Technique for Calibrating Network Models

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Technique for Calibrating Network Models

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Abstract: In calibrating a water distribution system model, the model user usually adjusts pipe roughness (e.g., Hazen-Williams C factor) or water use so that pressures and flows predicted by the model agree with values observed in the field. This paper presents formulas to assist the user in deciding whether to adjust C or water use and by how much. The key to using the formulas is to observe pressures in the system for at least two significantly different use rates. Such data are often collected during fire flow tests. A model is considered calibrated to the extent that it can predict the behavior of the water distribution system over a wide range of operating conditions and water use.

Introduction

A very important step in the development of a water distribution network model is the comparison of results predicted by the model with observations taken in the field. If the input for the model is correct, then predicted pressures and flows will match observed values. However, the data initially used to describe the network are usually not perfect, so some values must be changed in order for the predicted and observed results to agree. The question the model user must answer is, therefore: which values need to be changed and by how much?

To correct for inaccuracies in input data it is necessary to first understand the sources of these inaccuracies. These can be grouped into several categories: (1) Incorrect estimate of water use; (2) incorrect pipe carrying capacity; (3) incorrect head at constant head points (i.e., pumps, tanks, pressure reducing valves); or (4) poor representation of system in model (e.g., too many pipes removed in skeletizing the system).

In most cases it is possible to accurately determine the elevation of water in a tank or pressure at a pump during the time calibration data were collected, and unless major problems exist with the results caused by skeletizing the system, it is not standard practice to significantly change the network to be modeled by inserting additional pipes. Since it is almost impossible to accurately measure water use and pipe carrying capacity, these parameters are usually not modified to make model results agree with field observations. (Note that in this paper, the Hazen-Williams *C* factor is used to represent carrying capacity, although pipe roughness or Manning's *n* could also be used.)

If the model predicts too much head loss in a certain group of pipes, the user can either decrease the estimate of water use in that area or increase the *C* factor. The user is in a situation similar to a person attempting to adjust the color on a television set with two knobs. However, because of the effort and computer time involved in making a run,

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the user would like to correct the data with one step rather than by trial and error.

There are several schools of thought on how to calibrate models. The AWWA Research Committee on Water Distribution Systems (5) states that ". . . the major source of error in simulation of contemporary performance will be in the assumed loadings distributions and their variations." On the other hand, Eggener and Polkowski (2) state: "the weakest piece of input information is not the assumed loadings condition, but the pipe friction factor." Cesario (1) reported that data on pump lifts, valve positions, and pressure reducing valve settings are modified first, while loadings and pipe coefficients are the last variables to be changed.

How can a user tell which parameter to change? The answer is that if the user is only trying to make the model predict pressure under one operating condition, it does not matter. The user can always force the model to fit the observations for a single set of observations. If the wrong parameter is adjusted the model, however, will give poor results if compared to observations at a different flow rate (e.g., average flow versus fire flow), since the model was calibrated using compensating errors. The key to selecting the correct parameter to change is having field observations corresponding to more than one flow rate, while knowing pump pressures, tank elevations, and valve settings corresponding to that time.

In this article, the definition of calibration is presented and formulas for determining improved values of C and water use are developed and analyzed. The data required to use the formula are described and the formulas are applied to an example problem. Some practical aspects of calibration and implications of using the formulas are also analyzed.

Definition

Shamir and Howard (4) state that calibration "consists of determining the physical and operational characteristics of an existing system and determinimg the data [that] when input to the computer model will yield realistic results." The AWWA Research Committee on Distribution (5) used the word "verified" in place of "calibrated" but described a process of calibration: "System simulation is considered verified during preliminary analysis for design when calculated pressures are satisfactorily close to observed field gage readings for given field source send-out and storage conditions. If simulation is not satisfactory, the possibility of local aberrations, such as open boundary valves, is investigated. In the absence of other expected causative factors, the assumed local arterial network loads are adjusted until computed and observed field pressures are within reasonable agreement for various levels and extremes of demand, pumping, and storage." (Note that there should be agreement over a wide range of operating conditions. Problems occur, in part, because field observations are usually not made over a wide enough range of operating conditions.)

The following, more precise, definitions are proposed. *Calibration* of a water distribution model is a two step process consisting of: (1) Comparison of pressures and flows predicted with observed pressures and flows for known operating conditions (i.e., pump operation, tank levels, pressure reducing valve settings); and (2) adjustment of the input data for the model to improve agreement between observed and predicted valves. A model is considered *calibrated* for a set of operating conditions and water uses if it can predict flows and pressures with reasonable agreement. Calibration at one set of operating conditions and water use does not necessarily imply calibration in general, although confidence in the accuracy of results from the model should increase with an increase in the range of conditions for which the model is calibrated.

Reasonable Agreement

Quantifying what is meant by "reasonable agreement" is difficult since it depends on: (1) The quality of the pressure and elevation data used; and (2) the amount of effort the model user is willing to spend fine tuning the model. While a well calibrated pressure gage is accurate to ±3 ft (1 m) of pressure head, elevation data are of widely varying quality. In the worst case, it must be read from contour maps with 20 ft (6.1 m) intervals which, with even the best interpolations, is only accurate to ±7 ft (2.1 m). In most cases, better data on hydrant elevation are available so that static head can usually be determined to an accuracy of ±5 ft (±1.5 m) to ±10 ft (±3.1 m). [Velocity head is usually on the order of 1 ft (0.31 m) and is ignored in virtually all models of water distribution systems.] The problem is also complicated by the fact that tank levels, pump operation, and water use may change dramatically during the time the data are collected. This effect can be minimized by collecting data over a short period of time, taking a snapshot of the system, during which there are no changes in pump operation and during which tank levels are recorded continuously (or at least observed frequently). Data collected over several years without regard to pump operation or tank levels are useful only for the crudest verification of a network model. Systems with many wells and pressure reducing valves and a great deal of change in hydraulic grade line elevation across the system are generally much more difficult to calibrate than small systems with only one or two pumps or tanks, and no automatic valves.

Therefore, given a good data set, a model user should be able to achieve an average difference between predicted and observed head of ±5 ft (1.5 m) with a maximum difference of ±15 ft (5.0 m). With a poor data set, an average difference of ±10 ft (3.1 m) with a maximum difference of ±30 ft (10 m) is a reasonable target.

An alternative way of looking at calibration accuracy is in terms of head loss. If the system only has 10 ft (3 m) of head loss from the source to the node with the lowest head in the system, then it should be easy to achieve an accuracy of ±5 ft (1.5 m). If there are several hundred feet of head loss, an accuracy of ±20 ft (6 m) may be quite good.

Development of Equations for Correcting C and Q

The following information may be obtained in conjunction with routine fire flow testing.

1. The hydraulic grade line elevations at a given node (the hydrant)

corresponding to some lower flow rate, Q (hydrant closed), and higher flow rate, $Q + Q_f$ (hydrant open). These elevations may be defined as h_1 and h_2, respectively.

2. The flow at the hydrant, Q_f, during the flow test.
3. The hydraulic grade line elevation, H, at some nearly constant head location (e.g., pump, tank, pressure reducing valve).

Pertinent information that will generally be unknown includes: (1) The water use, Q, corresponding to h_1; and (2) the C factor.

For this situation it is possible to develop expressions to calculate the correct C factor for pipes serving a given area and the correct water use, Q for a given group of nodes in a network model. This development is presented in the following.

To develop a simple rule for calibration, represent a section of the system from a node with known head to the test hydrant as a single equivalent pipe. The head losses between the constant head point and the node at which the fire flow test was conducted can be expressed as

$$H_1 - h_1 = K_1\left(\frac{S}{C}\right)^{1.85} \quad \text{(1a)}$$

and $$H_2 - h_2 = K_2\left(\frac{S + Q_f}{C}\right)^{1.85} \quad \text{(1b)}$$

in which Q_f = difference in flow between high and low flow condition, in gallons per minute; H_1 = hydraulic grade line elevation at known head point for low flow, in feet; H_2 = hydraulic grade line elevation at known head point for high flow, in feet; h_1 = hydraulic grade line elevation at test node for low flow, in feet; h_2 = hydraulic grade line at elevation test node for high flow, in feet; K_1 = K for equivalent pipe for low flow; K_2 = K for equivalent pipe for high flow; S = water use at nodes significantly affecting hydrant test, in gallons per minute, $S = \Sigma_{i=1}^{m} Q_i$; and m = number of nodes affecting test.

There are four unknowns in the preceding equations, K_1, K_2, C, and S. K_1 and K_2 depend upon the diameters and lengths of the complicated piping network and are equal if there is no water use between the constant head point and the test hydrant. (Note that if the known head point is a tank, a pressure reducing valve that operates in the same node for high and low flow, or a pump with a flat head-characteristic curve, then it is acceptable to let $H_1 = H_2$.)

K_1 and K_2 can be estimated utilizing the user's initial estimates of C and Q (referred to as C_e and Q_e). Given the user's values of C_e and Q_e, the model can be used to predict the hydraulic grade line elevation as h_3 for flow Q_e, and h_4 for flow $Q_e + Q_f$. Expressions similar to Eq. 1 can be used to estimate K_1 and K_2 if the user's estimates, Q_e and C_e, are not too greatly in error:

$$K_1 = (H_1 - h_3)\left(\frac{C_e}{S_e}\right)^{1.85} \quad \text{(2a)}$$

$$K_2 = (H_2 - h_4)\left(\frac{C_e}{S_e + Q_f}\right)^{1.85} \quad \text{(2b)}$$

in which h_3 = model estimate of h_1 for C_e and S_e, in feet; h_4 = model estimate of h_2 for C_e and S_e, in feet; C_e = user's estimate of C; Q_e = user's estimate of Q; and $S_e = \Sigma_{i=1}^{m} Q_{ei}$.

Inserting the values of K_1 and K_2 into Eq. 1 and solving for S and C yields

$$S = \frac{Q_f}{\frac{b}{a}\left(1 + \frac{Q_f}{S_e}\right) - 1} = AS_e \qquad (3a)$$

$$C = \frac{Q_f C_e}{b(S_e + Q_f) - a\,S_e} = BC_e \qquad (3b)$$

$$\text{in which} \quad a = \left(\frac{H_1 - h_1}{H_1 - h_3}\right)^{0.54} \qquad (3c)$$

$$\text{and} \quad b = \left(\frac{H_2 - h_2}{H_2 - h_4}\right)^{0.54} \qquad (3d)$$

$$A = \frac{Q_f}{\frac{b}{a}(S_e + Q_f) - S_e} \qquad (3e)$$

$$B = \frac{Q_f}{b(S_e + Q_f) - aS_e} \qquad (3f)$$

Eqs. 3 can be used to calculate improved values of C and Q for calibration. The coefficients a and b are useful as indicators of the magnitude and the source of error in the initial estimates. (This is analyzed in more detail later in this paper.)

To adjust estimated use rates at individual nodes use

$$Q_i = AQ_{ei}, \quad i = 1, 2, \ldots m \qquad (4)$$

Similarly, the C-factor for many pipes in an area must be adjusted, so Eq. 3b is more correctly written as

$$C_j = BC_{ej}, \quad j = 1, 2, \ldots n \qquad (5)$$

in which n = number of pipes affected by tests.

The parameters, A and B, are actually correction factors for the use and C factors, respectively. As such, an A of 1.15 means that water use should be increased by 15% over the initial estimate, while a B of 0.8 means that the C factor in the affected pipes should be reduced by 20%.

Application

The technique described earlier works best when applied to pie shaped sectors emanating from the known head node. Where the water use changes dramatically (about an order of magnitude) along one of these sectors, it is necessary to subdivide into two or more tiers and solve each tier successively from the source as a known head node. The downstream end of the first tier would become the known head node for the second, and so on.

Ideally, the data required to use Eqs. 3 already exist in the files of the water utility. Q_f, h_1, and h_2 are routinely evaluated as part of fire flow tests. Utilities with even the crudest instrumentation record water elevation in tanks or pressure at pumps, so determining H should be relatively easy. In rare instances (1), telemetry data on the entire system are available in sufficient detail for calibration.

Usually not all of the data required are available so some fire flow tests should be conducted (3). The following should be kept in mind when collecting the data: (1) Conduct the tests on the perimeter of the skeletal distribution system (i.e., not near source or on lines not included in the model); and (2) use as large a test flow at the fire hydrant as possible.

The preceding rules are necessary to insure that $H_1 - h_1$ and $H_2 - h_2$ are large. If they are not, then small errors in determining h_1 and h_2 can result in large errors in Q and C. It will also be shown in the section titled "Analysis" that if Q_f is much smaller than S, it is difficult to determine the source of error (i.e., C_e or Q_e) in the initial estimate, and errors in measuring h_1 and h_2 are accentuated.

The method for calculating C and $\acute{Q}$ developed in this paper is based on the assumption that Q_f is known. If the difference in water use between high and low flow conditions is not known, then Eqs. 3 cannot be used. The beauty of these equations lies in the fact that the data required for calibration can be collected in a single test.

Application to Network Example

Consider the skeletal network shown in Fig. 1, with the exact values for diameter, length, C, and water use given in the figure and Tables 1–3.

Suppose a model user decides to model the pipes using a C factor of 115, based on the age and type of pipe, and the user incorrectly estimates water use at nodes 20 and 30 as 100 (0.0063), and 400 (0.0252) gpm (m^3/s). If the user knows the hydraulic grade line (HGL) elevation at the tank to be 200 ft (60.9) m), the model predicts the HGL elevation as shown in row 2 of Table 1. (The measured HGL elevations based on

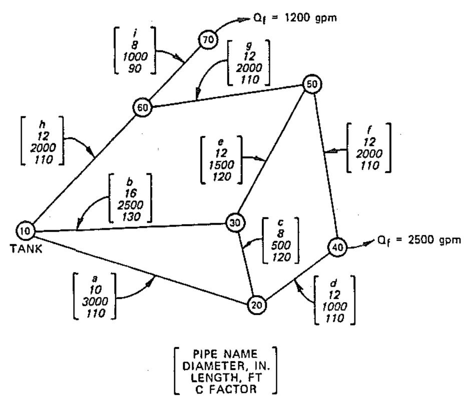

FIG. 1.—Example of Water Distribution System (gpm × 6.309 × 10^{-5} = m^3/s; in. × 25.40 = mm; and ft × 0.3048 = m)

TABLE 1.—Hydraulic Grade Line (HGL) Data for Network Example, in Feet

Water use (1)	Node 40		Node 70	
	Low flow HGL (2)	High flow (2,500 gpm) HGL (3)	Low flow HGL (4)	High flow (1,200 gpm) HGL (5)
Actual readings	181	150	173	64
Initial run	189	162	184	123
Corrected run	180	147	171	63

Note: ft × 0.3048 = m; gpm × 6.309 × 10^{-5} = m^3/s.

TABLE 2.—*C*-Factors for Network Example

Pipe (1)	Actual (2)	Initial (3)	Corrected (4)
a	100	115	115
b	130	115	115
c	120	115	115
d	110	115	115
e	120	115	115
f	110	115	115
g	110	115	115
h	110	115	83
i	90	115	83

TABLE 3.—Water Use for Network Example, in Gallons per Minute

Node (1)	Actual (2)	Initial (3)	Corrected (4)
20	500	150	205
30	2,000	400	548
40	500	500	685
50	1,500	1,500	2,055
60	1,000	1,000	1,000
70	400	400	400
Total	5,900	3,950	4,893

Note: gpm × 6.309 × 10^{-5} = m^3/s.

the exact data are given in row 1). With the data in rows 1–2, it is possible to calculate a and b for nodes near node 40 and 70 as

$$a(40) = \left(\frac{200 - 181}{200 - 189}\right)^{0.54} = 1.34 \quad (6a)$$

$$a(70) = \left(\frac{200 - 173}{200 - 184}\right)^{0.54} = 1.33 \quad (6b)$$

$$b(40) = \left(\frac{200 - 150}{200 - 162}\right)^{0.54} = 1.16 \quad (6c)$$

$$b(70) = \left(\frac{200 - 64}{200 - 123}\right)^{0.54} = 1.36 \quad \text{(6d)}$$

For this problem, nodes 20, 30, 40, and 50, and pipes *a*, *b*, *c*, *d*, *e*, and *f* are to be adjusted using the results from node 40. The C factor for pipe g is not adjusted since it does not significantly affect either hydrant test. For node 40, Eqs. 3 give

$$\left.\begin{aligned} A &= \frac{2{,}500}{\frac{1.16}{1.34}(2{,}550 + 2{,}500) - 2{,}550} = 1.37 \\ S &= 150 + 400 + 500 + 1{,}500 = 2{,}550 \end{aligned}\right\} \quad \text{(7a)}$$

$$B = \frac{2{,}500}{1.16(2{,}550 + 2{,}500) - 1.34(2{,}550)} = 1.02 \quad \text{(7b)}$$

The water use at nodes 20, 30, 40, and 50 are multiplied by *A* and are listed in Table 3, but since *B* is approximately one the C factors are not modified.

Next, *A* and *B* are calculated for the nodes (60 and 70) and pipes (*h* and *i*) influencing pressures around test node 70:

$$A = 0.95 \quad \text{(8a)}$$

$$B = 0.72 \quad \text{(8b)}$$

Since *A* is near one, it may not be necessary to change water use in that area (those numbers are already estimated correctly). However, the C factor for pipes *h* and *i* are multiplied by 0.72.

When the corrected values for C and Q are input to the model, the predicted pressures (Table 1 row 3) are much closer to the observed pressures (row 1). In general, the values of Q and C have improved but individual values of Q (node 50) and C (pipe *h*) in some cases actually become less accurate. (The total use is more accurate even though use at some individual nodes may be less accurate.)

Analysis

The parameters, *a* and *b*, as defined in Eqs. 3 are useful dimensionless indicators of the error in head loss at low and high flow conditions, respectively. Values less than one indicate that the model overestimated head loss as compared to observed head loss, while values greater than one indicate the model underestimated head loss. The model can be considered calibrated when $a \sim 1 \sim b$.

Values of *a* and *b* should generally be on the order of one. For example, if $a = 2$ or 0.5, the difference between observed and predicted head at the node would be 2.6 times the head loss which would indicate a very large error. The proper interpretations of *a* and *b* are summarized in Table 4.

While *a* and *b* provide insight into the nature of the error, Eqs. 3 must be used to actually determine the corrected values for the C factor and water use, Q. To give the reader an appreciation for the effect of *a* and

TABLE 4.—Interpretation of *a* and *b*

Value (1)	*a* (2)	*b* (3)
<1	Too much head loss predicted at low flow	Too much head loss predicted at high flow
=1	Head loss correct at low flow	Head loss correct at high flow
>1	Too little head loss predicted at low flow	Too little head loss predicted at high flow

b on the difference between the initial and corrected values of C and Q, the percent difference in C (i.e., $|C - C_e|/C_e$) and Q (i.e., $|Q - Q_e|/Q_e$) were calculated for a large array of values for *a* and *b* while letting C_e = 120, $Q_f = Q_e = S$ = 1,000 gpm (0.063 m^3/s), for a one node system.

Fig. 2 shows the affect of *a* and *b* on C. Within the area titled "Accept C," the change in C is less than 10% from C_e. The dashed line corresponds to a combination of *a* and *b* for which $C = C_e$. In this case, it corresponds to the line $b = (a + 1)/2$ which was determined from Eq. 3*a* for $C = C_e$. The region titled "Infeasible" corresponds to combinations of *a* and *b* for which the denominator in Eqs. 3 is negative.

Fig. 3 shows the affect of *a* and *b* on Q. Within the area titled "Accept Q," Q is within 10% of Q_e. The dashed line corresponds to $Q = Q_e$ and in this case is the line representing $a = b$. For $b >> a$, the water use estimate should be reduced, and for $b << a$, the water use estimate should be increased.

Figs. 2–3 were combined to yield Fig. 4 which summarizes the corrections to be made to C_e and Q_e to calibrate the model. Those who believe in achieving calibration by adjusting C are working under the assumption that $a = b$. Those who only adjust Q are assuming implicitly that

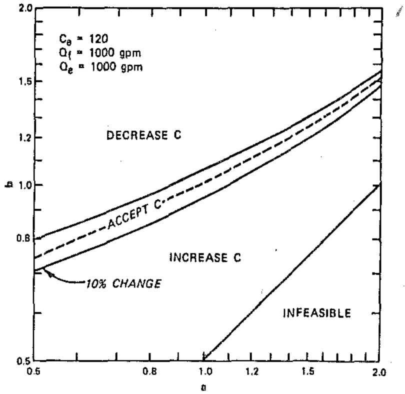

FIG. 2.—Effect of *a* and *b* on Hazen-Williams C (gpm × 6.309 × 10^{-5} = m^3/s)

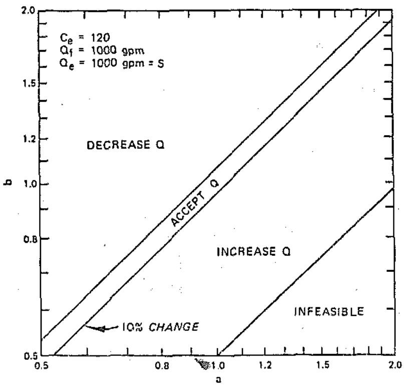

FIG. 3.—Effect of *a* and *b* on Water Use Estimate (gpm × 6.309 × 10^{-5} = m^3/s)

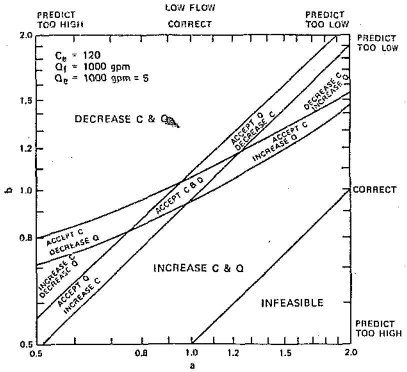

FIG. 4.—Summary of Corrections Required to Achieve Calibration (gpm × 6.309 × 10⁻⁵ = m³/s)

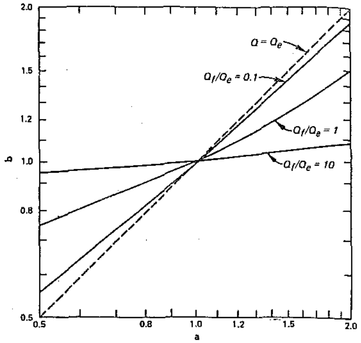

FIG. 5.—Effect of Q_f/Q_e on Lines Corresponding to $C = C_e$

$$b = \frac{aQ_e + Q_f}{Q_e + Q_f} \quad (9a)$$

or, for the data in Fig. 4:

$$b = \frac{(a + 1)}{2} \quad (9b)$$

There are a few special cases which provide insight into the adjustments needed for calibration:

1. If $a = b$ (similar error at high and low flow), adjust C.
2. If $a = 1$ and $b \neq 1$ (good calibration at low flow), adjust both Q and C by similar fraction.
3. If $a \neq 1$, $b = 1$, and $Q_f >> Q_e$ (good calibration at high flow, and low flow much less than high flow), adjust Q.

Statement 1 can be proven by setting $Q = Q_e$ in Eq. 3*a* and solving for b. Statement 2 can be proven by letting $a = 1$ in Eqs. 3 and showing that $(C - C_e)/C_e = (Q - Q_e)/Q_e$. Statement 3 can be proven by letting $C = C_e$ and $Q_e/Q_f = 0$ in Eq. 3*b* and solving for b.

The importance of insuring that Q_f is significant in comparison to Q_e (actually S when several nodes are affected) can be shown by plotting the line for $C = C_e$ for several values of Q_f/Q_e, as shown in Fig. 5. As Q_f/Q_e approaches zero, the line for $C = C_e$ approaches the $Q = Q_e$ line and it is not possible to determine if C or Q should be adjusted. In practical terms, as Q_f/Q_e becomes small, model results become increasingly sensitive to errors in measuring h_1 and h_2. Therefore, the accuracy of Eqs. 3 is greatest for large flows during fire flow tests.

Source of Errors in Initial Simulations

Ideally, the data used in the initial runs of a model are so accurate that there is no need to correct the input using Eqs. 3. In some cases, the corrections are small and are due to random error in estimating C or predicting water use at the time the calibration data were taken. When the corrections are large, the model user should not blindly change C or Q without trying to understand why the initial estimates, C_e and Q_e, were poor. Taking time to understand the source of the errors can provide valuable insights into the behavior of the water distribution system. This is shown in the following examples.

If the corrected C is much lower than the initial estimate (i.e., $C << C_e$), it is implied that the hydraulic carrying capacity of the mains is less than anticipated. While this may be due to an increase in pipe roughness with age, it may also be due to a closed or partially closed valve in a main. It is not uncommon during a modeling study to locate valves that have been mistakenly left closed.

A corrected C that is much greater than the initial estimate (i.e., $C >> C_e$) implies that the hydraulic carrying capacity of the mains is more than anticipated. This is usually due to not including important pipes in developing the skeletal model. This can be corrected by increasing C or including additional pipes in the model.

If the corrected water use is much less than the initial estimate (i.e., $Q << Q_e$), the model user should attempt to identify water users not in operation when the data were collected. For example, the field observations may have been made on a school holiday or workers at a factory near the test node may have been on strike that day.

If the corrected water use is much greater than the initial estimate (i.e., $Q >> Q_e$), the model user should look for unusual water uses. For example, was the municipal swimming pool being filled that day or was lawn watering use especially high because of a drought? The error may also be due to an illegal connection or a large main break.

Recommended Procedure

The recommended calibration procedure can be summarized as follows:

1. Prepare model data to the greatest accuracy possible.
2. Make initial run of model.
3. Measure H_1, H_2, h_1, h_2, and Q_f.
4. If error is acceptable, calibration is complete; if not, go to next step.
5. Calculate a and b.
6. Calculate corrected C and Q.
7. Rerun model.
8. If error is acceptable, calibration is complete.
9. Return to 5 (or if that is not successful, return to 3).

Summary

The formulas presented in this paper can greatly simplify the process of calibrating water distribution system models. The key to using the formulas is collecting pressure data for the system under both high and low water use conditions while recording pump and tank operation. This can be done by conducting fire flow tests.

In addition to the formulas for calculating C and Q, some qualitative guidance was developed for calibration. Given the results of the model for the initial estimates of C and Q:

1. Adjust C if there is similar error at high and low flow.
2. Adjust C and Q by similar amounts if the model is accurate at low flow but inaccurate at high flow.
3. Adjust Q if the model is accurate at high flow but inaccurate at low flow.

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

A = correction factor for water use, $Q_f/[(b/a)(S_e + Q_f) - S_e]$;
a = indicator of accuracy of calibration at low flow, $((H_1 - h_1)/(H_1 - h_3))^{0.54}$;
B = correction factor for C factor , $Q_f/[b(S_e + Q_f) - aS_e]$;
b = indicator of accuracy of calibration at high flow, $((H_2 - h_2)/(H_2 - h_4))^{0.54}$;
C = actual Hazen-Williams C factor;
C_e = initial estimate of C;
C_{ej} = initial estimate of C for pipe j;
C_j = correct value of C for pipe j;
H_1 = head at known head point at low flow, in feet;
H_2 = head at known head point at high flow, in feet;
h_1 = observed hydraulic grade line elevation at low flow, in feet;
h_2 = observed hydraulic grade line elevation at high flow, in feet;

h_3 = predicted hydraulic grade line elevation at low flow, in feet;
h_4 = predicted hydraulic grade line elevation at high flow, in feet;
K = head loss coefficient;
K_1 = value of K for low flow;
K_2 = value of K for high flow;
m = number of nodes to be corrected;
n = number of pipes to be corrected;
Q = actual water use, in gallons per minute;
Q_e = initial estimate of water use, in gallons per minute;
Q_{ei} = initial estimate of water use for node i, in gallons per minute;
Q_f = fire flow during hydrant test, in gallons per minute;
Q_i = actual water use at node i, in gallons per minute;
S = total water use in nodes affecting test, in gallons per minute = $\Sigma_{i=1}^{m} Q_i$; and
S_e = estimate of S, in gallons per minute.