CE 533
Hydraulic System Design I

Chapter 1
Gradually-Varied Flow

1.1 Introduction

- A control is any feature which determines a relationship between depth and discharge. The uniform flow itself may be thought of as a control, since from a resistance equation such as Manning's we may, given the depth, calculate the discharge.
- However, uniform flow is not, of course, associated with particular localized features in the channel, it is the state which the flow tends to assume in a long uniform channel when NO OTHER CONTROLS are present. If there are other controls they tend to pull the flow away from the uniform condition, and there will be a transition-which may be gradual or abrupt between the two states of flow.
- In this chapter, the gradually-varied flow will be considered.

Types of Channel Slopes

The channel slopes can be classified as follows:

- A mild slope is one on which uniform flow is subcritical,
- A steep slope is one on which uniform flow is supercritical,
- A critical slope is one on which uniform flow is critical.
- Let y_o and y_c designate the uniform and critical depths for a given discharge Q, respectively, then we can write that:

  Mild slope $\longrightarrow$ $y_o > y_c$

  Steep slope $\longrightarrow$ $y_o < y_c$

  Critical slope $\longrightarrow$ $y_o = y_c$
- The classification of the slope will depend on the roughness, on the magnitude of the slope itself, and to a lesser extent on the discharge. Same slope, depending on the roughness of the channel bottom can be mild, critical or steep.

Critical Slope, S_c

- On the other hand, a critical slope can be defined by using the Manning equation as:

$$S_c = \left(\frac{Qn}{A_c R_c} \right)^2, \text{ where } R_c = \frac{A_c}{P_c}$$

Where A_c, P_c, R_c are the area, wetted perimeter, and hydraulic radius of the flow computed by using the critical depth y_c, for the discharge Q, respectively.

Then, the classification of slopes become:

Mild slope $\rightarrow S_o < S_c$

Steep slope $\rightarrow S_o > S_c$

Critical slope $\rightarrow S_o = S_c$

Horizontal slope $\rightarrow S_o = 0$

Adverse slope $\rightarrow S_o < 0$

1.3 Basic Assumptions and Equations of GVF

There are two basic assumption used in GVF. These are:

1.In gradually-varied flow , we can use resistance equations such as Chezy's or Manning's to describe the state of flow provided that the slope S is interpreted as the slope of the total energy lines, S_f.

$$S_f = \frac{V^2}{C^2 R} \quad \text{or} \quad S_f = \left(\frac{Vn}{R^{2/3}}\right)^2$$

2.In gradually-varied flow, it may be assumed that the streamlines are almost straight and parallel. This means that the pressure distribution is hydrostatic, i.e.:

$$\mathbf{P = \gamma y}$$

- Now let’s examine the flow in order to obtain a complete description of the longitudinal flow variation within a non uniform transition region.
- The energy equation at any cross section is

$$\mathrm{H} = \mathrm{z} + \mathrm{y} + \frac{\mathrm{v}^2}{2\mathrm{g}}$$

the derivative with respect to x gives:

$$\frac{dH}{dx} = \frac{d}{dx}\left(z + y + \frac{v^2}{2g} \right) = S_f = \frac{V^2}{C^2 R}$$

or

$$\frac{d}{dx}\left(y + \frac{V^2}{2g} \right) = -\frac{dz}{dx} - S_f = S_o - S_f$$

Or it can be written in terms of specific energy E as:

$$\frac{dE}{dx} = S_o - S_f$$

On the other hand: $$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

For a given Q, E=E(y), and the derivative of E with respect to y is:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{agA^3}\frac{dA}{dy}$$

for an open-channel flow

dA= Tdy

Therefore dE/dy and dE/dx become:

$$\frac{dE}{\mathrm{d}y} = 1 - \frac{Q^2 T}{gA^3} = 1 - \mathrm{F}_{\mathrm{r}}^2$$

$$\frac{dE}{dx} = \frac{dE}{dy}\frac{dy}{dx} = S_o - S_f$$ or

$$\left(1 - F_r^2\right)\frac{dy}{dx} = S_o - S_f$$

It can also be written as:

$$\frac{dy}{dx} = \frac{S_o - S_f}{\left(1 - F_r^2\right)}$$

Therefore the equations of the gradually-varied flow are:

$$\frac{dE}{dx} = S_o - S_f \qquad \text{and}$$

$$\frac{dy}{dx} = \frac{S_o - S_f}{\left(1 - F_r^{\,2}\right)}$$

This is the differential equation for y=y(x), but it is NOT in general explicitly soluble, but many numerical methods have been developed for its solution, which will be considered later. Meanwhile we will consider certain general questions relating to the solution.

1.4 Water Surface Profiles (Longitudinal Profiles)

It is important to systematically classify the water surface profiles in a channel before computation of flow profiles is carried out. Such classification helps to get an overall understanding of how the flow depth varies in a channel. It also helps to detect any mistakes made in the flow computation.

The variation of water surface profile can be obtained without solving the equation of GVF:

$$\frac{dy}{dx} = \frac{S_o - S_f}{\left(1 - F_r^2\right)}$$

For a specified value of Q, both F_r and S_f are functions of the depth, y. In fact, both F_r and S_f will decrease as y increases. Recalling the definitions for the normal depth, y_o ,and the critical depth, y_c , the following inequalities can be stated.

Remember that: $$F_r = \frac{V_c}{\sqrt{gy_c}} = 1 \quad \Rightarrow \quad V_c = \sqrt{gy_c}$$

.

For a given discharge, Q, and depth of flow y:

IF $\left.\begin{array}{l} y \rangle y_c \\ V \langle V_c \end{array}\right\} F_r \langle 1$ and IF $\left.\begin{array}{l} y \langle y_c \\ V \rangle V_c \end{array}\right\} F_r \rangle 1$

Similarly' let's examine S_o and S_f wrt y_o and y:

in GVF: $S_f = \left(\frac{Vn}{R^{2/3}} \right)^2$ and for uniform flow $S_f = S_0$

Therefore, for a given discharge, Q, and depth of flow y:

IF $\left. \begin{matrix} y \rangle y_0 \\ V \langle V_0 \end{matrix} \right\} S_f \langle S_0$ and IF $\left. \begin{matrix} y \langle y_0 \\ V \rangle V_0 \end{matrix} \right\} S_f \rangle S_0$

A gradually varied flow profile is classified based on the channel slope, and the magnitude of flow depth, y in relation to y_0 and y_c . The channel slope is classified based on the relative magnitudes of the normal depth, y_0 and the critical depth, y_c.

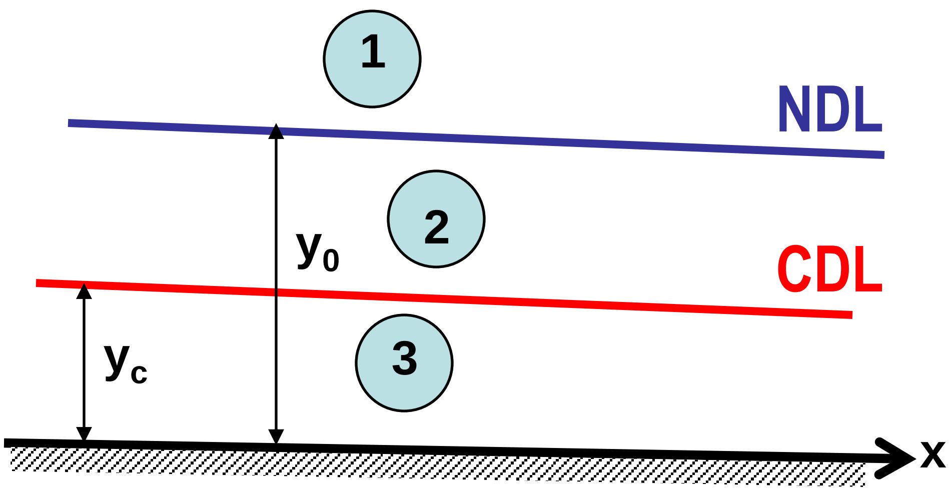
- $y_0 > y_c$: "Mild slope" (M)
- $y_0 < y_c$: "Steep slope" (S)
- $y_0 = y_c$: "Critical slope" (C)
- $S_0 = 0$: "Horizontal slope" (H)
- $S_0 < 0$: "Adverse slope" (A).

It may be noted here that slope is termed as "sustainable" slope when $S_0 > 0$ because flow under uniform conditions can occur for such a channel. Slope is termed as "unsustainable" when $S \leq 0$ since uniform flow conditions can never occur in such a channel.

Flow profiles associated with mild, steep, critical, horizontal, and adverse slopes are designated as M, S, C, H and A profiles, respectively.

•For a given discharge and channel, the normal depth y_0, and critical depth y_c can be computed. The flow area can be divided into 3 regions by the Normal Depth Line (NDL), the Critical Depth Line (CDL), and channel bottom.

•The space above the channel bed can be divided into three zones depending upon the inequality defined by equations:



$$\left.y > y_0\right\} \quad S_f < S_0$$
$$\left.y < y_0\right\} \quad S_f > S_0$$

$$\left.y > y_c\right\} \quad F_r < 1$$
$$\left.y < y_c\right\} \quad F_r > 1$$

- Therefore, the water surface profiles can be determined by using the equation of GVF together with these inequalities.

$$\frac{dy}{dx} = \frac{S_o - S_f}{\left(1 - F_r^2\right)}$$

$$y \gtrless y_c \qquad F_r \lessgtr 1$$

$$y \gtrless y_o \qquad S_f \lessgtr S_o$$

Mild Slope

Zone 1: $y>y_0>y_c$

$y > y_o \Rightarrow S_f < S_0$ and $y > y_c \Rightarrow F_r < 1$

Therefore:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} \Rightarrow \frac{dy}{dx} = \frac{+}{+} > 0 \quad \therefore \quad \text{y will increase in the flow direction.}$$

The lower limit is the normal depth line.

$$\text{As } y \to y_0 \quad S_f \to S_0 \quad \Rightarrow \frac{dy}{dx} \to 0$$

y will approach the NDL asymptotically.

The upper limit is ∞; as $y \to \infty$; $F_r \to 0$, and $S_f \to 0$

$$\therefore \quad \frac{dy}{dx} \to S_0 \quad \text{and water surface would look like:}$$

Zone 2: $y_0 > y > y_c$

$y < y_o \Rightarrow S_f > S_0$ and $y > y_c \Rightarrow F_r < 1$ *Therefore*:

$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} \Rightarrow \frac{dy}{dx} = \frac{-}{+} < 0 \therefore$ y will decrease in the flow direction.

The upper limit is the normal depth line.

As $y \rightarrow y_0$ $S_f \rightarrow S_0 \Rightarrow \frac{dy}{dx} \rightarrow 0$

y will approach the NDL asymptotically.

The lower limit is CDL, as $y \rightarrow y_c$, $F_r \rightarrow 1$

$\therefore \frac{dy}{dx} \rightarrow \infty$ means depth becomes vertical

This is physically impossible. Therefore, momentarily $S_f \rightarrow 0$

and hence $\frac{dy}{dx}$ has a finite value.

The water surface would look like:

Zone 3: $y_0 > y_c > y$

$y < y_o \Rightarrow \mathrm{S_f} > S_0$ and $y < y_c \Rightarrow \mathrm{F_r} > 1$ *Therefore* :

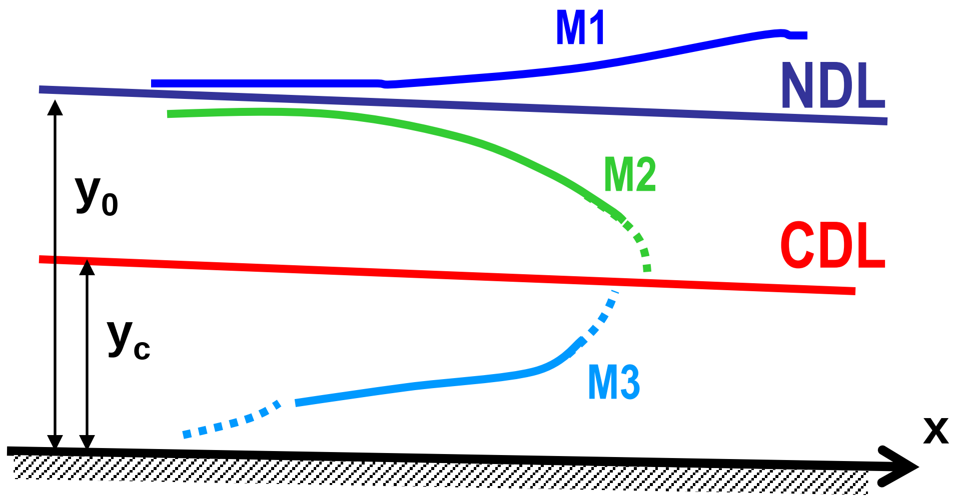
$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-}{-} > 0 \;\therefore\; \text{y will increase in the flow direction.}$$

The upper limit is the critical depth line.

As $\mathrm{y} \rightarrow \mathrm{y_c} \Rightarrow \dfrac{\mathrm{dy}}{\mathrm{dx}}$ has a finite value.

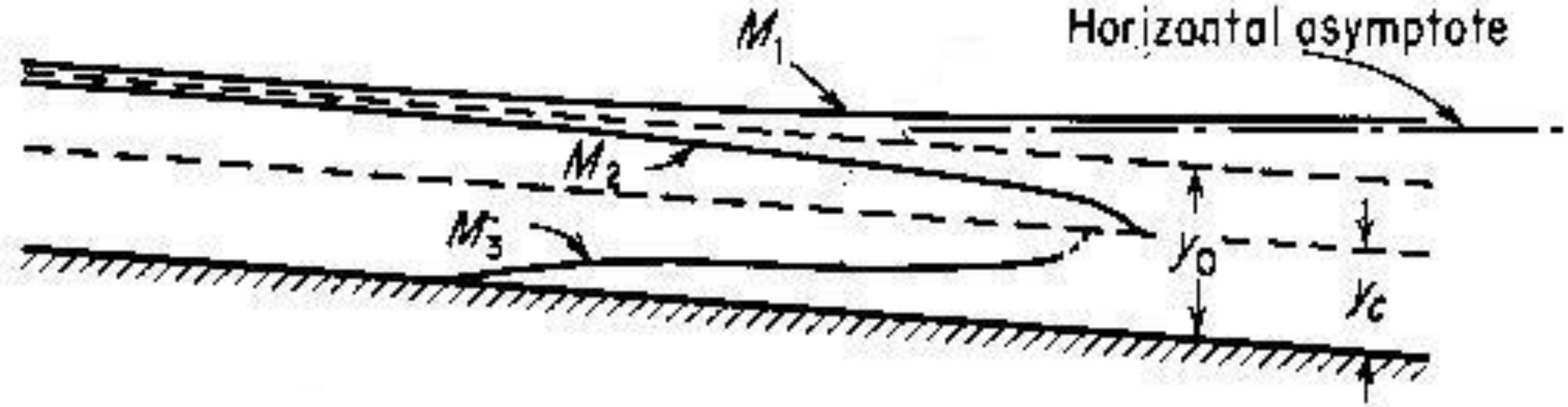
The lower limit is $\mathrm{y} \rightarrow 0$. This result is of little practical interest, since zero depth never actually occurs.

The water surface would look like :


M1
NDL
M2
CDL
M3
y0
yc
x

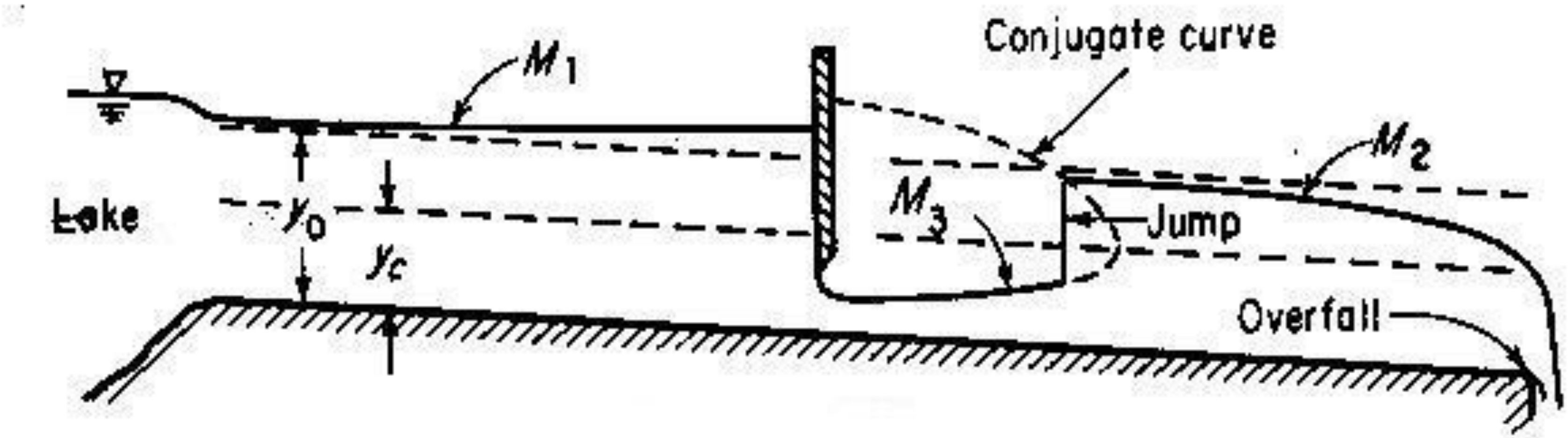
Mild Slope

On a mild slope, the possible water surface profiles are:



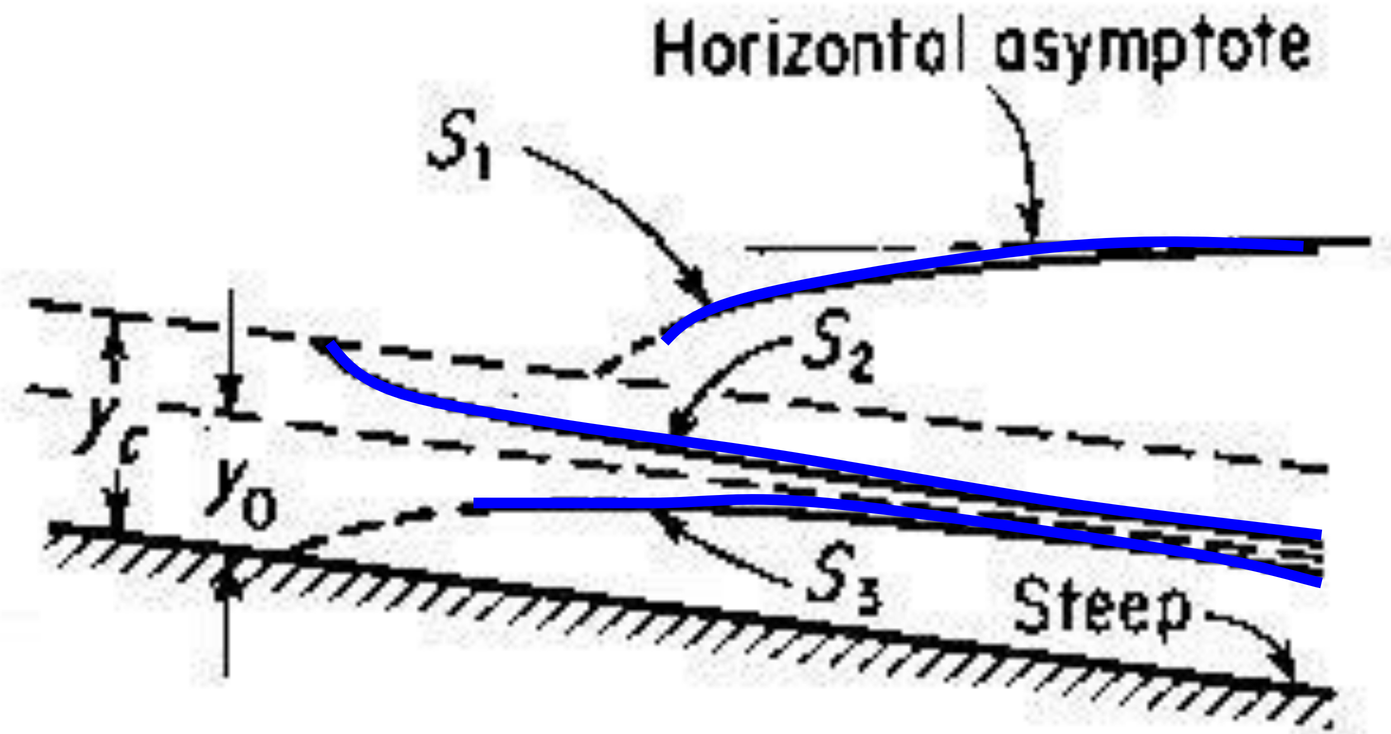
Water-surface profile on a mild slope

- Consider a long mild channel taking water from a lake, and ending with a free fall. The water surface profile would look like:



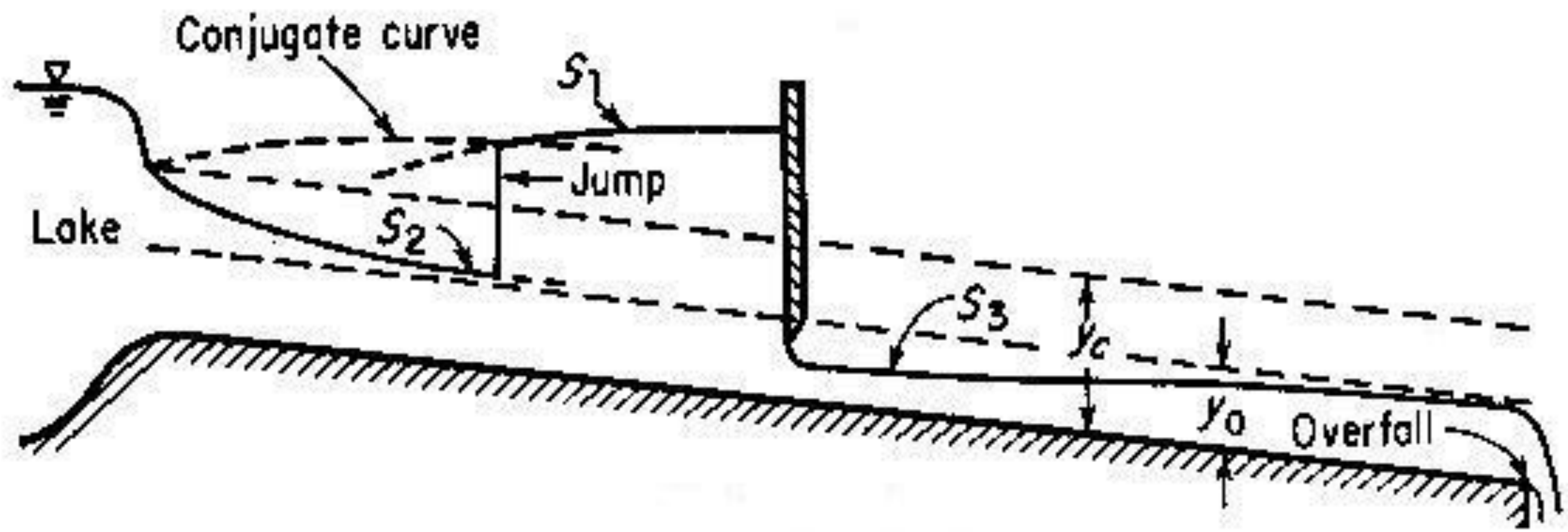
Steep Slope

- On a steep slope, the possible water surface profiles are:



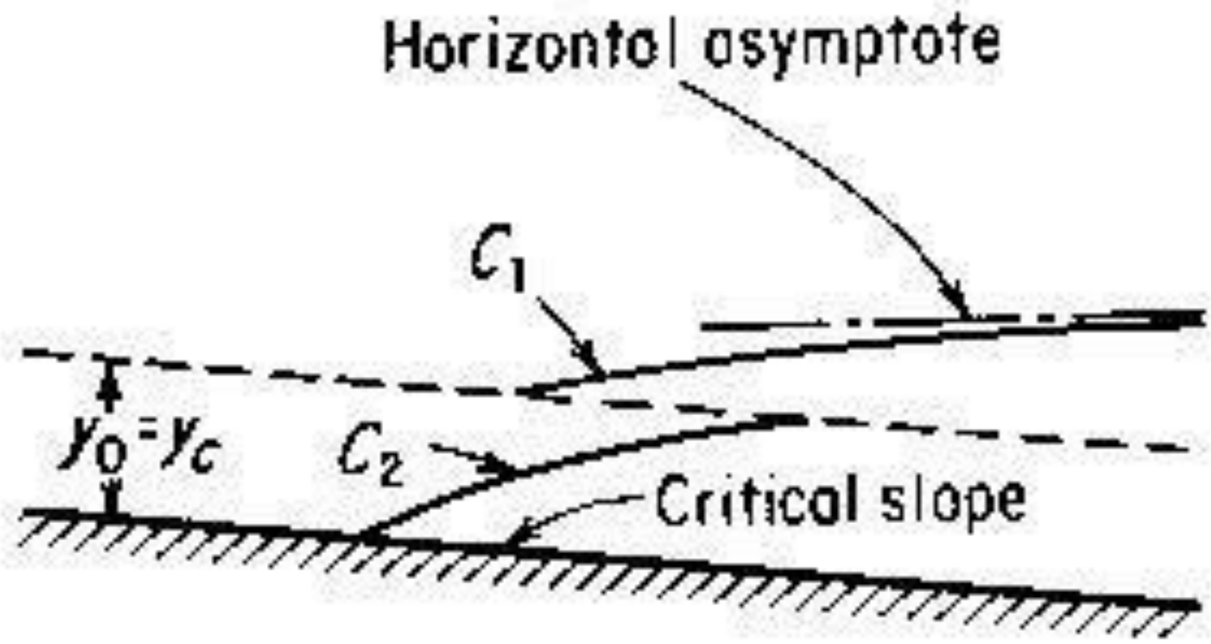
Water-surface profile on a steep slope

- Consider a long steep channel taking water from a lake, and ending with a free fall. The water surface profile would look like:



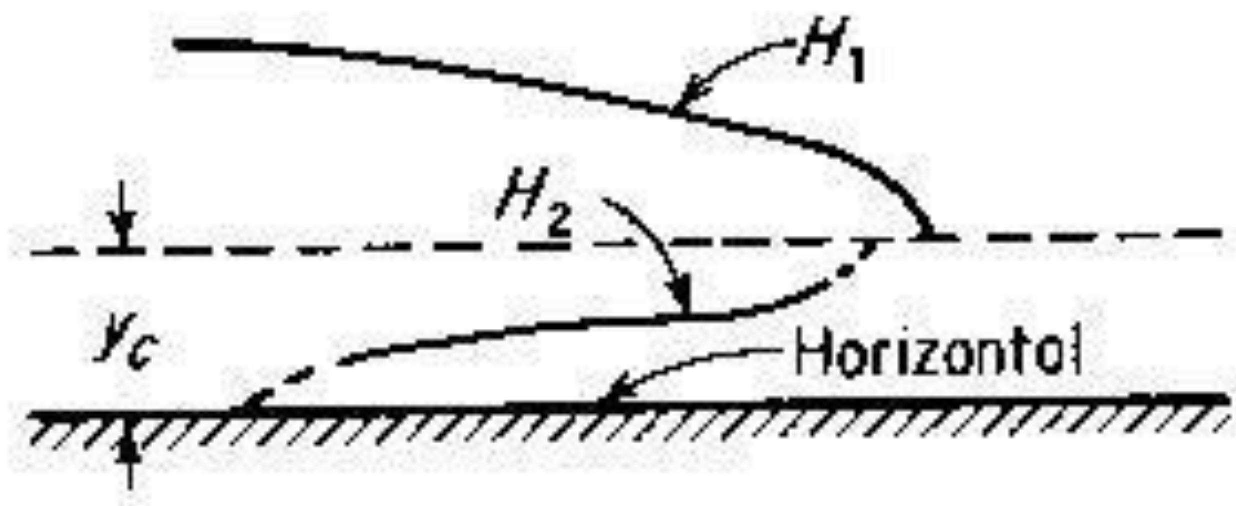
Critical Slope

On a critical slope, the possible water surface profiles are:



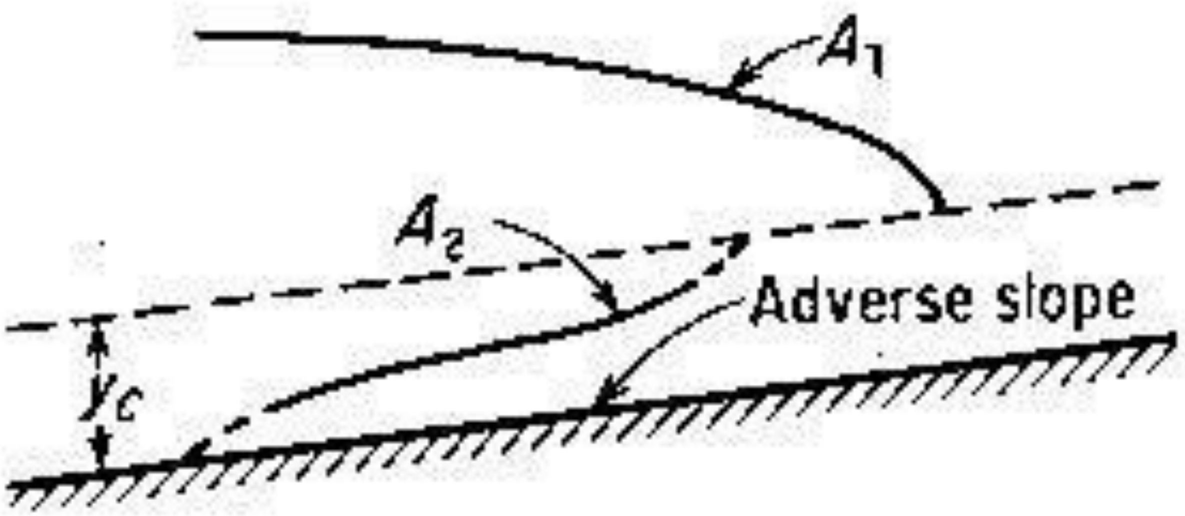
Horizontal Slope

On a horizontal slope, the possible water surface profiles are:



Adverse Slope

On an adverse slope, the possible water surface profiles are:



From the cases examined here, we can obtain certain principles that can be applied to all cases;

1.The sign of can be readily determined from the equation and the inequalities:

$$\frac{dy}{dx} = \frac{s_o - s_f}{1 - F_r^2}$$

$$\left.y > y_0\right\} \; S_f < S_0$$
$$\left.y < y_0\right\} \; S_f > S_0$$

$$\left.y > y_c\right\} \; F_r < 1$$
$$\left.y < y_c\right\} \; F_r > 1$$

2.When the water surface approaches the uniform depth line, it does so asymptotically.

3.When the water surface approaches the CDL, it meets this line at a fairly finite angle

4. If the curve includes a critical section, and if the flow is subcritical upstream (as in case of M_2 curve) then a that critical section is produced by a feature such as a free overfall. But if the flow is supercritical upstream (as in M_3 curve) the control cannot come from the critical section, and indeed such a section will probably not occur in reality but will be bypassed by a hydraulic jump.
5. Above all, every profile exemplifies the important principle that subcritical flow is controlled from downstream (e.g, M_1 and M_2 curves) and supercritical flow from upstream (e.g. the M_3 curve). In fact these profiles owe their existence to the action of upstream or downstream controls.

Occurence of Critical Flow

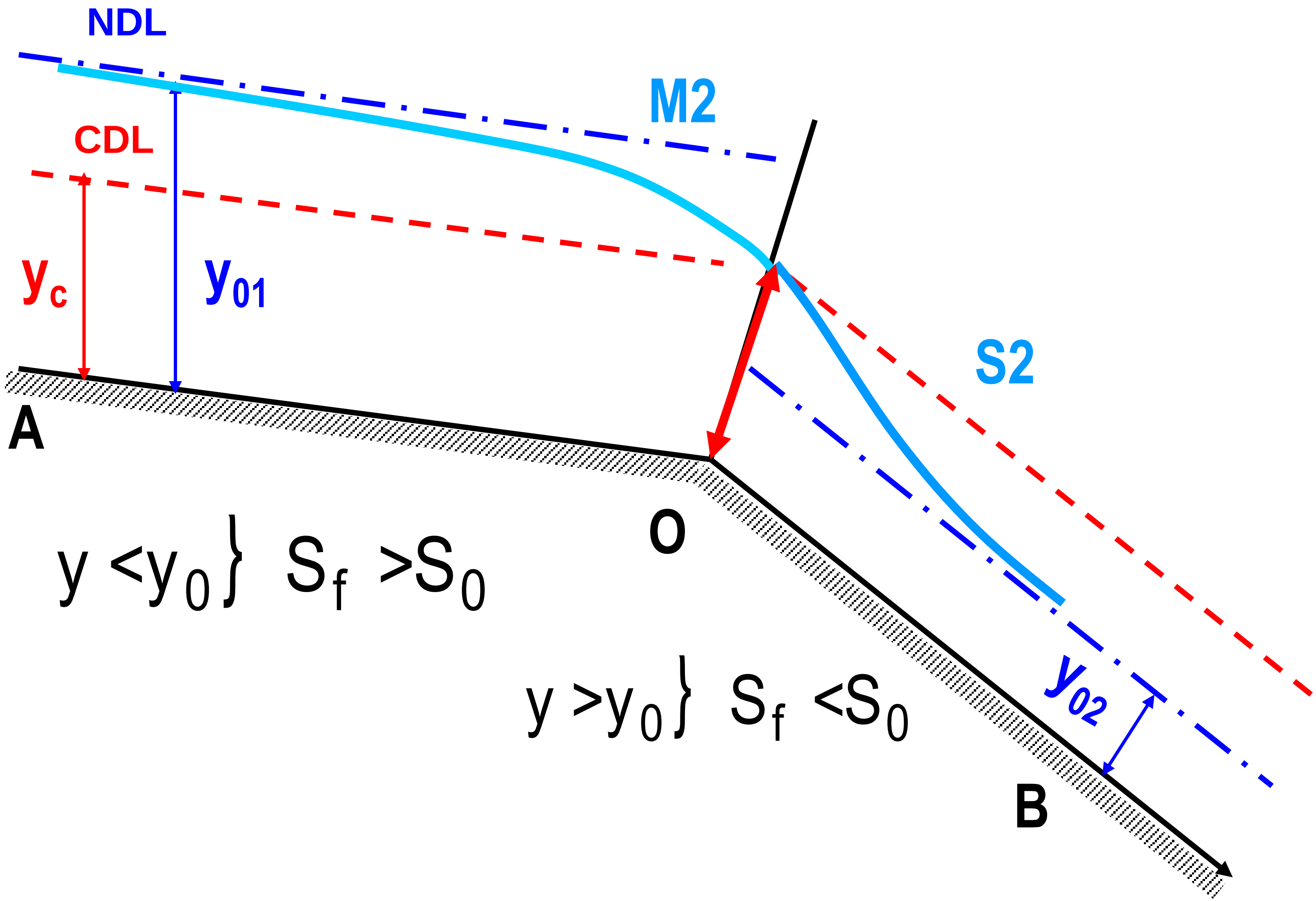
$$\left(1 - F_r^{\,2}\right)\frac{dy}{dx} = S_o - S_f$$

Consider a special case that S_0=S_f:

This means that either

$\frac{\mathrm{dy}}{\mathrm{dx}} = 0 \rightarrow$ **Uniform flow, or** $\mathrm{F_r} = 1 \rightarrow$ Any real physical meaning

- Consider a long channel of two sections: one of mild upstream and one of steep slope downstream.
- The flow will gradually change from subcritical at a great distance upstream to supercritical at a great distance downstream, passing through critical at some intermediate pt.
- In the transition region upstream of 0, that is between sections (A) and (O), the depth is less than y_{01} and the velocity is greater than uniform flow. On the other hand, between sections (O) and (B), the depth is greater than y_{02} and the velocity is less than uniform flow.



Therefore somewhere between sections (A) and (B), $S_f = S_0$. Hence $F_r = 1$, and flow will be critical. Flow will be critical at Section (O)..

- Critical flow occurs at the outflow from a lake into a steep channel or through a constriction in width: this last results shows that it will also occur at the head of a steep slope which is proceeded by a mild slope.

1.6 Interaction of Local features and Longitudinal Profiles

In treatment of elementary discharge problem (given S_0,n, and available specific energy, E, and determine Q), a difficulty arise which is a characteristic of nonuniform channel. The difficulty is: the engineer is given only a numerical value of channel bottom slope, and must decide himself/herself whether it is mild or steep. In order to determine it, one must know the discharge. This is not a serious difficulty. One can assume a type of slope and determine the discharge and then check the assumption and hence proceeds accordingly. However, a more general form of this problem is related with controls.

It has been discussed that controls are important as the points of origin for longitudinal profiles.

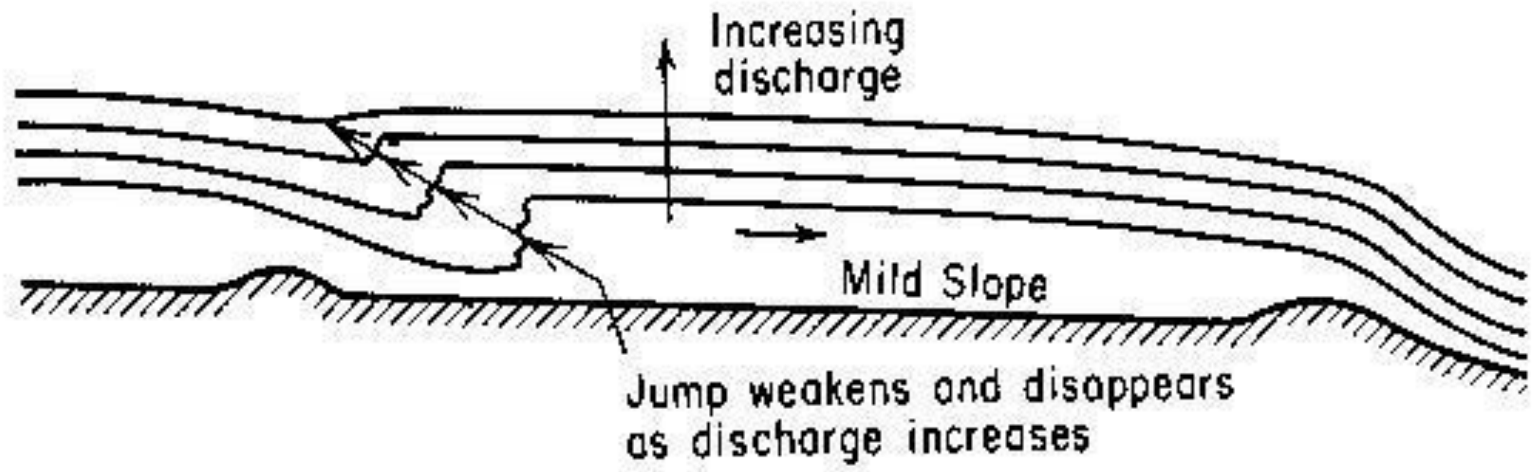
- But in previous discussions, the controls are all been nominated and their functioning described in advance. In practice this is never true: the engineer dealing with a specific problem is given only the description of certain channel features and must decide whether and how they will act as controls.

- In this respect, it is important to observe that while any control present will influence and help to determine the whole flow profile, the profile in its turn may be said to influence the control, in a sense that the form of the profile may determine whether a certain feature acts as a control or not.

- The most familiar example of this action is the “drowning” of a control as in case of flow profile behind a sluice gate; the S_1 curve behind it would fill all the upstream channel and drown the lake outlet, which would be no longer be a critical section.
- The general principle which emerges is that **a control may be drowned and deprived of its function by a stronger control downstream.**

A further example of this feature is shown in figure below.

For the lowest profile shown in this figure, both weirs are acting as critical flow controls: as the discharge increases the hydraulic jump moves upstream, becoming weaker as it does so, until finally it vanishes.

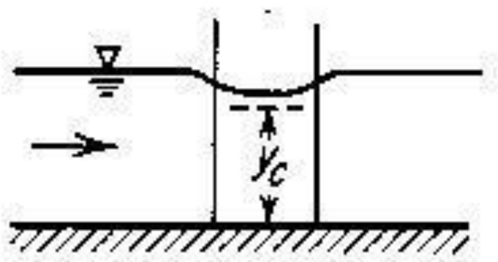


The only trace of it being a depression over the upstream weir. This weir is now drowned and flow over it is no longer critical.

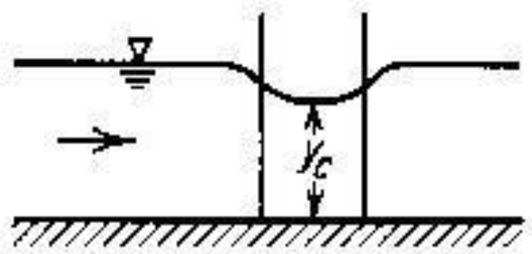
1.7 The Effect of a Choke on the Flow Profile

- The above discussion has dealt with the effect of a changing flow profile on a particular feature; it is also useful to consider this interaction in the converse way – i.e., to consider the effect on the flow profile of some feature as it is gradually converted into control by some continuous change in the discharge or in the geometry of the feature. This latter type of change is un-likely to occur in physical reality, but a consideration of its effect makes a useful, if artificial, exercise for a designer seeking to determine a suitable size fore some channel feature.

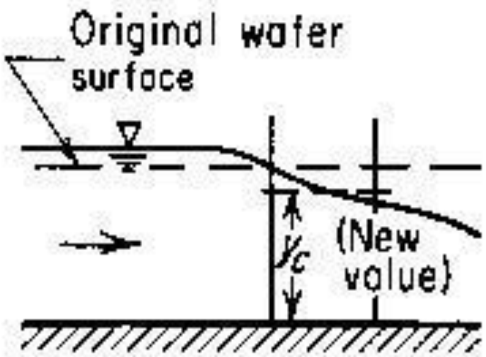
- An example of great practical interest is provided by a local width contraction (e.g; bridge piers or a culvert) in a long uniform channel of mild slope. Suppose that initially the contraction is not a very severe one and the flow can be passed through it without requiring more specific energy that the upstream flow possesses; i.e; without choking. The flow within the contraction is therefore subcritical, as is the uniform flow for a great distance upstream and downstream (See figure below; Fig. a). If now the contraction width is gradually reduced, a point is reached where the available specific energy is just sufficient to pass to the flow through the contraction in the critical condition (Fig.b). This is the threshold of the choking condition, where the contraction becomes a control.



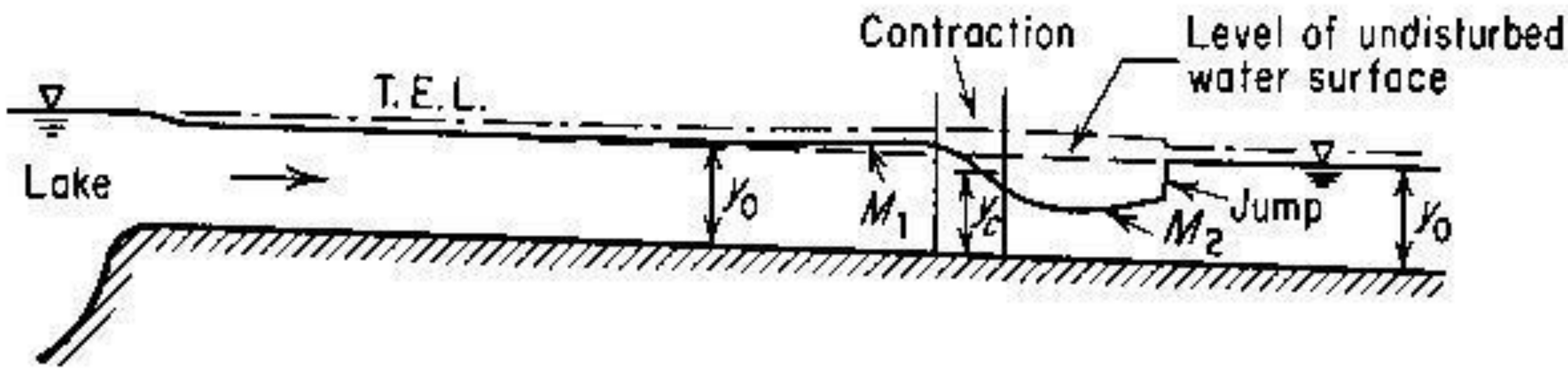
(a) Small contraction ratio



(b) Critical contraction ratio



(c) "Choke", or overcritical contraction ratio



(d) Longitudinal profile, condition (c)

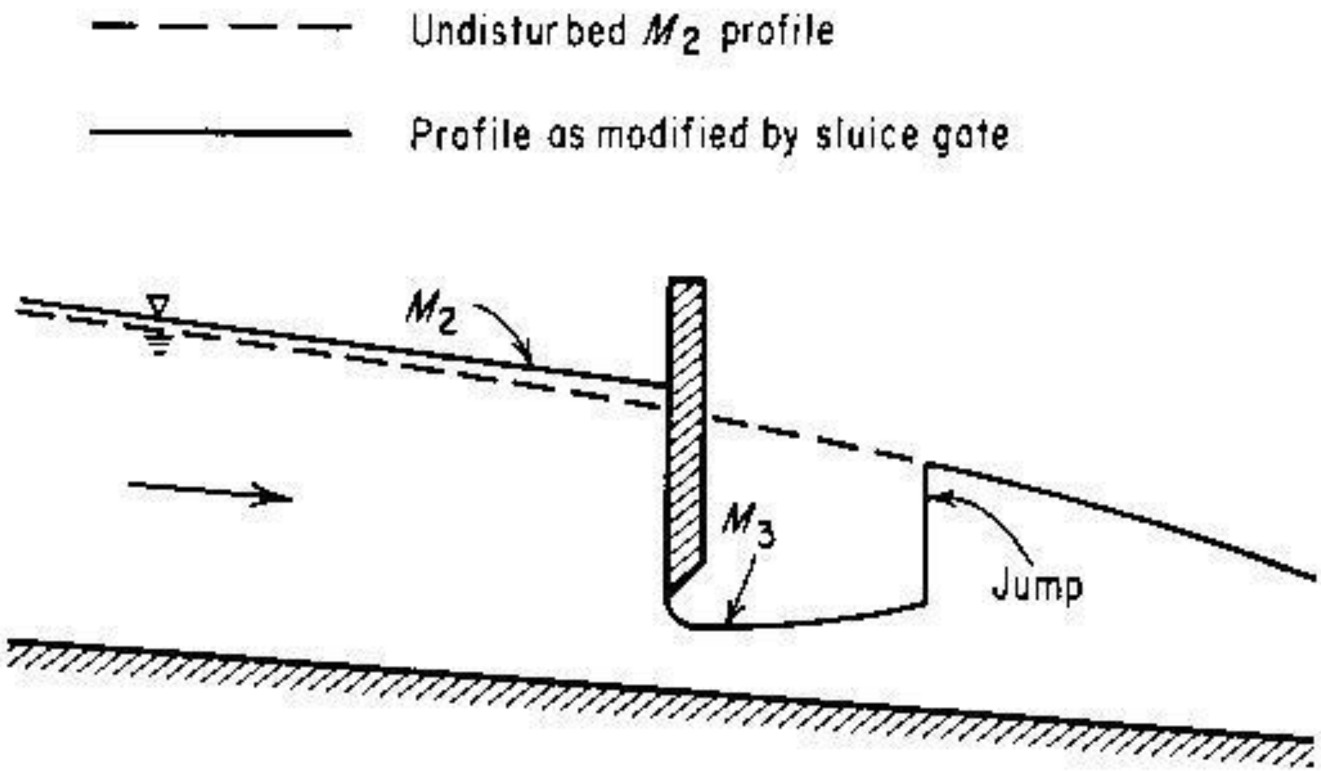
We now consider what happens if the contraction is narrowed even further. First, the flow within the contraction remains critical; this fact is of prime importance. Clearly there is no reason why this flow should return to sub-critical, for the condition which originally produced critical flow is now being pushed to even further extremes; on the other hand, the flow cannot pass to supercritical either. There is therefore no alternative to the maintenance of critical flow. Assuming for the moment that the discharge remains constant, it is seen that the discharge per unit width q within the contraction must increase, so that the critical depth y_c must also increase; it follows that the specific energy $E=3y_c/2$ will increase, within the contraction and upstream, so that the upstream depth must increase and an M_1 curve will appear upstream (Fig. c).

- This behavior accords well with the intuitive notion that a severe constriction in the channel will cause the water to “back up” or “head up” so as to force the required discharge through the constriction.
- We can now examine more critically the assumption that the discharge remains constant. Consider the channel as a whole, including the source of the flow, as in Fig. d. In this sketch is shown the whole the whole extent of the M_1 curve of Fig. c; if it “runs out”, as shown, before reaching the source, then the choking of the contraction has produced only a local disturbance which does not alter the discharge. On the other hand, if the contraction were made severe enough for the M_1 curve to reach right back to the lake, the discharge would be reduced somewhat.
- In order to calculate the amount of this reduction it is necessary to calculate the shape of the M_1 curve;

1.8 Specific Energy Changes Near Controls

- First, it is important to see that the choking and backing up shown in above Figs.(c) and (d) is independent of energy dissipation, and would occur even the walls of the contraction were streamlined so as to eliminate energy loss. Nevertheless, energy concepts are useful in discussing certain consequences of the choking process. It can be easily shown that in the M_1 curve the specific energy E increases in the downstream direction; it is this process which supplies the extra specific energy needed to pass the flow through the contraction. Further downstream, however, the flow must return to uniform, and to the appropriate value of E.

- The extra specific energy which was acquired upstream must therefore be given up, even if there is no energy loss in the contraction itself. And if there is no such energy loss, the required drop in E can occur only through the downstream development of supercritical flow (in which E decreases downstream) followed by a hydraulic jump. The total energy line therefore behaves as shown in Fig. d.
- Similar reasoning would apply to a control such as a sluice gate, shown in Figure below. The argument is not of course limited to the case where the undisturbed flow is uniform; in this figure the undisturbed flow (in the absence of the sluice gate) is an M_2 curve and the process of departure from and return to this curve is essentially similar to that shown for uniform flow in Fig. d.



Undisturbed M_2 profile
Profile as modified by sluice gate
M_2
M_3
Jump

- It is noteworthy that in the present case the upstream profile produced by the sluice gate is itself an M_2 curve, although at a higher level than the original one. Reduction of the sluice-gate opening would raise the upstream profile even further until it represented uniform flow; further reduction in the opening would produce an M_1 curve. Downstream of the hydraulic jump the profile is, of course, unaltered by the presence of the sluice gate.
- The preceding discussion has dealt with two types of control the barrier type, such as a weir or sluice gate, and the choked-constriction type, in which critical flow occurs within the constriction. Both have the effect, when placed on a mild slope, of forcing a rise in the upstream water level and total energy line, and this can occur without any energy dissipation at the control itself,

- But it is conceivable that the same backing-up effect could arise simply from some feature which causes energy dissipation without acting as a control. Typical of such features are obstacles such as bridge piers which dissipate energy but present only a moderate degree of contraction to the flow.
- Control of upstream flow is the essence of the action here discussed; it is therefore mainly applicable on mild slopes where the undisturbed flow is sub-critical. When the slope is steep the action of control is to create an S_1 curve upstream, which may move upstream and drown the source. The end result may therefore be similar to that on a mild slope, although the details of the mechanism are different.

1.9 Computation of Gradually Varied Flow

- The computation of gradually-varied flow profiles involves basically the solution of dynamic equation of gradually varied flow. The main objective of computation is to determine the shape of flow profile.
- Broadly classified, there are three methods of computation; namely:

1. The graphical-integration method,
2. The direct-integration method,
3. Step method.

The graphical-integration method is to integrate the . dynamic equation of gradually varied flow by a graphical procedure. There are various graphical integration methods. The best one is the Ezra Method.

The direct-integration method: Thje differentia equation of GVF can not be expressed explicitly in terms of y for all types of flow cross section; hence a direct and exact integration of the equation is practically impossible. In this method, the channel length under consideration is divided into short reaches, and the integration is carried out by short range steps.

- **The step method:** In general, for step methods, the channel is divided into short reaches. The computation is carried step by step from one end of the reach to the other.
- There is a great variety of step methods. Some methods appear superior to others in certain respects, but no one method has been found to be the best in all application. The most commonly ısed step methods are:

1. Direct-Step Method,
2. Standart-step Method.

1.9.1 The Direct Step Method (DSM)

- In direct step method, distance is calculated from the depth.
- It is only applicable to prismatic channels. The energy equation between two sections is:

$$z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g} + \overline{S}_f \Delta x$$

- Assume that $\boldsymbol{\alpha_1 = \alpha_2} = 1$ $z_1 - z_2 = S_0 \Delta x$, Then above equation can be written as:

$$S_0 \Delta x + E_1 = E_2 + \overline{S}_f \Delta x$$

- Solving for Δx:

-

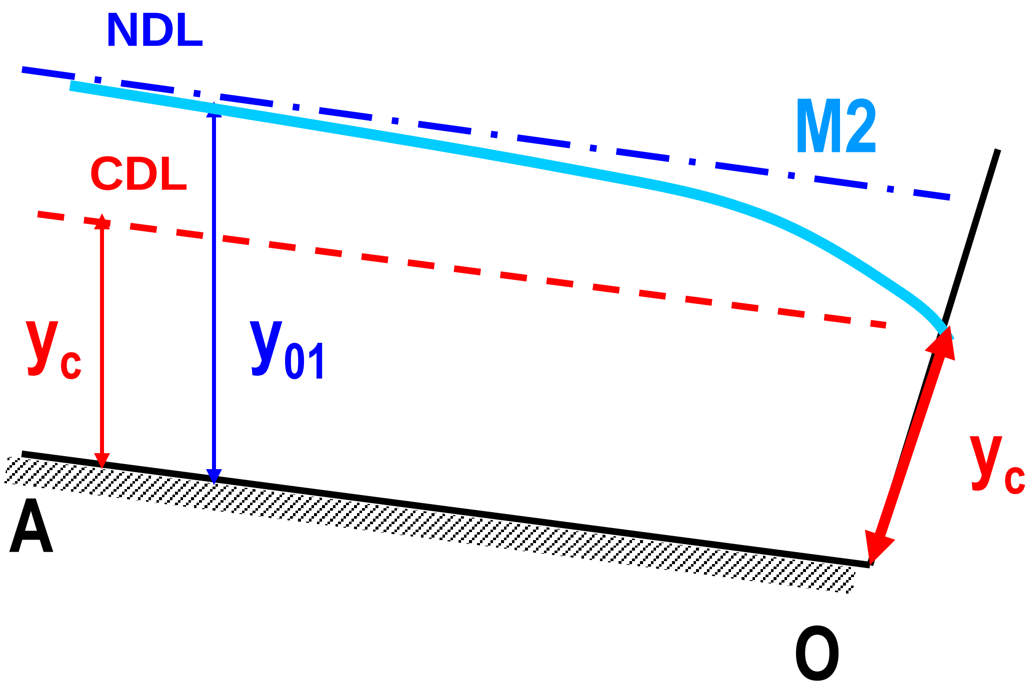
- The distance between two sections can be calculated from:

$$\Delta x = \frac{E_2 - E_1}{S_o - \overline{S}_f} = \frac{\Delta E}{S_o - \overline{S}_f} = \frac{\Delta\left(y + \alpha\frac{V^2}{2g}\right)}{S_o - \overline{S}_f}$$

- The slope of energy-grade line can be computed from Manning's Equation as:

$$S_f = \frac{n^2 V^2}{R^{4/3}} \qquad \overline{S}_f = \frac{1}{2}\left(S_{f_1} + S_{f2}\right)$$

Consider the M2 profile in a channel section. Suppose that we want to calculate the length of M2-profile for a given discharge Q, and channel section.



- We know that the depth will be changing between the limits $y_c \leq y < y_0$.
- On the other hand the distance between two sections can be written as:

$$\Delta x = \frac{E_2 - E_1}{S_o - \overline{S}_f} = \frac{\Delta E}{S_o - \overline{S}_f} = \frac{\Delta\left(y + \alpha\frac{V^2}{2g}\right)}{S_o - \overline{S}_f}$$ and

$$\overline{S}_f = \frac{1}{2}\left(S_{f1} + S_{f2}\right) \qquad S_f = \frac{n^2 V^2}{R^{4/3}}$$

- Start from a control section. In this case the control section is the downstream section where critical flow occurs.
- Nominate a series values of y, between the range of $y_c \leq y < y_0$
- Calculate the values of R, V, E, and S_f corresponding to these assumed depths.
- Calculate Δx for each interval between successive values of y

- **NOTE:**
- If flow is subcritical computation is from downstream towards upstream.
- If flow is supercritical computation is from upstream towards downstream.
- If one of the depth is uniform depth for the section under consideration, then 1% off value of normal depth y_0 must be taken. For example for profiles like M1 and S2 where normal depth is approached asymptotically from above 1.01 y_0, and for profiles such as M2 and S3, where normal depth is approached asymptotically from belove 0.99 y_0 must be taken.

The best thing is to prepare a table as follows:

y (m)	A (m^2)	P (m)	R=A/P (m)	U (m/s)	E (m)	S_f	$\overline{S}_f$	Δx (m)	x (m)
Y_c									
Y_1									
Y_2									
y_3									

1.9.2 The Standard-Step Method

In this method, the depth is calculated from distance.

Applicable to both nonprismatic and prismatic channels. It is a trial and error process

- Assume y_2, compute H_2: $$H_2 = z_2 + y_2 + \frac{V_2^2}{2g}$$
- Also compute: $$H_2' = H_1 + h_f = H_1 + \frac{1}{2}\left(S_{f1} + S_{f2}\right)\Delta x$$
- Where $$h_f = \frac{1}{2}\bar{S}_f \Delta x$$
- Compare H_2 to H_2':
- if $H_2 \approx H_2'$ then assumed y_2 is O.K.
- But, if $H_2 \neq H_1'$, then assume another value of y_2

Improve your initial estimate by this amount:

$$\Delta y_2 = \frac{H_e}{1 - Fr_2^2 + \frac{3}{2}\frac{s_{e_2}}{R_2}\Delta x} \qquad \text{where} \qquad H_e = H_2 - H_2'$$

For natural rivers, instead of depth y, it is preferable to use the height h of the water level above some fixed datum. This height, h=z+y, is known as the STAGE. Hence total head at a section can be written as:

$$H = y + z + \alpha\frac{V^2}{2g} = h + \alpha\frac{V^2}{2g}$$

- In nonprismatic channels, the hydraulic elements are no langer independent of the distance along the channel
- In natural channels, it is generally necessary to conduct a field survey to collect data required at all sections considered in the computation.
- The computation is carried on by steps from station to station where the hydraulic characteristics have been determined. In such cases the distance between stations is given, and the procedure is to determine the depth of flow at the stations.

1.9.3 Direct-Integration Methods

We have seen that the flow equation

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - F_r^2}$$

is true for all forms of channel section, provided that the Froude number F_r is properly defined by the equation:

$$F_r^2 = \frac{V^2}{g}\frac{T}{A} = \frac{Q^2}{g}\frac{T}{A^3}$$

and the velocity coefficient, α = 1, channel slope θ is small enough so that cos θ =1.

We now rewrite certain other elements of this equation with the aim of examining the possibility of a direct integration. It is convenient to use here the conveyance K and the section factor Z.

The Conveyance of a channel section, K:

If a large number of calculations are to be made, it is convenient to introduce the concept of “conveyance” of a channel in order to calculate the discharge. The “conveyance” of a channel indicated by the symbol K and defined by the equation

$$Q = KS^{1/2} \qquad \textbf{or} \qquad K = \frac{Q}{\sqrt{S}}$$

This equation can be used to compute the conveyance when the discharge and slope of the channel are given.

When the Chézy formula is used: $\mathrm{K} = \mathrm{CAR}^{1/2}$

where c is the Chézy’s resistance factor. Similarly when the Manning formula is used

$$\mathrm{K} = \frac{1}{\mathrm{n}} \mathrm{AR}^{2/3}$$

- When the geometry of the water area and resistance factor or roughness coefficient are given,

  One of the above formula can be used to calculate K. Since the Manning formula is used extensively in most of the problems, in following discussion the second expression will be used. Either K alone or the product Kn can be tabulated or plotted as a function of depth for any given channel section: the resulting tables or curves can then be used as a permanent reference, which will immediately yields values of depth for a given Q, S and n. This conveyance factor concept is widely used for uniform flow computation.

  Since the conveyance K is a function of the depth of flow y, it may be assumed that:

$$K^2 = C_1 y^N$$

  where

  C_1 = coefficient, and

  N = a parameter called hydraulic exponent

Taking the logarithms of both sides of above eq. And then differentiating with respect to y:

$$2\ell nK = N\ell ny + \ell nC_1$$

$$\frac{d(\ell nK)}{dy} = \frac{N}{2y} \rightarrow N = 2y\frac{d(\ell nK)}{dy}$$

On the other hand: from Manning's Eq.

$$K = \frac{1}{n}AR^{2/3}$$

Taking the logarithm of both sides and differentiating with respect to y:

$$\ell nK = \frac{2}{3}\ell nR + \ell nA - \ell nn$$

$$\frac{d(\ell nK)}{dy} = \frac{2}{3R}\frac{dR}{dy} + \frac{1}{A}\frac{dA}{dy}$$

- The derivative of hydraulic radius with respect to y:

$$\frac{dR}{dy} = \frac{1}{P}\frac{dA}{dy} - \frac{A}{P^2}\frac{dA}{dy} = \frac{T}{P} - \frac{R}{P}\frac{dP}{dy}$$

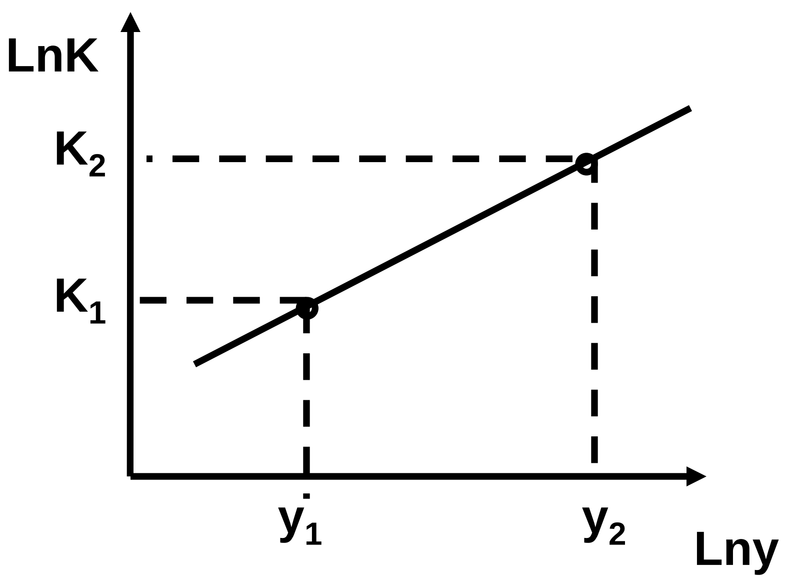
$$\frac{d(\ell nK)}{dy} = \frac{2}{3}\frac{1}{R}\left(\frac{T}{P} - \frac{R}{P}\frac{dP}{dy}\right) + \frac{T}{A} = \frac{2}{3}\frac{T}{A} - \frac{2}{3}\frac{1}{P}\frac{dP}{dy} + \frac{T}{A}$$

$$\frac{d(\ell nK)}{dy} = \frac{N}{2y} \rightarrow N = 2y\frac{d(\ell nK)}{dy}$$

$$\frac{d(\ell nK)}{dy} = \frac{1}{3A}\left(5T - 2R\frac{dP}{dy}\right)$$

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right)$$

- This is the general Eq. for the hydraulic exponent N. If the channel cross section is known N can be calculated accordingly provided that the derivative dP/dy can be evaluated. For most channels, except for channels with abrupt changes in cross-sectional form and for closed conduits with gradually closing top, a logarithmic plot of K as ordinate against the depth as abscissa will appear approximately as straight line. Thus if any two points with coordinates (K_1, y_1) and (K_2, y_2) are taken from the straight line, the approximate value of N may be computed by the following Eq.



$$N = 2\frac{\log(K_1 / K_2)}{\log(y_1 / y_2)}$$

- For wide rectangular channels: $R \approx y$
- The Chézy Equation gives the value of K as:

$$K^2 = C^2 A^2 R = C^2 b^2 y^2 y = C^2 b^2 y^3 \rightarrow N = 3$$

- On the other hand the Manning Equation gives the value of K as:

$$K^2 = \frac{1}{n^2} A^2 R^{4/3} = \frac{1}{n} b^2 y^2 y^{4/3} = \frac{1}{n^2} b^2 y^{10/3}$$

$$\rightarrow N = 10/3$$

The Section Factor: Z

- The Section Factor: Z is especially used for critical flow computation. However it becomes useful to transform the GVF equation into a form which can be integrated directly.
- We now consider the Froude number

$$F_r^2 = \frac{Q^2}{g}\frac{T}{A^3}$$

- Since this term equals unity at critical flow, then

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

- A_c, T_c are the values of A and T at critical flow..

- By definition we introduce the concept of section factor as

$$Z^2 = \frac{A^3}{T} \quad \textbf{or} \quad Z^2 = A^2\frac{A}{T} = A^2 D \quad \textbf{and} \quad Z = A\sqrt{D}$$

- The section factor for critical flow becomes $Z_c^2 = \frac{A_c^3}{T_c} = \frac{Q^2}{g}$

- or $Z_c = \frac{Q}{\sqrt{g}}$ for critical flow only.

Since the section factor z is a function of depth, the equation indicates that there is only one possible critical depth for maintaining the given discharge in a channel and similarly that, when the depth is fixed, there can be only one discharge that maintains the critical flow and makes the depth critical in that given channel section.

- Since the section factor z is a function of the depth of flow y, it may be assumed that

$$Z^2 = C_2 y^M$$

Where C_2 is a coefficient and M is a parameter called the hydraulic exponent. Taking the logarithms on both sides of above equation and then differentiating with respect to y:

$$\frac{d(\ell nZ)}{dy} = \frac{M}{2y} \rightarrow M = 2y\frac{d(\ell nZ)}{dy} = 2\frac{d(\ell nZ)}{d(\ell ny)}$$

Now taking the logarithms on both sides of Eq. $Z^2 = \frac{A^3}{T}$

$$2\ell nZ = 3\ell nA - \ell nT$$

Take the derivative with respect to y:

$$\frac{d(\ell nZ)}{dy} = \frac{3}{2A}\frac{dA}{dy} - \frac{1}{2T}\frac{dT}{dy} = \frac{3}{2A}T - \frac{1}{2T}\frac{dT}{dy}$$

- Then M becomes::

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right)$$

- This is a general equation for the hydraulic exponent M, which is a function of the channel section and the depth of flow.
- For a given channel section M can be computed directly from this expression, provided that the derivative dT/dy can be evaluated. However, approximate values of M for any channel section may be obtained from the following equation:

$$M = 2\frac{\log(Z_1 / Z_2)}{\log(y_1 / y_2)}$$

where z_1 and z_2 are section factors for any two depths y_1 and y_2 of the given section, i.e.:

$$Z_1^2 = \frac{A_1^3}{T_1} \quad \text{and} \quad Z_2^2 = \frac{A_2^3}{T_2}$$

For rectangular sections: A=by, and T=b, hence section factor becomes:

$$Z^2 = \frac{b^3 y^3}{b} = b^2 y^3 \Rightarrow M = 3$$

- Now, coming back to the computation of gradually varied flow; the term, (S_0-S_f)may be written as

$$S_0 - S_f = S_0\left(1 - \frac{S_f}{S_0}\right)$$

- On the other hand $K = \frac{Q}{\sqrt{S_f}}$ and for uniform flow $K_0 = \frac{Q}{\sqrt{S_0}}$

- Therefore $\frac{S_f}{S_0} = \frac{Q^2 / K^2}{Q^2 / K_o^2} = \frac{K_o^2}{K^2}$

- And hence $S_0 - S_f = S_0\left(1 - \frac{K_o^2}{K^2}\right)$

- Also, it is assumed that $K = C_1 y^N$ and $K_0 = C_1 y_0^N$

- Therefore

$$S_0 - S_f = S_0\left(1 - \frac{K_o^2}{K^2}\right) = S_0\left[1 - \left(\frac{y_o}{y}\right)^N\right]$$

- On the other hand:

$$F_r^2 = \frac{Q^2}{g}\frac{T}{A^3} = \frac{Q^2}{g}\frac{1}{Z^2} = \frac{Z_c^2}{Z^2}$$

- because for critical flow: $Z_c^2 = \dfrac{Q^2}{g}$
- Also it is assumed that $Z^2 = C_2 y^M$

Therefore Froude number can be written as:

$$F_r^{\,2} = \frac{C_2 y_c^M}{C_2 y^M} = \left(\frac{y_c}{y}\right)^M$$

Finally the Gradually Varied Flow Eq. takes the form:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^{\,2}} = S_0 \frac{1 - \left(\frac{y_o}{y}\right)^N}{1 - \left(\frac{y_c}{y}\right)^M}$$

Bresse's Method:

- Applicable only to wide rectangular channels. Assumptions:
- Wide rectangular channel,
- Chézy's formula is applicable and Chézy's C is constant.
- From Chezy's eq'n: , $K = CA\sqrt{R}$ A=by $R \approx y$
- Therefore $K^2 = C^2 A^2 R = C^2 b^2 y^2 y = C^2 b^2 y^3 \Rightarrow N = 3$

$$Z^2 = \frac{A^3}{T} = \frac{b^3 y^3}{b} = b^2 y^3 \Rightarrow M = 3$$

Therefore, the GVF Equation becomes:

$$\frac{dy}{dx} = S_o \frac{1 - \left(\frac{y_o}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3}$$

Manipulating this equation we can integrate it.

$$S_o dx = \left[\frac{1-\left(\frac{y_c}{y}\right)^3}{1-\left(\frac{y_o}{y}\right)^3} \right] dy$$

Adding and subtracting $\left(\frac{y_0}{y}\right)^3$

$$S_o dx = \left[\frac{1-\left(\frac{y_o}{y}\right)^3 + \left(\frac{y_o}{y}\right)^3 - \left(\frac{y_c}{y}\right)^3}{1-\left(\frac{y_o}{y}\right)^3} \right] dy$$

Rearrange it and multiply and divide by $\left(\frac{y}{y_0}\right)^3$

$$S_o dx = \left[1 - \frac{\left(\frac{y_c}{y}\right)^3 - \left(\frac{y_o}{y}\right)^3}{1-\left(\frac{y_o}{y}\right)^3} \cdot \frac{\left(\frac{y}{y_o}\right)^3}{\left(\frac{y}{y_o}\right)^3} \right] dy$$

$$S_o dx = \left[1 - \frac{\left(\frac{y_c}{y_o}\right)^3 - 1}{\left(\frac{y}{y_o}\right)^3 - 1}\right] dy$$

or

$$S_o dx = \left[1 - \frac{1 - \left(\frac{y_c}{y_o}\right)^3}{1 - \left(\frac{y}{y_o}\right)^3}\right] dy$$

Let $U = \dfrac{y}{y_o} \Rightarrow dy = y_o du$ change variables

- $$S_o dx = \left[1 - \frac{1 - \left(\frac{y_c}{y_o}\right)^3}{1 - U^3}\right] y_o du$$

Integrating

$$S_o x = y_o\left[u - \left(1 - \left(\frac{y_c}{y_o}\right)^3\right)\int \frac{du}{1-u^3}\right] + c$$

Let ϕ be the integral of $\phi = \int \frac{du}{1-u^3}$

$$\phi = \int \frac{du}{1-u^3} = \frac{1}{6} \ell n \frac{u^2+u+1}{(u-1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2u+1}$$

Therefore, the distance between any two section becomes:

$$L = x_2 - x_1 = \frac{y_o}{S_o} \left\{ (u_2 - u_1) \left[1 - \left(\frac{y_c}{y_o} \right)^3 \right] (\phi_2 - \phi_1) \right\}$$

Bakhmeteff Method

- Bakhmeteff improved Bresse's method as follows:

- $$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{s_o}-\mathrm{s_f}}{1-\mathrm{F_r^2}}=\mathrm{s_o}\frac{\left(1-\frac{\mathrm{s_f}}{\mathrm{s_o}}\right)}{1-\mathrm{F_r^2}}$$ Let $F_r^2=\beta\frac{S_f}{S_o}$

Therefore $\frac{dy}{dx}=S_o\frac{1-\frac{S_f}{S_o}}{1-\beta\frac{S_f}{S_o}}$ Then we can write that:

$$dx=\frac{1}{S_0}\frac{1-\beta\frac{S_f}{S_0}}{1-\frac{S_f}{S_0}}dy$$

On the other hand, we can wite that: $\frac{S_f}{S_0}=\left(\frac{y_o}{y}\right)^N$

Hence dx becomes:

$$dx = \frac{dy}{s_o} \frac{1-\beta\left(\frac{y_o}{y}\right)^N}{1-\left(\frac{y_o}{y}\right)^N}$$

Add and subtract $\left(\frac{y_0}{y}\right)^N$

$$dx = \frac{dy}{s_o} \frac{1-\beta\left(\frac{y_o}{y}\right)^N}{1-\left(\frac{y_o}{y}\right)^N} + \left(\frac{y_o}{y}\right)^N - \left(\frac{y_o}{y}\right)^N$$

and rearrange as:

$$S_0 dx = \left[1 - \frac{(1-\beta)\left(\frac{y_o}{y}\right)^N}{1-\left(\frac{y_o}{y}\right)^N}\right] dy$$

Now multiple and divide by $\left(\frac{y}{y_0}\right)^N$

$$S_0 dx = \left[1 - \frac{(1-\beta)\left(\frac{y_o}{y}\right)^N}{1-\left(\frac{y_o}{y}\right)^N} \frac{\left(\frac{y}{y_o}\right)^N}{\left(\frac{y}{y_o}\right)^N}\right] dy$$

and rearrange as:

- $$S_0 dx = \left[1 - \frac{(1-\beta)}{1-\left(\frac{y}{y_o}\right)^N}\right] dy$$

Let $U = \frac{y}{y_o} \Rightarrow dy = y_o du$

- Integrating

$$\int dx = \int \frac{y_0}{S_0}\left[1 - \frac{(1-\beta)}{1-u^N}\right] dy$$

$$x = \frac{y_0}{S_0}\left\{u - (1-\beta)\int \frac{du}{1-u^N}\right\}$$

Therefore, the distance between any two section becomes:

$$\mathrm{L} = \mathrm{x}_2 - \mathrm{x}_1 = \frac{\mathrm{y}_o}{\mathrm{s}_o}\{(\mathrm{u}_2 - \mathrm{u}_1) - (1-\beta)[\mathrm{F}(\mathrm{u}_2, \mathrm{N}) - \mathrm{F}(\mathrm{u}, \mathrm{N})]\}$$

- Here we are assuming that β is constant

$$F_r^2 = \beta \frac{S_f}{S_0} = \frac{Q^2 T}{gA^3} \Rightarrow \beta = \frac{S_0}{S_f} \frac{Q^2 T}{gA^3}$$

and hence

$$Q = CA\sqrt{RS_f}$$

$$S_f = \frac{Q^2}{C^2 A^2 R}$$

Therefore β becomes:

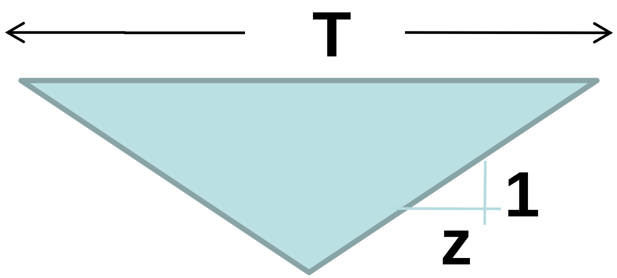
$$\beta = \frac{S_0 C^2 A^2 R}{Q^2} \frac{Q^2 T}{gA^2} = \frac{S_0 C^2 T}{gP} = \frac{S_0 C^2}{g} \frac{T}{P}$$

- If T/b is constant, then β will be constant.
- For example, for a rectangular channel, T=b, and P=b+2y, hence

$$\frac{T}{P} = \frac{b}{b+2y} = \frac{b}{b\left(1+2\dfrac{y}{b}\right)} = \frac{1}{1+2y/b}$$

- And T/b becomes constant only for wide rectangular channel.

- For a triangular channel: T=2zy



$$P = 2\sqrt{1+z^2}\,y$$

$$\frac{T}{P} = \frac{2zy}{2\sqrt{1+z^2}\,y} = \frac{z}{\sqrt{1+z^2}} = \text{constant}$$

- The integral in the equation is designated by F(u,N), that is

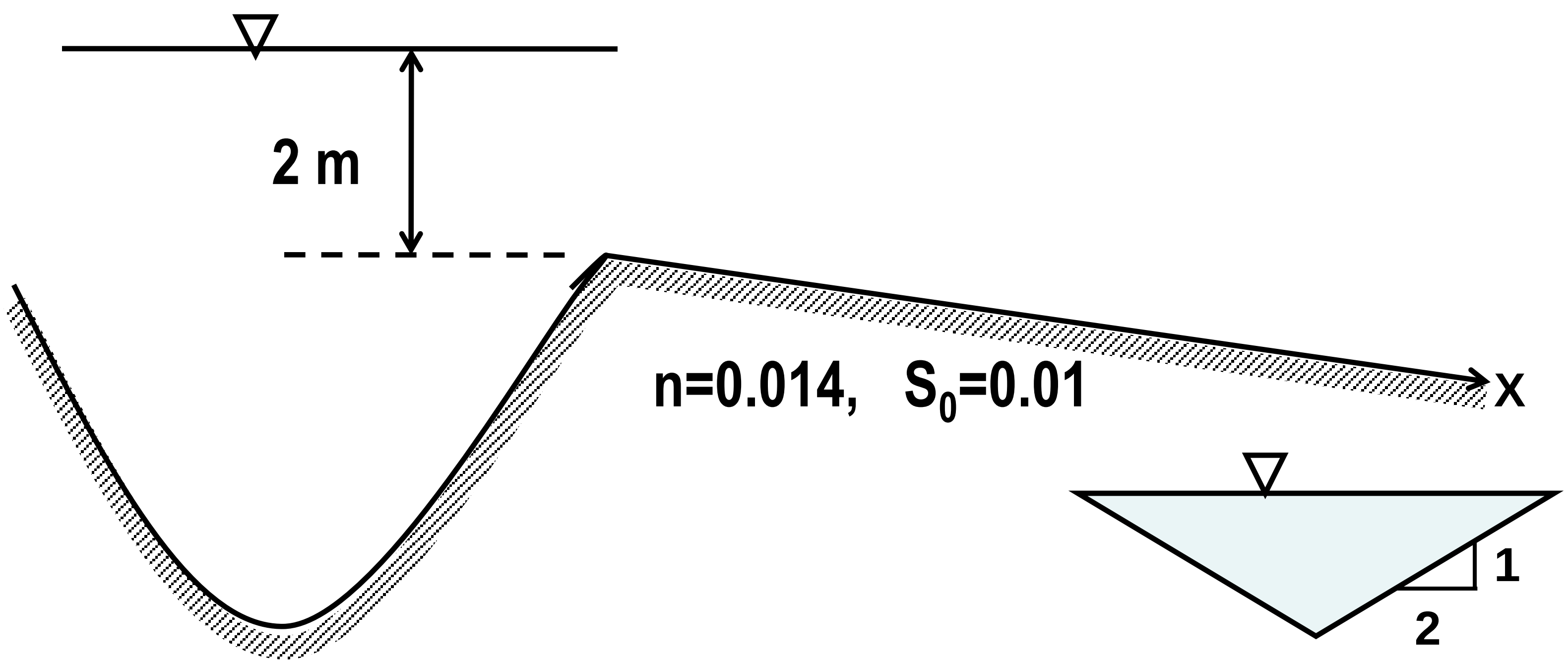
- $F(u, N) \equiv \int_{o}^{u} \frac{du}{1-u^{N}} =$ Varied-Flow Function

- The values of F(u,N) have been obtained numerically, and are given in tabular form for N ranging from 2.2 to 9.8.

Example on Bakhmeteff Method:

Water is taken from a lake by a triangular channel,with side slope of 1V:2H. The channel has a bottom slope of 0.01, and a Manning's roughness coefficient of 0.014. The lake level is 2.0 m above the channel entrance, and the channel ends with a free fall. **Determine :**

1. The discharge in the channel,
2. The water-surface profile and the length of it by using the direct-integration method.



a) To determine the discharge, assume that the channel slope is steep. Then at the head of steep slope, the depth is critical depth. Hence 2=Emin

$$A = 2y^2, \quad P = 2\sqrt{5}y, \quad R = \frac{A}{P} = \frac{y}{\sqrt{5}}, \quad T = 4y, \quad D = \frac{A}{T} = \frac{y}{2}$$

$$2 = E_C = y_C + \frac{D_C}{2} = y_C + \frac{y_C}{4} = \frac{5}{4}y_C \Rightarrow y_C = 1.6 \text{ m.}$$

$$A_c = 2x(1.6)^2 = 5.12\,\text{m}^2 \quad \frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(5.12)^3}{4x1.6} = 20.97$$

$$\Rightarrow Q = 14.34\,\text{m}^3/s \quad \text{let's compute the normal depth:}$$

$$Q = \frac{A_0}{n}\left(\frac{A_0}{P_o}\right)^{2/3}\sqrt{S_o} \Rightarrow 14.34 = \frac{2y_0^2}{0.014}\left(\frac{2y_0^2}{2y_0\sqrt{5}}\right)^{2/3}\sqrt{0.01}$$

$$y_0^{8/3} = 1.716 \Rightarrow y_0 = 1.225\,\text{m} < y_c = 1.6\,\text{m} \quad \therefore \text{steep slope \&}$$

$$Q = 14.34\,\text{m}^3/s$$

2. The water-surface profile and the length of it by using the direct-integration method.

- The water surface profile will be S2 type, and the depth of flow will change between the limits: $1.01y_0 \le y \le y_c$.
- Since T/P is constant for triangular channels, we can use Bakhmeteff's method. Let's compute the value of β:

$$\beta = \frac{S_0 C^2}{g}\frac{T}{P} \quad Q = CA_0\sqrt{R_0 S_0} \quad A_0 = 3.0\ \mathrm{m}^2$$

$$R_0 = 0.5476\ \mathrm{m},\ T = 4.9\ \mathrm{m},\ P = 5.48\ \mathrm{m}$$

$$C = 64.595 \quad \Rightarrow \quad \beta = \frac{S_0 C^2}{g}\frac{T}{P}$$

$$\beta = \frac{0.01x64.6^2x4.9}{9.81x5.48} = 3.80$$

- Let’s obtain the value of N

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right), \quad \mathrm{R} = \frac{\mathrm{y}}{\sqrt{5}}, \ \mathrm{P} = 2\mathrm{y}\sqrt{5}$$

$$N = \frac{2y}{3x2y^2}\left(5x4y - 2\frac{y}{\sqrt{5}}x2\sqrt{5}\right)$$

$$N = \frac{16}{3} = 5.333$$

$$\mathrm{L} = \mathrm{x}_2 - \mathrm{x}_1 = \frac{\mathrm{y}_o}{\mathrm{s}_o}\{(\mathrm{u}_2 - \mathrm{u}_1) - (1 - \beta)[\mathrm{F}(\mathrm{u}_2, \mathrm{N}) - \mathrm{F}(\mathrm{u}, \mathrm{N})]\}$$

- To obtain the values of F(u,N) we have to use tables with interpolation.

y (m)	**$u=y/y_0$**	**F(u,5.333)**
1.6	1.31	0.0811
1.237	1.01	0.622
Δ	-0.296	0.5411

- Interpolation:
- For u=1.30 and N=5 $\rightarrow$ F(1.30, 5)=0.100 and
- For u=1.32 and N=5 $\rightarrow$ F(1.32, 5)=0.093
- Hence by interpolation:
- For u=1.31 and N=5 $\rightarrow$ F(1.31, 5)=0.0965

- Similarly:
- For u=1.30 and N=5.4 $\rightarrow$ F(1.30, 5.4)=0.081 and
- For u=1.32 and N=5.4 $\rightarrow$ F(1.32, 5.4)=0.093
- Hence by interpolation:
- For u=1.31 and N=5.333 $\rightarrow$ F(1.31, 5)=0.0811

$$L = x_2 - x_1 = \frac{y_o}{s_o}\left\{\left(u_2 - u_1\right) - \left(1 - \beta\right)\left[F\left(u_2, N\right) - F\left(u, N\right)\right]\right\}$$

$$L = \frac{1.225}{0.01}\left\{\left(-0.296\right) - \left(1 - 3.8\right)0.5411\right\} = 149.34 \text{ m}$$

- Therefore, the length of S2 profile is 150 m.

Ven Te Chow Method

- Ven Te Chow improved Bakhmeteff's method for all types of cross sections as follows:

$$\frac{dy}{dx} = S_o \frac{1-\left(\dfrac{y_o}{y}\right)^N}{1-\left(\dfrac{y_c}{y}\right)^M}$$

The gradually-varied flow equation can be written as

$$dx = \frac{1}{S_o}\frac{1-(y_c/y)^M}{1-(y_o/y)^N}dy$$

multiply by both numerator and

denominator by $\left(\dfrac{\mathrm{y}}{\mathrm{y_o}}\right)^{\mathrm{N}}$ and second term in numerator by $\left(\dfrac{y_0}{y_o}\right)^M$

$$dx = \frac{1}{S_o}\left[\frac{(y/y_o)^N - (y/y_o)^N (y_c/y)^M (y_0/y_o)^M}{(y/y_o)^N - 1}\right]dy$$

$$dx = \frac{1}{S_o}\left[\frac{\left(\frac{y}{y_o}\right)^N \left(\frac{y_c}{y_o}\right)^M \left(\frac{y_o}{y}\right)^M - \left(\frac{y}{y_o}\right)^N}{1-(y/y_o)^N}\right]dy$$

Now let $u = \frac{y}{y_o} \rightarrow dy = y_o du$ then above equation becomes:
:

$$dx = \frac{y_o}{s_o}\left[\frac{\left(\frac{y_c}{y_o}\right)^M u^{N-M} - u^N}{1-u^N}\right]du$$

Adding and subtracting 1 to the numerator, dx becomes::

$$dx = \frac{y_o}{S_o}\left[\frac{1-u^N-1+\left(\frac{y_c}{y_o}\right)^M u^{N-M}}{1-u^N}\right]du$$

Rearrange to obtain

$$dx = \frac{y_o}{S_o}\left[1-\frac{1}{1-u^N}+\left(\frac{y_c}{y_o}\right)^M \frac{u^{N-M}}{1-u^N}\right]du$$

Integrate

$$x = \frac{y_o}{s_o}\left[u-\int_o^u \frac{du}{1-u^N}+\left(\frac{y_c}{y_o}\right)^M \int_o^u \frac{u^{N-M}}{1-u^N}du\right]+\text{const.}$$

- The first integral on the RHS of above eq. is designated by or

$$F(u,N) \equiv \int_{o}^{u} \frac{du}{1-u^N} = \text{Varied-Flow Function}$$

The second integral may also be expressed in the form of the varied-flow function. Let

$$v = u^{N/J} \qquad \text{and} \qquad J = \frac{N}{(N-M+1)}$$

$$u = v^{J/N} \qquad\qquad du = \frac{J}{N} v^{\frac{J-N}{N}} dv$$

$$\int_{o}^{u} \frac{u^{N-M}}{1-u^N} du = \frac{J}{N} \int_{o}^{v} \frac{v^{\frac{J}{N}(N-M)+\frac{J-N}{N}}}{1-v^J} dv = \frac{J}{N} \int_{o}^{v} \frac{dv}{1-v^J}$$

- Therefore:

$$x=\frac{y_o}{S_o}\left[u-\int_o^u\frac{du}{1-u^N}+\left(\frac{y_c}{y_o}\right)^M\frac{J}{N}\int_o^v\frac{dv}{1-v^J}\right]+\text{const.}$$

or

$$x=\frac{y_o}{S_o}\left[u-F(u,N)+\frac{J}{N}\left(\frac{y_c}{y_o}\right)^M F(v,J)\right]+\text{const.}$$

$$L=x_2-x_1=\frac{y_o}{S_o}\{(u_2-u_1)-[F(u_2,N)-F(u_1,N)]\}+$$

$$\frac{J}{N}\left(\frac{y_c}{y_o}\right)^M[F(v_2,J)-F(v_1J)]\ \}$$

- Where the subscripts 1 and 2 refer to sections 1 and 2 respectively. This eq. contains varied-flow functions, and its solution can be simplified by the use of the varied-flow-function table. This table gives values of F(u,N) for N ranging from 2.2 to 9.8. Replacing values of u and N by corresponding values of .F(v,J).
-
- In computing a flow profile, first the flow in the channel is analyzed, and the channel is divided into a number of reaches. Then the length of each reach is computed by the above Eq. from known or assumed depths at the ends of the reach. The procedure of the computation is as follows:

1. Compute the normal depth y_o and critical depth y_c from the given values of Q and s_o, n

$$Q = \frac{A}{n} R^{2/3} \sqrt{S_o} \Rightarrow y_o \qquad\qquad \frac{Q^2}{g}\frac{T}{A^3} = 1 \Rightarrow y_c$$

2. Determine the hydraulic exponents N and M for an estimated average depth of flow in the reach under consideration. It is assumed that the channel section under consideration has approximately constant hydraulic exponents

- If any two depths in the section are known then

$$N = 2\frac{\log(K_1/K_2)}{\log(y_1/y_2)} \qquad M = 2\frac{\log(z_1/z_2)}{\log(y_1/y_2)}$$

- N and M can be computed from above expressions or use the general formulas if dP/dy and dT/dy can be evaluated.

3. Compute J by

$$J = \frac{N}{N - M + 1}$$

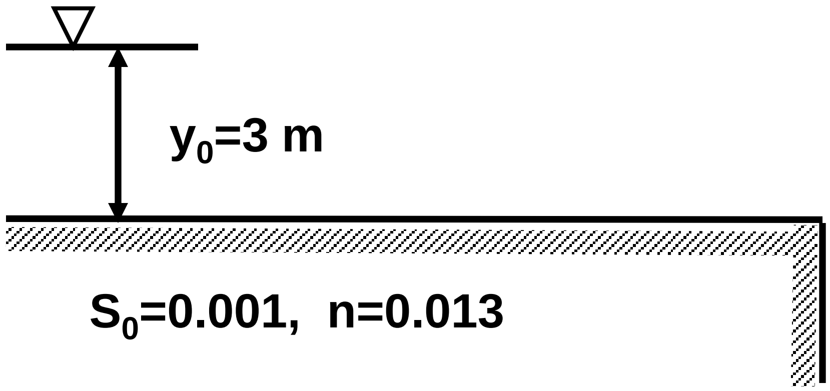
5. Compute values $u = {}^{y}\!/\!_{y_0}$ and $v = u^{N/J}$ at the two end sections of the reach
6. From the varied-flow-function table, find values of F(u,N) and F(v,J).
7. Compute the length of reach by the given equation obtained above.

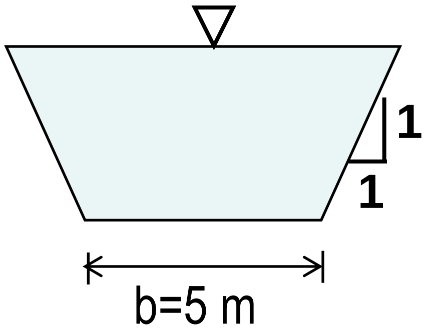
- In doing so a table can be prepared as follows

y	u	v	F(u,N)	F(v,J)
y_1				
y_2				
y_3				
Δ				

Example on Ven Te Chow's Method:

- Water flows at a uniform depth of 3 m. in a trapezoidal channel. The trapezoidal section has a bottom width of 5 m., and side slope of 1H:1V. The channel has a bottom slope of 0.001, and a Manning's roughness coefficient of 0.013. The channel ends with a free fall. **Determine** the water-surface profile and the length of it by using the direct-integration method.





First, determine the discharge, and then the type of the slope:

$$Q = \frac{A}{n} R^{2/3} \sqrt{S_0} \qquad A = by + zy^2 = 5x3 + 3^2 = 24\,\mathrm{m}^2$$

$$P = b + 2y\sqrt{1+z^2} = 5 + 2x3\sqrt{2} = 13.46\,\mathrm{m.}$$

$$\mathrm{R} = \frac{\mathrm{A}}{P} = 1.783\,\mathrm{m} \quad \mathrm{Q} = \frac{24}{0.013}(1.783)^{2/3}\sqrt{0.001} = 86\,\mathrm{m}^3/s$$

To determine critical depth y_c

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{\left(5y_c + y_c^2\right)^3}{5+2y_c} = \frac{86^2}{9.81} = 753.92$$

by trial and error $\quad \mathrm{y_c} = 2.6\,\mathrm{m}$

y_0=3 m > y_c=2.6 m Therefore it is mild slope. Hence M2 profile occurs.

The depth changes between the limits y_c = 2.6 m ≤ y ≤ 0.99y_0 = 2.97 m

- Let's compute the hydraulic exponents

$$N = 2\frac{\ln\left(K_2/K_1\right)}{\ln\left(y_2/y_1\right)} \qquad K = \frac{A}{n}R^{2/3} \qquad M = 2\frac{\ln\left(Z_2/Z_1\right)}{\ln\left(y_2/y_1\right)} \qquad Z = A\sqrt{\frac{A}{T}}$$

$$L = x_2 - x_1 = \frac{y_0}{S_0}\left\{(u_2 - u_1) - \left[F(u_2, N) - F(u_1, N)\right] + \left(\frac{y_c}{y_0}\right)^M \frac{J}{N}\left[F(v_2, J) - F(v_1, J)\right]\right\}$$

$$u = \frac{y}{y_0} \qquad \mathrm{v} = \mathrm{u}^{N/J}$$

Computation of Hydraulic Exponents N and M, n=0.013

y (m)	A (m^2)	P (m)	R (m)	T (m)	K	Z	N	M	J
2,60	19,76	12,35	1,60	10,2	2078,9	27,50	3,71	3,55	3,19
2,97	23,67	13,40	1,77	10,94	2660,7	34,82			

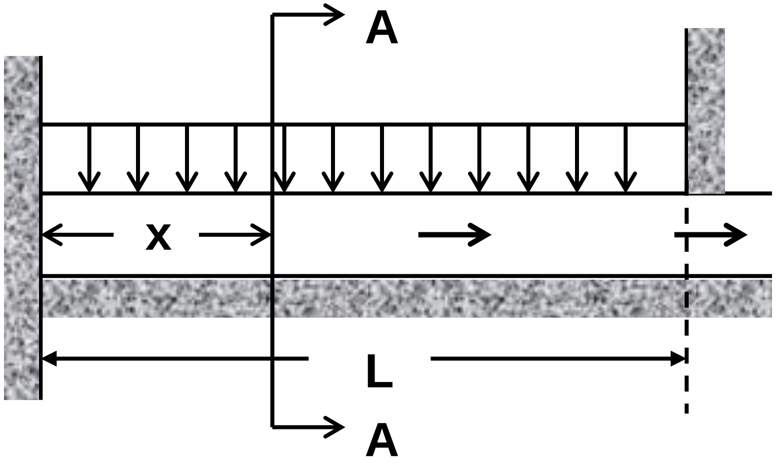
Computation of Length of M2 profile by using Direct-Integration Method in one step

y_0=3.0 m , y_c=2.6 m, n=0.013, $S_{0=}$0.001, N=3.7, M=3.6, J=3.2

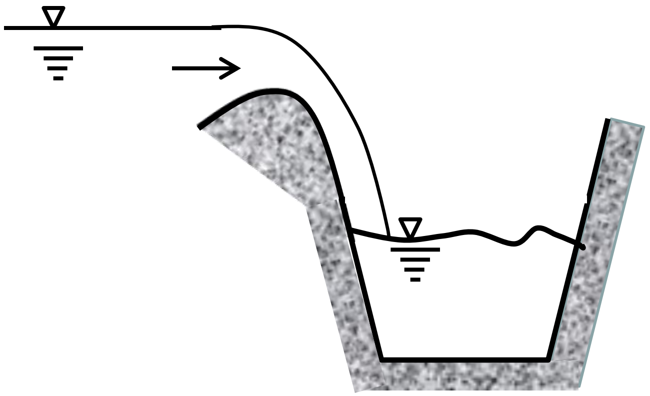
y (m)	u	v	F(u,3.7)	F(v,3.2)	L (m)
2,60	0,87	0,85	1,05	1,043	-341,781
2,97	0,99	0,99	1,7875	2,017	

L=342 m

1.5 Side-Channel Spillways



A side-channel carries water away from an overflow spillway in a channel parallel to the spillway crest as shown in figure.



The discharge over the entire width (L) of an overflow spillway can be determined by Eq.(1), and the discharge through any section of the side channel at a distance x from the upstream end of the channel is

$$Q_x = xCH_a^{3/2} \qquad \text{Eq.(1)}$$

The side-channel spillway must provide a slope steep enough to carry away the accumulating flow in the channel. However, a minimum slope and depth at each point along the channel is desired in order to minimize construction costs. For this reason, an accurate water surface profile for the maximum design discharge is important in the side-channel spillway design.

The flow profile in the side channel cannot be analyzed by the energy principle (i.e., gradually-varied flow profile) because of the highly turbulent flow conditions that cause excess energy loss in the channel.

However, an analysis based on the momentum between two adjacent sections, a distance of Δx apart, in the side channel,

$$\sum F = \rho(Q + \Delta Q)(V + \Delta V) - \rho QV \qquad \text{Eq.(2)}$$

where ρ is the density of water, V is the average velocity, and Q is the discharge at the upstream section. The symbol Δ signifies the incremental change at the adjacent downstream section.

The forces represented on the left-hand side of Eq.(2) usually include the weight component of the water body between the two sections in the direction of the flow $\left[(\rho g A \Delta x)\sin\theta\right]$, the unbalanced hydrostatic forces

$$\rho g A \bar{y}\cos\theta - \rho g(A+\Delta A)\left(\bar{y}+\Delta y\right)\cos\theta$$

and a friction force, F_f, on the channel bottom. Here A is the water cross-sectional area, $\bar{y}$ is the distance between the centroid of the area and the water surface, and θ is the angle of the channel slope.

The momentum equation may thus be written as

$$\rho g A \Delta x \sin\theta + \left[\rho g A \bar{y} - \rho g(A+\Delta A)(\bar{y}+\Delta y)\right]\cos\theta - F_f = \rho(Q+\Delta Q)(V+\Delta V) - \rho QV \quad \text{Eq.(3)}$$

Let $S_o = \sin\theta$ for a reasonably small angle, and $Q = Q_1,\ V+\Delta V = V_2,\ A = (Q_1+Q_2)/(V_1+V_2)$,

and $F_f = \gamma A S_f \Delta x$; the above equation may be simplified to

$$\Delta y = -\frac{Q_1(V_1+V_2)}{g(Q_1+Q_2)}\left(\Delta V + V_2\frac{\Delta Q}{Q_1}\right) + S_0\Delta x - S_f\Delta x \quad \text{Eq.(4)}$$

Where Δy is the change in water surface elevation between the two sections.

This equation is used to compute the water surface profile in the side channel.

The first term on the right-hand side represents the change in water surface elevation between the two sections resulting from the impact loss caused by the water falling into the channel. The middle term represents the change from the bottom slope, and the last term represents the change caused by friction in the channel. Relating the water surface profile to a horizontal datum, we may write

$$\Delta z = \Delta y - S_0 \Delta x = -\frac{Q_1 (V_1 + V_2)}{g(Q_1 + Q_2)}\left(\Delta V + V_2 \frac{\Delta Q}{Q_1}\right) - S_f \Delta x \qquad \text{Eq.(5)}$$

Note that when $Q_1 = Q_2$ or when $\Delta Q = 0$, Eq.(5) reduces to

$$\Delta z = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) - S_f \Delta x \qquad \text{Eq.(6)}$$

which is the energy equation for constant discharge in an open channel as derived in Computation of Gradually-Varied Flow.

Example

A 6-m overflow spillway discharge water into a side-channel spillway with a horizontal bottom slope. If the overflow spillway (C=2.043 $m^{1/2}$/s) is under a head of 1.28 meter, determine the depth change from the end of the side channel (after it has collected all of the water from the overflow spillway) to a point 1.5 meter upstream. The concrete (n=0.013) side channel is rectangular with a 3-m bottom. The water passes through critical depth at the end of the side channel.

Solution

The flow at the end of the side channel (Eq.(1)) is

$$Q = CL(H_a)^{3/2} = (2.043)(6)(1.28)^{3/2} = 17.75\ m^3/\mathrm{sec}$$

The flow at a point 1.5 meter upstream (Eq.(1)) is

$$Q = xC(H_a)^{3/2} = (4.5)(2.043)(1.28)^{3/2} = 13.31\ m^3/\mathrm{sec}$$

Solving for critical depth at the end of the channel, we have

$$q = Q/b = 17.75/3 = 5.92\ \mathrm{m}^3/s/m$$

$$y_c = \left[q^2/g\right]^{1/3} = \left[(5.92)^2/\{(9.81)\}\right]^{1/3} = 1.528\ \mathrm{m}$$

The solution method employs a finite-difference solution scheme (Eq.(4)) and an iterative processs can be employed to compute the upstream depth. The upstream depth (or depth change, Δy) is estimated, and Eq.(4) is solved for a depth change. The two depth changes are compared, and a new estimate is made if they are not nearly equal. Table... displays the solution. Because side-channel spillway profile computations involve implicit equations, computer algebra software (e.g., Mathcad, Maple, and Mathematic) or spreadsheet programs will prove very helpful.

ΔX	**Δy**	**y**	**A**	**Q**	**V**	**Q_1+Q_2**	**V_1+V_2**	**ΔQ**	**ΔV**	**R**	**S_f**	**Δy**
0		1.528	4.584	17.75	3.872							
1.5	-0.3	1.828	5.484	13.31	2.427	31.06	6.299	4.44	1.445	0.824	0.001289	-0.755
	-0.755	2.583	7.749	14.31	1.847	27.62	4.274	-1	0.580	0.949	0.000618	-0.093
ΔX	**Δy**	**y**	**A**	**Q**	**V**	**Q_1+Q_2**	**V_1+V_2**	**ΔQ**	**ΔV**	**R**	**S_f**	**Δy**
0		1.528	4.584	17.75	3.872							
1.5	-0.755	2.283	6.849	13.31	1.943	31.06	5.816	4.44	1.929	0.905	0.000729	-0.819
ΔX	**Δy**	**y**	**A**	**Q**	**V**	**Q_1+Q_2**	**V_1+V_2**	**ΔQ**	**ΔV**	**R**	**S_f**	**Δy**
0		1.528	4.584	17.75	3.872							
1.5	-0.819	2.347	7.041	13.31	1.890	31.06	5.763	4.44	1.982	0.915	0.00068	-0.825
ΔX	**Δy**	**y**	**A**	**Q**	**V**	**Q_1+Q_2**	**V_1+V_2**	**ΔQ**	**ΔV**	**R**	**S_f**	**Δy**
0		1.528	4.584	17.75	3.872							
1.5	-0.825	2.353	7.059	13.31	1.886	31.06	5.758	4.44	1.987	0.916	0.000675	-0.826
ΔX	**Δy**	**y**	**A**	**Q**	**V**	**Q_1+Q_2**	**V_1+V_2**	**ΔQ**	**ΔV**	**R**	**S_f**	**Δy**
0		1.528	4.584	17.75	3.872							
1.5	-0.826	2.354	7.062	13.31	1.885	31.06	5.757	4.44	1.987	0.916	0.000675	-0.826