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Optimal Urban Water Distribution Design

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A heuristic linear programming-based procedure has been developed for the least cost layout and design of water distribution networks. The methodology is capable of analyzing a wide range of demand pattern and pipe failure combinations. Hydraulic consistency is ensured throughout the procedure through the use of the Hardy-Cross network solver technique. The procedure can also be extended for use in the expansion or reinforcement of existing network systems. While the techniques used to reduce the size of the constraint set to enable the procedure to handle a wide range of loading conditions do not guarantee global optimality, a pragmatic "reasonable" optimum is achieved. The method is demonstrated by application to the design of a new network and the expansion of an existing network. In the expansion of the existing network problem the solution obtained was less expensive than any previously published solution.

INTRODUCTION

The application of operations research methodology to water distribution network design has received considerable attention over the last 15 years [e.g., *Karmeli et al.*, 1968; *Schaake and Lai*, 1969; *Deb and Sakar*, 1971; *Watanatada*, 1973; *Shamir*, 1974; *Alperovits and Shamir*, 1977; *Bhave*, 1979; *Quindry et al.*, 1981]. These earlier studies, and in particular those dealing with branched networks, assumed a given layout for the network and then designed the network components on the basis of the assumed layout. The studies by *Alperovits and Shamir* [1977], *Quindry et al.* [1979], and *Quindry et al.* [1981], however, analyzed looped water networks with the added ability to readjust flows within the assumed layout in an attempt to converge upon the least cost solution.

Only recently has the joint problem of least cost layout and component design of looped water distribution networks been addressed. While some models have been developed for municipal water networks [e.g., *Mays et al.*, 1976; *Martin*, 1980], they are not generally appropriate as they are mainly concerned with branched networks such as storm sewer collection systems. The complicating feature in the development of formal procedures capable of addressing the joint problem of layout and design is the strong interrelationship between the layout and component sizings.

Two methods which are capable of considering the layout and design problems have been published recently by *Rowell and Barnes* [1982] and *Morgan and Goulter* [1982]. In their study, *Rowell and Barnes* [1982] developed a model which addressed the problem through a two-level hierarchical approach. In this model a least cost branched layout is first determined. The looping required for reliability is then provided by the insertion of redundant pipes interconnecting the branches of the tree. However, as is shown by *Goulter and Morgan* [1984], the assumptions used in this procedure sacrifice hydraulic consistency in the search for the least cost solution.

Morgan and Goulter [1982] developed a model using two linked linear programs to solve the least cost layout and design of these looped systems. In this model one linear program solves the layout, while the other determines the least cost component sizes. The constraint set used to ensure looping in the layout part of the procedure requires every node to be connected by at least two pipes. However, while the model ensures hydraulic consistency at every stage, fulfillment of the looping constraint described above does not explicitly guarantee true redundancy. The lack of any truly successful models in this realm of joint layout and component design indicates a general need for development of new models in this area.

An additional consideration in water network design has arisen with the recent publicity concerning the deterioration of urban infrastructures across North America. The expansion or upgrading of existing systems has become an important issue in municipal engineering. A few models which are capable of analyzing the least cost upgrading or expansions of existing networks have recently been developed [e.g., *Quindry et al.*, 1981; *Gessler*, 1982]. Since most, if not all existing networks are looped systems, the approaches used in these models have been based on techniques which are capable of analyzing looped networks. The particular network on which these models are tested and compared is the New York Tunnels Expansion project which was first considered by *Schaake and Lai* [1969]. In each of the succeeding studies a lower cost system was found which fulfilled the expansion requirements.

The model developed in this study represents an alternative approach to the least cost layout and design of looped water distribution networks and the least cost expansion of existing networks. Before proceeding with the development of the methodology the objectives and requirements of such a model are reviewed.

OBJECTIVES OF A COMPREHENSIVE WATER DISTRIBUTION NETWORK MODEL

In developing models for design of urban water distribution networks there is a wide range of criteria and parameters which should be considered. The following list enumerates those criteria which are considered in this study.

1. The system must deliver a given flow at a specified pressure to any node in the system when one of the key pipes in the system is not functioning. Therefore at least two inde-

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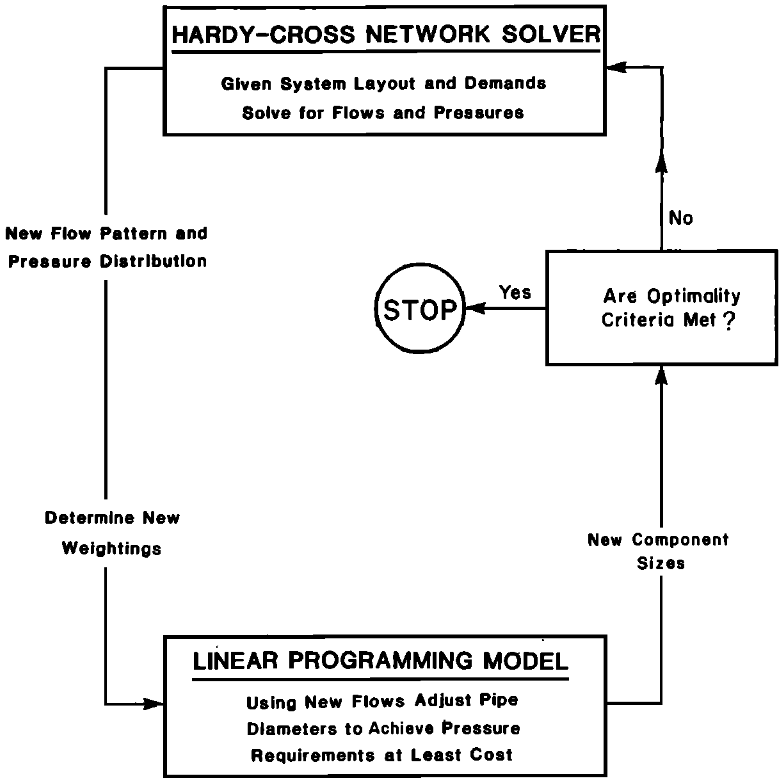

Fig. 1. Linkage between linear program and Hardy-Cross network solver.

pendent and adequate paths from a source to each node must be provided.

2. The system must deliver severe fire flow demands at adequate pressures. While these fire flow demands occur infrequently at the various nodes in the system, they may, however, be the constraining factor in the design of systems.

3. The method should be applicable to expansion of existing systems as well as design of new systems.

4. The method should incorporate a realistic cost function, preferably using standard unit costs as given by suppliers, e.g., cost per unit length of pipe.

An implication of criteria 1 and 2 given above is that there is a large number of possible demand patterns which have to be considered. The significance of this problem was addressed by *Templeman* [1982] in his discussion of *Quindry et al.* [1981]. In this discussion Templeman asserts that the use of optimization techniques actually tends to remove redundancy conditions by optimizing out of the network any capacity which is not required by that particular loading. In order to give proper consideration to the implied criteria of network reliability, resilience, and flexibility, Templeman then asserts that the design procedure should consider the ability of the network to serve fire fighting demands at all nodes. Even though the occurrence of simultaneous fires at all or even some nodes is not generally considered probable, there is still a wide range of fire fighting demand patterns to be considered, i.e., one demand pattern for a fire at each node.

Most of the studies which have been directed at analysis of looped systems have only considered one demand pattern. *Alperovits and Shamir* [1977] and *Quindry et al.* [1981] recognized this problem by considering two or three demand patterns. For a realistically sized network, however, consideration of only two or three patterns falls far short of a complete recognition of the flexibility-reliability problem. The model developed in this paper is capable of considering a very wide range of fire flows in a network.

MODEL DEVELOPMENT

General Description

The procedure is based upon a linear programing formulation linked to a network solver. The linear programing step of the model is used to design and/or modify pipe sizes, while the network solver step is used to balance flows and pressures. Within the linkages between these two steps is a means for removing uneconomical pipe locations. The development of the procedure for the single loading case will first be described. The model will then be extended for use under multiple loadings.

The first step of the procedure is the assumption of an initial flow pattern or pipe layout, and component sizes for the given loading pattern. Note that this layout should include all candidate links within the network. While the complete set of all candidate links can theoretically be very large, the physical conditions, e.g., street right of ways, topography, etc., generally restrict the actual number of candidate links to considerably smaller subset.

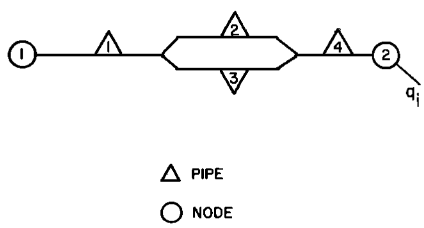

Fig. 2. Simple looped system.

The actual pressures and flows within the network resulting from the given pipe layout and known demands are then determined. Of the many methods available [see *Holloway and Chaudry*, 1983] to solve for these pressures, the Hardy-Cross technique was chosen due to the combination of its simplicity and generally widespread acceptance. It is, however, relatively simple to replace the Hardy-Cross technique with another network solver if network or other conditions warrant.

The flow pattern and pressure distribution provided by the network solver are the passed to a pipe design–modification procedure. If the pressures at some demand or diversion points are below some stipulated minimum, it is necessary to replace sections of pipe with pipe of larger diameter. The pipe modification procedure determines which sections of pipe should be replaced in order to raise the pressure to an acceptable minimum in a least cost manner.

Conversely, if the pressures are above the minimum allowable, then some sections of pipe may be replaced by pipes of smaller diameter. The decision in this case is, Which section of pipe should be replaced in order to bring about the greatest saving while maintaining minimum pressure requirements elsewhere? Pipe diameters which have been assumed initially or selected as replacement pipes in previous iterations are referred to as "designated" diameters. In general, replacement of these designated diameters with both smaller and larger diameters is needed, since some areas may be underdesigned, while other areas may be overdesigned. An overall reduction in cost may also be obtained by increasing the diameter of one pipe, therefore allowing a number of other pipes to be replaced by pipes of smaller diameter. The new configuration of the pipe sizes is passed back to the network solver to calculate true flows and pressures. The new pressures and flows are then passed back to the pipe modification step. The process is repeated iteratively as shown in Figure 1 until an "optimal" solution is converged upon. Details of the iterative process and the criteria for optimality are described in more detail in a later section, following the mathematical development of the objective function and constraint set.

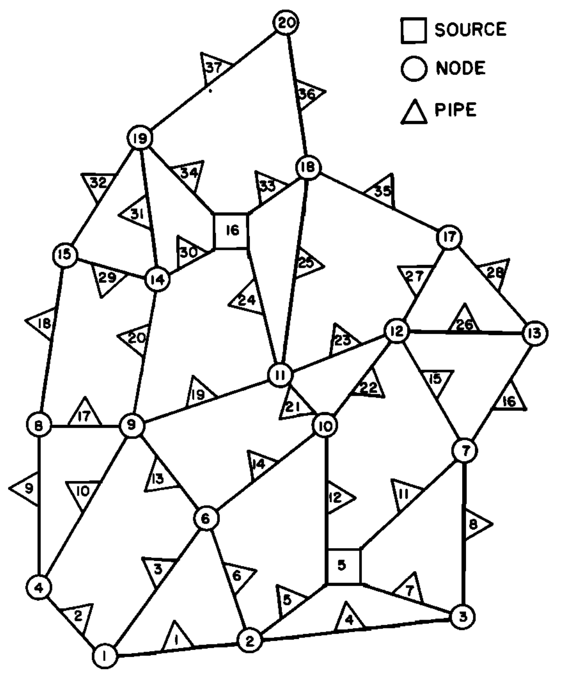

Fig. 3. Example network showing all possible pipe locations.

TABLE 1. Cost per Meter for Different Diameter Pipes

Layout and Design Example		New York City Tunnel Problem	
Diameter, meters	Cost, dollars per meter	Diameter, inches	Cost, dollars per foot
0.125	58.00	36	93.50
0.150	62.00	48	134.00
0.200	71.70	60	176.00
0.250	88.90	72	221.00
0.300	112.30	84	267.00
0.350	138.70	96	316.00
0.400	169.00	108	365.00
0.450	207.00	120	417.00
0.500	248.00	132	469.00
0.550	297.00	144	522.00
0.600	347.00	156	577.00
0.650	405.00	168	632.00
0.700	470.00	180	689.00
		192	746.00
		204	804.00

Imperial units are used here to facilitate direct comparison with previous studies. One inch equals 2.54 cm; 1 foot equals 30.48 cm.

The linear programming step of the procedure is formulated as follows.

Objective Function

$$\min \sum_{j=1}^{NL} (K_{jdr}X_{jdr} + K_{jds}X_{jds}) \tag{1}$$

where

K_{jdr} unit cost of changing, in link j, a pipe of the dth diameter to a pipe of the larger rth diameter, $K_{jdr} > 0$ (dollars per meter);

K_{jds} cost (saving) of changing, in link j, a pipe of the dth diameter to a pipe of the smaller sth diameter; $K_{jds} < 0$ (dollars per meter)

X_{jdr} and X_{jds} decision variables: length of pipe of dth diameter in link j replaced by pipe of rth or sth diameter, respectively (meters);

NL number of links in network;

C_j cost per unit length of pipe of jth diameter (hence $K_{jdr} = C_r - C_d$ and $K_{jds} = C_s - C_d$) (dollars per meter).

Constraints

1. Pressure constraints: these constraints ensure that the pressures at each demand point are adequate:

$$\sum_{j \in p_i} (G_{jdr}X_{jdr} + G_{jds}X_{jds}) \leq H_i - h_i \qquad \forall i \tag{2}$$

where G_{jdr} is the change in hydraulic gradient in link j caused by replacing unit length of pipe of dth diameter by unit length of pipe of larger rth diameter assuming constant flow (meters per minute):

$$G_{jdr} = J_{jr} - J_{jd} \tag{3}$$

G_{jds} is the change in hydraulic gradient in link j caused by replacing unit length of pipe at dth diameter by unit length of pipe of smaller sth diameter assuming constant flow (meters per minute):

$$G_{jds} = J_{js} - J_{jd} \tag{4}$$

and where

J_{jd} hydraulic gradient for pipe of dth diameter in link j meters per minute;

TABLE 2. Data for Layout and Design Example System

Link	Connecting Nodes From	Connecting Nodes To	Length, m	Initial Assumed Diameter, m
1	1	2	760.00	0.400
2	1	4	520.00	0.400
3	1	6	890.00	0.400
4	2	3	1120.00	0.400
5	2	5	610.00	0.400
6	2	6	680.00	0.400
7	3	5	680.00	0.400
8	3	7	870.00	0.400
9	4	8	860.00	0.400
10	4	9	980.00	0.400
11	5	7	890.00	0.400
12	5	10	750.00	0.400
13	6	9	620.00	0.400
14	6	10	800.00	0.400
15	7	12	730.00	0.400
16	7	13	680.00	0.400
17	8	9	480.00	0.400
18	8	15	860.00	0.400
19	9	11	800.00	0.400
20	9	14	770.00	0.400
21	10	11	350.00	0.400
22	10	12	620.00	0.400
23	11	12	670.00	0.400
24	11	16	790.00	0.400
25	11	18	1150.00	0.400
26	12	13	750.00	0.400
27	12	17	550.00	0.400
28	13	17	700.00	0.400
29	14	15	500.00	0.400
30	14	16	450.00	0.400
31	14	19	750.00	0.400
32	15	19	720.00	0.400
33	16	18	540.00	0.400
34	16	19	700.00	0.400
35	17	18	850.00	0.400
36	18	20	750.00	0.400
37	19	20	970.00	0.400

Cost of initial assumed system equaled $4,590,040.

H_i minimum allowable pressure head (meters);
h_i existing pressure head (meters);
p_i set of links on the path from the source to node i.

All other terms as defined previously.

2. Length constraints: these constraints ensure that the linear program does not replace more of the existing pipe than is available:

$$X_{jdr} \le L_j \tag{5}$$

$$X_{jds} \le L_j \tag{6}$$

where L_j is the length of link j (m). If the existing pipe in the link is made up of lengths of two different diameters, the length of pipe able to be replaced is less than or equal to the existing length in that link:

$$X_{jdr} \le l_{1j} \tag{7}$$

$$X_{jds} \le l_{2j} \tag{8}$$

where l_{1j} is the length of designated diameter of smaller diameter in link j (meters); and l_{2j} is the length of designated diameter of larger diameter in link j (meters).
and

$$l_{1j} + l_{2j} = L_j \qquad \forall j \tag{9}$$

All other terms are as previously described. Branched systems can be solved directly with the linear programming model as formulated. With a looped system, however, additional factors must be taken into account. If there are several paths to the demand node i from a source (or sources) with a fixed head (or heads), then all paths to that node must be considered. Change to one link on one of these several paths may not have the same importance to the change in pressure at node i as a change in another link in the same path or in a link in one of the other paths.

This condition is explained with reference to Figure 2 which shows a simple looped system. In order to reflect the relative contribution of changes in pipe diameter to the changes in pressure at node i, a weighting approach is used. Changes in links 1 and 4 have a direct one to one effect on the pressure at node 2. Designating the weighting attributed to link j with respect to its effect on node i as W_{ij} it can be seen that W_{21} and W_{24} should both equal 1.0.

The weightings given to links 2 and 3 must be less than 1.0, since links 2 and 3 do not handle all the flow between the reservoir and the outlet separately. The method used to weight these links is to calculate the percentage of flow drawn from node 1 that passes through link 2 and link 3. For example, if 65% of the flow is passing through link 2 and 35% is through link 3, then the weightings would be $W_{22} = 0.65$ and $W_{23} = 0.35$. A more complete description of the generalized weighting algorithm is given in the appendix.

As a result of the use of these weightings, (2) can be modified to consider the range of paths in a looped system;

$$\sum_{j \in P_i} (W_{ij} G_{jdr} X_{jdr} + W_{ij} G_{jds} X_{jds}) \le H_i - h_i \qquad \forall i \tag{10}$$

where P_i is the set of paths from node i to a fixed head source (each link is counted only once). All other terms are as previously described.

With a looped system, however, the linearity assumption expressed by (10) holds only approximately for the effect caused by changes in pressure, as the flows in the pipes do not remain constant as pipes are changed throughout the network.

TABLE 3. Flow Demands and Initial Pressure Assumptions at Each Node for Layout and Design Example

Node	Flow Demand, L/s	Minimum Head, m	Initial Head, m
1	165	75.00	80.00
2	220	74.00	90.00
3	145	73.00	90.00
4	165	72.00	70.00
5*	···	102.00	102.00
6	140	73.00	80.00
7	175	67.00	90.00
8	180	72.00	70.00
9	140	70.00	75.00
10	160	69.00	90.00
11	170	71.00	93.00
12	160	70.00	85.00
13	190	64.00	80.00
14	200	73.00	90.00
15	150	73.00	80.00
16*	···	96.00	96.00
17	165	67.00	80.00
18	140	70.00	90.00
19	185	70.00	90.00
20	165	67.00	70.00

*Nodes 5 and 16 are source nodes rather than demands.

The feedback to the network solver is therefore needed to calculate the true pressure and flows. This iterative process is described as follows.

Iterative Process and Optimality Criteria

The new flow pattern developed by the Hardy-Cross method given the pipe sizes provided by the linear program is now used to calculate the new weights for the links and the network is analyzed again by the linear programming model. This process is repeated iteratively as shown in Figure 1 until an optimal solution is converged upon.

It should be noted that the iterative nature of the procedure permits the progressive reduction in the sizes of pipes that are initially assumed to be part of the network. However, as formulated, the linear program is not capable of finally eliminating the uneconomical pipes. The pressure constraints represented by (10) consider the change in pressure at a given node caused by changes in the pipe diameters. These changes in pressure are based upon the assumption of fixed flow in the link for which the pipe is being selected. As the pipe sizes get progressively smaller for the same fixed flow, the hydraulic gradients become progressively more steep thus causing more severe effects on the pressure at the node for which the constraint has been prepared. For example, given the same flow in the link, a reduction in pipe diameter from 200 to 150 mm will have less effect on the pressure gradient than a reduction from 150 to 100 mm. The effect becomes more pronounced with each successive reduction of pipe diameter. In the limit, the reduction of an existing pipe to a zero diameter pipe, i.e., elimination of the pipe for a given flow causes an infinite pressure gradient in that link, thus making the whole problem infeasible. As formulated, the linear programming model is not capable of eliminating pipes for this reason. In order to provide the capability of eliminating uneconomical pipes, an additional condition is introduced as follows.

Each pipe in a network has a weighting associated with it. The weighting is determined, as is described in the appendix, during the development of the pressure constraints. The minimum pipe with the lowest weighting in any run is removed from the network, the flows redistributed using the Hardy-Cross technique, and the program run again. The minimum pipes in this analysis are taken to be those pipes whose diameters are equal to the smallest allowable size specified by the design engineer. If the network resulting from removal of this pipe is more economical than the previous solution then it becomes the new best result and the procedure continues as before. If the new network is more expensive that pipe is retained and the old best result is used. This consideration of minimum pipes continues at each iteration until the lowest weighting to be considered is greater than a specified value, e.g., 0.5. Once this situation is reached the regular pipe replacement and flow distribution phases of the approach continue as before.

The use of the pipe with the smallest weighting is based upon hydraulic principles in network flows. Small weightings for a link or pipe under a particular loading pattern shows that the contribution of that link to the pressure and flow at any node is relatively small. The removal of that pipe with the smallest weighting is likely to have the least effect on the overall pressure profile and flow pattern. The application of this criteria permits the removal of these links which are deemed uneconomical in the network as a whole.

Final "optimality" is considered to have been obtained when the linear program does not choose to replace any part of the network and the maximum weighting in any link in the system is greater than a previously specified value. While the use of such criteria does not guarantee true overall optimality, the review of the cost of the new solution after each pipe removal ensures that the cost will never increase. Furthermore, the criterion used in the choice of suitable pipes for removal is based on the combination of two parameters which recognize the interdependence of the flow patterns and the contribution of each link to the flow to any given node.

It should be noted that a property of this linear programming formulation is that when the optimal solution is found, none of the length constraints will be binding. The reason that the length constraints do not bind at optimality lies in the choice of X_{jds} and X_{jdr} as the decision variables. These variables do not represent actual lengths of pipe chosen for the network links but rather the length of pipe to be replaced to achieve a cheaper solution. Since at optimality, no pipes will be selected for replacement, the variables X_{jds} and X_{jdr} will be zero in the optimal problem and the appropriate length constraints nonbinding. The length constraints are, however, needed initially if the first assumption is far from optimal. Since the number of these constraints is dependent on the number of links rather than the number of demand patterns considered, the size of this constraint set will remain constant when the procedure is extended for the analysis of the multiple demand patterns situation described in the following sections.

TABLE 4. Pipe Breaks and Fire Flows for Each Demand Pattern Design in Layout and Design Example

Demand Pattern	Link Broken	Fire Flows: At Node	Fire Flows: Demand, L/s
1	1	1	70.0
2	2	4	70.0
3	3	1	70.0
4	4	3	70.0
5	5	2	70.0
6	6	6	90.0
7	7	3	70.0
8	8	7	70.0
9	9	4	70.0
10	10	4	70.0
11	11	7	90.0
12	12	10	90.0
13	13	9	90.0
14	14	6	90.0
15	15	12	50.0
16	16	13	70.0
17	17	8	70.0
18	18	8	70.0
19	19	9	90.0
20	20	9	90.0
21	21	11	100.0
22	22	12	50.0
23	23	12	50.0
24	24	11	100.0
25	25	11	100.0
26	26	13	70.0
27	27	17	70.0
28	28	13	70.0
29	29	15	70.0
30	30	14	120.0
31	31	19	120.0
32	32	15	70.0
33	33	18	120.0
34	34	19	120.0
35	35	17	70.0
36	36	20	120.0
37	37	20	120.0

TABLE 5. Results of First Interation of Multiple Demand Pattern Design for Layout and Design Example

	Pipe Sizes in Link		
Link	Diameter, m	Length, m	Maximum Link Weighting Over All Loading Patterns
1	0.200	350.98	1.000
	0.250	409.02	
2	0.150	250.77	0.703
	0.200	269.23	
3	0.200	753.42	0.894
	0.250	136.58	
4	0.200	1120.00	0.550
5	0.300	610.00	0.892
6	0.200	680.00	0.777
7	0.250	680.00	0.771
8	0.150	130.92	0.867
	0.200	739.08	
9	0.125	89.63	0.297
	0.150	770.37	
10	0.200	980.00	0.759
11	0.200	890.00	0.383
12	0.350	577.84	1.000
	0.400	172.16	
13	0.200	620.00	0.440
14	0.300	800.00	0.885
15	...	...	...
16	0.150	507.91	0.611
	0.200	172.09	
17	0.150	123.33	0.754
	0.200	356.67	
18	0.200	860.00	0.746
19	0.200	800.00	0.338
20	0.200	677.52	0.720
	0.250	92.48	
21	0.200	350.00	0.483
22	0.200	620.00	1.000
23	0.200	600.07	1.000
	0.250	69.63	
24	0.250	790.00	0.653
25	0.150	118.84	0.612
	0.200	1031.16	
26	0.150	123.94	0.686
	0.200	626.06	
27	0.150	150.86	0.759
	0.200	399.14	
28	0.150	700.00	0.389
29	0.200	500.00	0.952
30	0.250	314.92	0.955
	0.300	135.08	
31	0.200	647.03	0.643
	0.250	102.97	
32	0.200	720.00	1.000
33	0.250	540.00	0.848
34	0.300	606.71	1.000
	0.350	93.29	
35	0.150	466.31	0.808
	0.200	383.69	
36	0.200	546.60	1.000
	0.250	203.40	
37	0.200	569.24	1.000
	0.250	400.76	
Cost = $2,074,762			

Discussion of Formulation

A significant difference between this model and those which have been published previously for analysis of looped systems is the lack of a constraint set to ensure that the algebraic sum of head losses around each loop is equal to zero. In many models this constraint set is required to ensure hydraulic consistency within the network and to provide dual variables necessary for gradient search techniques used to change the flows or pressures in the network [e.g., *Alperovits and Shamir*, 1978; *Quindry et al.*, 1981]. In this model, however, the constraint set is not used for either purpose.

Hydraulic consistency is maintained through the use of the Hardy-Cross network solver as follows. If the linear program varies pipes at any iteration, the system becomes hydraulically inconsistent. The Hardy-Cross technique is then used to redistribute the flows. The new flow pattern and resulting pressure distribution, which are always consistent, are passed back to the linear program, and the possibility of additional pipe replacements is considered.

When the objective function for a particular iteration is equal to zero, corresponding to no change in the pipe sizes the optimal answer has been reached. Since there is no change in the pipe sizes in the network the current solution is optimal. Furthermore, since it was checked by the Hardy-Cross method prior to consideration by the linear program, the solution is also hydraulically consistent.

While gradient search techniques were initially considered, the computational problems associated with their use when the model is extended to handle a wide range of worst case combinations of fire flows and pipe breaks eliminated them from further consideration. This point is described in greater detail in the following section which describes the extension of the model to these multiple demand and pipe break patterns.

EXTENSION TO MULTIPLE DEMAND PATTERNS

If a system is designed for a single loading, then the most economical layout will be a branched system without any redundant or inefficiently utilized pipes. This bias toward branched systems is due to the economies of scale in which use of one large pipe represents a less expensive method of transporting water than two smaller pipes. If, however, a single pipe fails in a branched network, then no flow can reach the nodes downstream of that failure. This problem is generally alleviated by adding redundant pipes to ensure looping. These redundant pipes selected by other models [e.g., *Alperovits and Shamir*, 1977; *Quindry et al.*, 1981] often have small diameters, usually the minimum available diameter. Consequently, there is often no guarantee that in the case of a primary link failure the secondary path can deliver water at adequate pressure.

One solution to the problem of undersized "redundant" links is to design the system while considering all the worst case scenarios, i.e., worst case combinations of fire flows and pipe breaks. A method of doing this is to add a set of pressure constraints for each demand pattern. Formulations for multiple demand patterns have been previously described by *Schaake and Lai* [1969], *Alperovits and Shamir* [1977], and *Quindry et al.* [1981]. The major weakness with these formulations is that the additional constraints needed to consider multiple demand patterns rapidly make the problem computationally impractical. For example, if 40 constraints were needed for 1 demand pattern then three times as many, or 120, would be needed for 3 demand patterns.

Many of these constraints can, however, be eliminated because they are nonbinding. From generalized linear programming theory [*Hillier and Lieberman*, 1980, pp. 68–117] it can be shown that the maximum number of nonzero variables in the solution must equal the number of binding constraints. Equation 1 states that there are two variables for each link in the system, one representing the length of pipe to be replaced by a larger diameter pipe, the other representing the length of pipe to be replaced by a smaller diameter pipe. Since in a given link a pipe can only be either increased or decreased in size, it can be seen that there will be at most one nonzero decision variable (X_{jds} or X_{jdr}) associated with each link. In many cases both decision variables for a particular link will be zero. This condition results in the maximum number of nonzero decision variables being equal to the number of candidate links in the network. However, the maximum number of nonzero decision variables must be equal to the number of constraints. The maximum number of pressure constraints needed is therefore equal to the total number of links, NL.

The pressure constraints that are actually used for the multiple demand pattern case must be selected before the linear programming model is applied. The network solver step of the procedure first solves for the pressures and flows associated with each demand pattern. The amount by which each actual pressure is below or above the minimum pressure is calculated

$$a_{it} = h_{it} - H_i \qquad \forall i \text{ and } t \tag{11}$$

where a_{it} is the pressure head value at node i for demand pattern t (meters).

Starting with the critical node, i.e., that node at which the pressure is the furthest from the minimum acceptable level (most negative a_{it}), the pressure constraints to be used in the linear program are constructed by

$$\sum_{j \in P_i} (W_{ijt} G_{jtdr} X_{jdr} + W_{ijt} G_{jtds} X_{jds}) \le H_i - h_{it} \tag{12}$$

$$\forall i,\ t \in NL \text{ largest } (H_i - h_{it})$$

where $G_{jtdr} = J_{jtr} - J_{jtd}$; J_{jtd} is the hydraulic gradient for pipe d in link j when the flow in the pipe is Q_{jt} for demand pattern t (m/m); and W_{ijt} is the weighting for link j in path i under load pattern t. All other terms are as previously described.

This development of pressure constraints continues until NL constraints have been constructed. The linear program is then solved and the component sizes determined by the linear program are passed back to the Hardy-Cross network solver and the iterative process continued.

It should be noted that since all the right-hand sides of (12) are determined for each demand pattern by the Hardy-Cross network solver, it is also possible to consider simultaneous pipe breakages and fire flow conditions. This joint consideration of the pipe breakage and fire flow for a particular node is achieved by simply removing the broken pipe as a candidate flow path for the Hardy-Cross analysis and from the constraint set associated with achieving minimum allowable pressures for the node with the fire flow demand.

In the multiple demand pattern situation, each pipe has a number of different weightings associated with it, namely, one for each load pattern. The removal of minimum pipes discussed in relation to the single load pattern is now performed by the same approach but by using the lowest maximum weighting rather than the lowest weighting. The removal of the pipe with the smallest maximum weighting has the least effect on the overall pressure profiles and flow patterns.

As was shown earlier, the use of the Hardy-Cross network solver in the iterative process eliminates the need for loop constraints to maintain hydraulic consistency. It was also suggested that the consideration of a wide range of fire flow and pipe break combinations made the use of gradient technique computationally impractical. The reasoning behind this suggestion is as follows. Each fire flow pattern will have a unique set of loop constraints and pressure head constraints. The inclusion of the "worst case" pipe break for each fire flow pattern adds additional complexity to these loop constraints.

TABLE 6. Results of Final Iteration of Multiple Demand Pattern Design for Layout and Design Example

Link	Pipe Sizes in Link: Diameter, m	Pipe Sizes in Link: Length, m	Maximum Link Weighting Over all Loading Patterns
1	0.250	760.00	1.000
2	0.150 0.200	112.99 407.01	1.000
3	0.250 0.300	796.70 93.00	1.000
4	...	...	...
5	0.300 0.350	370.60 239.40	1.000
6	0.200	680.00	0.725
7	0.200 0.250	473.58 206.42	1.000
8	0.200 0.250	314.96 555.04	1.000
9	...	...	...
10	0.200 0.250	519.99 460.01	1.000
11	0.250	890.00	1.000
12	0.400	750.00	1.000
13	0.250	620.00	0.797
14	0.350 0.400	540.84 259.16	1.000
15	...	...	...
16	0.150 0.200	98.14 581.86	1.000
17	0.150 0.200	41.82 438.18	1.000
18	0.200 0.250	173.27 686.73	1.000
19	...	...	...
20	0.200	770.00	0.889
21	...	...	...
22	0.200 0.250	35.54 584.46	1.000
23	0.150 0.200	345.08 324.92	0.663
24	0.200 0.250	336.57 453.43	0.443
25	0.200	1150.00	0.588
26	0.200	750.00	1.000
27	0.150 0.200	98.81 451.19	1.000
28	...	...	...
29	0.200	500.00	1.000
30	0.250 0.300	6.36 443.64	0.973
31	0.150 0.200	81.59 668.41	0.562
32	0.200 0.250	713.83 6.17	1.000
33	0.250	540.00	1.000
34	0.300	700.00	1.000
35	0.150 0.200	39.23 810.77	1.000
36	0.200 0.250	538.20 211.80	1.000
37	0.200 0.250	625.46 344.54	1.000

Cost = $1,950,698

The use of the gradient search techniques would require the availability of all dual variables, both from the loop and pressure head constraints. The computational effort required to set up all the constraints for all load patterns, solve the linear program, and analyze the gradient term rapidly becomes computationally impractical even for a moderately sized system.

An important feature of this technique is the interaction between the two methods required to achieve optimality. Both

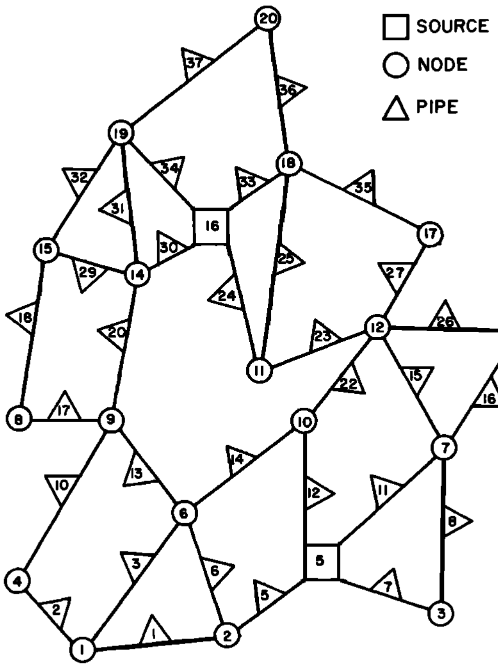

Fig. 4. Final layout for multiple demand patterns.

the network solver and the linear programming formulation use the results of the previous iteration. Iterative approaches involving linear programming have been used in previous studies by *Kally* [1972], *Alperovits and Shamir* [1978], and *Quindry et al.* [1981]. In the two latter studies the simplex tableau from the previous iteration must be saved in some form to maintain efficiency. The technique used in this paper is similar to that of Kally in that the previous solution is described by the "zero" simplex tableau.

Since the combination of the criteria for removal of pipe and the stepwise determination of the *NL* pressure constraints does not permit a claim for absolute optimality, the procedure must be classified as heuristic. However, the iterative nature of the problem permits crucial links to reveal themselves at any stage of the iterative procedure. The use of this simplified constraint set also permits the model to consider explicitly a wide range of flow demands without being overwhelmed by the impractically large numbers of constraints required for complete and total enumeration of all load patterns. In this fashion the model approaches the consideration of fire fighting demands at all nodes, as was suggested by *Templeman* [1982]. Furthermore, unlike the model of *Rowell and Barnes* [1982], this model ensures hydraulic consistency for all loading patterns and pipe combinations through the continued use of the network solver to distribute the flows and check the pressures for each loading pattern.

Extension of Model to Consider Network Expansion

One of the criteria used as a basis for the development of this model is that it should also be capable of analyzing expansion or upgrading of existing networks. The formulation described in the previous section can be used in the least cost design of network components necessary to upgrade an existing network. This extension to network expansion is achieved with minimal additional effort.

In an upgrading mode the existing pipe network becomes the initial assumed pipe network. This network is analyzed under the new flow or loading conditions, and the nodes with inadequate pressures are identified. The set of constraints associated with (12) is then formulated as before, with the following provision. The original formulation permits the replacement of "existing" or "chosen" pipe by either a smaller or a larger pipe. The rationale for this choice is that during the design phase the existing pipe is not actually in place. Hence a replacement of the existing pipe by a smaller pipe results in true savings during the construction phase.

In an upgrading situation replacement of an actual existing pipe by a smaller pipe results in increased capital cost. Consequently, if the initial or existing pipe is the true existing pipe, the X_{jds} variable representing reduction in pipe size can be dropped from the constraint sets and from the objective function. The existing pipe is then given a cost of zero, which reflects the cost of leaving it in the existing network.

If, however, in a subsequent iteration the model wishes to reduce the capacity of a particular link for which the capacity has been increased previously, allowance must be made for such a reduction. This allowance can be achieved by permitting all links which, at a particular iteration, have capacities above the initial capacity to have X_{jds} variables which allow pipe size reduction to the initial existing capacity. Other than these rather simple precautions, the upgrading mode of the model can be run in exactly the same manner as the design mode.

Model Application

In order to demonstrate the power and flexibility of the procedure, the model was applied to two different flow net-

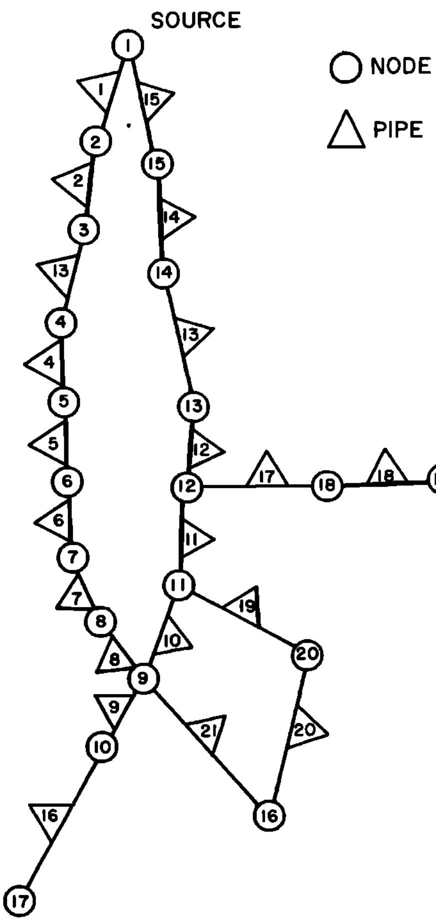

Fig. 5. Layout for New York water supply tunnels systems.

work problems. The first of these problems is a network layout and design situation, while the second is a network expansion problem.

Network Layout and Design

The example network layout and design chosen to demonstrate the model is shown in Figure 3. This network has 2 sources, 20 nodes, and 37 possible pipe locations. The pipe costs, link connections and lengths, and initial pipe size assumptions are shown in Tables 1 and 2 respectively. The minimum allowable and initial pressure distribution assumptions at each node are shown in Table 3.

This system is to be designed for a wide range of fire flow and pipe break combinations. The criterion of flexibility selected was that the system must give adequate pressure for each loading combination of a single pipe being out of order and a fire flow is present at the worst possible location. Each loading combination will represent a different location for the pipe break. This worst location for a fire flow will vary with the pipe that is assumed broken and should be picked by the design engineer. For this example the location and magnitude of each of these fire flows is given in Table 4. Using Tables 3 and 4, 37 different demand patterns, one for each node, are generated and the corresponding pressure constraints formulated.

The results of the first iteration through the linear program and network solver are shown in Table 5. The final column gives the maximum weighting assigned to each pipe during the construction of the constraints. The criterion used for removing a pipe is that the pipe must be the minimum size and have a maximum weighting less than 0.5. The minimum pipe (in this case taken to be pipe of 125 mm diameter) with the lowest maximum weighting is removed first. The program is then rerun. If the solution is cheaper this result is the new best result. The iterative process for pipe replacement is continued until all the lowest maximum weightings are greater than 0.5. The final results are shown in Figure 4 and Table 6. To obtain these results six iterations requiring a total of 6 min and 4.6 s of CPU time on the AMDAHL 5850 at the University of Manitoba were needed.

TABLE 7. New York City Network Layout Data

Link	Connecting Nodes From	Connecting Nodes To	Length, feet	Existing Diameter, inches
1	1	2	116,600.	180
2	2	3	19,800.	180
3	3	4	7,300.	180
4	4	5	8,300.	180
5	5	6	8,600.	180
6	6	7	19,100.	180
7	7	8	9,600.	132
8	8	9	12,500.	132
9	9	10	9,600.	180
10	11	9	11,200.	204
11	12	11	14,500.	204
12	12	13	12,200.	204
13	13	14	24,100.	204
14	14	15	21,100.	204
15	1	15	15,500.	204
16	10	17	26,400.	72
17	12	18	31,200.	72
18	18	19	24,000.	60
19	11	20	14,400.	60
20	16	20	38,400.	60
21	9	16	26,400.	72

Imperial units are used to facilitate comparison with previous studies. One foot equals 30.48 cm; 1 inch equals 2.54 cm.

TABLE 8. Solutions for New York City Expansion Problem

Link Location	Split Pipe Solution: Diameter of Added Pipe, inches	Split Pipe Solution: Length, feet	Discrete Pipe Solution: Diameter of Added Pipe, inches	Discrete Pipe Solution: Length, feet
7	144	9,600.00	144	9,600.00
16	96	23,432.08	96	26,400.00
	108	2,967.92		
17	96	31,200.00	96	31,200.00
18	72	57.99	84	24,000.00
	84	23,942.01		
19	48	4,527.76	60	14,400.00
	60	9,872.24		
21	72	3,492.89	84	26,400.00
	84	22,907.11		
	Cost = $38.9 million		Cost = $39.2 million	

Imperial units are used to facilitate comparison with previous studies. One foot equals 30.48 cm; 1 inch equals 2.54 cm.

Review of the results shows that even with the multiplicity of demand patterns (37 in all), the model was capable of ascertaining that six links (links 4, 9, 15 and 19, 21, 28) could be eliminated. The elimination of these links does not destroy the capability of the system to handle the predetermined fire flow and pipe break conditions, as these are checked at each iteration by the Hardy-Cross network solver.

Network Expansion Problem

This example demonstrates how the technique can be applied to expansion of an existing system, the New York City expansion problems [*Schaake and Lai*, 1969], and compares the results to previous studies. It should be noted that in keeping with previous studies this system will be analyzed with only one loading pattern and no pipe breaks. The full capability of this approach will not therefore be demonstrated in this example. The example merely serves as a means of comparing one capability of the model with previous approaches.

The cost and layout data of the New York City problem are described in Figure 5 and Tables 1 and 7, respectively. The imperial system of measurements was used to facilitate comparison with previous studies. The proposed method of expansion is the same as in the previous studies [*Schaake and Lai*, 1969], *Quindry et al.* [1981], and *Gessler* [1982], i.e., to reinforce the system by constructing tunnels parallel to the existing tunnels. For the initial conditions all reinforcing tunnels were assumed to be 84 inches in diameter.

The final results, shown in Table 8, indicate that in order to arrive at a least cost solution of $38.9 million, new pipes are added to only six links. A comparison between the cost of this solution and the cost of previously published solutions is given in Table 9. Two computer runs and 14.37 s CPU time were needed to obtain these results. It can be seen that the solution provided by this procedure is significantly cheaper than the solutions of *Schaake and Lai* [1969] and *Quindry et al.* [1981]. It is also cheaper than the solution provided by *Gessler* [1982]. This model, however, had a number of other significant advantages over Gessler's approach. The advantages are discussed below.

The solution developed by the procedure described in this

TABLE 9. Comparison of Solution of New York City Expansion Problem With Previous Studies

Study	Cost
Schaake and Lai [1969]	$78.1 million
Quindry et al. [1979]	$63.6 million
Gessler [1982]	$41.8 million
Split pipe solution	$38.9 million
Discrete pipe solution	$39.2 million

paper has discrete pipe diameters which span an entire link length in some cases and "split" pipes, i.e., two pipes of different diameter, spanning the link, in other cases. To compare this solution to that of *Gessler* [1982], who used only discrete pipes across the entire length of the links, the split pipes were replaced by a single diameter equal to the major portion of the present solution (see Table 8). The split configuration was tested using the Hardy-Cross technique, and the pressures were found to be adequate. The costs of these two solutions, i.e., the split pipe solution and the discrete pipe solution, are compared with the previous studies in Table 9. It can be seen that even the discrete pipe solution used for comparison with Gessler's solution is less expensive than the previous solutions.

While the cost of the solution provided by *Gessler* [1982] is of the same order as the cost of the solution generated by this model, it should be noted that Gessler's solution technique was based on a total enumeration approach and required significantly more computational effect than that required by the procedure introduced in this paper. The procedure therefore has two distinct advantages in that it provides a better solution with less computational effort.

The New York City tunnels problem has a number of local optima. Experience with the use of the model has shown that by starting the reinforcing at the smaller diameters, e.g., the smallest three or four "pipes," the procedure converges upon the given solutions. When starting with the larger diameter reinforcing sizes the model tends to converge upon more expensive local optima. The reasons for this situation are not fully understood. It is believed, however, that use of the smaller diameter forces the procedure to utilize reinforcing pipes at more than one location in the initial iterations. The procedure therefore has a greater flexibility in how it handles pipe diameter increases and decreases in these links in later iterations. It is suggested, however, that when using this approach the initial reinforcing alternatives are based upon the smaller diameters.

SUMMARY AND CONCLUSIONS

An iterative procedure capable of analyzing both layout and design of new systems and expansion of existing systems has been developed. One of the major advantages of the technique is that it ensures hydraulic consistency in each of the networks considered during the iterative procedure.

In the design of new systems a wide range of loading patterns and pipe failure combinations can be considered. The manner in which the large number of constraints associated with all possible load combinations is reduced to a manageable size does not permit a claim for global optimality. The procedure must therefore be classified as heuristic. The results obtained by the procedure, however, show that it is capable of efficiently producing economical solutions.

The method has been demonstrated as being applicable to the layout and design of a network of 2 sources, 20 nodes, and 37 links, and the expansion of an existing network. Comparison of the results produced by this procedure for a network expansion problem with the results of previous studies for the same problem shows that this procedure produces an inexpensive solution in an efficient manner.

The procedure is relatively simple, being based on two widely available and accepted techniques, namely, the Hardy-Cross Network solver and linear programming operations research technique. The ability of the model to use the very efficient simplex algorithm of the linear programming makes the procedure applicable to large systems. The method also utilizes realistic and easily accessible pipe cost functions using standard cost data, i.e., cost per unit length.

APPENDIX: WEIGHTING ALGORITHM

The weight assigned to each link j for each node i and demand pattern t in the pressure constraint equations, given by (12), is denoted by W_{ijt}. Since i and t remain constant for each equation throughout the weighting procedure, they are omitted for clarity in this example. The equation used to determine W_{ijt} is given below:

$$W_{ijt} = W_j = (Q_j/I_m) \times w_m \tag{A1}$$

where

Q_j the flow in link j;

I_m the sum of the inflow to node m;

w_m the weight of the node immediately downstream of link j.

The weight of the node m is calculated by

$$W_m = \sum_{j \in B} W_j \tag{A2}$$

where B is the set of all outflow links from the node m.

The procedure begins at the node i being constrained by (12) with $m = i$ and $w_m = 1.0$. The algorithm used to calculate these weights is demonstrated by application to the simple network shown in Figure A1. The flows in each link, which

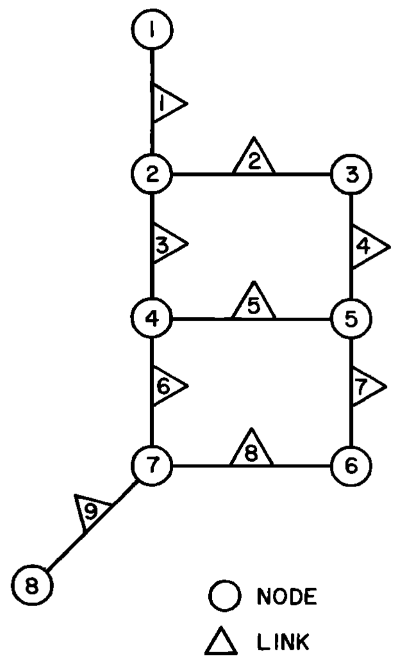

Fig. A1. Network for demonstration of weighting algorithm.

are obtained from the Hardy-Cross results, are shown in Figure A2. In this example, the constraint equation for the pressure at node 5 will be developed. The procedure begins immediately upstream of node 5. The total flow entering node 5 is calculated from the summation of all inflows, i.e., links 5 and 7 (160 + 190 = 350). The weights of links 5 and 7 are calculated from this total.

$$W_5 = (160/350) \times 1 = 0.46 \quad \text{(A3)}$$

$$W_7 = (190/350) \times 1 = 0.54 \quad \text{(A4)}$$

These weights are then assigned to the nodes upstream of the links, i.e., nodes 4 and 6. Node 4, for example, is assigned the weight 0.46 from the single outflow link from that node. Similarly, node 6 is assigned a weight of 0.54. The process then continues upstream. Under this formulation link 8 contributes 100% of the flow to node 6 via node 7. Link 8 has its weight calculated as follows:

$$W_8 = 1 \times (0.54) = 0.54 \quad \text{(A5)}$$

Link 3 contributes 50% of the flow to node 4 and is therefore assigned a weight calculated thus

$$W_3 = (0.5) \times (0.46) = 0.23 \quad \text{(A6)}$$

Likewise,

$$W_6 = (0.5) \times (0.46) = 0.23 \quad \text{(A7)}$$

It should be noted that the flow to node 5 from node 7 follows two distinct paths. The weighting at node 7 is therefore

$$W_7 = (0.23) + (0.54) = 0.77 \quad \text{(A8)}$$

The weights for each link and node are shown in Figure A2.

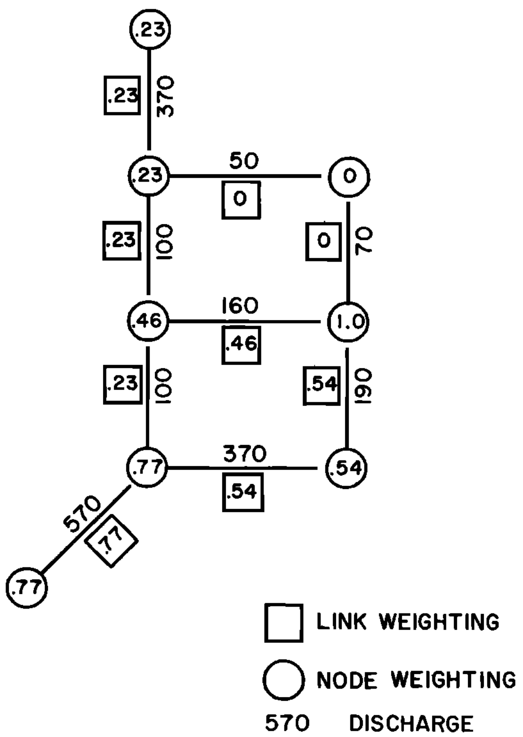

Fig. A2. Final flows and weights for network.

Links 2 and 4 have weights of zero, since none of the flow going to node 5 passes through these links.

This process is repeated for each possible load pattern.

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