Chapter Four : UNIFORM FLOW

- Concept of Uniform Flow
- Establishment of Uniform Flow
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  - MANNING's EQUATION
- Uniform Flow Computation
  - Discharge and Velocity
  - Hydraulic Radius
  - Normal Depth
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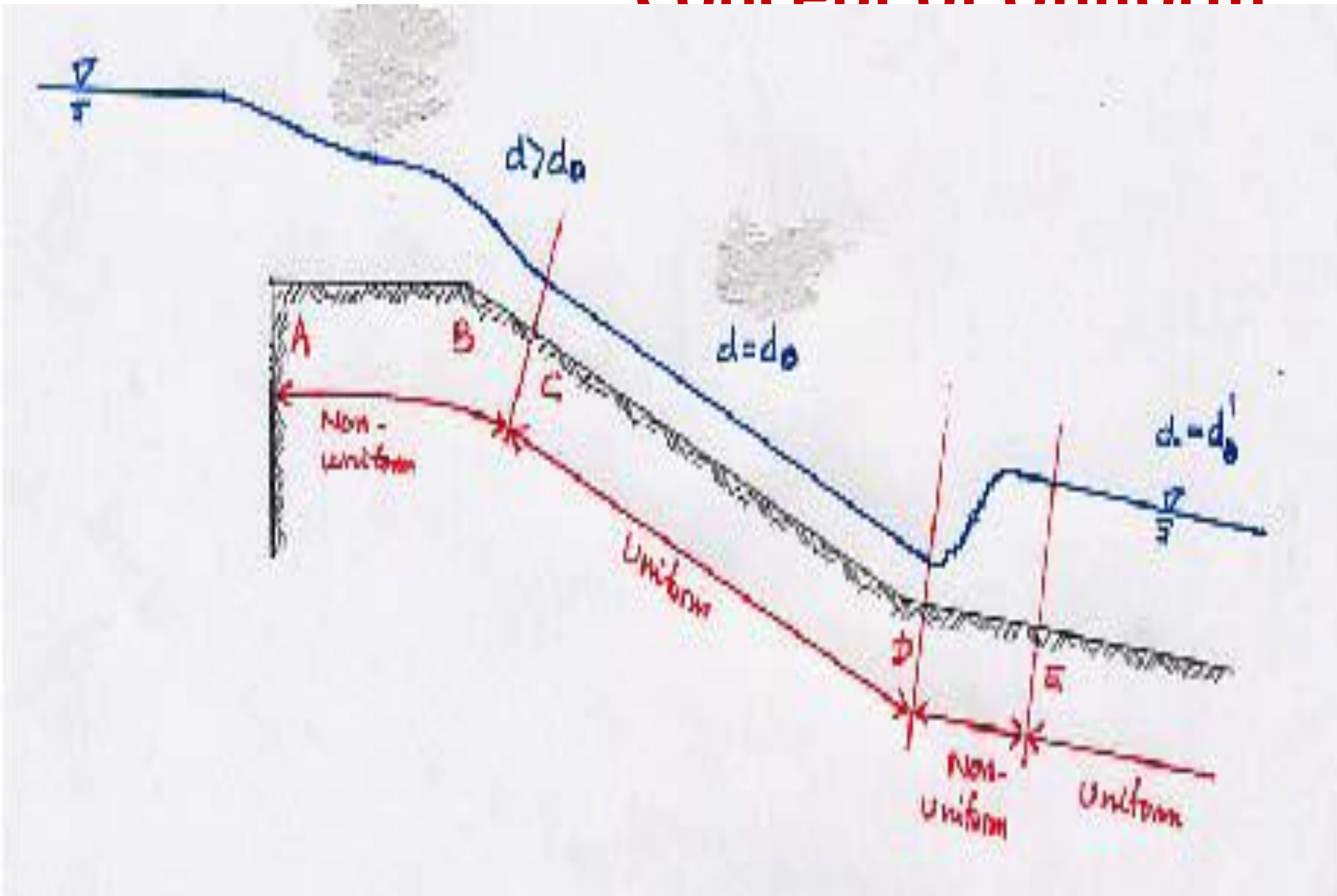
  - Compound section
  - Critical slope
- Design of Irrigation Canal

Concept of Uniform Flow

- Uniform flow is referring the **steady uniform flow**
- Steady flow is characterized by no changes in time.
- Uniform flow is characterized by the **water cross section and depth remaining constant** over a certain ***reach of the channel.***
- For any channel of given roughness, cross section and slope, there exists one and only one water depth, called the **normal depth , at which the flow will be uniform.**
- Uniform equilibrium flow can occur only in a straight channel with a constant channel slope and cross-sectional shape, and a constant discharge.
- The energy grade line S_f, water surface slope S_w and channel bed slope S_0 are all parallel, i.e. S_f=S_w=S_o

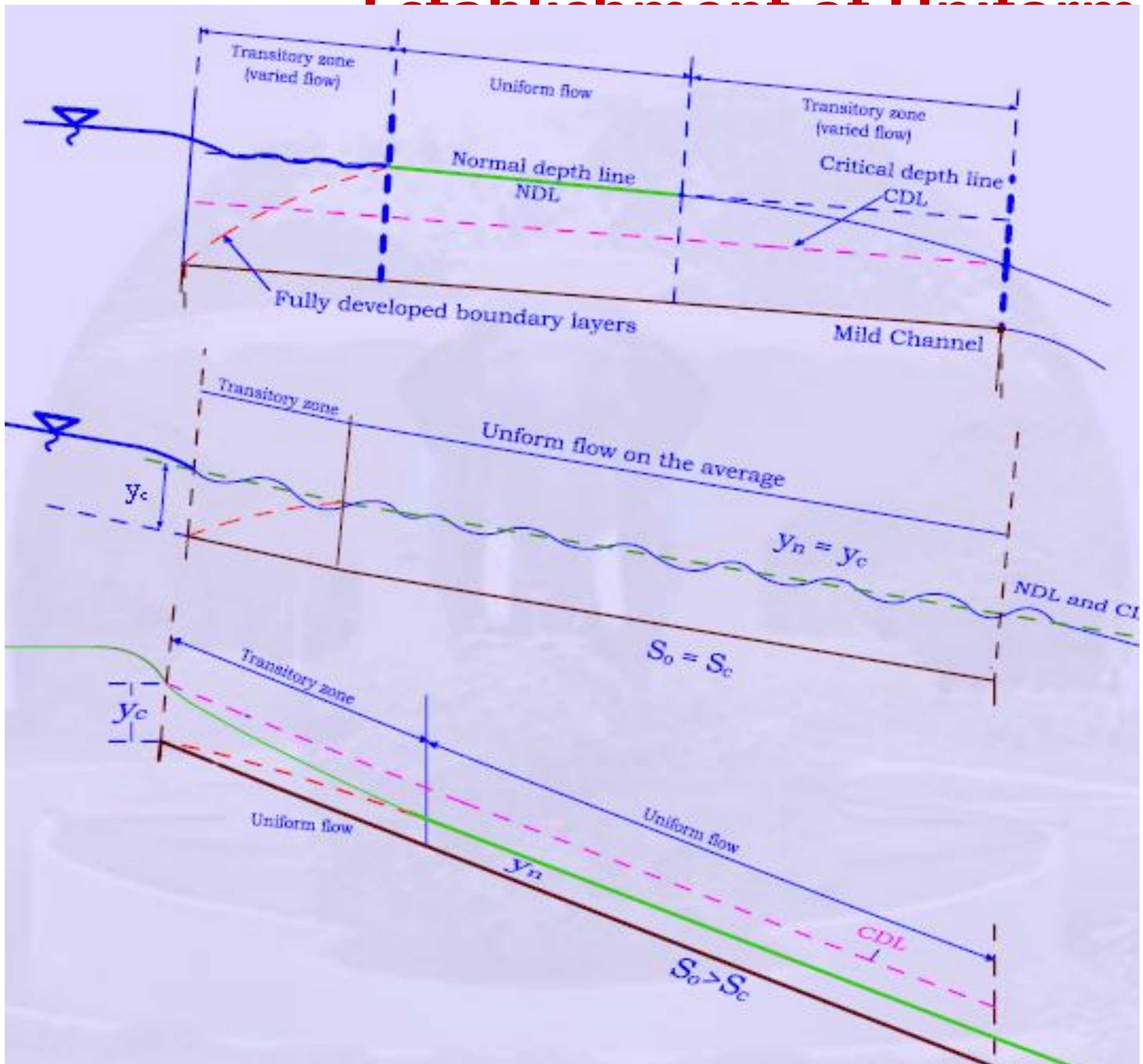
Concept of Uniform



Establishment of Uniform Flow

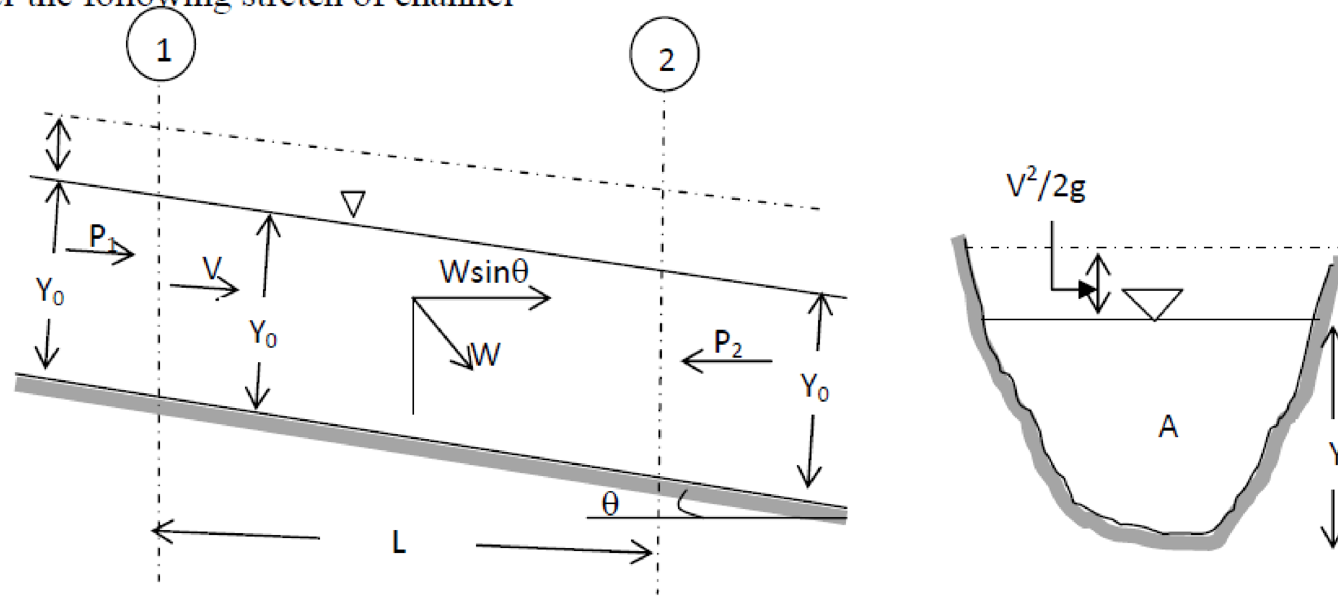
- When flow occurs in an open channel, resistance is encountered by the water as it flows downstream
- A uniform flow will be developed if the resistance is balanced by the gravity forces.
- If the water enters into a channel slowly, the velocity and the resistance are small, and the resistance is out balanced by the gravity forces, resulting in an accelerating flow in the upstream reach.
- The velocity and the resistance will gradually increase until a balance between resistance and gravity forces is reached. At this moment and afterward the flow becomes uniform.
- The upstream reach that is required for the establishment of uniform flow is known as the ***transitory zone***

Establishment of Uniform…



The Chezy Formula

Consider the following stretch of channel



- By definition there is no acceleration in uniform flow
- By applying the momentum equation to control volume encompassing sections 1 and 2, distance L apart as shown in the figure

The Chezy .....

$$P_1 - W\sin\theta - F_f - P_2 = M_2 - M_1 \text{ ------------------ } (4.1)$$

Where: P_1 and P_2 are the pressure forces and M_1 and M_2 are the momentum fluxes at section 1 and 2 respectively

W= weight to fluid in the control volume and

F_f = shear force at the boundary

Since the flow is uniform $P_1 = P_2$ and $M_1 = M_2$ also $W = \gamma AL$ and $F_f = \tau_0 PL$

Where τ_0 = average shear stress on the wetted perimeter of length P

γ= unit weight of water

Replacing sinθ by S0 (bottom slope) equation (4.1) become

$$\gamma ALS_0 = \tau_0 PL \Rightarrow \tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0 \text{ ----------------- } (4.2)$$

Where R= A/P = hydraulic radius , which is a length parameter accounting for the shape of the channel. And it plays a very important role in developing flow equations which are common to all shapes of channels.

The Chezy .....

$\tau_0 = k\rho V2,$

$$k\rho V^2 = \gamma R S_0 \Rightarrow V = C\sqrt{RS_0} \text{ ------------------------------ } (4.3)$$

The equation expressed in eq (4.3) is called Chezy Equation and the coefficient also called Chezy coefficient

Where $C = \sqrt{\frac{\gamma}{\rho}\frac{1}{k}}$ = a coefficient which depends on the nature of the surface

We know the Darcy –Weisbach equation in the pipe flow expressed as

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Where h_f = head loss due to friction in a pipe of diameter D and length L

f = Darcy-Weisbach friction factor

The Chezy .....

- For smooth pipes, f is found to be a function of the Reynolds number (Re= *VD/v) only.*
- For rough turbulent flows, f is a function of the relative roughness (ε/D) and types of roughness, which independent of the Reynolds number.
- Open Channel, we can be considered to be a conduit cut into two

$$h_f = f\frac{L}{4R}\frac{V^2}{2g} \Rightarrow V = \sqrt{\frac{8g}{f}}.\sqrt{R}.\sqrt{h_f / L} \quad \text{------------------} (4.4)$$

Note that for uniform flow in an open channel h_f/L = slope of the energy line = $S_f = S_0$,

$$C = \sqrt{8g / f} \quad \text{--} (4.5)$$

$$\left(\mathrm{Re} = \frac{4RV}{\nu}\right) and \left(\frac{4R}{\varepsilon_s}\right)$$

$$\frac{1}{\sqrt{f}} = 1.80 \log \mathrm{Re} - 1.5146 \quad \text{--} \quad (4.6)$$

And
$$\frac{1}{\sqrt{f}} = 1.14 - 2.0 \log\left(\frac{\varepsilon_s}{4R} + \frac{21.25}{\mathrm{Re}^{0.9}}\right) \quad \text{--} \quad (4.7)$$

Equation (4.7) is valid for $5000 \leq \mathrm{Re} \leq 10^8$ and $10^{-6} < \varepsilon_s/4R < 10^{-2}$

S.No	Surface	Equivalent Roughness (ε_s) in mm
1	Glass	3×10^{-4}
2	Very smooth concrete surface	0.15 – 0.30
3	Rough concrete	3.0 - 4.5
4	Earth Channels (straight, uniform)	3.0
5	Rubble masonry	6.0
6	Untreated channel	3.0 -10.0

Example 3.1:

A 2.0m wide rectangular channel carries water at 20oc at a depth of 0.5m. The channel is laid on a slope of 0.0004. Find the hydrodynamic nature of the surface if the channel is made of

A. Very smooth concrete

B. Rough concrete

C. Estimate the discharges in the channel in both case using chezy formula with Dancy-Weisbach f.

The MANNING'S Formula

The simplest resistance formula and the most widely used equation for the mean velocity calculation is the **Manning equation which has been derived by Robert Manning (1890)** ***by*** analyzing the experimental data obtained from his own experiments and from those of others. His equation is,

$$V = \frac{1}{n} R^{2/3} S_o^{1/2} \text{ -- (4.8)}$$

Where V = mean velocity

R= Hydraulic Radius

So = channel slope

n = Manning's roughness coefficient

if we equating Equations. (4.3) and (4.8), we get

$$CR^{1/2} S_o^{1/2} = \frac{1}{n} R^{2/3} S_o^{1/2}$$

$$C = \frac{R^{1/6}}{n} \text{ -- (4.9)}$$

Similarly we can be equating equations (4.5) and (4.9) we get

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n} \Rightarrow f = \left(\frac{n^2}{R^{1/3}}\right)(8g) \text{ ------------------------ (4.10)}$$

OTHERS RESISTANCE FORMULAE

1. Pavlovsik formula: $C = \dfrac{R^x}{n}$ in which $x = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}\left(\sqrt{n} - 0.10\right)$ and n= manning's coefficient. This formula appears to be in use in Russia
2. Bazin's formula $C = \dfrac{87.0}{1 + M/R}$, in which M = a coefficient dependent on the surface roughness
3. Ganguillet and Kutter Formula $C = \dfrac{23 + \dfrac{1}{n} + \dfrac{0.00155}{S_o}}{1 + \left[23 + \dfrac{0.00155}{S_o}\right]\dfrac{n}{\sqrt{R}}}$, in which n= manning's coefficient

MANNING'S Roughness Coefficient (n)

Factors Affecting Manning's Roughness Coefficient

- Surface Roughness
- Vegetation
- Channel Irregularity
- Channel Alignment
- Silting and Scouring
- Obstruction
- Size and Shape of the Channel
- Stage and Discharge
- Seasonal Change

Determination of Manning's Roughness Coefficient

– **Cowan Method**

$$n = (n_0 + n_1 + n_2 + n_3 + n_4) \times m \text{ --- } (4.11)$$

Where: n_0 is a basic value for straight, uniform, smooth channel in the natural materials involved,

n_1 is a value added to n_0 to correct for the effect of surface irregularities,

n_2 is a value for variations in shape and size of the channel cross-section,

n_3 is a value of obstructions,

n_4 is a value for vegetation and flow conditions, and

m is a correction factor for meandering of channel.

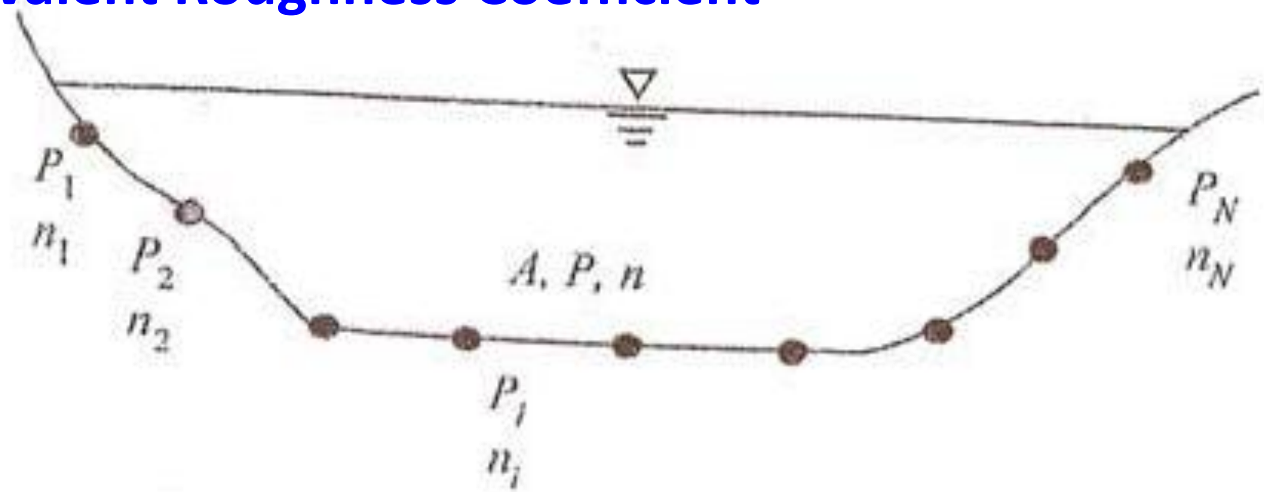
– **Empirical Formulae for n**

$$n = \frac{d_{50}^{1/6}}{21.1} \text{ --- } (4.12)$$

$$n = \frac{d_{90}^{1/6}}{26} \text{ --- } (4.13)$$

…….Coefficient (n)

Equivalent Roughness Coefficient



No	Investigators	n_e	Concept
1	Horton (1933); Einstein (1934)	$= \left[\frac{1}{P}\sum\left(n_i^{3/2}P_i\right)\right]^{2/3}$	Mean Velocity is constant in all subareas
2	Pavloskii (1931), Muhlhofer(1933) Einstein and Banks (1950)	$= \left[\frac{1}{P}\sum\left(n_i^2 P_i\right)\right]^{1/2}$	Total resistance force F is sum of subarea resistance force, ΣF_i
3	Lotter (1932)	$= \frac{PR^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}}$	Total discharge is sum of subarea discharge
4	Yen (1991)	$= \frac{\sum\left(n_i P_i\right)}{P}$	Total shear velocity is weighted sum of subarea shear velocity

Example 3.2

An earthen trapezoidal channel ($n = 0.025$) has a bottom width of 5.0 m, side slopes of 1.5 horizontal: 1 vertical and a uniform flow depth of 1.10 m. In an economic study to remedy excessive seepage from the canal two proposals,

a) to line the sides only and,

b) to line the bed only are considered.

If the lining is of smooth concrete ($n = 0.012$), calculate the equivalent roughness in the above two cases.

UNIFORM FLOW COMPUTATION

- **Discharge and Velocity**
- **Hydraulic Radius**
- **Normal Depth**
- **Hydraulically Efficient Cross-section**
- **Compound section**
- **Critical slope**

Problem Type	Given	Required
1	y_0, n, S_0, Geometric elements	Q and V
2	Q, y_0, n, Geometric elements	S_0
3	Q, y_0, S_0, Geometric elements	n
4	Q, n, S_0, Geometric elements	y_0
5	Q, y_0, n, S_0, Geometry	Geometric elements

Discharge and Velocity

Chezy Equation

$$V = C\sqrt{RS_0}\,, \quad Q = CA\sqrt{RS_o}$$

Manning equation $$V = \frac{1}{n} R^{2/3} S_o^{1/2}$$

$$Q = \frac{1}{n} A R^{2/3} S_o^{1/2} \Rightarrow Q = K\sqrt{S_o}$$

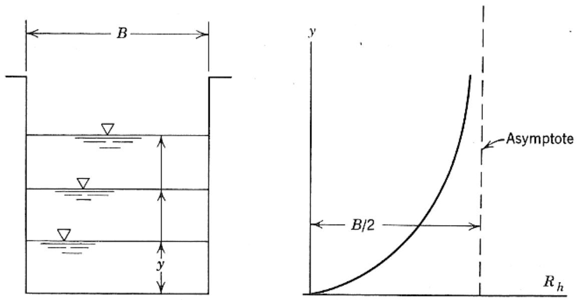
Where ***K= 1/nAR^2/3*** is called Conveyance of the channel

Or

If equating it as, nk= $AR^{2/3}$, it is called the section factor for uniform, flow

Hydraulic Radius

Deep Channel



$$A = By\,,\, P = B + 2y$$

$$R = \frac{A}{P} = \frac{By}{B + 2y}$$

$$R = \frac{B}{\frac{B}{y} + 2}$$

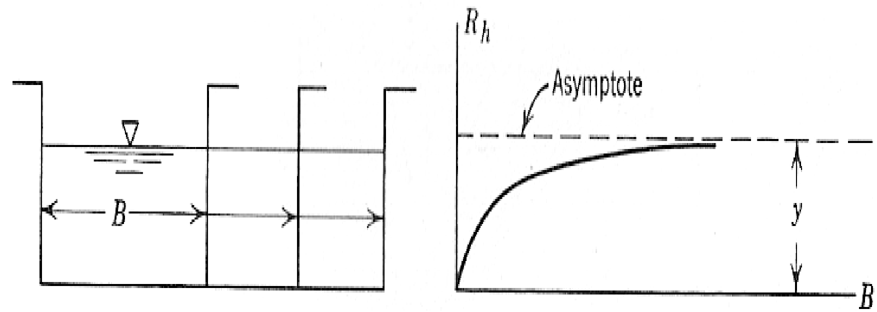
$$y = 0 \rightarrow R = 0$$

$$y \rightarrow \infty, R = \frac{B}{2}$$

for narrow deep cross-sections

R≈ B/2.

Wide Channel



$$R = \frac{By}{B + 2y}$$

$$R = \frac{y}{1 + \frac{2}{yB}}$$

$$B = 0 \rightarrow R = 0$$

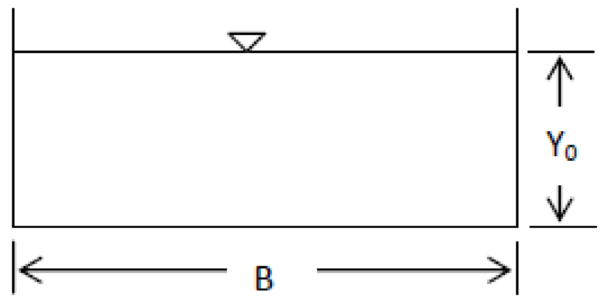
$$B \rightarrow \infty, R \rightarrow y$$

Wide ***shallow rectangular cross-sections***

R≈y

Normal Depth

Rectangular Channel



Area $A = By_o$

Wetted Perimeter $P = B + 2y_o$

Hydraulic Radius $R = A/P$

$$R = \frac{y_o}{1 + 2\frac{y_o}{B}}$$

wide Rectangular Channel ($y_0/B < 0.02$)

- Considering a unit width of a wide rectangular channel,

  $B = 1.0, \Rightarrow A = y_o$, and $Q/B = q$

  $R = y_o$

$$q = \frac{1}{n} y_o^{5/3} S_o^{1/2} \Rightarrow y_o = \left[\frac{qn}{\sqrt{S_o}}\right]^{3/5}$$

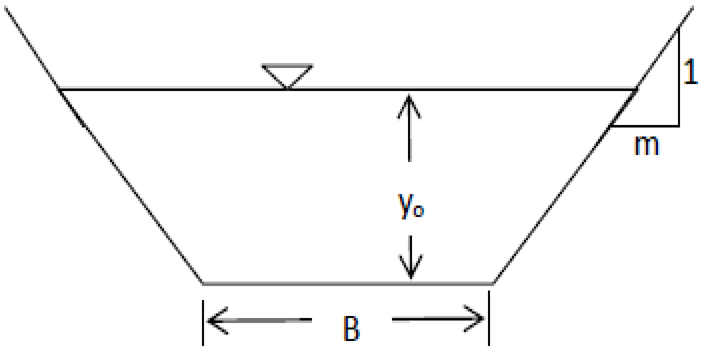
Rectangular Channel with $y_0/B > 0.02$)

$$\frac{Qn}{\sqrt{S_o}} = AR^{2/3}, \quad \text{and} \quad AR^{2/3} = \frac{(By_o)^{5/3}}{(B+2y_o)^{2/3}} = \frac{(y_o/B)^{5/3}}{(1+2y_o/B)^{2/3}} B^{8/3}$$

$$\frac{Qn}{\sqrt{S_o} B^{8/3}} = \frac{AR^{2/3}}{B^{8/3}} = \frac{(\eta_o)^{5/3}}{(1+2\eta_o)^{2/3}} = \phi(\eta_o) \qquad \text{Where } \eta_0 = \frac{y_0}{B}$$

Normal Depth

Trapezoidal Channel



Area = A = (B + myo)yo
Wetted Perimeter = P= (B +2yo√m2+1)
Hydraulic Radius

$$R = \frac{A}{P} = \frac{(B + my_o)y_o}{\left(B + 2\sqrt{m^2 + 1}y_o\right)}$$

$$\frac{Qn}{\sqrt{S_o}} = AR^{2/3} = \frac{(B + my_o)^{5/3} y_o^{5/3}}{\left(B + 2\sqrt{m^2 + 1}y_o\right)^{2/3}}$$

$$\frac{AR^{2/3}}{B^{8/3}} = \frac{Qn}{\sqrt{S_o}B^{8/3}} = \frac{(1 + m\eta_o)^{5/3} \eta_o^{5/3}}{\left(1 + 2\sqrt{m^2 + 1}y_o\right)^{2/3}} = \phi(\eta_o, m)$$

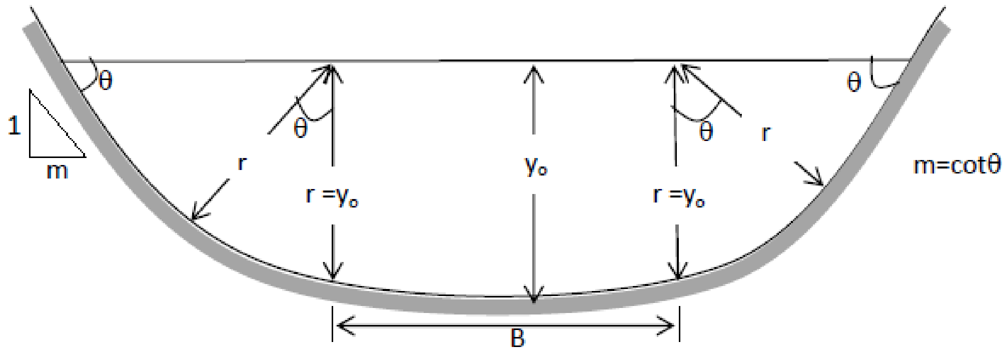
Where $\eta_0 = \frac{y_0}{B}$

Normal Depth

Lined Channel

Indian Standards **(IS: 4745 -1968)** consists of two standardized lined canal section

- Trapezoidal > 55m3/sec
- Triangular < 55m3/sec).



For standard trapezoidal case

Area = A = $By_o + my_o^2 + y_o^2\theta = (B + y_o\varepsilon)y_o$, where $\varepsilon = m + \theta = \left(m + \tan^{-1}\dfrac{1}{m}\right)$

Wetted perimeter =P = $B + 2my_o + 2y_o\theta = B + 2y_o\varepsilon$

Hydraulics radius =R= A/P = $\dfrac{(B + y_0\varepsilon)y_o}{B + 2y_o\varepsilon}$

$$Q = \frac{1}{n}(\varepsilon y_o^2)(y_o/2)^{2/3} S_o^{1/2}$$

Example 3.3

A standard lined trapezoidal cannel section is to be designed to convey 100m^3/sec of flow. The side slope is to be 1.5H:1V and the manning's coefficient n= 0.016. The longitudinal slope of the bed is 1 in 5000m. If a bed width of 10m is preferred what would be the normal depth?

The Hydraulic Efficient Channel Section

The best hydraulic (the most efficient) cross-section for a given Q, n, and S_0 is the one with a minimum excavation and minimum lining cross-section.

$$A = A_{min} \Rightarrow P = P_{min}.$$

In other case the best hydraulic cross-section for a given A, n, and S_0 is the cross-section that conveys ***maximum discharge.***

Thus the cross-section with the ***minimum wetted perimeter for a given discharge Q and area A is consider the best hydraulic cross-section***

The Hydraulic Efficient …..

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

$$Q = C' \times R^{2/3}$$

$$C' = const$$

$$Q = Q_{max} \rightarrow R = R_{max}$$

$$R = \frac{A}{P}$$

$$R = R_{max} \rightarrow P = P_{min}$$

$$V = \frac{Q}{A} \rightarrow \frac{Q}{A_{min}} = V_{max}$$

$$V = \frac{1}{n}S_0^{1/2}R^{2/3} = C \times R^{2/3}$$

$$V = V_{max} \rightarrow R = R_{max}$$

$$R = \frac{A}{P}$$

$$R = R_{max} \rightarrow P = P_{min}$$

Rectangular channel section

$$A = By = \text{Constant}$$

$$B = \frac{A}{y}$$

$$P = B + 2y = \frac{A}{y} + 2y$$

Since P=Pmin for the best hydraulic section, taking the derivative of P with respect to y

$$\frac{dP}{dy} = \frac{\frac{dA}{dy}\times y - A}{y^2} + 2 = 0$$

$$\frac{A}{y^2} = 2 \rightarrow A = 2y^2 = By$$

$$B = 2y$$

$$R = \frac{A}{P} = \frac{2y^2}{4y} = \frac{y}{2}$$

The Hydraulic Efficient …..

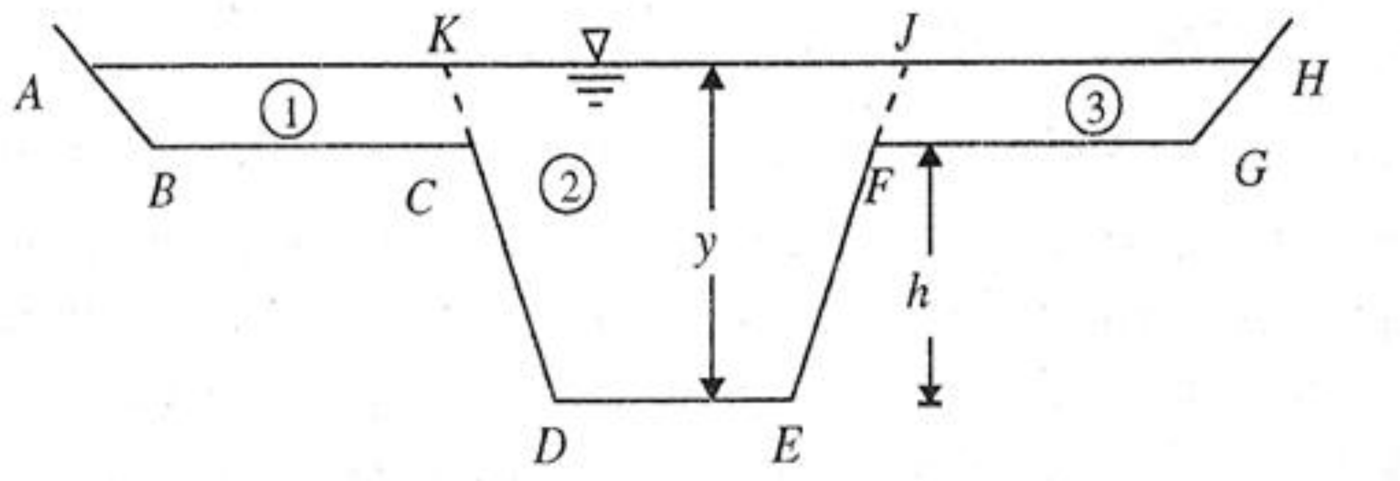
Channel Shape	A	P	B	R	L
Rectangle (half square)	$2y^2$	$4y$	$2y$	$\frac{y}{2}$	$2y$
Trapezoidal (half regular hexagon)	$\sqrt{3}y$	$2\sqrt{3}y$	$\frac{2}{\sqrt{3}}y$	$\frac{y}{2}$	$\frac{4}{\sqrt{3}}y$
Circular (semicircle)	$\frac{\pi}{2}y^2$	πy	-	$\frac{y}{2}$	$2y$
Triangle (vertex angle = 90^0)	y^2	$2\sqrt{3}y$	-	$\frac{y}{2\sqrt{2}}$	$2y$

(A = Area, P = Wetted perimeter, B = Base width,
R = Hydraulic radius, L = Water surface width)

Compound Sections

- combination of elementary sections
- Natural channels, such as rivers, have flood plains

Compound Sections …



- divided into subsections by arbitrary lines, extensions of the deep channel boundaries
- longitudinal slope to be same for all subsections
- subsections will have different mean velocities depending upon the depth and roughness of the boundaries
- If the depth of flow is confined to the deep channel only (y < h), calculation of discharge by using Manning's equation is very simple. However, when the flow spills over the flood plain (y > h), the problem of discharge calculation is complicated

Compound Sections …

Thus to compute the discharge when Y > h

- Calculate wetted perimeter for each sub-areas, but the imaginary divisions (FJ and CK in the Figure) are considered as boundaries for the deeper portion only
- Calculate the discharge as the sum of the partial discharges in the sub-areas

$$Q_p = \sum Q_i = \sum V_i A_i$$

- The discharge is also calculated by considering the whole section as one unit, (ABCDEFGH area in Figure), say Q_w
- The larger of the above discharges, Q_p and Q_w, is adopted as the discharge at the depth y

Critical slope (S_c)

Critical slope is the slope of a specified channel necessary to have uniform flow of a given discharge with critical depth as normal depth

$$Q = \frac{1}{n} A_c R_c^{2/3} S_c^{1/2}$$

$$S_c = \frac{n^2 Q^2}{A_c^2 R_c^{4/3}}$$

If we consider rectangular channel

$$S_c = \frac{n^2 g^{10/9}}{q^{2/9}}$$

If $S_c > S_o \Rightarrow Y_o > Y_c \Rightarrow$ Subcritical flow $\Rightarrow$ mild slope channel

If $S_c < S_o \Rightarrow Y_o < Y_c \Rightarrow$ Supercritical flow $\Rightarrow$ Steep slope channel

If $S_c = S_o \Rightarrow Y_o = Y_c \Rightarrow$ Critical flow $\Rightarrow$ Critical slope channel

Design of Irrigation Canal

- Designed based on rules and equations uniform flow

$$Q = \frac{1}{n} A R^{2/3} S_0^{0.5}$$

- A and R the geometric elements that have to consider in the design process

- General the design of the canal governed with the relation below which has six variables one dependent and other independent.

- Each variables should be known either explicitly or implicitly, or as inequalities, **mostly in terms of empirical relationships**

$$Q = f\left(n, y_0, S_0, B, m\right)$$

Design of Irrigation …

- **Roughness coefficient (n)**
  - Procedures for selecting *n are discussed and values of n can be taken from Table (4.4)*
- **Longitudinal Slope (S_o)**
  - The longitudinal slope is fixed on the basis of topography to command as much area as possible with the limiting velocities acting as constraints. Usually the slopes are of the order of 0.0001.
- **Canal Section (m)**
  - Normally a trapezoidal section is adopted. The side slope, expressed as *m horizontal: 1 vertical, depends on the type of canal, (i.e. lined or unlined, nature and type of soil* through which the canal is laid).
  - **m can be in the range of 0.25 -2 based on the material it is recommended 1.0 -1.5 for natural soil.**

Design of Irrigation …

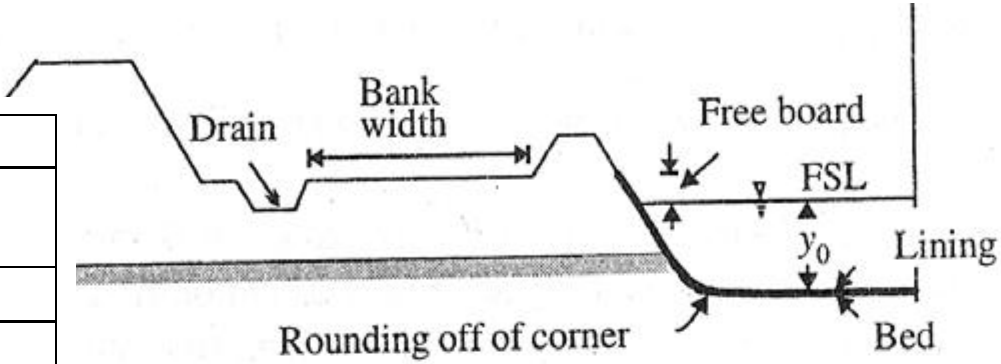
- **Width to Depth Ratio (B/y)**
  - The relationship between width and depth varies widely depending upon the design practice. If the hydraulically most-efficient channel cro ss-section is adopted,

$$m = \frac{1}{\sqrt{3}} \rightarrow B = \frac{2y_0}{\sqrt{3}} = 1.155y_0 \rightarrow \frac{B}{y_0} = 1.155$$

  - **For other section** $$\frac{B}{y_o} = 2(\sqrt{1+m^2} - m)$$
  - **Usually depths higher than 4.0 m is not recommended unless it is it is absolutely necessary.**

- **Free Board**

Discharge (m^3/sec)	Free board (m)	
	Unlined	Lined
$Q < 10.0$	0.50	0.60
$Q \geq 10.0$	0.75	0.75



Design of Irrigation …

- **Procedures**
  - Start with B/y ratio and compute width (B), area (A), and hydraulics radius (R) , interims of depth (y)
  - substitute the values in discharge equation
  - Solve the equation for y, if the value of y < 4 acceptable
  - Compute the area and the velocity, if the V < Vp acceptable
- If not take the maximum permissible velocity and then compute the area.
- Using the area compute the depth if it in the acceptable range

Nature of boundary	Permissible maximum velocity (m/sec)
Sandy soil	0.30 – 0.60
Black cotton soil	0.60 – 0.90
Hard soil	0.90 – 1.10
Firm clay and loam	0.90 – 1.15
Gravel	1.20
Disintegrated rock	1.50
Hard rock	4.00
Brick masonry with cement pointing	2.50
Brick masonry with cement plaster	4.00
Concrete	6.00
Steel lining	10.00

- Compute the width and the side slope to fix the design elements
- If not proceed to adjust the side slop of the canal
- If doesn't work test on the longitudinal slope

ASSIGMENT

Design a trapezoidal Irrigation channel to carry 75m3/sec. The maximum permissible slope is 0.0005.

a. If it lined with brick-in-cement mortar (n=0.013)
b. Unlined canal (n=0.02)

Take canal side slope m=1