# Flood – Frequency Analysis

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# introduction

- Hydrologic systems are sometimes impacted by extreme events, such as severe storms, floods, and droughts.
- The magnitude of an extreme event is inversely related to its frequency of occurrence, very severe events occurring less frequently than more moderate events.
- The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions.

- The results of flood flow frequency analysis can be used for many engineering purposes:

- for the design of dams, bridges, culverts, and flood control structures;
- to determine the economic value of flood control projects;
- to delineate flood plains and determine the effect of encroachments onthe flood plain.

# flood

- Is an unusually high stage in a river-normally the level at which the river overflowsw its banks and inundates the adjoining area causing significant amount of damage.

# Estimation methods of magnitude of peak flood

1. Rational method(<50km$^2$)
2. Emperical method
3. UH technique(<5000km$^2$)
4. Flood frquency studies

Use of particular method depends on

i. The desired objective
ii. The availiable data
iii. The importance of the project

# Concepts of statistics and probability

- The hydrologic processes are measured as

  1. Point Sample
  -Measurements made through time at a fixed location in space.*"Time Series".*

  2. Distributed Samples
  -Measurement made over a line or area in space at a specific point in time. *"Space Series".*

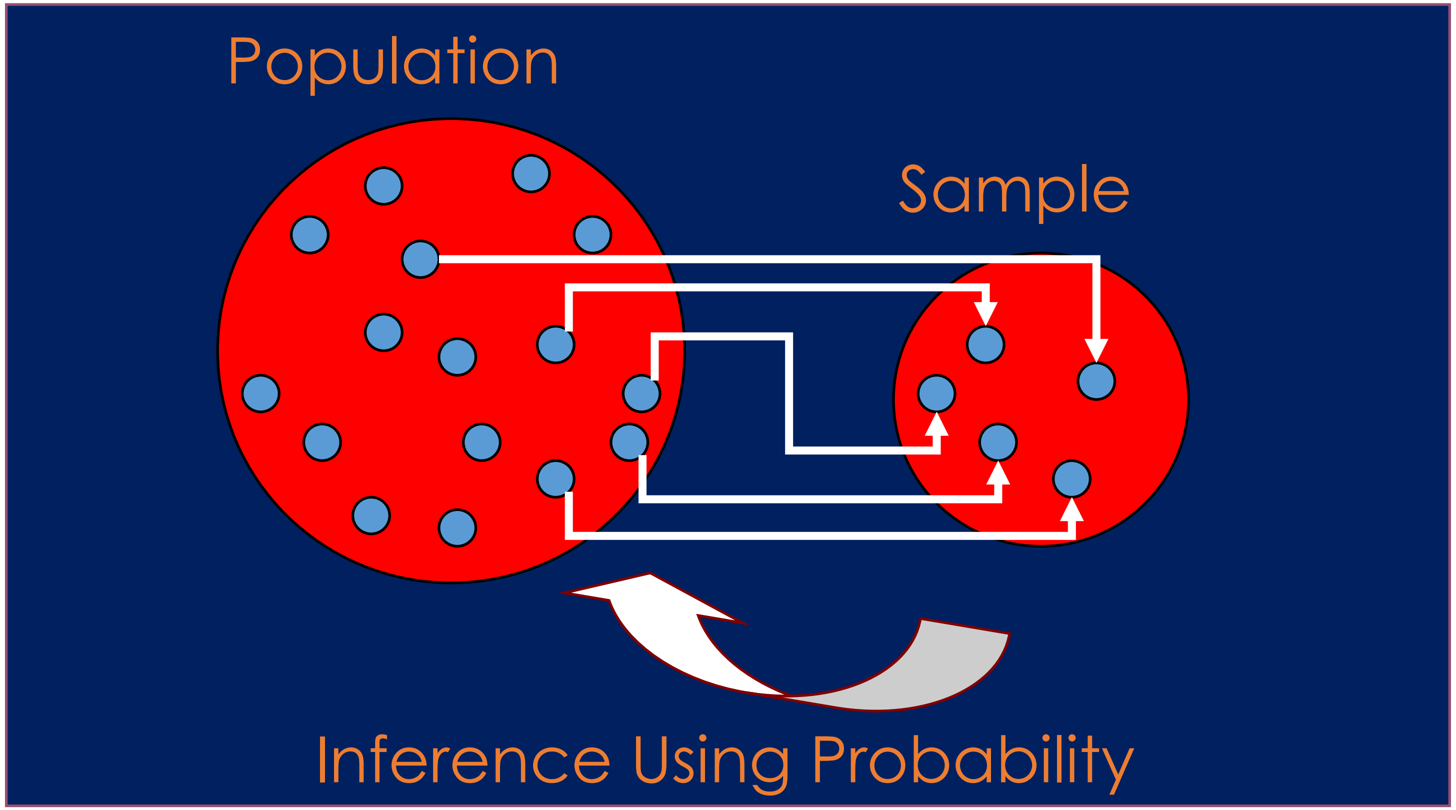
- The hydrologic processes evolve in space and time

# Concepts of statistics and probability

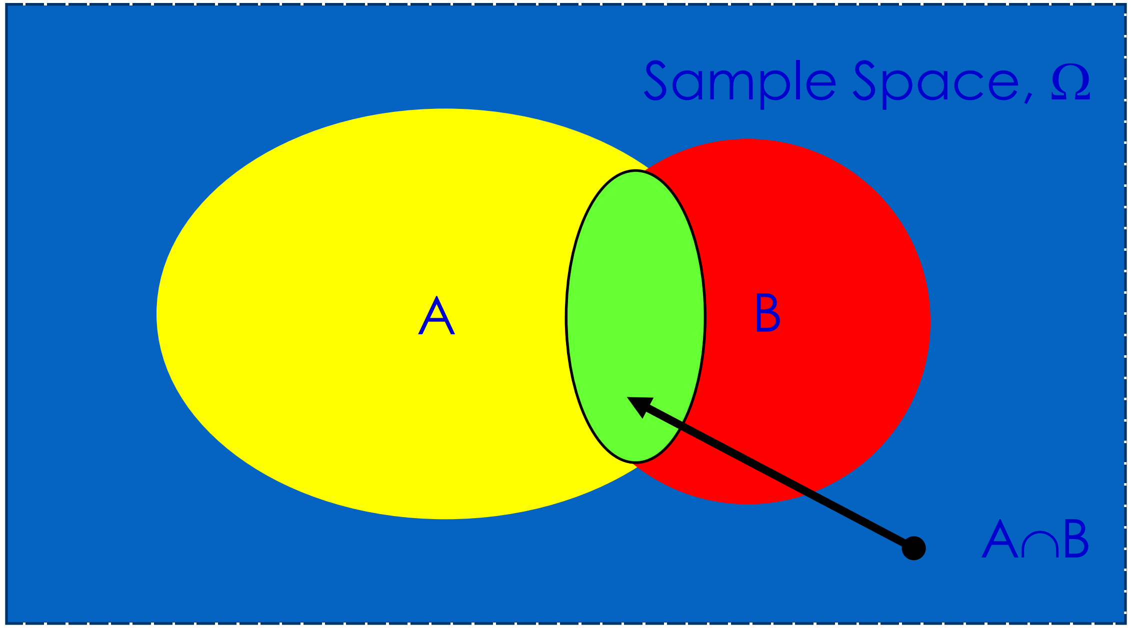
- The hydrologic process is partly predictable or "*Deterministic Process*".

- Some hydrologic process is partly unpredictable (random) or "*Stochastic Process*"

# Concepts of statistics and probability



# Concepts of statistics and probability




- P(A) = Probability of Event A
- P(A) = lim(nA/n) ; $n \rightarrow \infty$
- nA/n = relative frequency

- Sample Space; the set of all possible sample that could be drawn from the population.

- Event; a subset of sample space.

# Concepts of statistics and probability

- 1. Total Probability

    $P(A1)+P(A2)+\ldots\ldots\ldots+P(Am) = P(\Omega) = 1$

  2. Complementarity

    $P(A) = 1-P(B)$

  3. Conditional Probability

    -Dependent Events

      $P(A \cap B) = P(B/A)*P(A)$

      $P(B/A) = P(A \cap B)/P(A)$

    -Independent Events

      $P(B/A) = P(B)$

      $P(A \cap B) = P(B)*P(A)$

# example

The values of annual precipitation in College Station, Texas, from 1910 to 1970 are shown in table and plotted as a time series in the figure.

What is the probability that the annual precipitation R in any year will be less than 35 in? Between 35 and 45 in?

# example

| Year | 1910 | 1920 | 1930 | 1940 | 1950 | 1960 | 1970 |
|---|---|---|---|---|---|---|---|
| 0 | | 48.7 | 44.8 | 49.3 | 31.2 | 46.0 | 33.9 |
| 1 | 39.9 | 44.1 | 34.0 | 44.2 | 27.0 | 44.3 | 31.7 |
| 2 | 31.0 | 42.8 | 45.6 | 41.7 | 37.0 | 37.8 | 31.5 |
| 3 | 42.3 | 48.4 | 37.3 | 30.8 | 46.8 | 29.6 | 59.6 |
| 4 | 42.1 | 34.2 | 43.7 | 53.6 | 26.9 | 35.1 | 50.5 |
| 5 | 41.1 | 32.4 | 41.8 | 34.5 | 25.4 | 49.7 | 38.6 |
| 6 | 28.7 | 46.4 | 41.1 | 50.3 | 23.0 | 36.6 | 43.4 |
| 7 | 16.8 | 38.9 | 31.2 | 43.8 | 56.5 | 32.5 | 28.7 |
| 8 | 34.1 | 37.3 | 35.2 | 21.6 | 43.4 | 61.7 | 32.0 |
| 9 | 56.4 | 50.6 | 35.1 | 47.1 | 41.3 | 47.4 | 51.8 |

# solution

There are n = 79-11+1 = 69 data.

Let A be the event $R < 35.0$ in.

B be the event $R > 45.0$ in.

The numbers of values falling in these ranges are

$n_A = 23$

$n_B = 19$

So, $P(A) \approx 23/69 = 0.333$

$P(B) \approx 19/69 = 0.275$

The probability that the annual precipitation is between 35 and 45 in can be calculated

$$P(35.0 \leq R \leq 45.0) = 1 - P(R<35.0) - P(R>45.0)$$
$$= 1 - 0.333 - 0.275$$
$$= 0.392$$

# Frequency and probability functions

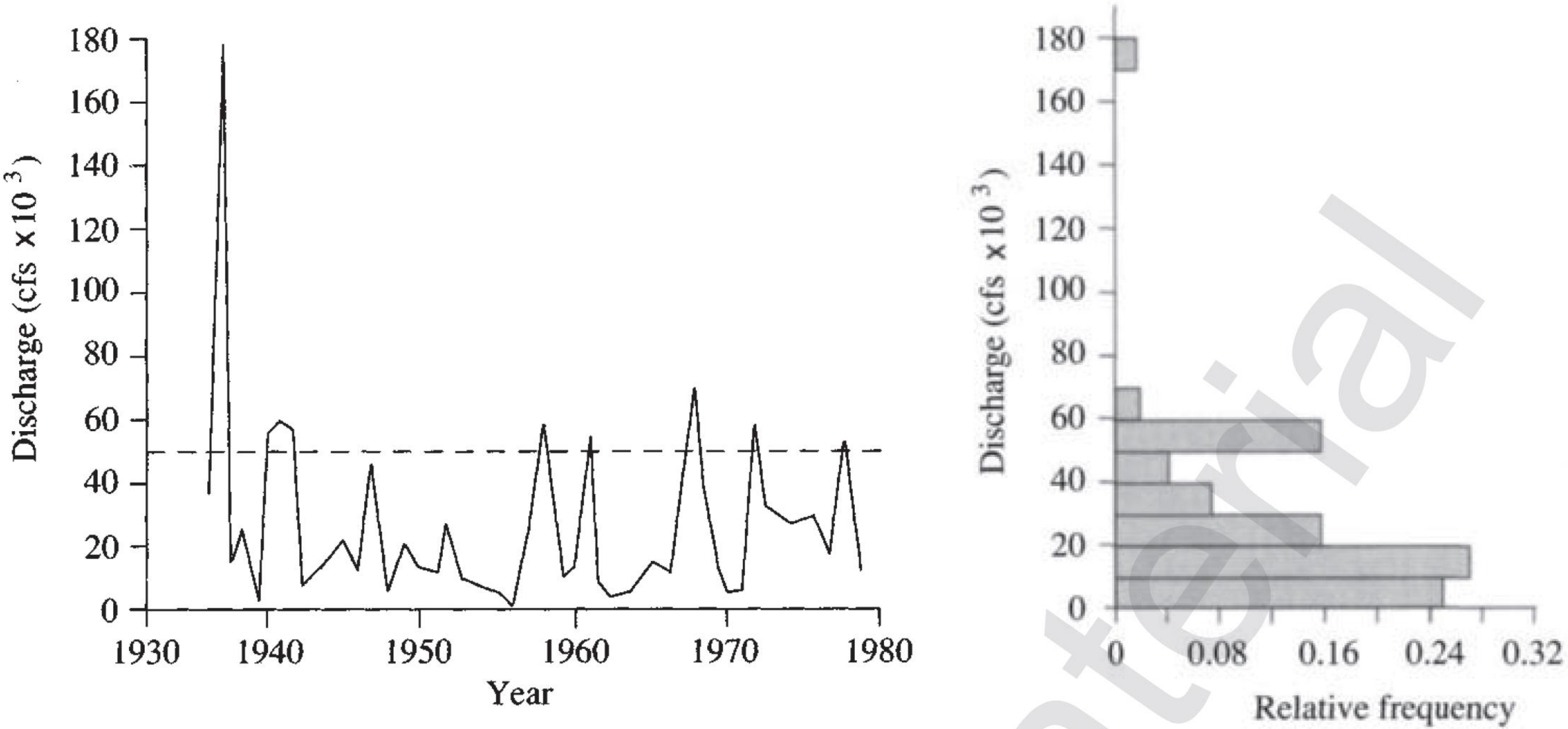
Annual series:-is a hydrologic series ,the values of annual maximum flood from a given catchment area over a large number of succesive years.

Probability of exceedence:-is the probability of a certain event being exceeded or equalled.

$$p= \frac{m}{N+1}$$

M=order of an event(put indecresing order)

N =total number of events in a data




($a$) Time series of annual maximum discharges

($b$) Relative frequency function

# Frequency and probability functions

- Reccurence interval(return period or frequency):T

$$T = \frac{1}{P}$$

General equation of hydrologic frequency analysis

$$X_T = \dot{X} + k\delta$$ ................ *chow(1951)*

$X_T$ value of the variate X of a random hydrologic series with return period T

$\dot{X}$=mean of the variate

*K=frequency factor which depends on T*

δ =standard deviation of the variate

# Gumbels method

- Gumbel defined a flood as the largest values of the 365 daily flows and the annual series of the flood flows constitute a series of largest values of flows.
- Probability of occurance of an event equal to or larger than a value $X_o$ is

$$P(X \geq x_o) = 1 - e^{-e^{-y}}$$

$$P(X \geq x_0) = 1 - e^{-e^{-y}} \quad (7.14)$$

in which $y$ is a dimensionless variable given by

$$y = \alpha(x - a)$$

$$a = \bar{x} - 0.45005\,\sigma_x$$

$$\alpha = 1.2825/\sigma_x$$

Thus $$y = \frac{1.2825\,(x - \bar{x})}{\sigma_x} + 0.577 \quad (7.15)$$

For N… ∞

where $\bar{x}$ = mean and $\sigma_x$ = standard deviation of the variate $X$. In practice it is the value of $X$ for a given $P$ that is required and as such Eq. (7.14) is transposed as

$$y_p = -\ln[-\ln(1 - P)] \quad (7.16)$$

Noting that the return period $T = 1/P$ and designating

$y_T$ = the value of $y$, commonly called the reduced variate, for a given $T$

$$y_T = -\left[\ln.\ln\frac{T}{T-1}\right] \quad (7.17)$$

or $$y_T = -\left[0.834 + 2.303\log\log\frac{T}{T-1}\right] \quad (7.17a)$$

Now rearranging Eq. (7.15), the value of the variate $X$ with a return period $T$ is

$$x_T = \bar{x} + K\sigma_x \quad (7.18)$$

where $$K = \frac{(y_T - 0.577)}{1.2825} \quad (7.19)$$

Note that Eq. (7.19) is of the same form as the ...

For practical use

$$x_T = \bar{x} + K\sigma_{n-1}$$

here $\sigma_{n-1}$ = standard deviation of the sample of size $N$

$$= \sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$$

$K$ = frequency factor expressed as

$$K = \frac{y_T - \bar{y}_n}{S_n}$$

which $y_T$ = reduced variate, a function of $T$ and is given by

$$y_T = -\left[\ln . \ln \frac{T}{T-1}\right]$$

$$y_T = -\left[0.834 + 2.303 \log \log \frac{T}{T-1}\right]$$

$\bar{y}_n$ = reduced mean, a function of sample size $N$ and is give
7.3; for $N \to \infty$, $\bar{y}_n \to 0.577$

$S_n$ = reduced standard deviation, a function of sample siz
given in Table 7.4; for $N \to \infty$, $\bar{y}_n \to 1.2825$

# Practical

*N* = sample size

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0.4952 | 0.4996 | 0.5035 | 0.5070 | 0.5100 | 0.5128 | 0.5157 | 0.5181 | 0.5202 | 0.5220 |
| 20 | 0.5236 | 0.5252 | 0.5268 | 0.5283 | 0.5296 | 0.5309 | 0.5320 | 0.5332 | 0.5343 | 0.5353 |
| 30 | 0.5362 | 0.5371 | 0.5380 | 0.5388 | 0.5396 | 0.5402 | 0.5410 | 0.5418 | 0.5424 | 0.5430 |
| 40 | 0.5436 | 0.5442 | 0.5448 | 0.5453 | 0.5458 | 0.5463 | 0.5468 | 0.5473 | 0.5477 | 0.5481 |
| 50 | 0.5485 | 0.5489 | 0.5493 | 0.5497 | 0.5501 | 0.5504 | 0.5508 | 0.5511 | 0.5515 | 0.5518 |
| 60 | 0.5521 | 0.5524 | 0.5527 | 0.5530 | 0.5533 | 0.5535 | 0.5538 | 0.5540 | 0.5543 | 0.5545 |
| 70 | 0.5548 | 0.5550 | 0.5552 | 0.5555 | 0.5557 | 0.5559 | 0.5561 | 0.5563 | 0.5565 | 0.5567 |
| 80 | 0.5569 | 0.5570 | 0.5572 | 0.5574 | 0.5576 | 0.5578 | 0.5580 | 0.5581 | 0.5583 | 0.5585 |
| 90 | 0.5586 | 0.5587 | 0.5589 | 0.5591 | 0.5592 | 0.5593 | 0.5595 | 0.5596 | 0.5598 | 0.5599 |
| 100 | 0.5600 | | | | | | | | | |

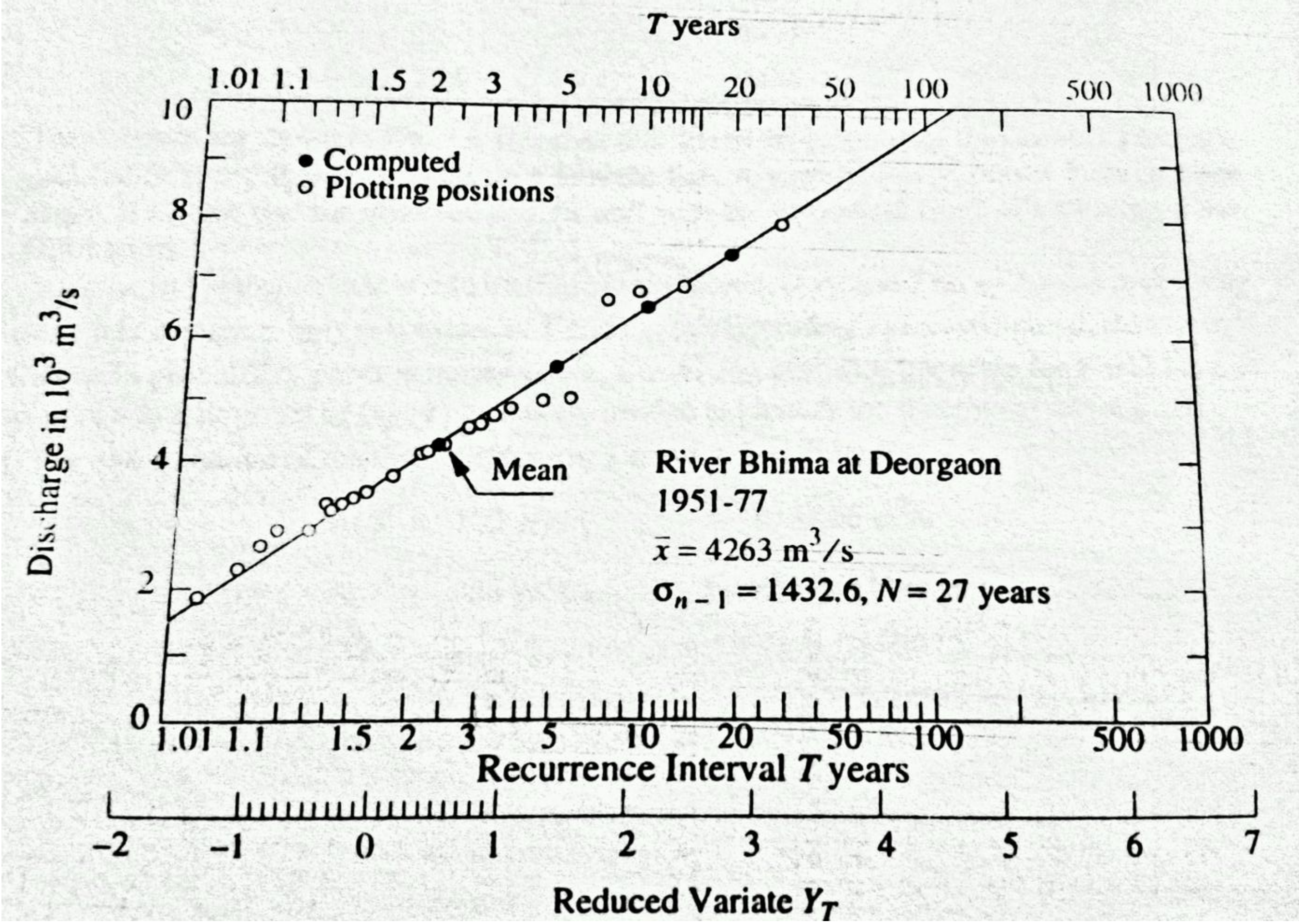
TABLE 7.4 REDUCED STANDARD DEVIATION $S_n$ IN GUMBEL'S EXTREME VALUE DISTRIBUTION

*N* = sample size

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0.9496 | 0.9676 | 0.9833 | 0.9971 | 1.0095 | 1.0206 | 1.0316 | 1.0411 | 1.0493 | 1.0565 |
| 20 | 1.0628 | 1.0696 | 1.0754 | 1.0811 | 1.0864 | 1.0915 | 1.0961 | 1.1004 | 1.1047 | 1.1086 |
| 30 | 1.1124 | 1.1159 | 1.1193 | 1.1226 | 1.1255 | 1.1285 | 1.1313 | 1.1339 | 1.1363 | 1.1388 |
| 40 | 1.1413 | 1.1436 | 1.1458 | 1.1480 | 1.1499 | 1.1519 | 1.1538 | 1.1557 | 1.1574 | 1.1590 |
| 50 | 1.1607 | 1.1623 | 1.1638 | 1.1658 | 1.1667 | 1.1681 | 1.1696 | 1.1708 | 1.1721 | 1.1734 |
| 60 | 1.1747 | 1.1759 | 1.1770 | 1.1782 | 1.1793 | 1.1803 | 1.1814 | 1.1824 | 1.1834 | 1.1844 |
| 70 | 1.1854 | 1.1863 | 1.1873 | 1.1881 | 1.1890 | 1.1898 | 1.1906 | 1.1915 | 1.1923 | 1.1930 |
| 80 | 1.1938 | 1.1945 | 1.1953 | 1.1959 | 1.1967 | 1.1973 | 1.1980 | 1.1987 | 1.1994 | 1.2001 |
| 90 | 1.2007 | 1.2013 | 1.2020 | 1.2026 | 1.2032 | 1.2038 | 1.2044 | 1.2049 | 1.2055 | 1.2060 |
| 100 | 1.2065 | | | | | | | | | |

# Gumbel probability plot

- Is an aid paper for convinient graphical representation of gumbels distribution.
- Usually used for graphical extrapolation



# Example(solution)

- Annual maximum recorded floods in a river for the period of 1951-1977 is given below.verify wheather the gumbel extreme-value distribution fit the recorded values.estimate the flood discharge with recurrence interval of
- 100 years
- 150 years

# Example 2

- Flood frequency computations for the river by using gumbel method yeilded the following results.

| return period | peak flood |
|---|---|
| 50 | 40809 |
| 100 | 46300 |

Estimate the flood magnitude with return period of 500years.

OLUTION : By Eq. (7.20),

$$x_{100} = \bar{x} + K_{100}\,\sigma_{n-1}$$

$$x_{50} = \bar{x} + K_{50}\,\sigma_{n-1}$$

$$(K_{100} - K_{50})\,\sigma_{n-1} = x_{100} - x_{50} = 46300 - 40809 = 5491$$

But $$K_T = \frac{y_T}{S_n} - \frac{\bar{y}_n}{S_n}$$

where $S_n$ and $\bar{y}_n$ are constants for the given data series.

$$\therefore \quad (y_{100} - y_{50})\frac{\sigma_{n-1}}{S_n} = 5491$$

By Eq. (7.22)

$$y_{100} = -[\ln\ \ln(100/99)] = 4.60015$$

$$y_{50} = -[\ln . \ln(50/49)] = 3.90194$$

$$\frac{\sigma_{n-1}}{S_n} = \frac{5491}{(4.60015 - 3.90194)} = 7864$$

For $T$ = 500 years, by Eq. (7.22),

$$y_{500} = -[\ln . \ln(500/499)] = 6.21361$$

$$(y_{500} - y_{100})\frac{\sigma_{n-1}}{S_n} = x_{500} - x_{100}$$

$$(6.21361 - 4.60015) \times 7864 = x_{500} - 46300$$

$$x_{500} = 58988, \quad \text{say } 59{,}000\ \text{m}^3/\text{s}$$

### Confidence Limits

Since the value of the variate for a given return period, $x_T$ determined by Gumbel's method can have errors due to the limited sample data used, an estimate of the confidence limits of the estimate is desirable. The confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only.

For a confidence probability $c$, the confidence interval of the variate $x_T$ is bounded by values $x_1$ and $x_2$ given by[6]

$$x_{1/2} = x_T \pm f(c)\, S_e \tag{7.23}$$

where $f(c)$ = function of the confidence probability $c$ determined by using the table of normal variates as

| $c$ in per cent | 50 | 68 | 80 | 90 | 95 | 99 |
|---|---|---|---|---|---|---|
| $f(c)$ | 0.674 | 1.00 | 1.282 | 1.645 | 1.96 | 2.58 |

$$S_e = \text{probable error} = b\,\frac{\sigma_{n-1}}{\sqrt{N}} \tag{7.23a}$$

$b = \sqrt{1 + 1.3\,K + 1.1\,K^2}$

$K$ = frequency factor give by Eq. (7.21)

$\sigma_{n-1}$ = standard deviation of the sample

# Estimation of design floods to spillway and other hydraulic structures

- A well-designed, constructed and operated dam can reduce flood risk in developed areas downstream by temporarily impounding flood waters and
- attenuating the observed peak flood flows in vulnerable low lying areas, even if the dam is not specifically designed for flood mitigation.

# Estimation of design floods to spillway and other hydraulic structures

- There are several potential causes of dam failure including
- hydrologic,
- hydraulic,
- geologic,
- seismic,
- structural,
- mechanical, and
- operational.

# Estimation of design floods to spillway and other hydraulic structures

- hydrologic failure modes are limited to the selection of the IDF for the hydrologic design of a dam to reduce risks to the public.
- However, decisions to invest in risk reduction actions should be made in the context of all risks at a dam to ensure that the proposed modifications are not providing a false sense of safety by not addressing other risks or failure modes which may be more likely.

TABLE 7.8 GUIDELINES FOR SELECTING DESIGN FLOODS, (CWC, INDIA)[1]

| S. no. | Structure | Recommended design flood |
|---|---|---|
| 1. | Spillways for major and medium projects with storages more than 60 $Mm^3$ | (a) PMF determined by unit hydrograph and probable maximum precipitation (PMP)<br>(b) If (a) is not applicable or possible flood–frequency method with T = 1000 years |
| 2. | Permanent barrage and minor dams with capacity less than 60 $Mm^3$ | (a) SPF determined by unit hydrograph and standard project storm (SPS) which is usually the largest recorded storm in the region<br>(b) Flood with a return period of 100 years.<br>(a) or (b) whichever gives higher value. |
| 3. | Pickup weirs | Flood with a return period of 100 or 50 years depending on the importance of the project. |
| 4. | Aqueducts | |
| | (a) Waterway | Flood with $T = 50$ years |
| | (b) Foundations and free board | Flood with $T = 100$ years |
| 5. | Project with very scanty or inadequate data | Empirical formulae |