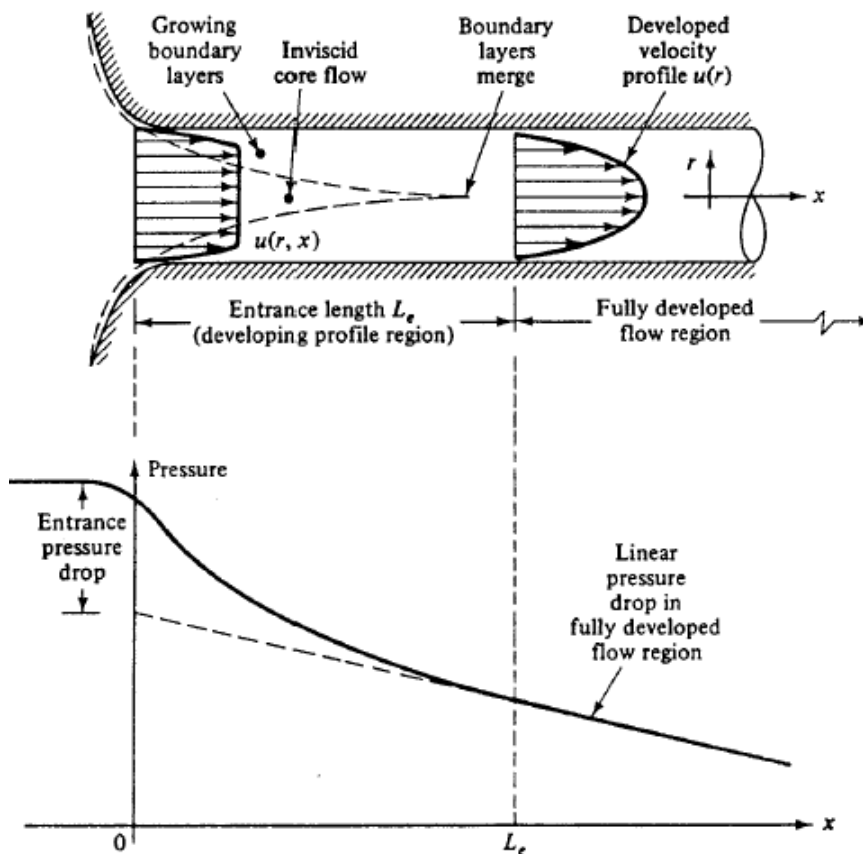


3. Closed Conduit Flow

Flow in closed conduits (pipe, if conduit is circular in section, and duct otherwise) differs from that of open channel flow in the mechanism that derives the flow. In the case of open channel flow, flow occurs due to the action of gravity. In closed-conduit flow, however, although gravity is important, the main driving force is the pressure gradient along the flow. The emphasis of this section will be on pipes.

Flow in pipes is an example of internal flow, i.e., the flow is bounded by the walls, in contrast to external flow where the flow is unbounded. For internal flows, the fluid enters the conduit at one point and leaves at the other. At the entrance to the conduit there appears what is known as **entrance region** within which the viscous boundary layer grows and finally at the downstream end of this region covers the entire cross section. The flow beyond the entrance region is said to have fully developed. The fully developed flow is characterized by a constant velocity profile (for a steady flow), a linear drop in pressure with distance, and a constant wall shear stress.



The entrance length is a function of Reynolds number and is given by relations below:

$$\frac{L_e}{d} \approx 0.06 Re$$

for laminar flow, and

$$\frac{L_e}{d} \approx 4.4 Re^{1/6}$$

for turbulent flow.

Where

$$Re = \frac{\rho v d}{\mu}$$

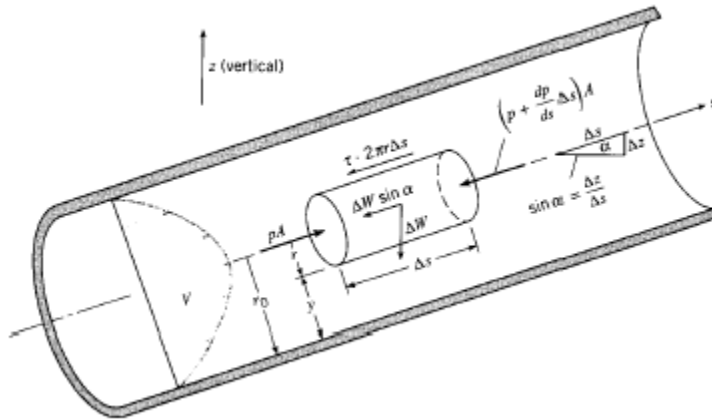
Laminar flow in pipes

Recall that flow can be classified into one of two types, **laminar** or **turbulent** flow (with a small transitional region between these two). The non-dimensional number, the Reynolds number, Re , is used to determine which type of flow occurs:

Laminar flow: $Re < 2000$

Transitional flow: $2000 < Re < 4000$

Turbulent flow: $Re > 4000$

Derivation of basic equations of steady laminar flow in pipes

Consider a case of steady laminar flow in a circular pipe shown below:

Since the flow is steady velocity distribution remains the same through out the length of the pipe. Hence acceleration of the flow is zero. Hence the sum of all forces for the fluid element shown should be zero.

$$pA - \left(p + \frac{dp}{ds} \Delta s \right) A - \tau * 2\pi r \Delta s - \Delta W \sin \alpha = 0 \quad \text{but } \Delta W = \gamma A \Delta s \text{ and } A = \pi r^2$$

$$\gamma \pi r^2 \Delta s \sin \alpha + 2\tau \pi r \Delta s + \frac{dp}{ds} \Delta s \pi r^2 = 0$$

$$\tau = -\frac{1}{2} \left(\frac{dp}{ds} + \gamma \sin \alpha \right) r \quad \sin \alpha = \frac{\Delta z}{\Delta s} = \frac{dz}{ds} \text{ and } p \text{ is a function of } s \text{ only}$$

$$\tau = -\frac{1}{2} \frac{d}{ds} (p + \gamma z) r$$

but for laminar flow $\tau = \mu \frac{dv}{dy}$

Substituting this and simplifying one obtains the relationship for velocity as:

$$V = -\frac{R^2 - r^2}{4\mu} \frac{d}{ds} (\gamma z + p)$$

Thus the velocity distribution in a circular pipe under laminar flow condition is parabolic, with maximum value at the center.

$$V_{max} = -\frac{R^2}{4\mu} \frac{d}{ds} (\gamma z + p)$$

For a horizontal pipe

$$V_{max} = -\frac{R^2}{4\mu} \frac{dp}{ds}$$

The discharge through the pipe is obtained as

$$Q = \int_0^R \left[-\frac{R^2 - r^2}{4\mu} \frac{d}{ds} (\gamma z + p) \right] 2\pi r \cdot dr = -\frac{d}{ds} (\gamma z + p) \frac{\pi R^4}{8\mu}$$

The average velocity,

$$\bar{V} = \frac{Q}{A} = -\frac{R^2}{8\mu} \frac{d}{ds}(\gamma z + p) = -\frac{D^2}{32\mu} \frac{d}{ds}(p + \gamma z) = \frac{V_{max}}{2}$$

$$\frac{d}{ds}(p + \gamma z) \text{ or } \frac{dH}{ds} = \frac{H_2 - H_1}{L} = -\frac{32\mu\bar{V}}{D^2}$$

$$h_f = \frac{\Delta H}{\gamma} = \frac{H_1 - H_2}{\gamma} = \frac{32\mu\bar{V}}{\gamma D^2} L$$

This is known as the Hagen –Poiseuille Formula for Laminar flow

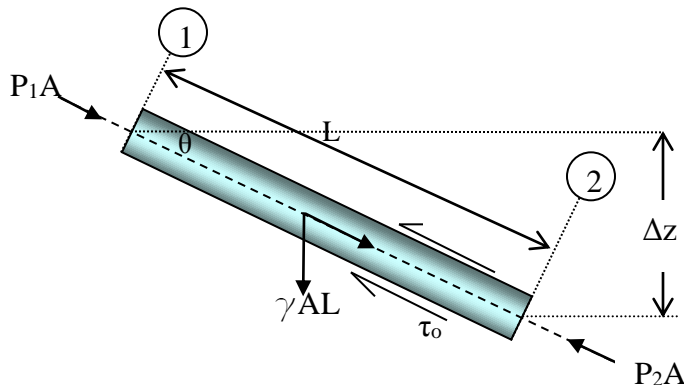
This equation for head loss due to friction is commonly written as

$$h_f = \frac{64}{Re} \frac{L}{D} \frac{V^2}{2g}$$

Turbulent Flow

In turbulent flow there is no longer an explicit relationship between mean stress and mean velocity gradient $\partial u/\partial r$ (because momentum is transferred more by the net effect of random fluctuations than by viscous forces). Hence, to relate quantity of flow to head loss we require an empirical relation connecting the wall shear stress and the average velocity in the pipe.

For turbulent flow, the boundary shear stress is taken as $\tau_o = \lambda \frac{\rho V^2}{2}$ and the derivation of the equation for the friction head loss proceeds in the same way as in the case of laminar flow. Consider a segment of an inclined circular pipe conveying a fluid of density ρ and viscosity μ ,



$$\sin \theta = \Delta z/L$$

For steady uniform flow, since there is no acceleration, $\Sigma F = m a = 0$

$$(P_1 - P_2)A + \gamma A \Delta z - \tau_o P L = 0, \quad \text{where } P \text{ is the wetted perimeter}$$

Substituting $\Delta P = (P_1 - P_2)$ and dividing the whole expression by A , one gets

$$\Delta P + \gamma \Delta z = \tau_o L/R \quad \text{where } R = A/P$$

$$\text{Hence } (\Delta P + \gamma \Delta z)/\gamma L = \frac{1}{2} \lambda \frac{V^2}{gR}$$

$$\text{But } (\Delta P + \gamma \Delta z)/\gamma = h_f$$

$$\text{Thus } \frac{h_f}{L} = \lambda \frac{V^2}{2gR}, \quad \text{for a pipe flowing full } R = D/4$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g}; \quad \text{where } f = 4\lambda = \text{friction factor for turbulent flow.}$$

The last equation for the friction loss in pipes is known as the *Darcy-Weisbach equation*. f is called the Darcy coefficient. This equation also applies for laminar flow with a substitution of $64/R_e$ for the friction factor. For a turbulent flow f is a function of the Reynolds number and the relative wall roughness of the pipe for turbulent flow.

A graphical summary of past experimental results has been presented by moody. This chart, known as the Moody diagram, is a plot of the friction factor as a function of Reynolds number and the relative roughness of the pipe wall, i.e. ϵ/D where ϵ is the roughness in consistent units.

An empirical equation for the friction coefficient is also given by Colebrook and White,

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{\epsilon}{3.7D} + \frac{2.51}{R_e \sqrt{f}} \right], \text{ which applies in both smooth and rough turbulent zones.}$$

Hazen-Williams Formula

The Hazen-Williams Formula has been developed specifically for use with water and has been accepted as the formula used for pipe-flow problems in North America. It reads

$$V = 0.849CR^{0.63} s^{0.54}$$

Where: V = average velocity of flow, (m/s)

R = hydraulic radius, m

S = slope of the energy gradient ($s = h_L/L$)

C = a roughness coefficient

This formula can be rearranged to give

$$\frac{h_L}{L} = \left(\frac{V}{0.849CR^{0.63}} \right)^{1.852} \quad \text{where } R = D/4 \text{ for pipes}$$

Local Losses (Minor Losses)

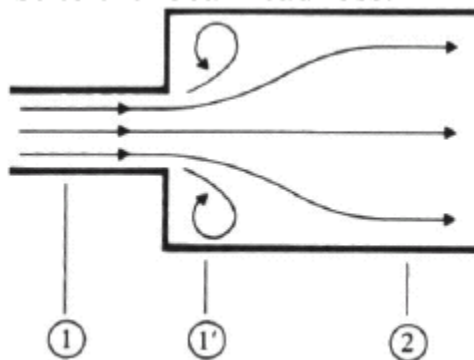
In addition to head loss due to friction there are always head losses in pipe lines due to bends, junctions, valves etc. Such losses are called Minor losses. For completeness of analysis these should be taken into account. In practice, in long pipe lines of several kilometers their effect may be negligible but for short pipeline the losses may be greater than those for friction.

Local losses are usually expressed in terms of the velocity head, i.e.

$$h_i = k_i \frac{V^2}{2g} \quad \text{where } k_i \text{ is the minor loss coefficient}$$

Losses at Sudden Enlargement

Consider the flow in the sudden enlargement, shown in figure below, fluid flows from section 1 to section 2. The velocity must reduce and so the pressure increases (as follows from Bernoulli). At position 1' turbulent eddies occur which give rise to the local head loss.



Apply the momentum equation between positions 1 and 2 to give:

$$P_1 A_1 - P_2 A_2 = \rho Q (V_2 - V_1)$$

Now use the continuity equation to remove Q . (i.e. substitute $Q = A_2 V_2$)

$$P_1 A_1 - P_2 A_2 = \rho A_2 V_2 (V_2 - V_1)$$

Rearranging gives $\frac{P_2 - P_1}{\rho g} = \frac{V_2}{g} (V_1 - V_2)$

Now apply the Bernoulli equation from point 1 to 2, with the head loss term h_L

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_L$$

And rearranging gives $h_L = \frac{V_1^2 - V_2^2}{2g} - \frac{P_2 - P_1}{\rho g}$

Combining the two expressions $h_L = \frac{V_1^2 - V_2^2}{2g} - \frac{V_2(V_1 - V_2)}{g}$

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

Substituting again for the continuity equation to get an expression involving the two

areas, (i.e. $V_2 = V_1 A_1 / A_2$) gives $h_L = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2g}$

This gives the expansion loss coefficient $k_e = \left(1 - \frac{A_1}{A_2}\right)^2$

When a pipe expands in to a large tank $A_1 \ll A_2$ i.e. $A_1/A_2 \approx 0$ so $k_e = 1$. That is, the head loss is equal to the velocity head just before the expansion into the tank.

In other situations such as bends, junctions, sudden contractions, valves and fittings determination analytical values for the loss coefficient is difficult. The loss coefficient is a function of the type of obstruction in the flow and its values are given as in the subsequent figures and tables.

The concept of equivalent pipe length

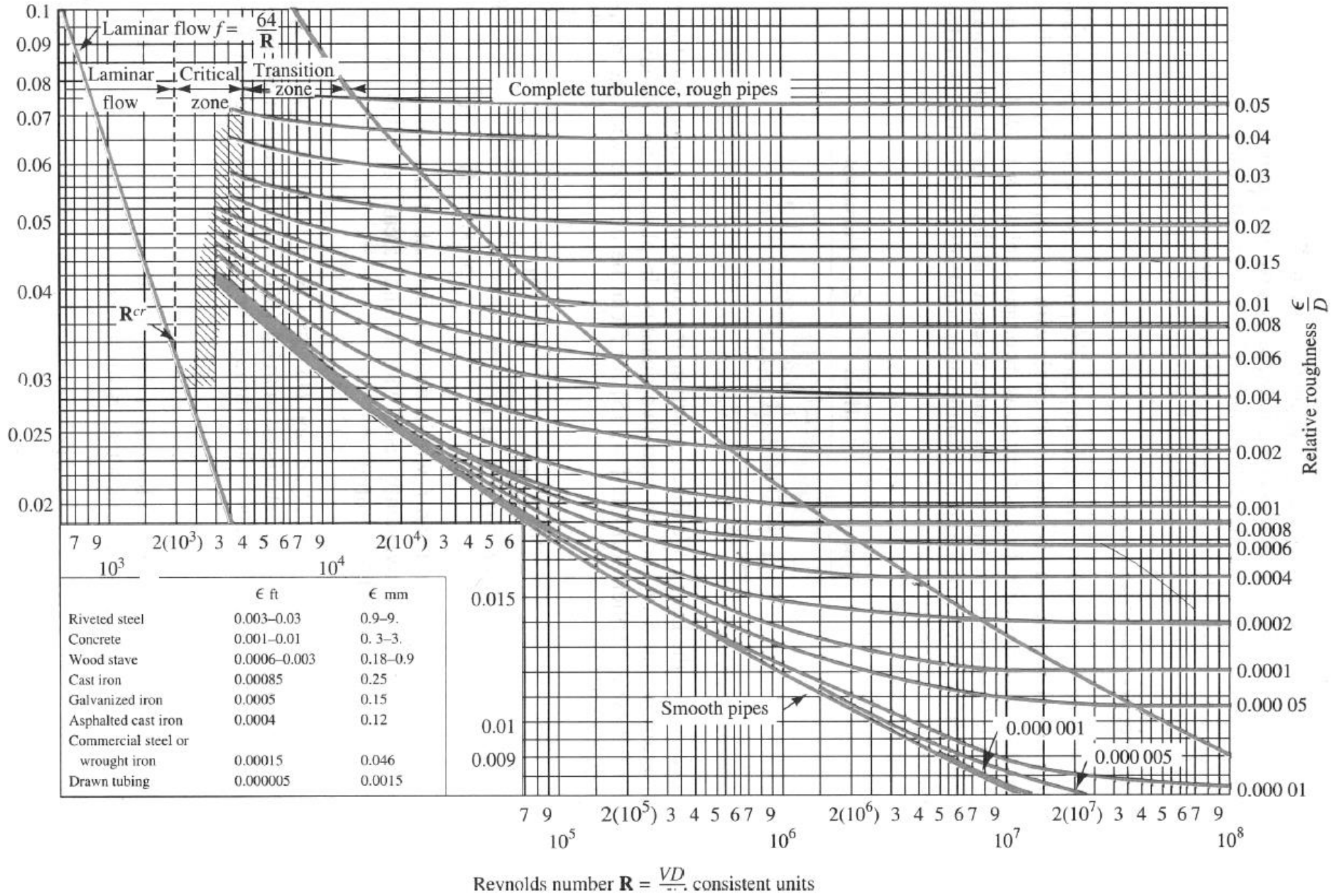
From the previous discussions it can be observed that all types of energy loss in pipes are expressed as a coefficient times the velocity head. Hence if one is interested in the energy loss alone, the minor losses can be expressed in terms of a friction loss over an equivalent length of the pipe. Hence the equivalent length corresponding to a fitting having the minor loss coefficient of k_1 can be obtained from

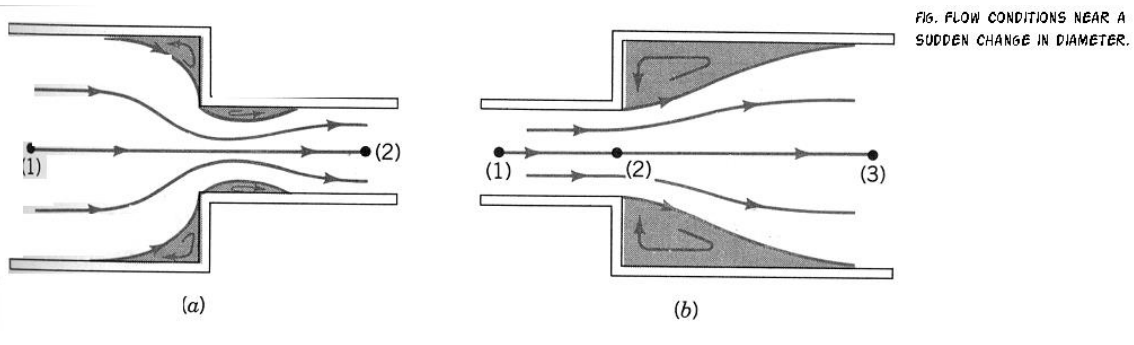
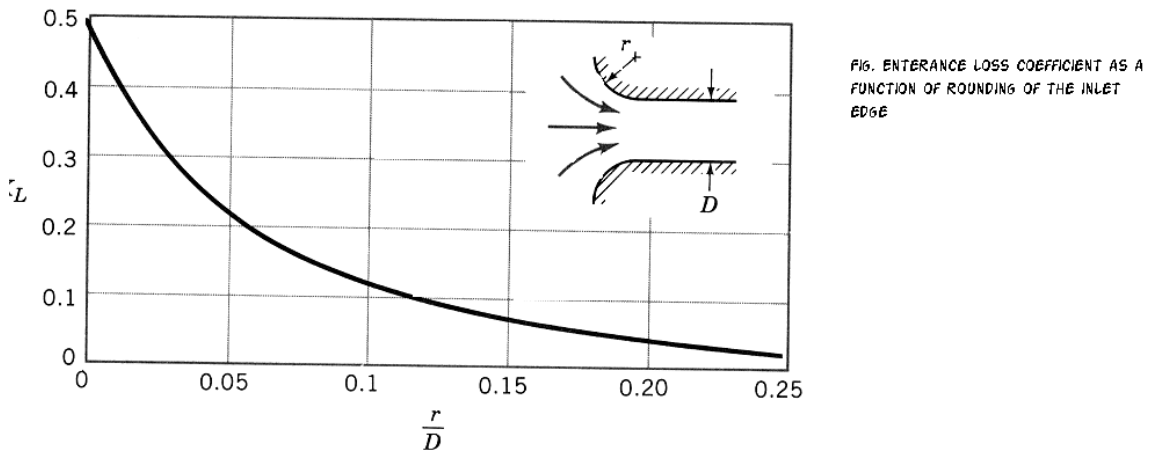
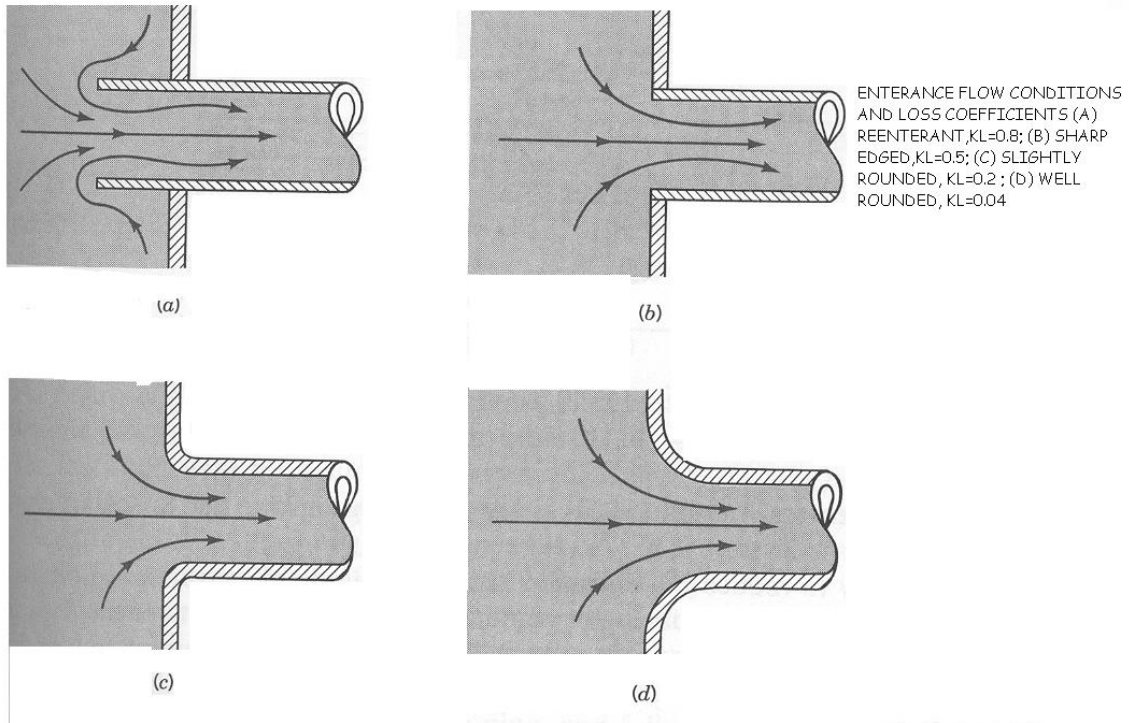
$$f \frac{Le}{D} \frac{V^2}{2g} = k_1 \frac{V^2}{2g}$$

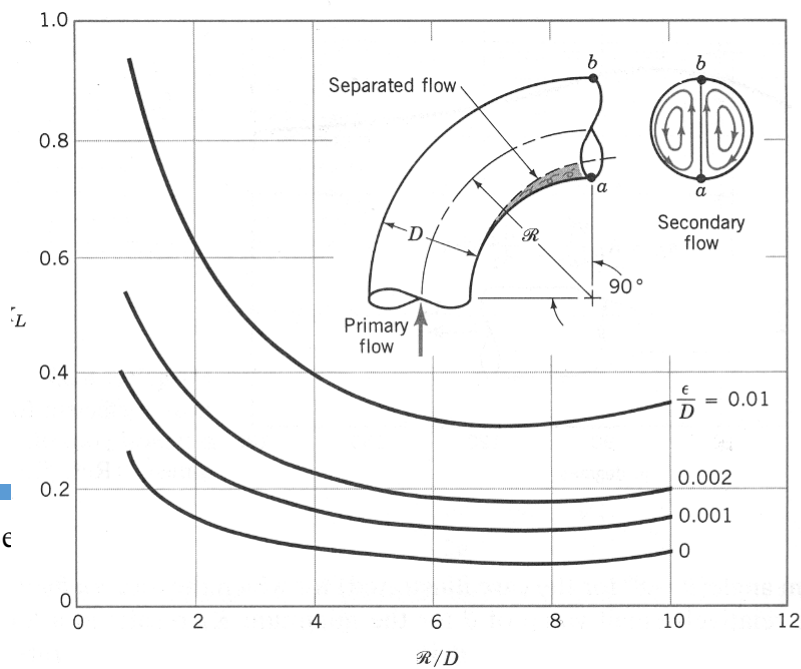
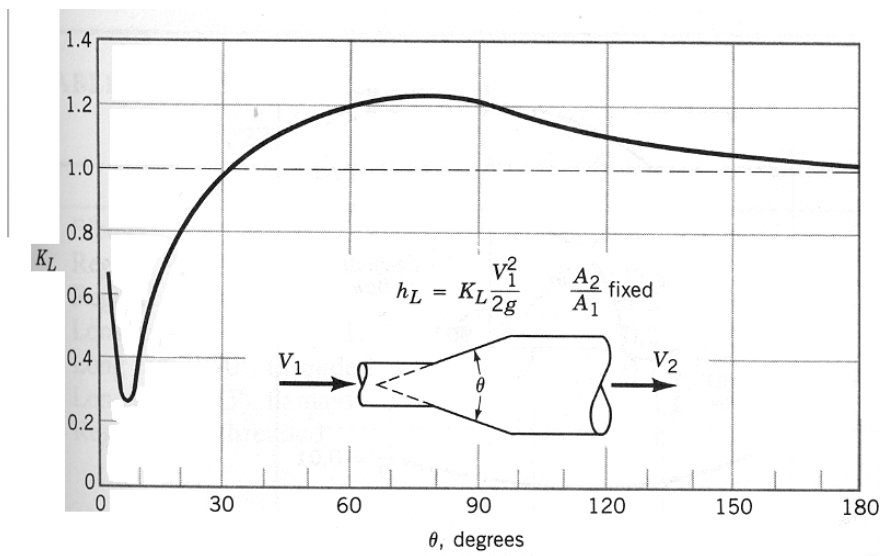
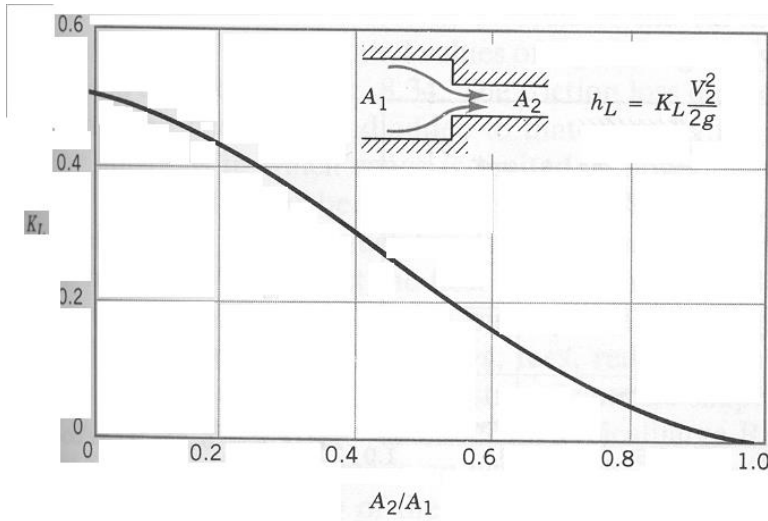
$$\Rightarrow Le = \frac{k_1}{f} D$$

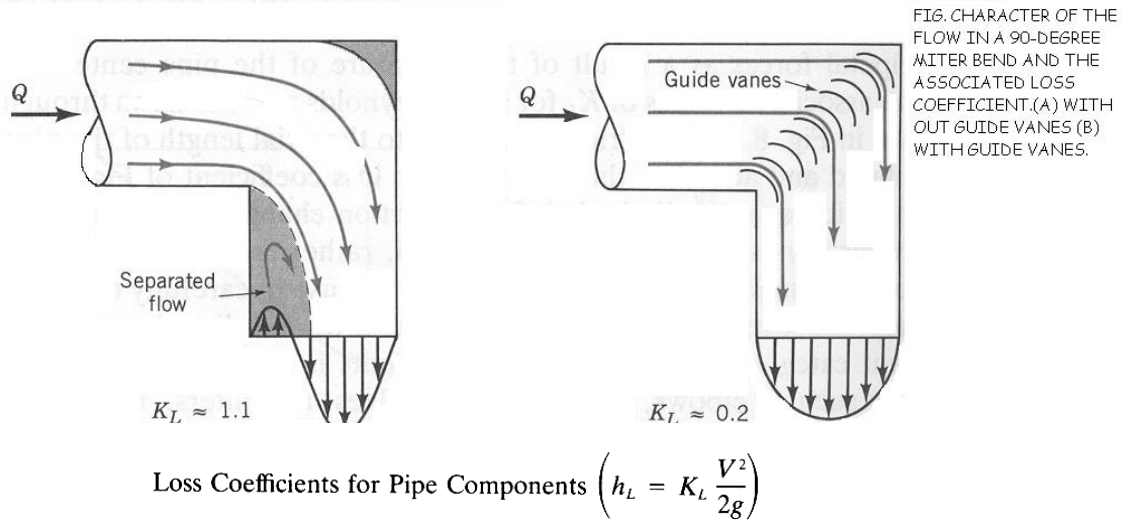
In the same way, if a pipe system consists of a series of two pipes having diameters D_1 and D_2 , and friction coefficients f_1 and f_2 , if the head loss is same in both pipe segments for the same Q , then two pipes are said to be equivalent. Equivalently the length of, say the second pipe, that produces the same total head loss as for the first pipe can be obtained from,

$$f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$
$$\Rightarrow L_2 = \frac{f_1}{f_2} \left(\frac{D_2}{D_1} \right)^5 L_1$$







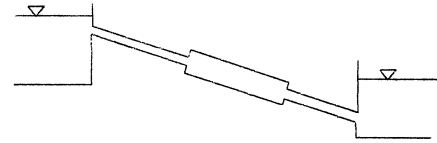


Component	K_L	
a. Elbows		
Regular 90°, flanged	0.3	
Regular 90°, threaded	1.5	
Long radius 90°, flanged	0.2	
Long radius 90°, threaded	0.7	
Long radius 45°, flanged	0.2	
Regular 45°, threaded	0.4	
b. 180° return bends		
180° return bend, flanged	0.2	
180° return bend, threaded	1.5	
c. Tees		
Line flow, flanged	0.2	
Line flow, threaded	0.9	
Branch flow, flanged	1.0	
Branch flow, threaded	2.0	
d. Union, threaded		
	0.08	
e. Valves		
Globe, fully open	10	
Angle, fully open	2	
Gate, fully open	0.15	
Gate, 1/4 closed	0.26	
Gate, 1/2 closed	2.1	
Gate, 3/4 closed	17	
Swing check, forward flow	2	
Swing check, backward flow	∞	
Ball valve, fully open	0.05	
Ball valve, 1/3 closed	5.5	
Ball valve, 2/3 closed	210	

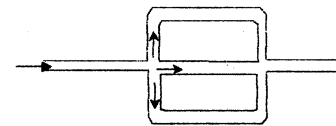
Multiple Pipe systems

In most practical pipe-flow problems the system constitutes multiple pipes joined in different ways. Such complex systems can be one or a combination of the following types

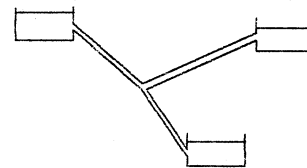
- i) **pipes in series:** here one pipe takes the fluid after the other so that the same flow rate passes through out the entire pipe system.



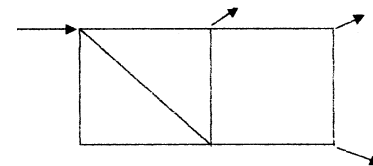
- ii) **pipes in parallel:** in parallel pipes two or more pipes branch from a point (node) and rejoin some distance downstream. Hence at the node the flow is divided into the pipes whereas the pipes flow under the same energy difference between the nodes.



- iii) **Branching pipes:** such pipes branch off from the main and may return to it. Typical example is pipes that convey flow from multiple reservoirs.



- iv) **Pipe networks:** such a system consists of pipes interconnected in such a way that the flow makes a circuit.



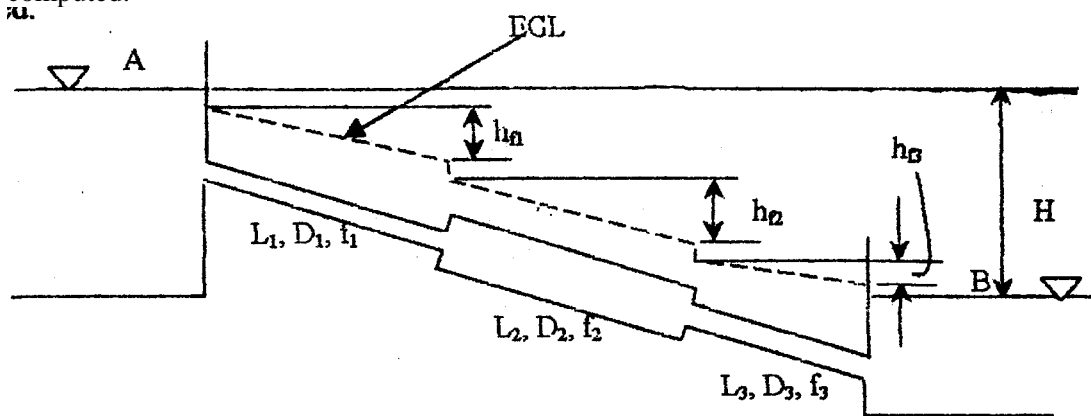
Pipes in series

In such a system the same flow passes through all the pipes involved and hence the usual problems are either:

To determine Q for a given head H , or

To determine the required head H to maintain a certain flow rate.

The latter problem is relatively simple as the friction coefficients for each pipe can easily be computed.



For datum through B. the energy equation including the loss terms takes the form:

$$Z_A + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = Z_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \Sigma h_{L(A-B)}$$

$$H = \Sigma h_{L(A-B)}$$

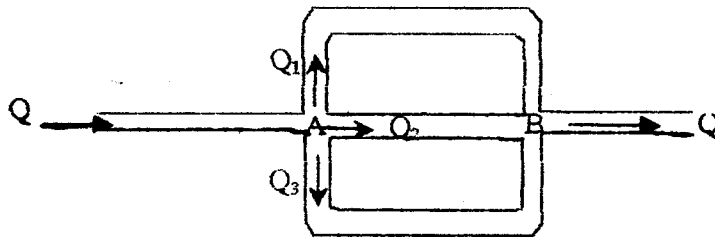
$$\Sigma h_{L(A-B)} = k_{en1} \frac{V_1^2}{2g} + f_1 \frac{L_1}{D_a} \frac{V_1^2}{2g} + k_{ex1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} + k_{c2} \frac{V_3^2}{2g} + f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} + k_{ex3} \frac{V_3^2}{2g}$$

Since the flow rate is the same through out the pipes, the above equation can be reduced to

$$H = \frac{16Q^2}{2g\pi^2} \left[\left(\frac{k_{en1}}{D_1^4} + f_1 \frac{L_1}{D_1^5} + \frac{k_{ex1}}{D_1^4} \right) + \left(f_2 \frac{L_2}{D_2^5} \right) + \left(\frac{k_{c2}}{D_3^4} + f_3 \frac{L_3}{D_3^5} + \frac{k_{ex3}}{D_3^4} \right) \right]$$

To determine the flow rate, since Re is not known, assume values of the friction coefficient for the pipes and compute the value of Q from the equation above. With this value of Q compute Re and based on ϵ/D determine f for each pipe. This iterative procedure is repeated until the assumed and computed values of the friction coefficient are closer to each other.

Pipes in parallel



In such arrangements the flow must satisfy:

$$i) Q = Q_1 + Q_2 + Q_3$$

$$ii) h_{f(A-B)} = h_{f1} = h_{f2} = h_{f3}$$

The common types of problems and the recommended procedures are given below.

i) to determine the discharge Q for a given head difference between A and B. Since in such a case, the head loss is known, one can write

$$h_{f1} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} \quad \text{and solve for } V_1 \text{ by trial}$$

Similar equations can be written for the other pipes that make up the system and solved for their respective velocities. The total discharge is the sum of the product of the velocities and the cross-sectional areas.

ii) the other problem is the determination of the distribution of the discharge among the pipes involved given the total flowrate. A step by step procedure for such problems is:

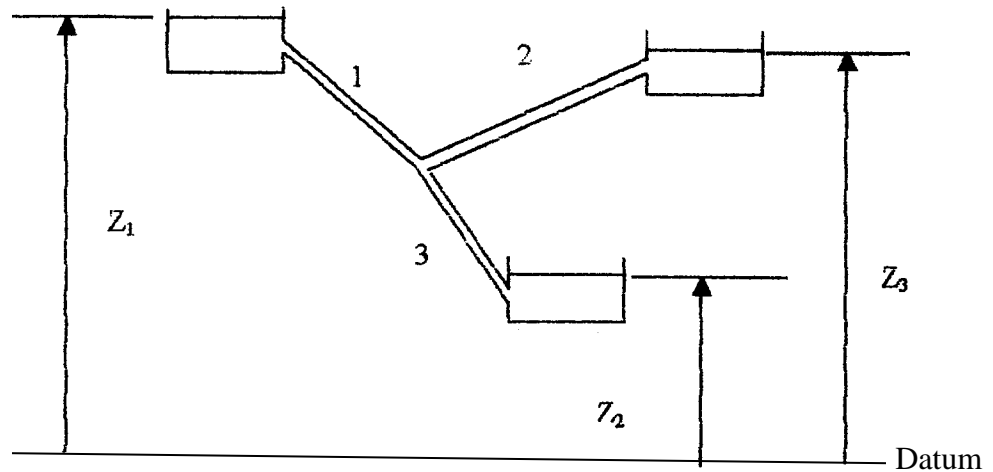
- Assume a likely discharge in one of the pipes, say pipe 1, as Q_1 and compute the head loss through the pipe.
- Using the computed value of the head loss in pipe 1, compute the discharge through the other pipes.
- Add the computed trial discharges in all the pipes and compare the sum with the given total discharge. If the sum is not equal to the total discharge, correct the trial discharges by adjusting them as given below

$$Q_1 = \frac{Q_1}{Q} Q \quad Q_2 = \frac{Q_2}{Q} Q \quad Q_3 = \frac{Q_3}{Q} Q$$

- Compute the head losses again and check if they are equal.

Branching pipes

Such an arrangement of pipes falls in neither of the above two (i.e. parallel or series) categories. The pipes do not also form a network of complete loops. A typical example is the three-reservoir problem shown in the figure.



The problem is often to find the flow rate (including the direction) in each pipe. As the elevation of the HGL at the junction is not known, the flow can not be readily computed. Hence the procedure for solution starts by assuming a value for this head at the junction. The flow rate in each pipe is then computed for the assumed head at the junction. The flow rates computed in such a way are then checked if they satisfy continuity. If the sum of the discharges in the pipes is less than zero (with flow away from the junction taken negative), then this means the assumed head is too high and it is reduced for the next trial. The procedure is repeated until the sum of the flow rates is very close to zero.

Pipe networks

Pipes that are interconnected in such a way that they make loops (or circuits) form a network. In such systems the flow in any of the pipes may come from different circuits and as such it is not simple to know the direction of flow by observation. Pipe network problems involve the analysis of existing systems, i.e. the determination of flow rate in each pipe, pressure at junctions (or nodes), the head losses in the pipes and the selection of appropriate material and size.

The solution of network problems always uses iterative procedures that make use of the following two facts:

- the flow into a junction must equal the flow out of the junction, i.e. at each node (and for the entire system) continuity must be satisfied,
- the algebraic sum of the head losses around any circuit must add up to zero.

Below is outlined a method (commonly known as the Hardy-Cross method. after Prof. Hardy Cross)

- by careful inspection of the network, assume a reasonable distribution of flow rate in the pipes so that continuity is satisfied at each node,
- compute the head loss in each pipe. For this either the Darcy-Weisbach equation can be used with the friction coefficient determined from the Moody diagram, or other methods as discussed below. The Darcy-Weisbach equation can be reformulated as

$$h_L = \frac{1}{2g} \left(f \frac{L}{D} + \sum k \right) V^2 = \frac{1}{2gA^2} \left(f \frac{L}{D} + \sum k \right) Q^2 = rQ^n$$

$$\text{where } r = \frac{1}{2gA^2} \left(f \frac{L}{D} + \sum k \right); n = 2$$

Industrial (commercial) pipe-friction formulas are also used in practice, which are generally of the form:

$$h_L = \frac{LRQ^n}{D^m} = rQ^n$$

where $r = \frac{LR}{D^m}$

R is a resistance coefficient, which, in the case of the Hazen-Williams formula is given as

$$R = 10.675/C,$$

n = 1.852, and m = 4.8704 and C depends upon the roughness and is given in the following table.

Pipe material and condition	C
Extremely smooth, straight pipes; asbestos-cement	140
Very smooth pipes; concrete; new cast iron	130
Wood stave; new welded steel	120
Vitrified clay; new riveted steel	110
Cast iron after years of use	100
Riveted steel after years of use	95
Old pipes in bad condition	60 to 80

Thus compute $\sum h_L = \sum rQ^n$ and check if the sum of the head losses is zero. If it is not the case then adjust the initially assumed flow rate as given below.

- o if the initially assumed discharge is Q_o and the adjustment that should be made to have sum of head losses zero is ΔQ , then head loss for the new discharge is given by $h_L = rQ^n = r(Q_o + \Delta Q)^n$, which can be expanded to get

$$h_L = r \left(Q_o^n + nQ_o^{n-1}\Delta Q + \frac{n(n-1)}{1*2} Q_o^{n-2}\Delta Q^2 + \dots \right)$$

for a small ΔQ , higher order terms of this value can be neglected and the equation approximated as

$$h_L = rQ_o^n + r.nQ_o^{n-1}\Delta Q, \text{ hence}$$

$$\sum h_L = \sum rQ_o^n + \sum r.n.Q_o^{n-1}\Delta Q = 0$$

from which

$$\Delta Q = \frac{\sum rQ_o^n}{\sum r.n.Q_o^{n-1}}$$

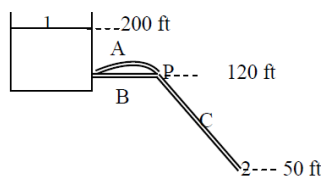
Note: $\sum rQ^n$ is the algebraic sum of the head losses with due regard to signs whereas $\sum r.n.Q^{n-1}$ is the arithmetic sum without any sign consideration.

- o the procedure is repeated until the discharges in the pipes satisfy the two conditions mentioned at the beginning, i.e. $\sum rQ^n = 0$ and continuity is satisfied at each node. When these are satisfied the successive values of the corrective discharge, ΔQ become very small.

Examples

1) Three pipes are interconnected. The pipes characteristics are as follows:

pipe	D(in.)	L(ft)	F(Moody)
A	6	2000	0.02
B	4	1600	0.032
C	8	4000	0.024



Find the rate at which water will flow in each pipe find also the pressure at point P. all pipe lengths are much greater than 1000 diameter, therefore minor losses may be neglected.

Solution:

Apply Bernoulli's between 1 & 2 through pipe A – C knowing that $P_1 = P_2 =$ atmospheric pressure, $v_1 = v_2 \sim 0$, $h_p = 0$

$$Z_1 - Z_2 = 150 = h_L$$

$$h_L = \frac{4f_1 L_1 v_A^2}{2gD_A} + \frac{4f_2 L_2 v_C^2}{2gD_C}$$

$$150 = \frac{0.02 \times 2000 \times v_A^2}{2g(6/12)} + \frac{0.024 \times 4000 \times v_C^2}{2g(8/12)} \quad \dots\dots\dots(1)$$

apply continuity equation,

$$Q_A + Q_B = Q_C$$

$$36v_A + 16v_B = 64v_C \quad \dots\dots\dots(2)$$

Also, $h_{LA} = h_{LB}$

$$80v_A^2 = 153.6v_B^2 \quad \dots\dots\dots(3)$$

Solving equations (1), (2) & (3)

$$v_A = 7.78 \text{ ft/s} \rightarrow Q_A = 1.53 \text{ ft}^3/\text{s}$$

$$v_B = 5.62 \text{ ft/s} \rightarrow Q_B = 0.49 \text{ ft}^3/\text{s}$$

$$v_C = 5.88 \text{ ft/s} \rightarrow Q_C = 2.02 \text{ ft}^3/\text{s}$$

As a check, $Q_A + Q_B = Q_C$

To find pressure at point P, apply Bernoulli's between 1 and P through pipe A or pipe B.

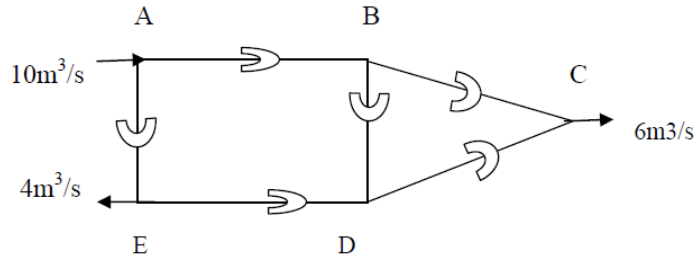
Bernoulli's through A; knowing that $P_{1\text{gage}} = 0$, $v_1 \sim 0$

$$\odot P_p = 4.75 \text{ ft}$$

As a check, apply Bernoulli's equation between 2 & P through C $\odot P_p = 4.75 \text{ ft}$.

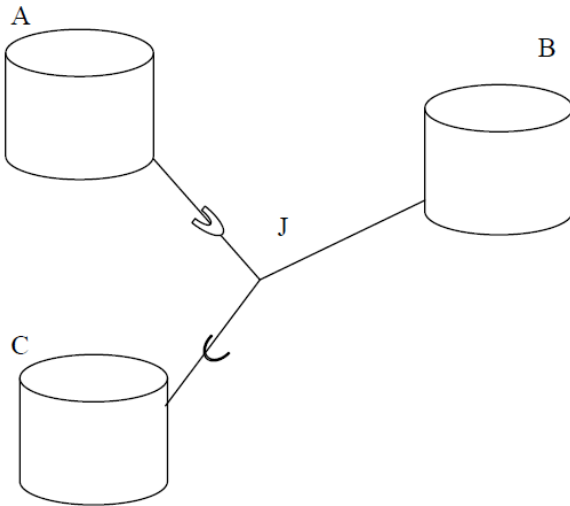
Exercises

- 1) Determine the flow rate in each pipe in the network below using hardy cross method.



Pipe	Length (l, m)	Diameter (ϕ , mm)	Friction factor (f)
AB	200	150	0.02
BC	150	150	
DC	150	150	
ED	200	150	
AE	250	150	
BD	250	150	

- 2) Reservoirs A, B and C have constant water levels of 150, 120 and 90 m respectively above datum and are connected by pipes to a single junction J at elevation 125 m. The length (L), diameter (D), friction factor (f) and minor-loss coefficient (K) of each pipe are given below. Calculate flow in each pipe?



Pipe	L (m)	D(m)	f	K
JA	1600	0.3	0.015	40
JB	1600	0.2	0.015	25
JC	2400	0.25	0.025	50