Chapter 2 - Summary

Method of Root finding

Why numerical method is used for root finding?

- if solving the equation $f(x) = 0$ directly or analytically is not so simple or possible

1. Bisection Method

- Simplest method
- need two initial guesses that bracketing the root

Advantage

- Since all subsequent guesses will bracketing the root, Convergence is guaranteed

Disadvantages

- Can be difficult to find suitable initial guesses
- Converges very slowly

Remark:- The method can give a false root if $f(x)$ is not continuous in the interval

2. Linear interpolation (Regula Falsi) Method

- need two initial guesses that bracketing the root

Advantages

- Since all subsequent guesses will bracketing the root, Convergence is guaranteed
- Closer to the root than bisection method

Disadvantages

- Can be difficult to find suitable initial guesses
- The approach of the root is one sided

Remark:- The method can give a false root if $f(x)$ is not continuous in the interval

3. Direct iteration Method

- Transform $f(x)$ to $g(x) = T(f(x),x)$
- Choose $T(f,x)$ such that $x = g(x)$ corresponds to $f(x) = 0$
  [i.e. needs to rearrange the function $f(x)$]
- Convergence requires $|C| < 1$ where Convergence factor $C \equiv g'(x^*) < 1$

Advantage

- It needs only one starting value/ initial point

Disadvantages

- If $g'(x) < 0$ then get oscillatory behavior
- If $|g'(x)| > 1$ then get divergence
- When the slope of $g(x)$ is too large near a fixed point of $g(x)$, then the method will fail to converge

Remark

- ✓ The choice of $g(x)$ is important to obtain convergence of the function $g(x)$
- ✓ Rate of convergence maximized if $T(f,x)$ chosen so that $g'(x)$ minimized
- ✓ Need to make sure $|g''|$ does not become too large!

4. Newton-Raphson Method

- It needs only one starting value/ initial point
- It needs the formula for the derivative of the function

Advantage

- Converges much more than the Bisection and Regula Falsi Methods

Disadvantages

- It requires the evaluation of both $f(x)$ and $f'(x)$ at each step
- If $f'(x)$ vanishes at an iteration point, then the method will fail to converge

5. Secant (chord) Method

- Similar to Newton-Raphson, but
  - Approximate $f'(x)$ by finite difference
  - Uses two points, not one
- The Formula is identical to Linear Interpolation, but
  - Use two newest points rather than those bracketing the root / i.e. the oldest point is always discarded in favor of the new/

Advantages

- Not need a formula for the derivatives
- Rapid convergent

Disadvantage

- Convergence is not guaranteed