CIVE2400 Fluid Mechanics
Section 2: Open Channel Hydraulics

1. Open Channel Hydraulics

1.1 Definition and differences between pipe flow and open channel flow

The flow of water in a conduit may be either *open channel flow* or *pipe flow*. The two kinds of flow are similar in many ways but differ in one important respect. Open-channel flow must have a *free surface*, whereas pipe flow has none. A free surface is subject to atmospheric pressure. In Pipe flow there exist no direct atmospheric flow but hydraulic pressure only.

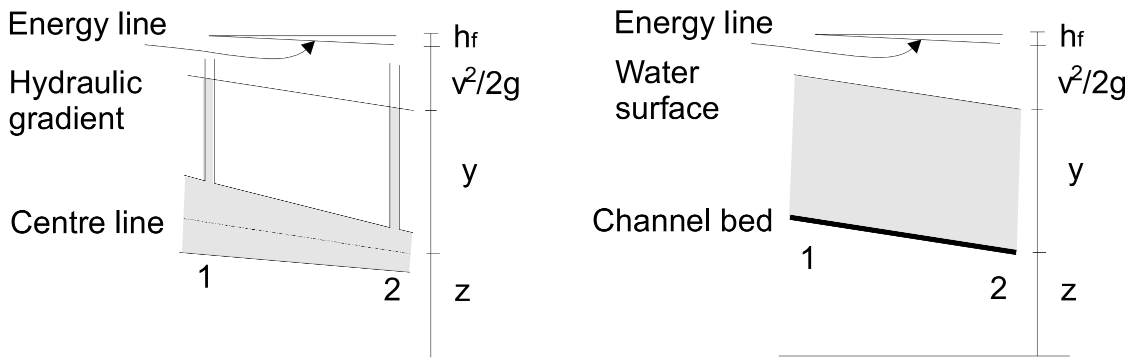


Figure of pipe and open channel flow

The two kinds of flow are compared in the figure above. On the left is pipe flow. Two piezometers are placed in the pipe at sections 1 and 2. The water levels in the pipes are maintained by the pressure in the pipe at elevations represented by the *hydraulics grade line* or *hydraulic gradient*. The pressure exerted by the water in each section of the pipe is shown in the tube by the height y of a column of water above the centre line of the pipe.

The total energy of the flow of the section (with reference to a datum) is the sum of the elevation z of the pipe centre line, the piezometric head y and the velocity head $V^2/2g$, where V is the mean velocity. The energy is represented in the figure by what is known as the *energy grade line* or the *energy gradient*.

The loss of energy that results when water flows from section 1 to section 2 is represented by h_f.

A similar diagram for open channel flow is shown to the right. This is simplified by assuming parallel flow with a uniform velocity distribution and that the slope of the channel is small. In this case the *hydraulic gradient* is the water surface as the depth of water corresponds to the piezometric height.

Despite the similarity between the two kinds of flow, it is much more difficult to solve problems of flow in open channels than in pipes. Flow conditions in open channels are complicated by the position of the free surface which will change with time and space. And also by the fact that depth of flow, the discharge, and the slopes of the channel bottom and of the free surface are all inter dependent.

Physical conditions in open-channels vary much more than in pipes – the cross-section of pipes is usually round – but for open channel it can be any shape.

Treatment of roughness also poses a greater problem in open channels than in pipes. Although there may be a great range of roughness in a pipe from polished metal to highly corroded iron, open channels may be of polished metal to natural channels with long grass and roughness that may also depend on depth of flow.

Open channel flows are found in large and small scale. For example the flow depth can vary between a few cm in water treatment plants and over 10m in large rivers. The mean velocity of flow may range from less than 0.01 m/s in tranquil waters to above 50 m/s in high-head spillways. The range of total discharges may extend from 0.001 l/s in chemical plants to greater than 10000 m^3/s in large rivers or spillways.

In each case the flow situation is characterised by the fact that there is a free surface whose position is NOT known beforehand – it is determined by applying momentum and continuity principles.

Open channel flow is driven by gravity rather than by pressure work as in pipes.

	Pipe flow	**Open Channel flow**
Flow driven by	Pressure work	Gravity (potential energy)
Flow cross section	Known, fixed	Unknown in advance because the flow depth is unknown
Characteristics flow parameters	velocity deduced from continuity	Flow depth deduced simultaneously from solving both continuity and momentum equations
Specific boundary conditions		Atmospheric pressure at the free surface

1.2 Types of flow

The following classifications are made according to change in flow depth with respect to time and space.

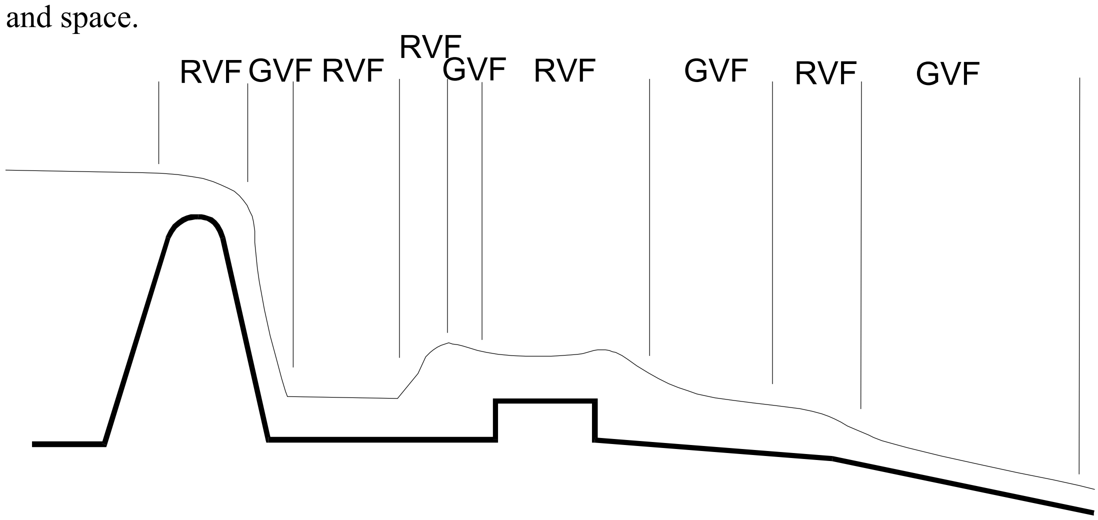


Figure of the types of flow that may occur in open channels

Steady and Unsteady: Time is the criterion.
Flow is said to be steady if the depth of flow at a particular point does not change or can be considered constant for the time interval under consideration. The flow is unsteady if depth changes with time.

Uniform Flow: Space as the criterion.
Open Channel flow is said to be *uniform* if the depth and velocity of flow are the same at every section of the channel. Hence it follows that uniform flow can only occur in prismatic channels. For steady uniform flow, depth and velocity is constant with both time and distance. This constitutes the fundamental type of flow in an open channel. It occurs when gravity forces are in equilibrium with resistance forces.

Steady non-uniform flow.
Depth varies with distance but not with time. This type of flow may be either (a) *gradually varied* or (b) *rapidly varied.* Type (a) requires the application of the energy and frictional resistance equations while type (b) requires the energy and momentum equations.

Unsteady flow
The depth varies with both time and space. This is the most common type of flow and requires the solution of the energy momentum and friction equations with time. In many practical cases the flow is sufficiently close to steady flow therefore it can be analysed as gradually varied steady flow.

1.3 Properties of open channels

Artificial channels
These are channels made by man. They include irrigation canals, navigation canals, spillways, sewers, culverts and drainage ditches. They are usually constructed in a regular cross-section shape throughout – and are thus prismatic channels (they don't widen or get narrower along the channel. In the field they are commonly constructed of concrete, steel or earth and have the surface roughness' reasonably well defined (although this may change with age – particularly grass lined channels.) Analysis of flow in such well defined channels will give reasonably accurate results.

Natural channels
Natural channels can be very different. They are not regular nor prismatic and their materials of construction can vary widely (although they are mainly of earth this can possess many different properties.) The surface roughness will often change with time distance and even elevation. Consequently it becomes more difficult to accurately analyse and obtain satisfactory results for natural channels than is does for man made ones. The situation may be further complicated if the boundary is not fixed i.e. erosion and deposition of sediments.

Geometric properties necessary for analysis
For analysis various geometric properties of the channel cross-sections are required. For artificial channels these can usually be defined using simple algebraic equations given y the depth of flow.

The commonly needed geometric properties are shown in the figure below
and defined as:

Depth (y) – the **vertical** distance from the lowest point of the channel section to the free surface.
Stage (z) – the vertical distance from the free surface to an arbitrary datum
Area (A) – the cross-sectional area of flow, normal to the direction of flow
Wetted perimeter (P) – the length of the wetted surface measured normal to the direction of flow.
Surface width (B) – width of the channel section at the free surface
Hydraulic radius (R) – the ratio of area to wetted perimeter (A/P)
Hydraulic mean depth (D_m) – the ratio of area to surface width (A/B)

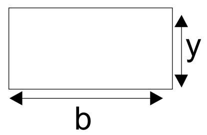
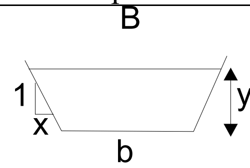
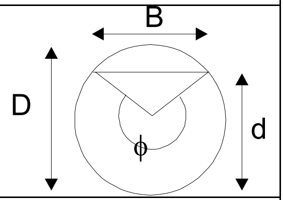
	Rectangle	Trapezoid	Circle
			
Area, A	by	$(b+xy)y$	$\frac{1}{8}(\phi - \sin\phi)D^2$
Wetted perimeter P	$b + 2y$	$b + 2y\sqrt{1+x^2}$	$\frac{1}{2}\phi D$
Top width B	b	$b+2xy$	$(\sin\phi/2)D$
Hydraulic radius R	$by/(b+2y)$	$\dfrac{(b+xy)y}{b+2y\sqrt{1+x^2}}$	$\dfrac{1}{4}\left(1 - \dfrac{\sin\phi}{\phi}\right)D$
Hydraulic mean depth D_m	y	$\dfrac{(b+xy)y}{b+2xy}$	$\dfrac{1}{8}\left(\dfrac{\phi - \sin\phi}{\sin(1/2\phi)}\right)D$

Table of equations for rectangular trapezoidal and circular channels.

1.4 Fundamental equations

The equations which describe the flow of fluid are derived from three fundamental laws of physics:
1. Conservation of matter (or mass)
2. Conservation of energy
3. Conservation of momentum

Although first developed for solid bodies they are equally applicable to fluids. A brief description of the concepts are given below.

Conservation of matter
This says that matter can not be created nor destroyed, but it may be converted (e.g. by a chemical process.) In fluid mechanics we do not consider chemical activity so the law reduces to one of conservation of mass.

Conservation of energy
This says that energy can not be created nor destroyed, but may be converted form one type to another (e.g. potential may be converted to kinetic energy). When engineers talk about energy "losses" they are referring to energy converted from mechanical (potential or kinetic) to some other form such as heat. A friction loss, for example, is a conversion of mechanical energy to heat. The basic equations can be obtained from the First Law of Thermodynamics but a simplified derivation will be given below.

Conservation of momentum

The law of conservation of momentum says that a moving body cannot gain or lose momentum unless acted upon by an external force. This is a statement of Newton's Second Law of Motion:

Force = rate of change of momentum

In solid mechanics these laws may be applied to an object which is has a fixed shape and is clearly defined. In fluid mechanics the object is not clearly defined and as it may change shape constantly. To get over this we use the idea of *control volumes*. These are imaginary volumes of fluid within the body of the fluid. To derive the basic equation the above conservation laws are applied by considering the forces applied to the edges of a control volume within the fluid.

1.4.1 The Continuity Equation (conservation of mass)

For any control volume during the small time interval δt the principle of conservation of mass implies that the mass of flow entering the control volume minus the mass of flow leaving the control volume equals the change of mass within the control volume.

If the flow is steady and the fluid incompressible the mass entering is equal to the mass leaving, so there is no change of mass within the control volume.

So for the time interval δt:

Mass flow entering = mass flow leaving

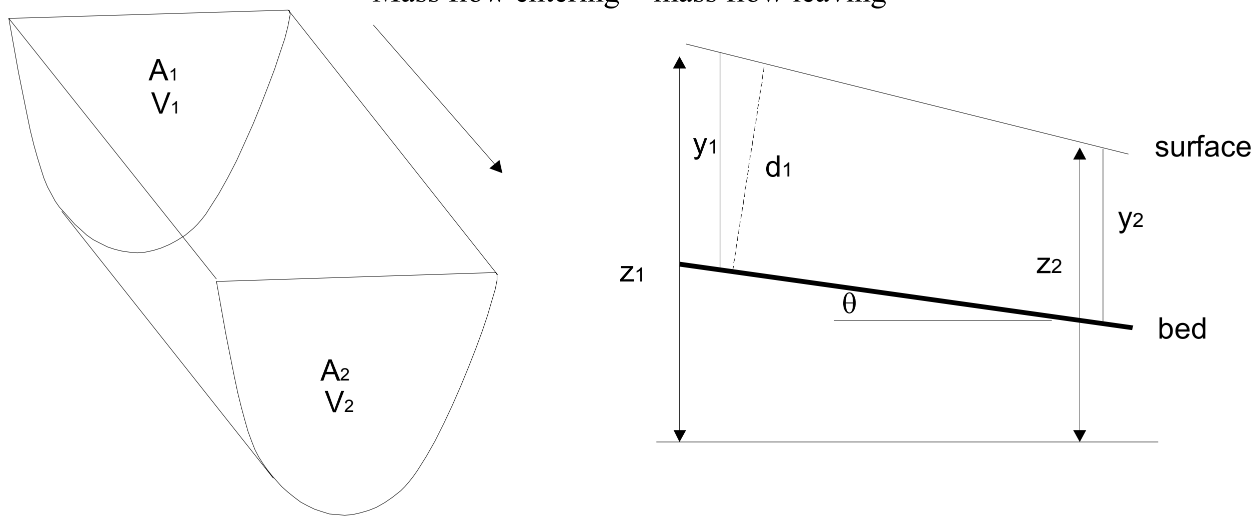


Figure of a small length of channel as a control volume

Considering the control volume above which is a short length of open channel of arbitrary cross-section then, if ρ is the fluid density and Q is the volume flow rate then
mass flow rate is ρQ and the continuity equation for steady incompressible flow can be written

$$\rho Q_{\text{entering}} = \rho Q_{\text{leaving}}$$

As, Q, the volume flow rate is the product of the area and the mean velocity then at the upstream face (face 1) where the mean velocity is u_1 and the cross-sectional area is A_1 then:

$$Q_{\text{entering}} = u_1 A_1$$

Similarly at the downstream face, face 2, where mean velocity is u_2 and the cross-sectional area is A_2 then:

$$Q_{\text{leaving}} = u_2 A_2$$

Therefore the continuity equation can be written as

$$u_1 A_1 = u_2 A_2$$

Equation 1.1

1.4.2 The Energy equation (conservation of energy)

Consider the forms of energy available for the above control volume. If the fluid moves from the upstream face 1, to the downstream face 2 in time δt over the length L.

The work done in moving the fluid through face 1 during this time is

$$\text{work done} = p_1 A_1 L$$

where p_1 is pressure at face 1
The mass entering through face 1 is

$$\text{mass entering} = \rho_1 A_1 L$$

Therefore the kinetic energy of the system is:

$$KE = \frac{1}{2} m u^2 = \frac{1}{2} \rho_1 A_1 L u_1^2$$

If z_1 is the height of the centroid of face 1, then the potential energy of the fluid entering the control volume is :

$$PE = mgz = \rho_1 A_1 L\, g\, z_1$$

The total energy entering the control volume is the sum of the work done, the potential and the kinetic energy:

$$\text{Total energy} = p_1 A_1 L + \frac{1}{2} \rho_1 A_1 L u_1^2 + \rho_1 A_1 L g z_1$$

We can write this in terms of energy per unit weight. As the weight of water entering the control volume is $\rho_1\, A_1\, L\, g$ then just divide by this to get the total energy per unit weight:

$$\text{Total energy per unit weight} = \frac{p_1}{\rho_1 g} + \frac{u_1^2}{2g} + z_1$$

At the exit to the control volume, face 2, similar considerations deduce

$$\text{Total energy per unit weight} = \frac{p_2}{\rho_2 g} + \frac{u_2^2}{2g} + z_2$$

If no energy is supplied to the control volume from between the inlet and the outlet then energy leaving = energy entering and if the fluid is incompressible $\rho_1 = \rho_1 = \rho$

So,

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 = H = \text{constant}$$

Equation 1.2

This is the Bernoulli equation.

Note:

1. In the derivation of the Bernoulli equation it was assumed that no energy is lost in the control volume - i.e. the fluid is *frictionless.* To apply to non frictionless situations some energy loss term must be included

2. The dimensions of each term in equation 1.2 has the dimensions of length (units of meters). For this reason each term is often regarded as a "head" and given the names

$$\frac{p}{\rho g} = \text{pressure head}$$
$$\frac{\text{u}^2}{2\text{g}} = \text{velocity head}$$
$$z = \text{velocity or potential head}$$

3. Although above we derived the Bernoulli equation between two sections it should strictly speaking be applied along a stream line as the velocity will differ from the top to the bottom of the section. However in engineering practise it is possible to apply the Bernoulli equation with out reference to the particular streamline

1.4.3 The momentum equation (momentum principle)

Again consider the control volume above during the time δt

$$\text{momentum entering} = \rho \delta Q_1 \delta t u_1$$
$$\text{momentum leaving} = \rho \delta Q_2 \delta t u_2$$

By the continuity principle : $\delta Q_1 = \delta Q_2 = \delta Q$
And by Newton's second law Force = rate of change of momentum

$$\delta F = \frac{\text{momentum leaving - momentum entering}}{\delta t}$$
$$= \rho \delta Q (u_2 - u_1)$$

It is more convenient to write the force on a control volume in each of the three, x, y and z direction e.g. in the x-direction

$$\delta F_x = \rho \delta Q (u_{2x} - u_{1x})$$

Integration over a volume gives the total force in the x-direction as

$$F_x = \rho Q(V_{2x} - V_{1x})$$

Equation 1.3

As long as velocity V is *uniform* over the whole cross-section.

This is the momentum equation for steady flow for a region of uniform velocity.

Energy and Momentum coefficients

In deriving the above momentum and energy (Bernoulli) equations it was noted that the velocity must be constant (equal to V) over the whole cross-section or constant along a stream-line. Clearly this will not occur in practice. Fortunately both these equation may still be used even for situations of quite non-uniform velocity distribution over a section. This is possible by the introduction of coefficients of energy and momentum, α and β respectively.

These are defined:

$$\alpha = \frac{\int \rho u^3 dA}{\rho V^3 A}$$

Equation 1.4

$$\beta = \frac{\int \rho u^2 dA}{\rho V^2 A}$$

Equation 1.5

where V is the mean velocity.

And the Bernoulli equation can be rewritten in terms of this mean velocity:

$$\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z = \text{constant}$$

Equation 1.6

And the momentum equation becomes:

$$F_x = \rho Q \beta(V_{2x} - V_{1x})$$

Equation 1.7

The values of α and β must be derived from the velocity distributions across a cross-section. They will always be greater than 1, but only by a small amount consequently they can often be confidently omitted – but not always and their existence should always be remembered. For turbulent flow in regular channel α does not usually go above 1.15 and β will normally be below 1.05. We will see an example below where their inclusion is necessary to obtain accurate results.

1.5 Velocity distribution in open channels

The measured velocity in an open channel will always vary across the channel section because of friction along the boundary. Neither is this velocity distribution usually axisymmetric (as it is in pipe flow) due to the existence of the free surface. It might be expected to find the maximum velocity at the free surface where the shear force is zero but this is not the case. The maximum velocity is usually found just below the surface. The explanation for this is the presence of secondary currents which are circulating from the boundaries towards the section centre and resistance at the air/water interface. These have been found in both laboratory measurements and 3d numerical simulation of turbulence.

The figure below shows some typical velocity distributions across some channel cross sections. The number indicates percentage of maximum velocity.

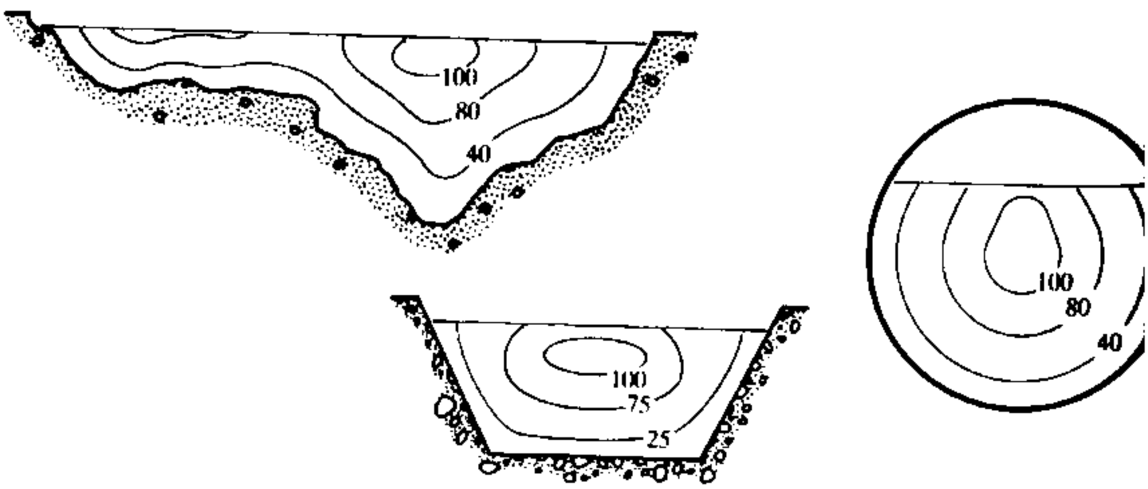


Figure of velocity distributions

1.5.1 Determination of energy and momentum coefficients

To determine the values of α and β the velocity distribution must have been measured (or be known in some way). In irregular channels where the flow may be divided into distinct regions α may exceed 2 and should be included in the Bernoulli equation.

The figure below is a typical example of this situation. The channel may be of this shape when a river is in flood – this is known as a *compound channel.*

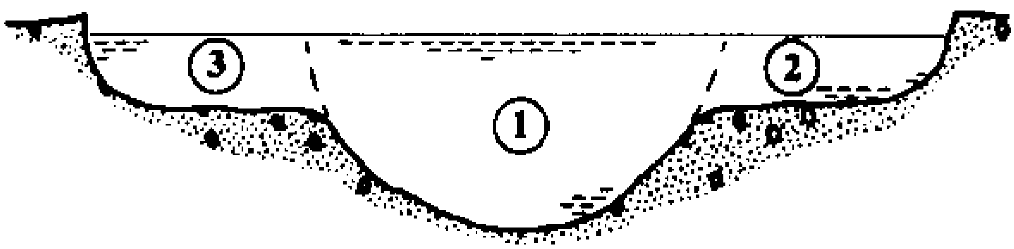


Figure of a compound channel with three regions of flow

If the channel is divided as shown into three regions and making the assumption that $\alpha = 1$ for each then

$$\alpha = \frac{\int u^3 dA}{\overline{V}^3 A} = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{\overline{V}^3 (A_1 + A_2 + A_3)}$$

where

$$\overline{V} = \frac{Q}{A} = \frac{V_1 A_1 + V_2 A_2 + V_3 A_3}{A_1 + A_2 + A_3}$$

1.6 Laminar and Turbulent flow

As in pipes , and all flow, the flow in an open channel may be either laminar or turbulent. The criterion for determining the type of flow is the *Reynolds Number*, *Re*.

For pipe flow

$$\mathrm{Re} = \frac{\rho u D}{\mu}$$

And the limits for reach type of flow are

Laminar: Re < 2000
Turbulent: Re > 4000

If we take the characteristic length as the hydraulic radius R = A/P then for a pipe flowing full R = D/4 and

$$\mathrm{Re}_{\mathrm{channel}} = \frac{\rho u R}{\mu} = \frac{\rho u D}{\mu 4} = \frac{\mathrm{Re}_{\mathrm{pipe}}}{4}$$

So for an open channel the limits for each type of flow become

Laminar: $\mathrm{Re}_{\mathrm{channel}} < 500$
Turbulent: $\mathrm{Re}_{\mathrm{channel}} > 1000$

In practice the limit for turbulent flow is not so well defined in channel as it is in pipes and so 2000 is often taken as the threshold for turbulent flow.

We can use the ideas seen for pipe flow analysis to look at the effect of friction. Taking the Darcy-Wiesbach formula for head loss due to friction in a pipe in turbulent flow

$$h_f = \frac{4fLV^2}{2gD}$$

and make the substitution for hydraulic radius R = D/4
And if we put the bed slope $S_o = L/h_f$ then

$$S_o = \frac{4fV^2}{2g4R}$$

and

$$\lambda = \frac{8gRS_o}{V^2} \quad f = \frac{2gRS_o}{V^2}$$

The *Colebrook-White* equation gives the f - Re relationship for pipes, putting in R=D/4 the equivalent equation for open channel is

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left(\frac{k_s}{14.8R} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right)$$

where k_s is the *effective roughness* height

A chart of the λ - Re relationship for open channels can be drawn using this equation but its practical application is not clear. In pipes this relationship is useful but due to the more complex flow pattern and the extra variable (R varies with depth and channel shape) then it is difficult to apply to a particular channel.

In practice flow in open channels is usually in the rough turbulent zone and consequently simpler friction formulae may be applied to relate frictional losses to velocity and channel shape.

1.7 Uniform flow and the Development of Friction formulae

When uniform flow occurs gravitational forces exactly balance the frictional resistance forces which apply as a shear force along the boundary (channel bed and walls).

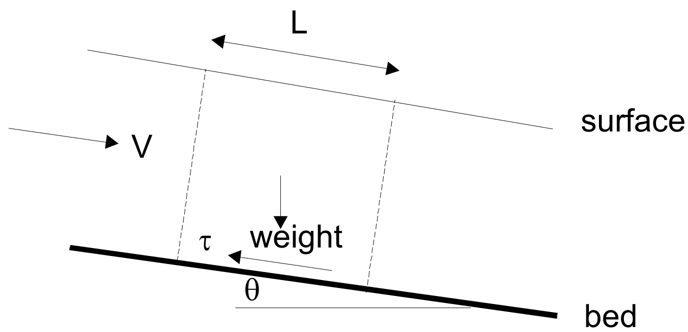


Figure of forces on a channel length in uniform flow

Considering the above diagram, the gravity force resolved in the direction of flow is

$$\text{gravity force} = \rho g A L \sin\theta$$

and the boundary shear force resolved in the direction of flow is

$$\text{shear force} = \tau_o P L$$

In uniform flow these balance

$$\tau_o P L = \rho g A L \sin\theta$$

Considering a channel of small slope, (as channel slopes for unifor and gradually varied flow seldom exceed about 1 in 50) then

$$\sin\theta \approx \tan\theta = S_o$$

So

$$\tau_o = \frac{\rho g A S_o}{P} = \rho g R S_o$$

Equation 1.8

1.7.1 The Chezy equation

If an estimate of τ_o can be made then we can make use of Equation 1.8.

If we assume the state of rough turbulent flow then we can also make the assumption the shear force is proportional to the flow velocity squared i.e.

$$\tau_o \propto V^2$$
$$\tau_o = KV^2$$

Substituting this into equation 1.8 gives

$$V = \sqrt{\frac{\rho g}{K} R S_o}$$

Or grouping the constants together as one equal to C

$$V = C\sqrt{RS_o}$$

Equation 1.9

This is the Chezy equation and the C the "Chezy C"

Because the K is not constant the C is not constant but depends on Reynolds number and boundary roughness (see discussion in previous section).

The relationship between C and λ is easily seen be substituting equation 1.9 into the Darcy-Wiesbach equation written for open channels and is

$$C = \sqrt{\frac{2g}{f}}$$

1.7.2 The Manning equation

A very many studies have been made of the evaluation of C for different natural and manmade channels. These have resulted in today most practising engineers use some form of this relationship to give C:

$$C = \frac{R^{1/6}}{n}$$

Equation 1.10

This is known as Manning's formula, and the *n* as Manning's *n*.

Substituting equation 1.9 in to 1.10 gives velocity of uniform flow:

$$V = \frac{R^{2/3} S_o^{1/2}}{n}$$

Or in terms of discharge

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_o^{1/2}$$

Equation 1.11

Note:
Several other names have been associated with the derivation of this formula – or ones similar and consequently in some countries the same equation is named after one of these people. Some of these names are; Strickler, Gauckler, Kutter, Gauguillet and Hagen.

The Manning's *n* is also numerically identical to the Kutter *n*.

The Manning equation has the great benefits that it is simple, accurate and now due to it long extensive practical use, there exists a wealth of publicly available values of *n* for a very wide range of channels.

Below is a table of a few typical values of Manning's *n*

Channel type	**Surface material and form**	**Manning's *n* range**
River	earth, straight	0.02-0.025
	earth, meandering	0.03-0.05
	gravel (75-150mm), straight	0.03-0.04
	gravel (75-150mm), winding	0.04-0.08
unlined canal	earth, straight	0.018-0.025
	rock, straight	0.025-0.045
lined canal	concrete	0.012-0.017
lab. models	mortar	0.011-0.013
	Perspex	0.009

1.7.3 Conveyance

Channel conveyance, *K*, is a measure of the carrying capacity of a channel. The *K* is really an agglomeration of several terms in the Chezy or Manning's equation:

$$Q = AC\sqrt{RS_o}$$

$$Q = KS_o^{1/2}$$

Equation 1.12

So

$$K = ACR^{1/2} = \frac{A^{5/3}}{nP^{2/3}}$$

Equation 1.13

Use of conveyance may be made when calculating discharge and stage in compound channels and also calculating the energy and momentum coefficients in this situation.

1.8 Computations in uniform flow

We can use Manning's formula for discharge to calculate steady uniform flow. Two calculations are usually performed to solve uniform flow problems.

1. Discharge from a given depth
2. Depth for a given discharge

In steady uniform flow the flow depth is know as ***normal depth***.

As we have already mentioned, and by definition, uniform flow can only occur in channels of constant cross-section (prismatic channels) so natural channel can be excluded. However we will need to use Manning's equation for gradually varied flow in natural channels - so application to natural/irregular channels will often be required.

1.8.1 Uniform flow example 1 - Discharge from depth in a trapezoidal channel

A concrete lined trapezoidal channel with uniform flow has a normal depth is 2m.
The base width is 5m and the side slopes are equal at 1:2
Manning's *n c*an be taken as 0.015
And the bed slope $S_0 = 0.001$

What are:
a) Discharge (Q)
b) Mean velocity (V)
c) Reynolds number (Re)

Calculate the section properties

$$A = (5 + 2y)y = 18m^2$$
$$P = 5 + 2y\sqrt{1 + 2^2} = 13.94m$$

Use equation 1.11 to get the discharge

$$\begin{aligned} Q &= \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} S_o^{1/2} = \frac{1}{0.015}\frac{18^{5/3}}{13.94^{2/3}} 0.001^{1/2} \\ &= 45m^3/s \end{aligned}$$

The simplest way to calculate the mean velocity is to use the continuity equation:

$$V = \frac{Q}{A} = \frac{45}{18} = 2.5m/s$$

And the Reynolds number (R=A/P)

$$\mathrm{Re}_{\text{channel}} = \frac{\rho u R}{\mu} = \frac{\rho u A}{\mu P} = \frac{10^3 \times 2.5 \times 18}{1.14 \times 10^{-3} \times 13.94} = 2.83 \times 10^6$$

This is very large - i.e. well into the turbulent zone - the application of the Manning's equation was therefore valid.

What solution would we have obtained if we had used the Colebrook-White equation?

Probably very similar as we are well into the rough-turbulent zone where both equations are truly applicable.

To experiment an equivalent k_s value can be calculated for the discharge calculated from $n = 0.015$ and y = 2m [$k_s = 2.225mm$] (Use the Colebrook-White equation and the Darcy-Wiesbach equation of open channels - both given earlier). Then a range of depths can be chosen and the discharges calculated for these *n and* k_s values. Comparing these discharge calculations will give some idea of the relative differences - they will be very similar.

1.8.2 Uniform flow example 2 - Depth from Discharge in a trapezoidal channel

Using the same channel as above, if the discharge is know to be 30m^3/s in uniform flow, what is the normal depth?

Again use equation 1.11

$A = (5+2y)y = 18m^2$

$P = 5 + 2y\sqrt{1+2^2} = 13.94m$

$$Q = \frac{1}{0.015}\frac{((5+2y)y)^{5/3}}{\left(5+2y\sqrt{1+2^2}\right)^{2/3}}0.001^{1/2}$$

$$30 = 2.108\frac{((5+2y)y)^{5/3}}{\left(5+2y\sqrt{1+2^2}\right)^{2/3}}$$

We need to calculate *y* from this equation.

Even for this quite simple geometry the equation we need to solve for normal depth is complex.

One simple strategy to solve this is to select some appropriate values of *y* and calculate the right hand side of this equation and compare it to Q (=30) in the left. When it equals Q we have the correct *y*. Even though there will be several solutions to this equation, this strategy generally works because we have a good idea of what the depth should be (e.g. it will always be positive and often in the range of 0.5-10 m).

In this case from the previous example we know that at Q = 45 m^3/s, $y = 2m$. So at Q = 30 m^3/s then y < 2.0m.

Guessed y (m)	Discharge Q (m^3/s)
1.7	32.7
1.6	29.1
1.63	30.1

You might also use the bisector method to solve this.

1.8.3 Uniform flow example 3 - A compound channel

If the channel in the above example were to be designed for flooding it may have a section like this:

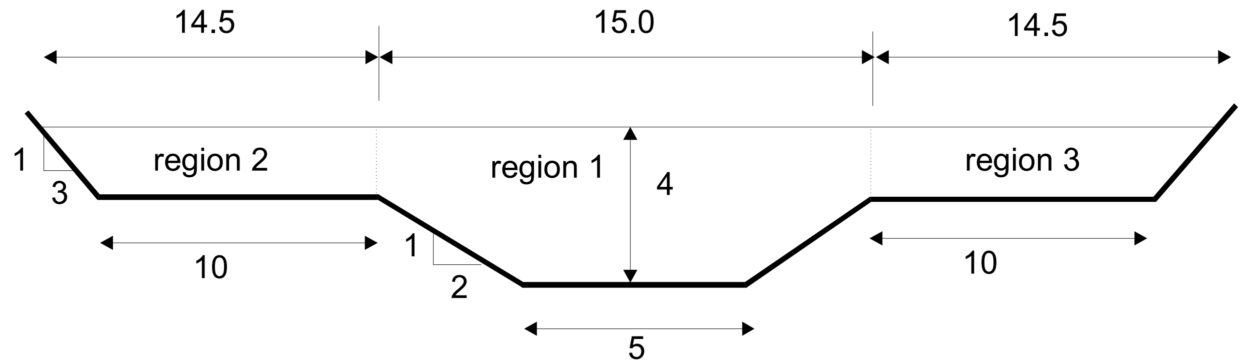


Figure of compound section

When the flow goes over the top of the trapezoidal channel it moves to the "flood plains" so the section allows for a lot more discharge to be carried.

If the flood channels are 10m wide and have side slopes of 1:3, and the Manning *n* on these banks is0.035, what are

a) the discharge for a flood level of 4m
b) the enery coefficient α

First split the section as shown in to three regions (this is arbitrary - left to the engineers judgement). Then apply Manning's formula for each section to give three discharge values and the total discharge will be $Q = Q_1 + Q_2 + Q_3$.

Calculate the properties of each region:

$$A_1 = \left(\frac{5+15}{2}\right)2.5 + (15 \times 1.5) = 47.5m^2$$

$$A_2 = A_3 = \left(\frac{10+14.5}{2}\right)1.5 = 18.38m^2$$

$$P_1 = 5 + \left(2\sqrt{5} \times 2.5\right) = 16.18m$$

$$P_2 = P_3 = 10 + \left(1.5\sqrt{10}\right) = 14.75m$$

The conveyance for each region may be calculated from equation 1.13

$$K_1 = \frac{47.5^{5/3}}{0.015 \times 16.18^{2/3}} = 6492.5$$

$$K_2 = K_3 = \frac{18.38^{5/3}}{0.035 \times 14.74^{2/3}} = 608.4$$

And from Equation 1.11 or Equation 1.12 the discharges

$$Q_1 = \frac{1}{0.015}\frac{47.5^{5/3}}{16.18^{2/3}}0.001^{1/2}$$

or

$$Q_1 = K_1 0.001^{1/2} = 205.3m^3/s$$

And

$$Q_2 = Q_3 = \frac{1}{0.035}\frac{18.38^{5/3}}{14.74^{2/3}}0.001^{1/2}$$

or

$$Q_2 = Q_3 = K_2 0.001^{1/2} = 19.2m^2/s$$

So

$$Q = Q_1 + Q_2 + Q_3 = 243.7m^3/s$$

The velocities can be obtained from the continuity equation:

$$V_1 = \frac{Q_1}{A_1} = 4.32m/s$$

$$V_2 = V_3 = \frac{Q_2}{A_2} = 1.04m/s$$

And the energy coefficient may be obtained from Equation 1.4

$$\alpha = \frac{\int u^3 dA}{\overline{V}^3 A} = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{\overline{V}^3 \left(A_1 + A_2 + A_3\right)}$$

where

$$\overline{V} = \frac{Q}{A} = \frac{V_1 A_1 + V_2 A_2 + V_3 A_3}{A_1 + A_2 + A_3}$$

Giving

$$\alpha = 1.9$$

This is a **very high** value of α and a clear case of where a velocity coefficient should be used.

Not that this method doe not give completely accurate relationship between stage and discharge because some of the assumptions are not accurate. E.g. the arbitrarily splitting in to regions of fixed Manning *n* is probably not what is occurring in the actual channel. However it will give an acceptable estimate as long as care is taken in choosing these regions.

1.9 *The Application of the Energy equation* for Rapidly Varied Flow

Rapid changes in stage and velocity occur whenever there is a sudden change in cross-section, a very steep bed-slope or some obstruction in the channel. This type of flow is termed **rapidly varied flow**. Typical example are flow over sharp-crested weirs and flow through regions of greatly changing cross-section (Venturi flumes and broad-crested weirs).
Rapid change can also occur when there is a change from super-critical to sub-critical flow (see later) in a channel reach at a hydraulic jump.

In these regions the surface is highly curved and the assumptions of hydro static pressure distribution and parallel streamlines do not apply. However it is possibly to get good approximate solutions to these situations yet still use the energy and momentum concepts outlined earlier. The solutions will usually be sufficiently accurate for engineering purposes.

1.9.1 The energy (Bernoulli) equation

The figure below shows a length of channel inclined at a slope of θ and flowing with uniform flow.

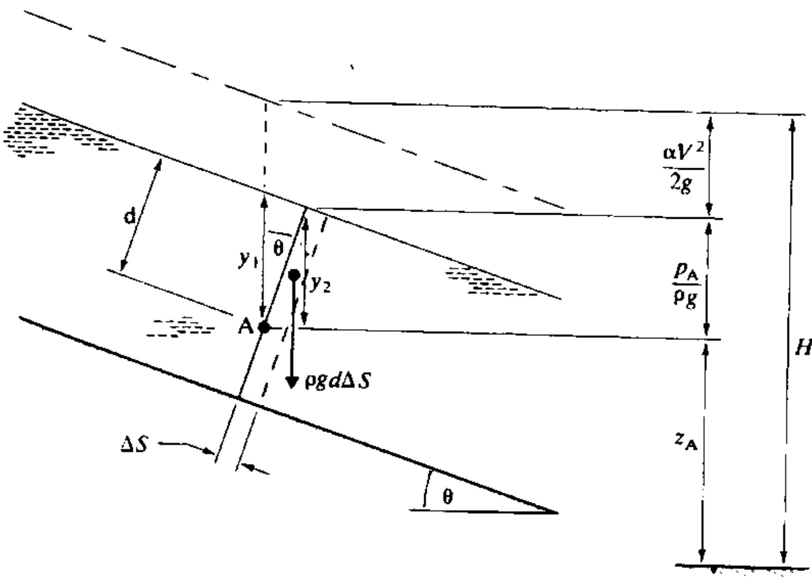


Figure of channel in uniform flow

Recalling the Bernoulli equation (1.6)

$$\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z = \text{constant}$$

And assuming a hydrostatic pressure distribution we can write the pressure at a point on a streamline, A say, in terms of the depth *d* (the depth measured from the water surface in a direction **normal** to the bed) and the channel slope.

$$p_A = \rho g d$$

[Note: in previous derivation we used *y* in stead of *d* – they are the same.]

In terms of the **vertical** distance

$$d = \frac{y_2}{\cos\theta} = y_1 \cos\theta$$

$$y_2 = y_1 \cos^2\theta$$

So

$$p_A = \rho g y_1 \cos^2\theta$$

So the pressure term in the above Bernoulli equation becomes

$$\frac{p_A}{\rho g} = y_1 \cos^2 \theta$$

As channel slope in open channel are very small (1:100 ≡ θ = 0.57 and $\cos^2\theta = 0.9999$) so unless the channel is unusually steep

$$\frac{p_A}{\rho g} = y_1$$

And the Bernoulli equation becomes

$$y + \frac{\alpha V^2}{2g} + z = \mathrm{H}$$

Equation 1.14

1.9.2 Flow over a raised hump - Application of the Bernoulli equation

Steady uniform flow is interrupted by a raised bed level as shown. If the upstream depth and discharge are known we can use equation 1.14 and the continuity equation to give the velocity and depth of flow over the raised hump.

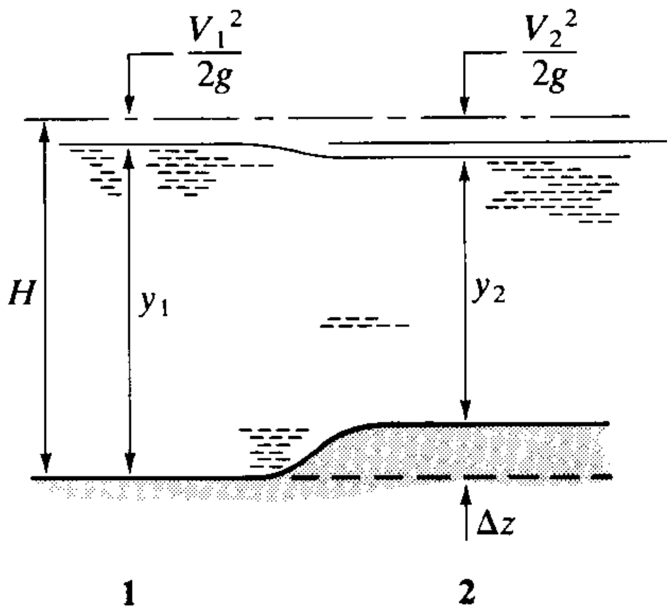


Figure of the uniform flow interrupted by a raised hump

Apply the Bernoulli equation between sections 1 and 2. (assume a horizontal rectangular channel $z_1 = z_2$ and take $\alpha = 1$)

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z$$

Equation 1.15

use the continuity equation

$$V_1 A_1 = V_2 A_2 = Q$$

$$V_1 y_1 = V_2 y_2 = \frac{Q}{B} = q$$

Where q is the flow *per unit width*.

Substitute this into the Bernoulli equation to give:

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z$$

rearranging:

$$2gy_2^3 + y_2^2\left(2g\Delta z - 2gy_1 - \frac{q^2}{y_1^2}\right) + q^2 = 0$$

Thus we have a cubic with the only unknown being the downstream depth, y_2. There are three solutions to this - only one is correct for this situation. We must find out more about the flow before we can decide which it is.

1.9.3 Specific Energy

The extra information needed to solve the above problem can be provided by the *specific energy equation.*

Specific energy, E_s, is defined as the energy of the flow with reference to the channel bed as the datum:

$$E_s = y + \frac{\alpha V^2}{2g}$$

Equation 1.16

For steady flow this can be written in terms of discharge Q

$$E_s = y + \frac{\alpha (Q/A)^2}{2g}$$

For a rectangular channel of width *b, Q/A = q/y*

$$E_s = y + \frac{\alpha q^2}{2gy^2}$$

$$(E_s - y)y^2 = \frac{\alpha q^2}{2g} = \text{constant}$$

$$(E_s - y) = \frac{\text{constant}}{y^2}$$

This is a cubic in y. It has three solutions but only two will be positive (so discard the other).

1.9.4 Flow over a raised hump - revisited. Application of the Specific energy equation.

The specific energy equation may be used to solve the raised hump problem. The figure below shows the hump and stage drawn alongside a graph of Specific energy E_s against y.

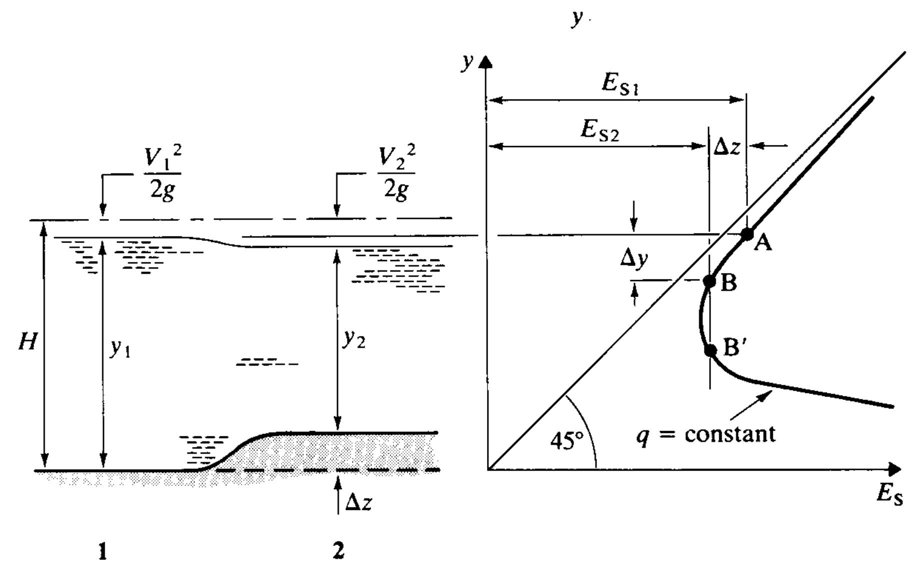


Figure of raised bed hump and graph of specific energy

The Bernoulli equation was applied earlier to this problem and equation (from that example) 1.15 may be written in terms of specify energy:

$$E_{s1} = E_{s2} + \Delta z$$

These points are marked on the figure. Point A on the curve corresponds to the specific energy at point 1 in the channel, but Point B **or** Point B' on the graph may correspond to the specific energy at point 2 in the channel.

All point in the channel between point 1 and 2 **must** lie on the specific energy curve between point A and B or B'. To reach point B' then this implies that $E_{S1} - E_{S2} > \Delta z$ which is not physically possible. So point B on the curve corresponds to the specific energy and the flow depth at section 2.

1.9.5 Example of the raised bed hump.

A rectangular channel with a flat bed and width 5m and maximum depth 2m has a discharge of 10m^3/s. The normal depth is 1.25 m. What is the depth of flow in a section in which the bed rises 0.2m over a distance 1m.
Assume frictional losses are negligible.

$$E_{s1} = E_{s2} + \Delta z$$

$$E_{s1} = 1.25 + \frac{\left(\frac{10}{1.25 \times 5}\right)^2}{2g} = 1.38$$

$$E_{s2} = y_2 + \frac{\left(\frac{10}{5 \times y_2}\right)^2}{2g} = y_2 + \frac{0.2039}{y_2^2}$$

$$1.38 = y_2 + \frac{0.2039}{y_2^2} + 0.2$$

$$1.18 = y_2 + \frac{0.2039}{y_2^2} = E_{s2}$$

Again this can be solved by a trial and error method:

y_2	E_{s2}
0.9	1.15
1.0	1.2
0.96	1.18

i.e. the depth of the raised section is 0.96m or the water level (stage) is 1.16m a drop of 9cm when the bed has raised 20cm.

1.10 Critical , Sub-critical and super critical flow

The specific energy change with depth was plotted above for a constant discharge *Q,* it is also possible to plot a graph with the specific energy fixed and see how *Q* changes with depth. These two forms are plotted side by side below.

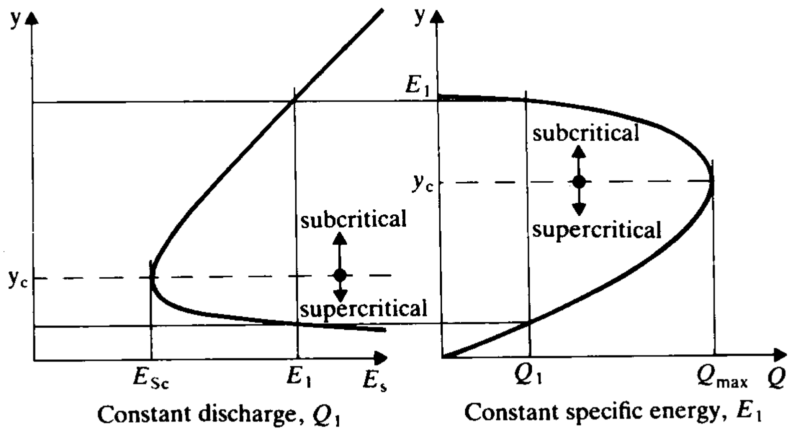


Figure of variation of Specific Energy and Discharge with depth.

From these graphs we can identify several important features of rapidly varied flow.

For a fixed discharge:

1. The specific energy is a minimum, E_{sc}, at depth y_c,
   This depth is known as *critical depth*.
2. For all other values of E_s there are two possible depths. These are called alternate depths. For
   subcritical flow $y > y_c$

   supercritical flow $y < y_c$

For a fixed Specific energy

1. The discharge is a maximum at critical depth, y_c.
2. For all other discharges there are two possible depths of flow for a particular E_s i.e. There is a sub-critical depth and a super-critical depth with the same E_s.

An equation for critical depth can be obtained by setting the differential of E_s to zero:

$$E_s = y + \frac{\alpha (Q/A)^2}{2g}$$

$$\frac{dE_s}{dy} = 0 = 1 + \frac{\alpha Q^2}{2g}\frac{d}{dA}\left(\frac{1}{A^2}\right)\frac{dA}{dy}$$

Since $\delta A = B\delta y$, in the limit $dA/dy = B$ and

$$0 = 1 - \frac{\alpha Q^2}{2g} B_c 2A_c^{-3}$$

$$\frac{\alpha Q^2 B_c}{g A_c^3} = 1$$

Equation 1.17

For a rectangular channel $Q = qb$, $B = b$ and $A = by$, and taking $\alpha = 1$ this equation becomes

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

as $V_c\, y_c = q$

$$V_c = \sqrt{g y_c}$$

Equation 1.18

Substituting this in to the specific energy equation

$$E_{sc} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$y_c = \frac{2}{3}E_{sc}$$

Equation 1.19

1.11 The Froude number

The Froude number is defined for channels as:

$$Fr = \frac{V}{\sqrt{gD_m}}$$

Equation 1.20

Its physical significance is the ratio of inertial forces to gravitational forces squared

$$Fr^2 = \frac{\text{inertial force}}{\text{gravitational force}}$$

It can also be interpreted as the ratio of water velocity to wave velocity

$$Fr = \frac{\text{water velocity}}{\text{wave velocity}}$$

This is an extremely useful non-dimensional number in open-channel hydraulics.

Its value determines the regime of flow – sub, super or critical, and the direction in which disturbances travel

Fr < 1 sub-critical
water velocity > wave velocity
upstream levels **affected** by downstream controls

Fr = 1 critical

Fr > 1 super-critical
water velocity < wave velocity
upstream levels **not affected** by downstream controls

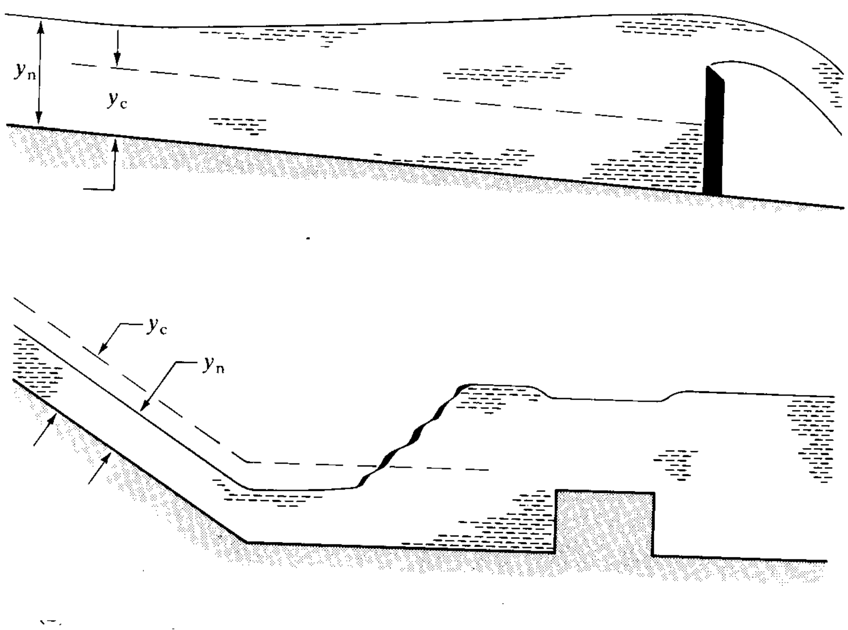


Figure of sub and super critical flow and transmission of disturbances

1.12 Application of the Momentum equation for Rapidly Varied Flow

The hydraulic jump is an important feature in open channel flow and is an example of rapidly varied flow. A hydraulic jump occurs when a super-critical flow and a sub-critical flow meet. The jump is the mechanism for the to surface to join. They join in an extremely turbulent manner which causes large energy losses.

Because of the large energy losses the energy or specific energy equation cannot be use in analysis, the momentum equation is used instead.

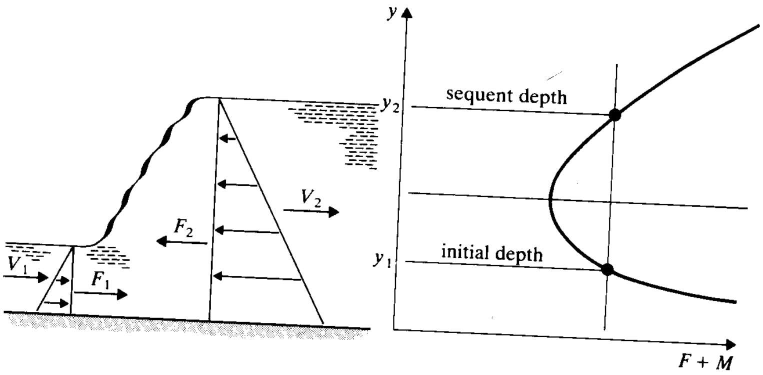


Figure of forces applied to the control volume containing the hydraulic jump

Resultant force in x- direction = $F_1 - F_2$
Momentum change = $M_2 - M_1$

$$F_1 - F_2 = M_2 - M_1$$

Or for a constant discharge

$$F_1 + M_1 = F_2 + M_2 = \text{constant}$$

For a rectangular channel this may be evaluated using

$$F_1 = \rho g \frac{y_1}{2} y_1 b \qquad F_2 = \rho g \frac{y_2}{2} y_2 b$$

$$M_1 = \rho Q V_1 \qquad M_2 = \rho Q V_2$$

$$= \rho Q \frac{Q}{y_1 b} \qquad = \rho Q \frac{Q}{y_2 b}$$

Substituting for these and rearranging gives

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$$

Equation 1.21

Or

$$y_1 = \frac{y_2}{2}\left(\sqrt{1 + 8Fr_2^2} - 1\right)$$

Equation 1.22

So knowing the discharge and either one of the depths on the upstream or downstream side of the jump the other – or conjugate depth – may be easily computed.

More manipulation with Equation 1.19 and the specific energy give the energy loss in the jump as

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

Equation 1.23

These are useful results and which can be used in gradually varied flow calculations to determine water surface profiles.

In summary, a hydraulic jump will only occur if the upstream flow is super-critical. The higher the upstream Froude number the higher the jump and the greater the loss of energy in the jump.

1.13 Gradually varied flow

In the previous section of rapidly varied flow little mention was made of losses due to friction or the influence of the bed slope. It was assumed that frictional losses were insignificant – this is reasonable because rapidly varied flow occurs over a very short distance. However when it comes to long distances they become very important, and as gradually varied flow occurs over long distances we will consider friction losses here.

In the section on specific energy it was noted that there are two depth possible in steady flow for a given discharge at any point in the channel. (One is super-critical the other depth sub-critical.) The solution of the Manning equation results in only one depth – the normal depth.

It is the inclusion of the channel slope and friction that allow us to decide which of the two depths is correct. i.e. the channel slope and friction determine whether the uniform flow in the channel is sub or super-critical.

The procedure is

i. Calculate the normal depth from Manning's equation (1.11)
ii. Calculate the critical depth from equation 1.17

The normal depth may be greater, less than or equal to the critical depth.
For a given channel and roughness there is only one slope that will give the normal depth equal to the critical depth. This slope is known as the critical slope (S_c).

If the slope is **less** than Sc the normal depth will be **greater** than critical depth and the flow will be **sub-critical** flow. The slope is termed *mild*.

If the slope is **greater** than Sc the normal depth will be less than critical depth and the flow will be **super-critical** flow. The slope is termed *steep*.

1.13.1 Example of critical slope calculation

We have Equation 1.11 that gives normal depth

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_o^{1/2}$$

and equation 1.17 that given critical depth

$$\frac{\alpha Q^2 B_c}{2gA_c^3} = 1$$

Rearranging these in terms of Q and equating gives

$$\frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_c^{1/2} = \sqrt{\frac{gA^3}{B}}$$

For the simple case of a wide rectangular channel, width B = b, A = by and P≅b. And the above equation becomes

$$S_c = \frac{gn^2}{y_c^{1/3}}$$

Equation 1.24

1.13.2 Transitions between sub and super critical flow

If sub critical flow exists in a channel of a mild slope and this channel meets with a steep channel in which the normal depth is super-critical there must be some change of surface level between the two. In this situation the surface changes gradually between the two. The flow in the joining region is known as gradually varied flow.

This situation can be clearly seen in the figure on the left below. Note how at the point of joining of the two channels the depth passes through the critical depth.

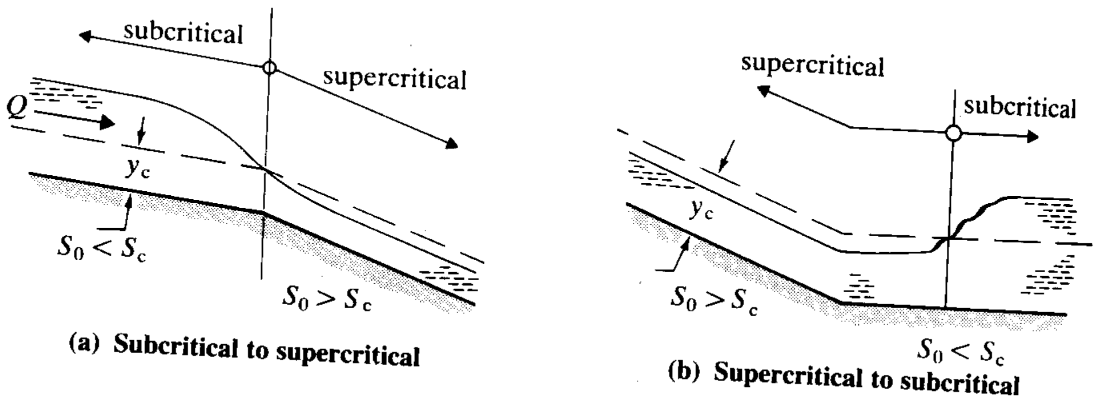


Figure of transition from sup to super-critical flow

If the situation is reversed and the upstream slope is steep, super critical flow, and the down stream mild, sub-critical, then there must occur a hydraulic jump to join the two. There may occur a short length of gradually varied flow between the channel junction and the jump. The figure above right shows this situation:

Analysis of gradually varied flow can identify the type of profile for the transition as well as the position hydraulic jumps.

1.14 The equations of gradually varied flow

The basic assumption in the derivation of this equation is that the change in energy with distance is equal to the friction loses.

$$\frac{dH}{dx} = -S_f$$

The Bernoulli equation is:

$$y + \frac{\alpha V^2}{2g} + z = \mathrm{H}$$

Differentiating and equating to the friction slope

$$\frac{d}{dx}\left(y + \frac{\alpha V^2}{2g}\right) = -\frac{dz}{dx} - S_f$$

Or

$$\frac{dE_s}{dx} = S_o - S_f$$

Equation 1.25

where S_o is the bed slope

We saw earlier how specific energy change with depth (derivation of Equation 1.17)

$$\frac{dE_s}{dy} = 1 + \frac{Q^2 B_c}{g A_c^3} = 1 + Fr^2$$

Combining this with equation 1.25 gives

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

Equation 1.26

This is the basic equation of gradually varied flow. It describes how the depth, y, changes with distance x, in terms of the bed slope S_o, friction S_f and the discharge, Q, and channels shape (encompassed in Fr and S_f).

Equations 1.25 and 1.26 are differential equations equating relating depth to distance. There is no explicit solution (except for a few special cases in prismatic channels). Numerical integration is the only practical method of solution. This is normally done on computers, however it is not *too* cumbersome to be done by hand.

1.15 Classification of profiles

Before attempting to solve the gradually varied flow equation a great deal of insight into the type of solutions and profiles possible can be gained by taking some time to examine the equation. Time spent over this is almost compulsory if you are to understand steady flow in open channels.

For a given discharge, S_f and Fr^2 are functions of depth.

$$S_f = \frac{n^2 Q^2 P^{4/3}}{A^{10/3}}$$

$$Fr^2 = \frac{Q^2 B}{g A^3}$$

A quick examination of these two expressions shows that they both increase with A, i.e. increase with y.

We also know that when we have uniform flow

$$S_f = S_o \qquad \text{and} \qquad y = y_n$$

So

$S_f > S_o$ when $y < y_n$

$S_f < S_o$ when $y > y_n$

and

$Fr^2 > 1$ when $y < y_c$

$Fr^2 < 1$ when $y > y_c$

From these inequalities we can see how the sign of dy/dx i.e. the surface slope changes for different slopes and Froude numbers.

Taking the example of a mild slope, shown in the figure below:

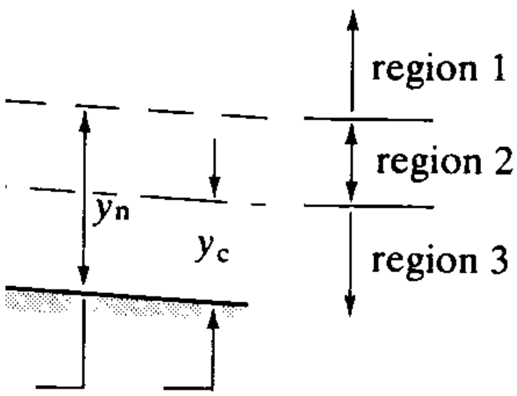


Figure of zones / regions

The normal and critical depths are shown (as it is mild normal depth is greater than critical depth). Treating the flow as to be in three zones:

i. zone 1, above the normal depth
ii. zone 2, between normal and critical depth
iii. zone 3, below critical depth

The direction of the surface inclination may thus be determined.

zone 1

$y > y_n > y_c$	$S_f < S_o$	$Fr^2 < 1$	→	dy/dx is positive, surface rising

zone 2

$y_n > y > y_c$	$S_f > S_o$	$Fr^2 < 1$	→	dy/dx is negative surface falling

zone 3

$y_n > y_c > y$	$S_f > S_o$	$Fr^2 > 1$	→	dy/dx is positive surface rising

The condition at the boundary of the gradually varied flow may also be determined in a similar manner:

zone 1
As $y \to \infty$ then S_f and $Fr \to 0$ and $dy/dx \to S_o$
Hence the water surface is asymptotic to a horizontal line for it maximum

As $y \to y_n$ then $S_f \to S_o$ and $dy/dx \to 0$
Hence the water surface is asymptotic to the line $y = y_n$ i.e. uniform flow.

zone 2

As for zone 1 as y approached the normal depth:
As $y \to y_n$ then $S_f \to S_o$ and $dy/dx \to 0$
Hence the water surface is asymptotic to the line $y = y_n$

But a problem occurs when y approaches the critical depth:
As $y \to y_c$ then $Fr \to 1$ and $dy/dx \to \infty$

This is physically impossible but may be explained by the pointing out that in this region the gradually varied flow equation is not applicable because at this point the fluid is in the rapidly varied flow regime.
In reality a very steep surface will occur.

zone 3

As for zone 2 a problem occurs when y approaches the critical depth:
As $y \to y_c$ then $Fr \to 1$ and $dy/dx \to \infty$

Again we have the same physical impossibility with the same explanation.
And again in reality a very steep surface will occur.

As $y \to 0$ then $dy/dx \to S_o$ the slope of bed of the channel*!*
The gradually varied flow equation is not valid here but it is clear what occurs.

In general, normal depth is approached asymptotically and critical depth at right angles to the channel bed.

The possible surface profiles within each zone can be drawn from the above considerations. These are shown for the mild sloped channel below.

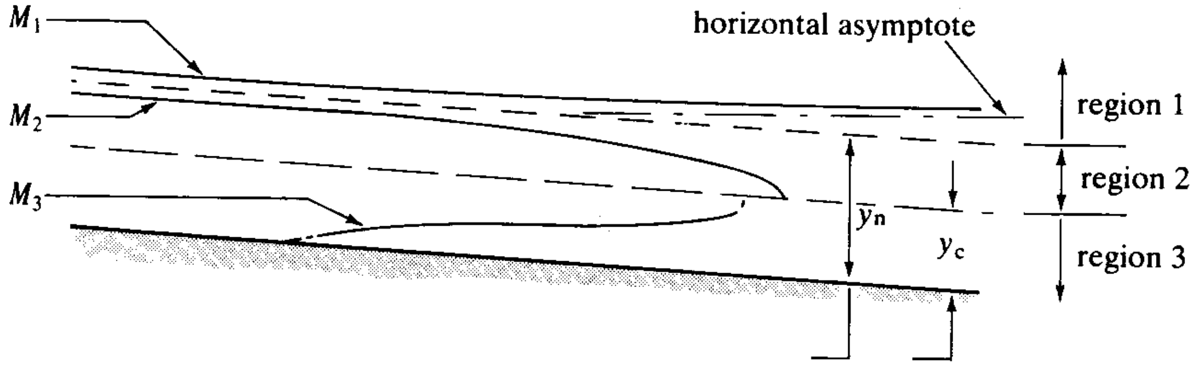


Figure of gradually varied flow surface profiles in a mild sloped channel

The surface profile in zone 1 of a mild slope is called an M1 curve, in zone 2 an M2 curve and in zone 3 an M3 curve.
All the possible surface profiles for all possible slopes of channel (there are 15 possibilities) are shown in the figure on the next page.

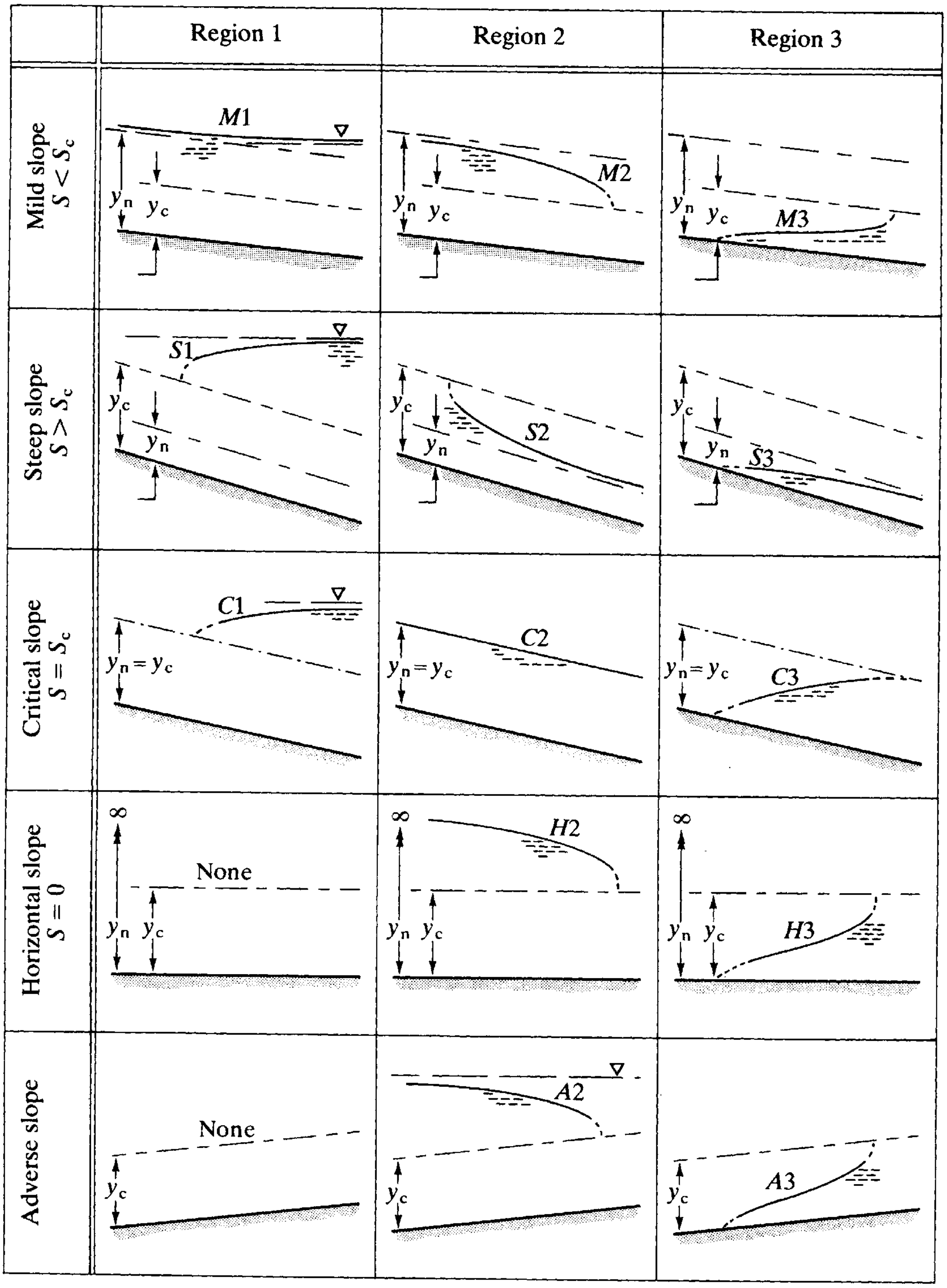


Figure of the possible gradually varied flow profiles

1.16 How to determine the surface profiles

Before one of the profiles discussed above can be decided upon two things must be determined for the channel and flow:

a) Whether the slope is mild, critical or steep. The normal and critical depths must be calculated for the design discharge
b) The positions of any *control points* must be established. Control points are points of known depth or relationship between depth and discharge. Example are weirs, flumes, gates or points where it is known critical flow occurs like at free outfalls, or that the flow is normal depth at some far distance down stream.

Once these control points and depth position has been established the surface profiles can be drawn to join the control points with the insertion of hydraulic jumps where it is necessary to join sub and super critical flows that don't meet at a critical depth.

Below are two examples.

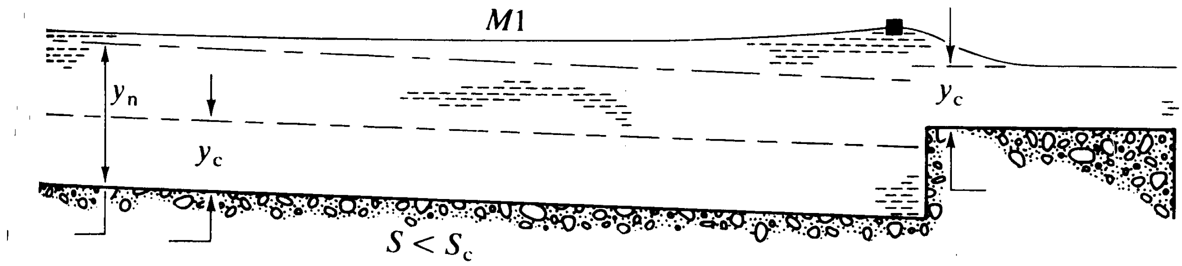


Figure of example surface profile due to a broad crested weir

This shows the control point just upstream of a broad crested weir in a channel of mild slope. The resulting curve is an M1.

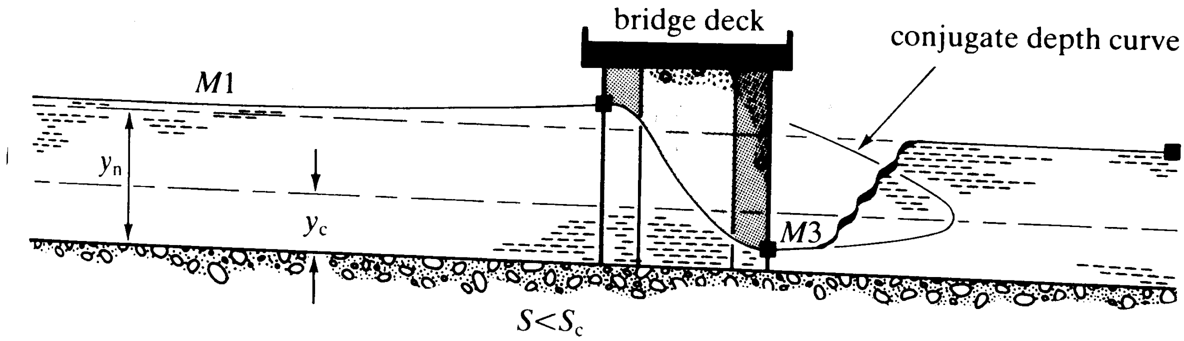


Figure of example surface profile through a bridge when in floo

This shows how a bridge may act as a control – particularly under flood conditions. Upstream there is an M1 curve then flow through the bridge is rapidly varied and the depth drops below critical depth so on exit is super critical so a short M3 curve occurs before a hydraulic jump takes the depth back to a sub-critical level.

1.17 Method of solution of the Gradually varied flow equation

There are three forms of the gradually varied flow equation:

$$\frac{dH}{dx} = -S_f$$

Equation 1.26

$$\frac{dE_s}{dx} = S_o - S_f$$

Equation 1.27

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

Equation 1.28

In the past direct and graphical solution methods have been used to solve these, however these method have been superseded by numerical methods which are now be the only method used.

1.17.1 Numerical methods

All (15) of the gradually varied flow profiles shown above may be quickly solved by simple numerical techniques. One computer program can be written to solve most situations.

There are two basic numerical methods that can be used

i. Direct step – distance from depth
ii. Standard step method – depth from distance

1.17.2 The direct step method – distance from depth

This method will calculate (by integrating the gradually varied flow equation) a distance for a given change in surface height.

The equation used is 1.28, which written in finite difference form is

$$\Delta x = \Delta y \left(\frac{1 - Fr^2}{S_o - S_f} \right)_{\text{mean}}$$

Equation 1.29

The steps in solution are:
1. Determine the control depth as the starting point
2. Decide on the expected curve and depth change if possible
3. Choose a suitable depth step Δy
4. Calculate the term in brackets at the “mean” depth ($y_{\text{initial}} + \Delta y/2$)
5. Calculate Δx
6. Repeat 4 and 5 until the appropriate distance / depth changed reached

This is really best seen demonstrated in an example. [See the example on the web site for this module: www.efm.leeds.ac.uk/CIVE/CIVE2400]

1.17.3 The standard step method – depth from distance

This method will calculate (by integrating the gradually varied flow equation) a depth at a given distance up or downstream.

The equation used is 1.27, which written in finite difference form is

$$\Delta E_s = \Delta x\left(S_o - S_f\right)_{\text{mean}}$$

Equation 1.30

The steps in solution are similar to the direct step method shown above but *for each Δx* there is the following iterative step:

1. Assume a value of depth y (the control depth or the last solution depth)
2. Calculate the specific energy E_{sG}
3. Calculate S_f
4. Calculate ΔE_s using equation 1.30
5. Calculate $\Delta E_{s\,(x+\Delta x)} = E_s + \Delta E$
6. Repeat until $\Delta E_{s\,(x+\Delta x)} = E_{sG}$

Again this is really best understood by means of an example. [See the example on the web site for this module: www.efm.leeds.ac.uk/CIVE/CIVE2400]

1.17.4 The Standard step method – alternative form

This method will again calculate a depth at a given distance up or downstream but this time the equation used is 1.26, which written in finite difference form is

$$\Delta H = -\Delta x\left(S_f\right)_{\text{mean}}$$

Equation 1.31

Where H is given by equation 1.14

$$y + \frac{\alpha V^2}{2g} + z = \mathrm{H}$$

The strategy is the same as the first standard step method, with the same necessity to iterate for each step.

1.18 Structures

1.19 Critical depth meters

The effect of a local rise in the bed level on the flow has been discussed earlier. It was shown that the depth would fall as the flow went over the rise. If this rise were large enough (for the particular velocity) the fall would be enough to give critical depth over the rise. Increasing the rise further would not decrease the depth but it would remain at critical depth. This observation may be confirmed by studying further the specific energy equation in a similar way to earlier.

The fact that depth is critical over the rise and that critical depth can be calculated quite easily for a give discharge is made use of in hydraulic structures for flow measurement. Actually it is the converse of the above, that the discharge can be easily calculated if the critical depth is know, that is most useful.

1.19.1 Broad-crested weir

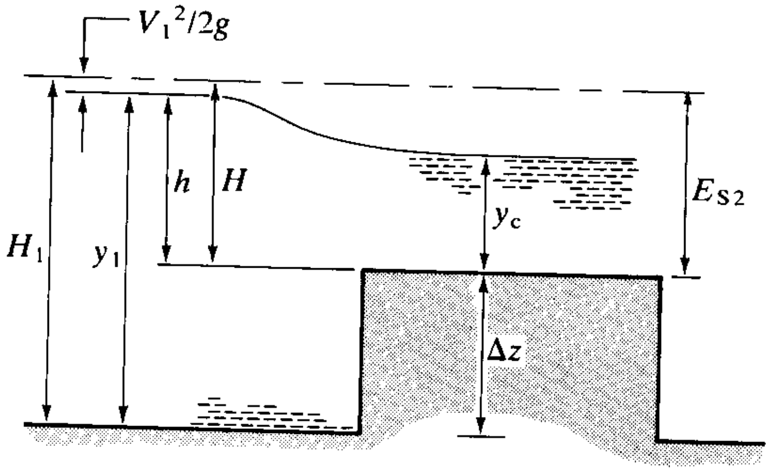


Figure of flow over a broad crested weir

Assuming that the depth is critical over the rise then we know

$$V_2 = V_c = \sqrt{g y_c}$$

$$Q = A V_c = b y_c \sqrt{g y_c}$$

where b is the width of the channel

Earlier Equation 1.19 it was shown that

$$y_c = \frac{2}{3} E_{s2}$$

Assuming no energy loss between 1 and 2

$$E_{s2} = h + \frac{V_1^2}{2g} = H$$

$$y_c = \frac{2}{3} H$$

So

$$Q = \frac{2}{3}\sqrt{\frac{2g}{3}} b h^{3/2}$$

$$Q = 1.705 b h^{3/2}$$

in practice there are energy losses upstream of the weir. To incorporate this a coefficient of discharge and of velocity are introduced to give

$$Q = C_d C_v 1.705 b h^{3/2}$$

1.19.2 Flumes

A second method of measuring flow by causing critical depth is to contract the flow. Similar specific energy arguments to those used for a raised bed can be used in analysis of this situation, and they come to similar predictions that depth will fall and not fall below critical depth.

It is also possible to combine the two by putting a raised section within the narrowed section.

These types of structures are known as flume, and the first type is known as a Venturi flume.

1.19.3 Venturi flume

Flumes are usually designed to achieve critical depth in the narrowest section (the throat) while also giving a very small afflux.

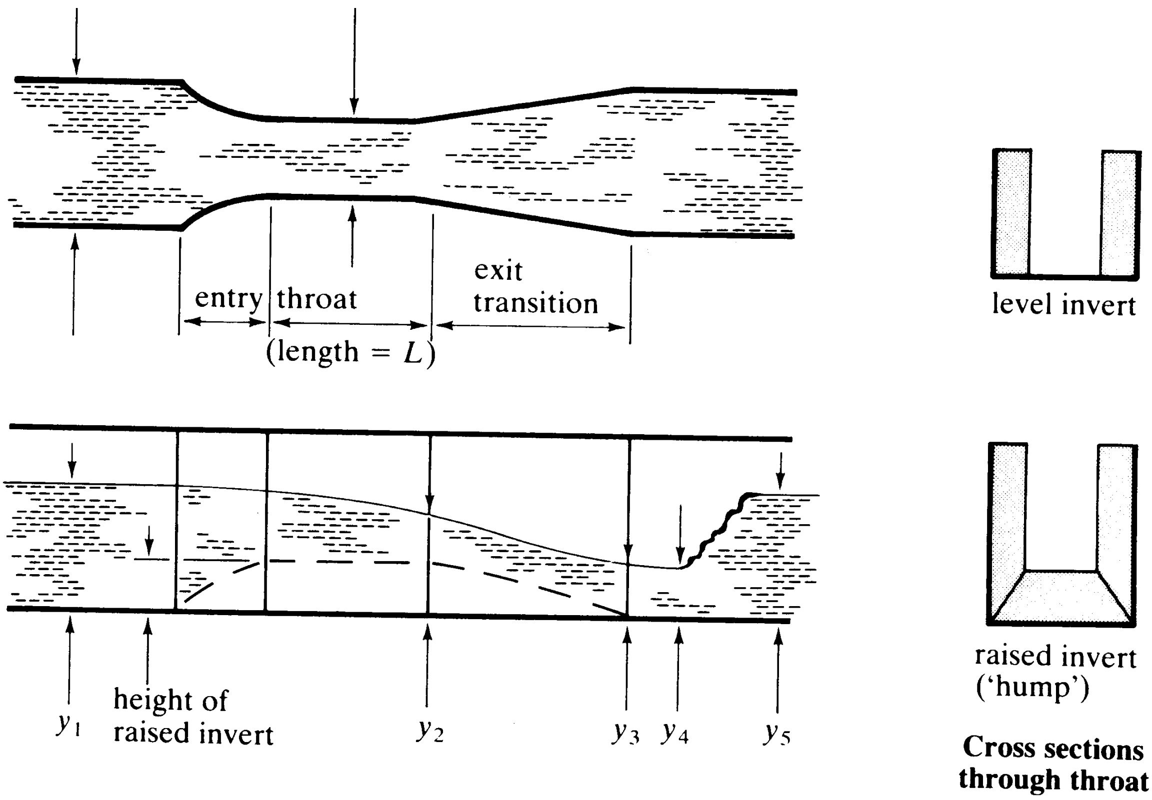


Figure of section through a Venturi flume

The general equation for the ideal discharge through a flume can be obtained from the energy and continuity considerations.

The energy equation gives:

$$E_s = y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

by continuity $V_1 = Q/by_1$ and $V_2 = Q/b_2y_2$

which when substituted into the energy equation gives

$$Q = by_1 \sqrt{\frac{2g(y_1 - y_2)}{(by_1 / b_2 y_2)^2 - 1}}$$

If critical flow is obtained in the throat then $y_2 = 2/3\ E_s$ which can be substituted to give (in SI units)

$$Q = 1.705 b_2 E_s^{3/2}$$

or introducing a velocity correction factor and a discharge coefficient

$$Q = 1.705 b_2 C_v C_d y_1^{3/2}$$