Chapter Four

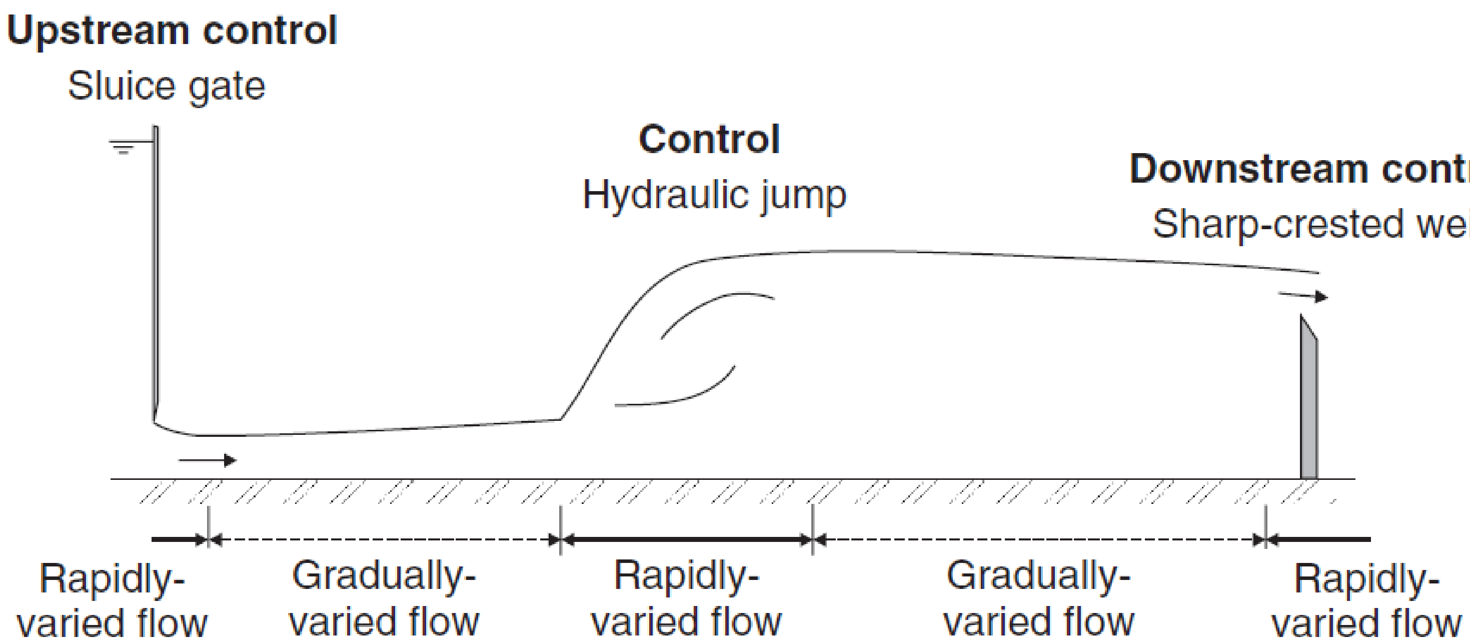
GRADUALLY-VARIED FLOW (GVF)

- GVF introduction
- Basic Assumptions
- Basic Differential Equation of GVF
- Classification of Flow Surface Profiles
- Features of Water Surface Profiles
- Analysis of Flow Profile
- Computation of GVF
    - Direct Integration
    - Numerical Analysis
        - Direct Step Method
        - Standard Step Method

1.Introduction :Gradually-varied flow (GVF)

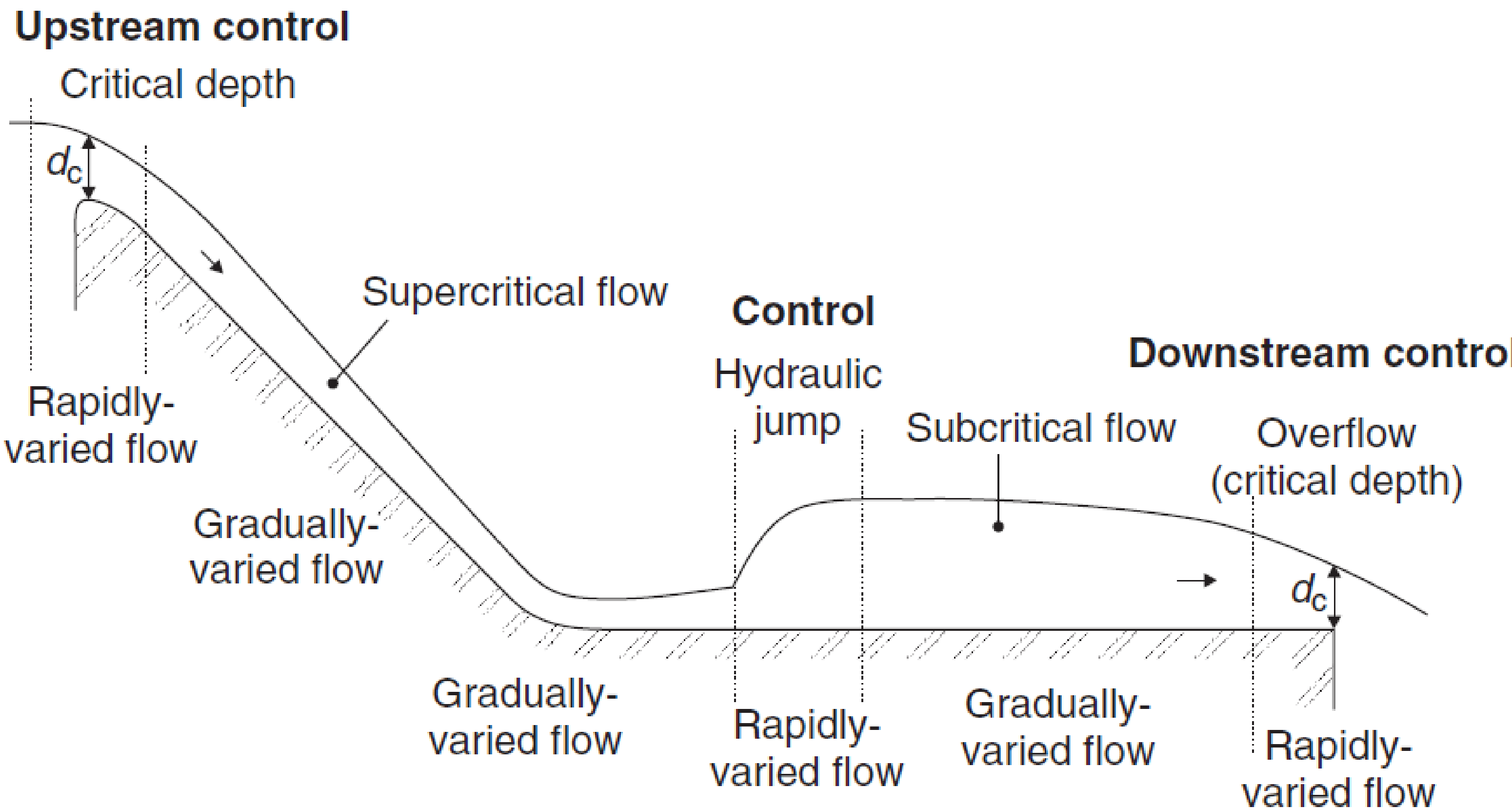
- The flow is a steady non-uniform flow
- The streamlines are parallel
  - Hydrostatic pressure distribution prevails over the channel section
- The velocity varies along the channel
  - The bed slope, water surface slope, and energy line slope will differ each other.
  - The fiction loss over the bed has significance
- Examples of GVF
  - The backwater produced by a dam or weir across a river
  - Drawdown produced at a sudden drop in a channel

EXAMPLE1



Upstream control
Sluice gate
Control
Hydraulic jump
Downstream control
Sharp-crested weir
Rapidly-
varied flow
Gradually-
varied flow
Rapidly-
varied flow
Gradually-
varied flow
Rapidly-
varied flow

EXAMPLE2



Upstream control
Critical depth
d_c
Rapidly-
varied flow
Supercritical flow
Gradually-
varied flow
Control
Hydraulic
jump
Subcritical flow
Downstream control
Overflow
(critical depth)
d_c
Gradually-
varied flow
Rapidly-
varied flow
Gradually-
varied flow
Rapidly-
varied flow

2. Basic Assumptions GVF

- considers only steady flows prismatic Channels.
- The head loss in a reach may be computed using an equation applicable to uniform flow having the same velocity and hydraulic mean radius of the section.
- Channel bottom slope is small. (i.e the depth of flow measured vertically is same as depth of flow measured perpendicular to channel bottom)
- The velocity distribution in the channel section is invariant. (the energy correction factor, α , is a constant and does not vary with distance.)
- The resistance coefficient is not a function of flow characteristics or depth of flow. It does not vary with distance.

Basic Assumptions….

Thus,

- if a GVF the depth of at any section is y, the energy line slope Se is given by

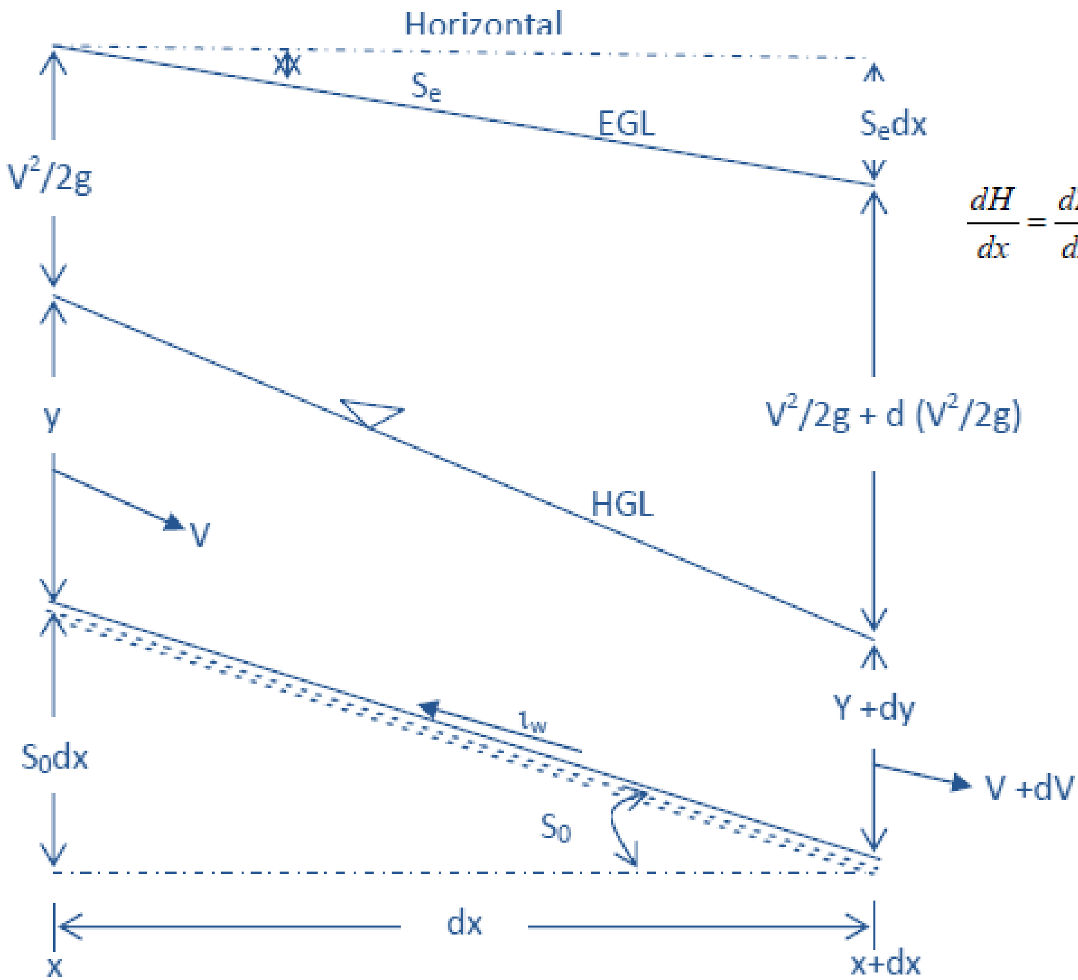
$$S_e = \frac{n^2 V^2}{R^{4/3}} \quad \text{---} \ (5.1)$$

- The flow satisfies the continuity and the energy equations with bottom friction losses included
- The two varied and unknown variables are velocity V(x) and Depth y(x), where x is distance along the channel.

3.Basic Differential Equation of GVF

Consider the total energy H of gradually varied flow in a channel of small slope and α =1.0 as

$$H = z + E = Z + y + \frac{V^2}{2g} \text{---------} (5.2)$$



$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dE}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \text{--} (5.3)$$

Basic Differential .....

In equation (5.3) each term has its own meanings and described as follows

$$\frac{dH}{dx}=\frac{dZ}{dx}+\frac{dE}{dx}=\frac{dZ}{dx}+\frac{dy}{dx}+\frac{d}{dx}\left(\frac{V^2}{2g}\right) \text{-- } (5.3)$$

- dH/ dx represents the energy slope. Since the total energy of the flow always decreases in the direction of motion, it is common to consider the slope of the decreasing energy line as positive and denoting it by Se.

$$\frac{dH}{dx}=-S_e \quad \text{-----------------------------------} (5.4)$$

- dZ/ dx : denotes the bottom slope. Similarly consider as positive and denoting with S_o

$$\frac{dZ}{dx}=-S_0 \quad \text{-----------------------------------} (5.5)$$

- dy/ dx : represents the water surface slope to the bottom of the channel

Basic Differential .....

So the energy equation of the two cross-sections can be rewritten as

$$\frac{V^2}{2g} + y + S_0 dx = S_e dx + \frac{V^2}{2g} + d\frac{V^2}{2g} + y + dy$$

$$\Rightarrow \qquad (S_0 - S_e)dx = dy + d\left(\frac{V^2}{2g}\right)$$

$$\Rightarrow \qquad (S_0 - S_e) = \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \text{-- } (5.6)$$

To eliminate the velocity derivative, differentiate the continuity equation

$$\frac{dQ}{dx} = 0 = A\frac{dV}{dx} + V\frac{dA}{dx} \text{ -- } (5.6)$$

But dA= Tdy, where T is the channel width at the surface, So Equation (5.7) become

$$\frac{dV}{dx} = -\frac{VT}{A}\frac{dy}{dx}$$

Basic Differential .....

So if we substitute the value dV/dx in equation (5.6) we obtain

$$\frac{dy}{dx}\left(1-\frac{V^2T}{gA}\right)=\left(S_0-S_e\right)$$

From the equation of Froude number we can see that V^2T/gA is the square of the Froude number of the local channel flow. The final desired form of the gradually varied flow equation is $\frac{dy}{dx}=\left(\frac{S_0-S_e}{1-Fr^2}\right)$. ------------------------------ (5.8)

It is also possible to express the basic differential equation of GVF with given Q as

$$\frac{dy}{dx}=\frac{S_0-S_f}{1-\frac{\alpha Q^2T}{gA^3}}$$ --------------------------------- (5.9)

The basic equation of GVF changes its sign according as the Froude number is subcritical or supercritical. The numerator also changes the sign according as S_0 is grater or less than S_e, which become the base for the flow surface profile classification.

Basic Differential .....

From eqn.(5.3) we can to drive another relation for the basic GVF equation, which is called differential –energy equation of GVF. And it is very important for numerical techniques for GVF profile computation

$$\frac{dE}{dx} = S_0 - S_e \quad \text{--} \; (5.10)$$