Classification of Flow Surface Profiles

Fitsume T.

Classification of Flow Surface Profiles

- Bottom slopes are classified as sustaining(S_o>0) and non-sustaining slopes($S_o \leq 0$).

Sustaining slopes

- Mild slope ($Y_o > Y_C$)
- critical slope ($Y_o = Y_C$)
- steep slope ($Y_o < Y_C$)

Non sustaining slopes

- Horizontal slope ($S_O = 0$)
- Adverse slope ($S_O < 0$)

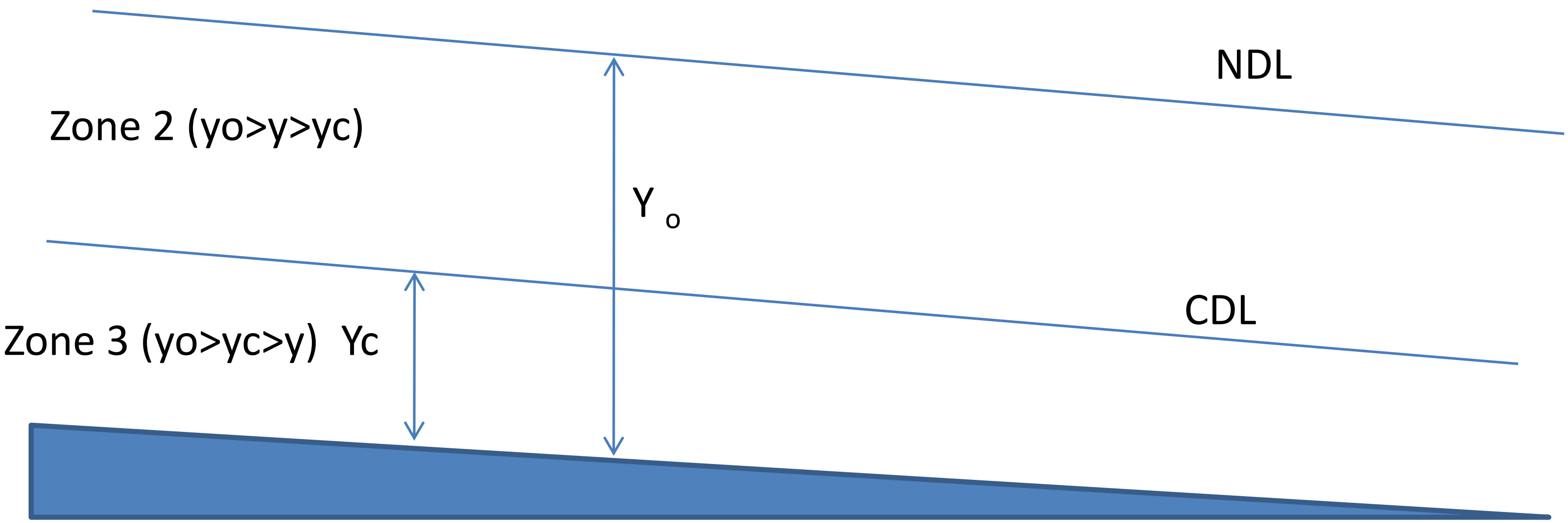
Number	Channel category	Symbol	Characteristic condition	Remark
1	Mild slope	M	$y_0 > y_c$	Subcritical flow at normal depth
2	Steep slope	S	$y_c > y_0$	Supercritical flow at normal depth
3	Critical slope	C	$y_c = y_0$	Critical flow at normal depth
4	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow
5	Adverse slope	A	$S_0 < 0$	Cannot sustain uniform flow

Sustaining slopes($Y_o > Y_c$)

$E_S = Y + V^2/2g$

M

Zone 1 (y>yo>yc)

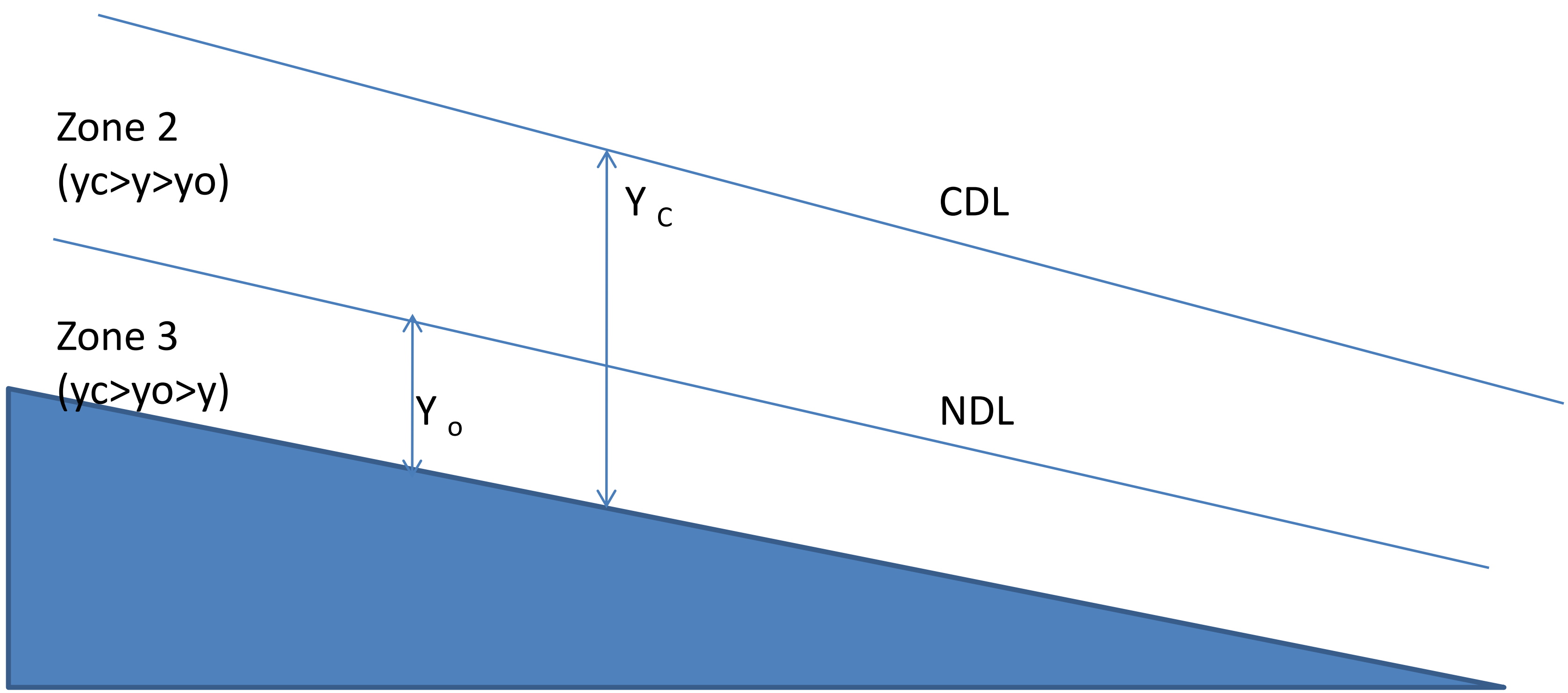


Sustaining slopes($Y_o < Y_c$)

$E_S = Y + V^2/2g$

S

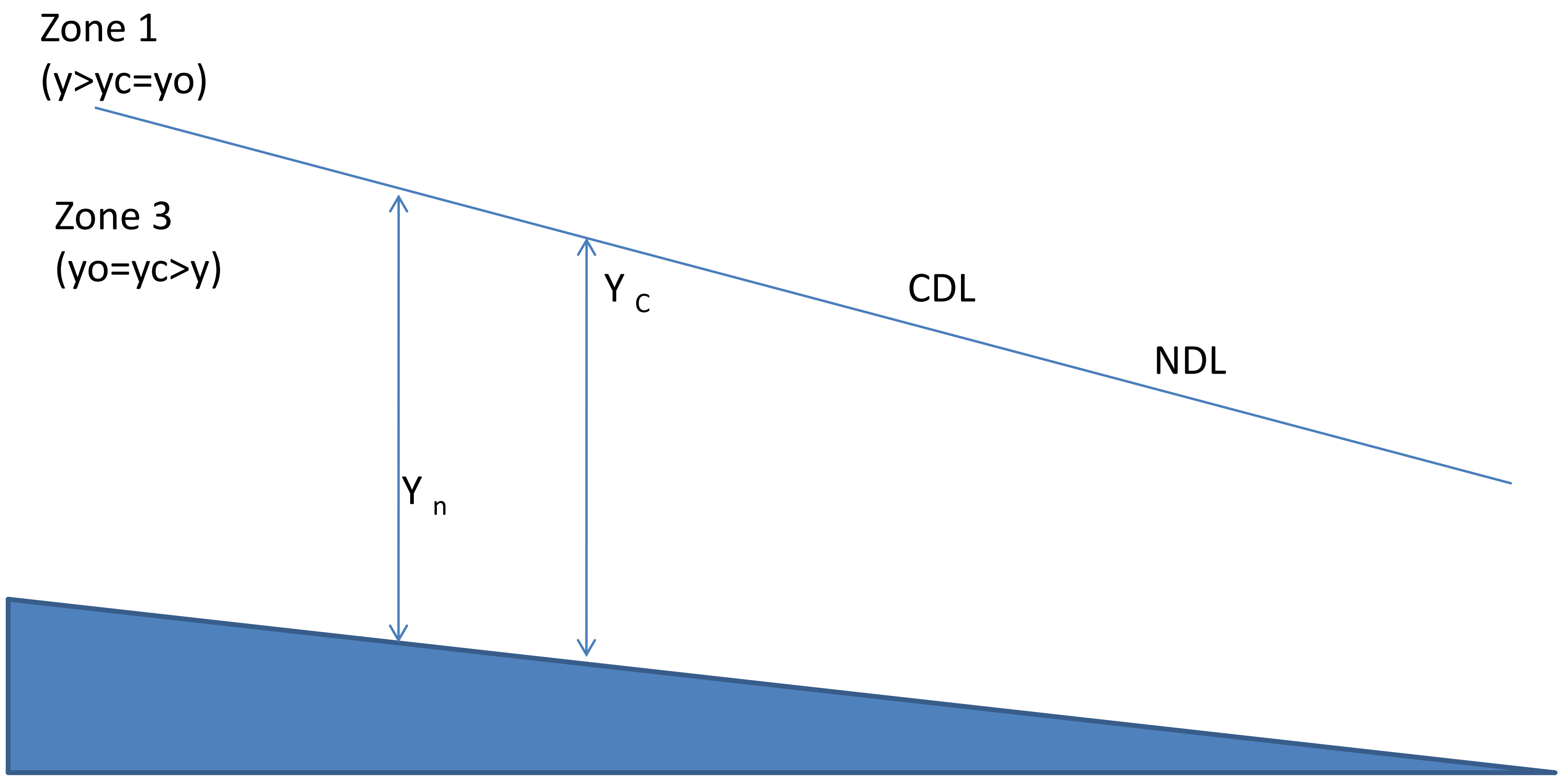
Zone 1 ($y > yc > yo$)



Sustaining slopes(Y_n=Y_c)

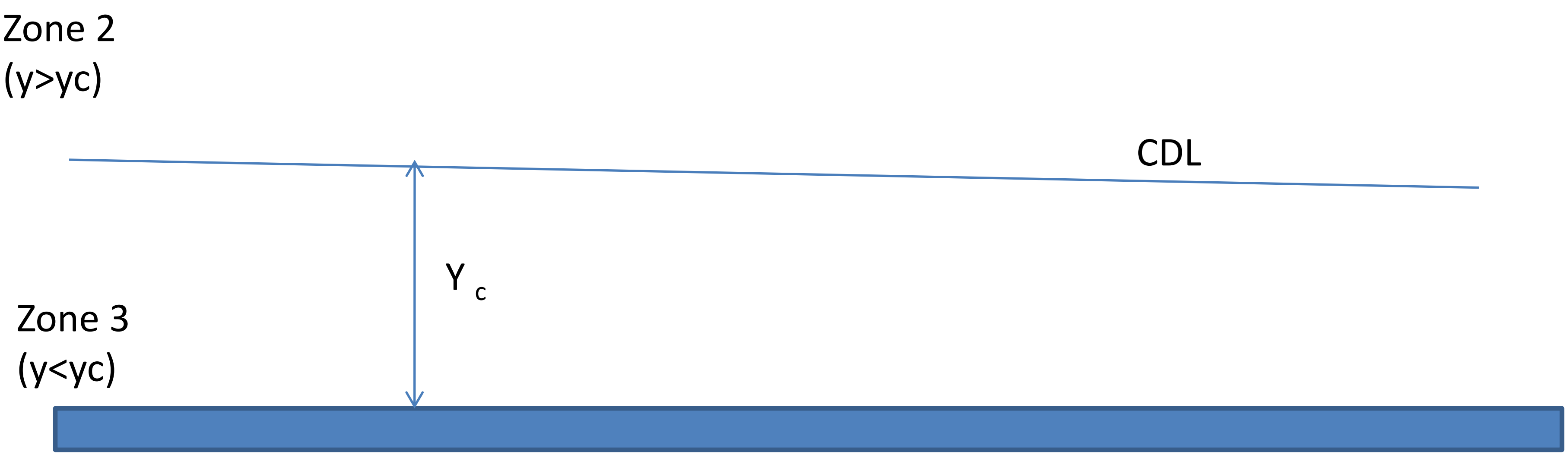
$E_S = Y + V^2/2g$

C



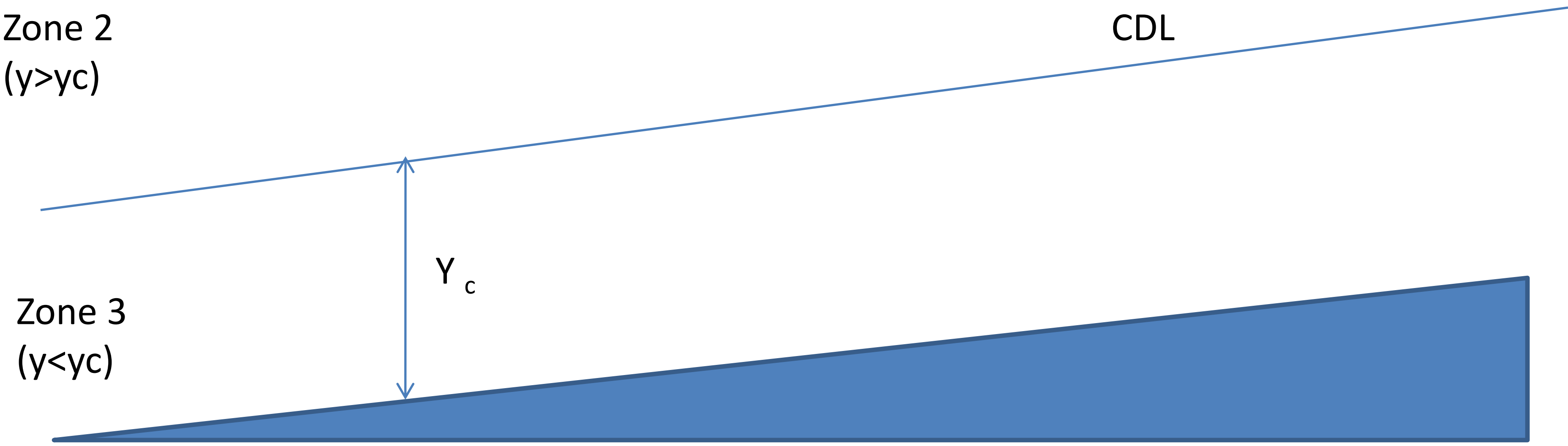
Non Sustaining slopes(S_o=0)

H



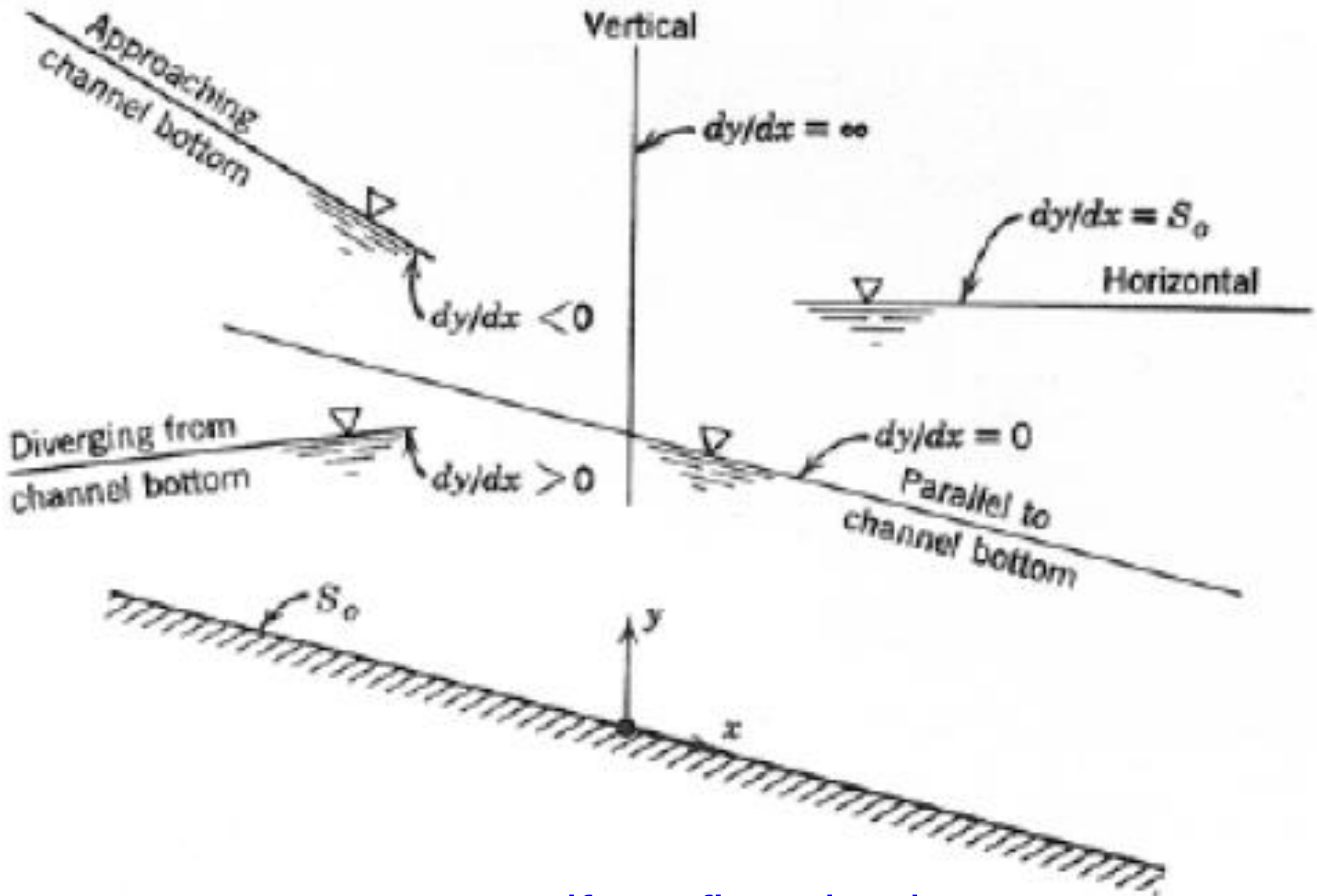
Non Sustaining slopes(S_o=0)

A



- Depending upon the channel category and region of flow, the water surface profiles will have characteristics shapes. Whether a given GVF profile will have an increasing or decreasing water depth in the direction of flow will depend upon the term dy/dx being positive (back water curve) or negative(drawdown curve).

$$\frac{dy}{dx} = \left(\frac{S_0 - S_e}{1 - Fr^2} \right).$$



y = Non-uniform flow depth.

$$y > y_0 \rightarrow S_e < S_0$$

$$y = y_0 \rightarrow S_e = S_0$$

$$y < y_0 \rightarrow S_e > S_0$$

$$y > y_c \rightarrow F_r < 1$$

$$y = y_c \rightarrow F_r = 1$$

$$y < y_c \rightarrow F_r > 1$$

y_0 = Uniform flow depth,

y_c = Critical flow depth,

y = Non-uniform flow depth

1. The water surface approaches the normal depth asymptotically

As $y \rightarrow y_0$, $V \rightarrow V_0$, $S_e = S_0$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{S_0 - S_0}{1 - F_r^2} = \frac{0}{cons} = 0$$

2. The water surface meets the critical depth line vertically.

As $y \rightarrow y_c$, $F_r^2 = 1$, $1 - F_r^2 = 0$,

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0 - S_e}{0} = \infty$$

3. The water surface meets a very large depth as a horizontal asymptote

As $y \rightarrow \infty$, $V = 0 \rightarrow F_r = 0 \rightarrow S_e \rightarrow 0$

$$\lim_{y \to \infty} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0}{1} = S_0$$

Based on this information, the various possible gradually varied flow profiles are grouped into twelve types

Channel Slope	Profile Type	Relation of y to y_n and y_c	S_f	F	Sign dy/dx
Mild	M_1	$y>y_n>y_c$	$<S_0$	<1	+
	M_2	$y_n>y>y_c$	$>S_0$	<1	−
	M_3	$y_n>y_c>y$	$>S_0$	>1	+
Steep	S_1	$y>y_c>y_n$	$<S_0$	<1	+
	S_2	$y_c>y>y_n$	$<S_0$	>1	−
	S_3	$y_c>y_n>y$	$>S_0$	>1	+
Critical	C_1	$y>y_n=y_c$	$<S_0$	<1	+
	C_3	$y_n=y_c>y$	$>S_0$	>1	+
Horizontal	H_2	$y_n>y>y_c$	$>S_0$	<1	−
	H_3	$y_n>y_c>y$	$>S_0$	>1	+
Adverse	A_2	$y>y_c$	$>S_0$	<1	−
	A_3	$y_c>y$	$>S_0$	>1	+

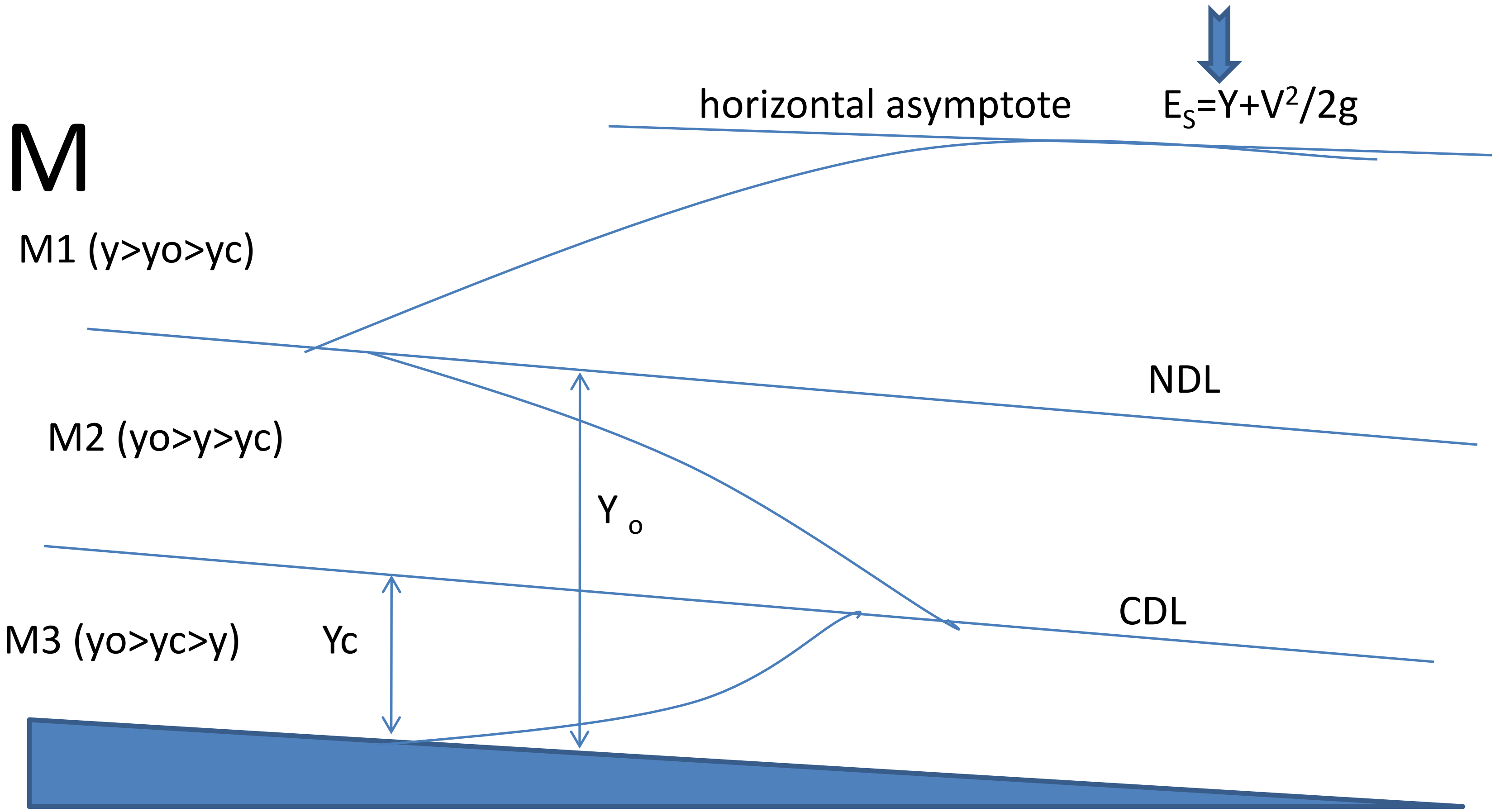
Draw down curve

Back water curve

Example 1

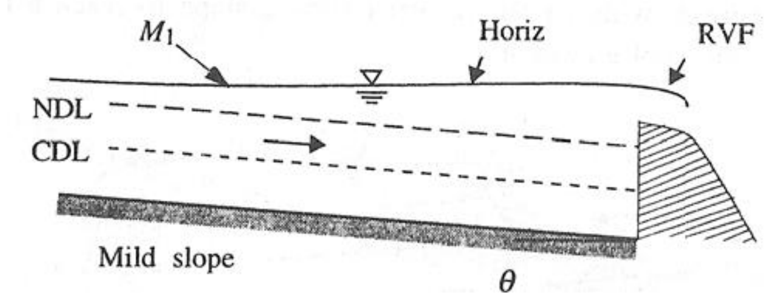
A rectangular channel with a bottom width of 4.0 m and a bottom slope of 0.0008 has a discharge of 1.50 m^3/sec. In a gradually varied flow in this channel, the depth at a certain location is found to be 0.30m. Assuming n = 0.016, Determine the type of GVF profile.

Features of Water Surface Profiles



- **M1 – Curve**

Water depth will increase in the flow direction



- Occurs when obstructions to flow, such as weirs, dams, control structures and natural features, or bends, produce Backwater curves.
- Sub critical flow with $y > y_0 > y_c$ and $Fr < 1 \Rightarrow (1 - Fr^2) > 0$
- Mild slope channel with $S_e < S_0 \Rightarrow S_0 - S_e > 0$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} \rightarrow \frac{dy}{dx} = \frac{+}{+} > 0$$

- water surface for the limit values (∞, y_0) are;

a). $Y \to \infty$, $V \to 0$, $Fr \to 0$, $(1-Fr^2) = 1$ and $Y \to \infty$, $V \to 0$, $Se \to 0$, $(S_o - S_e) = S_o$

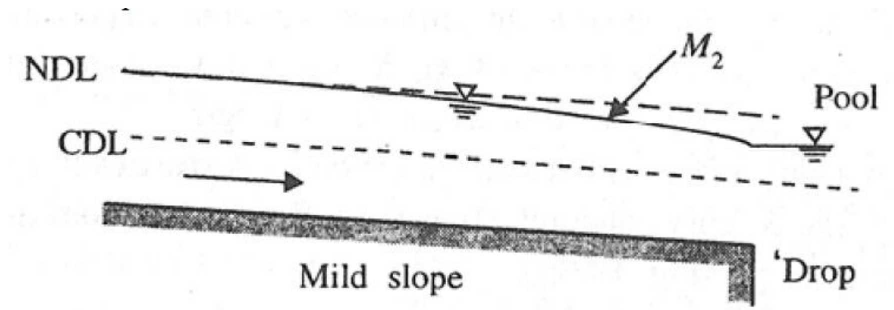
The water surface meets a very large depth as a horizontal asymptote.

b). $Y \to Yo$, $V \to Vo$, $Se \to So$, $(S_o - S_e) = 0$

The water surface approach the normal depth asymptotically

- **M2 – Curve**

Water depth will decrease in the flow direction



- Occurs at sudden drop of the channel, at constriction type of transitions and at the canal outlet into pools
- Water surface will be in Region 2
- Sub critical flow with $y_0 > y > y_c$ and $Fr < 1 \Rightarrow (1 - Fr^2) > 0$
- Mild slope channel with $S_e > S_0 \Rightarrow S_0 - S_e < 0$

$$\frac{dy}{dx} = \frac{S_o - S_e}{1 - Fr^2} = \frac{-}{+} = -$$

- water surface for the limit values (Y_0, Y_c) are;

a). $Y \rightarrow Yo$, $V \rightarrow Vo$, $Se \rightarrow So$, $(S_o - S_e) = 0$

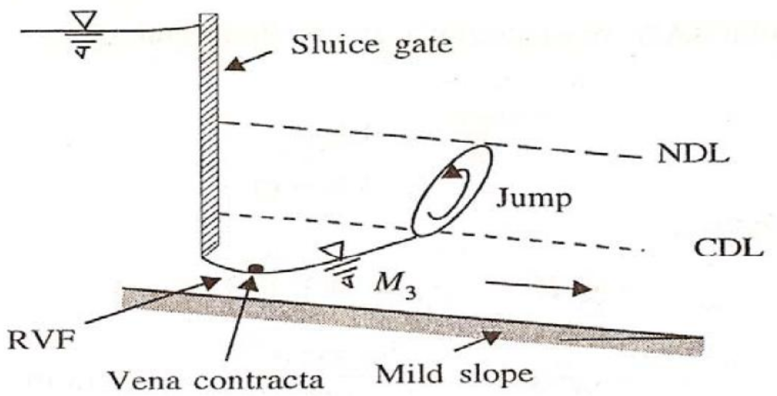
The water surface approach the normal depth asymptotically

b). $Y \rightarrow Yc$, $Fr \rightarrow 1$, $(1 - Fr^2) = 0$

The water surface meets the critical depth line Vertically .

- **M3 – Curve**

Water depth will increase in the flow direction



- Occurs when supercritical streams enters a mild slope channel .
- The flow is leading from a spillway or a sluice gate to a mild slope forms
- supercritical flow with $y_0 > y_c > y$ and $Fr > 1 \Rightarrow (1 - Fr^2) < 0$
- Mild slope channel with $S_e > S_0 \Rightarrow S_0 - S_e < 0$

$$\frac{dy}{dx} = \frac{S_o - S_e}{1 - Fr^2} = \frac{-}{-} = +$$

- water surface for the limit values (Y_0, Y_c) are;

a). Y→Yc, Fr =1, $(1-Fr^2)= 0$

The water surface meets the critical depth line Vertically .

b). Y→0 , V →∞, Se →So , $(S_o - S_e) = \infty$

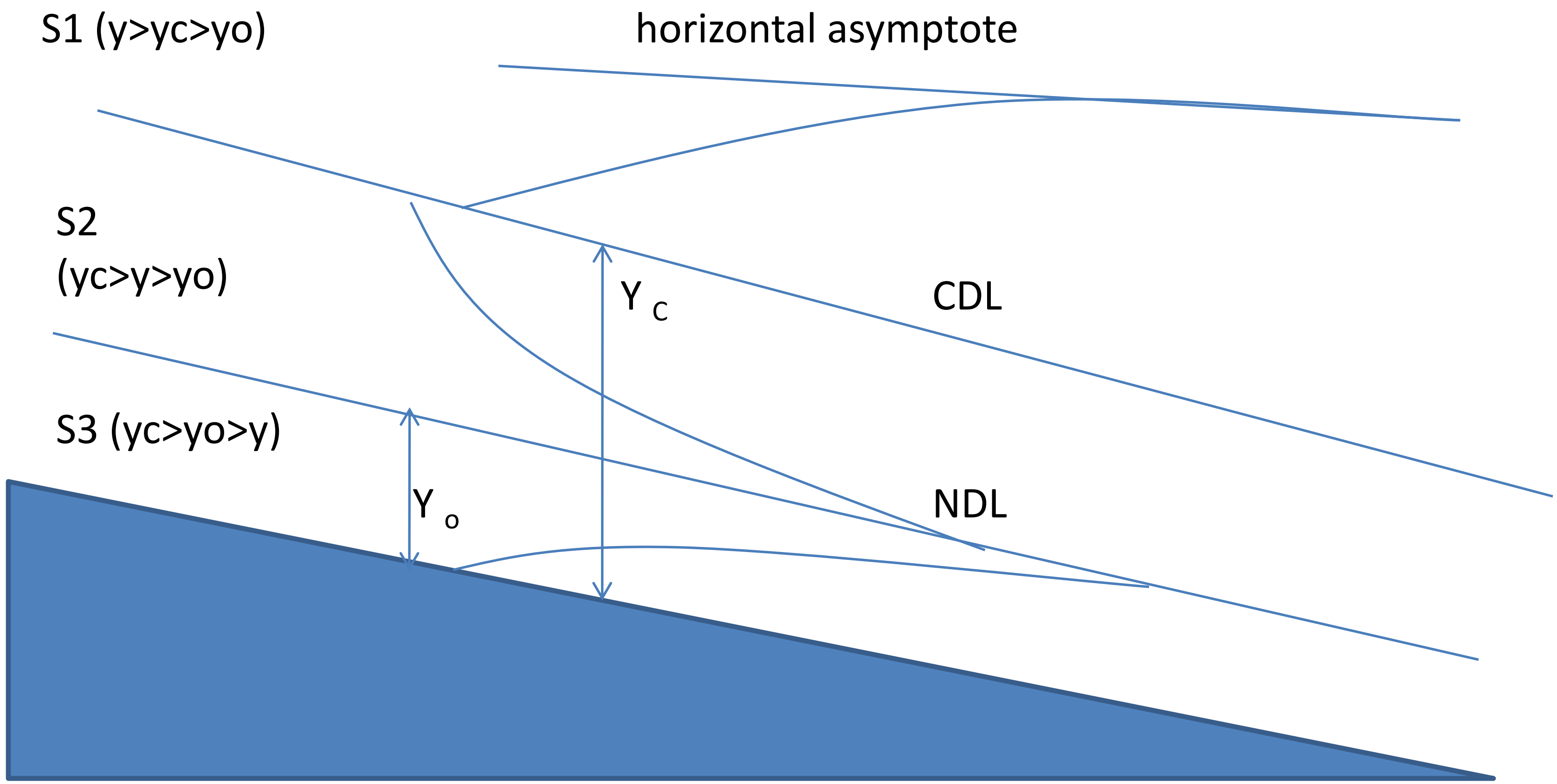
The water surface approach the bed with some angel, it may be taken as

$$S_o\left(\frac{y_o}{y_c}\right)^3$$

Features of Water Surface Profiles

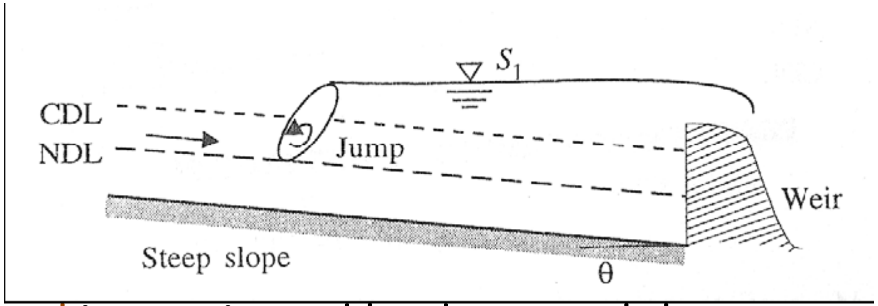
$E_S = Y + V^2/2g$

S



- **S1 – Curve**

Water depth will increase in the flow direction



- produced when flow from steep channel is terminated by deep pool that created by obstruction like weirs, or dams,
- At the beginning of the curve the flow changes from supercritical to subcritical flow through a hydraulic
- Supercritical flow with $y > y_c > y_0$ and $Fr > 1 \Rightarrow (1 - Fr^2) < 0$
- Step slope channel with $S_e > S_0 \Rightarrow S_0 - S_e < 0$

$$\frac{dy}{dx} = \frac{S_o - S_e}{1 - Fr^2} = \frac{-}{-} = +$$

- water surface for the limit values (∞, y_0) are;

a). $Y \rightarrow \infty$, $V \rightarrow 0$, $Fr \rightarrow 0$, $(1-Fr^2) = 1$ and $Y \rightarrow \infty$, $V \rightarrow 0$, $Se \rightarrow 0$, $(S_o - S_e) = S_o$

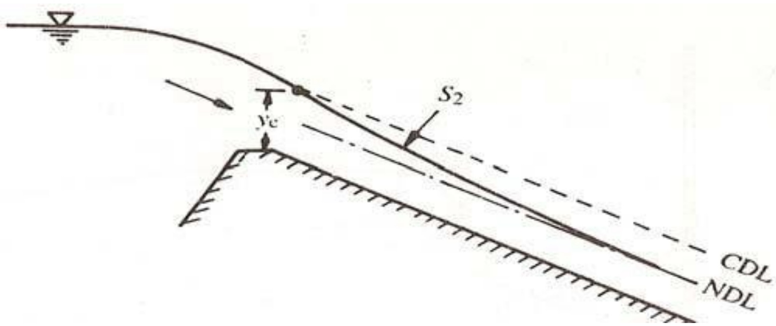
The water surface meets a very large depth as a horizontal asymptote.

b). $Y \rightarrow Yc$, $Fr \rightarrow 1$, $(1 - Fr^2) = 0$

The water surface meets the critical depth line Vertically

- **S2 – Curve**

Water depth will decrease in the flow direction



- Occurs at entrance region of Steep Channel leading from a reservoir and a brake grade
- Water surface will be in Region 2
- Sub critical flow with $y_c > y > y_o$ and $Fr > 1 \Rightarrow (1 - Fr^2) < 0$
- Steep slope channel with $S_e > S_0 \Rightarrow S_0 - S_e > 0$

$$\frac{dy}{dx} = \frac{S_o - S_e}{1 - Fr^2} = \frac{+}{-} = -$$

- water surface for the limit values (Y_0, Y_c) are;

a). Y→Yc, Fr →1, (1-Fr²)= 0

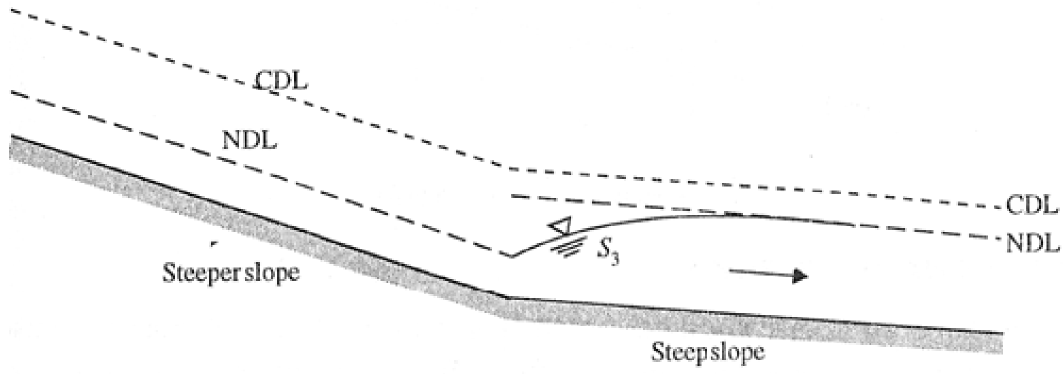
The water surface meets the critical depth line Vertically .

a). Y→Yo , V →Vo, Se →So , ($S_o - S_e$)=0

The water surface approach the normal depth asymptotically

- **S3 – Curve**

Water depth will increase in the flow direction



- Occurs when free flowfrom a sluice gate
- supercritical flow with $y_c > y_o > y$ and $Fr > 1 \Rightarrow (1 - Fr^2) < 0$
- Steep slope channel with $S_e > S_0 \Rightarrow S_0 - S_e < 0$

$$\frac{dy}{dx} = \frac{S_o - S_e}{1 - Fr^2} = \frac{-}{-} = +$$

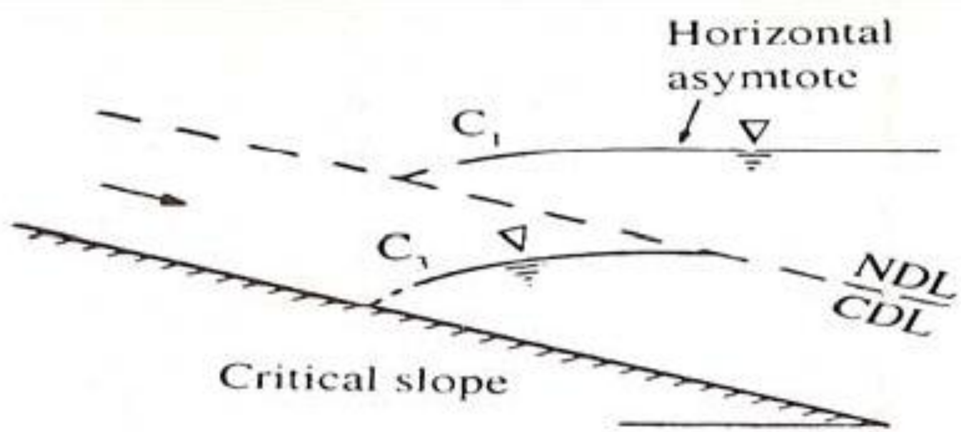
- water surface for the limit values (Y_0, Y_c) are;

Y→0 , V →∞, Se →So , ($S_o - S_e$)=∞

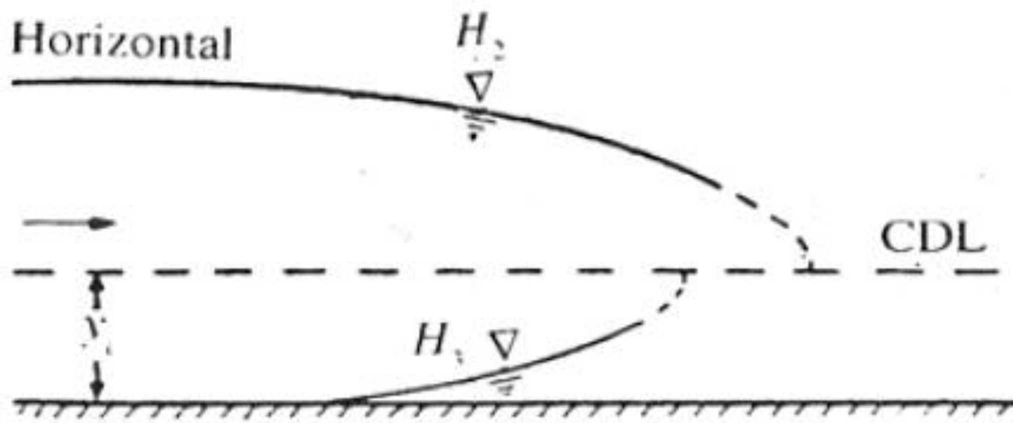
$$S_o\left(\frac{y_o}{y_c}\right)^3$$

The water surface approach the bed with some angel, it may be taken as

C – Curves



H – Curves



EXAMPLE 2

- A rectangular channel 6m wide conveys 100 m3/sec of water. The channel slope is 0.003 for the first reach and then a sudden change in the slope to 0.01 in the second reach. The manning n for the channel is 0.015.Sketch the water-surface profile in the channel.

Assignment 3

- Sketch the flow profile if the slopes in the first and second reaches of the channel in the example are interchanged.

Features of Water Surface Profiles

Control Sections

- A *control section is defined as a section in which **a fixed relationship** exists between the **discharge** and* **depth** of flow
  - *Weirs, spillways, sluice gates are some typical examples of structures which give rise to control sections.*
  - *The critical depth is also a control point. However, it is effective in a flow profile which changes from subcritical to supercritical flow.*
  - *In the reverse case of transition from supercritical flow to subcritical flow, a hydraulic jump is usually formed by passing the critical depth as a control point.*

Analysis of Flow Profile

- To determine the resulting water surface profile in a given case, one should be in a position to analyze the effects of various channel sections and controls connected in series.
  - **A break in grade from a mild channel to a milder channel**
    - It is necessary to first draw the critical-depth line (CDL) and the normal-depth line (NDL) for both slopes.
    - Since yc does not depend upon the slope for a taken Q = discharge, the CDL is at a constant height above the channel bed in both slopes.
    - The normal depth y_{01} for the mild slope is lower than that of the milder slope (y_{02}).
  - **Serial Combination of Channel Sections**
    - Draw the longitudinal section of the system.
    - Calculate the critical depth and normal depths of various reaches and draw the CDL and NDL in all reaches.
    - Mark all the controls, both the imposed as well as natural controls.
    - Identify the possible profiles.

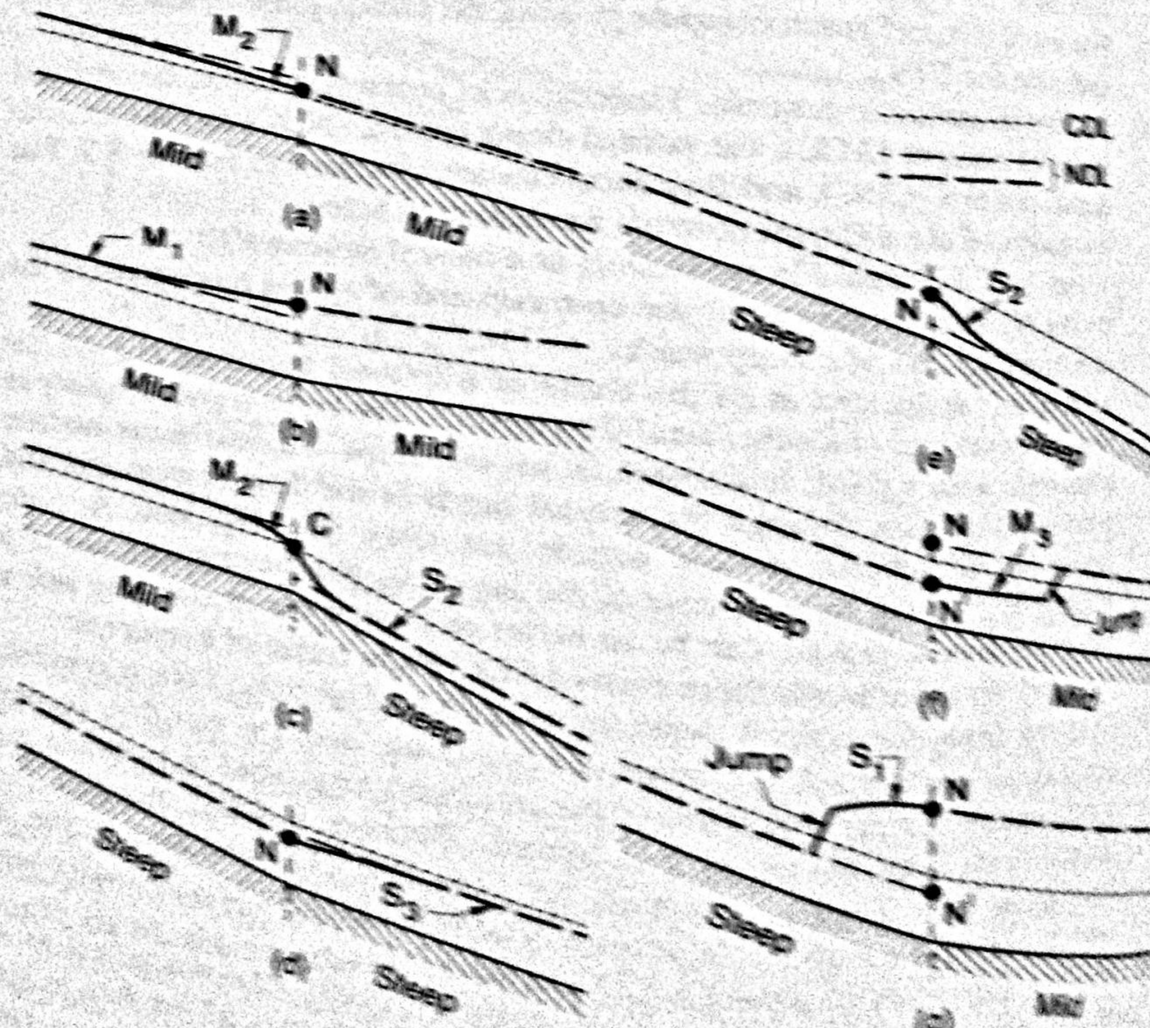


Figure 4-3. Flow profiles in channels with change in slope and roughness

Table 4-2. Types of Flow Profile in Prismatic Channels with a Change of Slope and Roughness

Reach 1 Slope	Reach 2 Slope	Possible Control(s)	Figure No.	Relative Magnitudes of Normal Depths and Specific Energies	Active Control	Flow Profiles	
						Reach 1	Reach 2
Mild	Mild	NDC at N CDC at C[#]	4-3a	$y_{n1} > y_{n2}$; $E_{n1} > E_{n2}$	NDC at N	M_2	UF
			4-3b	$y_{n2} > y_{n1}$; $E_{n2} > E_{n1}$	NDC at N	M_1	UF
Mild	Steep	CDC at C	4-3c	$y_{n1} > y_{n2}$ $E_{n1} > E_{n2}$ * or $E_{n1} < E_{n2}$ **	CDC at C	M_2	S_2
Steep	Steep	NDC at N	4-3d	$y_{n2} > y_{n1}$; $E_{n1} > E_{n2}$	NDC at N	UF	S_3
			4-3e	$y_{n1} > y_{n2}$; $E_{n2} > E_{n1}$	NDC at N	UF	S_2
Steep	Mild	NDC at N NDC at N′	4-3f	$y_{n2} > y_{n1}$; $(E_{n1} - \Delta E_{j1}) > E_{n2}$	NDC at N′	UF	M_3 & J
			4-3g	$y_{n2} > y_{n1}$; $E_{n2} > (E_{n1} - \Delta E_{j1})$	NDC at N	J & S_1	UF

Legend: J = Jump; UF = Uniform flow; ΔE_{j1} = Energy loss in the jump in reach 1.
*The loss of the specific energy within the M_2 curve is more than the gain of the specific energy within the S_2 curve.
**The loss of the specific energy within the M_2 curve is less than the gain of the specific energy within the S_2 curve.
[#] CDC is not shown in Figures 4-3a and 4-3b.