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Rational design of urban water supply and distribution systems

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Abstract: The design of urban water supply and distribution systems (WDSs) is conventionally based on a rigid-static analysis based on a number of simplifying assumptions, which often are not fulfilled during the operation of the systems. This fact often leads to a low performance of these systems with frequent failures, which disturb the activities and the life of the users. This paper proposes a dynamic analysis of the WDSs incorporating aspects of integration, sustainability, uncertainty and multicriteria analysis. More specifically the paper proposes the analysis of the entire system (from the source to the individual users) for the 24 hour operation (integration), for the conditions expected during the life long operation of the system (sustainability), taking into account the uncertain nature of the WDS variables (e.g. uncertain demand and pipe roughness) (uncertainty), and using a number of criteria such as economic, energy, greenhouse gases (GHG) emissions and other (multicriteria approach).

Key Words: Urban water, water distribution systems, water cycle, life cycle, energy consumption, GHG emissions, head-driven approach, uncertainty analysis

1. INTRODUCTION

The design of Municipal Water Supply and Distribution Systems (WDSs) is still practiced in professional studies using gross simplifying assumptions and a rigid-static analysis. As a result, the system is not tested under various conditions, which can be encountered during its operational life, and under different water demand scenarios. This may create frequent failures in meeting the actual demand or pressure requirements during the operation of the system operation in real life conditions. In the opposite direction the system is often over-designed and very costly.

During the recent years the scientific community has produced a large number of innovative methods which can offer significant improvements in the design of WDSs. Apart from the improvements of well known solvers for the analysis of WDSs (Jeppson, 1976; Larock et al., 2000; Lancey and Mays, 2000; Tsakiris and Spiliotis, 2010), more realistic description of the conditions of nodal demands, roughness of pipes and their variability can also offer significantly in the accuracy of the results derived (Xu and Goulter, 1996; Spiliotis and Tsakiris, 2012a).

Interesting reviews on the subject can be found in the books of Bhave (1991), Walski et al. (2003), Trifunovic (2006) and Swamee and Sharma (2008).

It is expected that even with the most conventional tools currently used in practice, some guidance for the assumptions and the range of values of some important parameters related to the materials used is still needed. In this context an attempt was recently made to produce and propose some practical guidelines for the analysis of WDSs, mainly targeted towards professional engineers, in a paper published recently in this journal (Tsakiris and Tsakiris, 2012). It was argued that the professional design could be improved by adopting some of the recent developments in the analysis of WDSs.

The existing methods for the analysis of the WDSs can be classified into five-generation classes (Tsakiris and Spiliotis, 2014). In short the five-generation classes and their basic characteristics are concisely presented in Table 1. Also some methods from the fourth and the fifth generation classes are briefly discussed in this paper.

As can be seen in Table 1, the recent developments for the analysis of WDSs (generation classes 3 and 4) incorporate:

a) The dependence of nodal outflow on the pressure at the node
b) The uncertainty analysis of nodal outflow and pipe roughness

Table 1. The five-generation classes methods for the analysis of WDSs.

Generation Class	Basic Characteristics	Representative References
1	Sequential solution of Q-equation or H-equations	Cross, 1936
2	A linear system is solved in any iteration for determining the ΔQ and Δh a. Linear b. Newton-Raphson	a. Wood and Charles, 1972 b. Shamir and Howard, 1968
3	Pipe equations are solved for Q and h simultaneously (e.g. Gradient method/EPANET)	a. Todini and Pilati, 1988
4	Analysis based on pressure dependent outflow (head driven simulation) a. h-N-R Hazen-Williams b. Linear flow along branches c. EPANET d. h-N-R Darcy-Weisbach	a. Tabesh et al., 2002 b. Giustolisi and Todini, 2009 c. Siew and Tanyimboh, 2012 d. Tsakiris and Spiliotis, 2014
5	Uncertainty is incorporated into the analysis a. Taylor series expansion around the mean (for Q or h) b. Fuzzy sets Q c. Fuzzy sets h	a. Xu and Goulter, 1996 b. Gupta and Bhare, 2007 c. Spiliotis and Tsakiris , 2012a

It is the opinion of the author that the engineering design could be improved if methods or elements of methods from the fourth and the fifth generation classes could be adopted for the analysis of WDSs. In the present paper however the design of WDSs is viewed in a wider frame. The main proposals (apart from the above) of this paper are to incorporate in the WDS analysis the entire system from the source to the end user (known as water cycle analysis) and the study of the system during its entire life cycle (life cycle analysis). In this context comparisons of hierarchically higher alternatives can be also compared whereas at the same time more realistic representation of the system parameters is used.

2. MAJOR ASSUMPTIONS

In the professional practice the conventional design of an Urban Water Supply and Distribution System is based on a number of simplifying assumptions, the most critical of which are:

a) The design discharge is based on the expected average demand of the estimated future population.
b) Empirical multipliers are used for estimating the maximum water demand.
c) The system is solved for steady state operation with the expected maximum demand.
d) The type of materials to be used is decided prior to the analysis of the system.
e) Pipe roughness and local energy losses are estimated using bibliographic data.
f) The internal diameters of the pipes are predetermined irrespective of the type of coating (e.g. internal coating in steel pipes has different depth if it is of epoxy resin or cement).
g) Energy losses are calculated with the expected maximum water demands concentrated at the nodes, uniformly and simultaneously distributed.

The major problem in this type of design is that all of the parameters involved are considered constant and reliable throughout the life cycle of the system.

It is of outmost importance that at least few additional tests and procedures could be implemented for narrowing the range of systematic errors in the operation of these systems. These can be:

a) The analysis of the entire system, «Water cycle analysis», from the source to the end user
b) The “dynamic” simulation of the system during a 24 hour expected operation.
c) The testing of the system for scenarios representing demand concentration at certain parts of the system (e.g. where concentration of population is expected to increase or extension of the system seems likely to occur in the future).
d) The description of the demand at the nodes and the pipe roughness by their range instead of constant values.

Some of these improvements will be discussed further below.

3. DESIGN CRITERIA

Conventionally during the design of a WDS, the principal criterion is the fulfilment of flow and head requirements in all parts of the urban area. In most of the cases the water distribution system is isolated and analysed separately from the water supply network (from the source to the service reservoir).

However, even if the system incorporates the water supply and the distribution systems together, the criterion which is used is to find the solutions which lead to the minimum total construction costs. Based on this single criterion the source of water, the location of the service reservoirs and the dimensions of the mains are selected.

Today, in the new complicated world, obviously, this single criterion seems very weak if our objective is to rationally design the system for most reliable and effective operation during its whole life.

Even in case of some type of cost optimisation, aiming at achieving the minimum construction cost, the result may be misleading. Just to note that the cost values used at the design stage (as in most public works) are logistic prices imposed by the responsible Ministry, whereas the prices at the construction and maintenance phases are the market values with all types of discounts and offers. That is to say that “optimisation” at the design stage searching for the min cost construction solution, does not offer substantially to an improved design of the WDS.

In our view the criteria which could be considered at the design phase (in a multicriteria framework), can be:

a) Total cost of construction, operation and maintenance in the life cycle of the project (LC)
b) Total energy consumption and emissions of GHG anticipated in the LC
c) Compatibility with the «Programme of Measures» of the Water Resources Management Plan in the River Basin District (WFD)
d) Effects of the project construction on the local and national economy
e) Sustainability of operation (e.g. source, environment, society, technology)
f) Adaptation capacity in man made changes and the anticipated climate change

4. WATER CYCLE ANALYSIS

As mentioned earlier, the water supply and distribution system can be studied as an entity following the flow of water from the source to the end users. This type of approach is known as «water cycle analysis».

In this approach the entire system can be divided into a number of components such as:

- Extraction of water at the source

- Conveyance to the service reservoir and the treatment plant
- Treatment of water
- Distribution of water
- Wastewater collection and conveyance to the wastewater treatment plant
- Wastewater treatment and disposal

For each component all the required information can be gathered for the total water demand of the municipality for each year. As known the total water demand comprises the residential, the commercial, industrial, irrigation and recreational water demand. Based on this demand, as it develops during the life cycle of the project, all other important variables are estimated.

If for instance we want to quantify the energy consumption and the greenhouse gas (GHG) emissions of the system, three categories may be identified (Arora et al., 2013):

- Embodied energy and embodied GHG emissions (that is the total energy and GHG emissions associated with the production of materials used in the equipment selected).
- Energy use and GHG emissions for transport and construction (that is the energy and GHG emissions for the transport of equipment and the construction).
- Operational energy use and operational GHG emissions (the energy use and GHG emissions during the entire operational life of the system)

The figures obtained for each category are transformed (if necessary) to values corresponding to the entire life cycle of the project due to the different time reference of each category. For easier comparison purposes the energy and the GHG emissions are expressed per thousand m^3 supplied.

An interesting example from a case study in Melbourne was presented recently by Arora et al. (2013).

From this study we selected the final table in which five different alternative water supply sources are compared based on the energy consumption and the GHG emissions per thousand of m^3 of water supplied. The alternatives examined are conventional water sources, rainwater, stormwater, recycled water and desalinated water. The results are presented in Table 2 per thousand of m^3 supplied.

Table 2. Energy and GHG emissions for the alternative water supply sources (Arora et al., 2013).

Alternative Water Supply Services	Energy use E (MWh/1000m^3)	GHG emission factor G (Mg/1000m^3)
Conventional	1.63	1.34
Rainwater	3.39	3.96
Stormwater	2.80	3.17
Recycled water	2.42	2.66
Desalinated water	6.78	5.77

Although this table has been based on local data some important, though indicative, remarks can be derived. For instance desalination, which is easily proposed in many cases (e.g. islands) and may be characterized as a “less costly” solution, is remarkably inferior solution in terms of the total energy use and the total GHG emissions.

5. LIFE CYCLE ANALYSIS

The analysis, which is proposed in this paper, is the life cycle analysis of the entire WDS. The system is tested for a representative 24-hour operation using all the data as they are influenced by the elapsed time. For instance, instead of the present and the design population after 30 years (P_0 and P_{30}, respectively), a function of P(t) is considered covering the entire operational life (e.g.

design life) of the system. In the same way we can devise functions of the expected roughness coefficient, r(t), the mean daily demand per capita, q(t), the energy consumption, e(t), the GHG emissions, GHG(t) etc, as functions of time. It is important to note that the values of the above functions give estimates of the most likely values (mean values) at each year, whereas the true values are distributed around each mean value.

Therefore, the analysis can be performed for the most expected values or other selected values around the mean. Fig. 1 shows an illustrative example of the most important parameters for the design as functions of time. In the same figure, the "confidence bands", indicating the range of values around the mean, are also presented.

The problem of dispersion, which reflects the uncertainty of the estimation of each determinant under review, will be dealt with in the next section, taking as an example the uncertainty of nodal demand and the pipe roughness.

To simplify the procedure and decrease the number of scenarios examined, the test of the system can be performed for several time horizons within the life cycle of the system. For example, for a life cycle of 30 years, the performance tests can be applied for the year 0, 5, 10, 15, 20 and 25, assuming constant conditions for the 5 year period following each test year.

Needless to say that there are several other ways to decrease the number of scenarios examined.

6. HEAD-DRIVEN METHODS

The head–driven methods incorporate in the analysis the dependence of the nodal outflow on the pressure at the node. Many authors have presented versions of this category of methods (e.g. Tabesh et al., 2002; Giustolisi and Todini, 2009; Nazif et al., 2010; Tabesh et al., 2011; Tsakiris and Spiliotis, 2014). Here only the latter method is briefly presented. Interested readers can find details in the original papers and discussions on the subject.

For the solution of the equations of the system, the h-Newton-Raphson solver is used as it was revised by Spiliotis and Tsakiris (2011; 2012b). The Darcy-Weisbach equation and the Colebrook-White equation are used for the calculation of linear energy losses and the friction coefficient f, respectively. In fact the explicit equation of Swamee and Jain (1976) is used avoiding iterations. Among the various types of outflow dependence on pressure, the quadratic dependence of outflow seems nearer to reality. Obviously in this method the Jacobian matrix is modified. Special constraints are also proposed for avoiding instability in the computational solution. Finally, comparison with other head-driven methods showed that the proposed method converges in a smaller number of iterations.

In an illustrative example presented by the authors of the above paper, applied to a small WDS with 7 nodes and 3 loops, the following results are derived (Table 3). It can be observed that in the head-driven method, the demand of the nodes 4 and 7 is not satisfied (the outflow is smaller than the planned). Obviously, this fact cannot be traced by the conventional method.

7. UNCERTAINTY ANALYSIS

As known, the hydraulic system analysis involves three vectors representing the nodal heads, the pipe flows and the diameters of the pipes constituting the branches of the system. The selection of these vectors should satisfy the two fundamental principles: a) Continuity and b) Energy conservation. That is (a) the algebraic sum of flows of the branches meeting at a node and external flows at each node is zero and (b) the algebraic sum of head losses (due to friction) or head gains (generated by pumps) around any loop of the system is zero (Tsakiris, 2013).

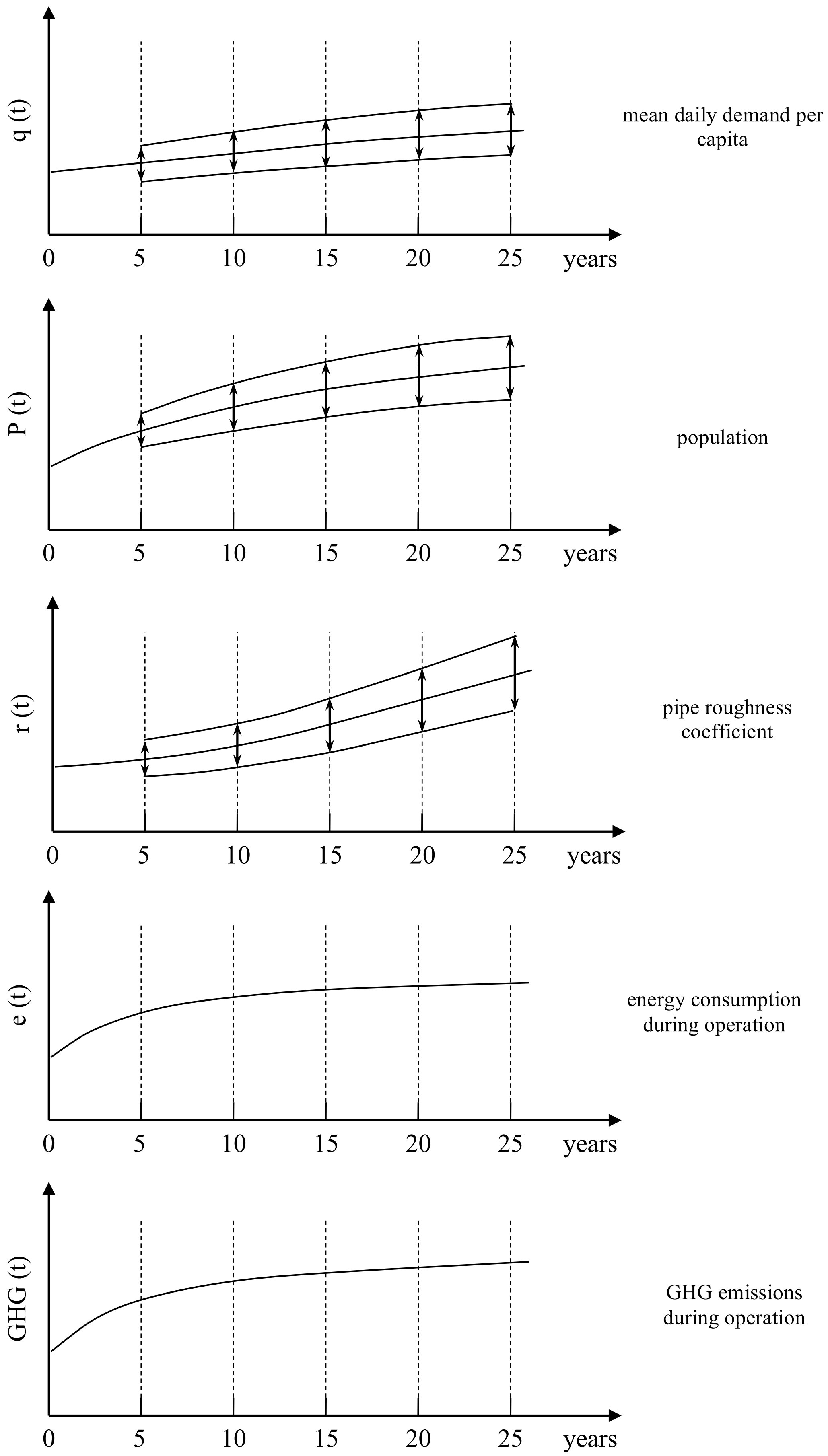

Figure 1. Indicative time dependence of some important parameters in the design of a municipal water supply and distribution system.

Table 3. Comparison of the results of the proposed head-driven method with the conventional method.

Nodes	q (L/s)	h_{min} (m-asl)	h(m-asl) (conventional)	q (L/s)	h(m-asl) (head-driven)
1	15.00	162.00	207.79	15.00	207.96
2	15.00	164.00	206.56	15.00	206.73
3	20.00	167.00	204.96	20.00	204.99
4	35.00	192.00	201.75	30.31	202.50
5	40.00	162.00	202.66	40.00	203.13
6	30.00	172.00	202.10	30.00	202.75
7	30.00	192.00	201.63	25.86	202.40

If diameters and external flows (e.g. nodal demands) are known, the hydraulic analysis seeks to determine the unknown nodal heads. In a hydraulic system with N nodes and M loops a system of N+M-1 equations can be written which, in a compact form (nodal formulation), can be written (Xu and Goulter, 1996):

$$F(H,C) + Q_{ext} = 0 \tag{1}$$

where H, C and Q_{ext} are vectors of the unknown nodal heads, known pipe roughness factors and external flows, respectively.

Since $F(H,C)$ is non-linear, iterative techniques are employed which start with a rather arbitrary initial guess of the system variables and gradually reach the state by which the continuity equation is satisfied.

The non-linear character of the hydraulic model creates a high complexity for incorporating uncertainty and imprecision of the system parameters into the analysis of the system. To overcome this difficulty a first order Taylor series expansion at some characteristic values of nodal heads (such as the mean values), external flows (nodal demands) and pipe roughness factors can be used (e.g. Xu and Goulter, 1996). By doing so after some algebraic calculations, Xu and Goulter (1996) reached a linearised hydraulic model represented by the following equation:

$$H = \bar{\bar{H}} + AQ_{ext} + BC \tag{2}$$

where $A= -J^{1}$, $B=AJ_C$ and $\bar{\bar{H}} = \bar{H} - A\bar{Q}_{ext} - B\bar{C}$, J being the Jacobian matrix, and J_C the sensitivity matrix with respect to change of pipe roughness.

Based on the above linearised hydraulic model, the authors studied the uncertainty and imprecision of the two major parameters such as the nodal external flows (or nodal demands) and the pipe roughness factors.

In this paper the above linearised model was used to demonstrate the various methods for studying the uncertainty of the design parameters. Three methods for the study of the uncertainty are presented following the analysis of Xu and Goulter. Namely, the Interval analysis, the Probability analysis and the Fuzzy set approach.

7.1 Interval Analysis

As known interval analysis represents uncertainty as a "grey" number (interval instead of a grisp number) for which only the upper and lower bounds are of interest. In fact, the problem of this type of analysis seeks for the grey numbers for the nodal heads if the grey numbers of nodal demands and the roughness factors are known. Using the linearised Equation 2 for the mid-value points of grey nodal demands and grey pipe roughness factors the grey nodal heads are expressed as:

$$\tilde{H} = \bar{\bar{H}} + A\tilde{Q}_{ext} + B\tilde{C} \tag{3}$$

where $\tilde{H}$, $\tilde{Q}_{ext}$ and $\tilde{C}$ are vectors of grey nodal heads, grey nodal demands and grey roughness factors, respectively.

Therefore, for each nodal head the upper and lower bounds are calculated by:

$$Max(H_i) = \bar{\bar{H}} + Max\left(\sum_{j=1}^{N} a_{ij}\,\tilde{Q}_{ext\,j} + \sum_{k=1}^{M} b_{ik}\,\tilde{C}_k\right) \tag{4}$$

$$Min(H_i) = \bar{\bar{H}} + Min\left(\sum_{j=1}^{N} a_{ij}\,\tilde{Q}_{ext\,j} + \sum_{k=1}^{M} b_{ik}\,\tilde{C}_k\right) \tag{5}$$

where a_{ij} and b_{ik} are the elements of sensitivity matrices A and B, respectively.

The above equations show that the interval of nodal heads are linear functions of the upper and lower bounds of the grey nodal demands and grey pipe roughness factors.

Xu and Goulter (1996) summarized their method by presenting three major steps to study uncertainly in water distribution systems.

1. Calculate the mid-values of nodal heads from mid-values of nodal demands and pipe roughness factors through a deterministic solver
2. Obtain the sensitivity matrices A and B from the deterministic solution
3. Calculate the interval for each nodal head.

7.2 Probability Analysis

As known, in probability analysis the involved parameters are considered as random variables following a certain probability density function (e.g. normal, uniform etc.). Therefore, if the characteristics of the pdf of nodal demands and pipe roughness factors (for instance the mean and the standard deviation) are known, the unknowns are the same characteristics of the nodal heads.

Using the same simplified linearised hydraulic model and assuming that the sensitivity matrices represent the weighted sum of random nodal demands and random roughness factors, the mean and the standard deviation of random nodal heads are calculated. The standard deviation can be better estimated by using the approach of Yen et al. (1986):

$$\sigma_{H_i} = \pm\sqrt{\sum_{j=1}^{N} a_{ij}^2\sigma_{Q_{ext_j}}^2 + \sum_{k=1}^{M} b_{ik}^2\sigma_{C_k}^2} \quad \text{for j=1(1) N and k=1(1) M} \tag{6}$$

where $\sigma_{Q_{ext_j}}^2$ and $\sigma_{C_k}^2$ are the variances of the random demand at node *j* and the random roughness factor for pipe *k*, respectively.

By this simplified approach the authors avoid using the joint probability distribution of both nodal demands and roughness factors, which lead to complicated procedures.

To summarise, the algorithm of the probability approach is as follows:

1. Solve for the estimation of mean values of nodal heads, using $\bar{H}$, using $\bar{Q}_{ext}$ and $\bar{C}$, respectively
2. Obtain the sensitivity matrices A and B
3. Calculate the standard deviation of the random nodal heads (Eq. 6)

Since the standard deviation of the input parameters cannot be easily estimated, several values of standard deviations derived from the engineering judgment may be tested performing a type of sensitivity analysis.

7.3 Fuzzy set Approach

In the fuzzy set approach the input parameters are represented by the so called membership functions instead of crisp numbers. The membership function represents a rather crude possibility function which determines the degree of belief in the realization of certain values of the parameter.

Triangular or trapezoidal types (mostly symmetrical) membership functions are usually used. In the case of the incorporation of uncertainties of hydraulic parameters (nodal demand and pipe roughness factors) in the determination of the uncertainty of the nodal heads, the objective is to find the membership function of the nodal heads.

In common with the previous methods, the linearised hydraulic model of Xu and Goulter is used. In conclusion the fuzzy nodal heads are calculated as follows:

1. Perform deterministic network simulation using the mid-values of nodal demands and pipe roughness factors.
2. Calculate the matrices A and B.
3. Estimate the critical points of the expected membership function using fuzzy arithmetic.

Another fuzzy set approach using triangular membership functions and an analytical framework for solving the system equations was used to study the uncertainty of nodal demands (Spiliotis and Tsakiris, 2012a). Results from this latter study are presented below.

8. NUMERICAL EXAMPLES OF UNCERTAINTY ANALYSIS

Xu and Goulter used a simple network to demonstrate the uncertainty analysis. They applied the empirical Hazen-Williams head loss equation. Lengths, diameters, nodal demands and Hazen-William coefficients were considered known as the most expected values. A 10% of the average (initially selected) variability around the mean value was assumed for both the nodal demands and the Hazen-Williams coefficients.

The results of the interval analysis produced nodal head variations (dependent on the node) up to 24% of the mean value.

Similarly the probability analysis produced the standard deviation for each nodal head, assuming coefficients of variation for nodal demands and Hazen-Williams coefficient equal to 0.10 and 0.05, respectively. The maximum standard deviation observed was 1.28m and occurred at the node 6 with average nodal head 46.66m.

Finally by using membership functions of symmetrical trapezoidal shape with 5% and 10% of the mean variability for a-cut 1.0 and 0, respectively, the membership functions of the nodal heads were derived. For the a-cut equal to 1, the highest deviation observed is 6.52 which occurred at the node 11. All the results of the example are presented in tables in the original paper.

As mentioned above, another comprehensive study for the uncertainty analysis of the nodal demands was presented by Spiliotis and Tsakiris (2012a). The authors used a fuzzy set approach and additional analytical tools to describe uncertainty of the nodal demand (Darcy-Weisbach head loss equation, Swamee and Jain explicit equation and Newton-Raphson solver etc). They presented a numerical example of a WDS which appears in Fig. 2 with data presented in Table 4 and 5. Triangular membership function was selected to describe the nodal demand. The variability of nodal demand was assumed 20% (1st scenario) and 40% (2nd scenario) of the mean value, the same for all the nodes of the system (a-cut=0). The results showing the membership functions of the heads at each node are presented in Fig. 3 for two nodal demand variability scenarios of 20 and 40% of the mean nodal demand.

As can be seen from Fig. 3, a range of values of head is derived, dependent on the a-cut selected. As expected the maximum range of nodal heads appears for the 40% variability of nodal demands and a-cut=0.

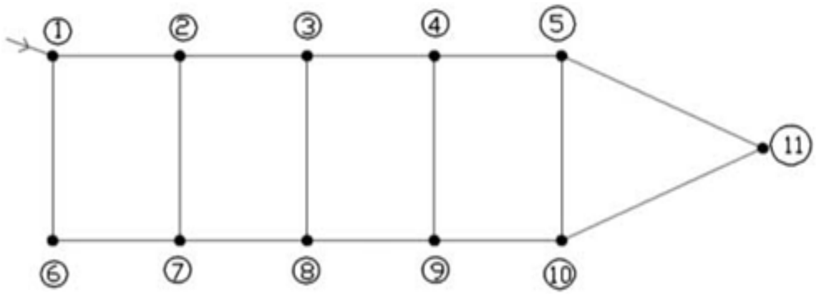

Figure 2. The water distribution system analysed (after Spiliotis and Tsakiris, 2012a).

Table 4. Internal diameter and length of each branch of the system.

Pipe	Internal Diameter (mm)	Length (m)
1-2	352.6	800
2-3	312.8	800
3-4	277.6	800
4-5	198.2	800
5-11	176.2	1400
1-6	312.8	1200
6-7	312.8	800
7-8	246.8	800
8-9	220.4	800
9-10	198.2	800
10-11	176.2	1400
2-7	220.4	1200
3-8	123.4	1200
4-9	110.2	1200
5-10	110.2	1200

Table 5. Nodal demand central value and ground elevation of each node.

Node	Nodal water demand (L/s)	Ground elevation (m a.s.l.)
2	30	142
3	25	145
4	30	144
5	25	149
6	25	141
7	25	142
8	25	140
9	25	150
10	25	150
11	40	135

9. CONCLUSIONS

The professional design of the municipal water supply and distribution systems is still based on gross assumptions using solvers of the early generation methods. During the recent decades a number of methods for analysing these systems (known as 3rd and 4th generation methods) have been proposed using more realistic representations of the determinants involved in the design. Among them, the head driven methods and the methods incorporating the uncertainty of the major variables are the most important.

This paper supports the adoption of these methods in the professional design of water distribution systems. Further the paper proposes the integrative design by incorporating in the analysis the entire water supply and distribution system from the source to the end user (spatial integration-water cycle analysis), and the analysis of the entire life of the system (temporal integration-life cycle analysis).

Finally, apart from the min cost criterion, new criteria are proposed for the comparison of various alternatives i.e. the total energy used and the total Greenhouse Gas (GHG) emissions. These additional criteria are also useful for assisting towards the mitigation and adaptation efforts related to the climate change.

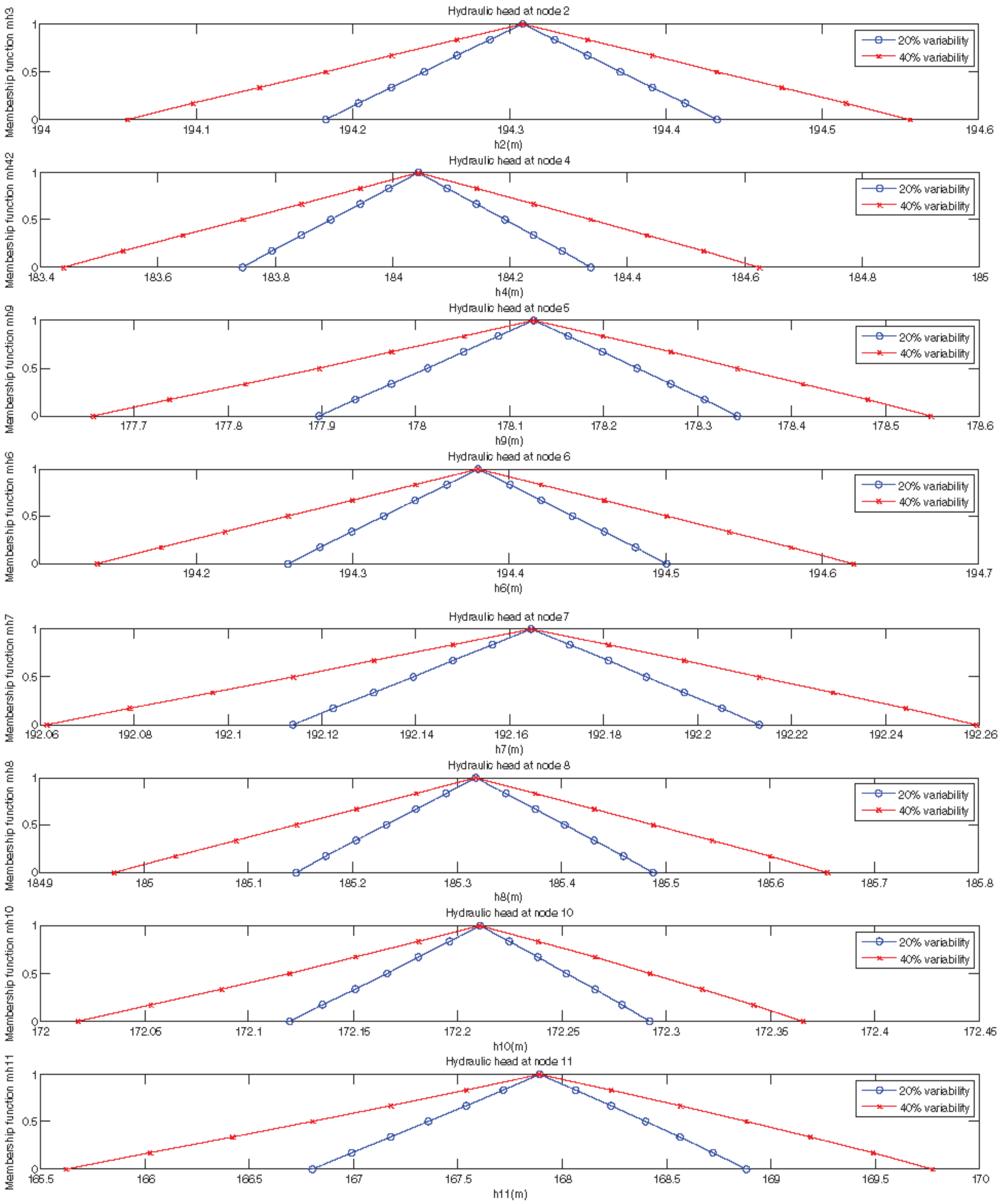

Figure 3. The membership functions of modal heads for both scenarios of 20 and 40% variability of nodal demand.

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