Water Distribution Network Analysis Using Excel

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Abstract: The analysis of water distribution networks has been and will continue to be a core component of civil engineering water resources curricula. Since its introduction in 1936, the Hardy Cross method has been used in virtually every water resources engineering text to introduce students to network analysis. The technique gained widespread popularity primarily because it is amenable to manual calculation techniques. However, the same subtle elegance that facilitates manual calculations often obscures the primary engineering and physical principles of water distribution systems relative to the nuances of algorithm implementation. Herein, the authors illustrate the application of commonly available spreadsheet software (*MicroSoft Excel*) to more concisely and effectively solve typical undergraduate network distribution problems using linear theory. Application development is much more efficient and straightforward than the corresponding Hardy Cross implementation enabling students to concentrate upon the engineering system and relevant design issues. The technique presented utilizes commonly available technology and is presented as a supplement to alternatives discussed in recent literature.

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Introduction

As an applied science, there exists a natural tension between the study of fundamental scientific theory and instruction in the application of analysis and design methodologies within undergraduate engineering curricula. Most engineering courses are structured to emphasize the relevant physical, chemical, and biological processes that are then reinforced by studying specific problem solving skills applied to systems of engineering interest. In the area of water resources engineering, analysis commonly results in nonlinear differential or algebraic equations or systems of equations. Consequently, the level of application complexity and realism introduced to undergraduates is often limited by the students' computational capability. Instructors must diligently balance the need to emphasize the engineering system physics versus instruction in numerical methods used to solve resulting mathematical equations. Student comprehension of basic concepts that govern complex engineering systems is often impeded by cumbersome computational procedures.

Effective instruction in hydraulic design and piping system analysis has been the subject of several recent publications. The December 2001 issue of the *Journal of Hydraulic Engineering* was devoted to the topic of teaching hydraulic design. A common theme among many of these articles was a desire to increase the level of realism associated with engineering systems introduced. Jewell (2001) discussed the use of a commercial equation solver to facilitate hydraulic design instruction. Weiss and Gulliver (2001) discussed the use of spreadsheets to analyze various hydraulic design projects. They illustrated that using the spreadsheet as a tool to analyze practical engineering problems not only teaches valuable engineering analysis skills but also enhances students' computer skills and helps prepare them for the challenges that they will face professionally. Hodge and Taylor (2002) presented a set of *Mathcad* procedures applied to analyze various piping system applications. The *Mathcad* procedures provided a consistent framework for analyzing and solving common piping-system applications. Huddleston (2002) discussed the use of spreadsheet tools to introduce students to fundamental concepts of computational fluid dynamics by using an illustration from open-channel hydraulics.

This study examines the use of *Excel*, a commonly available spreadsheet package, to analyze a water distribution network. The linear theory method is applied to develop the network equations and *Excel* is used to solve the nonlinear system of equations. The application of this technology is an efficient way to enable undergraduate students to solve a relatively complex engineering system while minimizing the computational burden. Built-in linear and nonlinear system functions are commonly available in commercial spreadsheet programs, providing to teachers and students an affordable alternative to more complicated or expensive software.

This technique enables students to analyze realistic applications while still requiring manual development of the governing equations to reinforce the underlying engineering principles. Use

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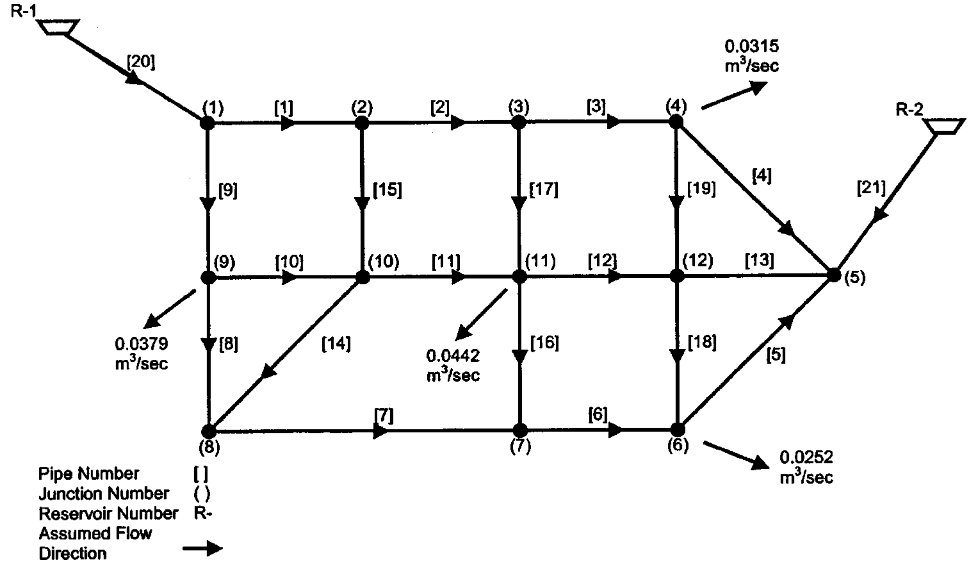

Fig. 1. Hydraulic network for example problem

of the software removes the necessity of analyzing the implementation and performance of numerical algorithms. Deletion of this material can be appropriate for an introductory course in which the primary objective is to teach students about the design, analysis and operation of a water distribution system with the study of specific numerical algorithms deferred to other courses.

Background

As presented in numerous references (e.g., Mays 2001), the steady-state flow distribution of an incompressible fluid through a piping network is governed by a one-dimensional energy and mass balance. Following the notation of Mays (2001), conservation of mass can be enforced at each of the N_J nodes within the network as

$$\sum Q_{\text{in}} - \sum Q_{\text{out}} = 0 \tag{1}$$

where Q_{in} and Q_{out} denote nodal demands and pipe flows into and out of the junction node. For each loop i of the N_L loops that comprise the network, conservation of energy requires that the sum of energy head loss h_L in each of the I_P pipes and energy gain H_{pump} across each of the J_P pumps in the loop must balance the net change in energy head ΔE_{FGN} as

$$\sum_{j=1,I_P} h_{L,j} - \sum_{k=1,J_P} H_{\text{pump},k} - \Delta E_{\text{FGN}} = 0 \tag{2}$$

For closed loops the change in energy grade is zero while the change in energy grade for loops between fixed grade nodes is prescribed.

The system of equations resulting from application of Eq. (1) to each junction node and Eq. (2) to each primary loop and independent set of fixed grade nodes, describes the flow distribution throughout the network. The resulting system of algebraic equations can be solved by a number of numerical algorithms, including Hardy Cross, Newton–Raphson, and the linear theory method (Mays 2001).

Based upon experience teaching this material, it is observed that students can quickly grasp the primary physics represented by Eqs. (1) and (2). When implementing a solution algorithm, such as Hardy Cross, students spend an inordinate amount of time struggling with implementation details and they are burdened by the quantity of computations required to converge to the network solution. This time devoted to nuances of the solution algorithm detracts from the students' overall understanding of water distribution systems and restricts the level of application complexity that can be introduced.

Network Piping System

Consider the hydraulic network described in Fig. 1, which is a variation of the network presented by Wood and Charles (1972). This example contains 12 nodes and eight loops which exceeds the usual expectations of a class assignment via manual calculations. As an illustration, the example is modified from the original reference by replacing the specified inflows with fixed grade nodes and utilizing the Darcy–Weisbach friction model. All piping materials are assumed to be cast iron. Specified junction demands and the assumed positive flow direction are indicated on the figure. The Darcy–Weisbach friction model is applied throughout and the friction factor is approximated using the Swamee and Jain (1976) formulation. The kinematic viscosity of water is prescribed as $1.0037\text{E}-6\ \text{m}^2/\text{s}$. Reservoir R-1 is at an elevation 3.66 m above the elevation of Reservoir R-2. Pipeline data are provided in Table 1.

The piping network is analyzed by developing a system of equations that represent the conservation of mass enforced at each of the 12 junctions, conservation of energy for each of the eight network loops, and conservation of energy between the two fixed grade nodes. This yields a system of 21 nonlinear, algebraic equations to solve simultaneously for the 21 unknown volumetric flow rates. Wood and Charles (1972) and Mays (2001) summarize the complete system of equations for this problem subject to specified inflow rates in lieu of the indicated fixed grade nodes.

Conservation of mass must be enforced at each of the twelve pipe junctions. For example, conservation of mass applied at junction J-9 yields

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Table 1. Pipeline Data for Example Problem.

Pipe number [ ]	Diameter (m)	Length (m)	Darcy-Weisbach relative roughness factor roughness (m)	Resulting flow rate (m^3/s)
[1]	0.305	457.2	0.00026	0.0558
[2]	0.203	304.8	0.00026	0.0400
[3]	0.203	365.8	0.00026	0.0165
[4]	0.203	609.6	0.00026	−0.0103
[5]	0.203	853.4	0.00026	−0.0087
[6]	0.203	335.3	0.00026	0.0126
[7]	0.203	304.8	0.00026	0.0150
[8]	0.203	762.0	0.00026	0.0097
[9]	0.203	243.8	0.00026	0.0480
[10]	0.152	396.2	0.00026	0.0004
[11]	0.152	304.8	0.00026	0.0108
[12]	0.254	335.3	0.00026	−0.0074
[13]	0.254	304.8	0.00026	−0.0160
[14]	0.152	548.6	0.00026	0.0053
[15]	0.152	335.3	0.00026	0.0157
[16]	0.152	548.6	0.00026	−0.0024
[17]	0.254	365.9	0.00026	0.0236
[18]	0.152	548.6	0.00026	0.0040
[19]	0.152	396.2	0.00026	−0.0047
[20]	1.000	25.0	0.00026	0.1037
[21]	1.000	25.0	0.00026	0.0351

$$Q_9 - Q_8 - Q_{10} - 0.0379\frac{\mathrm{m}^3}{\mathrm{s}} = 0 \tag{3}$$

Conservation of energy applied to representative loop Eq. (1) yields

$$K_1Q_1|Q_1| + K_{15}Q_{15}|Q_{15}| - K_{10}Q_{10}|Q_{10}| - K_9Q_9|Q_9| = 0 \tag{4}$$

Similarly, conservation of energy applied between the two fixed grade nodes yields

$$K_{20}Q_{20}|Q_{20}| + K_1Q_1|Q_1| + K_2Q_2|Q_2| + K_3Q_3|Q_3| + K_4Q_4|Q_4| - K_{21}Q_{21}|Q_{21}| - 3.66\ \mathrm{m} = 0 \tag{5}$$

In the preceding equations, Q_i and K_i denote, respectively, the volumetric flow rate and the friction coefficient for pipe i. Applying the Darcy–Weisbach friction model to the ith pipe, K_i is defined as

$$K_i = \frac{f_iL_i}{2gD_iA_i^2} \tag{6}$$

where L=length (m); D=diameter (m); A=area (m^2); and f= Darcy–Weisbach friction factor.

The resulting system of 21 linear and nonlinear, algebraic equations for the 21 unknown flow rates, $Q_1 - Q_{21}$ can be solved easily within a modern spreadsheet program. For example, the *Excel* function *Solver* (Gottfried 2000) is ideal for solving small nonlinear systems of equations such as this. The student merely needs to apply the function after defining the nonlinear system as one equation plus twenty constraints. The embedded solution algorithm (Newton's method variant) is robust and will converge to a solution if the user provides a reasonable initial guess for the unknowns. In essence, this constitutes an implementation of the linear theory method without the complexity of implementing a specific numerical algorithm to solve the nonlinear system of equations.

Students' prior familiarity with spreadsheet applications is a significant advantage to this approach. It is straightforward for students to associate a spreadsheet cell location with each of the conservation equations that must be satisfied. The significance of each conservation statement is reinforced since students must explicitly define the corresponding spreadsheet formula in order to successfully solve the problem. For reference, the resulting network distribution flow rates are included in Table 1.

Conclusions

The example presented illustrates the use of a commonly available spreadsheet package as a tool to facilitate instruction in water resource engineering. The *Excel* solution procedure is presented in a manner that is consistent with the governing conservation statements. This, coupled with students' preexisting familiarity with the spreadsheet package, enables students to focus on understanding of the engineering system rather than cumbersome computational procedures. Obviously, learning the skills necessary to implement and analyze the behavior of selected numerical algorithms is also an important aspect of the development and application of computational models. The instructional technique described herein merely provides the instructor with an alternative that allows deferral of much of the numerical analysis material to other courses.

Computer technology plays a significant role in engineering education. Determining how and at what level to introduce technology within the curricula is a significant challenge to educators. A productive role exists for scientific calculators, equation solvers, mathematics packages, spreadsheet applications, commercial analysis software, and programming assignments. The selection of the appropriate tool is dependent upon the course context and available technology infrastructure. The *Excel* illustration is presented as a bridge that enables students to analyze more realistic applications while still requiring enough manual development to reinforce the underlying engineering principles.

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