ground slab.

No adhesion, $\beta = 0$, ($h_1 = 0$, $p_1 = 0$), ($h_2 = 0.550$, $p_2 = 18 \times 0.550 = 9.9$)

The passive resistance is

$$= 2c\,B\,h + 0.5\,B\,h\,(p_1 + p_2) + \beta\,L\,B$$

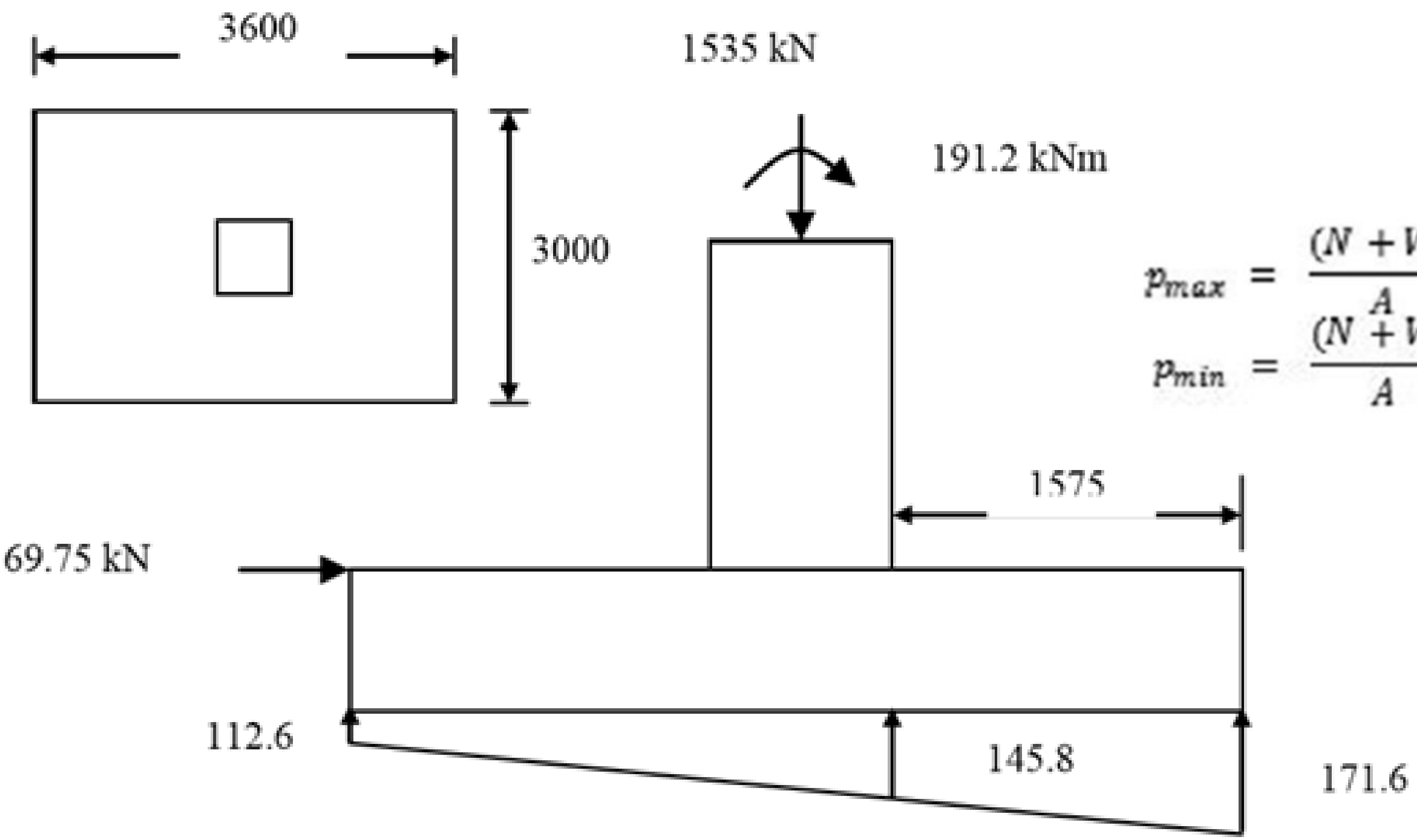
$$= \{2 \times 60 \times 3.0 \times 0.550\} + \{0.5 \times 3.0 \times 0.5 \times (0 + 9.9)\} + 0$$

$$= 198 + 7.4 = 205.4\ \text{kN}$$

Factored horizontal load = $(1.35 \times 35) + (1.5 \times 15) = 69.75$ kN

Passive resistance > 69.75 kN

# Worked Example




$$p_{max} = \frac{(N+W)}{A} + \frac{(M+Hh)}{Z}$$

$$p_{min} = \frac{(N+W)}{A} - \frac{(M+Hh)}{Z}$$

# Worked Example

c. Design of moment reinforcement

   I. ***Long-span moment steel***

Axial load $N = (1.35 \times 770) + (1.5 \times 330) = 1535$ kN

Horizontal load $H = (1.35 \times 35) + (1.5 \times 15) = 69.75$ kN

Moment $M = (1.35 \times 78) + (1.5 \times 34) + (0.5 \times 69.75) = 191.2$ kNm

Maximum pressure $= 1535/10.8 + 191.2/6.48 = 171.6$ kN/m$^2$

Minimum pressure $= 1535/10.8 - 191.2/6.48 = 112.6$ kN/m$^2$

At the face of the column pressure is

Pressure $= 112.6 + (171.6 - 112.6) \times (3.6 - 1.575)/3.6 = 145.8$ kN/m$^2$

Moment at the face of the column is

$$M_y = 145.8 \times 3.0 \times 1.575^2/2 + 0.5(171.6 - 145.8) \times 3.0 \times 1.575 \times (2/3) \times 1.575$$
$$= 606.5 \text{ kNm}$$

# Worked Example

If the cover is 40 mm and 16 mm diameter bars are used, the effective depth for the bottom layer is

$$d = 550 - 40 - 16/2 = 502 \text{ mm}$$

$$k = M/(bd^2 f_{ck}) = 606.5 \times 10^6/(3000 \times 502^2 \times 30) = 0.027 < 0.196$$

$$\frac{z}{d} = 0.5[1.0 + \sqrt{(1 - 3\frac{k}{\eta})}]$$

$$z/d = 0.98$$

$$A_s = M/\{0.87 f_{yk} z\}$$

$$f_{yk} = 500,\ f_{yd} = 500/1.15 = 435 \text{ MPa}$$

$$A_s = 606.5 \times 10^6/(435 \times 0.98 \times 502) = 2834 \text{ mm}^2$$

# Worked Example

$A_{s,\,min} = 0.26\times(f_{ctm}/f_{yk}) \times bd \geq 0.0013\ bd$

$f_{yk} = 500$ MPa,

$f_{ctm} = 0.3 \times f_{ck}^{0.67} = 0.3 \times 30^{0.67} = 2.9$ MPa.

$b = 3000$ mm, $d = 502$ mm

$$A_{s,\,min} = 0.26\times(2.9/500) \times 3000 \times 502 \geq 0.0013 \times 3000 \times 502$$

$$A_{s,\,min} = 2271\ mm^2 < 2834$$

Provide 15H16. $A_s =$ **3016** $nm^2$.

$$0.75\,(c + 3d) = 0.75\,(450 + 3 \times 502) = 1467\ mm,$$

$$L/2 = 3000/2 = 1500\ mm$$

$$0.75\,(c + 3d) < \ell_x$$

# Worked Example

The difference between 1467 mm and 1500 mm is small enough to be ignored and steel can be distributed uniformly. Provide 15 bars at 200 mm centres to give a total steel area of 3016 mm$^2$.

***II. Short-span moment steel***

$$\text{Average pressure} = 0.5 \times (171.6 + 112.6) = 142.1 \text{ kN/m}^2$$

$$\text{Moment } M_x = 142.1 \times 3.6 \times 1.275^2/2 = 415.8 \text{ kNm}$$

Using H12 bars,

$$\text{Effective depth } d = 550 - 40 - 16 - 12/2 = 488 \text{ mm}$$

$$k = M/(bd^2 f_{ck}) = 415.8 \times 10^6/(3600 \times 488^2 \times 30) = 0.016 < 0.196$$

$$\frac{z}{d} = 0.5[1.0 + \sqrt{(1 - 3\frac{k}{\eta})}]$$

z/d= 0.99

# Worked Example

$A_s = M/\{0.87\ f_{yk}\ z\}$

$f_{yk} = 500,\ f_{yd} = 500/1.15 = 435$ MPa

$A_s = 415.8 \times 10^6/(435 \times 0.99 \times 488) = 1979\ mm^2$

$A_{s,\ min} = 0.26\times (f_{ctm}/f_{yk}) \times bd \geq 0.0013\ bd$

$f_{ctm} = 0.3 \times f_{ck}^{\ 0.67} = 0.3 \times 30^{\ 0.67} = 2.9$ MPa, $f_{yk} = 500$ MPa,

$b = 3600$ mm, $d = 488$ mm

$A_{s,\ min} = 0.26\times (2.9/500) \times 3600 \times 488 \geq 0.0013 \times 3600 \times 488$

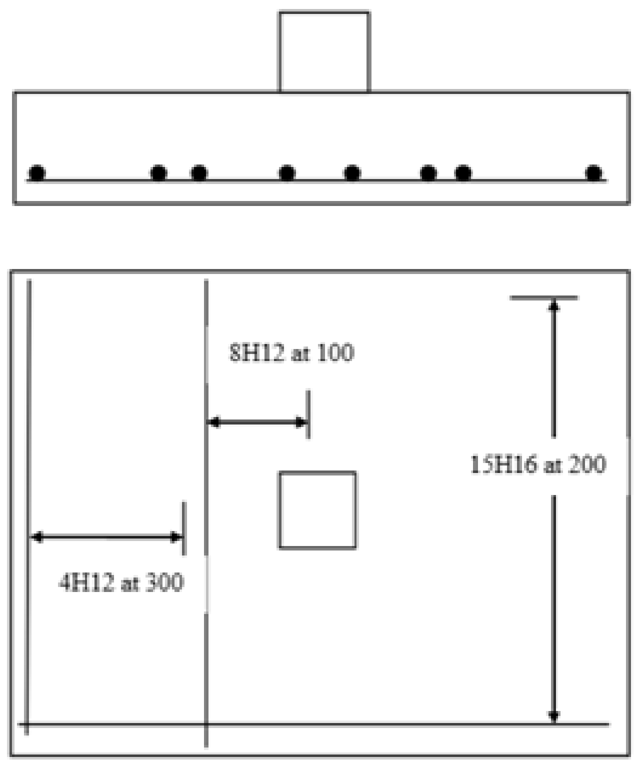
$A_{s,\ min} = 2649\ mm^2 > 1979\ mm^2$

Provide 24H12. $A_s$ provided = 2714 $mm^2$

# Worked Example

$$0.75(c + 3d) = 1436 < (\ell_x = 1800 \text{ mm})$$

Place two-thirds of the bars (16 bars) in the central zone **1914** ım wide. Provide 16H12 at **120 mm** over a width of 1500 mm. In the outer strips **843** nm wide provide 4H12 at **200 mm** centres.



# Worked Example

d. ***Vertical shear***

I. ***Long-span*** @ $d = 502$ mm

$$\text{Pressure} = 112.6 + (171.6 - 112.6) \times (3.6 - 1.575 + 0.502)/3.6 = 154.0\ \text{kN/m}^2$$

Shear at a distance d from the face of the column is

$$V_{Ed} = 0.5(154.0 + 171.6) \times 3.0 \times (1.575 - 0.502) = 524.1\ \text{kN}$$

$$v_{Ed} = 524.1 \times 10^3 / (3000 \times 502) = 0.35\ \text{MPa}$$

$$V_{Rd,c} = [C_{Rd,c}\, k\{100\rho_1 f_{ck}\}^{1/3} \qquad v_{min} = 0.035 k^{1.5} \sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \le 2.0$$

$$k = 1 + \sqrt{(200/502)} = 1.63 \le 2.0$$

# Worked Example

$$C_{Rd,c} = \frac{0.18}{(\gamma_c = 1.5)} = 0.12$$

$$\rho_1 = \frac{A_{sl}}{b_w d} \leq 0.02 \qquad A_{sl} = 15H16 = 3016\ mm^2,$$

$$= 3016/\,(3000 \times 502) = 0.002 \leq 0.02$$

$$C_{Rd,c} \times k \times (100 \times \rho_1 \times f_{ck})^{0.33} = 0.12 \times 1.63 \times (100 \times 0.002 \times 30)^{0.33} = 0.36$$

$$v_{min} = 0.035 \times k^{1.5} \times \sqrt{f_{ck}} = 0.035 \times 1.63^{1.5} \times \sqrt{30} = 0.40 > 0.36$$

$$v_{Rd,c} = 0.40\ MPa$$

$$(v_{Ed} = 0.35) < (v_{Rd,c} = 0.40)$$

No shear reinforcement is required.

# Worked Example

d. ***Vertical shear***

   II. ***Short-span***

Average pressure = 0.5(171.6 + 112.6) = 142.1 kN/m$^2$

The average pressure acts over an area of dimensions

$$\{(3000 - 450)/2 - 488 = 787 \text{ mm}\} \times 3600 \text{ mm}$$

$$V_{Ed} = 142.1 \times 3.6 \times 0.787 = \textbf{402.6 kN}$$

$$v_{Ed} = \textbf{402.6} \times 10^3 / (3600 \times 488) = 0.24 \text{ MPa}$$

$$V_{Rd,c} = [C_{Rd,c}\, k\{100\rho_1 f_{ck}\}^{1/3}$$

$$C_{Rd,c} = 0.18/(\gamma_c = 1.5) = 0.12,\ k = 1 + \sqrt{(200/488)} = 1.64 \leq 2.0$$

$$\rho_1 = A_{sl}/(b_w d) = 2714/(3600 \times 488) = 0.0016 \leq 0.02 \qquad A_{sl} = 24H12 = 2714 \text{ mm}^2,$$

# Worked Example

$$C_{Rd,c} \times k \times (100 \times \rho_1 \times f_{ck})^{0.33} = 0.12 \times 1.64 \times (100 \times 0.0016 \times 30)^{0.33} = 0.33$$

$$v_{min} = 0.035 \times k^{1.5} \times \sqrt{f_{ck}} = 0.035 \times 1.64^{1.5} \times \sqrt{30} = 0.40 > 0.33$$

$$v_{Rd,c} = 0.40 \text{ MPa}$$

$$(v_{Ed} = 0.24) < (v_{Rd,c} = 0.40)$$

No shear reinforcement is required.

# Worked Example

e. ***Punching shear and maximum shear***

Check punching shear around column perimeter:

$$\text{Column perimeter, } u_0 = 2(c_1 + c_2) = 1800 \text{ mm}$$

$$d = 495 \text{ mm}$$

Column axial force = 1535 kN

$$v_{Rd,\,max} = 0.3\times (1 - f_{ck}/250) \times f_{cd} = 0.3 \times (1 - 30/250) \times (30/1.5) = 5.28 \text{ MPa}$$

Shear stress around column perimeter $= 1535 \times 10^3 / (1800 \times 495)$

$= 1.72 \text{ MPa} < (v_{Rd,\,max} = 5.28\text{MPa})$

Thickness of the slab is acceptable.

# Worked Example

Check punching shear on perimeters at Nd from the face of the column, where $1 \leq N \leq 2$.

Average $d = 0.5\ (502 + 488) = 495$ mm

Average pressure = $(171.6 + 112.6)/2 = 141.8$ kN/m$^2$

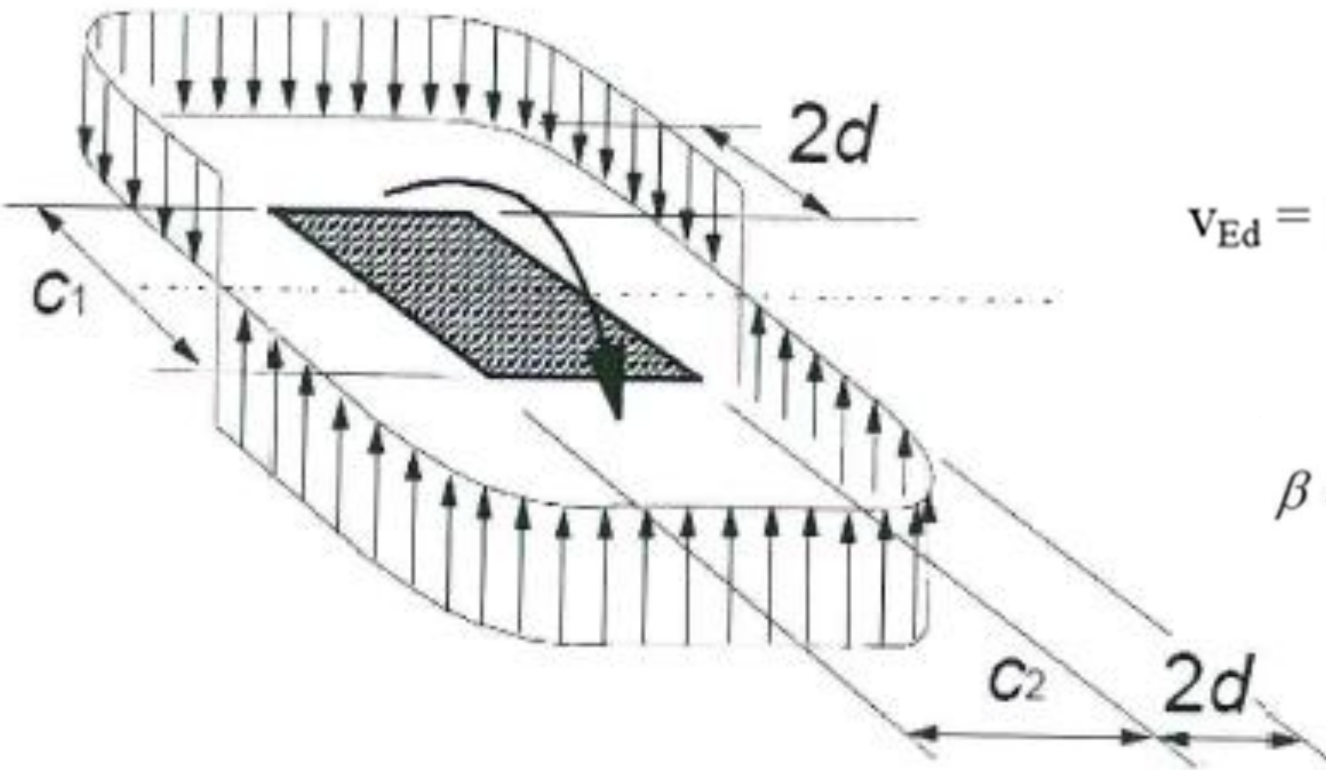
Area inside the perimeter $A = c_1 \times c_2 + 2 \times (c_1 + c_2) \times Nd + \pi \times (Nd)^2$

where $c_1 = c_2 = 450$ mm.

Upward thrust from base pressure = $141.8 \times A$ kN

Perimeter length, $u = 2(c_1 + c_2 + \pi \times Nd)$

# Worked Example




Shear distribution due to an unbalanced moment

$$v_{Ed} = \beta \times (V_{Ed} / (u_1 \times d))$$

$$\beta = 1 + k \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1}$$

# Worked Example

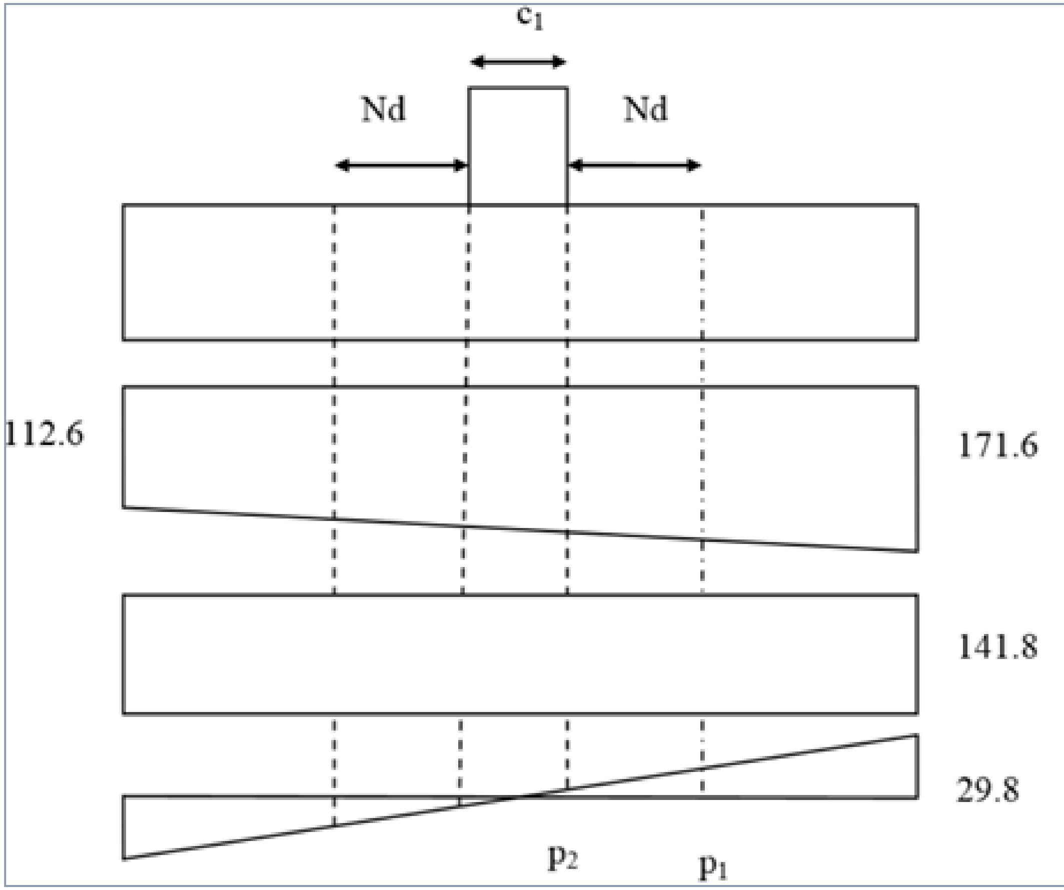
Pressure at column face is

$p_2 = 29.8 \times (c_1/L) = 3.688\ kN/m^2$

Pressure at punching shear perimeter,

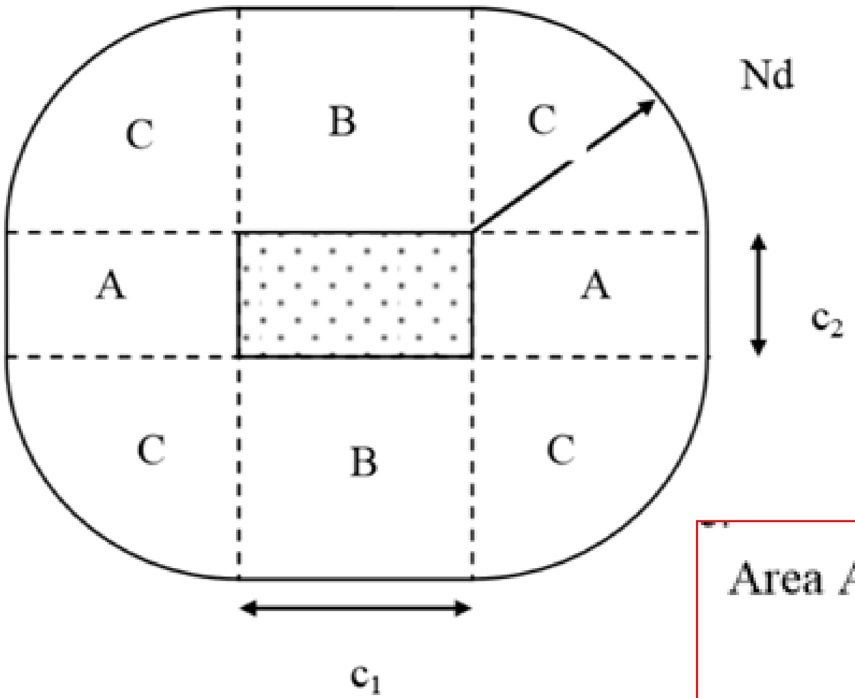
$p_1 = 29.8 \times (c_1 + 2\ Nd)/L\ kN/m^2$

where $c_1 = 450$ mm, $L = 3600$ mm.



# Worked Example

Moment caused by the linear pressure distribution in the three areas




$$\text{Area A: } M_a = \frac{2}{3} \times p_1 \times (0.5c_1 + Nd)^2 \times c_2$$

$$\text{Area B: } M_b = \frac{1}{3} \times p_2 \times c_1^2 \times Nd$$

# Worked Example

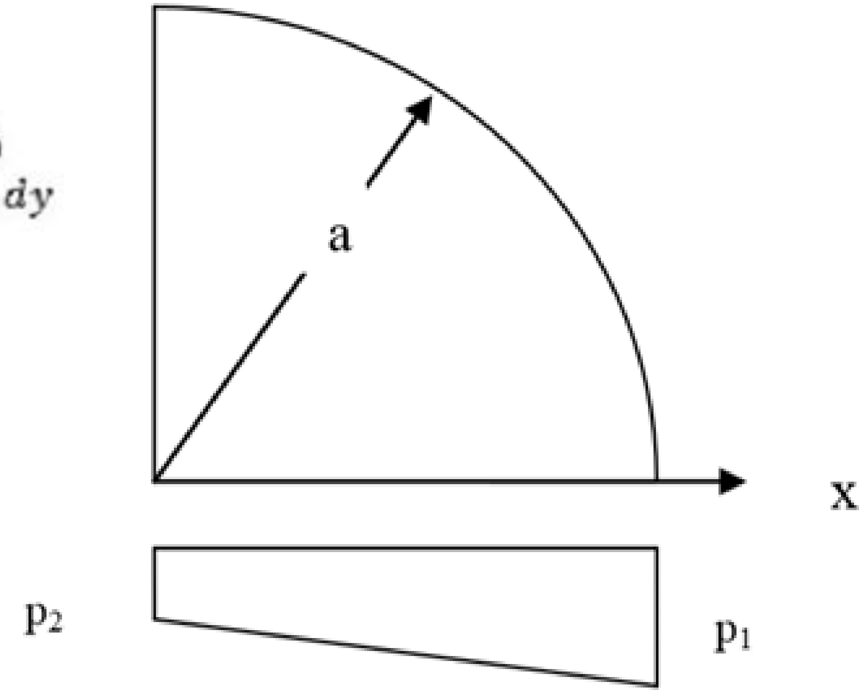
$$F = \int_{x=0}^{x=a} \{p_2 + (p_1 - p_2) \times \frac{x}{a}\}\, dx \int_{y=0}^{y=\sqrt{(a^2 - x^2)}} dy$$

$$F = \frac{a^2}{12} \times [(3\pi - 4)p_2 + 4\, p_1]$$

$$M_y = \int_{x=0}^{x=a} \{p_2 + (p_1 - p_2) \times \frac{x}{a}\}\, x\, dx \int_{y=0}^{y=\sqrt{(a^2 - x^2)}} dy$$

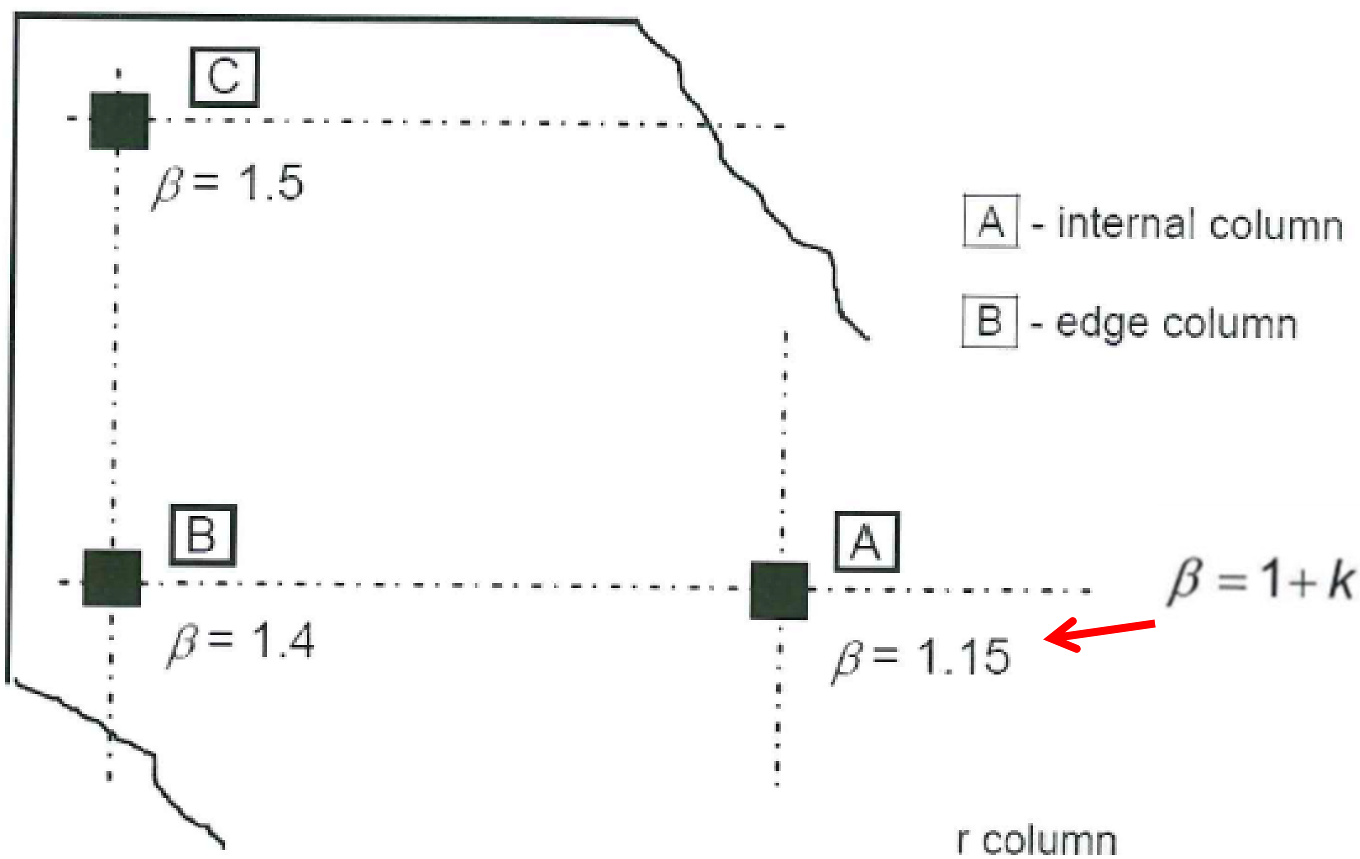
$$M_y = \frac{a^3}{48} \times [(16 - 3\pi)p_2 + 3\pi\, p_1]$$

$$M_c = 4M_y + 2 \times F \times c_1$$



# Worked Example

**Note:** Recommended values for $\beta$ are given in Figure 6.21N.




$$\beta = 1 + k\frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1}$$

# Worked Example

$V_{Ed,\,red}$ = Column axial force – upward thrust from base pressure

$M_{Ed,\,red}$ = Moment on the column – $(M_a + M_b + M_c)$

$$v_{Ed} = \beta \times (V_{Ed} / (u_1 \times d)$$

$$u_1 = 2(c_1 + c_2 + \pi \times Nd)$$

$$\beta = 1 + k \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1}$$

| $c_1/c_2$ | $\leq 0.5$ | 1.0 | 2.0 | $\geq 3.0$ |
|---|---|---|---|---|
| k | 0.45 | 0.60 | 0.70 | 0.80 |

$W_1$ corresponds to a distribution of shear $W_i = \int_0^{u_i} |e| \, dl$

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4 c_2 d + 16 d^2 + 2\pi d c_1$$

$$W = c_1 c_2 + 2 c_2 a + 0.5 c_1^2 + 4a^2 + \pi c_1 a$$

# Worked Example

| Nd | $p_1$ | A | N-Soil | $V_{Ed,red}$ | $M_a$ | $M_b$ | $M_c$ | M-soil | $M_{Ed,red}$ | $W_1$ | $u_1$ | $v_{Ed}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 495 | 11.8 | 1.9 | 265 | 1270 | 1.8 | 0.1 | 2.6 | 4.6 | 187 | 2.4 | 4.9 | 0.62 |
| 545 | 12.6 | 2.1 | 300 | 1235 | 2.2 | 0.1 | 3.5 | 5.9 | 185 | 2.7 | 5.2 | 0.56 |
| 594 | 13.4 | 2.4 | 338 | 1197 | 2.7 | 0.1 | 4.6 | 7.4 | 184 | 3.1 | 5.5 | 0.51 |
| 644 | 14.2 | 2.7 | 378 | 1157 | 3.2 | 0.2 | 5.9 | 9.3 | 182 | 3.4 | 5.8 | 0.46 |
| 693 | 15.0 | 3.0 | 420 | 1115 | 3.8 | 0.2 | 7.5 | 11.5 | 180 | 3.8 | 6.2 | 0.42 |
| 743 | 15.9 | 3.3 | 465 | 1070 | 4.5 | 0.2 | 9.4 | 14.0 | 177 | 4.2 | 6.5 | 0.39 |
| 792 | 16.7 | 3.6 | 511 | 1024 | 5.2 | 0.2 | 11.6 | 17.0 | 174 | 4.6 | 6.8 | 0.35 |
| 842 | 17.5 | 3.9 | 560 | 975 | 6.0 | 0.2 | 14.2 | 20.3 | 171 | 5.1 | 7.1 | 0.32 |
| 891 | 18.3 | 4.3 | 611 | 924 | 6.8 | 0.2 | 17.1 | 24.2 | 167 | 5.5 | 7.4 | 0.29 |
| 941 | 19.1 | 4.7 | 664 | 871 | 7.8 | 0.2 | 20.6 | 28.6 | 163 | 6.0 | 7.7 | 0.26 |
| 990 | 19.9 | 5.1 | 720 | 816 | 8.8 | 0.2 | 24.5 | 33.5 | 158 | 6.5 | 8.0 | 0.23 |

**N-soil = 141.8 x A**

**M-soil = $M_a + M_b + M_c$**

**$V_{Ed,red}$=1535 – N-soil**

**$M_{Ed,red}$=191.2 – M-soil**

# Worked Example

$C_{Rd,c} = 0.18/(\gamma_c = 1.5) = 0.12,$

$k = 1 + \sqrt{(200/495)} = 1.64 \leq 2.0,$

Average $100A_s/(bd) = \sqrt{(0.20 \times 0.16)} = 0.18$

$C_{Rd,c} \times k \times (100 \times \rho_1 \times f_{ck})^{0.33} = 0.12 \times 1.64 \times (0.18 \times 30)^{0.33} = 0.35$

$v_{min} = 0.035 \times k^{1.5} \times \sqrt{f_{ck}} = 0.035 \times 1.64^{1.5} \times \sqrt{30} = 0.40 > 0.35$

$v_{Rd,c} = 0.40$ MPa

$$v_{Rd} = v_{Rd,c} \times (2d/a) = 0.40 \times (2d/a)$$

At 'a' = d, $v_{Rd} = 0.80$ MPa which is greater than 0.62 MPa

At a = 2d, $v_{Rd} = 0.40$ MPa which is greater than 0.23 MPa

No shear reinforcement is required.

# Worked Example