# FOUNDATION ENGINEERING I

CEng 3204

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# CHAPTER FOUR

**Retaining Walls:** *Analysis and Design*
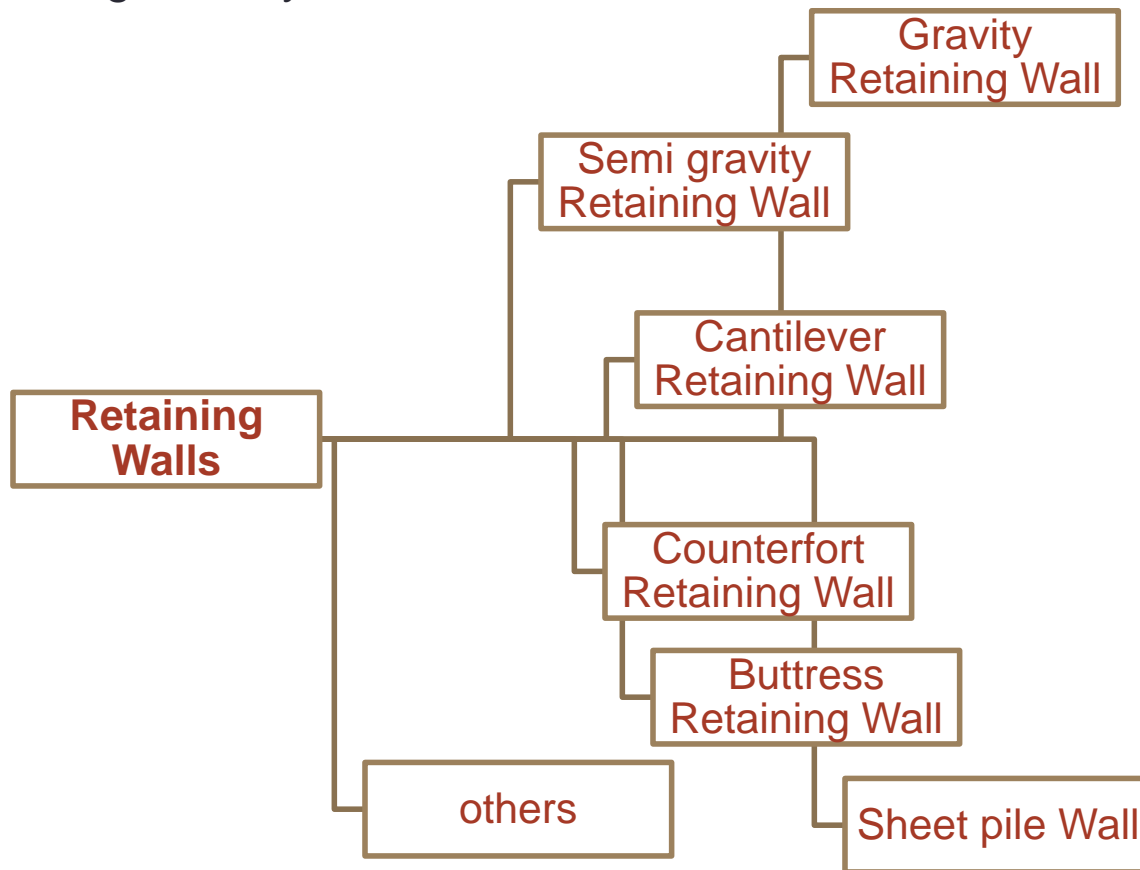
# Retaining Walls

- Retaining walls are structures used to retain mainly earth but also other materials which would not be able to attain its natural slope.

- Various types of retaining are widely used in civil engineering works ranging from their use in road and rail construction to support cuts and fills where space is limited to prevent the formation of appropriate side slopes, to the construction of marine structures such as docks, harbors, and jetties.
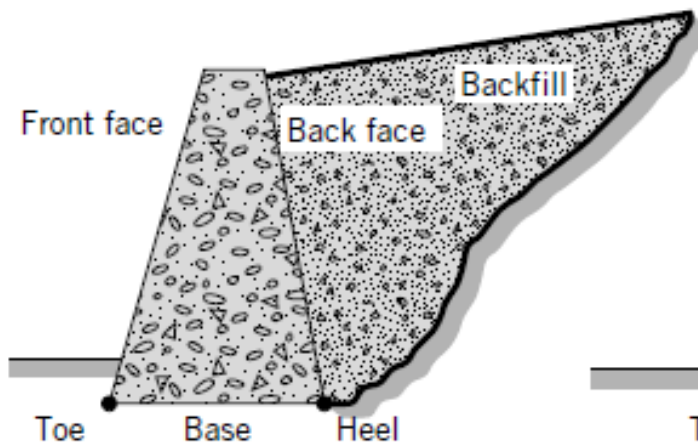
# Retaining Walls

# Retaining Walls

Various types of retaining walls are available based on the method of achieving stability.

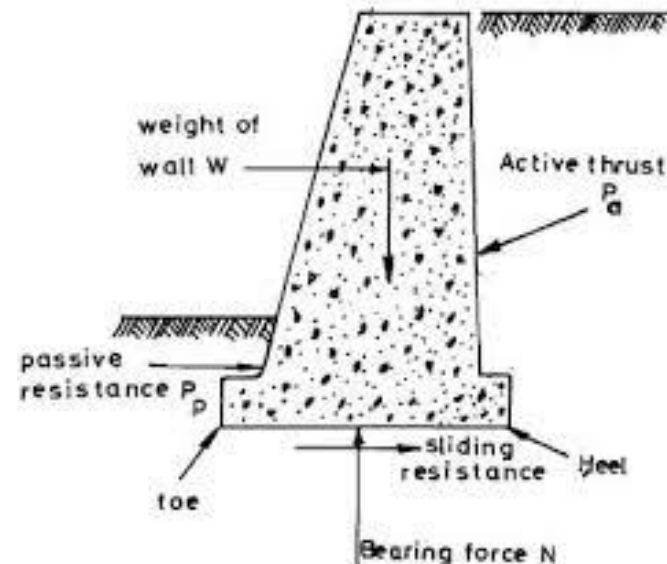| | |
|---|---|
| | Gravity Retaining Wall |
| | Semi gravity Retaining Wall |
| | Cantilever Retaining Wall |
| **Retaining Walls** | Counterfort Retaining Wall |
| | Buttress Retaining Wall |
| | others |
| | Sheet pile Wall |

# Retaining Walls

## Gravity Retaining Walls

- Resists the earth pressure exerted by backfill by its own self weight.
- Stability of wall depends on their weight.
- Usually built in stone masonry, and occasionally in plain concrete.
- Proportioned so that **no tension** is developed anywhere.
- Plain concrete gravity walls are not used for heights exceeding about 3m.

(a) Gravity retaining wall

# Retaining Walls

## Semi gravity Retaining Wall

- Similar to gravity walls but a small amount of reinforcements are added to reduce the mass of concrete.
- Resists external forces by the combined action of self weight, weight of soil above footing and the flexural resistance of the wall component.
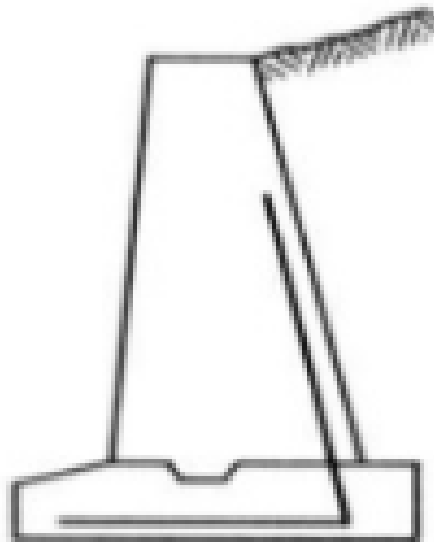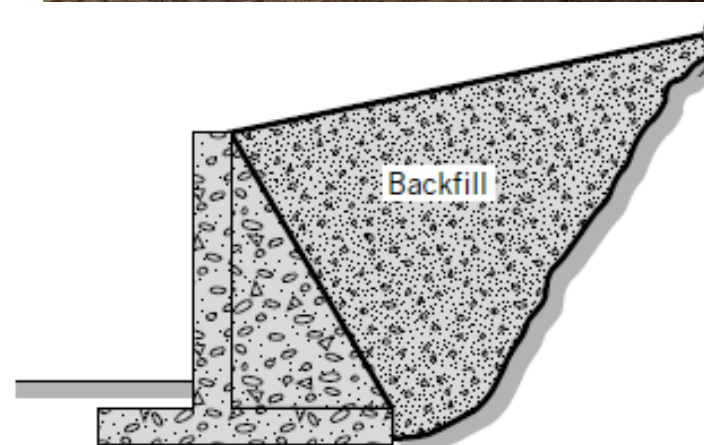
# Retaining Walls

**Cantilever Retaining Wall**

- Reinforced concrete walls that utilize cantilever action to retain mass of earth or any material behind them.
- It consists of vertical stem, and a base slab, made up of two distinct regions, a heel slab and a toe slab.
- Economical up to a height of 8m.
- Stability is provided by the weight of structure and earth on an inner base or the weight of the structure only when the base is constructed externally
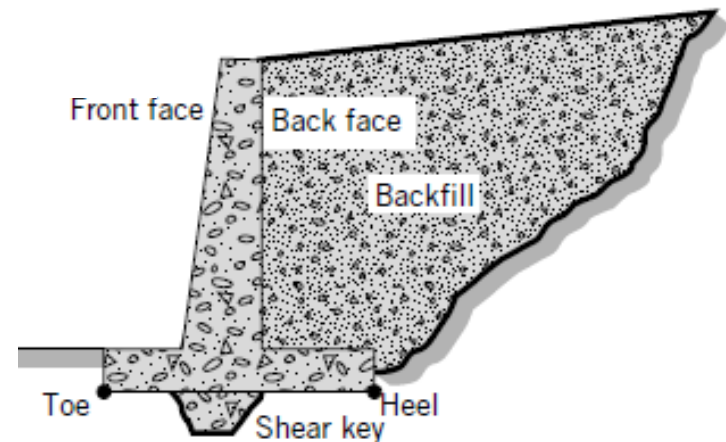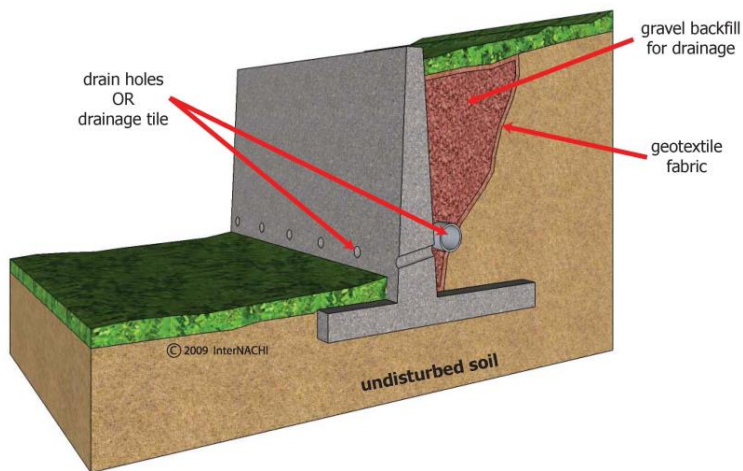
# Retaining Walls

## Counterfort Retaining Wall

- For large heights, in a cantilever retaining wall, the bending moments developed in the stem, heel slab and toe slab become very large and require large thickness.

- The bending moment can be considerably reduced by introducing transverse supports, called **counter-forts.**

- Stability is provided by the weight of the structure and earth on the base
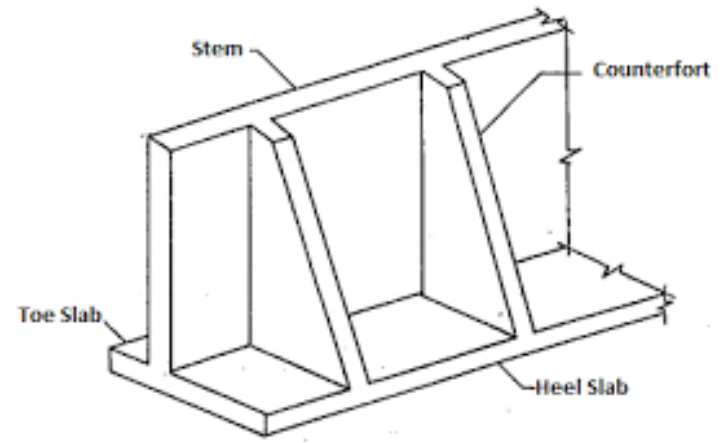
# Retaining Walls

## Buttress Retaining Wall

- Similar to counterfort retaining walls but the bracing is in front of the wall subjected to compressive force instead of tension.

- Stability of provided by the weight of the structure.
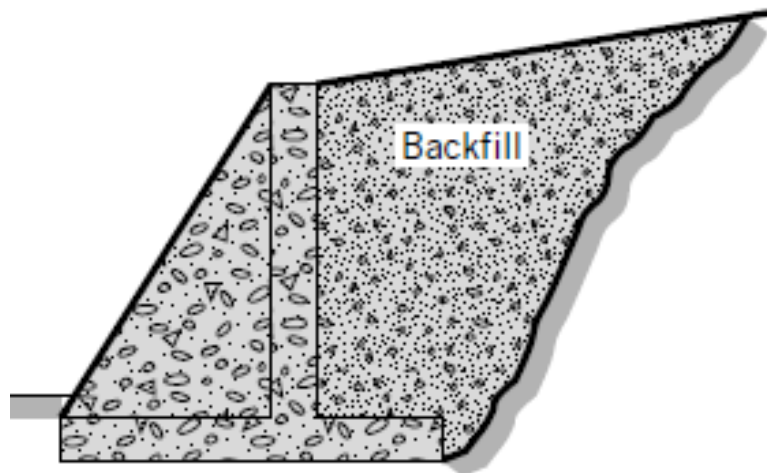


(d) Buttress wall

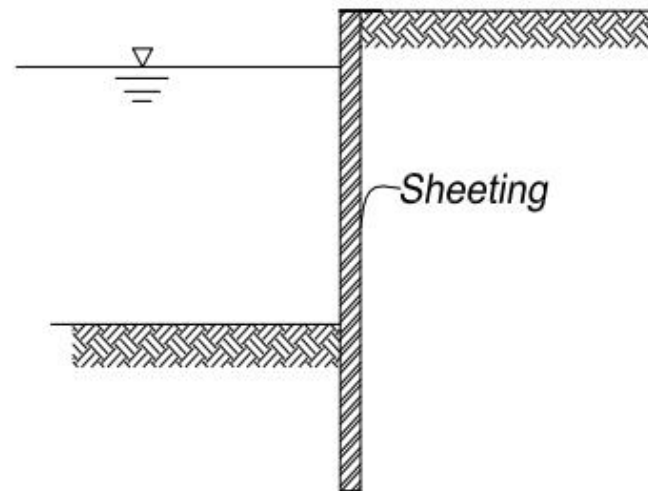# Retaining Walls

## Sheet Pile Wall

• Sheet pile walls are thin retaining walls constructed retain earth, water, or any other filling materials.

• Stability is achieved by passive earth pressure resistance.

Sheet pile wall(cantiliver type)

# Retaining Walls

- The design of retaining walls to EBCS 7 involves checking that the ground beneath the wall has sufficient:

  o  bearing resistance to withstand inclined, eccentric actions

  o  sliding resistance to withstand horizontal and inclined actions

  o  stability to avoid toppling

  o  stiffness to prevent unacceptable settlement or tilt

Verification of ultimate limit states (ULSs) is demonstrated by satisfying the inequalities:

$$V_d \leq R_d$$

$$H_d \leq R_d + R_{pd}$$
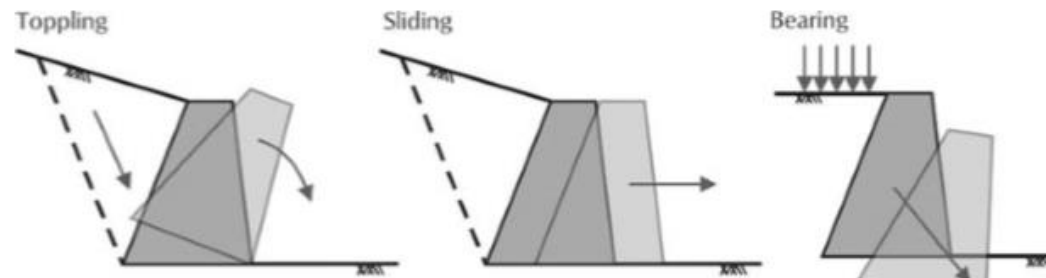
$$M_{Ed,dst} \leq M_{Ed,stb}$$

# Retaining Wall Analysis and Design

1. Initial sizing/proportioning of the wall

2. Stability Failure Analysis and Design.

3. Ground Failure Analysis and Design.

4. Bending Failure and detailing.
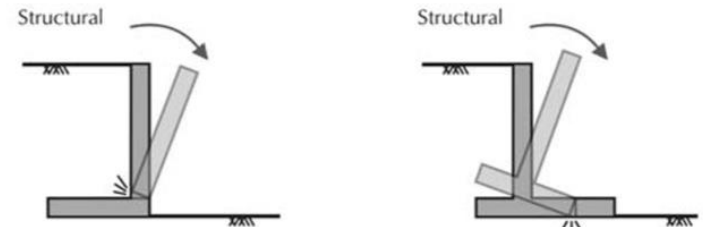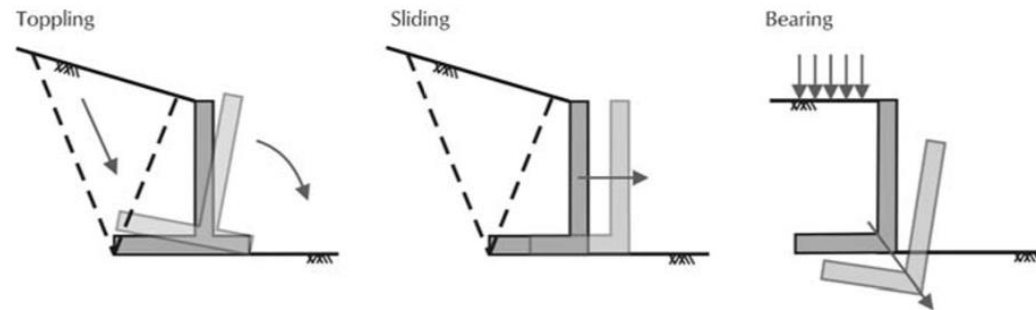
# Proportioning of Retaining Walls
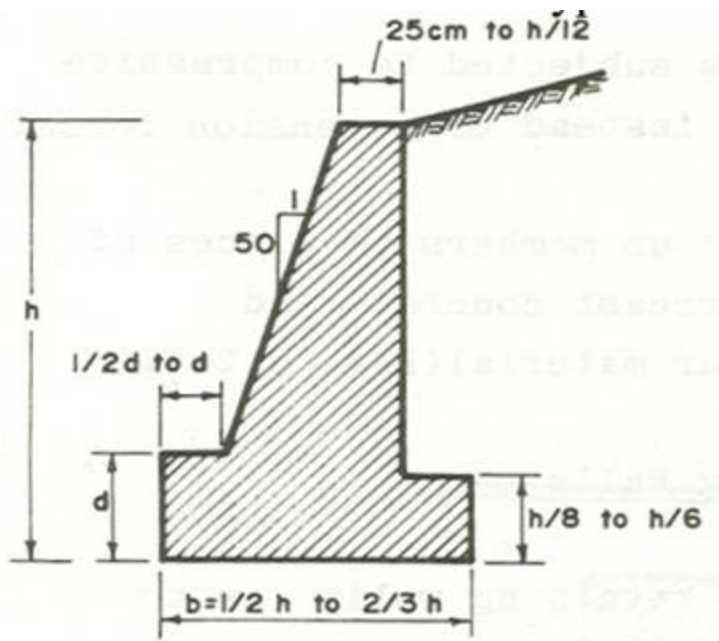


a) Gravity wall
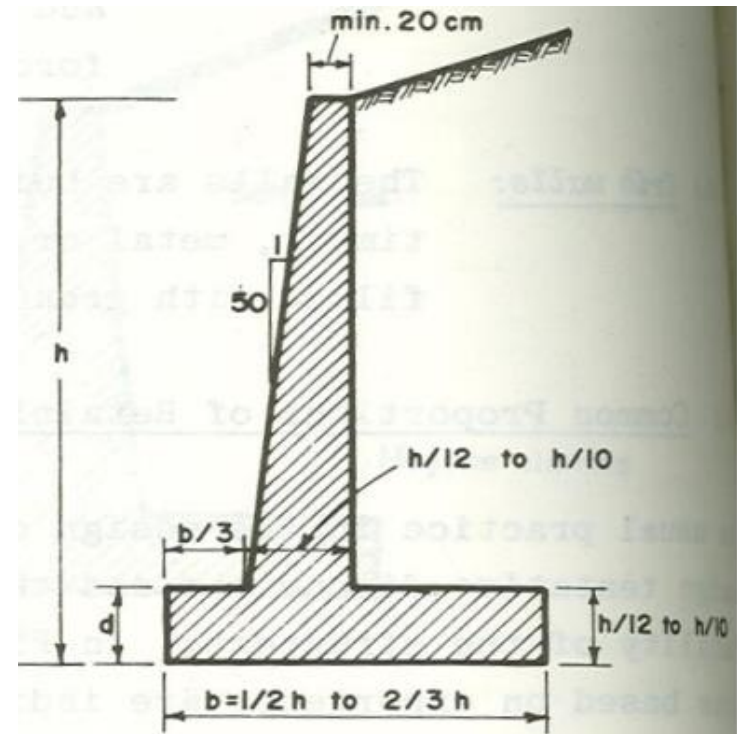
b) Cantilever wall

# Proportioning of Retaining Walls
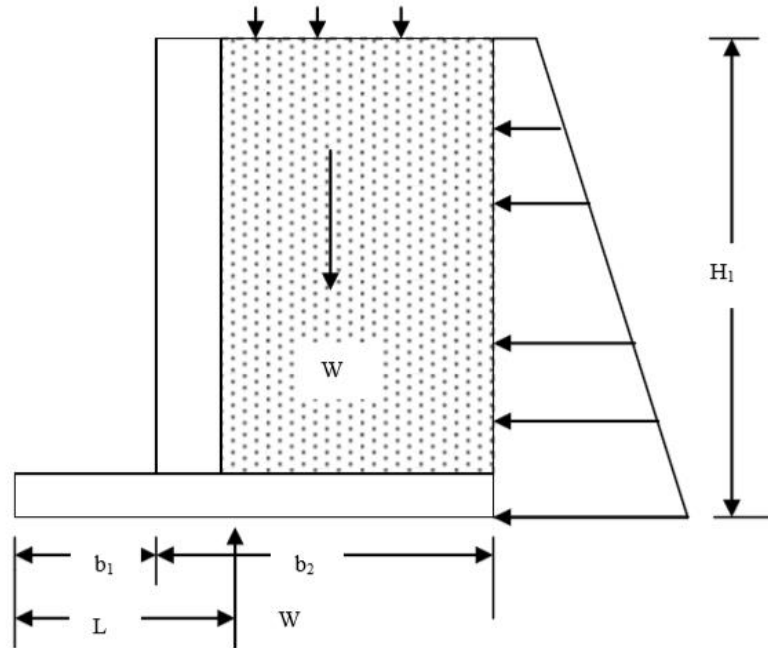


Ignoring the difference in unit weight between soil and concrete and the weight of the toe slab of width $b_1$, for a unit length of wall the total gravity load W is approximately given by

$$W = \gamma \, b_2 \, H_1 + q \, b_2$$

The total horizontal force $P_1$ is given by

$$P_1 = 0.5 \, K_a \gamma \, H_1^2 + K_a \, q \, H_1$$

where q = surcharge in kN/m$^2$ and $K_a$ = coefficient of active earth pressure.

# Proportioning of Retaining Walls

Ignoring any contribution from passive earth pressure and using load factors of $\gamma_Q = 1.5$ on $P_1$ as it is an adverse load and $\gamma_{G, inf} = 1.0$ on W as it is a beneficial load, for resistance against sliding,

$$\mu\, W \geq 1.5\, P_1$$

Substituting for W and $P_1$,

$$\{\mu\, \gamma\, b_2\, H_1 + q\, b_2\} \geq (\gamma_Q = 1.5) \times \{0.5\, K_a\, \gamma\, H_1^2 + K_a\, q \times H_1\}$$

Simplifying,

$$\boxed{\frac{b_2}{H_1}\{\mu + \frac{q}{\gamma H_1}\} \geq 1.5\, K_a\{0.5 + \frac{q}{\gamma H_1}\}}$$

Taking moments about the toe of the wall,

$$W\,(b_1 + b_2/2) - 0.5\, K_a \gamma\, H_1^2\,(H_1/3) - K_a\, q\, H_1\,(H_1/2) = W\, L$$

$$L = b_1 + 0.5\, b_2 - b_2\, K_a\,(H_1/b_2)^2\{1/6 + q/(2\,\gamma H_1)\}/\,[1 + q/(\gamma H_1)]$$

Eccentricity e of W with respect to the centre of the base is

$$e = 0.5(b_1 + b_2) - L$$

$$e = (1/6)\, b_2\, K_a\,(H_1/b_2)^2\{1 + 3q/(\gamma H_1)\}/\,[1 + q/(\gamma H_1)] - 0.5\, b_1$$

For no tension to develop at the heel, W must lie in the middle third of the base. Therefore

$$e \leq (b_1 + b_2)/6$$

$$\boxed{\frac{b_1}{b_2} \geq 0.25\, K_a\,(\frac{H_1}{b_2})^2[\frac{1 + 3\frac{q}{\gamma H_1}}{1 + \frac{q}{\gamma H_1}}] - 0.25}$$

# Stability Failure Analysis and Design.

- Design against **overturning**
  1. **Gravity Wall**
     - Resisting ≤ Acting
     - Resisting: self weight, $G_k$
     - Acting: Active earth pressure, $H_k$, Surcharge, $S_k$
     - $\gamma_{uf} H_k . y \leq \gamma_f G_k . x$



Resultant force $H_k$

Friction force $\mu G_k$
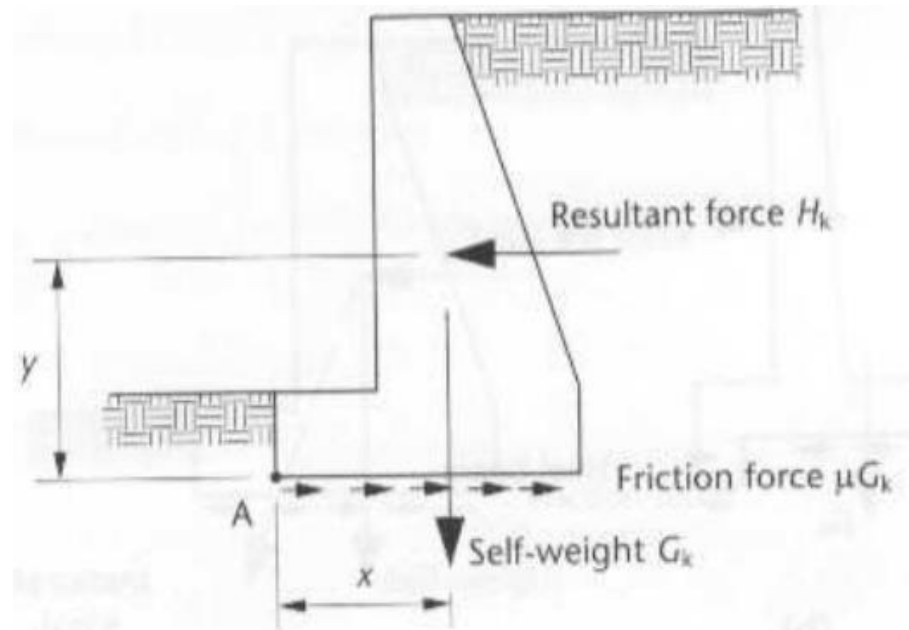
Self-weight $G_k$

  2. **Cantilever Wall**
     - Resisting ≤ Acting
     - Resisting: Self weight of wall, weight of soil on the base
     - Acting: Active earth pressure, surcharge

# Stability Failure Analysis and Design.

- Design against **overturning**
  2. **Cantilever Wall**
     - Cohesion less Soil, c=0

- Cohesive soil, $\phi=0$

$$p = K_a (\gamma z + q)$$

$$K_a = \frac{(1 - \sin \phi)}{(1 + \sin \phi)} \qquad P_1 = \frac{1}{2} K_a \gamma H_1^2 + K_a q H_1$$

$$p = \gamma z + q - 2c$$
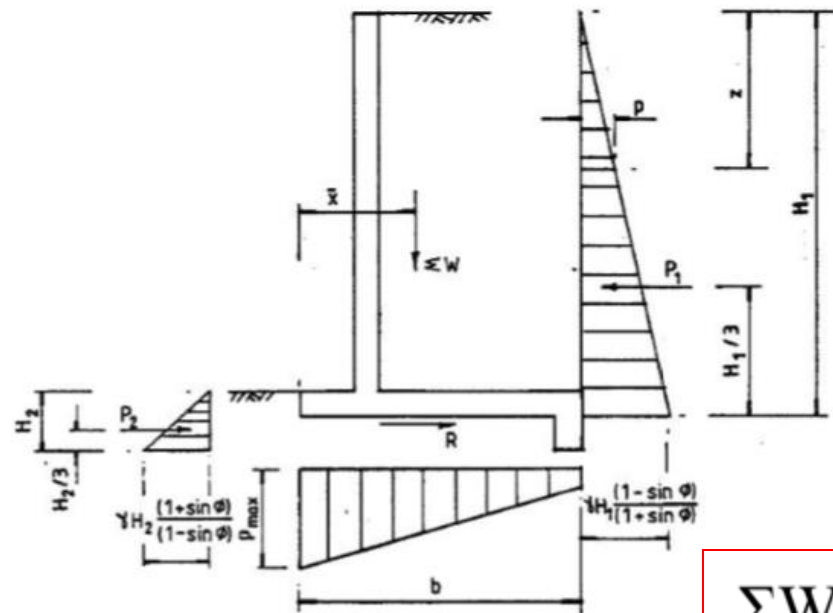


Fig. 12.2(a) Earth pressure: cohesion less soil (c = 0).
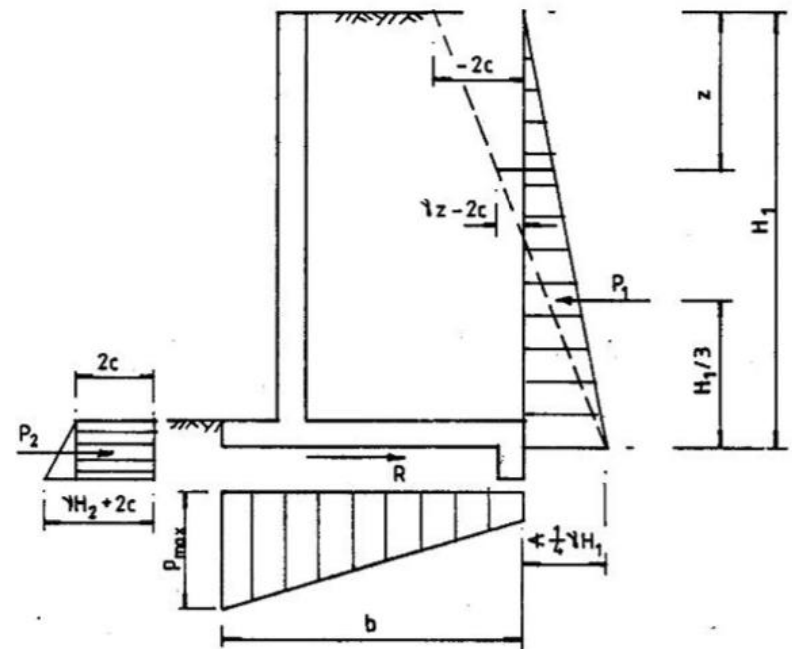


Fig. 12.2 (b) Earth pressure: cohesive soil ($\varphi = 0$).

$$\Sigma Wx \geq \gamma_f P_1 H_1/3$$

# Stability Failure Analysis and Design.

- Design against **sliding**
  1. **Gravity Walls**
     - Resisting: Friction and Adhesion, passive pressure
     - Acting: active pressure and surcharge
  2. **Cantilever Wall**
     - Cohesion less soil
       - Resisting: Friction, $R = \mu.\Sigma W$, $\mu = \tan\phi$, passive pressure, $P_2$

$$P_2 = \frac{1}{2}\gamma H_2^2 \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

     - Cohesive soil
       - Resisting: Adhesion, $R = \beta.b$, $\beta = 0.5$ to $0.7C$, passive pressure, $P_2$

$$P_2 = 0.5\gamma H_2^2 + 2cH_2$$

$$\gamma_f P_1 < P_2 + R$$

# Ground Failure Analysis and Design.

- Design against **bearing failure**

$$A = b \ m^2 \qquad Z = b^2/6 \ m^3.$$

If $\Sigma M$ is the sum of the moments of all vertical forces $\Sigma W$ about the centre of the base and of the active pressure on the wall, then

$$\Sigma M = \Sigma W(x - b/2) - P_I H_1/3$$

$x$ = the centre of gravity of vertical loads *from* the *toe* of the wall.

passive pressure in front of the base has again been neglected.

$$P_{max} = \frac{\Sigma W}{A} + \frac{\Sigma M}{Z}$$

This should not exceed the safe bearing pressure on the soil.
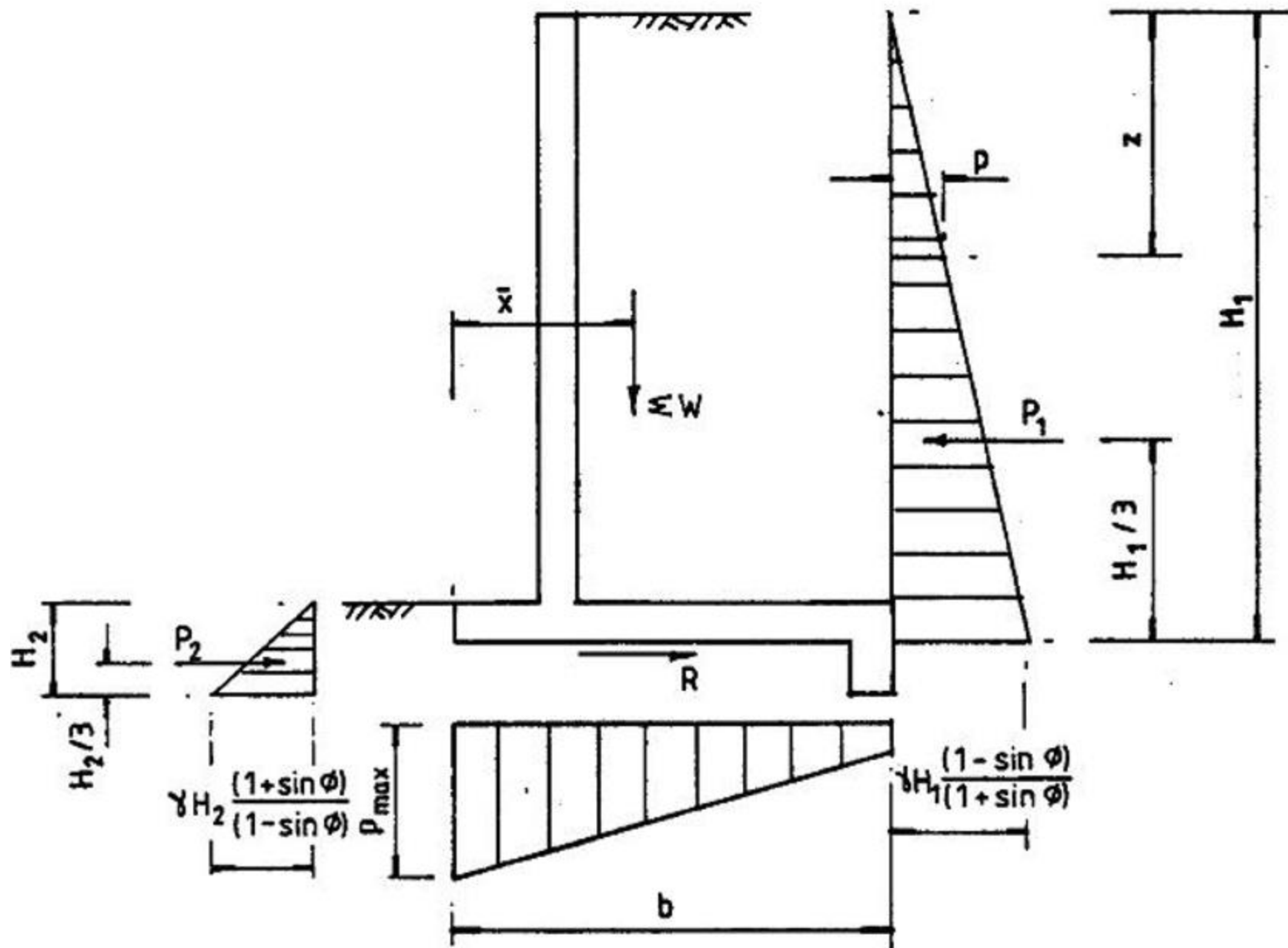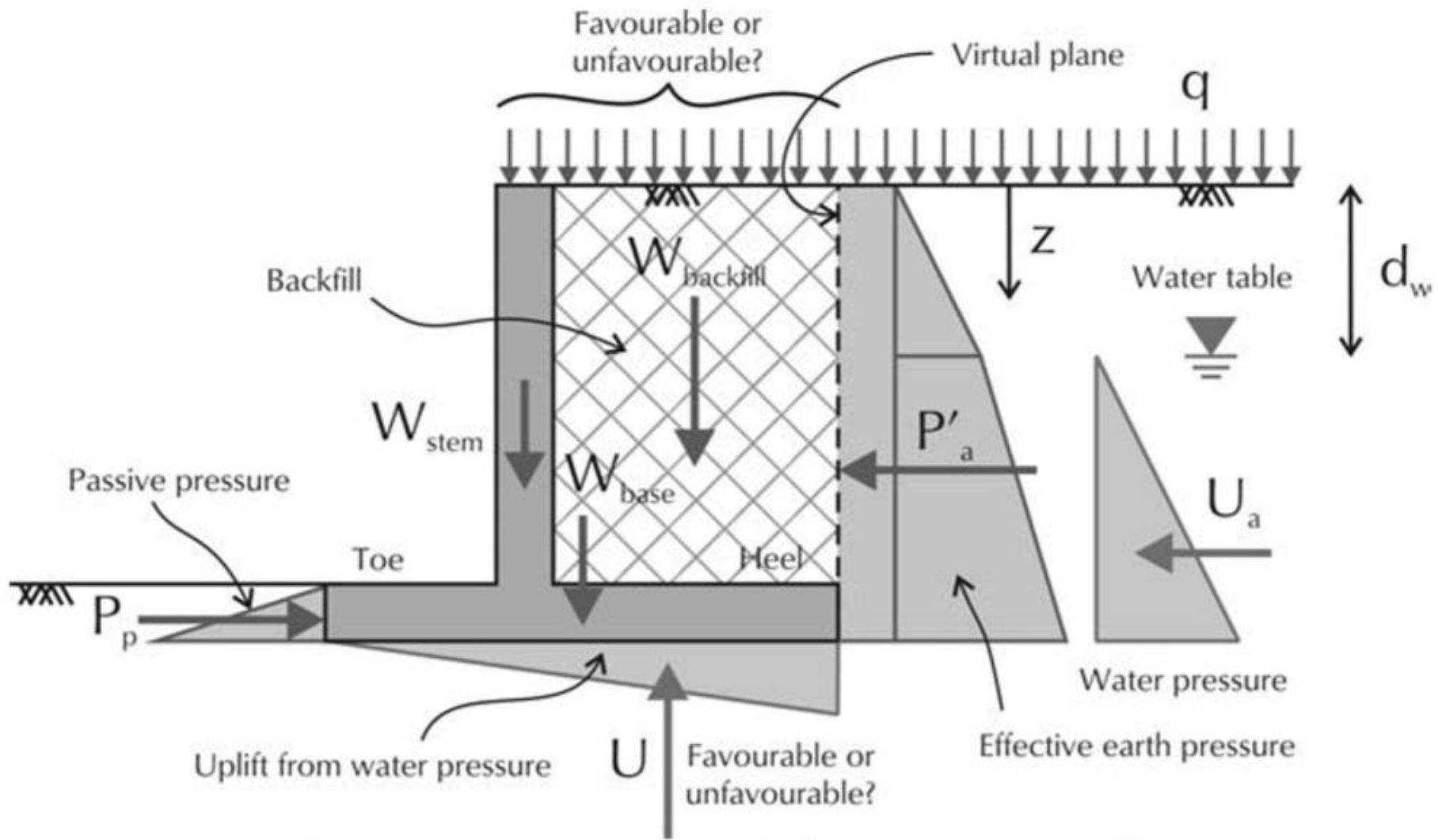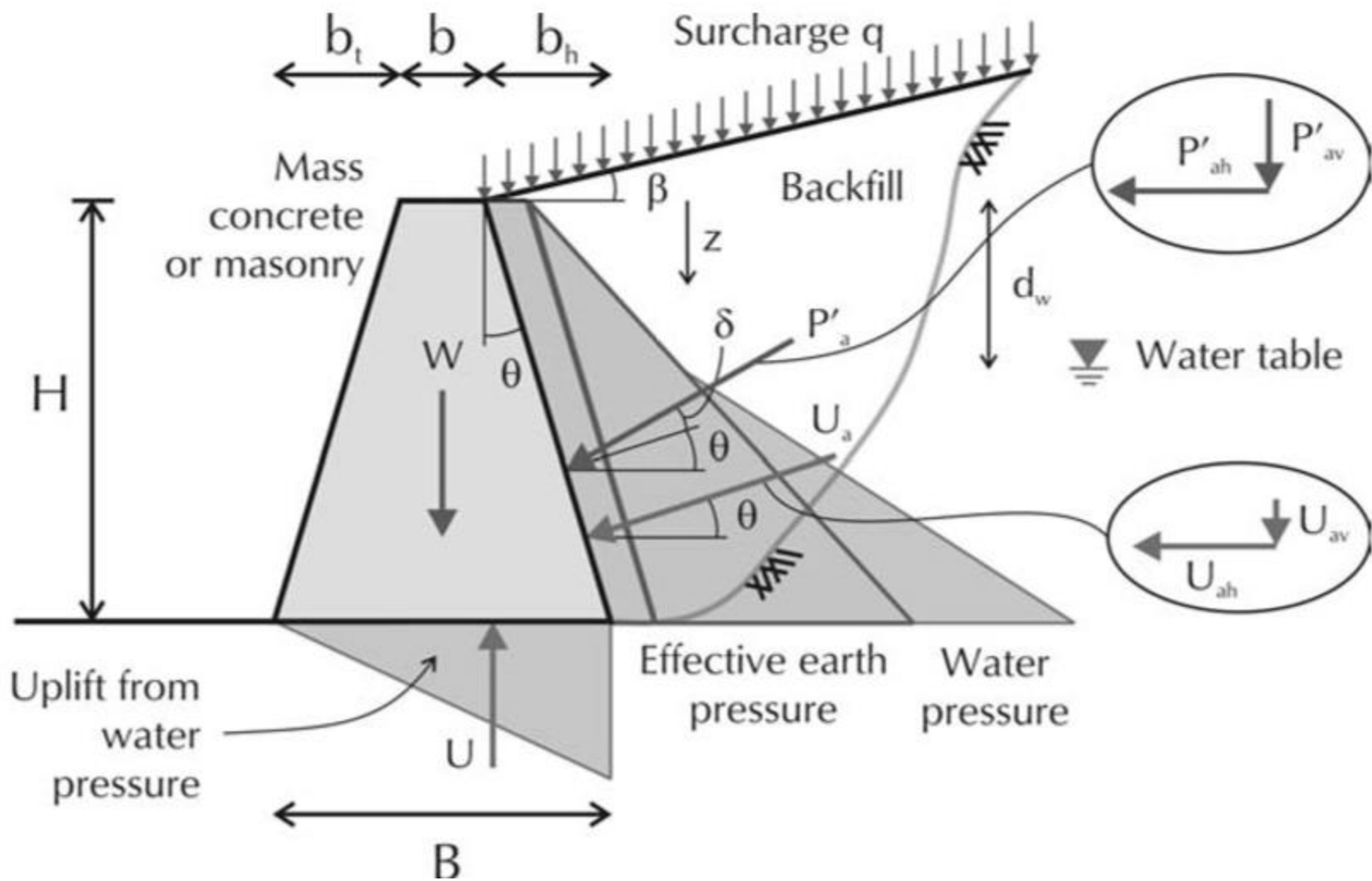
# Bending Failure and Detailing

- **Structural design**
  - For the wall and base.
  - Based on this re-bars are provided based on the flexure formula and max. moment on each component.
  - Not necessary for gravity walls.
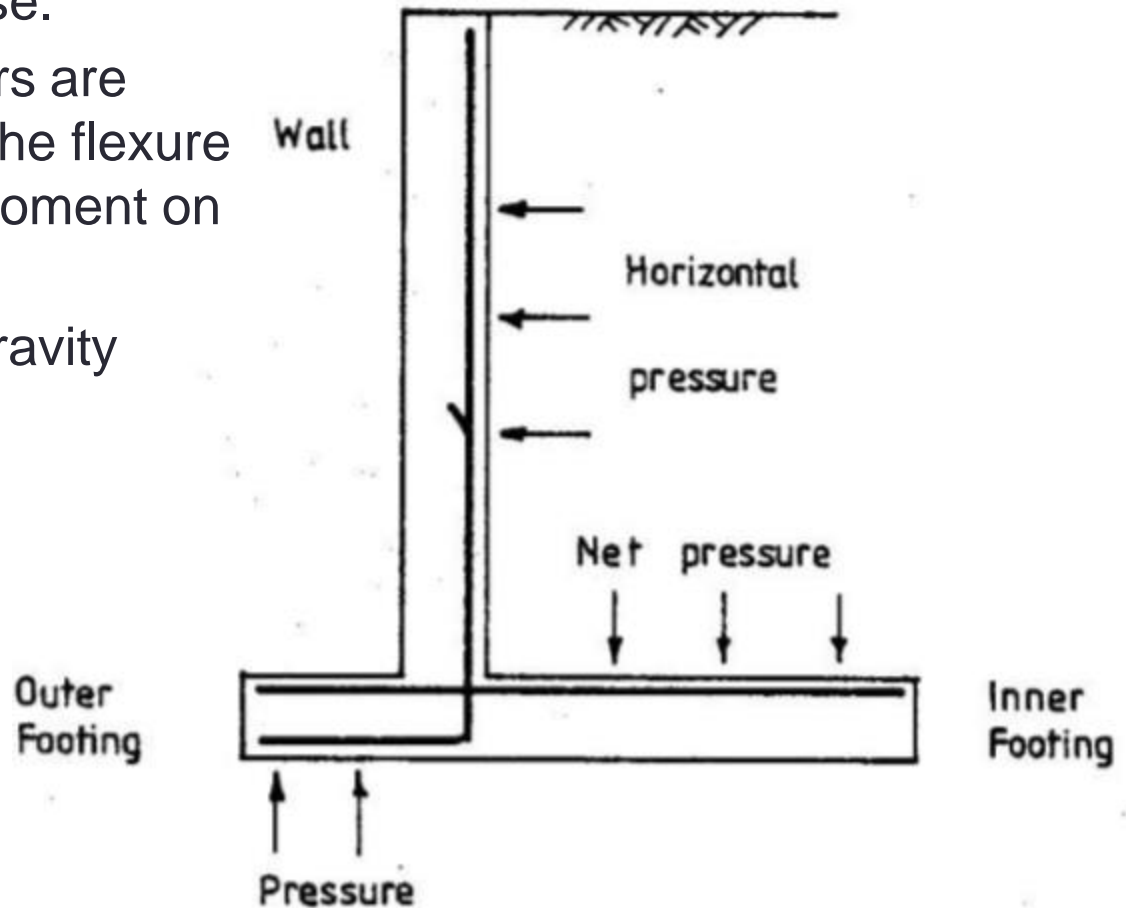
# Example

Design a cantilever retaining wall to support a bank of earth 3.5 m high. The top surface is horizontal behind the wall but it is subjected to a dead load surcharge of 15 kN/m$^2$. The soil behind the wall is well-drained sand with the following properties:

$$\text{Unit weight } \gamma = 18.0 \text{ kN/m}^3$$
$$\text{Angle of internal friction } \varphi = 30°$$

The material under the wall has a safe bearing pressure of 100 kN/m$^2$. The coefficient of friction $\mu$ between the base and the soil is 0.5. Design the wall using $f_{ck}$ = 30 MPa concrete and $f_{yk}$ = 500 MPa reinforcement.

Active earth pressure coefficient:
$$K_a = (1 - \sin \varphi)/ (1 + \sin \varphi) = (1 - 0.5)/ (1 + 0.5) = 0.3333$$

**(b) Check preliminary sizing**

**(i) Check minimum stem thickness**
For 1 m length of the wall, bending moment M at the base of the cantilever is
$$M = 0.5 \, K_a \gamma \, H^2 \, (H/3) + K_a \, q \, H \, (H/2)$$
Substituting $K_a = 0.333$, $\gamma = 18.0$ kN/m$^3$, $H = 3.5$ m, $q = 15$ kN/m$^2$,
$$M = 0.5 \times 0.3333 \times 18.0 \times 3.5^2 \times 3.5/3 + 0.3333 \times 15 \times 3.5 \times 3.5/2$$
$$M = 42.88 + 30.62 = 73.50 \text{ kN m/m}$$
In order for there to be no need for compression steel,
$$M < 0.196 \, bd^2 \, f_{ck}$$
Taking $b = 1000$ mm, $f_{cu} = 30$ N/mm$^2$,

$$d > \sqrt{\frac{M}{(0.196 \times b \times f_{ck}}} = \sqrt{\frac{73.50 \times 10^6}{0.196 \times 1000 \times 30}} = 112$$

Take a value of d much larger than this to reduce the amount of steel required. However it should not be so large that minimum steel requirement is greater than the calculated steel area.  0Assume total stem thickness of 250 mm.  Same thickness is assumed for the base slab as well.

## (ii) Check resistance to sliding

$$H_1 = 3.5 + 0.25 = 3.75 \text{ m},$$
$$q/ (\gamma H_1) = 15.0/ (17.6 \times 3.75) = 0.227$$

$$K_a = 0.333$$
$$\mu = 0.5$$

$$\frac{b_2}{H_1}\{\mu + \frac{q}{\gamma H_1}\} \geq 1.5 K_a \{0.5 + \frac{q}{\gamma H_1}\}$$

$$(b_2/H_1)\{0.5 + 0.227) \geq 1.5 \times 0.333 \times (0.5 + 0.227)$$
$$b_2 /H_1 \geq 0.50$$
$$b_2 \geq 1.875 \text{ m},$$
$$\text{Take } b_2 = 2.05 \text{ m},$$
$$b_2 / H_1 = 0.55$$

## (iii) Check eccentricity

$$b_2 = 2.05,$$
$$H_1/b_2 = 1.829$$
$$q/(\gamma H_1) = 0.227$$
$$K_a = 0.333$$

$$\frac{b_1}{b_2} \geq 0.25\, K_a\, (\frac{H_1}{b_2})^2 [\frac{1+3\,\dfrac{q}{\gamma H_1}}{1+\dfrac{q}{\gamma H_1}}] - 0.25$$

$$(b_1/b_2) \geq 0.25 \times 0.333 \times (1.829)^2\, [\{1+3 \times 0.227\}/\{1+0.227\}] - 0.25$$
$$(b_1/b_2) \geq 0.132,$$
$$b_1 \geq 0.27 \text{ m, Take } b_1 = 0.8 \text{ m.}$$

The proposed arrangement of the wall is shown in Fig.        Wall and base thicknesses are assumed to be 250 mm. A 0.6 m nib has been added under the wall to assist in the prevention of sliding.

Surcharge: 15 kN/m$^2$

5.0 kN/m$^2$

250

3500

250

600

800

1800
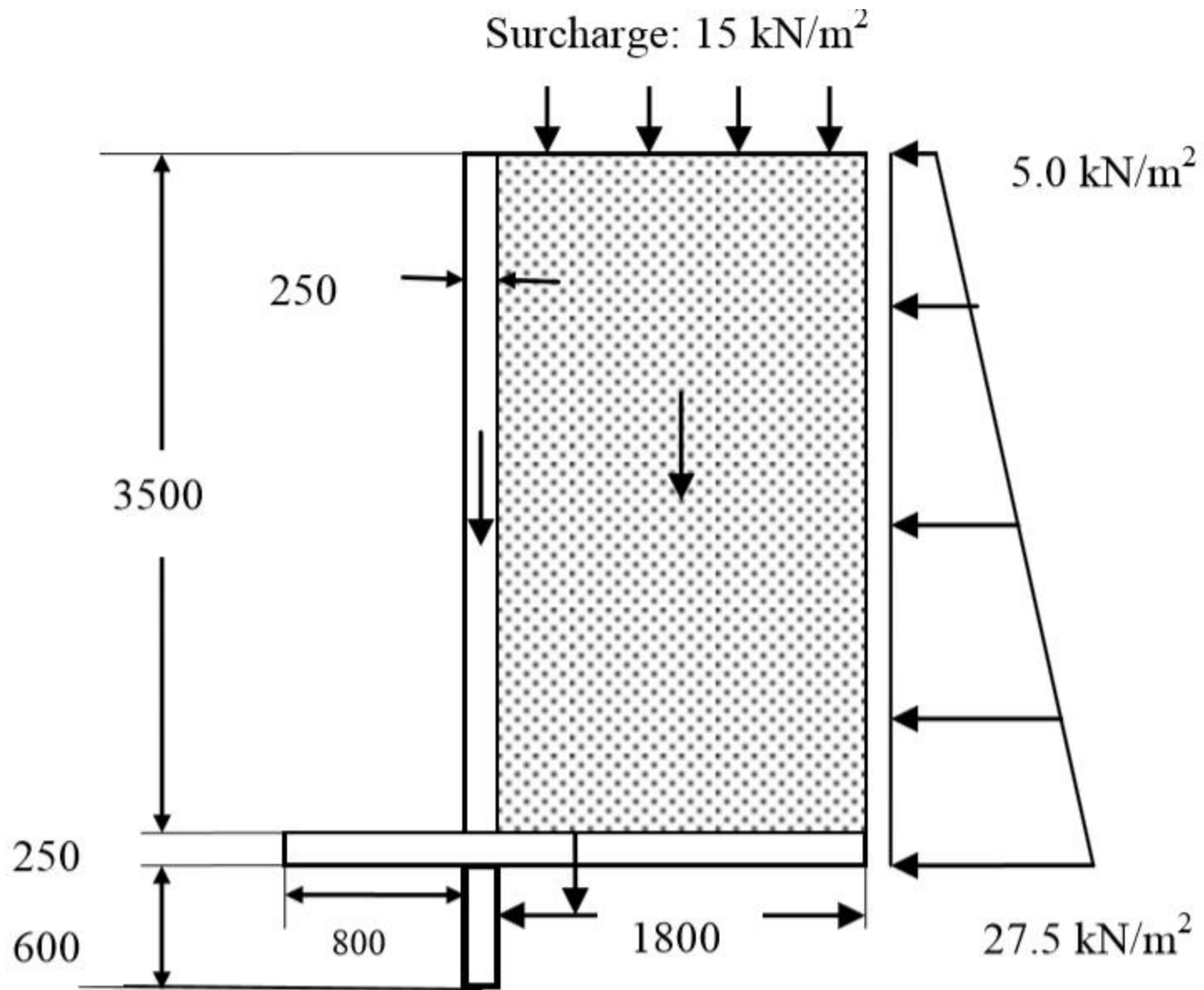
27.5 kN/m$^2$

## (c) Wall stability

Consider 1 m length of wall. The horizontal pressure at depth $z$ from the top is

$$p = K_a (\gamma z + q) = 0.333(18.0\, z + 15)$$

The horizontal pressure at the base (z = 3.75 m) = 27.5 kN/m$^2$

The horizontal pressure at the top (z = 0) = 5 kN/m$^2$.

The weight of wall, base and earth and the corresponding moments about the toe of the wall for stability calculations are given in Table        Clockwise moments are taken as positive.

| Type of Load | Load (kN) | Distance to centroid from A, m | Moment about A (kNm) |
|---|---|---|---|
| **HORIZONTAL (Active earth pressure)** | | | |
| Surcharge | 5 × 3.75 = 18.75 | 3.75/2 = 1.875 | –35.16 |
| Triangular | 0.5 × 3.75 × (27.5 – 5) = 42.19 | 3.75/3 = 1.25 | –52.73 |
| Σ | 18.75 + 42.19 = 60.94 | | –35.16 – 52.73 = –87.89 |
| | | | |
| **VERTICAL (Gravity)** | | | |
| Wall + Nib | (3.75 + 0.6) × 0.25 × 25 = 27.19 | 0.8 + 0.25/2 = 0.925 | 25.15 |
| Base | 2.85 × 0.25 × 25 = 17.81 | 2.85/2 = 1.425 | 25.38 |
| Back fill | 1.8 × 3.5 × 18.0 = 113.40 | 0.8 + 0.25 + 1.8/2 = 1.95 | 221.13 |
| Surcharge | 15 × 1.8 = 27 | 0.8 + 0.25 + 1.8/2 = 1.95 | 52.65 |
| Σ | 185.40 | | 324.31 |

# (i) Maximum soil pressure

$$\text{Width of base } b = 2.85 \text{ m}$$

For 1 m length of wall, area

$$A = 2.85 \text{ m}^2$$

$$\text{Section modulus } Z = 2.85^2/6 = 1.35 \text{ m}^3$$

Taking moments of all forces about the toe A, the centroid of the base pressure from A is at a distance L.

$$L \times 185.40 = 324.31 - 87.89 = 236.42$$

$$L = 236.42/185.40 = 1.275 \text{ m}$$

$$\text{Eccentricity, } e = B/2 - L = 2.85/2 - 1.275 = 0.15 < 2.85/6$$

Hence no tension is developed at C.

The base is acted on by

$$\text{Vertical load} = 185.40 \text{ kN}$$

$$\text{Moment } M = 185.40 \times e = 27.81 \text{ kNm.}$$

The maximum soil pressure at end of the toe slab calculated for *service load* is

$$185.40/ (A = 2.85) + 27.81/ (Z = 1.35) = 8565 \text{ kN/m}^2 < 100 \text{ kN/m}^2$$

This is satisfactory, as the maximum pressure is less than the safe bearing capacity of soil.

## (ii) Stability against overturning

The stabilizing (beneficial) moment due to gravity loads about the toe A of the wall has a partial safety factor $\gamma_{G, \text{inf}} = 1.0$ and the disturbing (adverse) moment due to horizontal loads has a partial safety factor $\gamma_Q = 1.5$. The net stabilizing moment is

$$(324.31 \times 1.0 - 87.89 \times 1.5) = 192.48 > 0$$

The wall is considered safe against overturning.

## (iii) Resistance to sliding

The forces resisting sliding are the friction under the base and the passive resistance for a depth of earth of 850 mm to the top of the base. The gravity loads are beneficial loads but the horizontal load is an adverse load. Ignoring the passive pressure, for the wall to be safe against sliding

$$(\mu = 0.5) \times \{(\gamma_{G, \text{inf}} = 1.0) \times 185.40\} > \{(\gamma_Q = 1.5) \times 60.0\},$$
$$\text{i.e. } 92.70 > 90.0$$

The resistance to sliding is satisfactory. There was no need for the nib but is included for additional protection. No reliance is placed on passive earth pressure.

**(iv) Overall comment**: The wall section is satisfactory. The maximum soil pressure under the base controls the design.

**(d) Structural design of wall, heel and toe slabs**

**(1) Cantilever wall slab**

**(a) Bending design:**
At serviceability limit state, the horizontal pressure at the base ($z = 3.5$ m) is
$$p = K_a (\gamma z + q) = 0.333(18.0 \times 3.5 + 15) = 26.0 \text{ kN/m}^2$$
At the top ($z = 0$) is 5 kN/m$^2$.
$$\text{Average pressure} = 0.5 \times (26.0 + 5.0) = 15.50 \text{ kN.m}^2$$

At ultimate limit state using $\gamma_Q = 1.5$, at the base of the cantilever, shear force V and moment M are
$$V = 15.50 \times 3.5 \times (\gamma_f = 1.5) = 81.38 \text{ kN}$$
$$M = \{(26.0 - 5.0) \times 0.5 \times 3.5 \times 3.5/3 + 5.0 \times 3.5 \times 3.5/2\} \times (\gamma_Q = 1.5)$$
$$M = 110.25 \text{ kNm/m}$$

$$M = 110.25 \text{ kNm/m}$$

Assume that the cover is 40 mm and the diameter of the bars is 16mm. Effective depth d is

$$d = 250 - 40 - 8 = 202 \text{ mm}$$

$$k = M/ (bd^2 f_{ck}) = 110.25 \times 10^6/ (1000 \times 202^2 \times 30) = 0.09 < 0.196$$

$$\frac{z}{d} = 0.5[1.0 + \sqrt{(1 - 3\frac{k}{\eta})}]$$

$$k = 0.09, \eta = 1.0, z/d = 0.927$$

$$A_s = 110.25 \times 10^6/ (0.927 \times 202 \times 0.87 \times 500) = 1353 \text{ mm}^2/\text{m}$$

Provide 12 mm diameter bars at 80 mm centres to give a steel area of $(\pi/4 \times 12^2) \times (1000/80) = 1414 \text{ mm}^2/\text{m}$.

Check minimum steel. From equation (9.1N) of the code,

$$A_{s, min} = 0.26 \times (f_{ctm}/f_{yk}) \times b \times d \geq 0.0013 \text{ b} \times \text{d}$$

$$f_{ctm} = 0.3 \times f_{ck}^{0.67} = 0.3 \times 30^{0.67} = 2.9 \text{ MPa}, f_{yk} = 500 \text{ MPa},$$

$$b = 1000 \text{ mm}, d = 202 \text{ mm}$$

$$A_{s, min} = 0.26 \times (2.9/500) \times 1000 \times 202 \geq 0.0013 \times 1000 \times 202$$

$$A_{s, min} = 305 \text{ mm}^2$$

provided steel is greater than the minimum percentage of steel.

Moment at SLS = Moment at ULS/ ($\gamma_Q$ = 1.5) = 110.25/1.5 = 73.5 kNm/m

Stress in steel at SLS = ($M_{SLS}/M_{ULS}$) × ($A_{s, reqd}/A_{s, Provided}$) × $f_{yd}$

= (73.5/110.25) × (1353/1414) × (500/1.15) = 277 MPa

Check maximum bar diameter and spacing of steel permitted: For steel stress at SLS of 280 MPa and for a maximum crack width of 0.3 mm, maximum spacing allowed from code Table 7.3N is 150 mm and from code Table 7.2N maximum bar size is 12mm. The provided steel area satisfies both criteria.

## (b) Curtailment of flexural steel

Determine the depth z from the top where the spacing of 12 mm bars can be doubled to 160 mm. Steel area at 160 mm c/c is equal to $(\pi/4 \times 12^2) \times (1000/160) = 707$ mm$^2$/m.

The corresponding moment of resistance is approximately

$$M = 0.5 \times 110.25 = 55.13 \text{ kNm}$$

This moment occurs at a depth z from top given by

$$55.13 = 1.5 \times K_a (\gamma z^3/6 + 15 \times z^2/2)$$
$$55.13 = 1.5 \times 0.333 \times (18.0 \times z^3/6 + 15 \times z^2/2)$$

Solving by trial and error, z = 2.67 m,

$$d = 250 - 40 - 6 = 204 \text{ mm}$$
$$M = 55.13$$
$$k = 55.13 \times 10^6/ (1000 \times 204^2 \times 30) = 0.044 < 0.196$$
$$\frac{z}{d} = 0.5[1.0 + \sqrt{(1 - 3\frac{k}{\eta})}]$$
$$k = 0.044, \eta = 1.0, z/d = 0.966$$

$$A_s = 55.13 \times 10^6/ (0.966 \times 202 \times 0.87 \times 500) = 650 \text{ mm}^2/\text{m}$$
$$A_s = 650 \text{ mm}^2/\text{m} > (A_{s, \text{ min}} = 305 \text{ mm}^2/\text{m})$$

From Table 5.5, Chapter 5, the required anchorage length for $f_{ck} = 30$ MPa is 36 bar diameters which is equal to $36 \times 12 = 432$ mm. For anchorage requirements, bars are to extend an anchorage length beyond the theoretical cut off point. Therefore alternate bars need to continue up to a distance from top of

$$= 2670 - 432 = 2238 \text{ mm}$$

Stop bars off bars at a distance from base equal to
$$= 3500 - 2238 = 1262 \text{ mm, say } 1300 \text{ mm.}$$

## (c) Shear check

At serviceability limit state, the horizontal pressure p at d from the base is

$$p = K_a (\gamma z + q) = 0.333[18.0 \times (3.5 - 0.202) + 15] = 24.80 \text{ kN/m}^2$$

$$\text{At the top } (z = 0). \quad p = 0.333[18.0 \times 0 + 15] = 5 \text{ kN/m}^2$$

$$\text{Average pressure} = 0.5 \times (24.80 + 5.0) = 14.90 \text{ kN.m}^2$$

At ultimate limit state using $\gamma_Q = 1.5$, at d from the base of the cantilever, shear force $V_{Ed}$ is

$$V_{Ed} = 14.90 \times (3.5 - 0.202) \times (\gamma_Q = 1.5) = 73.71 \text{ kN/m}$$

$$v_{Ed} = 73.71 \times 10^3 / (1000 \times 202) = 0.37 \text{ MPa}$$

$$\rho_1 = A_{sl} / (b_w d) \le 0.02$$

$$100\,\rho_1 = 100 \times 1414 / (1000 \times 202) = 0.70 < 2.0$$

$$C_{Rd,\,c} = 0.18 / (\gamma_c = 1.5) = 0.12, \; k = 1 + \sqrt{(200/202)} = 1.995 \le 2.0,$$

$$v_{Rd,\,c} = C_{Rd,\,c} \times k \times (100 \times \rho_1 \times f_{ck})^{0.3333} \ge (v_{min} = 0.035 \times k^{1.5} \times \sqrt{f_{ck}})$$

$$= 0.12 \times 1.995 \times (0.70 \times 30)^{0.3333} \ge (v_{min} = 0.035 \times 1.995^{1.5} \times \sqrt{30})$$

$$= 0.66 \ge 0.54$$

$$v_{Rd,\,c} = 0.66 \text{ MPa} > (v_{Ed} = 0.37 \text{ MPa})$$

The shear stress is satisfactory.

## (d) Distribution steel

Clause 9.3.1(2) of the code recommends that in slabs, secondary reinforcement of not less than 20% of the principal reinforcement and at a spacing less than or equal to 3.5 h or 400 mm whichever is lesser.

In this case main steel is 1413 $mm^2/m$. 20% of this value is 283 $mm^2/m$. 10 mm bars at 275 mm provide a steel area of 286 $mm^2/m$. h = 250 mm, 3.5 h = 875 mm.

For crack control on the outer face, provide 10 mm diameter bars at 275 mm centres each way. For ease of construction it is better to provide steel fabric.

## (2) Inner footing (heel slab)

In order to determine the appropriate load factors to be used, it is necessary to consider the effect of gravity loads and earth pressure loads on the bending moment caused in the heel slab.

From Table        gravity loads provide:

Vertical load = 184.40 kN,

Moment about the Toe A = 324.31 kNm (Clockwise)

The centroid of the base pressure due to *gravity loads only* from A is at a distance L.

$L \times 184.40 = 324.31$, giving L = 1.76 m,

Eccentricity, e = 2.85/2 − L = −0.334

The base is acted on by a

Vertical load = 184.40 kN

Moment M = 184.40 × e = 61.60 kNm (clockwise)

$184.40 / (A = 2.85) = 64.70 \ kN/m^2$

$M / (Z = 1.35) = 45.62 \ kN/m^2$

On the top of the heel slab there is surcharge of 15 kN/m$^2$ and a height of soil equal to 3.5 m and self weight of slab of 250 mm. The total downward load is

$$\{15 + 3.5 \times (\gamma = 18.0) + 0.25 \times 25\} = 84.25 \text{ kN/m}^2$$

The bending moment on the base due to horizontal earth pressure is

$$M = 87.89 \text{ kNm/m (anticlockwise)}$$
$$M / (Z = 1.35) = 65.10 \text{ kN/m}^2$$

The pressures due to gravity loads and horizontal earth pressure are shown in Fig.      .

The net effect of gravity loads is to produce tension on the bottom of the slab while the base pressure due to horizontal loads produces tension on the top of the slab. Therefore gravity loads are beneficial loads with a load factor of $\gamma_{G, inf} = 1.0$ while earth pressure loads are adverse with a load factor of $\gamma_{Q,} = 1.5$ to be applied. Using these load factors, the base pressure at right and left ends of the base slab are

$$\text{Left end} = 64.70 - 45.62 + 65.10 \times 1.5 = 116.73 \text{ kN/m}^2$$
$$\text{Right end} = 64.70 + 45.62 - 65.10 \times 1.5 = 12.67 \text{ kN/m}^2$$

The base pressure at the junction of the heel slab and cantilever is

$$= 12.67 + (116.73 - 12.67) \times (1.8/2.85) = 78.20 \text{ kN/m}^2$$

Fig.      shows the forces acting on the heel slab.

## (a) Bending design
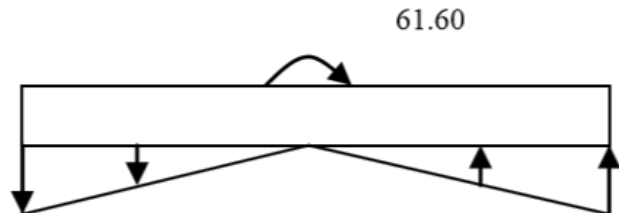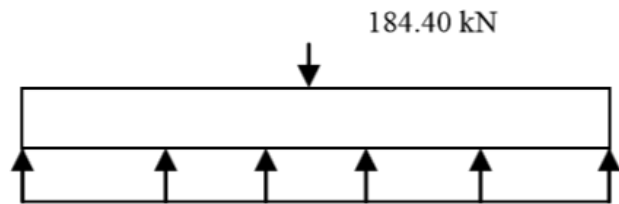
Referring to Fig.     moment M at the face of the wall is

$$M = 0.5 \times (84.25 - 12.67) \times 1.8^2 - 0.5 \times (78.20 - 12.67) \times 1.8 \times 1.8/3$$
$$= 80.57 \text{ kN m/m}$$
$$k = M/ (bd^2 \, f_{ck}) = 80.57 \times 10^6/ (1000 \times 202^2 \times 30) = 0.066 < 0.196$$
$$\frac{z}{d} = 0.5[1.0 + \sqrt{(1 - 3\frac{k}{\eta})}]$$
$$k = 0.044, \ \eta = 1.0, \ z/d = 0.948$$
$$A_s = 80.57 \times 10^6/ (0.948 \times 202 \times 0.87 \times 500) = 967 \text{ mm}^2/\text{m}$$

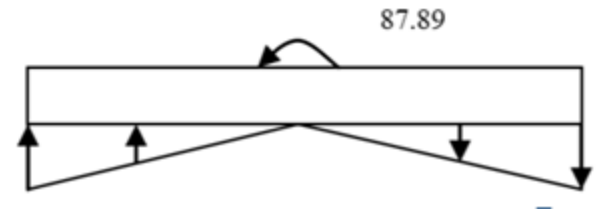184.40 kN

Moment due to gravity loads

64.70

61.60

Moment due to earth pressure

87.89

65.10

45.62

Check for minimum steel from equation (9.1N) of the code.

$$A_{s, min} = 0.26 \times (f_{ctm}/f_{yk}) \times b \times d \geq 0.0013 \ b \times d$$

$$f_{ctm} = 0.3 \times f_{ck}^{0.67} = 0.3 \times 30^{0.67} = 2.9 \ \text{MPa}, \ f_{yk} = 500 \ \text{MPa},$$
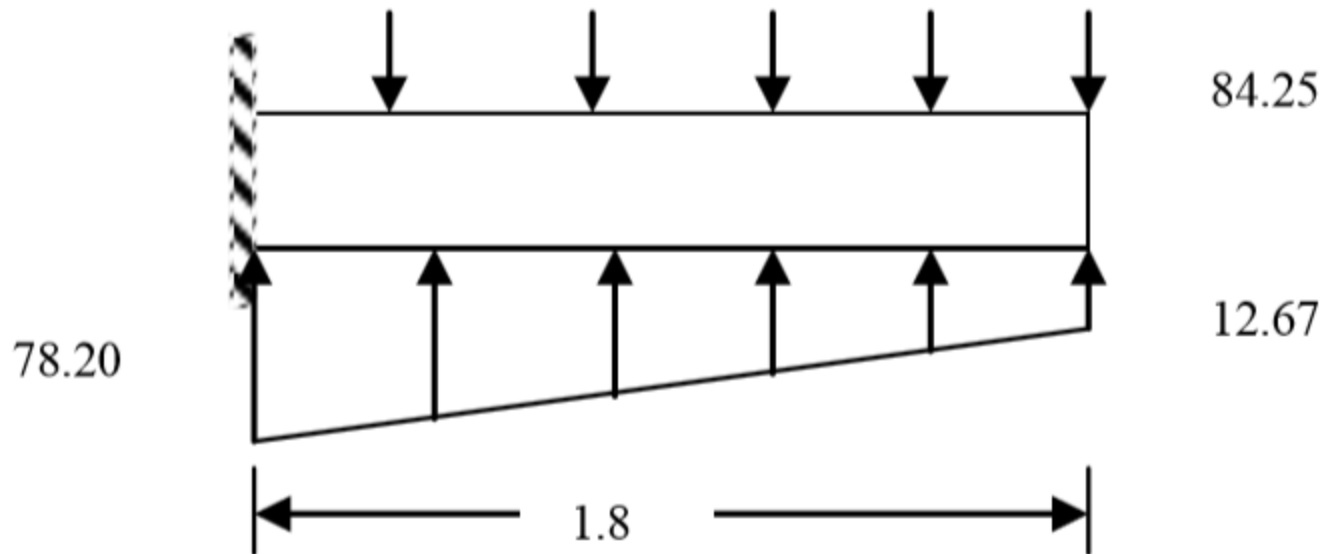
$$b = 1000 \ \text{mm}, \ d = 202 \ \text{mm}$$

$$A_{s, min} = 0.26 \times (2.9/500) \times 1000 \times 202 \geq 0.0013 \times 1000 \times 202$$

$$A_{s, min} = 305 \ \text{mm}^2$$

provided steel is greater than the minimum percentage of steel.
Provide 12 mm bars at 100 mm centre. $A_s = 1130 \ \text{mm}^2/\text{m}$.



84.25

12.67

78.20

1.8

Moment at SLS: Using unit load factors for all loads,

$$\text{Left end} = 64.70 - 45.62 + 65.10 = 84.18 \text{ kN/m}^2$$
$$\text{Right end} = 64.70 + 45.62 - 65.10 = 45.22 \text{ kN/m}^2$$

The base pressure at the junction of the heel slab and cantilever is

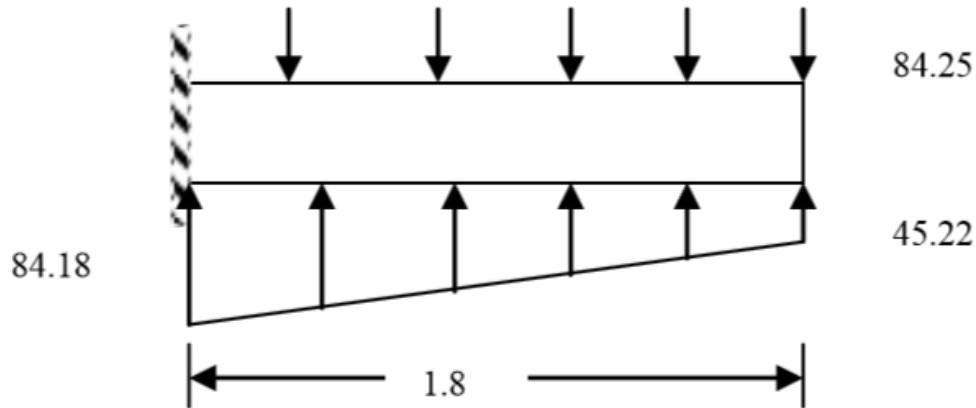$$45.22 + (84.18 - 45.22) \times (1.8/2.85) = 69.83 \text{ kN/m}^2$$



84.25

45.22

84.18

1.8

Fig. 12.8 Pressures in kN/m² acting on heel slab at SLS.

Referring to Fig. 12.8, moment M at the face of the wall is

$$M = 0.5 \times (84.25 - 45.22) \times 1.8^2 - 0.5 \times (84.18 - 45.22) \times 1.8 \times 1.8/3$$
$$= 42.54 \text{ kN m/m}$$

Stress in steel at SLS $= (M_{SLS}/M_{ULS}) \times (A_{s, reqd}/A_{s, Provided}) \times f_{yd}$
$$= (42.54/80.57) \times (967/1130) \times (500/1.15) = 197 \text{ MPa}$$

Check maximum bar diameter and spacing of steel permitted. For steel stress at SLS of 200 MPa and for a maximum crack width of 0.3 mm, maximum spacing allowed from code Table 7.3N is 250 mm and from code Table 7.2N maximum bar size is 25 mm. The provided steel area satisfies both criteria.

## (b) Shear Check

The base pressure at d from the junction of the heel slab and cantilever is

$$= 12.67 + (116.73 - 12.67) \times \{(1.8 - 0.202)/2.85\} = 71.02 \text{ kN/m}^2$$

Referring to Fig.        the shear force $V_{Ed}$ at d from the base slab-wall junction is

$$V_{Ed} = \{(84.25 - 12.67) - 0.5 \times (71.02 - 12.67)\} \times (1.8 - 0.202)$$

$$= 67.76 \text{ kN /m}$$

$$v_{Ed} = 67.76 \times 10^3 / (1000 \times 202) = 0.32 \text{ MPa}$$

$$\rho_1 = A_{sl} / (b_w \, d) \le 0.02$$

$$100 \, \rho_1 = 100 \times 1130/ (1000 \times 202) = 0.56 < 2.0$$

$$C_{Rd, c} = 0.18/ (\gamma_c = 1.5) = 0.12, \; k = 1 + \sqrt{(200/202)} = 1.995 \le 2.0,$$

$$v_{Rd, c} = C_{Rd, c} \times k \times (100 \times \rho_1 \times f_{ck})^{0.3333} \ge (v_{min} = 0.035 \times k^{1.5} \times \sqrt{f_{ck}})$$

$$= 0.12 \times 1.995 \times (0.56 \times 30)^{0.3333} \ge (v_{min} = 0.035 \times 1.995^{1.5} \times \sqrt{30})$$

$$= 0.61 \ge 0.54$$

$$v_{Rd, c} = 0.61 \text{ MPa} > (v_{Ed} = 0.32 \text{ MPa})$$

The shear stress is satisfactory.

## (c) Distribution steel

Clause 9.3.1(2) of the code recommends that in slabs, secondary reinforcement of not less than 20% of the principal reinforcement at a spacing less than or equal to 3.5 h or 400 mm whichever is lesser.

In this case main steel is 1130 mm$^2$/m. 20% of this value is 226 mm$^2$/m. H10 bars at 325 mm provide a steel area of 242 mm$^2$/m. h = 250 mm, 3.5 h = 875 mm.

For crack control on the outer face provide H10 bars at 325 mm centres each way. For ease of construction it is better to provide steel fabric.  ,

**(3) Outer Footing (toe slab)**
As shown in Fig.      both gravity and horizontal loads acting on the base slab produce tension on the bottom of the slab.  Therefore both loads are adverse and take a load factor of $\gamma_{G,\,sup}$ = 1.35 and $\gamma_Q$ = 1.50.  The only beneficial load is due to self weight.  Using these load factors, the base pressure at right and left ends of the base slab are
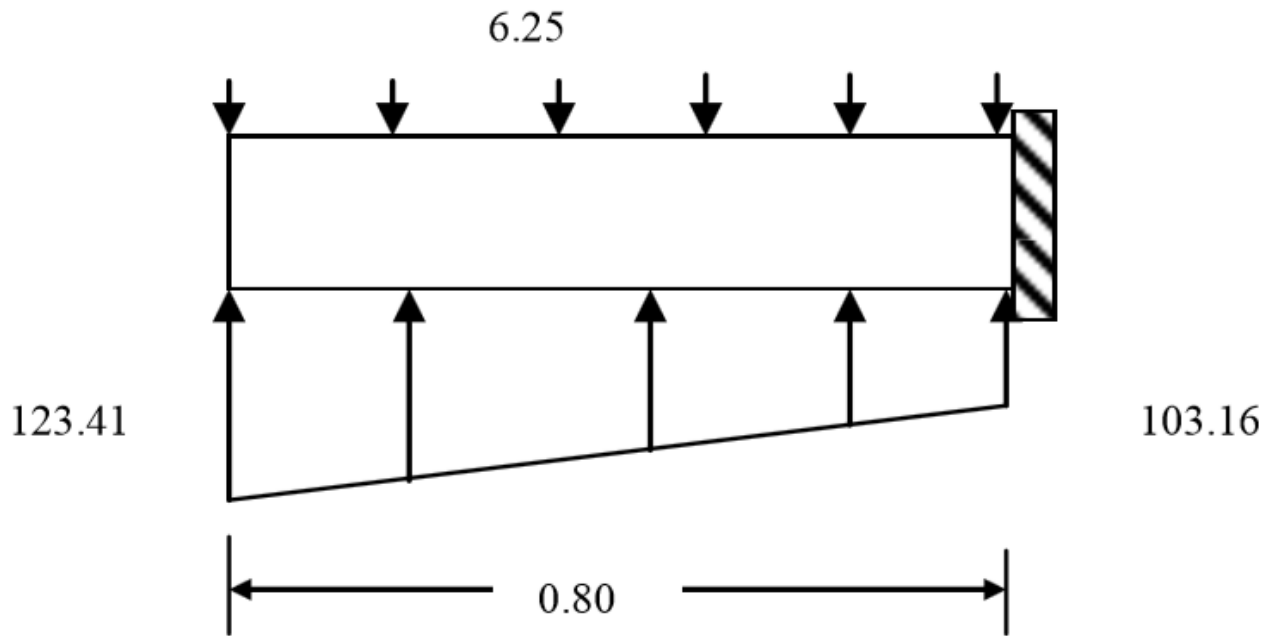
$$\text{Left end} = (64.70 - 45.62) \times 1.35 + 65.10 \times 1.5 = 123.41 \text{ kN/m}^2$$
$$\text{Right end} = (64.70 + 45.62) \times 1.35 - 65.10 \times 1.5 = 51.28 \text{ kN/m}^2$$

The base pressure at the junction of the toe slab and cantilever
$$= 51.25 + (123.41 - 51.25) \times (2.85 - 0.80)/2.85 = 103.16 \text{ kN/m}^2$$
$$\text{Self weight load} = 0.25 \times 25 = 6.25 \text{ kN/mm}^2$$

6.25

123.41

103.16

0.80

The moment at the face of the wall is:

$$M = 0.5 \times (103.16 - 6.25) \times 0.8^2 + 0.5 \times (123.41 - 103.16) \times 0.8 \times (2/3) \times 0.8$$
$$= 35.33 \text{ kN m/m}$$

Reinforcement from the wall which is designed for a moment of 110.25 kNm/m will be anchored in the toe slab and will provide the moment steel here. From the required anchorage length for $f_{ck} = 30$ MPa is 36 bar diameters which is equal to $36 \times 12 = 432$ mm. This will be provided by the bend and a straight length of bar along the toe slab.

**Shear stress:**

The base pressure p at d from the junction of the toe slab and cantilever s

$$p = 51.25 + (123.41 - 51.25) \times (2.85 - 0.80 + 0.202)/2.85 = 108.27 \text{ kN/m}^2$$

$$V_{Ed} = \{(108.27 - 6.25) + (123.41 - 108.27) \times 0.5\} \times (0.8 - 0.202) = 65.35 \text{ kN}$$

The flexural steel and the dimensions of the toe slab are same as for the stem which is safe for shear force of 73.71 kN. This is satisfactory. The distribution steel is 10 mm diameter bars at 240 mm centres.

## (4) Nib

The passive earth pressure coefficient $K_p = 1/K_a = 3.0$.
The earth pressures at the top and bottom of the nib are

$$\text{Top: } K_p \, \gamma \, z = 3 \times 18.0 \times 0.25 = 13.5 \text{ kN/m}^2$$
$$\text{Bottom: } K_p \, \gamma \, z = 3 \times 18.0 \times 0.85 = 45.90 \text{ kN/m}^2$$

Referring to Fig.        the shear and moment in the nib using a load factor of

$\gamma_Q = 1.5$ are as follows:

$$V = 1.5 \times (13.5 + 45.90) \times 0.6/2 = 26.73 \text{kN}$$
$$M = 1.5 \times \{13.5 \times 0.6^2/2 + (45.90 - 13.5) \times 0.5 \times 0.6 \times (2/3) \times 0.6\}$$
$$= 9.48 \text{ kNm/m}$$

The values are quite small. The minimum reinforcement is 305 mm$^2$/m. Provide 10 mm diameter bars at 250 mm centres ($A_s = 314$ mm$^2$/m) to lap onto the main wall steel. The distribution steel is 10 mm diameter bars at 250 mm centres.

Square mesh

H12 at 160

H12 at 80

H12 at 100