Lecture Notes - Sediment Transport – The Sediment Problem

At the end of the first lecture, we noted that our steep, nonlinear transport model depended on τ^*, which is the ratio of the bed shear stress τ to the weight of the sediment grains per unit bed area. The key variable governing the sediment weight is grain size *D*. We know that the range of grain sizes in a typical stream bed is very large and that these sizes may be sorted spatially and vertically and that this sorting might vary with time and with flow. To estimate transport rates, we have to come to terms with how to represent grain size. But, recall that when we solved our example transport model, Meyer-Peter & Müller, for q_s (eqn. 19 in Lecture 1), we found that the relation did not contain *D* explicitly. So, am I just making this stuff up when I claim that *D* is important? No, because M-PM (and other transport models) contain *D* implicitly because grain size has a dominant effect on the critical stress τ_c. So, our first task in this lecture is determining τ_c. Along the way, we will have to think about how many sizes we wish to be concerned with when we are dealing with a stream bed containing a very wide range of sizes. For whatever choice of size ranges, we will need a means of estimating τ_c or its surrogate τ_r.

It turns out the absence of *D* is a key feature of M-PM and other general transport models. Not including *D* explicitly in this model (and others we will develop) facilitates development of a general transport model that applies to sediments of any grain size, as well as to different grain sizes mixed within the same sediment.

1. The Difference Between τ_c and τ_r

So far, we have been introduced the critical shear stress τ_c and the reference shear stress τ_r. It can be easy to confuse them. The first, τ_c, is more of an abstract concept than something that can be readily measured. Nominally, it is the value of τ at which transport begins. Because it is a boundary, it is impossible to measure directly. If you observe grains moving, $\tau > \tau_c$ and if no grains are moving, $\tau < \tau_c$. But that begs the question of how long one should watch the bed, and how much of the bed one should watch, in order to determine whether grains are moving or not. When the flow is turbulent (meaning that τ at any point is fluctuating in time), the answers to these questions are not easy to answer. If our goal is to predict transport rate, the practical alternative, introduced in the first lecture, is to use the reference shear stress τ_r, which is the value of τ associated with a very small, predetermined transport rate. In the first lecture, we set this transport rate as $W^* = 0.002$. With measured transport rates over a range of small τ, it is a straightforward thing to determine τ_r. We will do this in the next lecture. By its definition, τ_r is associated with a small amount of transport, so τ_r should be slightly larger than τ_c.

2. The Different Applications of Critical Shear Stress

Applications of the general concept of incipient motion can be divided into two broad categories. The first is that τ_c(or τ_r) serves as an intercept, or threshold, in a sediment transport relation (e.g. in the Meyer-Peter & Muller relation). The presence of τ_c (or τ_r) in transport relations introduces the characteristic concave-down trend. In this case, we are not really worried about the entrainment of any grain in particular; we just need to know the flow at which transport begins. This is the purpose for which the reference shear stress τ_r was developed. In the second case, we *are* interested in the entrainment of individual grains. For example, we might be interested in flushing fines from the subsurface of a gravel-bed river in order to improve spawning habitat. Or we might be interested in the stability of bed and bank material in cases where channel stability depends on the material not moving at all. In this case, we are interested in the entrainment of individual grains or, more generally, the proportion of grains on the bed surface that are entrained. We might ask "At what discharge do 90% of the surface grains become entrained, thereby providing access to the subsurface and some flushing action?" Or, "At what discharge does 1% of the surface grains become entrained, thereby indicating that our rip-rap channel is beginning to fall apart?"

The difference between these two applications of incipient motion can be illustrated with their characteristic field methods. As an intercept in a transport relation, we would determine τ_r by measuring transport rate, and determining the value of τ at which the transport rate is equal to a small reference value. We discussed this in the first lecture and will return to it in the next. In contrast, the simplest way to measure actual bed entrainment is to use tracer grains. These might be painted rocks that are placed on the bed surface (generally, we try to replace an *in situ* grain with a painted grain of the same size, to provide a more realistic indication of the flow producing movement). If the streambed (or a portion of it is dry), it is even easier to just spay paint the bed itself, although this may raise aesthetic or legal objections. After a flow has passed over the bed, you return to see how many painted rocks remain. Tracers provide an excellent (and easy) way of measuring entrainment (did the grains move at all?), but it is difficult to determine transport rates from tracers, because a transport estimate would require relocating a very large fraction of the tracers and determining how far they moved. This helps to illustrate the difference between the two incipient motion concepts. Entraiment of (say) 50% of the grains on the bed does not tell you what the transport rates were. And measurement of the transport rate does not tell you how many of the surface grains were entrained. A significant transport rate could be produced by a few hyperactive grains, while most of the grains on the bed surface don't move at all.

A related concept is *partial transport*, which is defined as the condition in which only a portion of the grains on the bed surface ever move over the duration of a transport event. We could define partial transport in terms of all surface grains (e.g. 50% of the surface grains move over the transport event) or on a size –by-size basis (e.g. 90% of the 2-8mm grains move, 50% of the 8-32mm grains move, and only 5% of the >32mm grains move over the transport event). The scope and nature of partial transport was defined in the laboratory (Wilcock and McaArdell, 1997) and has been show to represent transport conditions in the field, even under large flow events (Haschenburger and Wilcock, 2003).

Beyond its importance in terms of defining bed stability and subsurface flushing, partial transport would appear to have important consequences for defining frequency and intensity of benthic disturbance in the aquatic ecosystem.

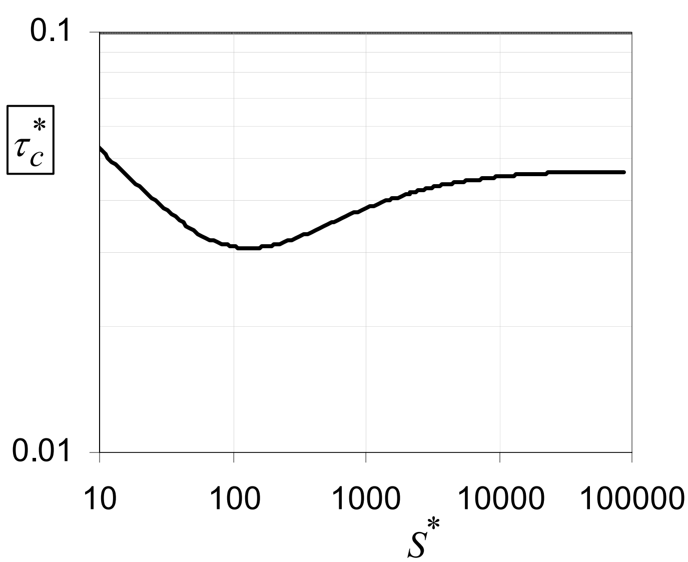
3. Basic Critical Shear Stress

The Shields Curve

The dimensional analysis we did in the first lecture led to the result that dimensionless transport rate depended on four dimensionless variables, the Shields Number τ^*, a dimensionless viscosity S^*, the relative density s, and the relative flow depth D/h. To remind you

$$S^* = \frac{\sqrt{(s-1)gD^3}}{\mu / \rho} \text{ and } s = \frac{\rho_s}{\rho} \qquad (1)$$

If we simply argue that the variables that determine transport rate are the same as those that determine whether grains are moving or not, then a dimensional analysis of the incipient motion problem is nearly the same as one for the transport problem: we just replace q_s with a "motion/no motion" binary variable. Incipient grain motion should be described by some relation between τ^*_c, S^*, s, and D/h. If, as we did before, we limit ourselves to typical values of s (2.65±5%) and flow depths more than a few times D, we end up with a relation between τ^*_c and S^*. For unisize sediments, this is represented by the widely known Shields diagram.



The trend on the diagram can be represented by the function

$$\tau_c^* = 0.105(S^*)^{-0.3} + 0.045\exp\left[-35(S^*)^{-0.59}\right] \qquad (2)$$

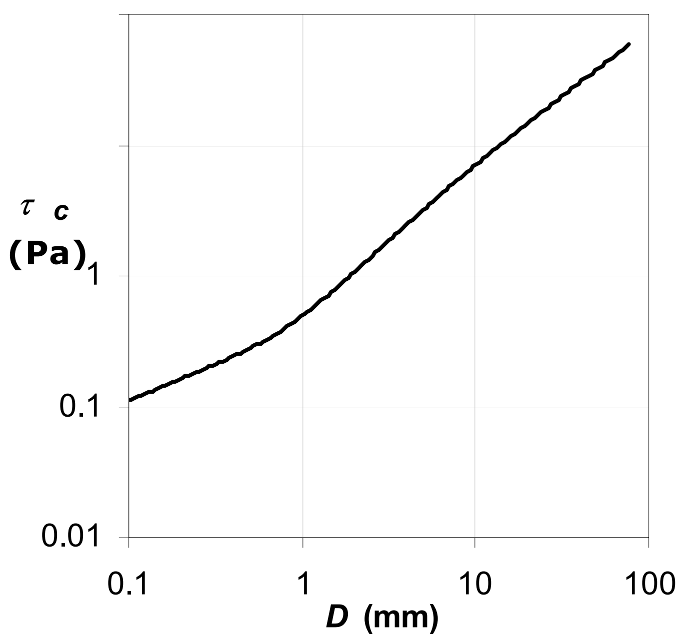
which allows τ^*_c to be determined without having to look values up on the diagram. The variation of τ^*_c with S^* demonstrates the effect of fluid viscosity on grain movement, as we alluded to in the first lecture. We note that τ^*_c approaches a constant value of about 0.045 for $S^* > 10{,}000$. This is of particular interest for us, because we are interested in coarse-bedded streams. Using the definition of τ^*_c, we see that $\tau^*_c = 0.045$ corresponds to

$$\tau_c = 0.045(s-1)\rho g D \tag{3}$$

or, using $s = 2.65$ and $\rho g = 9810$ kg m^{-2} s^{-2},

$$\tau_c = 0.73D \tag{4}$$

for τ_c in Pa and D in mm. This linear trend is clear when the Shields diagram is plotted as τ_c in Pa and D in mm



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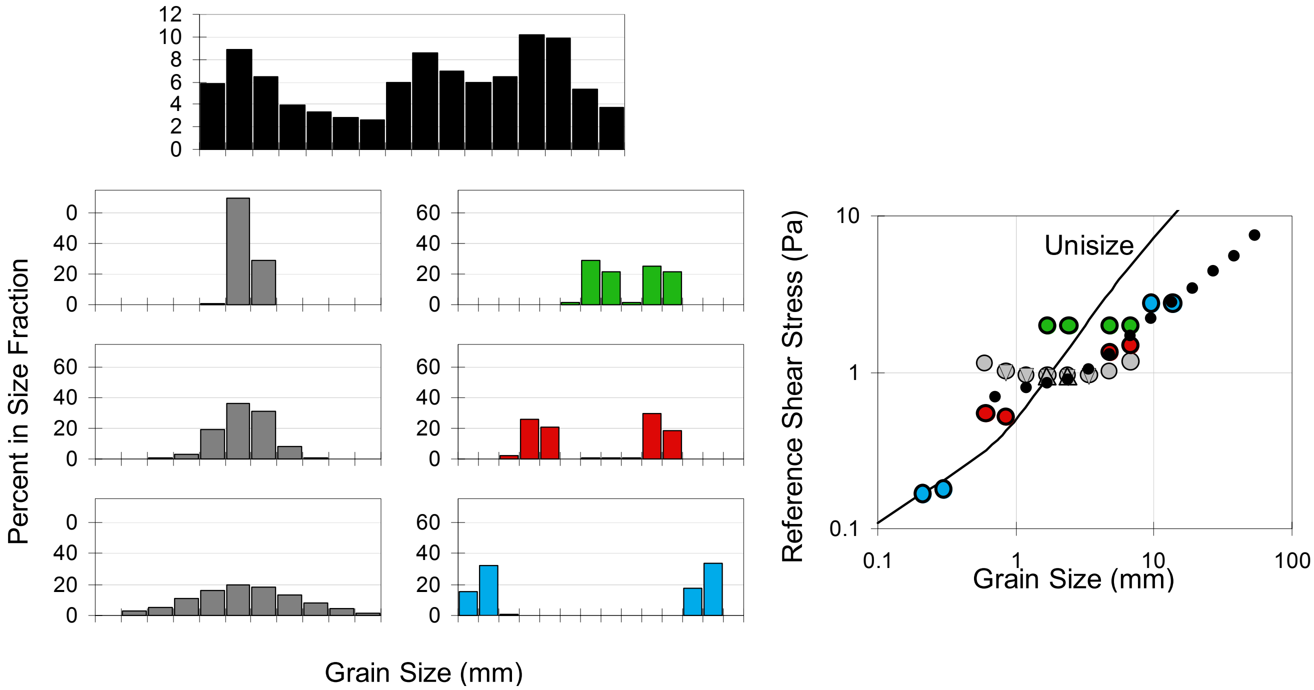
4. Incipient Motion of Mixed-Size Sediments

What if the bed material contains a range of sizes? For sediments that are not too widely sorted with a unimodal size distribution (e.g. a mixture of medium sand to pea gravel with D_{50} around 2mm, or a gravel with D_{50} around 32mm and little sand, it turns out that all of the different sizes in the mixture have just about the same value of τ_c. This is a key element of the condition of "equal mobility", which was hotly debated 10-15 years ago. Basically, what happens is that the tendency of larger grains to be harder to move (as reflected in. (3) and the dimensional Shields curve above) is almost exactly counterbalanced by the effect of mixing the different sizes together in the same sediment. When placed in a mixture, smaller grains will be harder to move than when in a unisize bed and larger grains will be easier to move. For the equal mobility case, one practical question remains: if all sizes have the same τ_c, what is it? It turns out that the Shields curve provides a pretty good indication of τ_c if the median grain size D_{50} is used for D. Thus, you can approximate τ_c for the entire mixture, as well as for individual size fractions, using the Shields Diagram and $D = D_{50}$.



Reference Shear Stress for different unimodal and bimodal sediments

The Effect of Sand

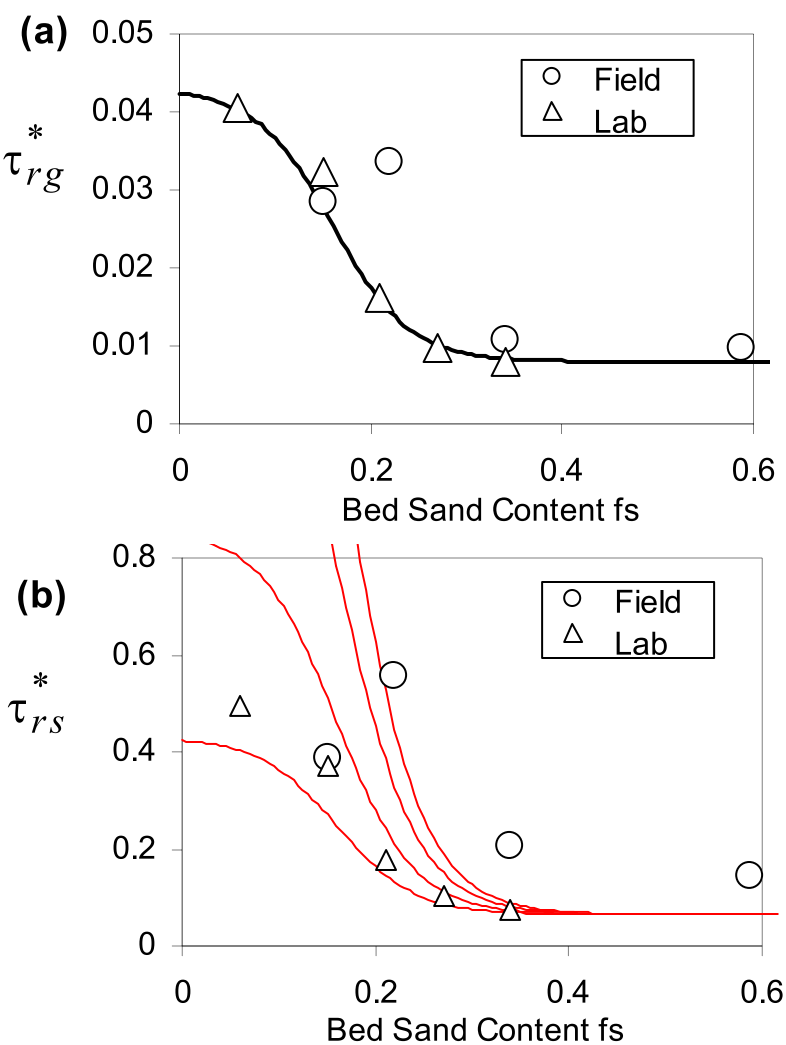
While investigating the incipient motion conditions of a wide range of sediments, it became apparent that a group of sediments did not fit the general pattern. These were gravel beds with more than a few percent sand. These sediments typically have a prominent mode in the gravel size range and another mode in the sand size range and we call the size distribution bimodal. What was observed is that τ_c for the sand fractions tended to be much smaller than for the gravel fractions (i.e. the sand begins moving at smaller flows than the gravel, violating the equal mobility condition) and that τ_c for the gravel fractions tended to be smaller in sandy mixtures than in mixtures with little sand. The sand content affected τ_c for *both* the sand and gravel fractions.

The bimodal nature of the size distributions and the previous observations that τ_c did not vary much within unimodal mixtures that were predominantly all sand or all gravel suggested that perhaps the problem could be solved by considering the mixtures as being composed of two fractions: sand and gravel. This had the possibility of not only allowing each fraction to have a different τ_c but also would allow the effect of sand content on τ_c of the gravel to be explained in terms of the proportion of sand in the bed. It turns out that such a two fraction approach for describing the bed material size distribution captures these points very well as shown by the plots of τ^* for the reference shear stress of the gravel and sand fractions for five different lab sediments and four different field cases (Wilcock, 1997; Wilcock and Kenworthy, 2002).

Although we will talk in the next lecture about how this result and its associated transport model were developed, its useful to discuss the basic trends to complete our discussion of incipient motion. The trends follow a pattern that fits nicely with our general understanding of transport. For the gravel, τ^*_{rg} approaches a standard unisize value of 0.045 as the sand content goes to zero. As sand content becomes large, τ^*_{rg} approaches a minimum of about 0.01, a value observed in different kinds of lab experiments. Most striking is the decrease in τ^*_{rg} over a range in sand content between about 10% and 30%. Over this range, the bed undergoes a transition from being *framework supported* (meaning that the bed consists of a framework of gravel clasts) to being *matrix supported* (meaning that the coarse grains are “floating: in a matrix of sand). This change in bed composition is clearly related to an associated change in transport behavior.

The trend in τ^*_{rs} is also clear, but more complex. As sand content approaches 1, τ^*_{rs} approaches a standard unisize value, just as for the gravel. As sand content approaches zero, we can expect that this small amount of sand will settle down among the gravel grains and the sand entrainment will only occur when the gravel moves. Thus, $\tau_{rs} = \tau_{rg}$. By the definitions of τ^*_{rs} and τ^*_{rg}, $\tau^*_{rs} = \tau^*_{rg}(D_g/D_s)$, so τ^*_{rs} will depend not only on τ^*_{rg}, but also (D_g/D_s). The multiple lines in the lower diagram are for different values of (D_g/D_s).

Repeating a point I am making throughout these lectures: the relation between transport rate and τ is very steep and nonlinear over the typical range observed in gravel-bed rivers. This means that getting τ_c correct is essential for an accurate prediction of transport rate in terms of excess shear stress (τ - τ_c or τ/τ_c). Alternatively, uncertainty in τ_c can take its share of the blame for the large error typically associated with predicted transport rates. And, importantly, if something (like the sand content of the bed) causes τ_c to change, the effect on transport rate will be very large. We will return to this in the next lecture.



Variation of reference Shields Number for (a) gravel and (b) sand fractions of five laboratory sediments and four field cases.

5. A two-fraction approach to transport in gravel-bed streams

Although the range of grain sizes in a gravel-bed river is very large, and there are now transport models capable of predicting the transport rate of many finely-divided size fractions (we will consider one in the next lecture), such predictions require detailed information on the bed composition, which is rarely available. A two-fraction approach, as suggested by the difference in behavior between fine and coarse bed material load, provides an approach that has both conceptual and practical advantages. Its conceptual underpinnings derive from the essential simplification of equal mobility, revised to state that the sizes within two separate, but related fractions—sand and gravel—are equally mobile. A two-fraction estimate allows sand and gravel to move at different rates, thereby permitting change in bed grain size due to changes in the relative proportion of sand and gravel, if not due to the changes in the representative grain size of either fraction. This provides a means of predicting the variation in the fines content of the bed, which may often be more variable than that of the coarse fraction, and whose passage, intrusion, or removal may be a specific environmental or engineering objective.

A two-fraction approach facilitates developing an estimate of the grain-size of an entire river reach. Areas with similar fines content may be mapped and combined to give a weighted average proportion of sand for the reach giving an integral measure of grain size with reasonable effort. This provides a superior description of the bed compared to an unsupported extrapolation from detailed sampling at only a few locations.

A two fraction approach provides a ready means of representing the interaction between the fine and coarse components of the bed material. Laboratory studies (Wilcock, Kenworthy, and Crowe, 2001; Curran and Wilcock, in review) show that the addition of sand to a gravel bed or to the sediment supply can increase gravel transport rates by orders of magnitude (this is indicated by the four-fold decrease in τ^*_{rg} in the two-fraction curve above). This effect is not captured in previous transport models. Because there are a variety of situations in which the supply of fine bed material can be increased (e.g. fire, reservoir flushing, dam removal, urbanization), an accurate and practical basis for addressing these situations is clearly needed.

In the next lecture, we will discuss more fully how sand content affects transport rates and how the incipient motion result discussed here was developed in the context of developing a two-fraction transport model (Wilcock, 1998; Wilcock and Kenworthy; 2002). In the lecture after that, we will exploit the advantages of the two-fraction concept in coming up with a practical approach to estimating transport rates in gravel-bed streams. (Wilcock, 2001).