RAPIDLY VARIED FLOW

Fitsume T.

contents

RVF

- Curved Stream lines
- High turbulence
- Discontinuous profile
- Pressure distribution is not hydrostatic
- Boundary friction is very small (insignificant)

RVF

- When RVF occurs in a sudden-transition structure, the physical characteristics of the flow are basically fixed by **the boundary geometry of the structure as well as by the state of the flow**

- When rapid changes in water area occur in RVF, **the velocity distribution coefficients α and β are usually far greater than unity** and cannot be accurately determined

- The separation zones, eddies, and rollers that may occur in RVF tend to **complicate the flow pattern and to distort the actual velocity distribution** in the stream

Hydraulic jump

supercritical flow changes to subcritical flow

–water surface rises abruptly,

–surface rollers are formed,

–intense mixing occurs,

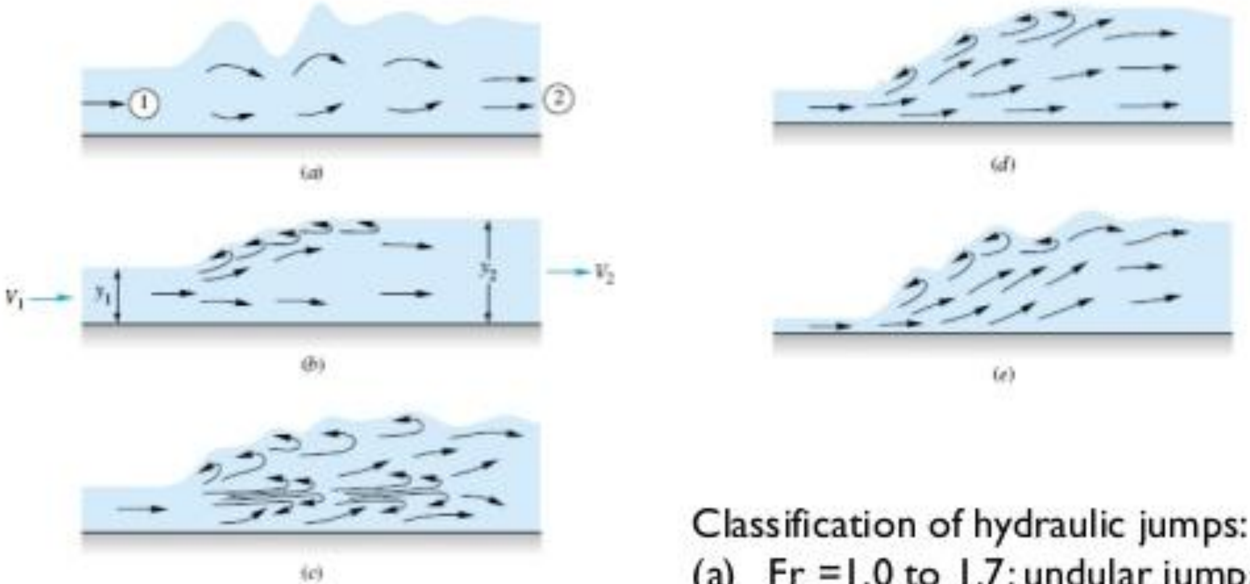
–air is entrained, and usually a large amount of energy is dissipated

Practical applications of the hydraulic jump

- *To dissipate energy in water flowing*
- *To recover head or raise the water level*
- *To increase the weight on an apron and thus reduce uplift pressure*
- *To increase the discharge of a sluice by holding back tail-water*
- *To mix chemicals*
- *To aerate the water*

Types of Hydraulic Jump

- Based of the Froude number hydraulic jumps are classified as
- **For F_{r1} = 1,** the flow is critical, and hence no jump can form.



Classification of hydraulic jumps:

(a) Fr =1.0 to 1.7: undular jumps;

(b) Fr =1.7 to 2.5: weak jump;

(c) Fr =2.5 to 4.5: oscillating jump;

(d) Fr =4.5 to 9.0: steady jump;

(e) Fr =9.0: strong jump.

undular Jump

- **For F_{r1} = 1 to 1.7,** the water surface shows undulations, and the jump is called an **undular Jump**.

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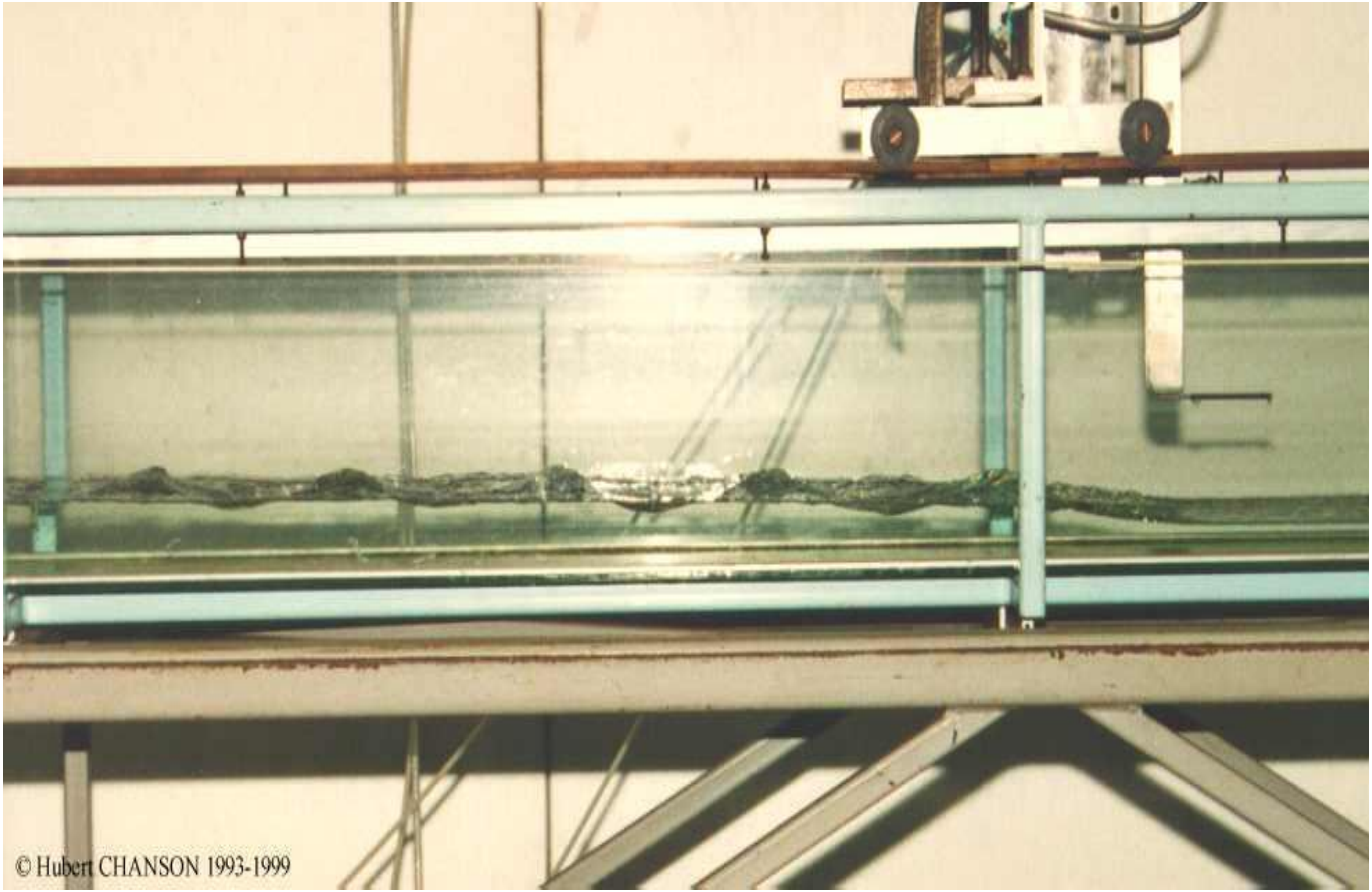

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weak jump

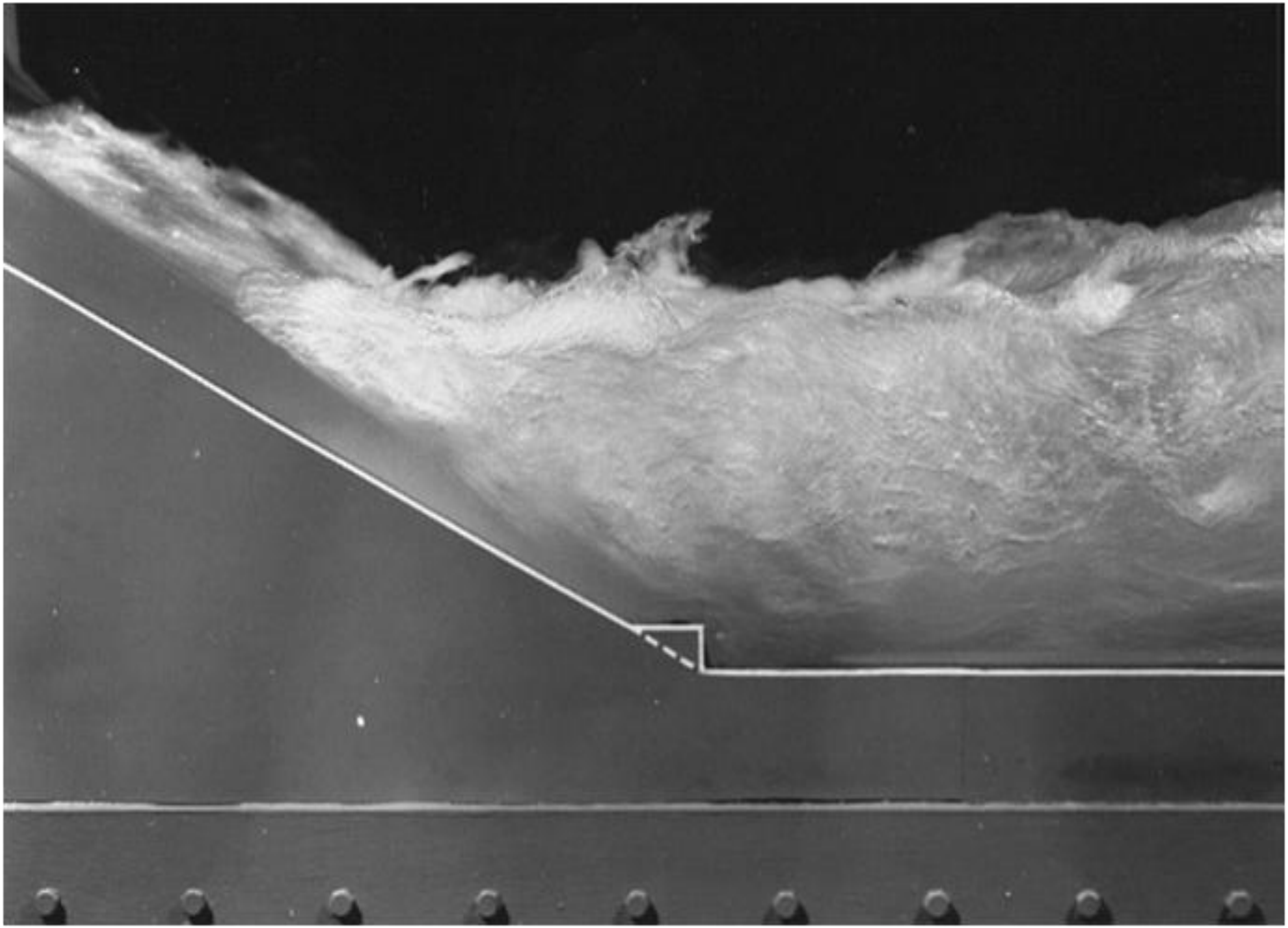
- ***For*** **F_{r1} = 1.7 to 2.5**, a series of small rollers develop on the surface of the jump, but the d/s water surface remains smooth. The velocity throughout is fairly uniform. This jump is called a

  weak jump

oscillating jump.

- **For F_{r1} = 2.5 to 4.5**, there is an oscillating jet entering the jump bottom to surface and back again with no periodicity. Each oscillation produces a large wave of irregular period which, very commonly in canals, can travel for miles doing unlimited damage to earth banks and ripraps. This jump is an

  oscillating jump.



steady jump

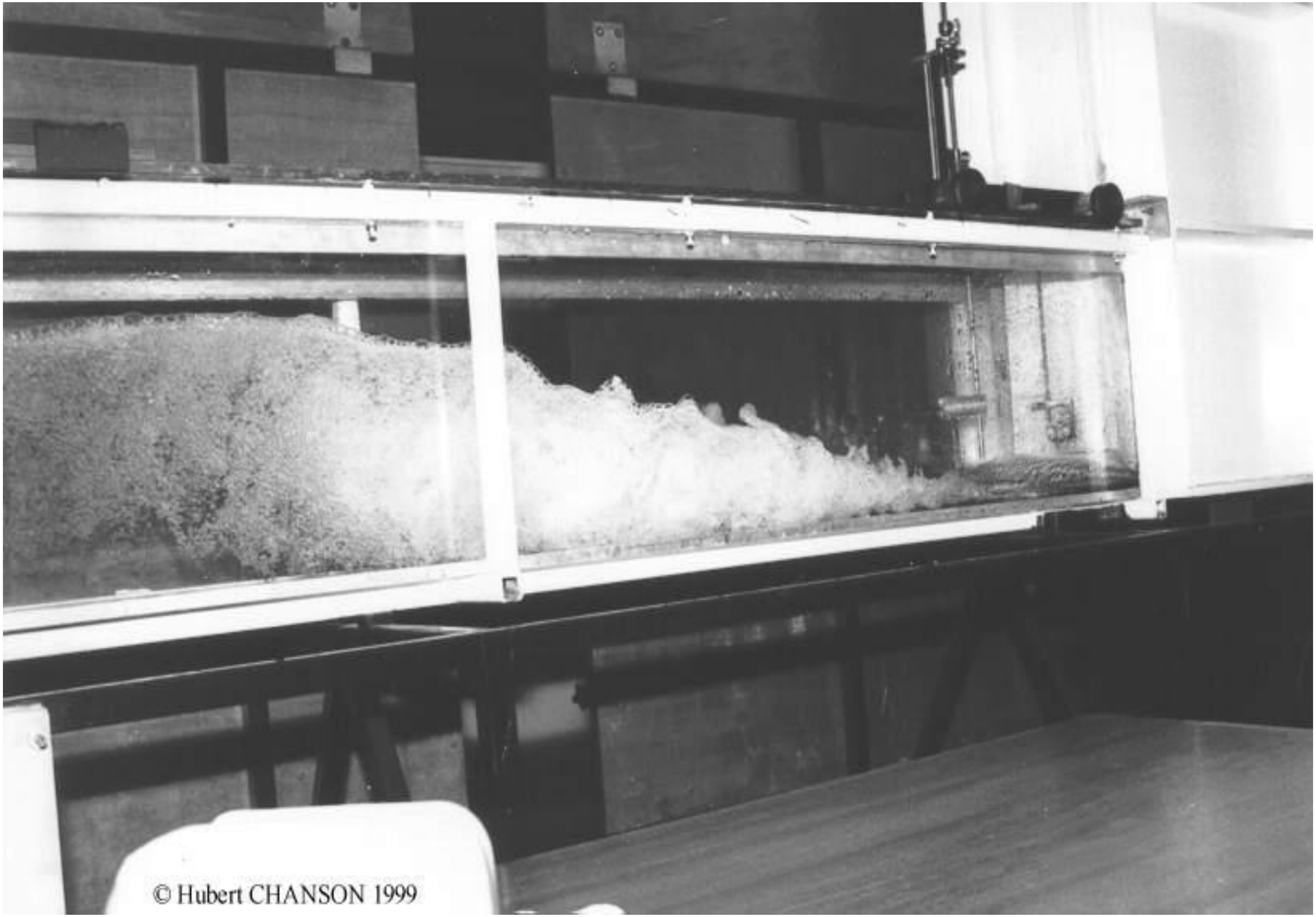
- **For F_{r1} = 4.5 to 9.0,** the downstream extremity of the surface roller and the point at which the high-velocity jet tends to leave the flow occur at practically at the same vertical section. The action and position of this jump are least to variation in tail-water depth. The jump is well-balanced and the performance is at its best. The energy dissipation ranges from 45 to 70%. This jump is called a

  steady jump

strong jump

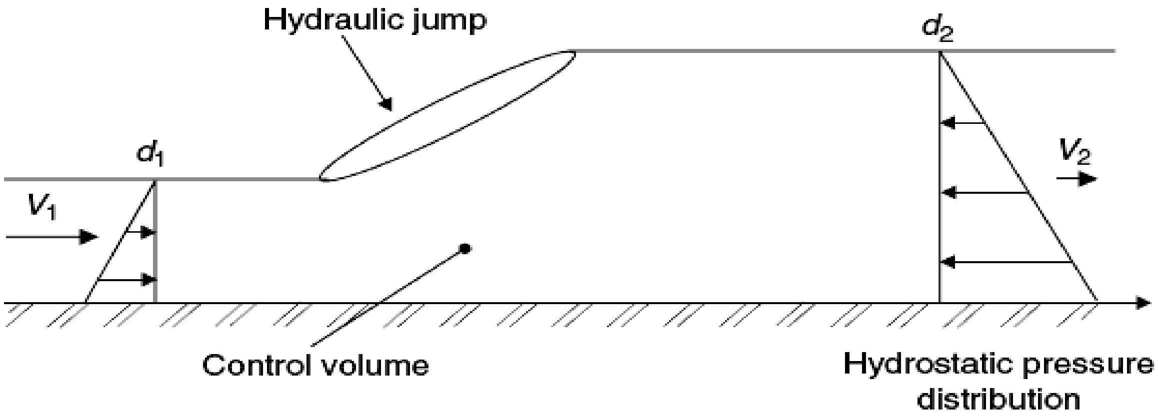
- **For F_{r1} = 9.0** and larger, the high-velocity jet grabs intermittent slugs of water rolling down the front face of the jump, generating waves downstream, and a rough surface can prevail. The jump action is rough but effective since the energy dissipation may reach 85%. This jump is called a

strong jump



- open channel\FT\dam overflowing & hydraulic jump.mp4

Hydraulic Jump in a Horizontal Rectangular Channel



- In most open channel flow problems involving jumps, **one of the two depths y_1 or y_2 would be known**, and we need to calculate the second one.
- Because the energy loss due to **hydraulic jump is usually significant and unknown**, we **cannot use the energy equation** to determine the unknown depth rather we use **the momentum equation** is written between the two sections.
  - **Considering a horizontal rectangular open channel of constant channel width *B***
  - **neglecting the shear stress at the channel bottom,**
  - **the resultant of the forces acting in the longitudinal direction are the result of hydrostatic pressure at the ends of the control volume**

Hydraulic Jump …..

- Applying the momentum equation in the *x*-direction to the control volume

$$\sum F_x = m(V_2 - V_1)$$

- The forces are the hydrostatic forces on each end of the system; thus the following is obtained $$P_1 A_1 - P_2 A_2 = (\rho V_2 A_2)V_2 - (\rho V_1 A_1)V_1$$

Or $$P_1 A_1 + \rho Q V_1 = P_2 A_2 + \rho Q V_2$$

- Where P_1 and P_2 are the pressures at the centroids of the respective areas A_1 and A_2. For rectangular channel of unit width where $P_1 = \gamma y_1/2$, $P_2 = \gamma y_2/2$, $Q = q$, $A_1 = y_1$, and $A_2 = y_2$,

$$\gamma \frac{y_1^2}{2} + \rho q V_1 = \gamma \frac{y_2^2}{2} + \rho q V_2$$

- But $q = v_1 y_1 = v_2 y_2$, so substitute v2 =q/y2, and v1=q/y1 and rewrite the above equation as

$$\frac{\gamma}{2}(y_1^2 - y_2^2) = \frac{\gamma q^2}{g}\left(\frac{1}{y_1} - \frac{1}{y_2}\right)$$

- The preceding equation can be further manipulated to yield

$$\frac{1 y_2}{2 y_1}\left(1 + \frac{y_2}{y_1}\right) = \frac{q^2}{g y_1^3} = F_1^2$$

Hydraulic Jump …..

- The term on the left-hand side of the equation is the Froude number F_{r1} Hence, we can write
$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\,F_{r1}^2 = 0$$
- By using the quadratic formula, it is solved for y_2/y_1 in terms of the u/s Froude number. This yields an equation for *depth ratio* across a hydraulic jump: or it is also called **Sequent depth ratio**
- $$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8\,F_{r1}^2} - 1\right) \quad \textbf{or} \quad y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\,F_{r1}^2} - 1\right)$$

Once we determine the flow depths upstream and downstream of the hydraulic jump, we can use the energy equation to calculate the head loss due to the jump as
$$h_{LJ} = \left(y_1 + \frac{q^2}{2gy_1^2}\right) - \left(y_2 + \frac{q^2}{2gy_2^2}\right)$$

- This equation is manipulated to obtain
$$h_{LJ} = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

Basic Characteristics of Hydraulic Jump

- **Sequent depth ratio**

$$\frac{y_2}{y_1} = \tfrac{1}{2}\left(\sqrt{1 + 8F_1^2} - 1\right)$$

or

$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1 + 8\,F_{r2}^2} - 1\right)$$

- **Energy Loss**

$$h_{LJ} = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

or

$$\Delta E = E_1 - E_2 = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

The ratio $\Delta E/E_1$ is known as the *relative loss*

- **Efficiency :** The ratio of the specific energy after the jump to that before the jump is defined as the **efficiency of the jump**. It is given by

$$\frac{E_2}{E_1} = \frac{(8F_1^2 + 1)^{3/2} - 4F_1^2 + 1}{8F_1^2(2 + F_1^2)}$$

This equation indicates that the efficiency of a jump is a dimensionless function, depending only on the Froude number of the approaching flow. The relative loss is equal to $1 - E_2/E_1$. It is also a dimensionless function of F_{r1}.

Basic Characteristics of Hydraulic Jump

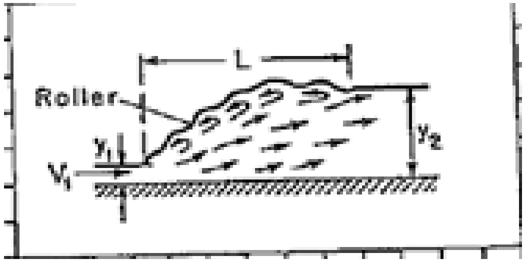
- **Height of Jump :** The difference between the depths after and before the jump is the *height of the jump, or* **$h_j = y_2 - y_1$.**
- If it is expressing as a ratio with respect to the initial specific energy, it is known the ***relative height***

$$\frac{h_j}{E_1} = \frac{y_2}{E_1} - \frac{y_1}{E_1} \quad \textbf{or} \quad \frac{h_j}{E_1} = \frac{\sqrt{1 + 8F_{r1}^2} - 3}{F_{r1}^2 + 2}$$

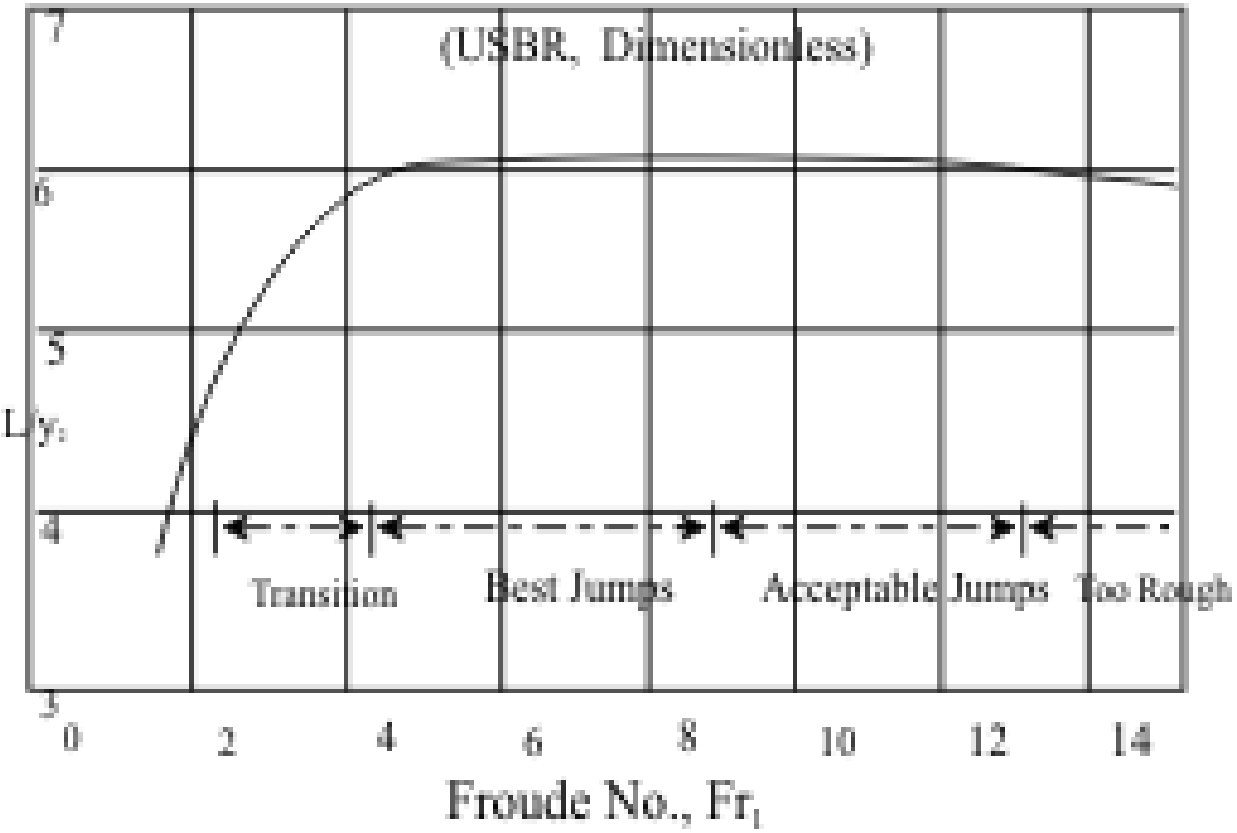
- **Length of a Jump :** may be defined as the distance measured from the front face of the jump to a point on the surface immediately d/s from the roller. This length cannot be determined easily by theory, but has been investigated experimentally
- The experimental data on L_j can be plotted conveniently with F_{r1} against dimensionless ratios $L/(y_2 - y_1)$, L/y_1, or L/y_2. For practical purposes, however, the plot of F_{r1} Vs L/y_2 is desirable,

- Thus the experimental determination of

$f(F_1)=L_j /y_2$ for higher value of **$F_1 > 5.0$,**
the relative Jump length **Lj/y2 become constant**
and it can be estimated **$Lj = 6.9(y_2-y_1)$**



Lengths of Hydraulic Jumps



Basic Characteristics of Hydraulic Jump

Profile of the Jump

- The jump profile is required **to determine the weight of water in a dissipater** in order **to counteract the uplift force** if the basin floor is laid on a permeable foundation. While designing **the height of the side walls**, the water profile is required.
- Bakhmetoff and Metzke who were the first to investigate systematically the longitudinal elements of the jump, took the end of the jump as the section of maximum surface elevation before the drop off caused by the channel conditions downstream.
- They also represented the surface profile of the jump by dimensionless curves for various F_1 values.
- Hager [1991a] developed the following empirical relationship for the flow depth, *y,* at distance, *x*, from the beginning of the jump

$$y = \tanh(1.5X)$$

Where $X = \frac{x}{L_r}$ and $Y = \frac{(y - y_1)}{(y_2 - y_1)}$ and Lr = length of the roller

Basic Characteristics of Hydraulic Jump

- Subramanya and Rajaratnam have shown that the jump profile can be expresses in non-dimensional manner as

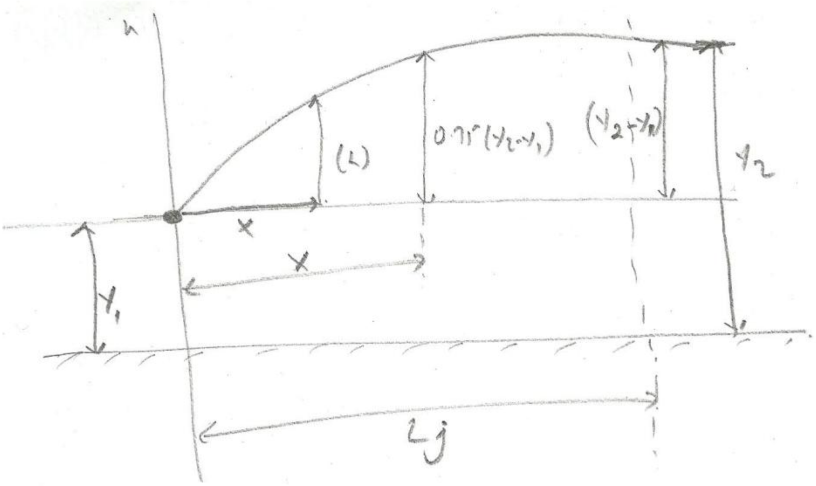
$$\eta = f(\lambda) \text{Where } \eta = \frac{h}{0.75(y_2 - y_1)} \quad \text{and} \quad \lambda = \frac{x}{X}$$

Where X is the length scale defined as the value of x at which

$$h = 0.75(y_2 - y_1)$$

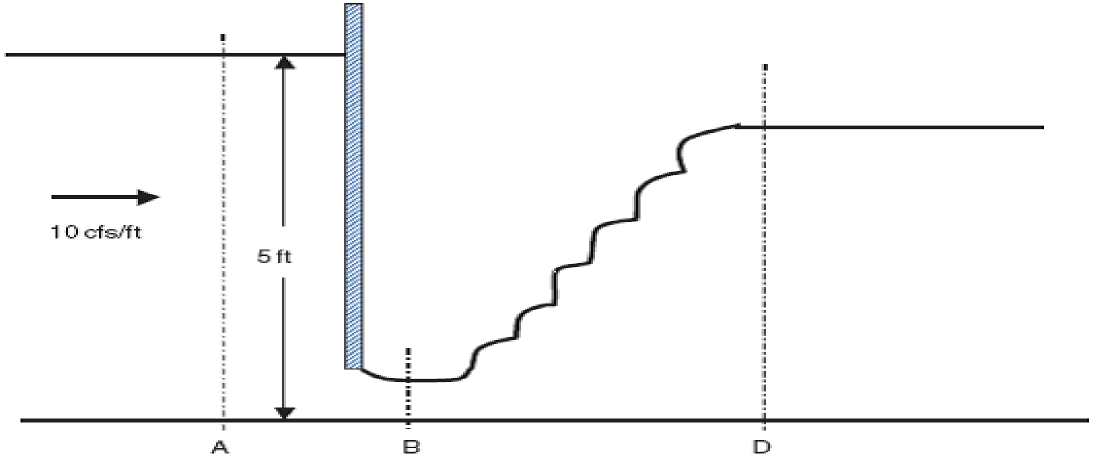
$$\text{X is given by } \frac{X}{y_1} = 5.08F_1 - 7.82$$

The coordinates of the profile are (x, h) with the boundary condition that x=0, at h=0 and x=Lj at h= $(y_2 - y_1)$. In general, h= f(x, F_1)



Example 6.1:

The rectangular channel shown in the figure below is nearly horizontal, and it carries a discharge, q = 0.95m^3/s/m. The flow depth upstream of the sluice gate is 1.5 m. A hydraulic jump occurs on the downstream side of the sluice gate. Determine the flow depth at sections B and D, and the energy loss due to the jump.



Hydraulic Jumps in Horizontal non-rectangular Channel

- The jump in such channels is characterized by a **lateral expansion of the jet** (if the channel width is increasing from bottom to top as is usually the case) in addition to the expansion in the vertical direction
- The conjugate depth ratio in trapezoidal, parabolic, and triangular channels is less than that of a rectangular channel at the same Froude number.
- Considering a frictionless, horizontal prismatic channel, in the absence of external forces (except the pressure force) the **momentum equation between the toe and the heel** (or between sections 1 and 2) of the jump can be written as

  P1- P2 = M_1 - M_2 =

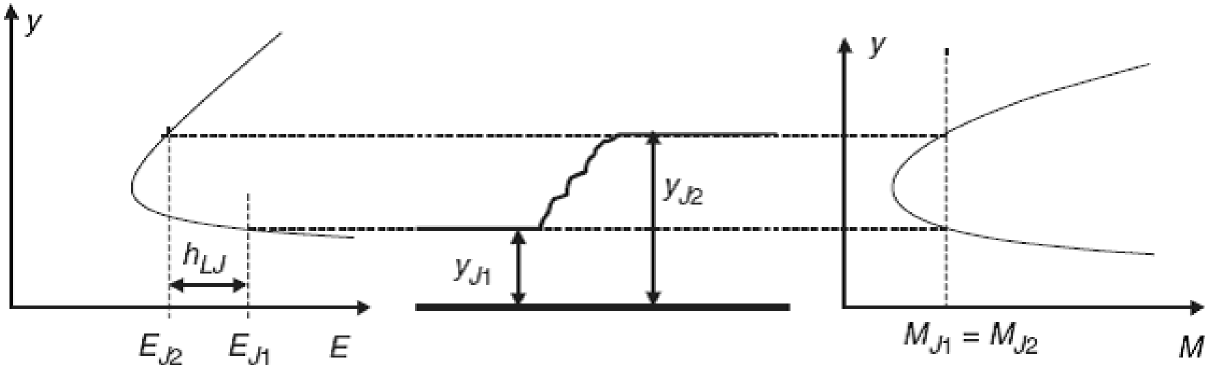
- Where A =area of cross section y= depth of the center of gravity of the area Re arrange $P_1+M_1 = P_2M_2$ = Constant = P +M

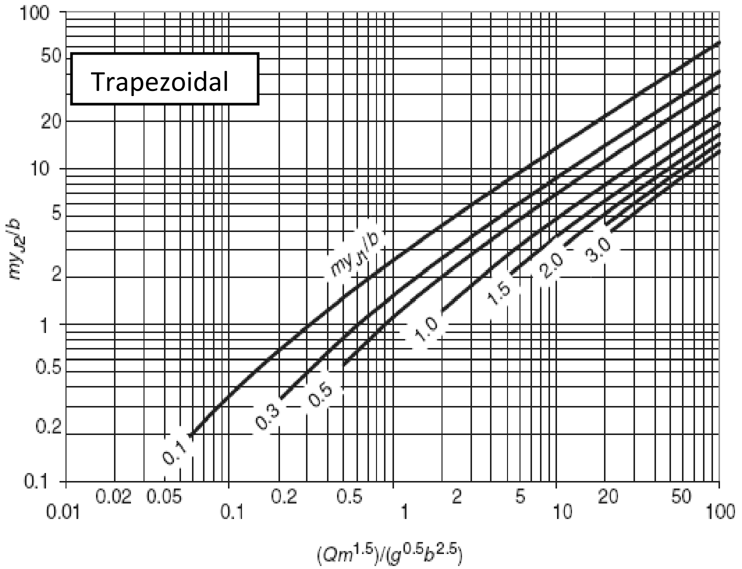
$$\gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 = \rho Q_2 V_2 - \rho Q_1 V_1$$

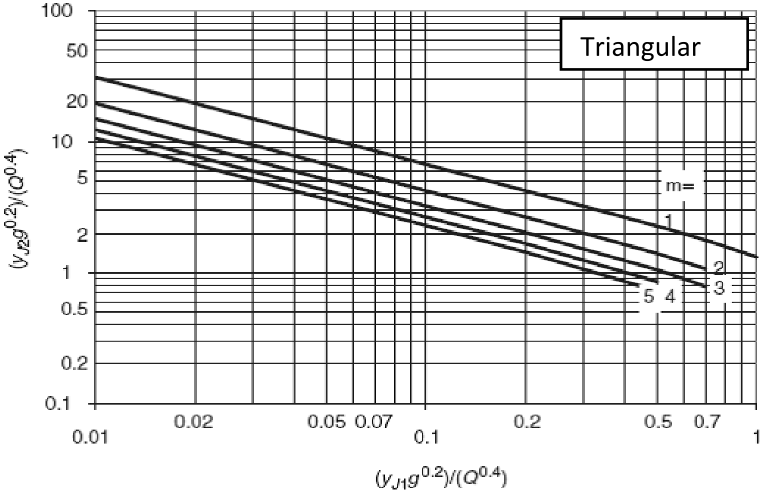
$$\frac{P+M}{\gamma} = P_s = \frac{Q^2}{g\,A} + A\bar{y}$$

- Where Ps is the specific momentum (force)
  - This equation is valid provided that the pressure distributions at sections 1 and 2 are hydrostatic.
  - Thus, for a given channel shape, Q and y_1, the value of M_1 can be calculated. The value of y_2 which makes M_2 equal to M_1 can then be determined by trial-and-error.

Hydraulic Jumps in Horizontal non-rectangular Channel







Example 6.2

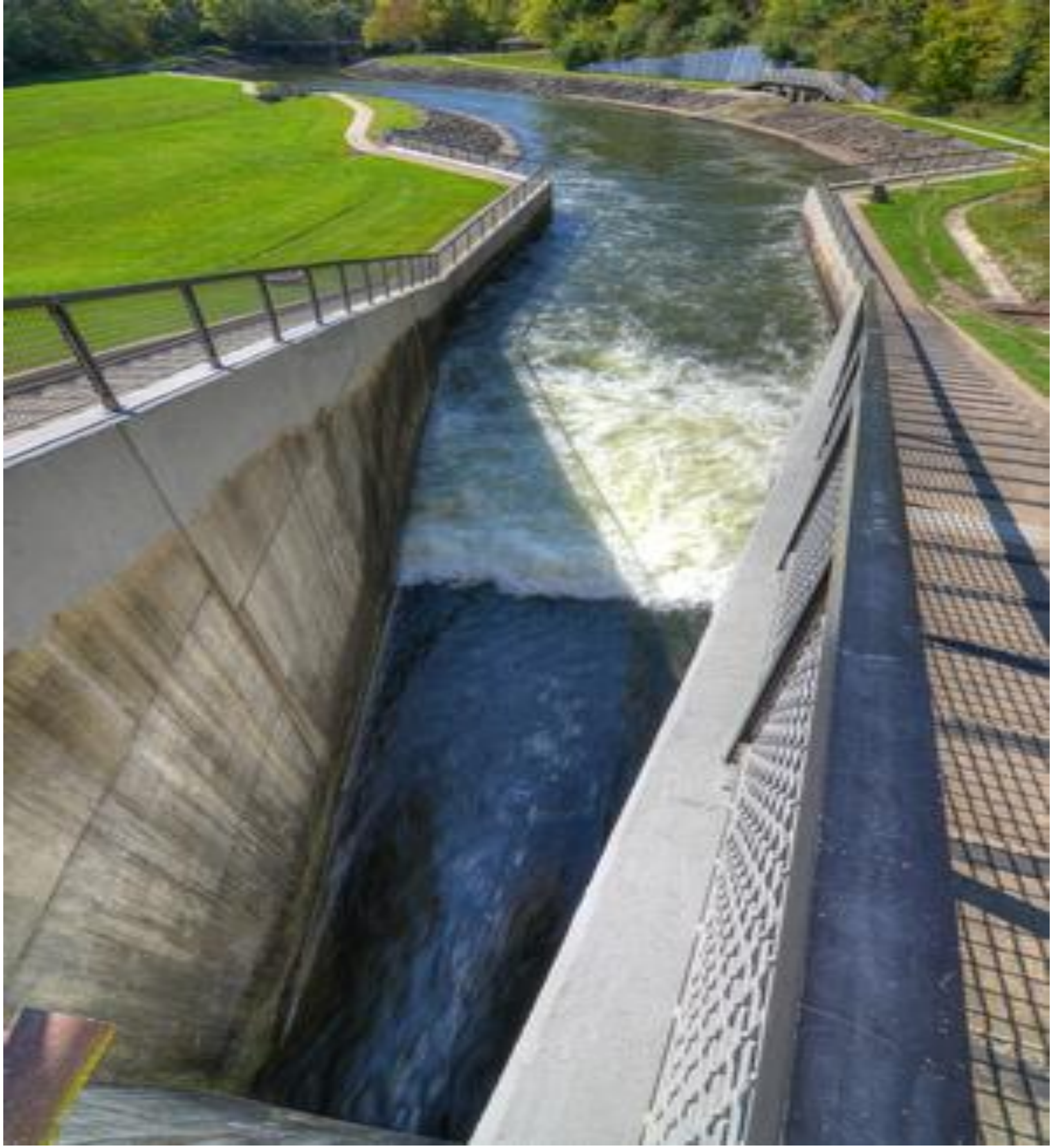
The trapezoidal channel has a bottom width of B = 1.80 m, side slopes of m = 2, and it carries a discharge of 8.5 m^3/s. A hydraulic jump occurs in this channel. The flow depth just before the jump is y_1 = 0.30 m. Determine the depth after the jump.

Hydraulic Jumps as Energy Dissipater

- From a practical viewpoint, hydraulic jump is a useful means of dissipating excess energy in supercritical flow.
  - Its merit is in preventing possible erosion below overflow spillways, chutes, and sluices, for it quickly reduces the velocity of the flow on the paved apron to a point where the flow becomes incapable of scouring the downstream channel bed.
- The hydraulic jump used for energy dissipation is usually confined partly or entirely to a channel reach known as the *stilling basin.*
- In practice, the stilling basin is designed with accessories to control the jump in the basin.
  - These accessories shorten the range within which the jump will take place and hence reduce the cost.
  - They also improve the dissipation function of the basin and stabilize the jump.

- **Position of Hydraulic Jump:** Hydraulic jump is formed at a location where the flow depths upstream and downstream of the jump satisfy the equation for the sequent depth ratio.

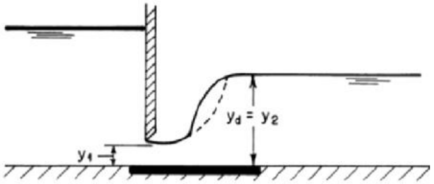
- Let the flow depth at the sluice outlet be y_1 and the sequent depth corresponding to this depth be y_2. There are several different possibilities for the formation of jump, depending upon on the tailwater depth, y_d.



- Stilling basin 1
- Stilling basin 2
- Stilling basin 3

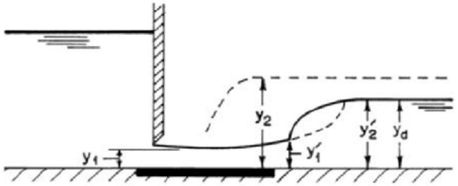
Hydraulic Jumps as Energy Dissipater

- Case A : Tail-water depth (y_d) equal to the sequent depth (y_2)
  - It is an ideal one for scour protection purposes.
  - needs device to control the position of the jump



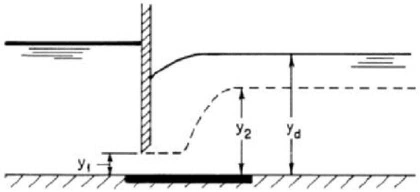
(a) $y_d = y_2$

- Case B : Tail-water depth (y_d) less than the sequent depth (y_2)
  - Jump repelled from the scour-resisting apron
  - It should be avoided in design



(b) $y_d < y_2$

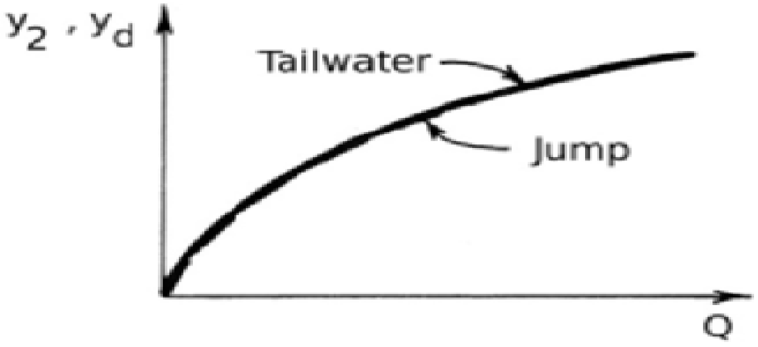
- Case C : Tail-water depth (y_d) grater than the sequent depth (y_2)
  - The jump forced to upstream and becoming a submerged jump
  - It is the safest case in design



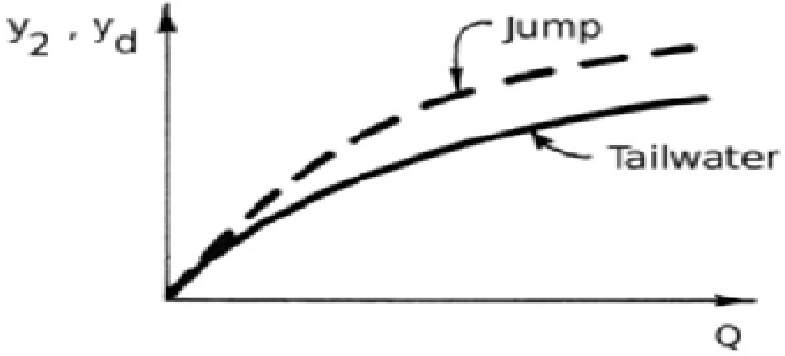
(c) $y_d > y_2$

Hydraulic Jumps as Energy Dissipater

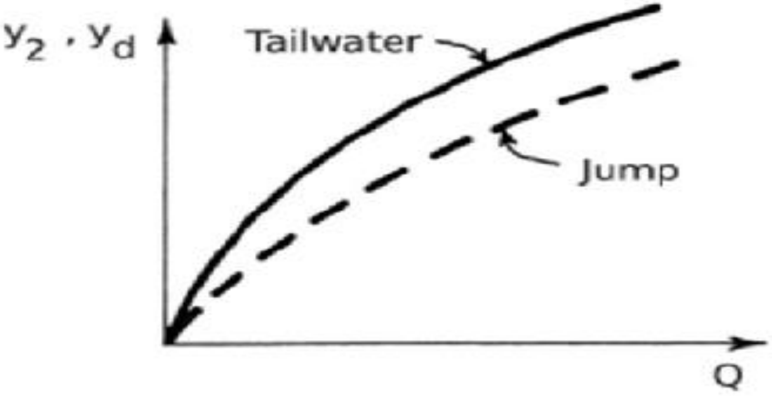
- **Tail-water Conditions:** Tail-water level plays a significant role in the formation of jump at a particular location.
- The tail-water fluctuates owing to changes in discharge in the channel.
- The tail-water rating curve is usually available as a relation between tail-water stage y_d and discharge Q.
- In a similar way, a jump rating curve may be constructed to show the relation between the sequent depth y_2 and Q.
- Depending upon these two curves, five different flow situations are possible.


y_2 , y_d
Tailwater
Jump
Q

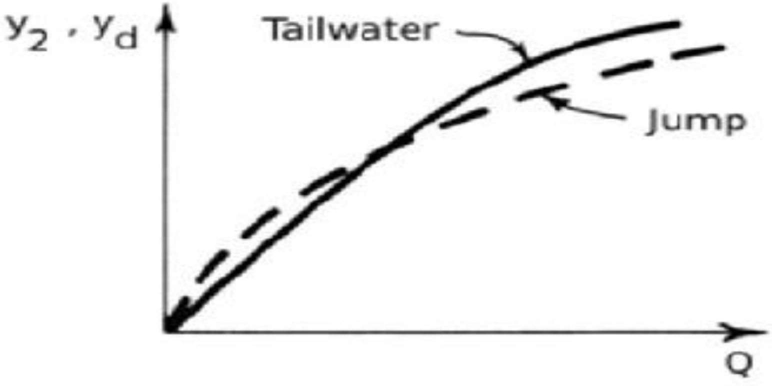
(a)


y_2 , y_d
Jump
Tailwater
Q

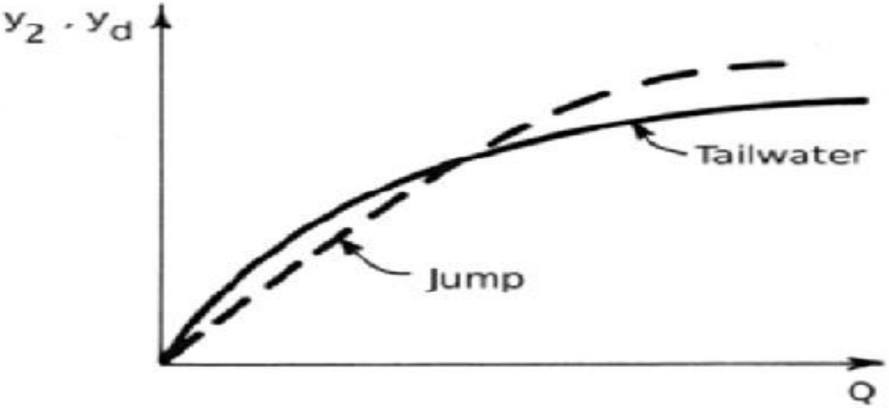
(b)


y_2 , y_d
Tailwater
Jump
Q

(c)


y_2 , y_d
Tailwater
Jump
Q

(d)


y_2 , y_d
Tailwater
Jump
Q

(e)

Hydraulic Jumps as Energy Dissipater

- **Class A:** represents the ideal condition in which the two rating curves always coincide. This means the jump forms at the desired place on the apron at all discharges.

- **Class B:** In this case the jump forms at a certain place far downstream. An effective method of ensuring that the jump will occur on the protected apron is to use sills to create a stilling basin.

- **Class C:** The jump may be controlled at the desired location by providing a drop in the channel bottom or by letting the jump form on a sloping apron.

- **Class D:** The tail-water curve is below the jump curve at low discharges and above it for higher discharges. The stilling basin may be designed so that the jump is formed in the basin at low rates of discharges and the jump moves on to a sloping apron at higher discharges.

- **Class E:** This is opposite to case (d) in the sense that the tail-water curve is above the jump curve at low discharges and below the jump curve at high discharges. An effective method to ensure a jump is to increase the tail-water depth sufficiently high by providing a stilling pool, thus forming a jump at high discharges.

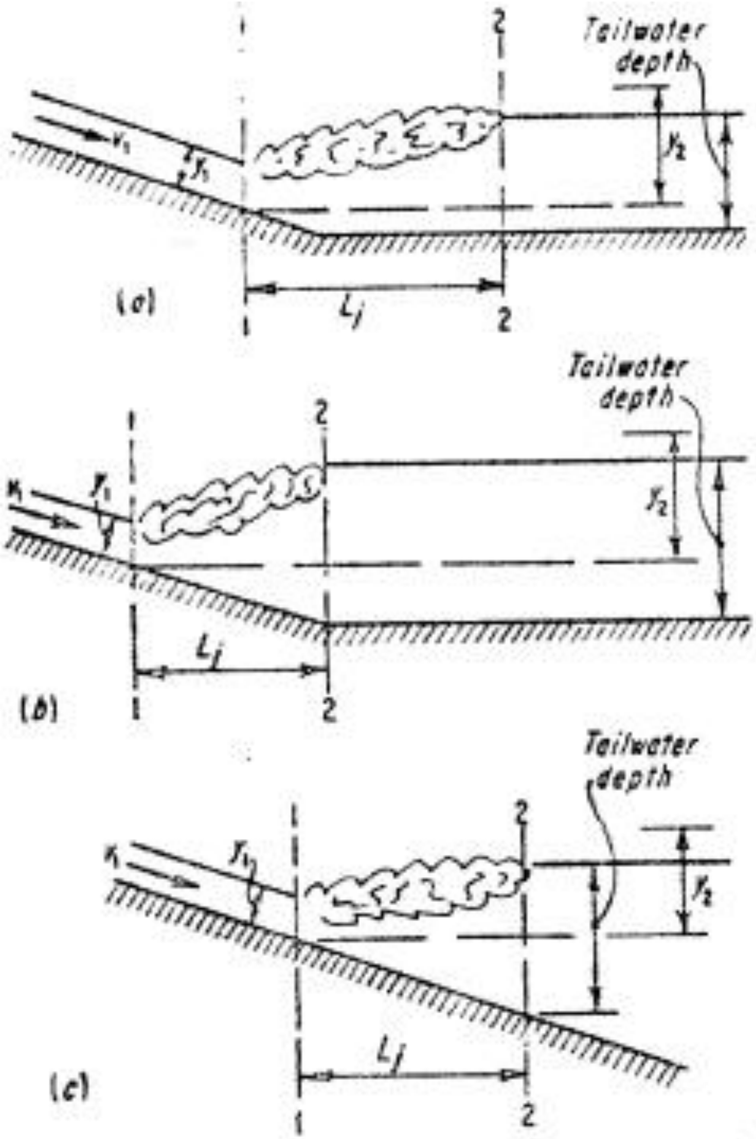


FIG. 6-1. Occurrence of the hydraulic jump on a sloping apron: (a) case I, (b) case II, (c) case III.

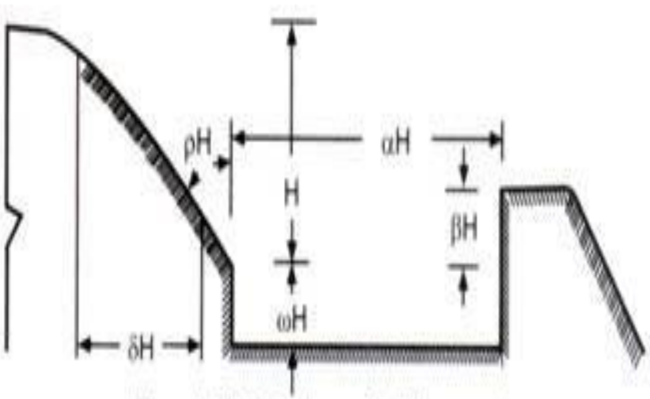


Fig. 19.17. Design of stilling pool

Exercise 1

- Water flows in wide rectangular channel of q=10m3/s/m and y1=1.25m. If the flow undergoes a hydraulic jump, compute
  - sequent depth (y_2)
  - Velocity at sequent section (v_2)
  - Froude number at sequent (F_2)
  - Dissipation loss (h_f)
  - The percentage of dissipation
  - The power dissipated per unit width
  - The tempreture rise due to dissipation if

    C_p =4200J/kg.k