MODULE - 8 LECTURE NOTES – 2

WATER DISTRIBUTION SYSTEMS

INTRODUCTION

The main purpose of water distribution network is to supply water to the users according to their demand with adequate pressure. Water distribution systems are composed of three major components: pumping stations, storage tanks and distribution piping. These systems are designed according to the loading conditions i.e., pressure and demand at nodal points. The loading conditions may include fire demands, peak daily demands or critical demands when the pipes are broken. A reliable design should consider all the loading conditions including the critical conditions. In this lecture we will discuss the simulation and optimization models for the design and analysis of water distribution networks.

COMPONENTS OF WATER DISTRIBUTION SYSTEMS

Various components of water distribution systems are:

(i) Pipes: These are the principal elements in the system. The flow or velocity is usually described using Hazen – Williams equation

$$V = 1.318 C_{HW} R^{0.63} S_f^{0.54} \tag{1}$$

where V is the average flow velocity. C_{HW} is the Hazen – Williams roughness coefficient, R is the hydraulic radius and S_f is the slope.

In terms of headloss h_L, the above equation can be expressed as,

$$h_L = \frac{KLQ^{1.852}}{C_{HW}^{1.852} D^{4.87}} = K_p Q^{1.852} \tag{2}$$

where L is the length of the pipe, D is the diameter and Q is the flow rate.

Headloss can also be determined using Darcy – Weisbach equation as

$$h_L = f \frac{L}{D} \frac{V^2}{2g} = \frac{8fL}{\pi^2 g D^2} Q^2 = K_p Q^2 \tag{3}$$

where f is the friction factor (determined from Moody's diagram) and g is the acceleration due to gravity.

(ii) Node: Junction nodes are connections of pipes to transfer the water. The diameter of pipe is changed at these nodes. Fixed grade nodes is where pressure and elevation are fixed i.e., reservoirs, tanks etc.

(iii) Valves: These are used to vary the head loss or to control the flow.

(iv) Tanks: It stores water and acts as a buffer by storing water at low demands and releasing at high demands.

(v) Pumps: Used to increase the energy

SIMULATION OF NETWORK

The flow distribution through a network should satisfy the conservation of mass and conservation of energy. Consider the network structure in Figure 1 with 6 pipes and 5 nodes.

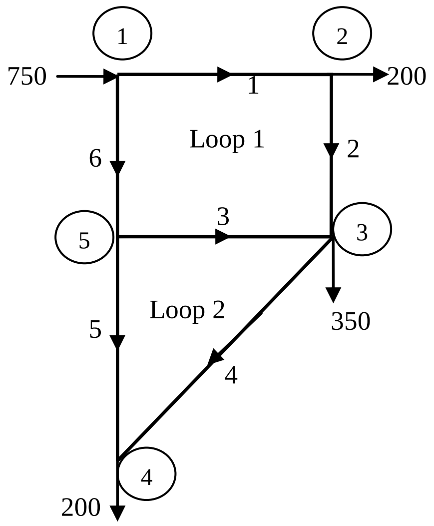


Fig. 1

Conservation of mass: Flow at each junction nodes must be conserved

$$\sum Q_{in} - \sum Q_{out} = Q_{ext} \quad (4)$$

where Q_{in} and Q_{out} are the flows in and out of the node respectively and Q_{ext} is the external supply or demand.

Conservation of energy: For each loop, energy must be conserved i.e., sum of head losses should be zero.

$$\sum h_{L_{i,j}} - \sum H_{pump} = 0 \quad (5)$$

where $h_{Li,j}$ is the head loss in the pipe connecting nodes i and j and H_{pump} is the energy added by the pump (if any). $h_{Li,j}$ can be determined using either eqn. 2 or 3.

Energy must be conserved between the fixed grade nodes which are points of known head (elevation plus pressure head).

$$\Delta E_F = \sum h_{L_{i,j}} - \sum H_{pump} \tag{6}$$

If the number of pipes in the network is N_L, number of junction nodes is N_J and number of fixed grade nodes is N_F, then total number of equations will be $N_L + N_J + (N_F - 1)$.

The set of equations obtained can be solved by any iterative techniques like Hardy-Cross method, linear theory method and Newton - Raphson method.

<u>**Hardy-Cross method**</u>

In this method, the loop equation (eqn. 5) in terms of flow is used. The loop equations are transformed into so called ΔQ equations in the form

$$\sum_{i,j} K_{p,i,j} \left(Q_{i,j} + \Delta Q_{i,j}\right)^n = 0 \tag{7}$$

Here head loss is determined from equation 2 or 3.

Eqn. 7 is rewritten to account the direction of flow as

$$\sum h_{L,i,j} = \sum_{i,j} K_{p,i,j} \left(Q_{i,j} + \Delta Q_{i,j}\right)^n sign\left(Q_{i,j} + \Delta Q_{i,j}\right) = 0 \tag{8}$$

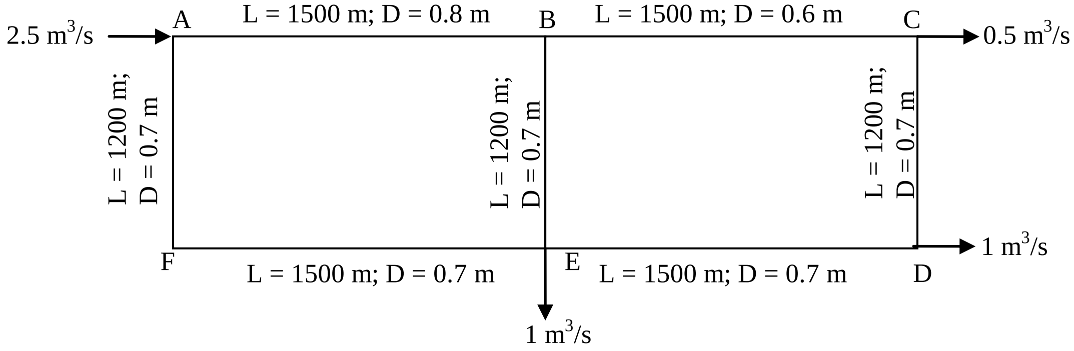
This equation can be finally expressed as

$$\Delta Q_p = -\frac{\sum_{i,j} K_{p,i,j} Q_{i,j}{}^n}{\sum_{i,j} \left| n K_{p,i,j} Q_{i,j}{}^{n-1} \right|} \tag{9}$$

First a flow distribution is assumed across the network. Then the correction ΔQ as given in eqn. 9 is applied in a particular loop p. The numerator in eqn. 9 is the algebraic sum of headloss in loop p taking care of the sign of the flow. If clockwise flows are taken positive, then the corresponding headlosses are positive. The same is applicable while applying correction also i.e., ΔQ_p is added to flows in the clockwise direction and subtracted from flows in counterclockwise direction.

Example:

Consider the pipe network shown below. The friction factor = 0.2. Determine the flow rate in each pipe.



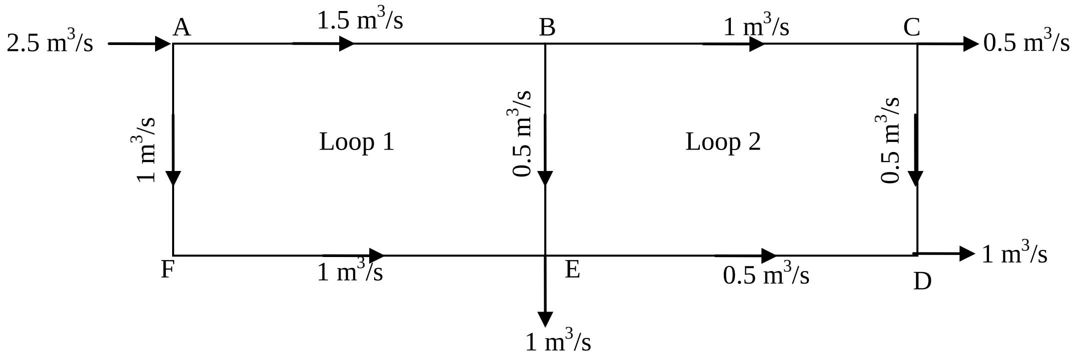
Solution:

Step 1: Determine the K values in eqn. 3

$$h_L = \frac{8fL}{\pi^2 g D^2} Q^2 = KQ^2 \text{ where } K = \frac{8fL}{\pi^2 g D^2}$$

$K_{AB} = 3.88$; $K_{BC} = 6.89$; $K_{FE} = K_{ED} = 5.06$; $K_{AF} = K_{BE} = K_{CD} = 4.05$

Step 2; Assume initial flows in each pipe as shown below



Step 3: Consider loop 1 and calculate ΔQ according to eqn. 9; n = 2. Consider anticlockwise flows positive.

$$\Delta Q_1 = -\frac{1}{2}\frac{K_{AF}Q_{AF}^2 + K_{FE}Q_{FE}^2 - K_{BE}Q_{BE}^2 - K_{AB}Q_{AB}^2}{K_{AF}Q_{AF} + K_{FE}Q_{FE} - K_{BE}Q_{BE} - K_{AB}Q_{AB}}$$
$$= -\frac{1}{2}\frac{4.05\times 1^2 + 5.06\times 1^2 - 4.05\times 0.5^2 - 3.88\times 1.5^2}{4.05\times 1 + 5.06\times 1 + 4.05\times 0.5 + 3.88\times 1.5}$$
$$= 0.0187$$

Step 4: Consider loop 2 and calculate ΔQ

$$\Delta Q_2 = \frac{1}{2}\frac{4.05\times 0.5^2 + 5.06\times 0.5^2 - 4.05\times 0.5^2 - 6.89\times 1^2}{4.05\times 0.5 + 5.06\times 0.5 + 4.05\times 0.5 + 6.89\times 1}$$
$$= 0.2088$$

Step 5: Flows for next iteration

Q_{AF} = 1 + 0.0187 = 1.0187

Q_{FE} = 1 + 0.0187 = 1.0187

Q_{BE} = 0.5 – 0.0187 + 0.2088 = 0.6901

Q_{AB} = 1.5 - 0.0187 = 1.4813

Q_{ED} = 0.5 + 0.2088 = 0.7088

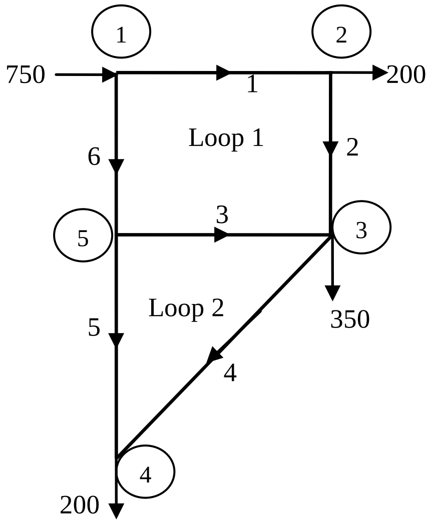
Q_{CD} = 0.5 – 0.2088 = 0.2912

Q_{BC} = 1 – 0.2088 = 0.7912

Repeat steps 2 to 5 with new flows till ΔQ is insignificant.

Linear theory method

Linear theory is more efficient when compared to Hardy-Cross method. Here we will demonstrate through the example network given below.



The equations for linear theory are:

<u>Conservation of mass:</u>

Node 1: $Q_1 + Q_6 = 750$

Node 2: $Q_1 - Q_2 = 200$

Node 3: $Q_2 + Q_3 - Q_4 = 350$

Node 4: $Q_4 + Q_5 = 200$

Node 5: $Q_6 - Q_3 - Q_5 = 0$

Among these 5 eqns. only 4 need to be used to avoid redundancy

<u>Conservation of energy:</u>

Loop 1: $K_{12}\, Q_{12}^2 + K_{23}\, Q_{23}^2 - K_{35}\, Q_{35}^2 - K_{51}\, Q_{51}^2 = 0$

Loop 2: $K_{53}\, Q_{53}^2 + K_{34}\, Q_{34}^2 - K_{45}\, Q_{45}^2 = 0$

Linearising the above eqns. using $k = K_p\, Q$, the eqns. can be written as

Loop 1: $k_{12}\, Q_{12} + k_{23}\, Q_{23} - k_{35}\, Q_{35} - k_{51}\, Q_{51} = 0$

Loop 2: $k_{53}\, Q_{53} + k_{34}\, Q_{34} - k_{45}\, Q_{45} = 0$

These 2 eqns. along with the 4 mass conservation eqns. can be solved to obtain 5 unknown discharges.

OPTIMIZATION OF WATER DISTRIBUTION SYSTEMS

Simulation of distribution networks as discussed above helps to determine the hydraulic parameters such as pressure heads, tank levels etc. These models are unable to determine the optimal or minimum cost system. In addition to the cost minimization, the typical goals of water distribution systems problem in designing pipe system can be:

A) Meeting the household demands.

B) Meeting the required water pressure at all nodes of the distribution system.

C) Optimal positioning of valves.

Therefore, designing water distribution system is a multiobjective problem, which is also characterized by nonlinearity resulting from the simulation model.

Since the main purpose of a water distribution system is to supply according to the demands with adequate pressure, a typical optimization problem will be to minimize the system's cost while meeting the demands at required pressures. Hence optimization problem can be stated as:

Minimize : Total cost (Capital cost + Energy cost for pumping water throughout the system)

Subject to:

(i) Hydraulic constraints

(ii) Water demand constraints

(iii) Pressure requirements