Deterministic versus Stochastic Design of Water Distribution Networks

Orazio Giustolisi[1]; Daniele Laucelli[2]; and Andrew F. Colombo[3]

Abstract: The paper describes a procedure for the robust design of water distribution networks which incorporates the uncertainty of nodal water demands and pipe roughness in a multiobjective optimization scheme aimed at minimizing costs and maximizing hydraulic reliability. The methodology begins with a deterministic system design in order to generate a set of optimal networks that serves as the initial population for subsequent multiobjective stochastic design. This approach does not depend on the choice of multiobjective optimizer (for example, a multiobjective genetic algorithm is used here) and can drastically reduce the number of "stochastic" runs needed for searching robust solutions. A collection of probability density functions based on the β function is introduced and applied to modeling variable uncertainty according to different physical requirements. The approach is tested in a case study involving a real network, illustrating its computational advantages.

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Introduction

Risk-based management of water distribution systems (WDS) encourages utilities to perceive reliability assessment as a useful tool for achieving effective management of new and existing networks. Walski (2001) stressed the need for developing new network design strategies, not only for addressing the minimization of pipe costs, but also the maximization of network reliability. This is usually interpreted as the provision of nodal head in excess of that which is established as a minimum within the network. The term "reliability" can be thought of as a system's ability to demonstrate adequate performance during both normal and unusual operating conditions (Xu and Goulter 1999) and is usually studied by considering two general classes of failures (Farmani et al. 2005): mechanical and hydraulic. The former refers to system component failure (such as pipe breaks, blockage, valve immobilization, pumping station interruptions, etc.), whose occurrence depends on appurtenance and device reliability and is thus closely related to rehabilitation/maintenance plans. Hydraulic failure refers to unforeseen alterations in nodal demands and pipe roughness, or in the inability to cope effectively with these.

Change in nodal demands often unfolds simultaneously with capacity deterioration due to aging and either process can result in pressure at one or more nodes falling below an acceptable level. Babayan et al. (2005) defined "robustness of the network" as the ability to adequately supply customers despite fluctuations in some, or all, of the design parameters (i.e., nodal demands, pipe roughness, etc.). Network robustness is dependent on the variability and cross correlations (Farmani et al. 2005) assumed for nodal demands and pipe roughness (Lansey et al. 1989; Xu and Goulter 1999; Babayan et al. 2005; Kapelan et al. 2005). Thus, the design challenge is to develop a strategy able to produce dependable results when faced with uncertainty (that is, incorporating robustness into the design approach). The stochastic least-cost WDS design problem was first conceived and solved as a single-objective formulation by Lansey et al. (1989). It was then interpreted as a constrained minimization problem and solved using the generalized reduced gradient 2 (GRG2) technique. Xu and Goulter (1999) developed an approach in which the first-order reliability method (FORM) was used and the optimization was performed by GRG2. Calculations proved too laborious and the method was time consuming even for small networks (Savic 2004). Moreover, the GRG2 optimization procedure assumes the decision variables (i.e., pipe diameters) as continuous, which is unrealistic. Tolson et al. (2004) tried to improve this approach combining a genetic algorithm (GA)-based optimization scheme with a method for estimating WDS reliability based on FORM. The approach necessitates repetitive calculation of the first-order derivatives and matrix inversions in order to calculate uncertainties and becomes computationally demanding even for small networks, sometimes inviting numerical problems. Babayan et al. (2005) avoided recourse to a sampling-based method using single objective GA linked to an integration-based uncertainty quantification technique. They assumed some probability density functions for nodal demand fluctuations (uncertainty) and defined a set of critical nodes (i.e., those that did not satisfy pressure requirements) which were used for the evaluation of the fitness function. However, the actual level of robustness cannot be specified explicitly in the problem formulation, but is calculated once the optimization process has converged to the final solution. The

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above-mentioned stochastic WDS design methodologies have one common limitation: optimization is formulated and solved as a constrained single-objective problem, thus resulting in a single optimal solution.

Recently, Kapelan et al. (2005) proposed a multiobjective (MO) optimization approach for solving WDS design under uncertainty. They modeled the uncertain variables (i.e., nodal demands and pipe roughness) by means of normal and uniform probability density functions (PDFs), respectively, which are calculated using the Latin hypercube (LH) sampling technique (McKay et al. 1979). The chosen method is the Robust NSGAII, which is based on the nondominated Sorting Genetic Algorithm II (NSGAII) (Deb et al. 2002). The procedure exploits a small number of samples for fitness evaluation, leading to significant computational savings and producing a robust Pareto optimal front of solutions.

This paper proposes a multiobjective approach to the WDS design problem, considering nodal demands and pipe roughness as uncertain variables. The optimal design procedure is conceived and formulated for use with an optimizer based on a standard GA. In particular, the optimized multiobjective genetic algorithm (OPTIMOGA), as in Giustolisi et al. (2004), is featured. The proposed strategy performs a deterministic design (i.e., constrained least-cost design procedure) as the first step and then, using the deterministic solutions as initial population, solves the robust design problem multiobjectively, implementing the minimization of design costs and the maximization of WDS robustness as objective functions. As explained subsequently, this approach can offer significant computational savings. Further, network robustness is defined based on the worst-performing (i.e., critical) node, after the evaluation of hydraulic performances of all the network nodes. Finally, a collection of PDFs is tested with the goal of modeling system uncertainty in different ways. Save for the normal distribution, they are all based on the β function (Mood et al. 1974) with each applied PDF being defined on a bounded domain and evaluated for a different range. The methodology is verified in a case study of a real planned network for an industrial area in an Apulian town (Southern Italy).

Network Simulation Model for Pipe Sizing

The paper assumes the demand-driven formulation given in Todini and Pilati (1988). However, it should be recalled that nodal demands are usually treated as constants for each simulation even if, in reality, they change. To reflect this, their fluctuations are here accounted for by some representative PDF (Kapelan et al. 2005; Babayan et al. 2005; Giustolisi et al., 2005). Therefore, a network comprising n_p pipes carrying unknown flows, n_n nodes with unknown pressure heads, and n_0 nodes with known pressure head (reservoirs) can be described as follows:

$$\begin{bmatrix} \mathbf{A}_{pp} & \mathbf{A}_{pn} \\ \mathbf{A}_{np} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ \mathbf{H} \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_{p0}\mathbf{H}_0 \\ \mathbf{q} \end{bmatrix} \tag{1}$$

where $\mathbf{Q}=[Q_1,Q_2,\ldots,Q_{np}]^T=[n_p,1]$ column vector of the computed pipe flows; $\mathbf{H}=[H_1,H_2,\ldots,H_{nn}]^T=[n_n,1]$ column vector of the computed nodal total heads; $\mathbf{H}_0=[H_{01},H_{02},\ldots,H_{0n0}]^T=[n_0,1]$ column vector of the known nodal total heads; and $\mathbf{q}=[q_1,q_2,\ldots,q_{nn}]^T=[n_n,1]$ column vector of the nodal demands, which here are assumed to vary according to some PDF.

In the mathematical system (1), $\mathbf{A}_{pp}$ represents a $[n_p,n_p]$ diagonal matrix whose elements are defined as $\mathbf{A}_{pp}(i,i)=R_i|Q_i|^{n-1}$, whereas $\mathbf{A}_{pn}=\mathbf{A}_{np}^T$ and $\mathbf{A}_{p0}$ are topological incidence submatrices, of size $[n_p,n_n]$ and $[n_p,n_0]$, respectively, derived from the general topological matrix $\bar{\mathbf{A}}_{pn}=[\mathbf{A}_{pn}|\mathbf{A}_{p0}]$ of size $[n_p,n_n+n_0]$, defined as in Todini and Pilati (1988). R_i is the pipe hydraulic resistance that is a function of pipe roughness, diameter, and length, whereas n is an exponent which takes into account the actual flow regime and adopted head loss relationship (here $n=2$ is used). In this work, pipe roughness (and thus pipe hydraulic resistance R_i) is assumed to vary according to some PDF.

Single-Objective Optimization

The classical problem of network design centers on the selection of pipe diameters, given the pressure head at nodes $\mathbf{H}$, computed from system (1) for fixed deterministic demands and roughness (dependent on diameters), and it is constrained by a minimum nodal pressure head for supplying demand. The problem is usually formulated with a single objective: minimum pipe cost. For instance, system (1) is typically completed with the following equations (Savic and Walters 1997):

$$f(D_1,D_2,\ldots,D_{n_p}) = \sum_{i=1}^{n_p} C(D_i,L_i) \rightarrow \min \tag{2}$$

$$H_j \geqslant P_j^{\min} + Z_j, \quad j=1,\ldots,n_n \tag{3}$$

where the former deals with financial cost minimization [dependent on pipe diameter (D_i) and length (L_i)] and the latter pertains to service level constraints ($P_j^{\min}$ is the minimum nodal pressure head and Z_j its elevation above datum) that must be satisfied in order to supply required nodal demands. Thus, pipe diameters are the decision variables of the optimization problem and their appropriate selection is the specific goal of the design task. The formulation in Eqs. (1)–(3) leads to a complex (nonlinear) combinatorial optimization environment as diameter choices are discrete. Savic and Walters (1997) demonstrated that GA are an efficient way to solve this kind of problem as they undertake a wide exploration of the solution space, implying a high probability of exposing the global optimum.

Robust WDS Design and Multiobjective Formulation

Recently, Kapelan et al. (2005) proposed a MO optimization approach for solving WDS design under uncertainty. They modeled the uncertain variables (i.e., nodal demands and pipe roughness) by means of Gaussian and uniform PDFs, assuming some correlation among the samples used. The LH technique was applied to calculate the PDFs in order to reduce the number of samples (at least equal to 30 for each individual in each generation). However, the key element in this work lies in the optimization method. They used the robust NSGAII, based on the NSGAII (Deb et al. 2002), which is able to evaluate each individual's fitness, spreading the sampling over a number of generations. The objective functions were pipe cost minimization and network robustness maximization, which is computed as the fraction (i.e., percentage) of the total number of samples for which the minimum pressure head requirement is met simultaneously at all nodes in the network. The methodology produces a robust Pareto front of optimal solutions which is then subjected to a final validation by applying a Monte Carlo (MC) simulation to the optimal solutions and deleting those that turn out to be dominated after the final check.

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The procedure makes use of a small number of samples for fitness evaluation, leading to significant computational savings if compared to the full sampling approach (the procedure without the final MC simulations takes at least 27 min to return the Pareto front).

Proposed Approach

This paper introduces an alternative to the procedure of Kapelan et al. (2005) which involves the development and testing of a robust MO design strategy for WDS design employing a general purpose MO genetic algorithm (MOGA) as optimizer (OPTIMOGA, Giustolisi et al. 2004). The innovative aspects of the procedure with respect to recent contributions (Kapelan et al. 2005; Babayan et al. 2005; Tolson et al. 2004) include:

1. The use of a set of β PDFs (Giustolisi et al. 2005) in order to model nodal demand and roughness uncertainties with some variety. This permits incorporation of a bounded PDF, which, coupled with the LH sampling technique, can allow a better random sample stratification leading to more accurate estimation of the empirical, nodal head PDF tails (Mood et al. 1974; Fishman 1996), which is important when evaluating WDS design robustness. Moreover, the proposed tests on real data of different PDFs (including the normal) can show the technical consistency of β PDFs, allowing engineers to choose from among them, according to their own knowledge and experience, those best suited to represent the features of uncertain parameters. The normal PDF is here defined on a bounded domain in order to avoid numerical and physical problems related to its original unbounded properties.
2. The evaluation of robustness of each optimal design solution is made with respect to the network's critical node (i.e., the worst-performing node), as the probability that its stochastic nodal head (H_j) is higher than the service level ($P_j^{\min}+Z_j$), assuming the conservative hypothesis that the nodal heads could vary according to a normal PDF. Thus, robustness maximization is an objective function integrated in the optimization procedure, whereas in previous works it is usually calculated at the end of single-objective optimization. As subsequently clarified, the features of the analyzed network (i.e., few nodes, similar water demands, etc.) allow for the evaluation of network robustness with reference to the critical node without compromising the procedure's efficiency. Larger and more complex networks might need a third objective function looking at a number of critical nodes or critical zones (not contiguous critical nodes) which can be affected by service failures.
3. The design procedure is twofold: first, the problem is solved within the context of a least-cost deterministic approach using the minimization of costs and pressure deficit on the network's critical node as objective functions; second, robust design is performed as a dual-objective optimization problem (cost versus network robustness) employing the deterministic solutions as an initial population. This assumption, to be clarified subsequently, is justified by evidence that the deterministic solutions (i.e., network configurations) are close to, or at least belong to, the final Pareto front of robust/stochastic solutions. The robust design phase uses the minimization of costs and the maximization of network robustness as objective functions. Overall, the entire approach can significantly reduce computational effort.

Nodal Demand and Pipe Roughness Uncertainty

Traditionally, both nodal demands q_j and pipe hydraulic resistance R_i have been treated as fixed and known parameters for design and performance evaluation. Clearly, this is not ideal as the values of these parameters are not usually known with accuracy, especially when entertaining long-term projections. To overcome this challenge, all future nodal demands and roughness parameters will be treated as uncertain variables whose values are governed by a PDF and, to simplify things, no correlation between any two random variables will be modeled (Kapelan et al. 2005). However, generally speaking, correlation among nodal demands can be assumed, for example, due to some extent on uncertain factors that affect the system as a whole, such as hot, dry weather, which can result in significant extra consumption (i.e., garden watering, drinking, etc.), thus increasing or decreasing demand at all nodes simultaneously (Kapelan et al. 2005). Assuming a normal PDF with unit mean and standard deviation σ, i.e., $N(1,\sigma)$, the following can be written for nodal demands and pipe roughness:

$$
\begin{aligned}
q_j^{\text{unc}} &\in N(q_j,\sigma_j q_j) = q_j N(1,\sigma_j)\\
R_i^{\text{unc}} &\in N(R_i,\sigma_i R_i) = R_i N(1,\sigma_i)
\end{aligned}
\tag{4}
$$

In Eq. (4), the $N(1,\sigma)$ PDF plays the role of a proportionality coefficient for the uncertain variables (i.e., σ is related to the relative uncertainty level of the corresponding variable). Uncertainty quantification is performed here using the sampling approach based on MC methodology. Specifically, the LH technique is employed as the variance reduction method for limiting the number of samples required. To avoid consideration of unrealistic extreme values in the case of a normal PDF, a special beta PDF is used (Giustolisi et al. 2005)

$$\beta(a,b) = \frac{x^{a-1}(1-x)^{b-1}}{\text{beta}(a,b)} \tag{5}$$

where $\beta(a,b)$ stands for the beta PDF with parameters a and b, and beta (a,b) is the beta function (Mood et al. 1974). In Eq. (5), the value of the beta PDF is defined for x in the range $[0, 1]$. The values of a and b can be chosen so that the probability distribution has the required mean value m and standard deviation σ

$$
\begin{aligned}
m &= \frac{a}{a+b}\\
\sigma &= (a+b)^{-1}\sqrt{\frac{ab}{(a+b+1)}}
\end{aligned}
\tag{6}
$$

By manipulating the values of a and b, the beta PDF can be used to generate different types of PDF bounded in a range that is defined by the user. The idea of assuming the shape, mean, and range (in spite of the standard deviation) of the PDF is technically sound. PDFs of different shape can be related to information about uncertainty and the variable's range is an easier parameter to select. Thus, four beta PDFs have been constructed with the same standard deviation, deriving from a particular normal PDF. This paper takes into consideration the distribution $N(0.5,\sigma)$, where σ is computed so that the normal PDF is constrained to the range $[0, 1]$, for example, the upper and lower limits corresponding to the cumulative values of 0.001 and 0.999. In this way, the standard deviation of the normal PDF is equal to $(\sigma = m/3.0902) \Rightarrow (0.1618 = 0.5/3.0902)$, with 3.0902 being the value associated with the cumulative PDF at 0.999. By Eq. (6), the

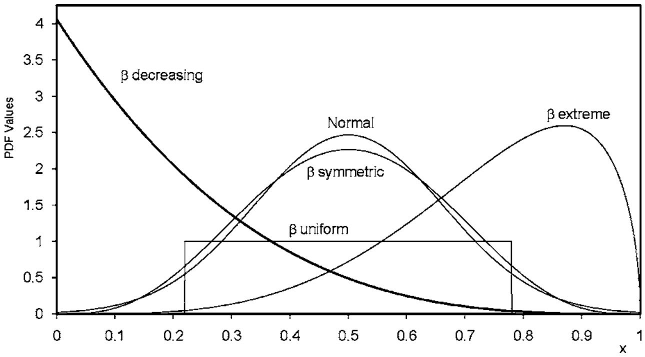

Fig. 1. Diagrams of the PDFs used within the optimization problem

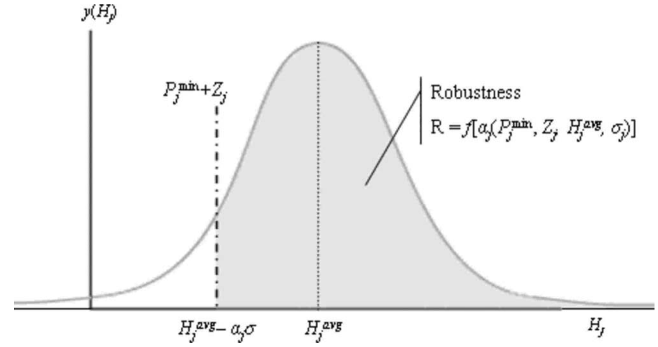

Fig. 2. Gaussian PDF assumed for robustness evaluation

following different beta PDFs have been built, all constrained to have their standard deviation equal to 0.1618 (see Fig. 1).

1. β symmetric is a beta PDF $B(a=b=4.2748)$, having a bell shape similar to the normal PDF, assuming an average value equal to 0.5. In spite of the normal PDF, it has the advantage of being bounded within a certain range, thus overcoming troubles related to the sampling of the empirical, nodal head PDF tails.
2. β extreme is a beta PDF $B(a=4.6216, b=1.5405)$ having a mean value equal to 0.75 and a shape that apportions more probability to values above the mean. This could be useful because, within the fixed range of variability, further peak conditions for the uncertain variable can be assumed for design purposes with higher probability than the mean value.
3. β uniform is a beta PDF $B(a=1, b=1)$ having the shape of a uniform PDF and mean value equal to 0.5, defined in the range [0.2198; 0.7802] in order to have standard deviation equal to 0.1618. It could be useful when there is no reliable statistical information.
4. β decreasing is a beta PDF $B(a=1, b=4.0554)$ having an exponential decreasing shape bounded in the range [0; 1] and a mean value equal to 0.1977. This PDF could be useful for simulating roughness trends related to pipe deterioration. This structural status of pipes is related to material and age, but there are other factors (i.e., intrusion of roots, internal corrosion, sediments, etc.) which can alter roughness in a manner differently than predicted using average values reported in the literature. At least a maximum value for pipe hydraulic resistance related to material and operating conditions can be assigned. Actually, the scope of pipe hydraulic resistance variation is minimally bounded by the values for new pipes and can be estimated by assuming a decreasing probability of larger values until the upper limit, which is related to the assumed range, is reached.

After defining the beta PDF, a MC sampler can be used to sample $\beta(a, b)$ with a generator based on the rejection and inversion method (Devroye 1986). As in other studies (Kapelan et al. 2005), the LH technique is applied here because it is a powerful variance reduction approach that makes the MC statistically meaningful by substantially decreasing the number of samples due to its enhanced random sample stratification. This feature can improve the efficiency of simulations, especially in large networks, saving much computational time with respect to other sampling-based methods (Giustolisi et al. 2005). For example, only 1,000 samples for performing the MC simulation were used in this work. The LH methodology is also applied to the uniform PDF required by the rejection and inversion method. After sampling $\beta(a, b)$, the following transformation can be applied to ensure the uncertain value of flow (q_j^{unc}) belongs to the interval $[q_j-(q_j \cdot m \cdot rg); q_j+(q_j \cdot m \cdot rg)]$

$$q_j^{\text{unc}} = q_j + q_j[x(B(a,b)) - m]rg \in [q_j - (q_j \cdot m \cdot rg), q_j + (q_j \cdot m \cdot rg)] \quad (7)$$

where rg=assumed variable range for the adopted beta PDF; m its mean; and q_j=assumed deterministic value for demand at node j. For pipe hydraulic resistance, the following transformation can be applied to ensure that the uncertain value of hydraulic resistance of ith pipe (R_i^{unc}) belongs to the interval $[R_i, R_i+R_i \cdot rg]$, treating the β decreasing function as the sole PDF,

$$R_i^{\text{unc}} = R_i + R_i[x(B(a,b))]rg \in [R_i, R_i + (R_i \cdot rg)] \quad (8)$$

Finally, it is worth noting that rg for the beta PDF plays the same role as σ for the normal PDF but is better able to deal with ranges than statistical variables when describing the uncertainty of hydraulic parameters.

Evaluation of Robustness

Assuming nodal demands as uncertain variables, according to a fixed PDF, means that the computed hydraulic variables $(\mathbf{Q}, \mathbf{H})$, see Eq. (1), become stochastic. Therefore, it is possible to take into account the standard deviation and average value of H node by node; thus, assuming a normal PDF as representative of the fluctuation of total head at node j (H_j) due to demand uncertainty, it is possible to write that

$$H_j(H_j^{\text{avg}}, \sigma) \geqslant P_j^{\min} + Z_j, \quad j = 1, \ldots, n_n \quad (9)$$

where H_j^{avg}=average value of total head calculated at node j and $\sigma[H_j(\mathbf{q})]$ its standard deviation at the node (which depends on all the nodal demand fluctuations). The assumption of a normal PDF to model the total nodal head uncertainty can be considered as a working hypothesis, rendering it possible to assume different PDFs. Taking into account Eq. (9), the design constraint in Eq. (3) can be rewritten as follows:

$$H_j^{\text{avg}} - \alpha_j \sigma[H_j(\mathbf{q})] \geqslant P_j^{\min} + Z_j, \quad j = 1, \ldots, n_n \quad (10)$$

In Eq. (10), the parameter α_j can be used for the evaluation of network robustness (how many times the stochastic nodal head $H_j(H_j^{\text{avg}}, \sigma[H_j(\mathbf{q})])$ will be higher than or equal to the minimum service level $P_j^{\min}+Z_j$, see Fig. 2). For example, if $\alpha_j=1.282$ the robustness is 90%, whereas if $\alpha_j=1$ it is 68% and if $\alpha_j=-1.282$ the robustness is 10%. Thus, the proposed approach evaluates network robustness looking at the fulfillment of Eq. (10) at the most critical node (i.e., that node resulting with the lowest value

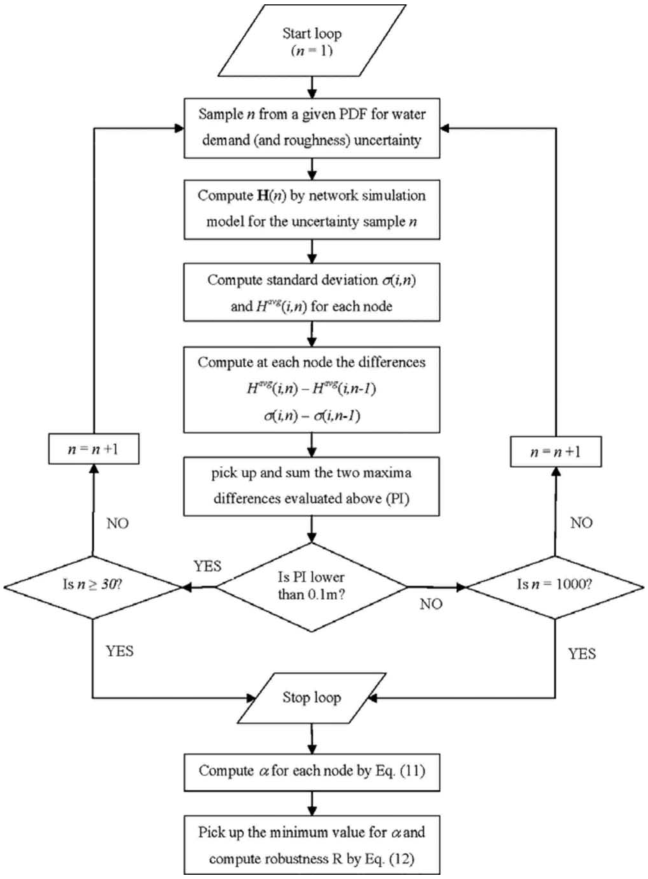

Fig. 3. Robustness evaluation procedure

of α), expressing it as the probability that its total nodal head H_j is higher than the service level $(P_j^{\min}+Z_j)$. From Eq. (10), α_j can be calculated as

$$\alpha(P_j^{\min},Z_j,H_j^{\text{avg}},\sigma)=\frac{H_j^{\text{avg}}-(P_j^{\min}+Z_j)}{\sigma} \tag{11}$$

From the perspective of WDS robustness, the aim of the proposed approach is to achieve the greatest value of α for the most critical node in a given network, thus achieving a greater probability (robustness) that the actual pressure H_j will be greater than $(P_j^{\min}+Z_j)$. Eq. (11) indicates that this can be achieved by ensuring the average total nodal head H_j^{avg} is as high, and its standard deviation as low, as possible (that is, obtaining a slender normal PDF, see Fig. 2).

The proposed MO approach seeks design cost minimization and network robustness (R) maximization, here cast as objective functions. The former has its formulation in Eq. (2), whereas the latter can be constructed as follows:

$$R[\%]=f\left(\max\{\min_{j\in[1,nn]}[\alpha(P_j^{\min},Z_j,H_j^{\text{avg}},\sigma)]\}\right) \tag{12}$$

The optimization goal is to maximize robustness of the network's most critical node, which implies maximizing the relevant α_j whose calculation is performed by Eq. (11) once the design constraint in Eq. (10) is evaluated for all nodes (see also Fig. 3 as further explanation). Finally, for the sake of simplicity, the above-presented equations have been reported assuming as uncertain only the nodal demands. As more clearly articulated in the case study, the present work also assumes uncertain pipe roughness, which itself influences total nodal head fluctuations.

Multiobjective Optimization Procedure

The MO procedure involves two related stages: (1) the deterministic phase, which consists of a least-cost network design, addressed to minimize costs and preserve service level $P_j^{\min}$ in a MO framework, without accounting for water demand and roughness uncertainty; and (2) the stochastic phase, consisting in a MO robust network design, taking into consideration the nodal demand and roughness uncertainty, and optimizing network design costs versus network robustness, as defined earlier. The algorithm used herein for multiobjective optimization is OPTIMOGA, which has been recently developed and applied to both test problems (Giustolisi et al. 2004) and applications (Giustolisi et al. 2006). The problem in Phase (1) is expressed by means of the mathematical system in Eq. (1), applying the constraint in Eq. (3) and assuming $P_j^{\min}$ as minimum value for the pressure head at nodes (level of service). Therefore, employing OPTIMOGA, the procedure returns a Pareto front of nondominated solutions using as objective functions the minimization of design cost, see Eq. (2), and minimization of the difference between the service level $(P_j^{\min}+Z_j)$ and total head H_j at the critical node. This second objective function can be defined on a certain range (PDR), thus admitting a lower pressure head limit at the critical node.

The stochastic phase is then performed as a MO robust design exercise which implements cost minimization, see Eq. (2), and robustness maximization, see Eq. (12), using as initial population the results of the deterministic phase. The aim of this choice is to speed up the robust design procedure, bearing in mind that the adopted MC sampler, even if improved by the LH technique, can require substantial computation time (Kapelan et al. 2005). From a GA optimization standpoint, the authors' intention is to streamline the stochastic procedure by setting out with an initial population that is closer to the final Pareto front, thus avoiding the great number of network evaluations (due to the adopted MC sampling) that are performed when the evolving population is far from the convergence. Looking at the problem also under an engineering perspective, this is achieved using the optimal results of the deterministic phase (least-cost networks), which are basically good solutions having low values of robustness. It is also worth noting that the proposed approach facilitates comparison between deterministic and robust solutions, highlighting those mains that exert a dominant influence on network reliability. This phase of the procedure employs OPTIMOGA as the MOGA optimizer. For each individual in each generation, a maximum number of samples ($S_{\max}$) of the assumed PDF for nodal demand and roughness uncertainty are performed, leading to the evaluation of $(H_j^{\text{avg}},\sigma_j)$ for each node by means of network simulations [see Eq. (1)]. Then, α_j is computed by Eq. (11) at each node and, next, the most critical node of the network is identified as that with the lowest value. Finally, each individual in the evolving population is assigned its network robustness from α pertaining to the most critical node using the inverse of the normal cumulative density function. The entire procedure, which is repeated every generation for each network/individual, is summarized in Fig. 3. The solutions are evaluated within the MOGA paradigm by means of a rank-based fitness assignment, as proposed by Fonseca and Fleming (1993) and implemented in OPTIMOGA. As shown in Fig. 3, the procedure applies a stopping criterion for the sampling process by assuming that if the mean head value and standard deviation do not vary sufficiently as the sampling process unfolds (i.e., less than a fixed approximation), the loop is stopped. However, the sampling procedure assumes a minimum number of samples be taken, imposing an initial threshold of S_{ini} for the first

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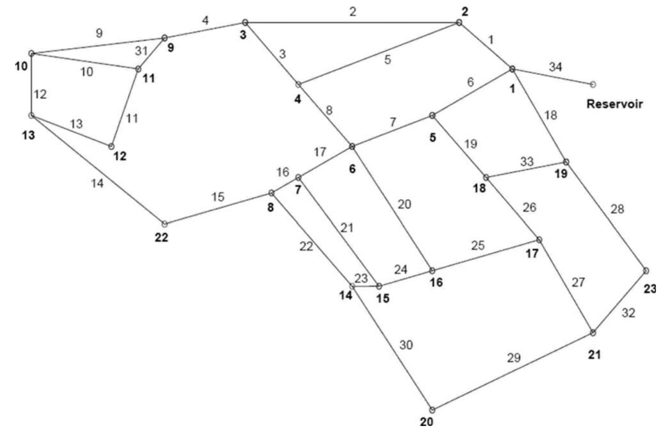

Fig. 4. Apulian network layout

individual to be evaluated within a generation. Fig. 3 then changes dynamically during the computation of all the individuals in the population, being set to the updated number of samples necessary to stabilize the initial oscillations of $H^{avg}(i,n)$ and $\sigma(i,n)$, with respect to the assumed approximation. Despite this, the number of iterations is constrained to the range $[S_{min}; S_{max}]$ to ensure an adequate number of samples when avoiding an endless loop. This contrivance leads to a marked reduction in the number of iterations, eschewing useless sampling and enjoying significant computational benefits.

Finally, during the MOGA run, the evolving Pareto front can be bounded between two extreme values of network robustness (R), implying the related limit of the parameter α. This choice is driven by the practical benefit of the returned diagrams; actually, networks having robustness beyond an upper limit can be considered too expensive to warrant further consideration, whereas networks with very low robustness should not be perceived as appreciably more reliable because they remain closely comparable to the deterministic solutions. In the following section, further descriptions are provided on the used limits and motivations.

Case Study

The case study is conceived in order to verify and demonstrate the methodology as applied to a real network. The featured system is the distribution network of an Apulian town (Southern Italy) whose layout is depicted in Fig. 4 with the corresponding data provided in Table 1. Each pipe represented in Fig. 4 is listed according to its identification number (Pipe number), and is accompanied by its start and end nodes as well as its length (L_i). All nodes are denoted by their identification number (Node number) and also appear in Table 1, which reports the relevant deterministic demands (q_j) (used as mean values for the uncertainty simulation) and elevations (Z_i). The node without an identification number corresponds to the sole reservoir of the network, with the head reported in Table 1 being the water level H_{01}. Table 2 lists the available pipe diameters, each coupled with its unit cost (expressed in Euros per meter) and its deterministic unit resistance coefficient,

$$\frac{R_i}{L_i} = u = \frac{\beta}{D_i^5} = \frac{8{,}57 \times 10^{-4}\left(1 + \frac{2\gamma}{\sqrt{D_i}}\right)}{D_i^5} \tag{13}$$

as evaluated by Bazin's formula (Bazin and Darcy 1865; Manning 1891), using $\gamma = 0.12$. This coefficient is used as the mean value for uncertainty evaluation.

As explained earlier, the proposed procedure is conceived as a general one, with a certain number of parameters to be set. Referring to Phase 1 (deterministic phase) the adopted minimum pressure head value (P_j^{min}) is equal to 10 m, whereas the range of definition for the used second objective function (PDR) is [0;2], thus admitting a lower pressure head limit of 8 m at the critical node. For Phase 2 (stochastic phase), the assumed maximum number of samples to be taken (S_{max}) is 1,000, whereas the lower limit for samples (S_{min}) is equal to 30. This last choice derives from the assumption that mean values and standard deviations evaluated on, at least, 30 samples can be reasonably considered as sufficiently statistically reliable (Benjamin and Cornell 1970). Finally, the assumed initial threshold for the number of samples (S_{ini}) is equal to 100.

During Phase 2 optimization, as noted earlier, the evolving Pareto front can be bounded between two extreme values of network robustness. The assumed lower bound is 10% [the design constraint in Eq. (10) is satisfied at the most critical node with a confidence limit at least equal to 10%], which implies that the relevant value for α is -1.282 (see Fig. 2). The upper bound is imposed in order to confine robustness evaluation to what is technically suitable, so the investigation assumes a maximum value set to $R = 90\%$, corresponding to α equal to 1.282 (see Fig. 2), as the cost of solutions rises exponentially for robustness above 90%, as noted by other authors (Kapelan et al. 2005; Tolson et al. 2004). During the MOGA run, networks exhibiting robustness beyond the assumed range are penalized. For example, a value of R higher than 90% [$\alpha > 1.282$] implies that the relevant network is assumed to have a robustness of 90%, thus there is incentive to retain the design if it is cheaper; otherwise, when network robustness falls below 10% [$\alpha < -1.282$], the solution is very cheap, but at the same time too low in robustness to be accepted (even lower than deterministic solution).

As explained, the proposed approach consists of two phases: the deterministic least-cost design of the network and the robust/stochastic design that directly involves network robustness within the optimization. Both of these harness OPTIMOGA (Giustolisi et al. 2004) as an optimizer, using different additional objective functions (minimization of pressure deficit at the critical node for Phase 1, maximization of network robustness for Phase 2) coupled with minimization of design costs. OPTIMOGA starts with a population made up of POP_{ini} individuals, POP_{ini} being set as 40 in this case. Each individual/chromosome is made up of a number of genes that equals the number of network links in the problem at stake, each gene being representative of the assumed diameter for the network configuration. Genes are comprised between 0 and 9, according to what is reported in Table 2. The genetic operators used are a multipoint crossover (with a probability of 40%), with a number of potential swapping points equal to the number of genes contained in the chromosomes. Care is taken in not swapping between two individuals' genes representing the same digit. A global mutation is initially used, each gene can be mutated with a 10% probability and assuming values ranging between 0 and 9. Afterwards, when the algorithm is in an exploitative phase of the Pareto front and it is not advisable to scatter the solutions in the objective space, a sort of local mutation is adopted, consisting in a small change of each mutating gene (i.e., +1 or −1 with respect to its original value). The selection of the mating pool is pursued with respect to the size of the best-found evolving Pareto front, by means of a rank-based fitness assignment (Fonseca and Fleming 1993). Finally, runs can be ended in two ways: by a stopping criterion based on the number of generations achieved, or by a criterion based on the number of

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Table 1. Apulian Network Data

Pipe				Node		
Pipe number	Start node	End node	L_i (m)	Node number	q_i (L/s)	Z_i (m)
1	1	2	348.5	1	10.863	6.4
2	2	3	955.7	2	17.034	7.0
3	3	4	483.0	3	14.947	6.0
4	3	9	400.7	4	14.280	8.4
5	2	4	791.9	5	10.133	7.4
6	1	5	404.4	6	15.350	9.0
7	5	6	390.6	7	9.114	9.1
8	6	4	482.3	8	10.510	9.5
9	9	10	934.4	9	12.182	8.4
10	11	10	431.3	10	14.579	10.5
11	11	12	513.1	11	9.0072	9.6
12	10	13	428.4	12	7.5745	11.7
13	12	13	419.0	13	15.200	12.3
14	22	13	1,023.1	14	13.550	10.6
15	8	22	455.1	15	9.226	10.1
16	7	8	182.6	16	11.200	9.5
17	6	7	221.3	17	11.469	10.2
18	1	19	583.9	18	10.818	9.6
19	5	18	452.0	19	14.675	9.1
20	6	16	794.7	20	13.318	13.9
21	7	15	717.7	21	14.631	11.1
22	8	14	655.6	22	12.012	11.4
23	15	14	165.5	23	10.326	10.0
24	16	15	252.1	Reservoir	0	$H_{01}=36.4$
25	17	16	331.5			
26	18	17	500.0			
27	17	21	579.9			
28	19	23	842.8			
29	21	20	792.6			
30	20	14	846.3			
31	9	11	164.0			
32	23	21	427.9			
33	19	18	379.2			
34	24	1	158.2			

solutions in the Pareto optimal set. In this case study the former was adopted, using 1,000 generations for the deterministic phase and 200 generations for the stochastic phase. More details about OPTIMOGA features can be found in Giustolisi et al. (2004).

Table 2. Structural and Economic Features of Diameters into the Apulian Network

GA coding	Nominal diameter	R_i/L_i	Cost (€/m)
1	100	265.15	240.1
2	150	18.565	387.78
3	180	9.8824	435.66
4	200	5.6291	483.84
5	225	3.0681	542.34
6	250	1.6390	610.90
7	300	0.8668	690.24
8	325	0.4605	780.19
9	350	0.2466	881.55

The analyzed case study applies the above-described procedure to the robust design of the network assuming three uncertainty scenarios: (1) only the nodal demands are assumed to be uncertain; (2) and (3) both the nodal demands and roughness are considered uncertain, with different assumed pipe roughness uncertainty. In Case (1), the β symmetric, β extreme, β uniform, and Gaussian PDF are used and compared. In Cases (2) and (3), the only PDF considered for nodal demands is the β symmetric PDF, whereas the β decreasing PDF is employed for pipe roughness. The scenarios considered different values of variable range *rg* (i.e., 20, 40, 60, 80, and 100% for nodal demands and 20 and 40% for pipe roughness) with respect to the average assumed values for nodal demands and to the initial values for roughness (i.e., values for new pipe), implying different bounded domains for the definition of any PDF. It is worth noting that some authors (Xu and Goulter 1999; Leonard et al. 2002; Tolson et al. 2004) have already used this kind of approach in order to simulate uncertainty in future demands and roughness. The ranges of the variables employed in this paper are reasonably higher (demands) or comparable (roughness) with those reported in literature.

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Table 3. Results of the Robust Design Optimization Assuming the Nodal Demand and the Pipe Roughness Uncertainty

Case study	PDF	Economic cost of the network layout (increasing cost percentage with respect to the deterministic solution)				
		20%	40%	60%	80%	100%
Case (1) (nodal demand)	Gaussian	€ 7,035,800 (1.21%)	€ 7,120,100 (2.42%)	€ 7,193,900 (3.48%)	€ 7,309,700 (5.15%)	€ 7,402,800 (6.49%)
	β extreme	€ 7,014,800 (0.91%)	€ 7,095,100 (2.06%)	€ 7,186,900 (3.38%)	€ 7,289,000 (4.85%)	€ 7,396,400 (6.40%)
	β uniform	€ 7,049,400 (1.41%)	€ 7,125,700 (2.50%)	€ 7,188,200 (3.40%)	€ 7,308,100 (5.13%)	€ 7,409,200 (6.58%)
	β symmetric	€ 7,003,200 (0.74%)	€ 7,069,600 (1.70%)	€ 7,161,900 (3.02%)	€ 7,233,000 (4.05%)	€ 7,323,200 (5.34%)
Case (2) (demand and roughness 20%)	β symmetric β decreasing	€ 7,177,300 (3.25%)	€ 7,236,100 (4.09%)	€ 7,375,000 (6.09%)	€ 7,421,400 (6.76%)	€ 7,584,600 (9.10%)
Case (3) (demand and roughness 40%)	β symmetric β decreasing	€ 7,322,900 (5.34%)	€ 7,381,200 (6.18%)	€ 7,443,100 (7.07%)	€ 7,556,000 (8.69%)	€ 7,696,900 (10.72%)

Note: The best deterministic solution (that fully satisfies the network pressure requirement) has a design cost of 6,951,600 €.

Results and Discussion

The numerical results of the scenarios in terms of economic costs for different variable ranges and PDFs are reported in Table 3. However, it contains only the network costs for the solutions (i.e., diameter configurations) with the highest assumed value of network robustness (i.e., 90%). This means that solutions reported in Table 3 satisfy the pressure requirement at the critical node with a probability of 90% (i.e., the confidence limit of the design constraint in Eq. (10), assuming that the computed nodal heads vary according to a normal PDF). Table 3 also reports (in parentheses) the increasing cost (expressed as a percentage) of robust solutions with respect to the best deterministic solution, which fully satisfies the network pressure requirement in Eq. (3). The total design cost of this solution is € 6,951,600 and it serves to emphasize the difference between the above-described deterministic solution and the most robust solution for any differently analyzed case, whereas any intermediate comparison among deterministic and robust network configurations can be made according to the specific economic needs of the user.

Looking at Case (1) in Table 3, one can notice that all the analyzed PDFs used for simulation of nodal demand uncertainty retrieve similar results in terms of final costs for each of the considered uncertainty categories. This could be interpreted as a general consistency of the proposed procedure even if, within the limits of assumed small network (if compared to other real urban networks), results obtained by different PDFs cannot be read as an indication of suitability for any particular distribution. However, according to what was explained earlier, the β PDFs still remain preferable, if looking at the procedure from a sampling-efficiency standpoint.

For Cases (2) and (3) the writers have assumed the β decreasing PDF for roughness uncertainty, using only two values for roughness variability [i.e., 20% for Case (2) and 40% for Case (3)] because large variability ranges can lead to physically infeasible pipes roughness (Giustolisi et al. 2005). For a variable range of 20% (40%) the extreme value of pipe roughness becomes 1.2 (1.4) times the assumed value (i.e., roughness for new pipes, as reported in the literature). Thus, the PDF shapes for Cases (2) and (3) could be representative of the state of network deterioration after many years. Considering the results in Table 3, what was observed for Case (1) can be reasonably confirmed for Cases (2) and (3).

Now, comparing the best deterministic solution (i.e., that used as a reference for the evaluation of results in Table 3), and the best robust solution in the most extreme case [i.e., Case (3) with a range of 100% for water demands and a range of 40% for pipe roughness], it is interesting to observe what difference exists in terms of diameter changes due to the robust design procedure. Looking at Table 4 and bearing in mind the list of the diameters in Table 2, one notices that 11 pipes (33% of all network pipes) have increased in diameter. By analyzing both the network layout in Fig. 5 and Table 4, the solution returned by the automatic procedure reveals the presence of three essential pathways along which the pipe diameters must be increased in order to improve the performance of certain critical nodes, which are basically those situated at the periphery of the network (i.e., Nodes 12, 13, 17, and 20), with respect to the future variation of uncertain parameters. Moreover, the escalating cost of such robust solutions relative to the best deterministic result should be underscored. In the case study, the increase in financial cost is about 734,400 euros (11%), a relatively modest sum considering the appreciable increase in network robustness that is experienced. This specific result could arise from the small extent of the analyzed network and of the assumed uncertainty, but could be more significant for a larger and more complex system.

Taking into account the results in Table 4, especially the modest increases in diameters associated with the robust solution, it seems that the MOGA design based on a least-cost approach (de-

Table 4. Comparison between Diameters of the Best Robust Solution and Deterministic Solution

Pipe number	Deterministic solution	Robust solution	Diameter increase
2	300	350	50
4	300	325	25
7	300	350	50
11	100	150	50
16	180	225	45
17	200	225	25
18	300	325	25
20	150	200	50
28	250	325	75
29	180	200	20
32	250	300	50

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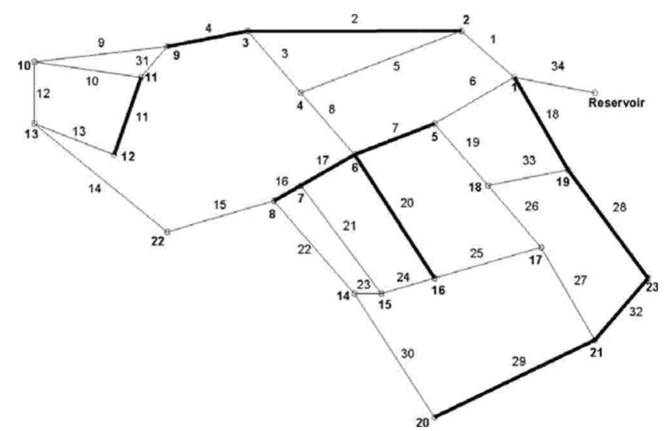

Fig. 5. Variations in the network diameters due to the robust design (pipes in bold have been changed)

terministic phase) is consistent and leads to quite reliable solutions, justifying the use of the deterministic Pareto front as the initial population for a subsequent robust analysis (filled circles in Fig. 6). By evaluating the optimal deterministic solutions subsumed within the robust approach that considers variable uncertainty, the solutions in Fig. 6 appear to be essentially robust network configurations, albeit with lower values of robustness (Fig. 6 reports the final Pareto front for Case (3)—range 100% and, for the sake of clarity, the *y* axis reports the values of 1-R instead of *R*). Moreover, the shadowed area between the dotted lines in Fig. 6 contains all the Pareto fronts obtained performing the optimization run on the same case study but starting from a completely random initial population, thus avoiding the dual-phase methodology. Looking at the Pareto fronts in Fig. 6, it seems clear that the proposed approach to stochastic design (by using deterministic design solutions as starting population) leads to a good optimal solution with respect to the optimization process. This observation supports use of deterministic solutions as starting populations for launching the robust design procedure instead of commencing from a randomly generated initial population.

Applying the optimal solution from a deterministic network design as the initial population for MOGA robust design, together with the improvements to the sampling process (i.e., Latin hyper-

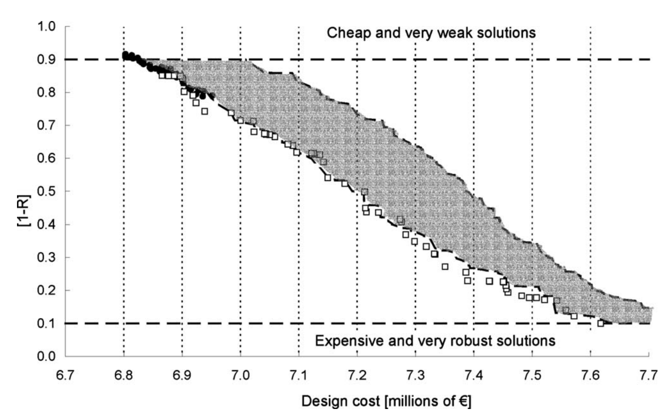

Fig. 6. The robust Pareto front (squares), the deterministic Pareto front/initial population (circles), and all the robust Pareto fronts obtained by performing many optimization runs with initial random population (shadowed area)

cube, use of bounded PDFs, etc.), leads to a more efficient design procedure that also proves to significantly reduce computational burden. In fact, the 1,000 generations executed during the deterministic phase took about 35,000 network calculations in almost 2 min (on a 0.9 GHz Intel Centrino PC), whereas the 200 generations performed for the robust phase involved roughly 4×10^5 network calculations, taking 15 min for the most laborious example [i.e., Case (3) using a wider range of nodal demands]. On the contrary, starting from a randomly generated initial population (that is, without using the deterministic solution as the starting point for MOGA), and performing the same number of generations, 2.3×10^7 network calculations were required. Therefore, the computational time increases by more than 70 min. Further, the near-optimal Pareto fronts obtained using different randomly generated initial populations fell into a large band (see Fig. 6), being strongly dependent on the starting point. Hence, the robust design using the deterministic sizing solutions as a departure point enjoys consistent results with respect to the optimization procedure's dependence on initial population and can reduce computational effort by about 98% in terms of the number of required network simulations.

A final observation is that a strict design constraint concerning service levels can render the WDS more robust with respect to unexpected variations in demand and even pipe roughness. Therefore, mostly for small networks, the robust design problem could be approached either in a deterministic scenario with an efficient MOGA optimizer and higher minimum pressure head requirements or by combining the GA search with an efficient MC simulation of the uncertain variables, as proposed here.

Conclusions

This paper proposes a refined approach to the robust design of WDS which consists of a sampling-based methodology implemented within a MO optimization environment. The technique can simulate nodal demand and pipe roughness uncertainty with any PDF function; the case study compared the traditionally adopted normal distribution with the β PDF family (symmetric, extreme, and uniform). The incorporation of LH admits a smaller amount of sampling, improving simulation efficiency with respect to traditional sampling-based methods. The optimization scheme is based on a MOGA optimizer (OPTIMOGA, Giustolisi et al. 2004), but any optimization tool can be used without significant modification to the basic approach. Unlike other similar methods (Kapelan et al. 2005), the proposed strategy does not link its computational savings (i.e., low number of samples and/or generations) to the particular way the optimization is performed. Its innovative aspect stems from the implementation of a double-step procedure: the design problem is first solved according to least-cost network design (deterministic phase) subject to a pressure constraint and returns a Pareto front of optimal solutions. The chosen solution then serves as an initial population for the robust design procedure, leading to the final Pareto front of optimal robust solutions in no more than 200 GA generations. This assumption is supported by the robustness of the deterministic optimal solutions (though with lower values of robustness), also leading to noticeable computational savings; the whole procedure (deterministic plus robust design) involves 1,200 generations, taking no more than 42,500 iterations.

Another key point is that the entire procedure permits the simultaneous realization of two major objectives: overall network robustness can be improved and the most important mains in

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terms of network reliability may be identified from the difference in the deterministic and stochastic solutions. Results illustrate the procedure's effectiveness in yielding information of practical engineering value. Despite this, further applications and refinements are warranted; in particular, implementation of other PDFs for simulating uncertain input variables and testing different nodal head distributions. In evaluating the promising results already yielded by using the β PDFs, and in wishing to exploit available knowledge about water demand behavior at specific nodes, it could be interesting to try different β PDFs for different nodes, bearing in mind that, if no specific knowledge is available, the β uniform PDF can work well. Further investigations could be performed for larger networks to further assess the methodology's computational requirements. It is also possible to expand the problem formulation to incorporate correlation among uncertain variables, aiming to reproduce what is widely assumed as technically reasonable. Moreover, additional/alternative objective functions, such as the maximization of average nodal robustness within the network or use of three or more objective functions, could be used in order to cope with scenarios in which there is no obvious single critical node but several that are easily prone to failure, and retain the procedure's structure. Thus, the approach could be a potentially reliable decision support tool in the right applications, furnishing decision makers with an optimized trade-off (Pareto optimal) curve between system design cost and robustness that exposes opportunities for realizing cost savings that only minimally impact robustness.

Notation

The following symbols are used in this paper:

$\bar{\mathbf{A}}_{pn}$ = general topological matrix;
$\mathbf{A}_{pn}, \mathbf{A}_{np}, \mathbf{A}_{p0}$ = topological incidence submatrices;
$\mathbf{A}_{pp}$ = diagonal matrix whose elements are $R_i|Q_i|^{n-1}$;
a, b = parameters of the beta function or beta probability density function;
D_i = diameter of the ith pipe of the network;
$\mathbf{H}$ = vector of total network heads;
$\mathbf{H}_0$ = vector of total fixed (i.e., known) network heads;
H_j = total head at the jth node of the network;
H_j^{avg} = average value of total head calculated at node j;
i = matrix index for pipes;
j = matrix index for nodes;
L_i = length of the ith pipe of the network;
m = mean value of beta probability density function;
n = head loss equation exponent and number of Latin hypercube samples;
n_n = total number of network nodes;
n_p = total number of network pipes;
n_0 = total number of nodes with known pressure head (reservoirs);
P^{min} = minimum assumed nodal pressure head (i.e., fixed level of service);
PDR = range of definition of the second objective function in the procedure's Phase 1;
$\mathrm{POP_{ini}}$ = number of individuals into the initial population for the MOGA optimization;
$\mathbf{Q}$ = vector of pipe flows;
Q_i = flow in the ith pipe of the network;
$\mathbf{q}$ = vector of nodal demands (i.e., varying according to some PDF);
q_j = nodal water demand at node j;
q_j^{unc} = the uncertain value of water demand at node j;
R = network robustness;
R_i = hydraulic resistance of ith pipe;
R_i^{unc} = uncertain value of hydraulic resistance of ith pipe;
rg = range of the beta PDF for uncertainty simulation;
S_{ini} = initial threshold for the number of samples for the uncertainty simulation;
S_{max} = maximum threshold for the number of samples for the uncertainty simulation;
S_{min} = minimum threshold for the number of samples for the uncertainty simulation;
x = independent variable of beta probability density function;
Z_j = elevation of the jth node of the network above datum;
α_j = coefficient related to the confidence limit of Eq. (10), assuming a Gaussian PDF;
σ = standard deviation value of probability density functions; and
0 = zero matrix.

Operators

$(\cdot)^T$ = vector/matrix transpose operator;
$\mathrm{beta}(a,b)$ = beta function;
$C(\cdot)$ = adopted cost function for network design;
$N(m,\sigma)$ = normal function having mean m and standard deviation σ; and
$\beta(a,b)$ = beta probability density function.

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