AAiT

Addis Ababa
Institute of Technology
School of Civil and
Environmental Engineering

Water Distribution Modelling
Lecture By Fiseha Behulu (PhD)

Lecture-2:
Basic Principles of Pipe Flow (Hydraulics)

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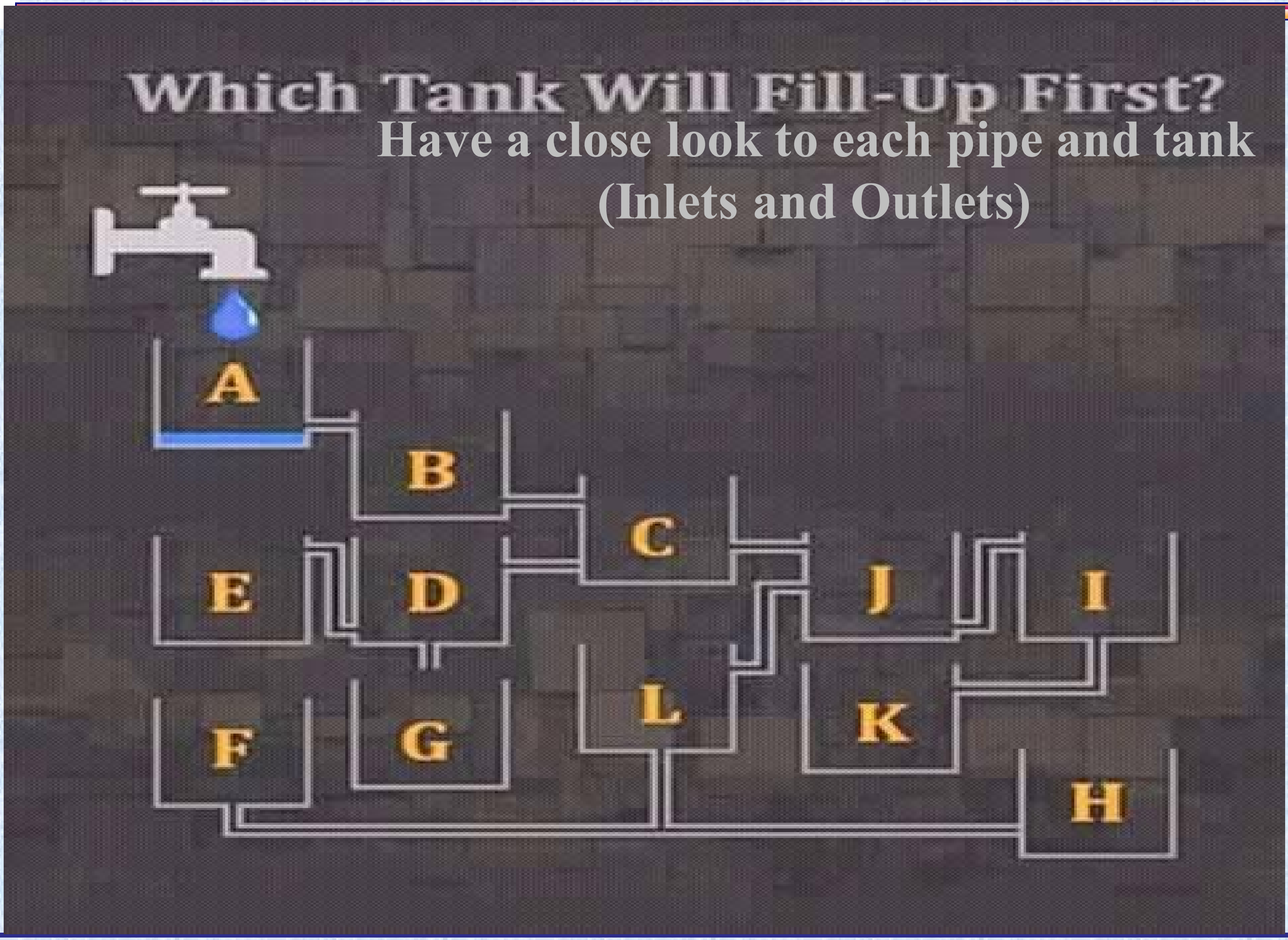
Contents of the Course

1. **Components of Water Supply**
2. **Basic Principles of Pipe Flow (Hydraulics)**
3. The Concept of Modeling
4. Model Calibration
5. Optimization in WDS
6. Water Hammer Theory
7. Water Supply Project Design (Application of Tools)

Basic Principles of Pipe Flow (Hydraulics)

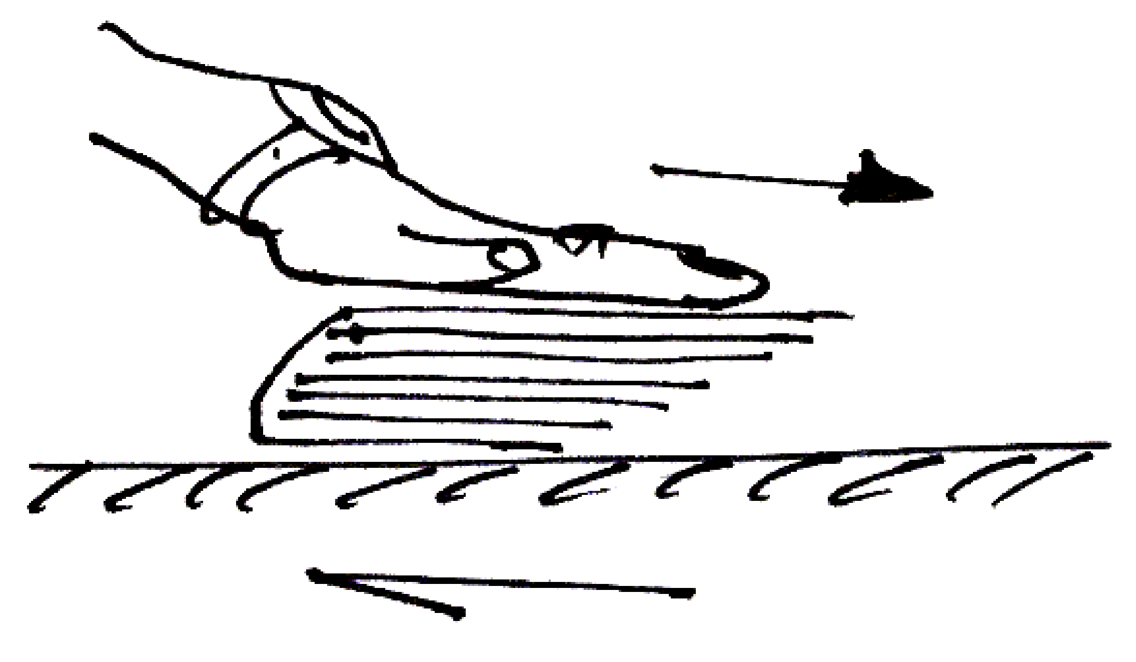
- Fluid Properties- Brief revision
- Fluid Statistics and Dynamics
- Energy Concept
- Friction Losses
- Minor Losses
- Network Hydraulics
- Water quality modeling

Brainstorming



Have a close look to each pipe and tank (Inlets and Outlets)

Properties of Fluid



A fluid is any substance that deforms continuously when subjected to shear stress, no matter how small the shear stress is.

The intermolecular cohesive forces are large in a solid, smaller in a liquid and extremely small in a gas.

Properties of Fluid

Quantity	Symbol	Dimensions
Density	ρ	ML^{-3}
Specific Weight	γ	$ML^{-2}T^{-2}$
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	$L^{2}T^{-1}$
Surface tension	σ	MT^{-2}
Bulk modules of elasticity	E	$ML^{-1}T^{-2}$

These are _fluid____ properties!

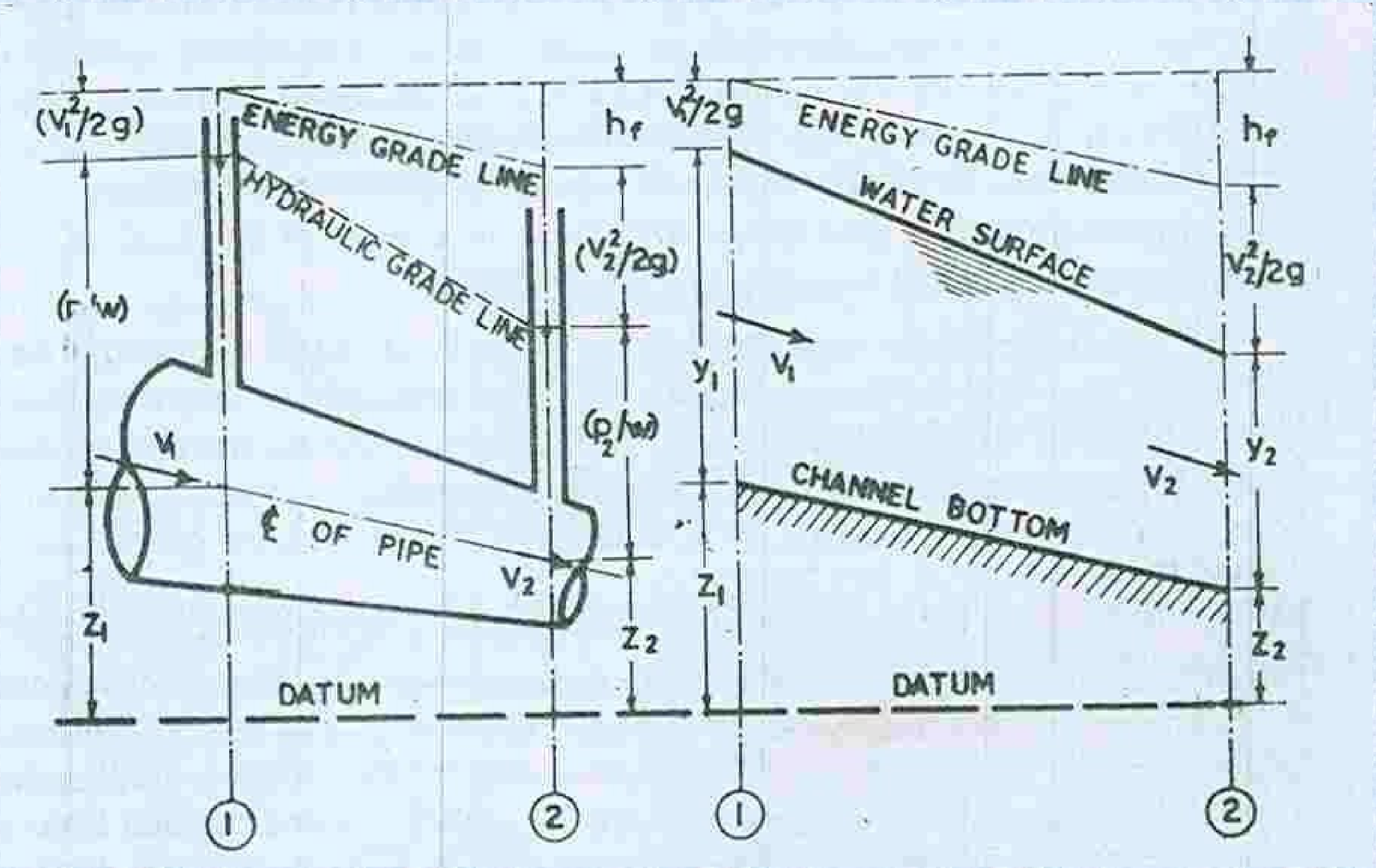
Please Refer your **Hydraulics** course from Undergraduate program

Pipe Flow Analysis

Objectives

- To understand laminar and turbulent flow in pipes and the analysis of fully developed flow
- Able to calculate the major and minor losses associated with pipe flow
- In order calculate and design the sizes of the pipes

Introduction



Comparison of open channel flow and pipe flow

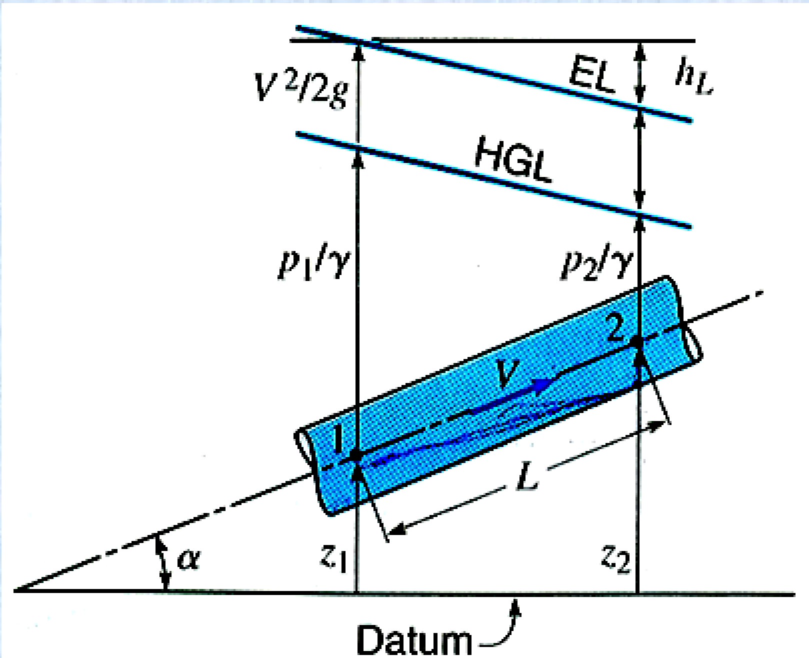
Introduction

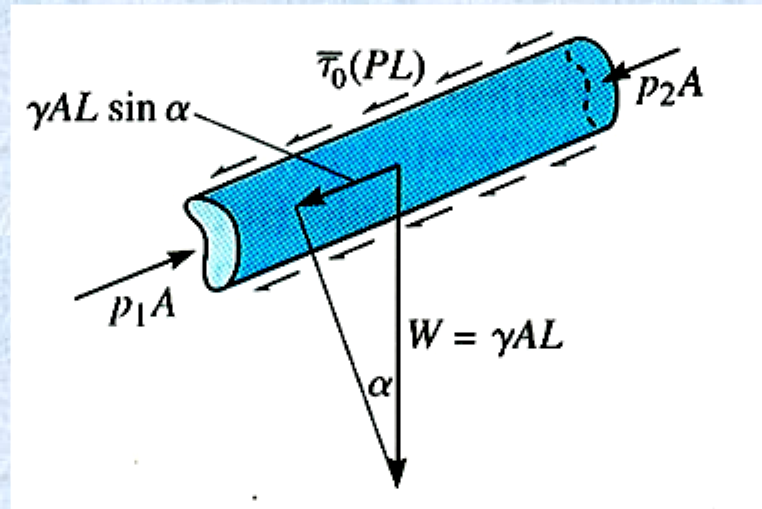
- Water is conveyed from its source, normally in **pressure pipelines**, to water treatment plants where it enters the distribution system and finally arrives at the consumer. *In addition oil, gas, irrigation water, sewerage can be conveyed by pipeline system.*

- The effect of friction is to decrease the pressure, causing a pressure *'loss'* compared to the ideal, frictionless flow case.

- The loss will be divided into ***major losses*** (due to friction in fully developed flow in constant area portions of the system) & ***minor losses*** (due to flow through valves, elbow fittings & frictional effects in other non-constant –area portions of the system).

Major loss (friction) in pipes





$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$p_1 A - p_2 A - \gamma AL \sin\alpha - \bar{\tau}_o PL = 0$$

$$\sin\alpha = \frac{(z_2 - z_1)}{L}$$

$$h_L = \bar{\tau}_o \frac{PL}{\gamma A} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

Friction in Circular Conduits (Pipe) flowing full

$$h_L = C_f 4 \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{V^2}{2g}$$

- for both smooth-walled and rough walled conduits. It is known as ***pipe –friction equation***, and commonly referred to as the ***Darcy-Weisbach*** equation
- Friction factor, *f*, is dimensionless and is also some function of Reynolds number

Pipe friction equations

- Darcy's Weisbach Equation

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \qquad \therefore\ f = \phi_1\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

- Hazen William Equation

$$h_f = \frac{10.675L}{D^{4.8704}}\left(\frac{Q}{C}\right)^{1.852} \quad \text{SI units}$$

- Chezy's Equation
- Manning's Equation

Reynolds Number

- Pipe flow regimes depends on the following factors:
  - *geometry,*
  - *surface roughness,*
  - *flow velocity,*
  - *surface temperature,* and *type of fluid,* among other things.

- After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the **ratio of *inertial forces* to *viscous forces*** in the fluid. This ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as

Laminar and Turbulent Flow

- ***Laminar flow* - in laminar flow the particles of **fluid move in an orderly manner & the steam lines** retain the same relative position in successive cross section. Laminar flow is associated with low velocity of flow and viscous fluids.

- ***Turbulent flow* - Here the fluid particles flow in a **disorder manner occupying different relative positions in successive cross section**. Turbulent flow is associated with high velocity flows.

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\mathrm{avg}} D}{\nu} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$

$$\begin{aligned} \mathrm{Re} &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim \mathrm{Re} &\lesssim 4000 && \text{transitional flow} \\ \mathrm{Re} &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

Pipe friction equations

1. For laminar flow type $f = 64\frac{v}{DV} = \frac{64}{\mathrm{Re}}$

2. For Transition flow type $f = \{-2\log_{10}[\frac{(\varepsilon/D)}{3.7} + \frac{2.51}{\mathrm{Re}(f^{1/2})}]\}^{-2}$

3. For *hydraulically turbulent smooth pipes (e= 0)* such as glass, copper,

$$f = \frac{0.3164}{\mathrm{Re}^{0.25}} \quad (4{,}000 < Re < 100{,}000)$$

Blasius equation

$$\frac{1}{\sqrt{f}} = 2\log\left(\mathrm{Re}/\sqrt{f}\right) - 0.8$$

Von Karman's and Prandtl equation for Re upto $3*10^6$

4. For Complete turbulence rough pipe flow type $f = [1.14 + 2\log_{10}(\frac{D}{\varepsilon})]^{-2}$

Pipe friction equations

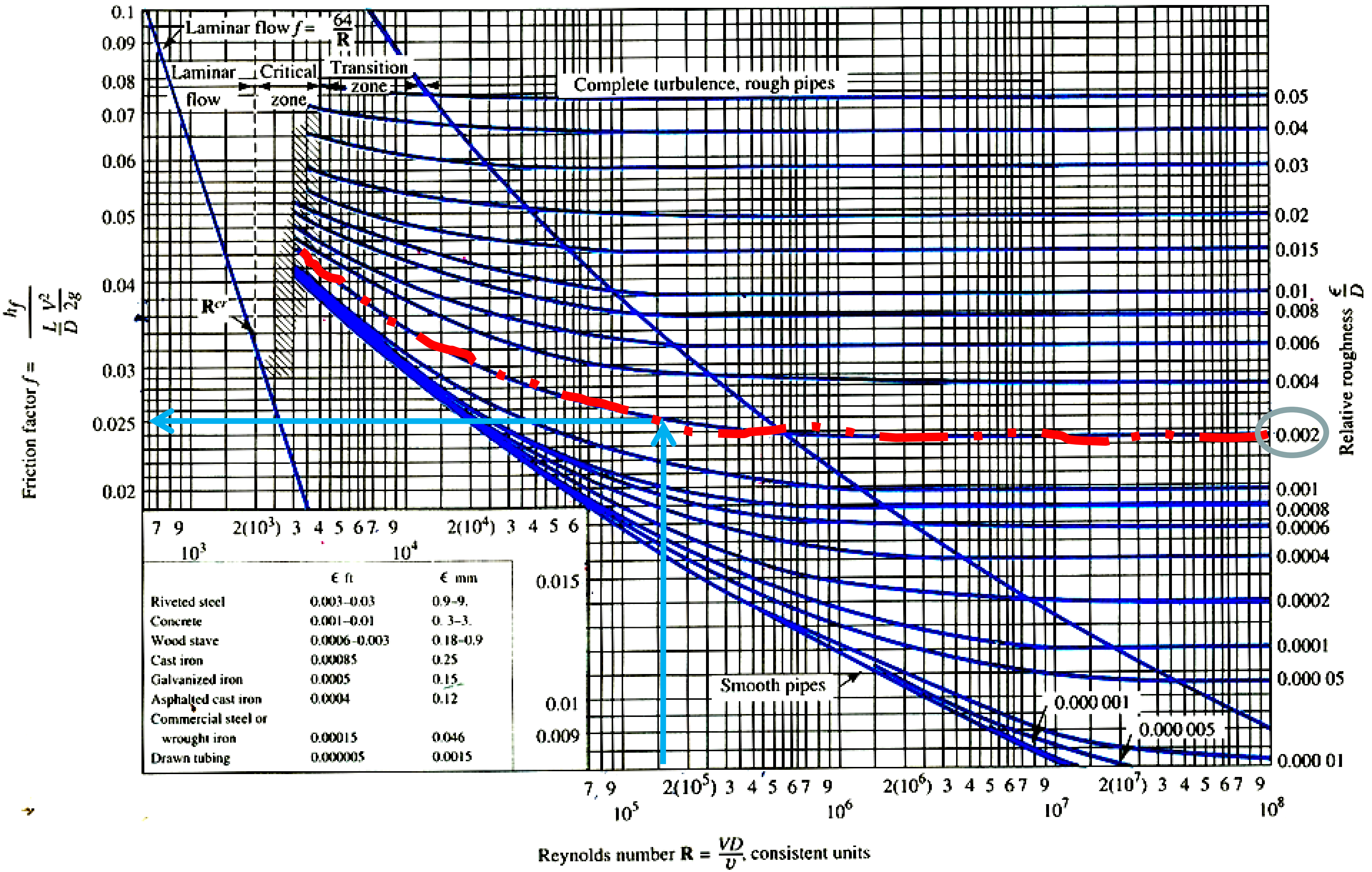
For all pipes, a general empirical formula by Colebrook - White is given by:

$$\frac{1}{\sqrt{f}} = -0.86 \ln\left(\frac{e}{3.71D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

The above equation is awkward to solve

- In 1944, Leewis F. Moody plotted the Darcy–Weisbach friction factor into what is now known as the Moody chart and diagrams are available to give the relation between f, Re, and e / D.

MOODY CHART


Laminar flow $f = \frac{64}{R}$
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes
Friction factor $f = \frac{h_f}{\frac{L}{D}\frac{V^2}{2g}}$
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
R_{cr}
Relative roughness $\frac{\epsilon}{D}$
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.000 05
0.000 01
0.000 001
0.000 005
Smooth pipes
7 9 10^3 $2(10^3)$ 3 4 5 6 7 9 10^4 $2(10^4)$ 3 4 5 6
7 9 10^5 $2(10^5)$ 3 4 5 6 7 9 10^6 $2(10^6)$ 3 4 5 6 7 9 10^7 $2(10^7)$ 3 4 5 6 7 9 10^8
Reynolds number $R = \frac{VD}{\upsilon}$, consistent units
€ ft | € mm
Riveted steel 0.003–0.03 0.9–9.
Concrete 0.001–0.01 0.3–3.
Wood stave 0.0006–0.003 0.18–0.9
Cast iron 0.00085 0.25
Galvanized iron 0.0005 0.15
Asphalted cast iron 0.0004 0.12
Commercial steel or wrought iron 0.00015 0.046
Drawn tubing 0.000005 0.0015

Pipe roughness

pipe material	pipe roughness ε (mm)
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18 - 0.6
rivet steel	0.9 - 9.0
corrugated metal	45
PVC	0.12

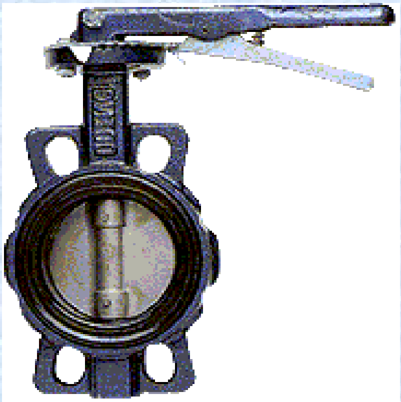
Questions

- Can the Darcy-Weisbach equation and Moody Diagram be used for fluids other than water? ___Yes___
- What about the Hazen-Williams equation? ___No___
- Does a perfectly smooth pipe have head loss? ___Yes___
- Is it possible to decrease the head loss in a pipe by installing a smooth liner? ___Yes___

Minor Losses in the Pipes

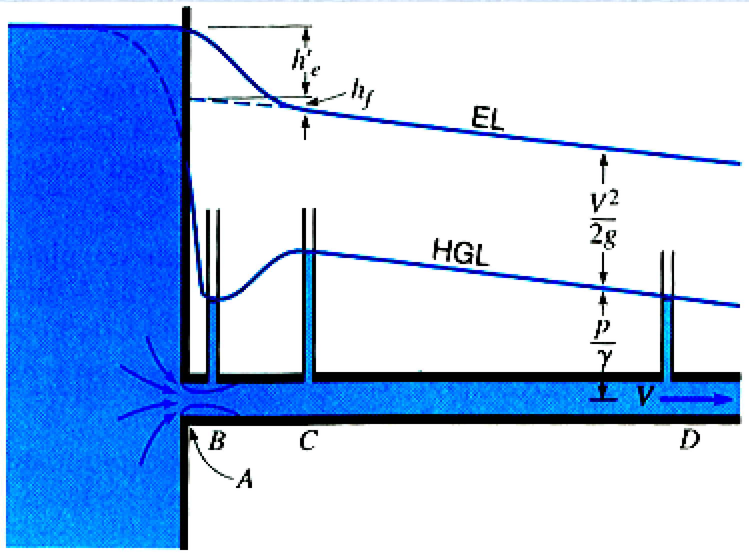
- Loss due to the local disturbances of the flow conduits such as changes in cross section, projecting gaskets, elbows, valves, and similar items are called ***minor Losses***.

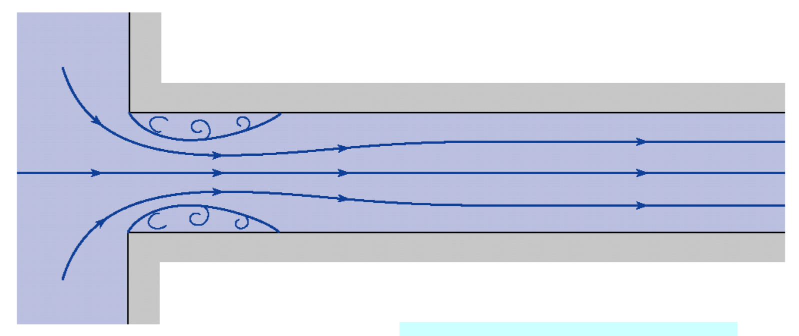
- In the case of a very long pipe or channel, these losses may be insignificant in comparison with the fluid friction in the length considered.



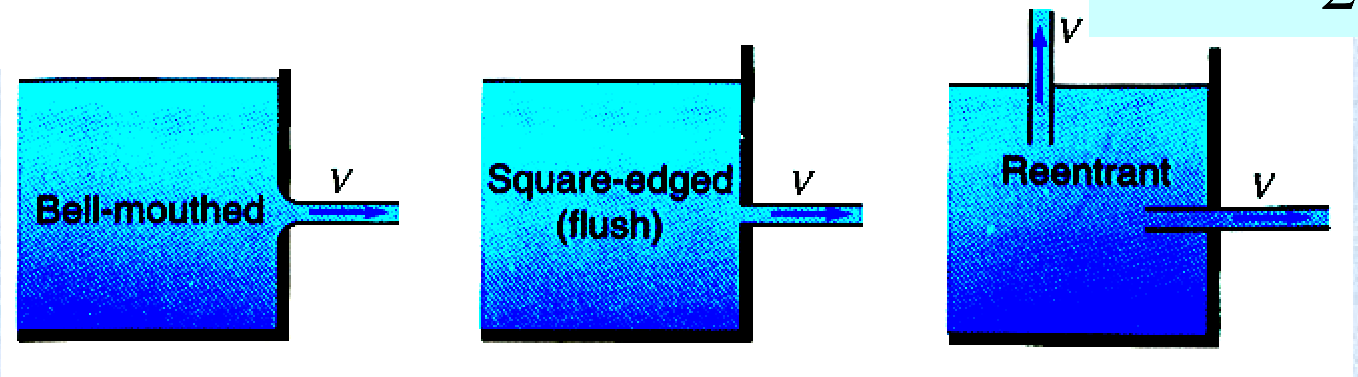
$$k\frac{V^2}{2g}=\frac{f(ND)}{D}\frac{V^2}{2g}$$

Loss of Head at Entrance





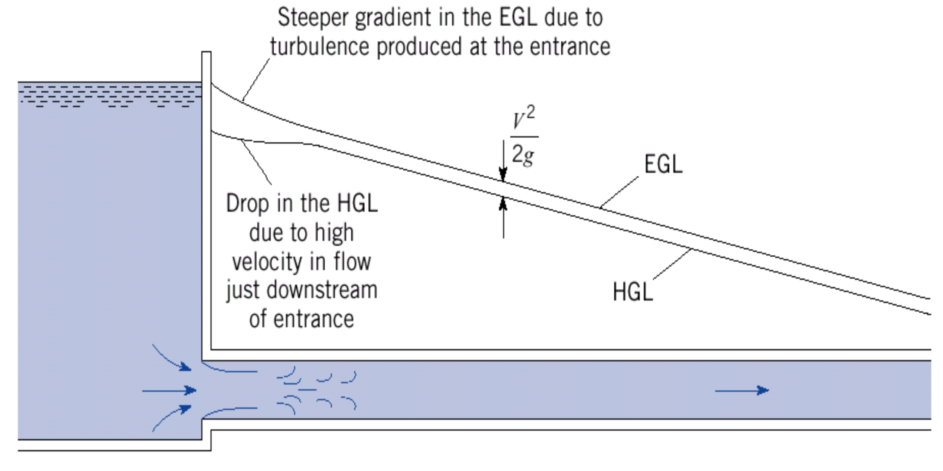
$$h_e' = k_e \frac{V^2}{2g}$$

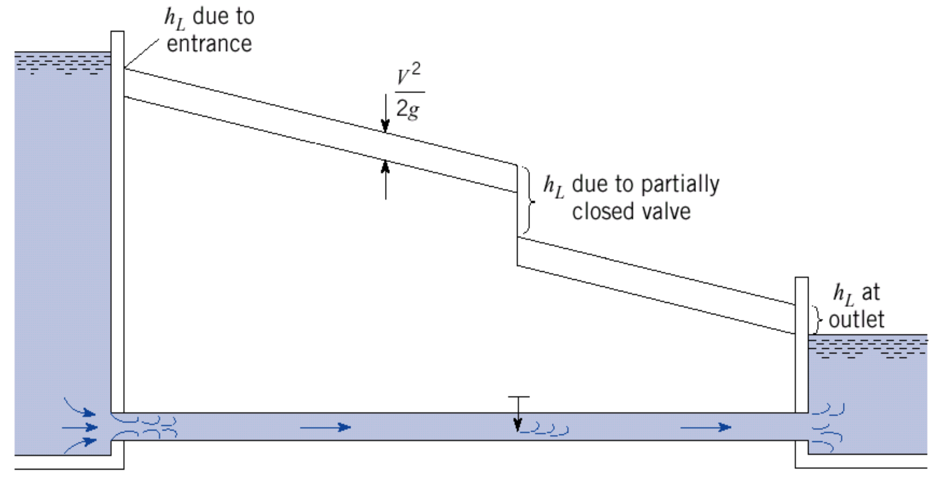


(a) $k_e = 0.04$ (b) $k_e = 0.5$ (c) $k_e \approx 0.8$

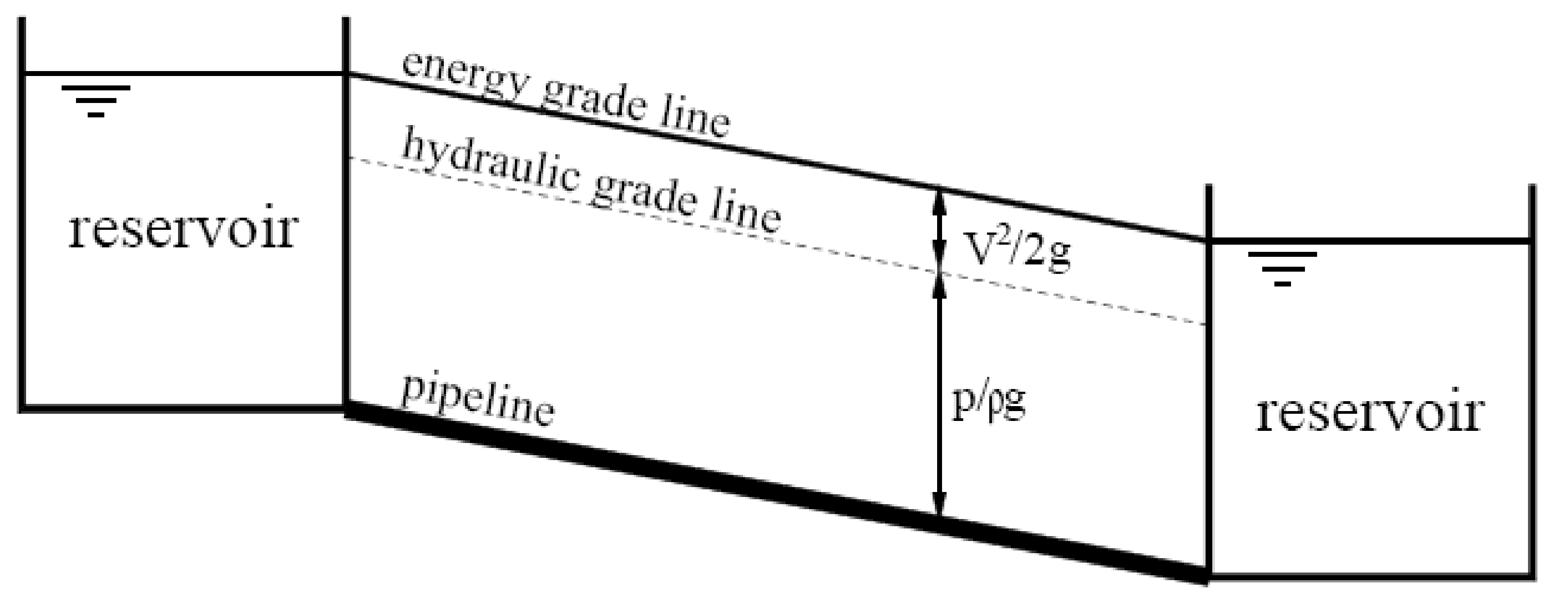
EGL & HGL for Losses in a Pipe

- Entrances, bends, and other flow transitions cause the EGL to drop an amount equal to the head loss produced by the transition.
- EGL is steeper at entrance than it is downstream of there where the slope is equal the frictional head loss in the pipe.
- The HGL also drops sharply downstream of an entrance

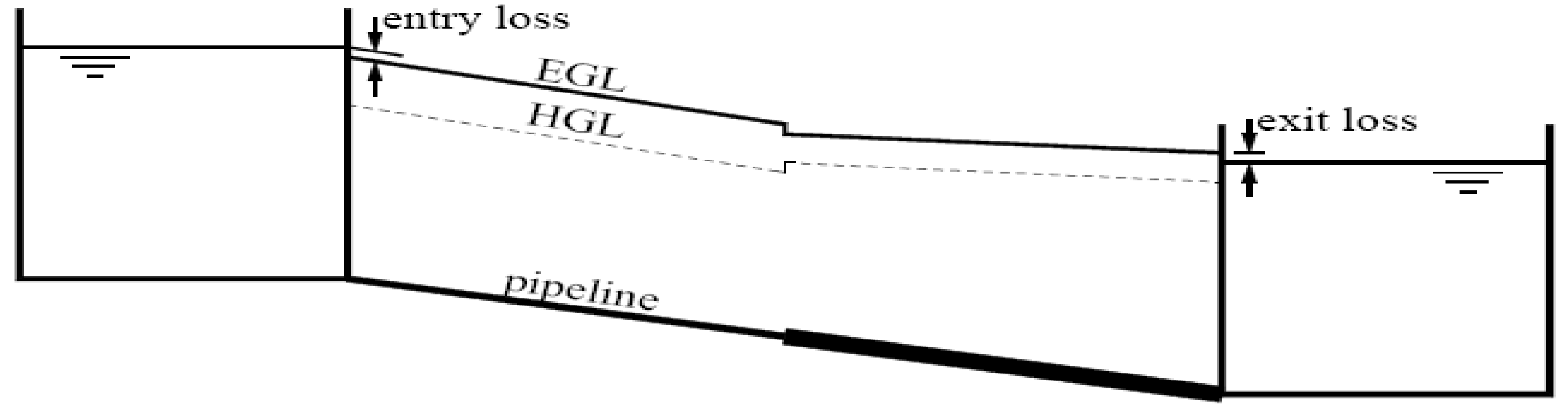


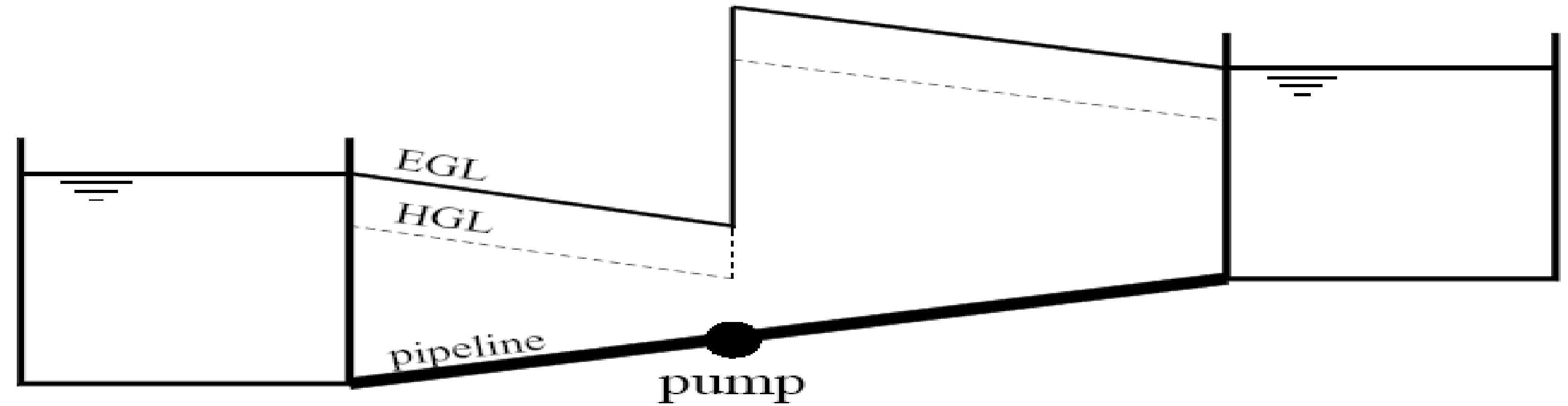


EGL & HGL for Losses in a Pipe

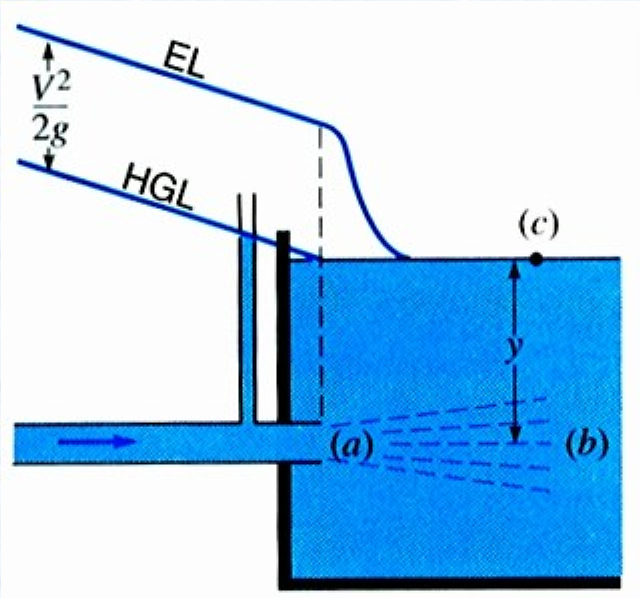


EGL & HGL for Losses in a Pipe





Loss of head at submerged discharges: (leave of pipe), ($h_{d'}$)



$$H_a = y + 0 + V^2/2g$$

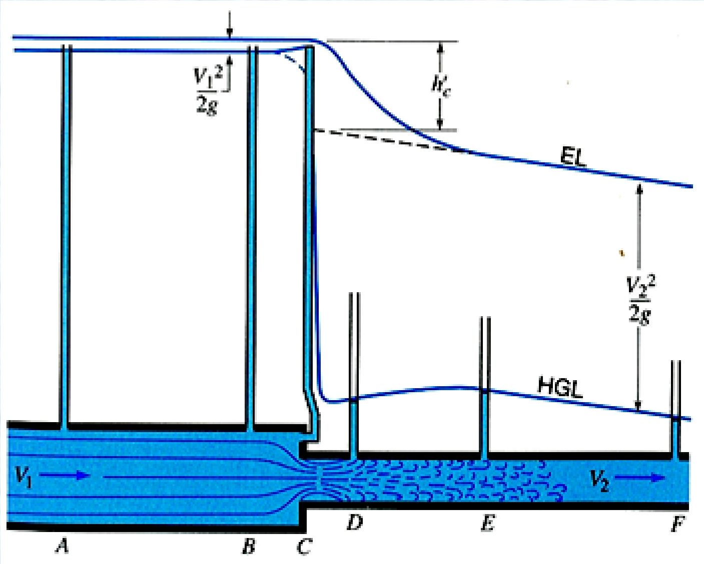
$$H_c = 0 + y + 0$$

$$h_d^{'} = H_a - H_c = \frac{V^2}{2g}$$

k_c

Loss Due to Contraction

- ***Sudden Contraction***



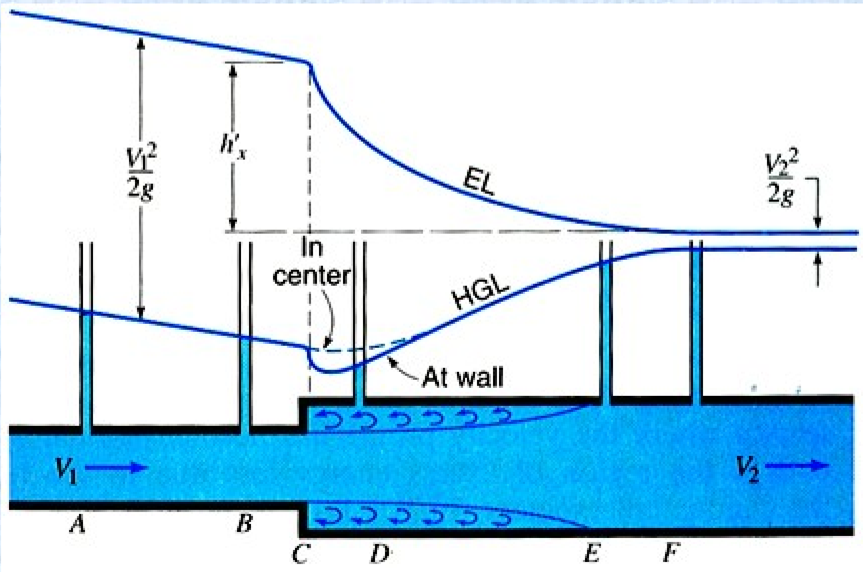
$$h_c' = k_C \frac{V_2^2}{2g}$$

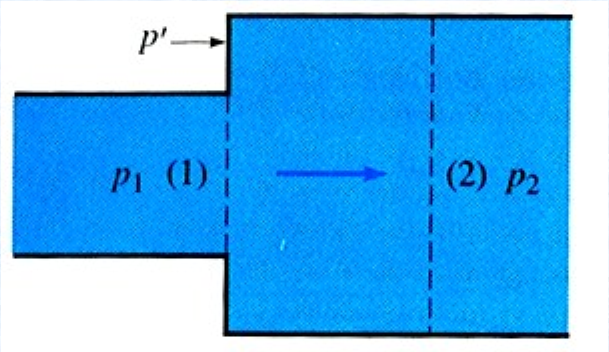
Losses due to gradual contraction the value of Kc = 0.05 – 0.10

Losses coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k_c	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

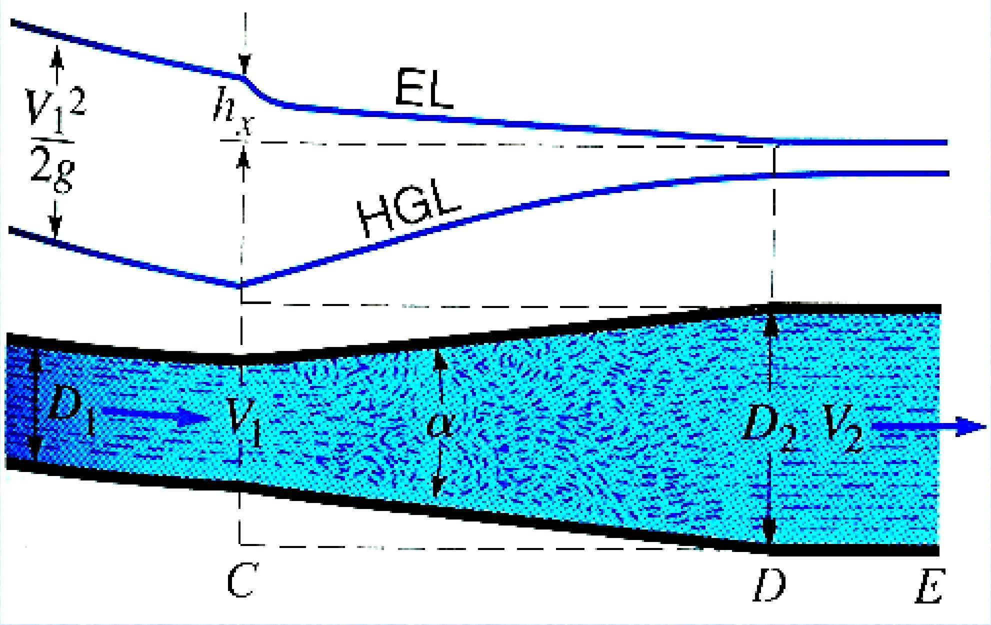
Loss due to sudden expansion





$$\therefore h_e = \frac{(V_1 - V_2)^2}{2g} = \left(\frac{D_2^2}{D_1^2} - 1\right)^2 \frac{V_2^2}{2g}$$

Gradual Expansion



$$h_e' = K' \frac{(V_1 - V_2)^2}{2g}$$

K'-is a function of cone angle α.

K'	0.4	0.6	0.95	1.1	1.18	1.09	1.0	1.0
α	20^0	30^0	40^0	50^0	60^0	90^0	120^0	180^0

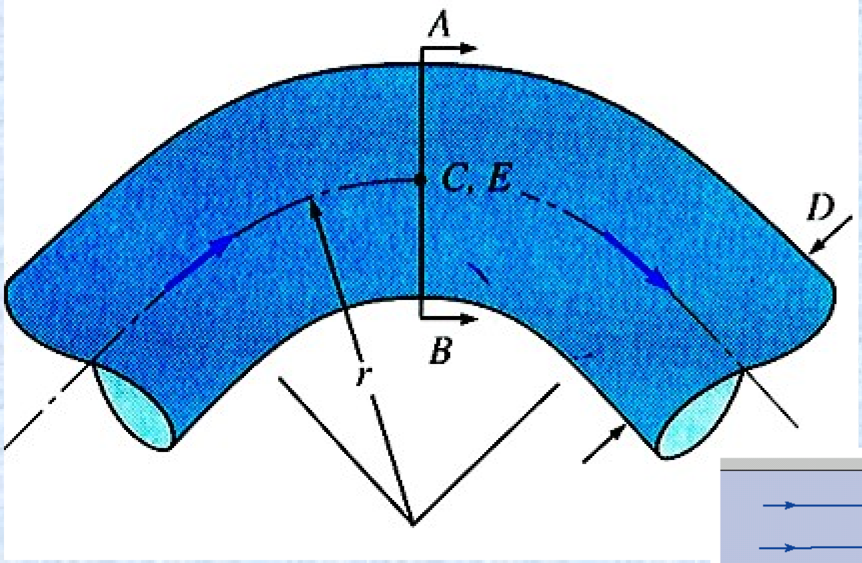
Loss in pipe fittings

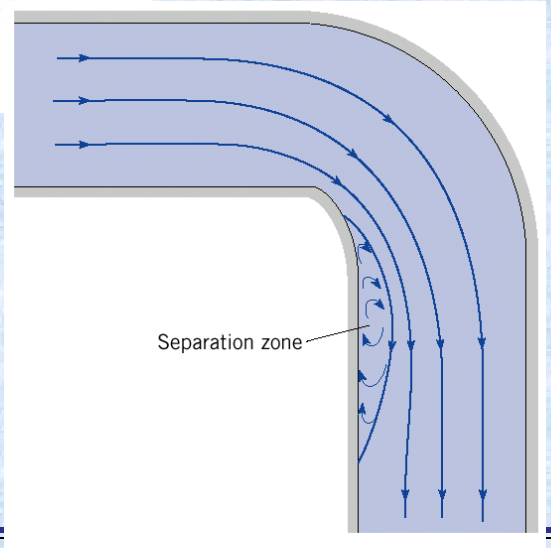
The values of "K_f" depends on the type of fittings

$$h_f = k_f \frac{V^2}{2g}$$

Fitting	K
Globe valve, wide open	10
Angle valve, wide open	5
Close –return bend	2.2
T-through side outlet	1.8
Short-radius elbow	0.9
Medium radius elbow	0.75
Long radius elbow	0.60
Gate valve, wide open	0.19
Half open	2.06
Pump foot value	5.60
Standard branch flow	1.80

Losses in bend & Elbow





$$h_b = k_b \frac{V^2}{2g}$$

Solution of single – pipe flow problems

The total head losses between two points is the sum of the pipe friction loss plus the minor losses, or

$$h_L = h_{Lf} + \sum h'$$

h_L – total head loss

h_{Lf} – major head loss $\quad h_{Lf} = f \frac{L}{D} \frac{V^2}{2g}$

$\sum h'$ - total minor loss

Pipe flow problems

- The above equation (h_L) relates four variables. Any one of these may be unknown quantity in practical flow situation. These are:

i.	L, Q, D	known	h_L	unknown
ii.	h_L, Q, D	known	L	unknown
iii.	h_L, Q, L,	known	D	unknown
iv.	h_L, L, D,	known	Q	unknown

Example

- A 100m length of smooth horizontal pipe is attached to a large reservoir. What depth, d, must be maintained in the reservoir to produce a volume flow rate of $0.03 m^3/sec$ of water? The inside diameter of the smooth pipe is 75mm. The inlet of the pipe is square edged. The water discharges to the atmosphere. Assume that density of the fluid is 1000kg/cubic meter and $\mu = 10^{-3} kg/m.s$

- **Solution**

$$\left(\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + Z_1\right) - \left(\frac{p_2}{\gamma} + \frac{v_2^2}{2g} + Z_2\right) = h_{LT}$$

- $h_{LT} = h_{Lf} + h_{Lm}$

$$h_{LT} = f\frac{L}{D}\frac{V^2}{2g} + k\frac{V^2}{2g}$$

- But $P_1 = P_2 = P_{atm}$, $V_1 \cong 0$, $V_2 = V$, $Z_2 = 0$ (measured from the center of the pipe line, then $Z_1 = d$.

$$h_{LT} = d - \frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g} + k\frac{V^2}{2g}$$

$$d = \frac{v^2}{2g}\left[f\frac{L}{D} + K + 1\right]$$

$$V_2 = V = \frac{Q}{A_2} = \frac{4Q}{\Pi D_2^2}, then$$

$$d = \frac{8Q^2}{\pi^2 D^4 g}\left[f\frac{L}{D} + k + 1\right]$$

Example…

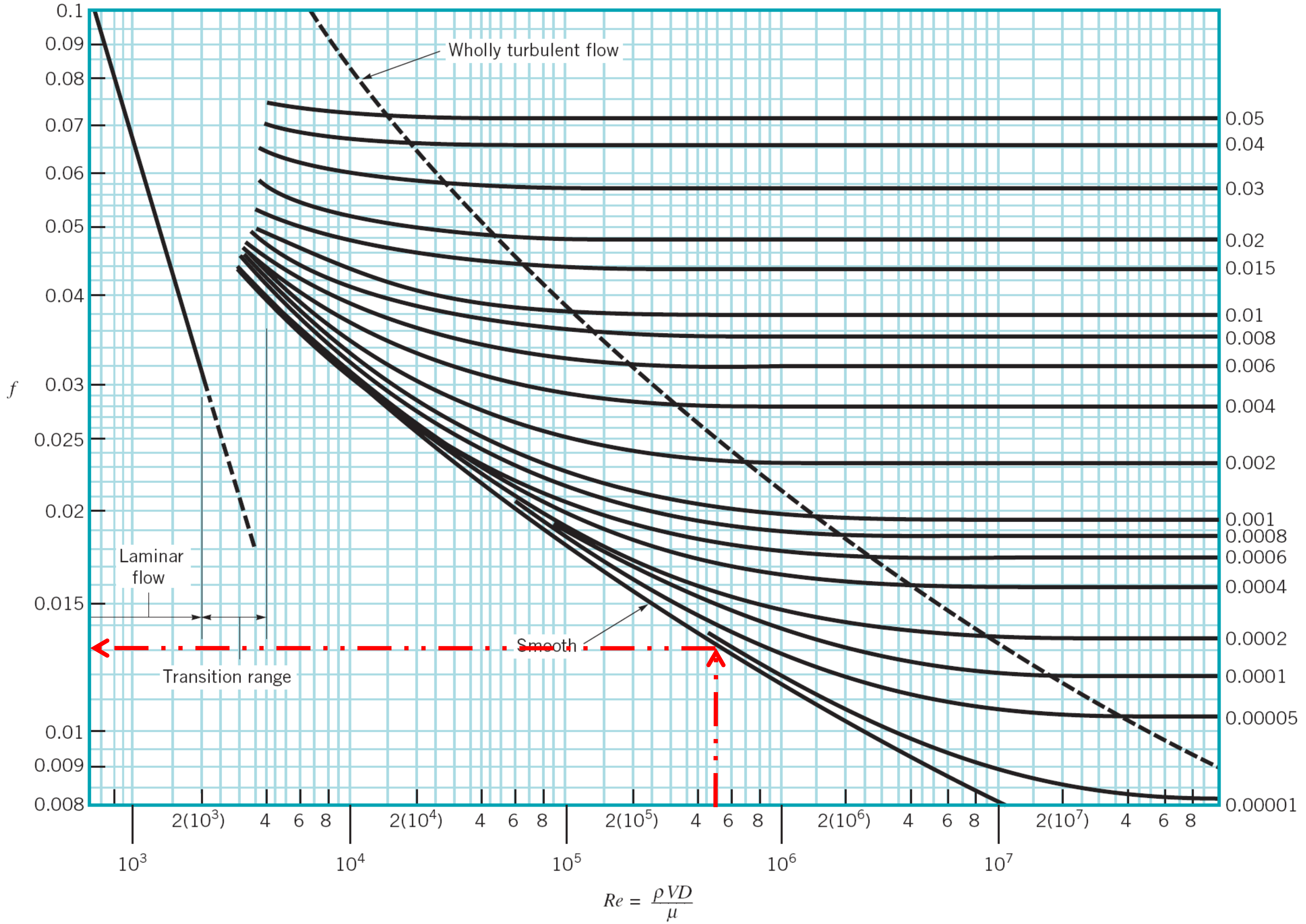
$$Re = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4}{\pi} * \frac{1000 * 0.03}{1x10^{-3} * 0.075} = 5.10x10^{5}$$

- For smooth pipe from Moody diagram, f = 0.0131, then k = 0.5 for square-edged.

$$d = \frac{8}{\pi^2} * \frac{(0.03)^2}{(0.075)^4 * 9.81} * \left[0.0131 * \frac{100}{0.075} + 0.5 + 1\right]$$

$$d = 44.6m$$

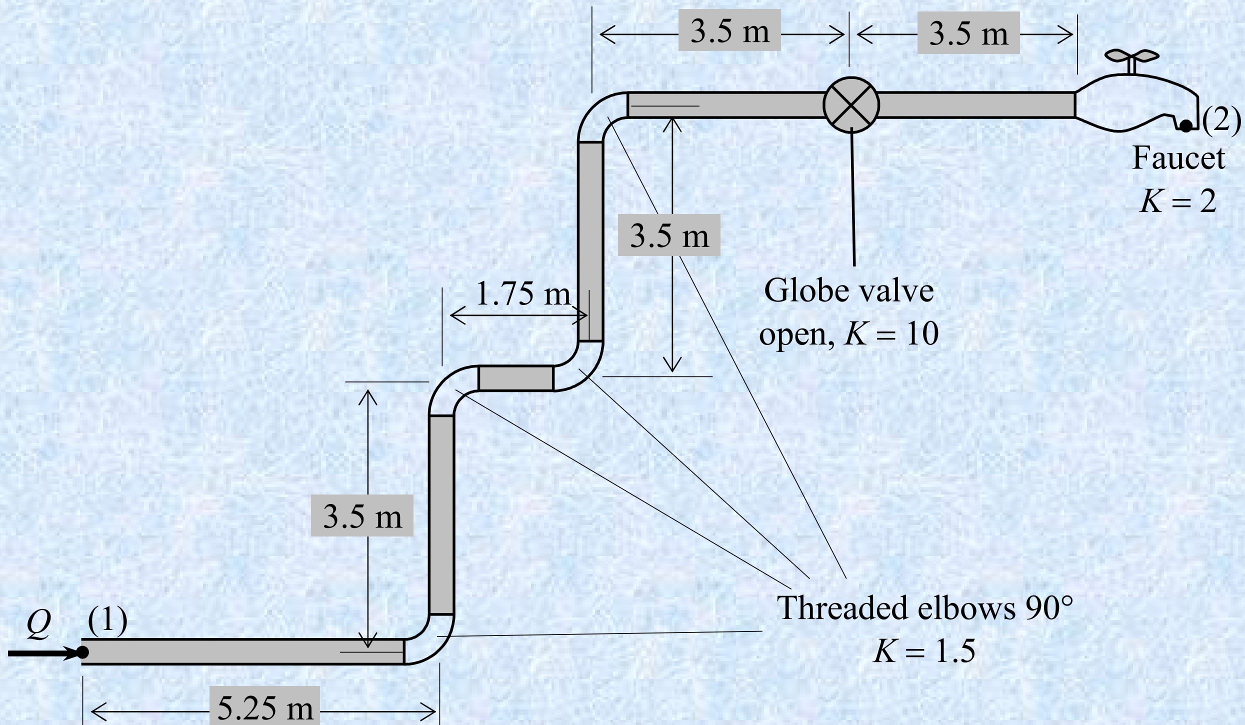
MOODY CHART

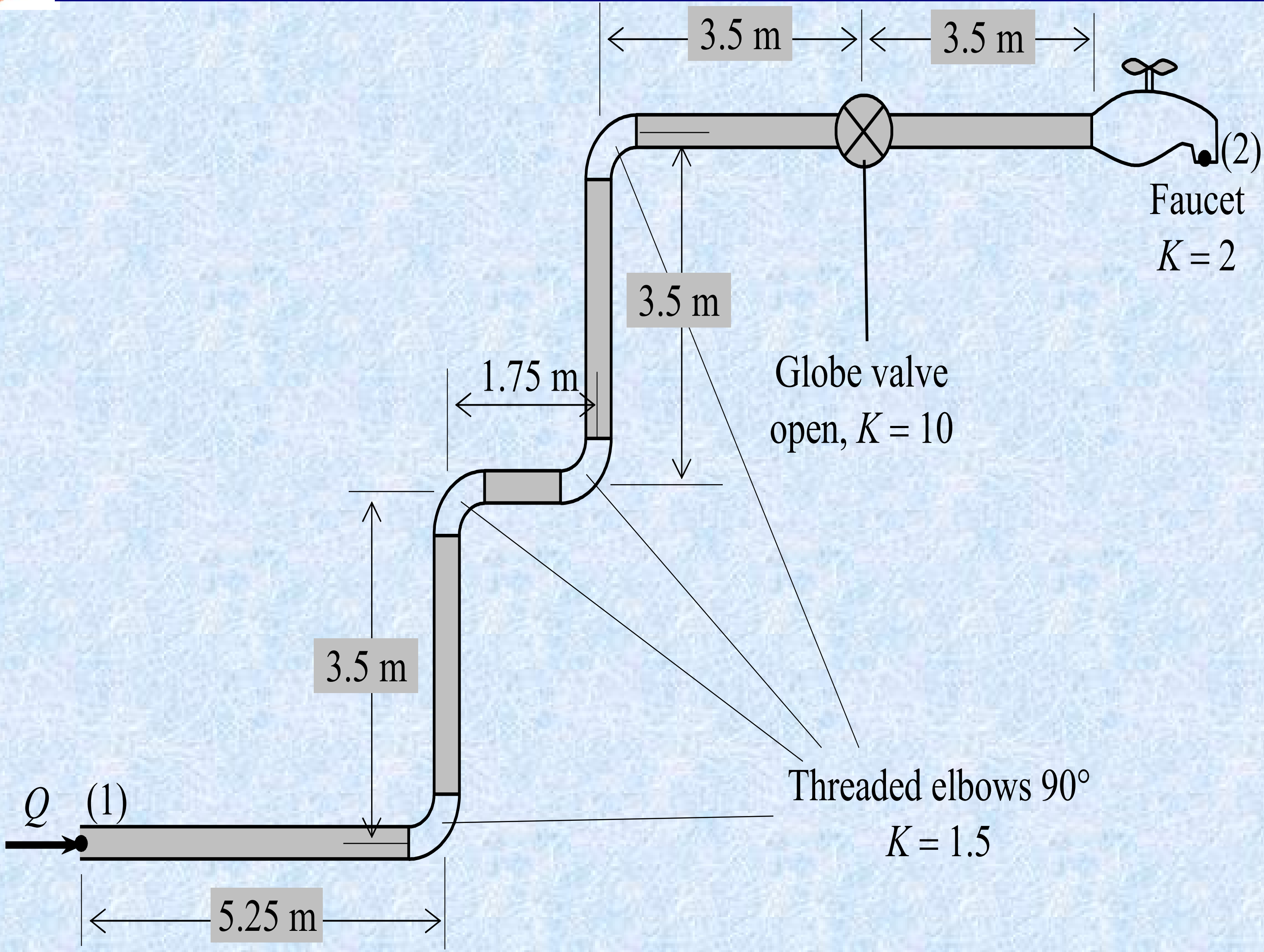


Wholly turbulent flow
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
f
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
ε/D
Laminar flow
Transition range
Smooth
2(10^3) 4 6 8 2(10^4) 4 6 8 2(10^5) 4 6 8 2(10^6) 4 6 8 2(10^7) 4 6 8
10^3 10^4 10^5 10^6 10^7
Re = ρVD/μ

Example

- Water flows from the ground floor to the second level in a three-storey building through a 20mm diameter pipe (drawn-tubing, $\varepsilon = 0.0015$ mm) at a rate of 0.75 liter/s. The layout of the whole system is illustrated in Figure below. The water flows out from the system through a valve with an opening of diameter 12.5 mm. Calculate the pressure at point (1).




3.5 m
3.5 m
(2)
Faucet
K = 2
3.5 m
Globe valve
open, K = 10
1.75 m
3.5 m
Threaded elbows 90°
K = 1.5
Q
(1)
5.25 m

Solution

From the modified Bernoulli equation, we can write

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 + \rho g h_L$$

In this problem, $p_2 = 0$, $z_1 = 0$. Thus,

$$p_1 = \frac{1}{2}\left(V_2^2 - V_1^2\right) + \rho g z_2 + \rho g\left(h_1 + h_m\right)$$

The velocities in the pipe and out from the faucet are respectively

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4\left(0.75\times10^{-3}\right)}{\pi(0.020)^2} = 2.387\, m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2} = \frac{4\left(0.75\times10^{-3}\right)}{\pi(0.012)^2} = 6.631\, m/s$$

The Reynolds number of the flow is

$$\mathrm{Re} = \frac{\rho V d}{\mu} = \frac{(998)(2.387)(0.020)}{1.12\times10^{-3}} = 42{,}546$$

Solution

The roughness $\varepsilon/d = 0.0015/20 = 0.000075$. From the Moody chart, $f \approx 0.022$ (or,0.02191 via the Colebrook formula). The total length of the pipe is

Hence, the friction head loss is $\ell = 5.25 + 4(3.5) + 1.75 = 21\text{m}$

The total minor loss is $h_f = f\frac{\ell}{d}\frac{V_1^2}{2g} = (0.022)\frac{21}{0.02}\frac{2.387^2}{2(9.81)} = 6.71\text{m}$

$$h_m = \sum K\frac{V_1^2}{2g} = [4(1.5) + 10 + 2]\frac{2.387^2}{2(9.81)} = 5.23\text{m}$$

$$\Delta h_{\omega\tau} = h_f + h_m = 6.71 + 5.23 = 11.94\text{m}$$

Therefore, the pressure at (1) is

$$p_1 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) + \rho g z_2 + \rho g\left(h_1 + h_m\right)$$

$$= \frac{1}{2}(998)\left(6.631^2 - 2.387^2\right) + (998)(9.81)(3.5 + 3.5)$$

$$+ 998(9.81)(6.71 + 5.23)$$

$$= 205k\text{Pa}$$

Example

Consider a water flow in a pipe having a diameter of D = 20 mm which is intended to fill a 0.35 liter container. Calculate:

(a) the minimum time required if the flow is laminar,

(b) the maximum time required if the flow is turbulent.

Use density ρ = 998 kg/m3 and dynamic viscosity μ = 1.12×10–3 kg/m·s

Solution:

(a) For laminar flow, use Re $=\rho VD/\mu$ = 2300: $V = \frac{2300\mu}{\rho D} = \frac{2300(1.12\times10^{-3})}{(998)(0.020)} = 0.118\, m/s$

$$t = \frac{\forall}{Q} = \frac{4\forall}{\pi D^2 V}$$

Hence, the minimum time t is $= \frac{4(0.35\times10^{-3})}{\pi(0.02)^2(0.118)} = \underline{9.45 s}$

b) For turbulent flow, use Re = $\rho VD/\mu$ = 4000: $V = \frac{4000\mu}{\rho D} = \frac{4000(1.12\times10^{-3})}{(998)(0.020)} = 0.224\, m/s$

$$t = \frac{\forall}{Q} = \frac{4\forall}{\pi D^2 V}$$

Hence, the minimum time t is $= \frac{4(0.35\times10^{-3})}{\pi(0.02)^2(0.224)} = \underline{4.96 s}$

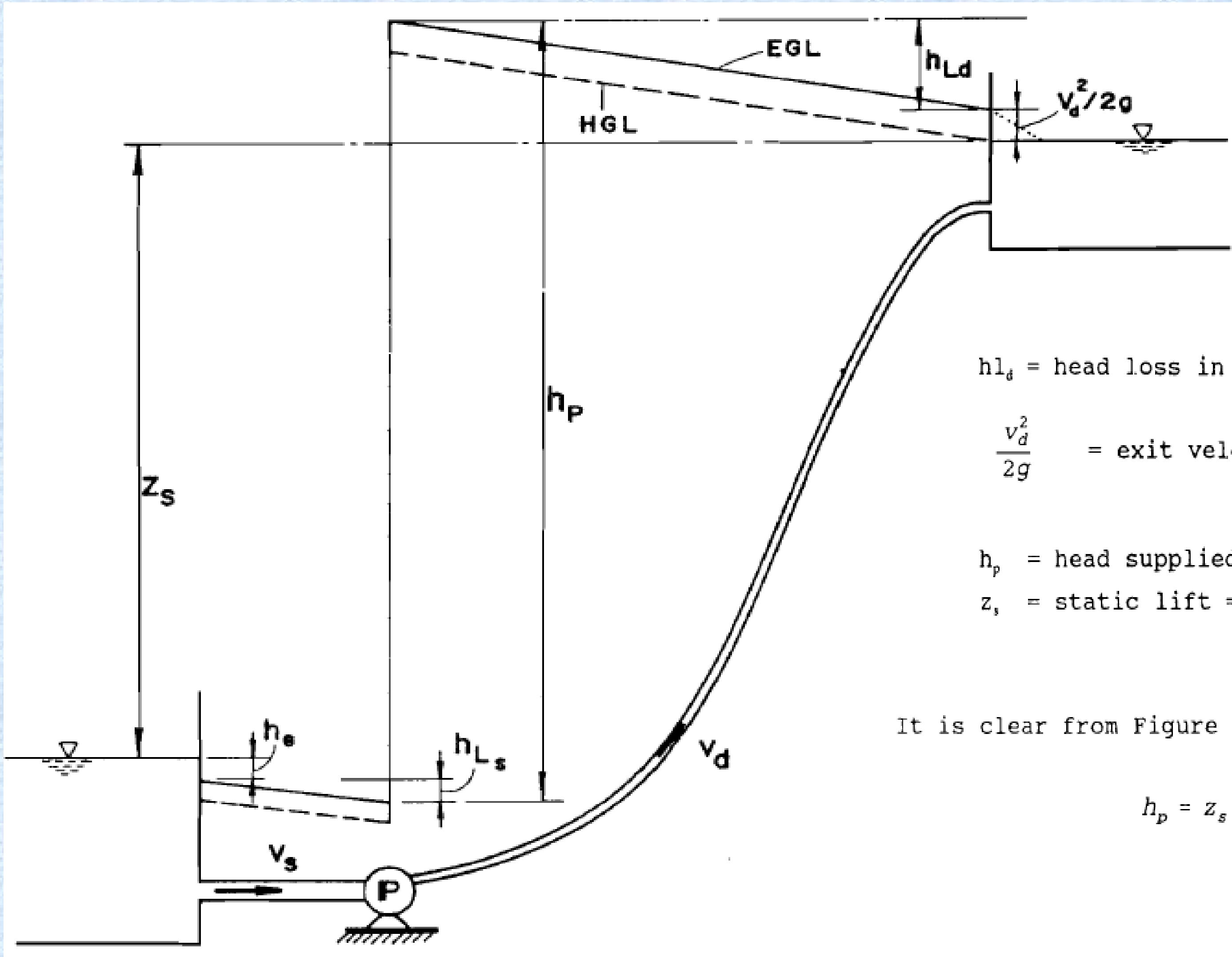
Pipe line with Pump or Turbine

- The pump lifts the fluid a height
- the power delivered to the liquid by the pump is $(\Delta Z + \sum h_l)$
- The power required to run the pump is greater than this, depending on the efficiency of the pump. The total pumping head, hp, for this case is:
- $\gamma Q\left(\Delta Z + \sum h_L\right)$
- If the pump discharges a stream through a nozzle, kinetic energy head of $V_2^2/2g$ is required. Total pumping head is:-

$$h_p = \Delta Z + \sum h_L .$$

$$h_p = \Delta Z + \frac{V_2^2}{2g} + \sum h_L$$

Pipeline with a pump



hl_d = head loss in delivery pipe

$\frac{v_d^2}{2g}$ = exit velocity head = exit head loss

h_p = head supplied by the pump

z_s = static lift = level difference between the reservoirs.

It is clear from Figure 5.9 that:

$$h_p = z_s + h_e + hl_s + hl_d + \frac{v_d^2}{2g}$$

Relation between Q & h_L in pipe

- Using Hazen William equation, it is possible to develop a relationship between head loss, hL, that occur in a pressurized pipe, and the flow rate, Q, flowing through this pipe.

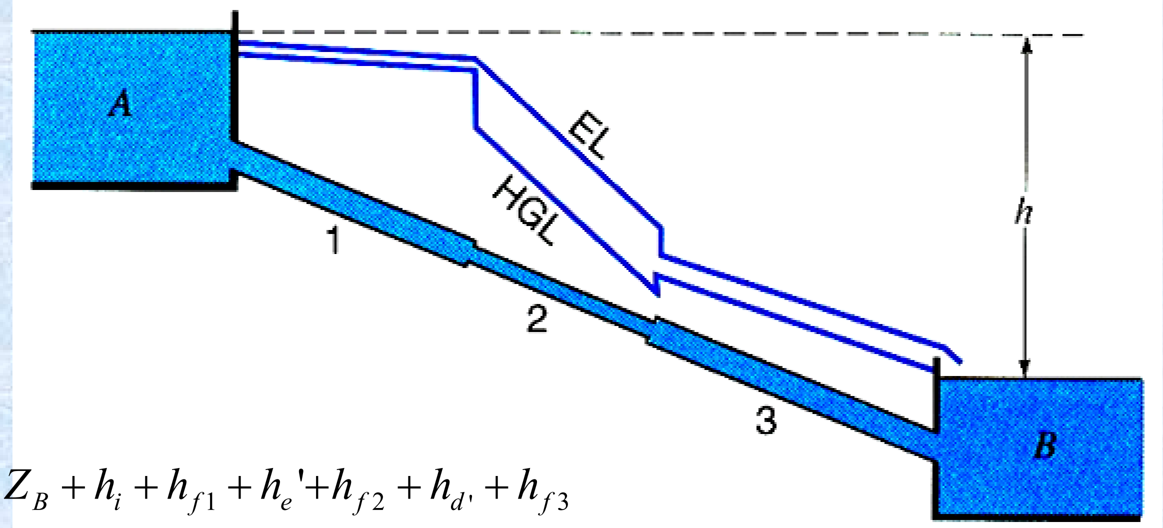
$$Q = Av = \left(\frac{\pi d^2}{4}\right) \times 0.849 C_{HW} \left(\frac{d}{4}\right)^{0.63} s^{0.54}$$

$$h_L = KQ^{1.852} = KQ^m$$

$$K = \frac{10.697L}{d^{4.871} C_{HW}^{\;\;1.852}}$$

Pipeline system

2. Pipes in Series



$$Q_1 = Q_2 = Q_3$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

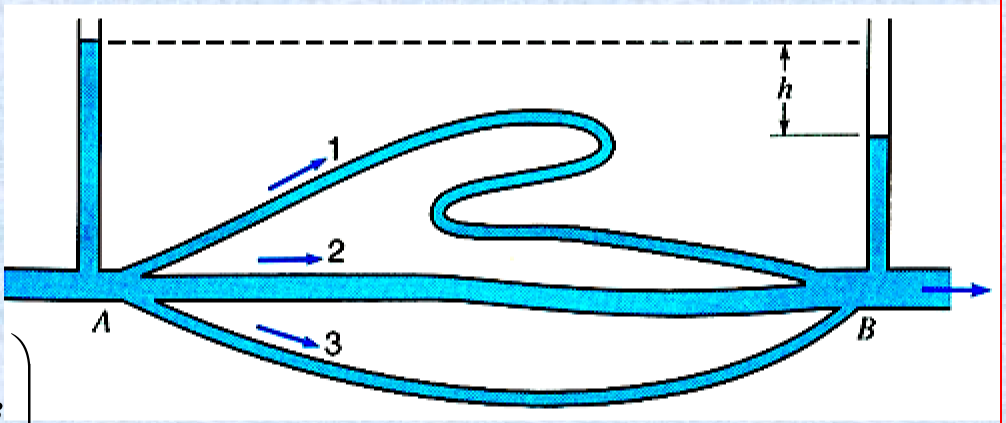
$$\frac{p_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_i + h_{f1} + h_e' + h_{f2} + h_{d'} + h_{f3}$$

$$h + 0 + 0 = 0 + 0 + 0 + k_i \frac{V_1^2}{2g} + f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + k_c \frac{V_2^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + \frac{(V_2 - V_3)^2}{2g} + f_3 \frac{L_3}{D_3}\frac{V_3^2}{2g} + k_{exit}\frac{V_1^2}{2g}$$

From quantity eqn.: $V_1 D_1^2 = V_2 D_2^2 = V_3 D_3^2$

$$h = \frac{V_1^2}{2g}\{k_i + f_1 \frac{L_1}{D_1} + \left[1 - \left(D_1 / D_2\right)^2\right]^2 + f_2 \frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4 + \left(\frac{D_1}{D_2}\right)^4\}$$

2. Pipes in Parallel



$$h_{f1} = h_{f2} = h_{f3} = \frac{P_A}{\gamma} + Z_A - \left(\frac{P_B}{\gamma} + Z_B\right)$$

$$Q = Q_1 + Q_2 + Q_3$$

- **Two types of problems occur:**

1. If the head loss between A and B is given, Q is determined.
2. If the total flow Q is given, then the head loss and distribution of flow are determined.

Equivalent pipes

- to replace the length of all the pipes in terms of **equivalent lengths** of any one given size, one which figures predominantly in the system
- $\mathbf{L_e}$ of pipe of certain diameter $\mathbf{D_e}$ which carry the same discharge and dissipate same energy or head $\mathbf{h_f}$ as the one with length L and diameter D.

$$\therefore h_{f1} = f_1 \frac{L_1}{D_1^5} \frac{8Q_1^2}{\pi^2 g}$$

$$h_{f2} = f_2 \frac{L_2}{D_2^5} \frac{8Q_2^2}{\pi^2 g}$$

$$h_{f1} = h_{f2} \qquad Q_1 = Q_2$$

$$\therefore \frac{f_1 L_1}{D_1^5} = \frac{f_2 L_2}{D_2^5} \Rightarrow L_2 = L_1 \frac{f_1}{f_2} \left(\frac{D_2}{D_1} \right)^5$$

Problem 1 in parallel connection

- The pressure and datum heads at A and B are known, to compute the discharge

$$h_f = \frac{P_A}{\gamma} + Z_A - \left(\frac{P_B}{\gamma} + Z_B \right)$$

- Q_1, Q_2, and Q_3 will be computed and then summed up in order to get the Q value

Problem 1 in parallel connection

- **Q is given, then h_f, and Q_1, Q_2, and Q_3 required**

1. assume a discharge Q_1' through pipe 1
2. solve for h_f' using assumed discharge Q_1' using equation

$$h_f = \left(f_1 \frac{L_1}{D_1} + \sum K \right) \frac{V_1^2}{2g}$$

3. Similarly, using h_f' compute Q_2' Q_3'
4. Now it is assumed that for the same energy loss to occur in the three different loops, that the total discharge Q should be divided in the same proportion as Q_1' Q_2' and Q_3'

$$Q_1 = \left(\frac{Q_1''}{\sum Q''} \right) Q \qquad Q_2 = \left(\frac{Q_2''}{\sum Q''} \right) Q \qquad Q_3 = \left(\frac{Q_3''}{\sum Q''} \right) Q$$

5. Check the correctness of the procedure by computing h_f1, h_f2, and h_f3 for the three different loops which should be the same. (1% tolerable)

Pipe Connection

Series connection

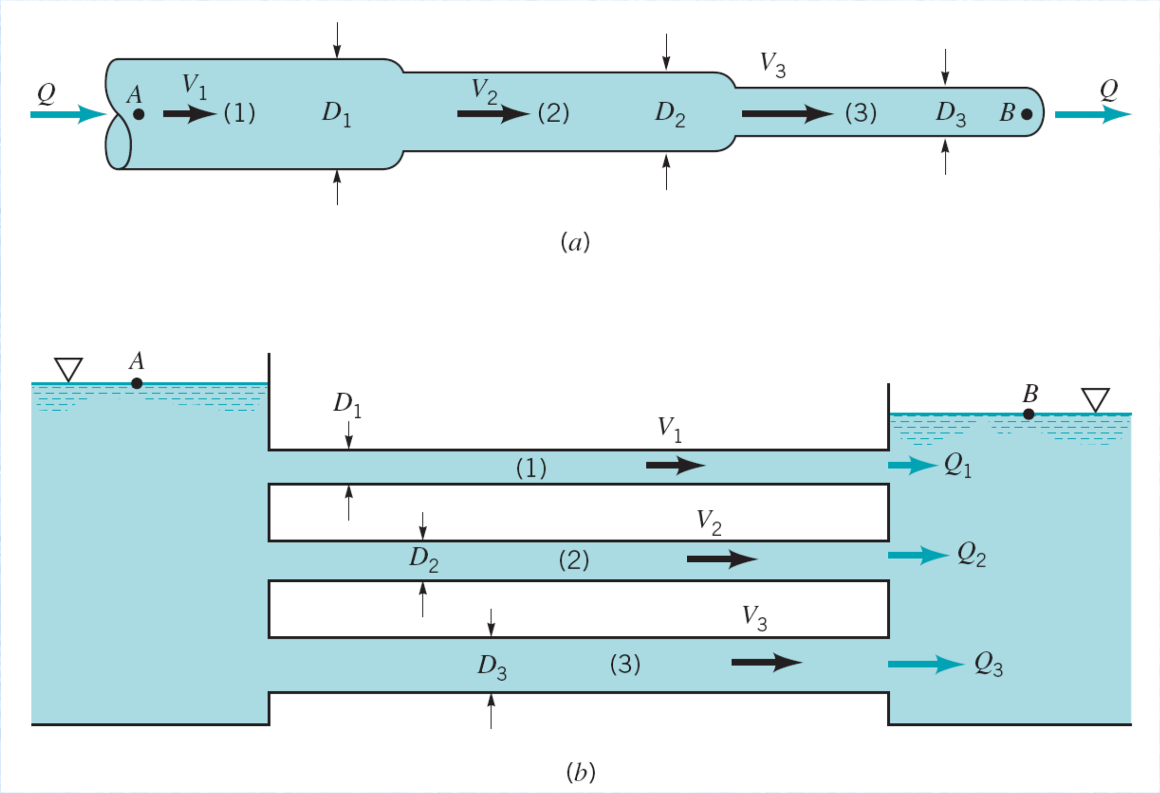
$$Q_1 = Q_2 = Q_3$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

Parallel Connection

$$Q = Q_1 + Q_2 + Q_3$$

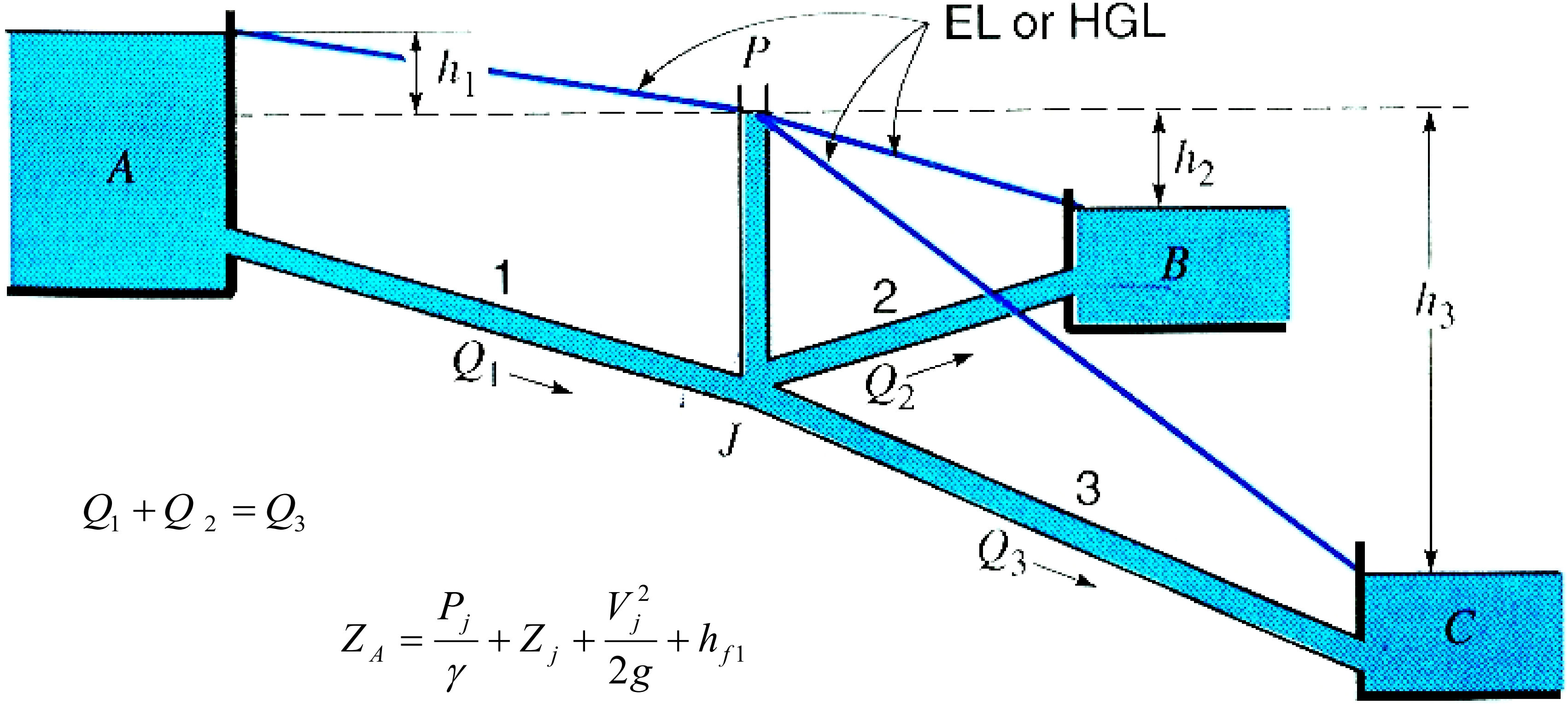
$$h_{L_1} = h_{L_2} = h_{L_3}$$



Exercise

- Three pipes were connected between two points A and B to carry 0.3m^3/sec. The point A and B lie 30m and 25m above a given datum, respectively. The pressure at A is maintained at 600Kpa. The pipe 1 is 100m long and 0.3m is diameter, the pipe 2 is 750m long and 0.2m is diameter and pipe 3 is 200m long and 0.4m is diameter. Assume all the pipes to be smooth. Determine the flow in each pipe and the pressure at B. Take kinematice viscosity of water = $10^{-6}m^2$/sec.

Branching pipes



$$Q_1 + Q_2 = Q_3$$

$$Z_A = \frac{P_j}{\gamma} + Z_j + \frac{V_j^2}{2g} + h_{f1}$$

$$h_{f1} = Z_A - \left(\frac{P_j}{\gamma} + Z_j \right)$$

Branching pipes

- Case I – Given all L, D, Elev A & Elev B, Q_1
  - Required - Elev C and Q_2, Q_3
- Case II – Given all L, D, Elev A & Elev C, Q_2
  - Required – Elev B, and Q_1 , Q_3
- Case III – Given all L, D, and Elevations
  - Required – Q_1, Q_2, and Q_3

Case I – Given all L, D, Elev A & Elev B, Q_1 Required - Elev C and Q_2, Q_3

1. Assume a proper value of ***f*** and calculate h_{f1} for a given L_1, D_1, and Q_1

2. Determine the elevation of J and hence the head difference between J and second reservoir H_{J2} which is also equal to the head loss in pipe 2 i.e. h_{f2}

3. Calculate the discharge for the third reservoir and the corresponding head loss using Darcy's equation, the surface elevation can then be determined

Case II – Given all L, D, Elev A & Elev C, Q_2 Required – Elev B, and Q_1 , Q_3

- Since Q_2 is given, the difference $Q_1 - Q_3$ is known. Similarly, it is seen from the previous figure that $h_{f1} + h_{f3}$ is also given. These relations are solved simultaneously for their component parts in one of the two ways.

a) Assume successive h_{f3} using trial values of Q_1 and Q_3. The computed values of h_{f1} and h_{f3} should satisfy elevation at junction J is common for all.

b) Assume successive elevation of J satisfy the second relation, determine Q_1, and Q_2 (using Darcy's equation until the first relations is also satisfied.

Case III – Given all L, D, and Elevations Required – Q_1, Q_2, and Q_3

- No flow in pipe 2 (Elevation of J and B are same)

- Find h_{f1} and h_{f3}, if $Q_1 > Q_3$, the flow is going into reservoir B and if $Q_1 < Q_3$ the flow is going out of reservoir B.

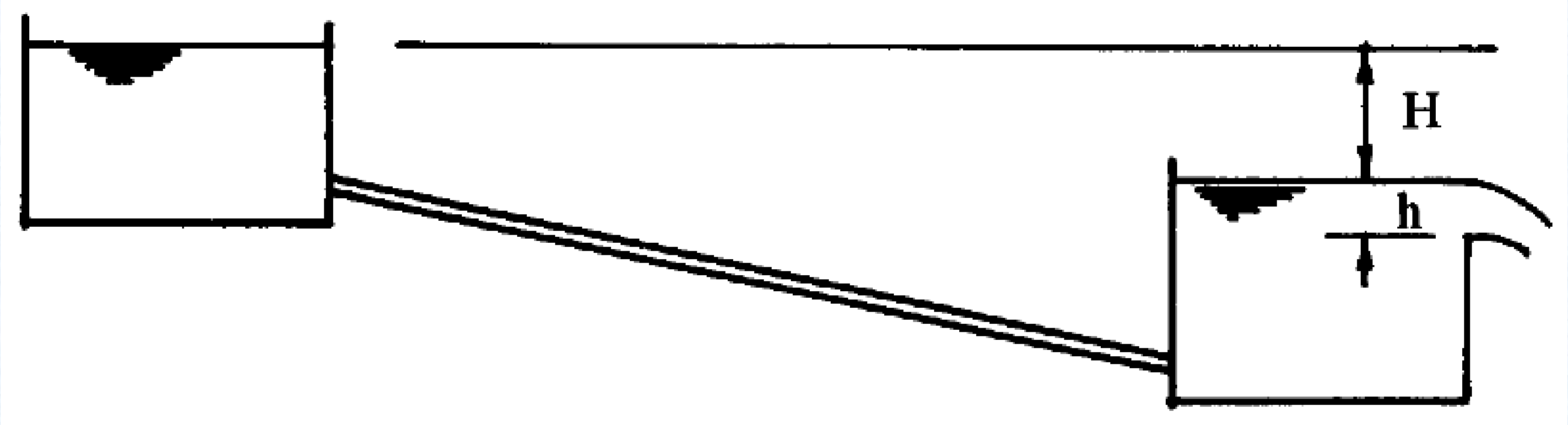
- Once the direction of Q_2 is determined, another trial elevation of piezometric head at J is assumed and h_{f1}, h_{f2} and h_{f3} are computed;
- then Q_1, Q_2, and Q_3 are determined and the equation of continuity is satisfied. If the flow into the junction is too great, a higher piezometric head at J is assumed, which will reduce the inflow and increase the outflow

Exercise

- A reservoir A with its surface 60m above datum supplies water to a junction J through 30cm diameter pipe 1500m long. From the junction, a 225cm diameter pipe 800m long feeds reservoir B, in which the surface is 30m above datum, while another pipe 400m long and 20cm diameter feeds another reservoir C. The water level in the reservoir C stands at 15m above datum. Calculate the discharge to each reservoir. Assume f for each pipe as 0.03.

Example

- A constant head tank delivers water through a uniform pipeline to a tank, at a lower level, for which the water discharges over a rectangular weir. Pipeline length 20.0m, diameter 100mm, roughness size 0.2mm. Length of weir crest 0.25m, discharge coefficient 0.6, crest level 2.5m below water level in header tank. Calculate the steady discharge and the head of water over the weir crest. Use minor head coefficient k of 1.5



Solution

For pipeline, $H = \frac{1 \cdot 5\, V^2}{2g} + \frac{\lambda\, L V^2}{2g\, D} = (2 \cdot 5 - h)$ (i)

or $H = \frac{Q^2}{2g\, A^2}\left(1 \cdot 5 + \frac{\lambda\, L}{D}\right) = (2 \cdot 5 - h)$ (ii)

Discharge over weir: $Q = \frac{2}{3} C_D \sqrt{2g}\, B\, h^{3/2}$ (iii)

$$\text{i.e. } Q = \frac{2}{3} \times 0 \cdot 6 \times \sqrt{19 \cdot 62} \times 0 \cdot 25 \times h^{3/2}$$

$$= 0 \cdot 443\, h^{3/2}$$

$$\text{i.e. } h = \left(\frac{Q}{0 \cdot 443}\right)^{2/3} \quad \text{(iv)}$$

Solution

Then in (ii) $\frac{Q^2}{2g\,A^2}\left(1{\cdot}5 + \frac{\lambda\,L}{D}\right) = 2{\cdot}5 - \left(\frac{Q}{0{\cdot}443}\right)^{2/3}$

$$\text{or}\quad \frac{Q^2}{2g\,A^2}\left(1{\cdot}5 + \frac{\lambda\,L}{D}\right) + \left(\frac{Q}{0{\cdot}443}\right)^{2/3} = 2{\cdot}5 \qquad \text{(v)}$$

Since λ is unknown this equation can be solved by trial or interpolation i.e. inputting a number of trial Q values and evaluating the left-hand side of equation (v):

$$H_1 = \frac{Q^2}{2g\,A^2}\left(1{\cdot}5 + \frac{\lambda\,L}{D}\right) + \left(\frac{Q}{0{\cdot}443}\right)^{2/3}$$

For the same values of Q, the corresponding values of h are evaluated from equation (iv).

For each trial value of Q, the Reynolds number is calculated and the

Solution

friction factor obtained from the Moody diagram, for $\frac{k}{D} = 0{\cdot}0002$. See table below.

whence $Q = 0{\cdot}0213\ m^3/s$ (21·3 l/s) when $H_l = 2{\cdot}5$ m

and $h = 0{\cdot}132$ m.

$Q\ m^3/s$	Re	λ	H_l (m)	h(m)
0·010	$1{\cdot}13 \times 10^5$	0·0250	0·617	0·08
0·015	$1{\cdot}69 \times 10^5$	0·0243	1·287	0·105
0·018	$2{\cdot}03 \times 10^5$	0·0241	1·810	0·118
0·020	$2{\cdot}25 \times 10^5$	0·0241	2·215	0·126
0·022	$2{\cdot}48 \times 10^5$	0·0240	2·655	0·135

Reading Assignment

- Pipeline systems and network analysis
- Check valve and pressure reducing valve

Design procedures for Complex (Looped) Pipe Networks

- Hardy Cross Method
- Newton-Raphson Method
- Linear Theory Methods
- EPANET
- WaterCAD

Hardy Cross Method

- This is an iterative procedure based on initially estimated flows in pipes
- The method is based on the following basic **equations of continuity of flow** and **head loss that should be satisfied**
- The steps are as follows:

Hardy Cross Method

Step-1: Assume the best distribution of flow that satisfies continuity by careful examination of the network.

- *The flow entering a node must be equal to the flow leaving the same node*

$$\sum Q_i = q_j \qquad \text{for all nodes } j = 1, 2, 3, \ldots, j_L.$$

Where Q_i *the discharge in pipe i meeting at node (junction) j, and q is nodal withdrawal at node j*

Hardy Cross Method…

Step-2: Calculate the head loss, h_f, in each pipe.

$h_f = k_i(Q)^2$ … **Darcy Weisbach**

$h_f = k_i(Q)^{1.85}$ … **Hazzen- Weliam**

- The algebraic sum of the heads around a closed loop must be zero.

$$\sum_{loop\ k} h_f = 0 \qquad \text{for all loops } k = 1,2,3,\dots k_i$$

$$\sum_{loop\ k} K_i Q_i |Q_i| = 0 \qquad K_i = \frac{8 f_i L_i}{\pi^2 g D_i^5} \qquad h_f = \frac{8 f L Q^2}{\pi^2 g D^5}$$

- For a loop, take head loss in the clockwise flows as positive and in the anti-clockwise flows as negative

Hardy Cross Method…

- In general, it is not possible to satisfy the condition for head loss with initially assumed pipe discharges satisfying nodal continuity equation.
- Therefore, discharges are modified so that $\sum h_L$ becomes closer to zero
- The modified pipe discharges are determined by applying a correction ΔQ to the initially assumed pipe flows

$$\sum_{\text{loop } k} K_i(Q_i + \Delta Q_k)|(Q_i + \Delta Q_k)| = 0$$

Hardy Cross Method…

Step-3: Calculate the correction factor for each loop by Expanding the equation and neglecting second power of ΔQ_k and simplifying it, the following equation is obtained:

$$\Delta Q = -\frac{\sum rQ_o|Q_o|^{n-1}}{\sum rn|Q_o|^{n-1}} = -\frac{\sum h_f}{n\sum \frac{h_f}{Q_o}}$$

$$\Delta Q_k = -\frac{\sum_{\text{loop } k} K_i Q_i |Q_i|}{2\sum_{\text{loop } k} K_i |Q_i|}. \qquad = -\frac{\sum h_f}{n\sum \frac{h_f}{Q_o}}$$

Formulae for flow correction, ΔQ

$$\Delta Q = \frac{-\sum HL}{2\sum(\frac{HL}{Q})} \quad \text{for Darcy-weisbach}$$

$$\Delta Q = \frac{-\sum HL}{1.85\sum(\frac{HL}{Q})} \quad \text{for Hazen-Williams}$$

The overall procedure for the looped network analysis can be summarized as;

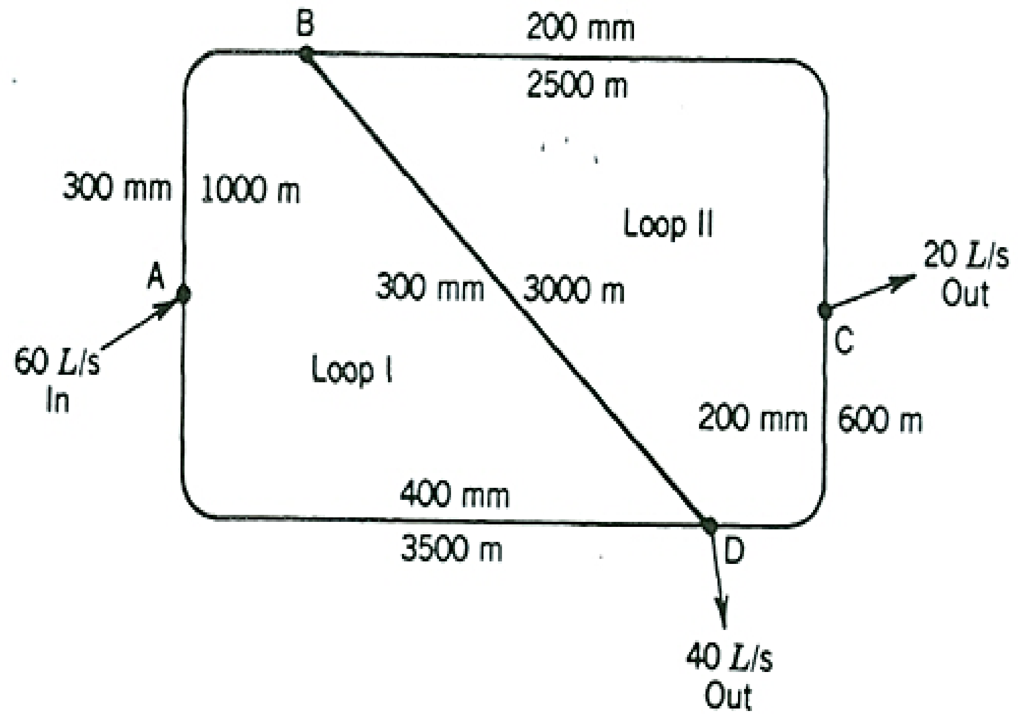
- **Number** all the **nodes pipe, links** and **loops**

- Apply **nodal continuity equation** at all the nodes to obtain **pipe discharges** ***(start by assuming an arbitrary discharge in one of two pipes joining and apply continuity equation to obtain discharge in the other pipe).***
- Adopt a **sign convention** that a pipe discharge is positive if it flows clockwise direction, otherwise negative
- **Estimate diameters** for the initially assumed flow rates
- **Calculate head loss** in the pipes as a function of the flow rate Q, the diameter, length and roughness of a pipe, .
- Work out to satisfy the **head loss requirement** ***(determine ΔQ for each loop and by using ΔQ value, new estimated flows are calculated).***
- **Iterate /repeat procedure**/ until $\Delta Q \approx 0$ i.e, negligibly small corrections (ΔQ), ($\sum h_L=0$)

Example

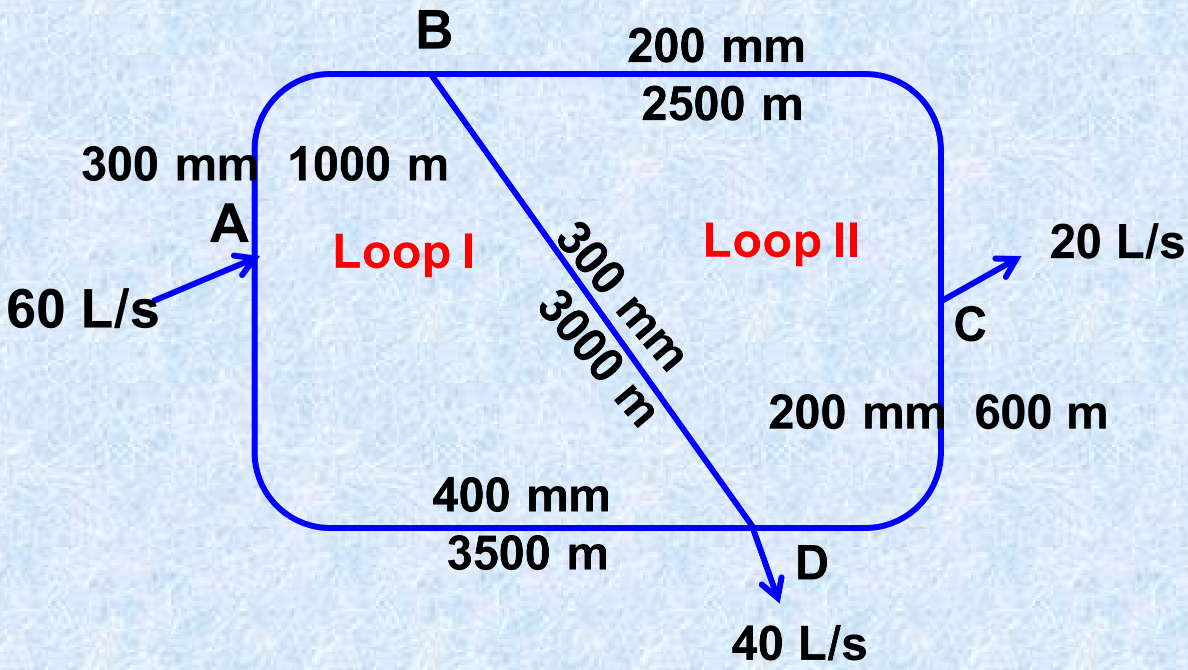
Determine the discharge in each of the pipes using Hard-Cross Method

Pipe	Diameter, mm	Length, m
AB	300	1000
BC	200	2500
BD	300	3000
AD	400	3500
DC	200	600



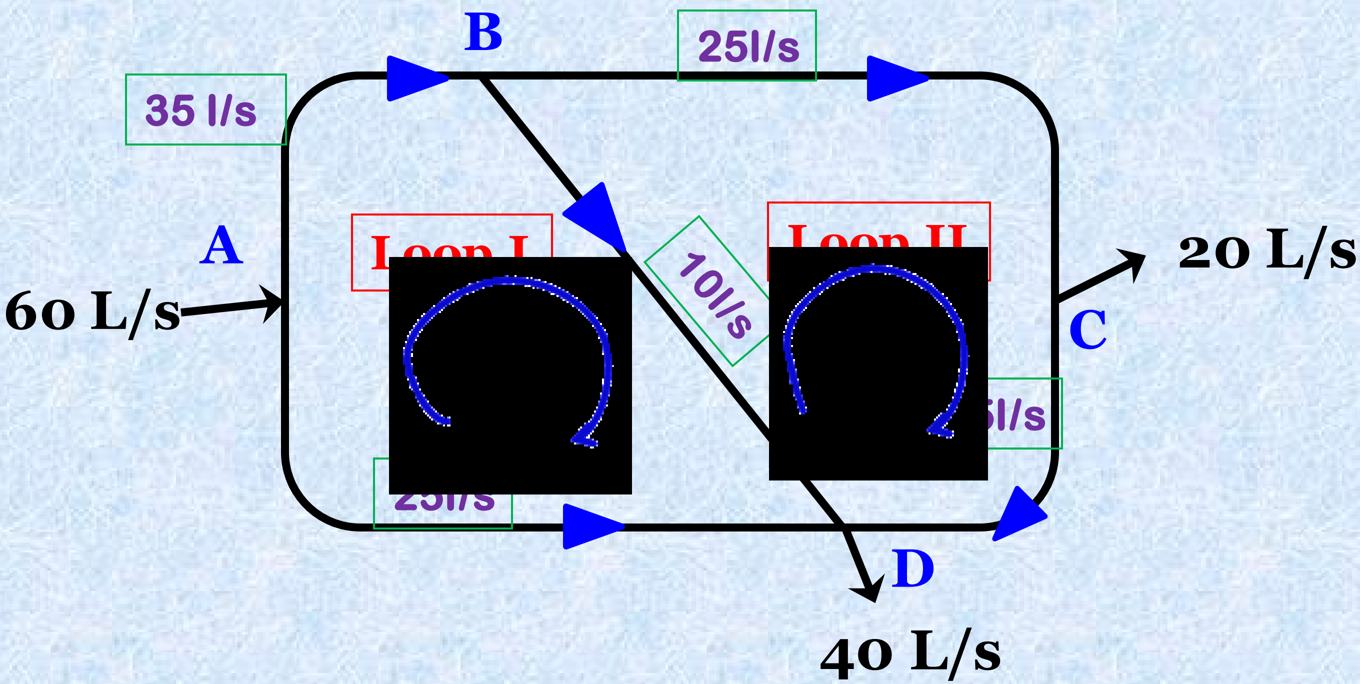
Example

Determine the flow rates in all the pipes of the network using Hardy Cross Method. Take C=100



Solution:

Trial discharges from continuity



Solution:

Iteration 1

$$h_f = 10.7\left(\frac{Q}{C}\right)^{1.85}\left(\frac{L}{D^{4.87}}\right)$$

Loop	Pipe	D, mm	L, m	Q, L/s	S, m/m	h_L, m	h_L/Q	$Q + \Delta Q$
I (first try)	AB	300	1000	35	0.0015	1.5	0.043	29
	BD	300	3000	10	0.000 15	0.45	0.045	4
	AD	400	3500	−25	−0.0002	−0.7	0.028	−31
						1.25	0.116	

$$\text{I: } \Delta Q_I = -\frac{1.25}{1.85 \times 0.116} \approx -6 \text{ L/s}$$

Loop	Pipe	D, mm	L, m	Q, L/s	S, m/m	h_L, m	h_L/Q	$Q + \Delta Q$
II (first try)	BC	200	2500	25	0.0055	13.75	0.55	12
	CD	200	600	5	0.0003	0.18	0.036	−8
	BD	300	3000	−4	neglect	neglect	neglect	−17
						13.9	0.586	

$$\text{II: } \Delta Q_I = -\frac{13.9}{1.85 \times 0.586} \approx -13 \text{ L/s}$$

Solution:

Iteration 2

Loop	Pipe	D, mm	L, m	Q, L/s	S, m/m	h_L, m	h_L/Q	$Q + \Delta Q$
I (second try)	AB	300	1000	29	0.001	1.00	0.034	24
	BD	300	3000	17	0.0004	1.20	0.071	12
	AD	400	3500	−31	−0.0003	−1.05	0.034	−36
						1.15	0.139	

$$\text{I: } \Delta Q_2 = -\frac{1.15}{1.85 \times 0.139} \approx -5 \text{ L/s}$$

Loop	Pipe	D, mm	L, m	Q, L/s	S, m/m	h_L, m	h_L/Q	$Q + \Delta Q$
II (second try)	BC	200	2500	12	0.0015	3.75	0.313	8
	CD	200	600	−8	−0.000 75	−0.45	0.056	−12
	BD	300	3000	−12	−0.0002	−0.60	0.050	−16
						2.7	0.419	

$$\text{II: } \Delta Q_2 = -\frac{2.7}{1.85 \times 0.419} \approx -4 \text{ L/s}$$

Solution:

- *Iteration 3*

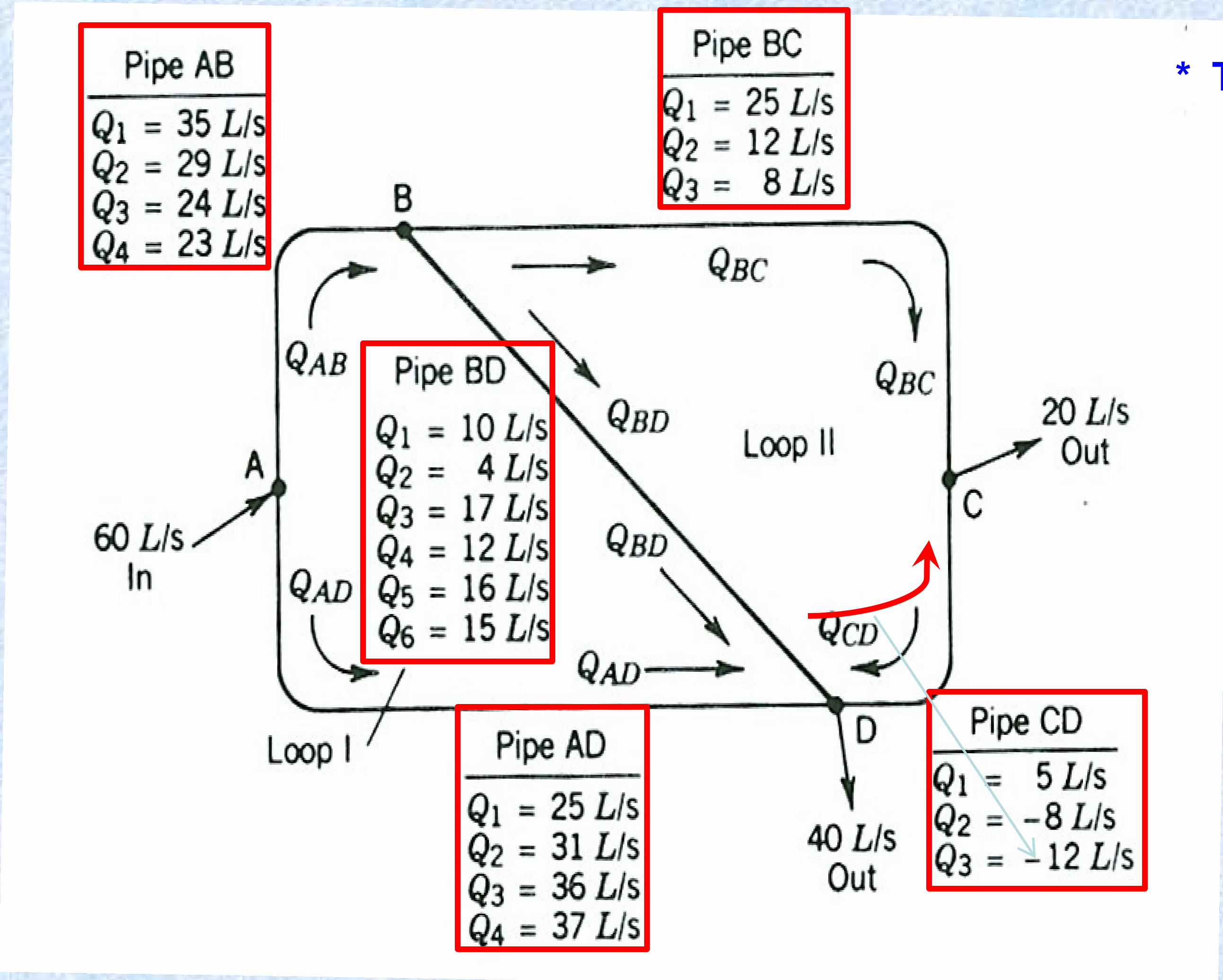
Loop	Pipe	D, mm	L, m	Q, L/s	S, m/m	h_L, m	h_L/Q	$Q + \Delta Q$
I (third try)	AB	300	1000	24	0.0007	0.7	0.029	23
	BD	300	3000	16	0.000 34	1.02	0.064	15
	AD	400	3500	−36	−0.0004	−1.4	0.039	−37
						0.32	0.132	

$$\text{I: } \Delta Q_3 = -\frac{0.32}{1.85 \times 0.132} \approx -1 \text{ L/s}$$

Loop	Pipe	D, mm	L, m	Q, L/s	S, m/m	h_L, m	h_L/Q	$Q + \Delta Q$
II (third try)	BC	200	2500	8	0.0007	1.75	0.219	
	CD	200	600	−12	−0.0015	−0.9	0.075	
	BD	300	3000	−15	−0.0003	−0.9	0.06	
						−0.05	0.354	

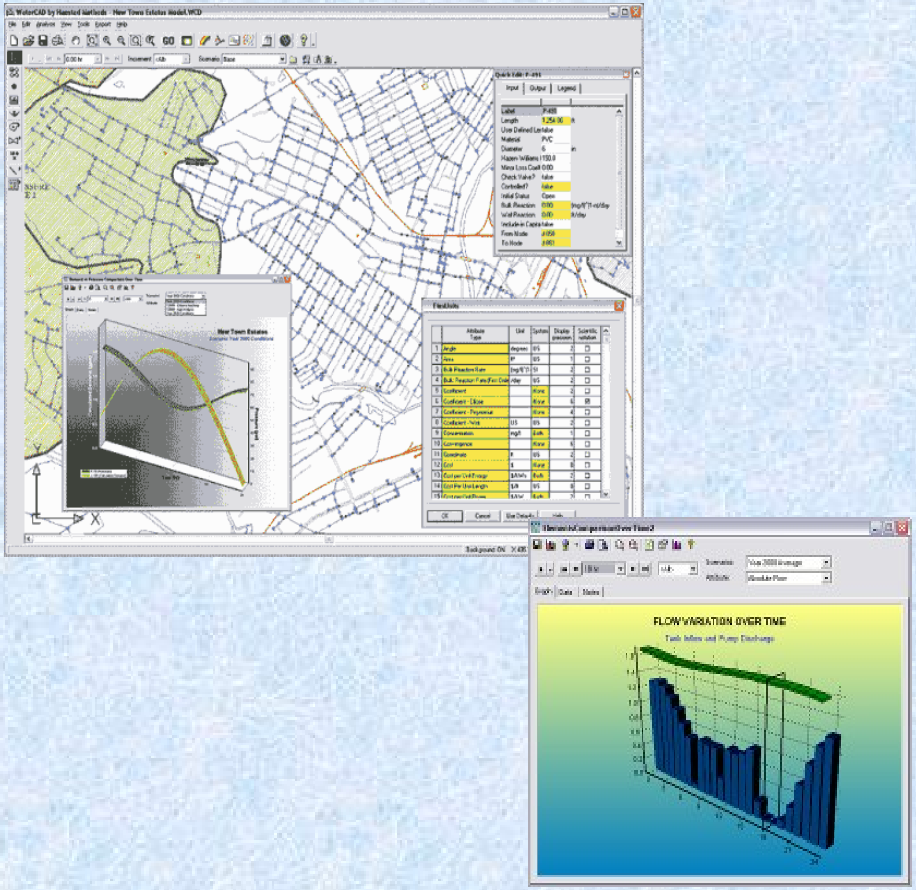
$$\text{II: } \Delta Q_3 = \frac{-0.05}{1.85 \times 0.354} \approx 0.07 \text{ (negligible)}$$

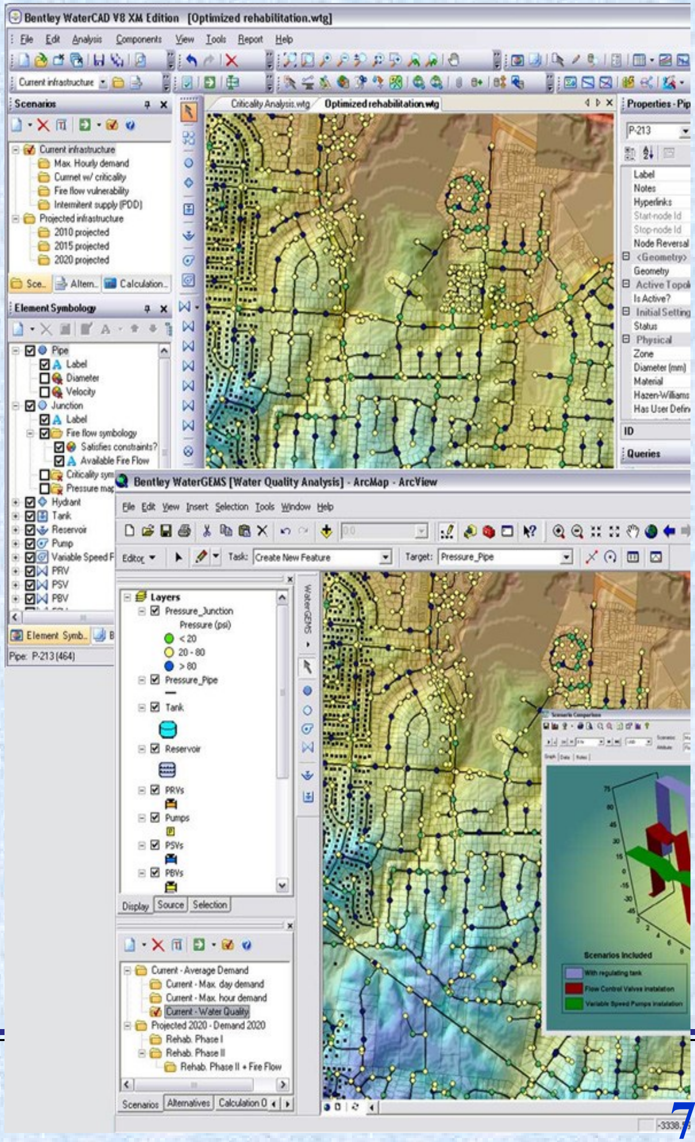
Final Solution



* Then residual head and velocity can be checked against design criterion

Water Distribution Modeling





WDS Simulation

- *Simulation:* the process of imitating the behavior of one system through the functions of another.
- refers to the process of using a mathematical representation of the real system, called a *model*.

WDS Simulation

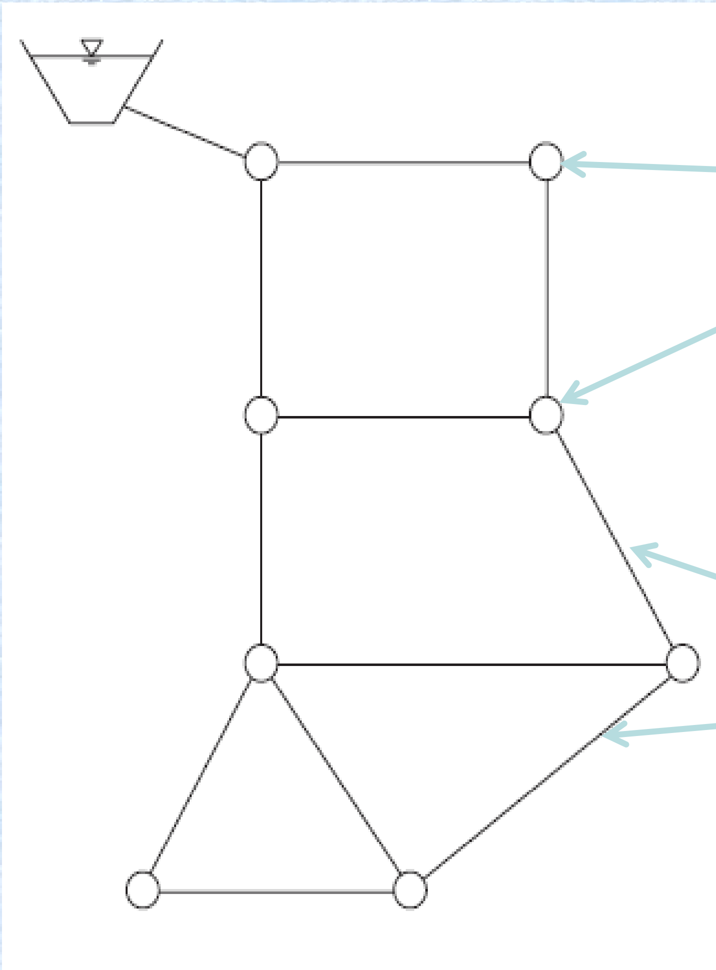
- **Steady-State:** a snapshot in time and are used to determine the operating behavior of a system under static conditions.

- **Extended Period Simulation (EPS):** used to evaluate system performance over time

Application of WDMs

- Long-range master planning, both new development and rehabilitation
- Fire protection studies
- Water quality investigations
- Energy management
- System design
- Daily operational uses including operator training, emergency response, and
- troubleshooting

Model Representation



Nodes

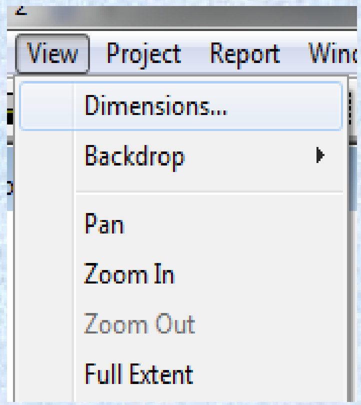
Links

Network Elements

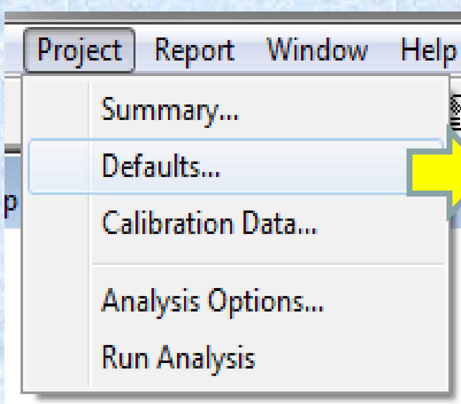
Element	Type	Primary Modeling Purpose
Reservoir	Node	Provides water to the system
Tank	Node	Stores excess water within the system and releases that water at times of high usage
Junction	Node	Removes (demand) or adds (inflow) water from/to the system
Pipe	Link	Conveys water from one node to another
Pump	Node or link	Raises the hydraulic grade to overcome elevation differences and friction losses
Control Valve	Node or link	Controls flow or pressure in the system based on specified criteria

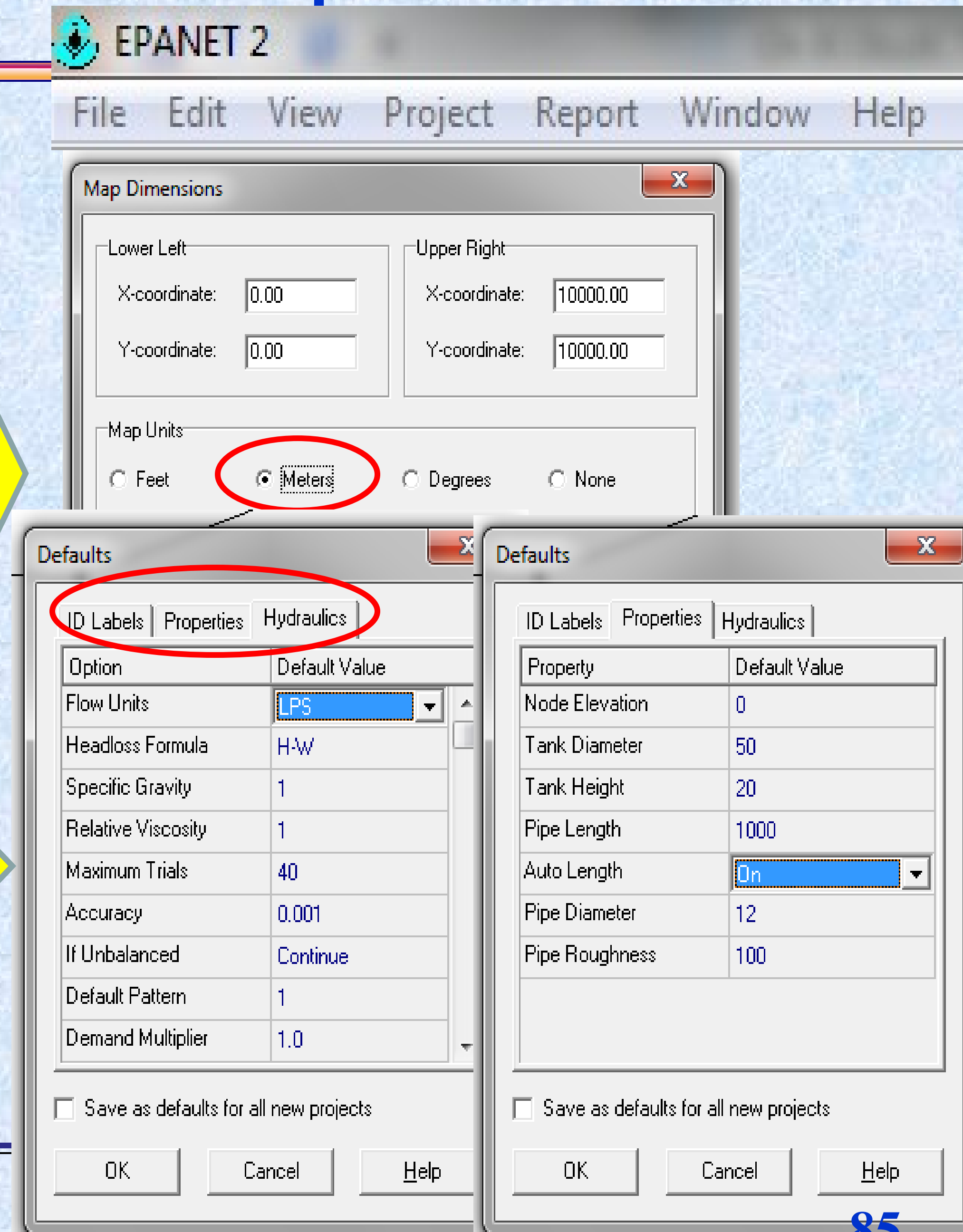
EPAnet steps

- Project setup
  - View >



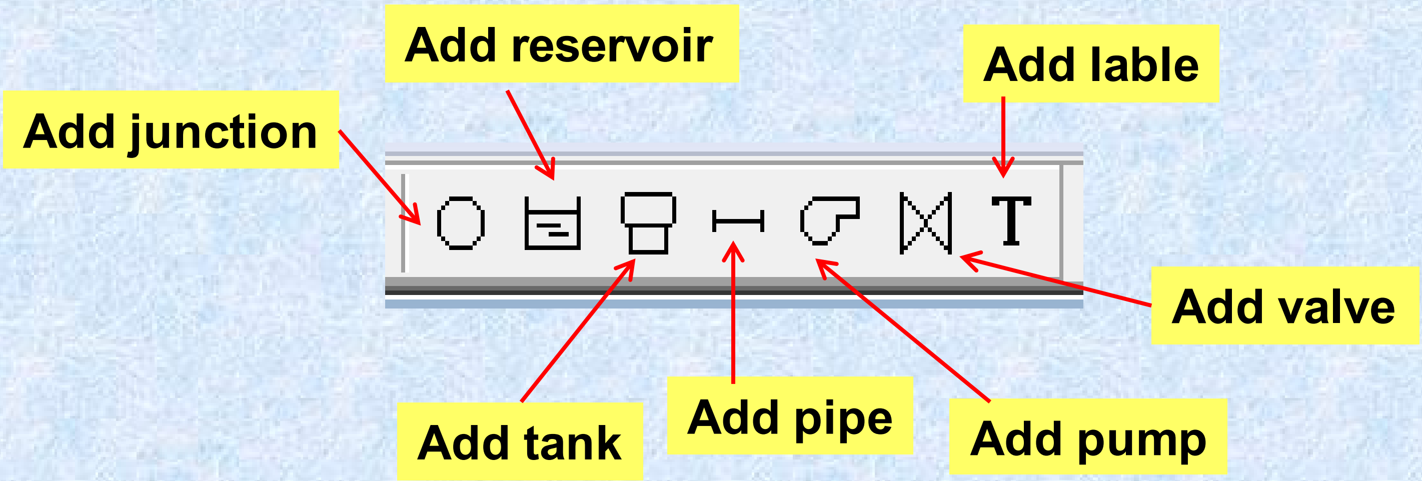
  - Project >





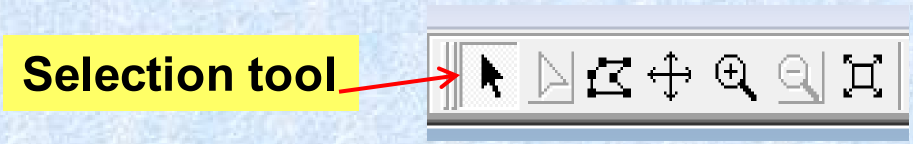
EPAnet steps …

- Model building
  - First construct nodes (Reservoirs, tanks, junctions)
  - Then connect links (pumps, pipes, valves)

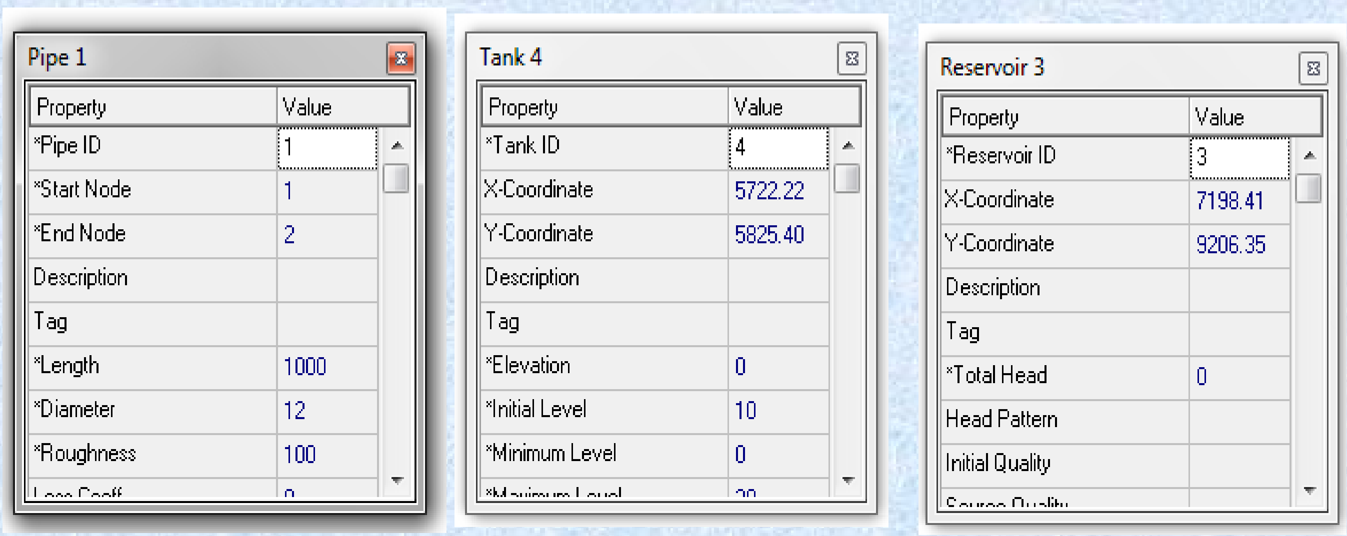


EPAnet steps …

- Input data
  - Double click element using selection tool



  - Modify data in the property box



Pipe 1

Property	Value
*Pipe ID	1
*Start Node	1
*End Node	2
Description	
Tag	
*Length	1000
*Diameter	12
*Roughness	100

Tank 4

Property	Value
*Tank ID	4
X-Coordinate	5722.22
Y-Coordinate	5825.40
Description	
Tag	
*Elevation	0
*Initial Level	10
*Minimum Level	0

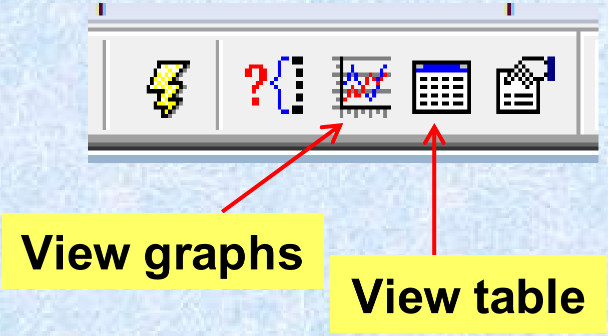
Reservoir 3

Property	Value
*Reservoir ID	3
X-Coordinate	7198.41
Y-Coordinate	9206.35
Description	
Tag	
*Total Head	0
Head Pattern	
Initial Quality	

EPAnet steps …

- Run the model

- View result



The following needs to be considered in your Project (Assignment #3)

- Demand Analysis (Including Population Forecasting)
- Preliminary network layout (GIS/ Google Earth)
- Distribution system design (In WaterCAD)
- Typical building sanitary system design (Optional)
- Report
  - Text
  - Drawing
  - EPANET model & result
  - WaterCAD model & result

Design procedures for Complex (Looped) Pipe Networks

- **Hardy Cross Method**
- Newton-Raphson Method
- Linear Theory Methods
- EPANET
- WaterCAD

Hardy Cross Method (Revised)

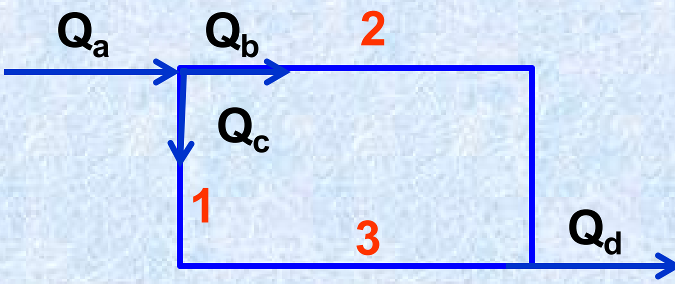
- The method is based on two basic principles:

1. **Conservation of Mass** (Continuity Equation)

   Inflow = Outflow at nodes

   $$Q_a = Q_b + Q_c$$

   The algebraic sum of the flow rates in the pipe meeting at a junction, together with any external flows is zero



2. **Conservation of Energy** (Head Loss Equation)

   Summation of head loss in a closed loop is Zero

   $$\sum \boldsymbol{h_l(loop)} = \mathbf{0}$$

   $$\sum K(Q + \Delta Q)^n) = 0$$

Hardy Cross Method (Revised)…

- The relationship between head loss and discharge must be maintained for each pipe:

$$h_f = \frac{fLV^2}{2gD}$$

$$V^2 = \left(\frac{Q}{A}\right)^2 = \left(\frac{Q}{\frac{\pi d^2}{4}}\right)^2$$

$$V^2 = \frac{16 * Q^2}{\pi^2 D^2}$$

Darcy-Weisbach Equation

$h_l(pipe)$= KQ^n n = 2; $K = \frac{8fL}{g\pi^2 D^5}$

Exponential Friction Formula (Hazen-Williams Equation)

$h_l(pipe)$= KQ^n n = 1.85; $K = \frac{10.67}{C^{1.85}\ D^{4.87}}$

Hardy Cross Method (Revised)…

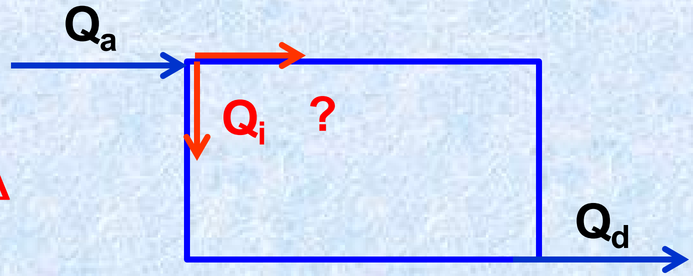
- Derivation of Hardy Cross Method (**For the first Principle**)

Actual Discharge = Q

Assumed Discharge = Q_i

Correction Discharge = Δ

$Q = Q_i + \Delta$



$$\sum K((Q_i+\Delta)^n)=0$$

$$\sum K Q_i^{\,n} + \sum nK\Delta Q_i^{\,n-1} + \sum \frac{n-1}{2} nK\Delta^2 Q_i^{\,n-2} + \ldots = 0$$

Binomial Expansion

$$\sum \mathrm{KQ_i}^{\mathrm{n}} + \sum \mathrm{nK\Delta Q_i}^{\mathrm{n-1}} = 0$$

For Small values of Δ

$$\Delta = -\frac{\sum K Q_i^{\,n}}{\sum nKQ_i^{\,n-1}} = -\frac{\sum h_l}{n\sum \frac{h_l}{Q_i}}$$

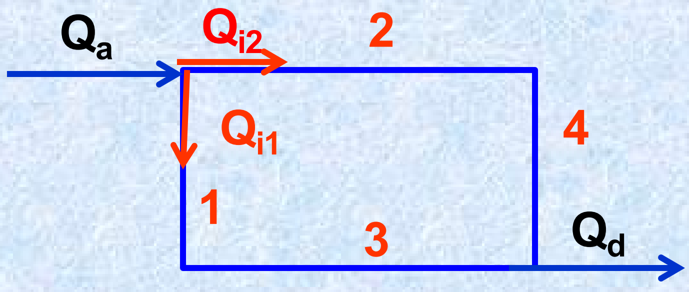
n = 2 for Darcy Weisbach

n = 1.85 for Hazen Williams

Hardy Cross Method (Revised)…

- Derivation of Hardy Cross Method (For the second Principle)

$$\sum h_l(loop) = 0$$

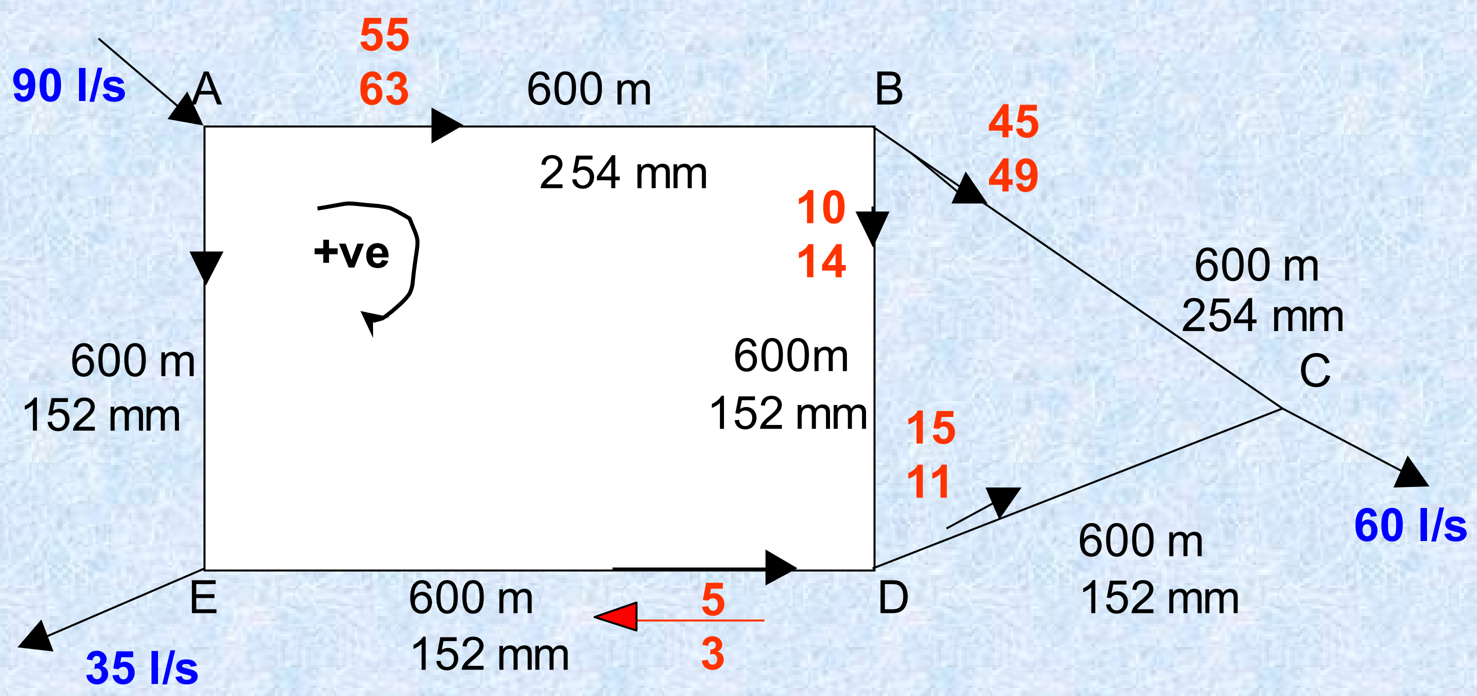


$$h_f(1) + h_f(2) + h_f(3) + h_f(4) = 0$$

- Note that the clockwise water flows are positive while the anti-clockwise ones are negative.
- Positive and negative flows give rise to positive and negative head losses respectively

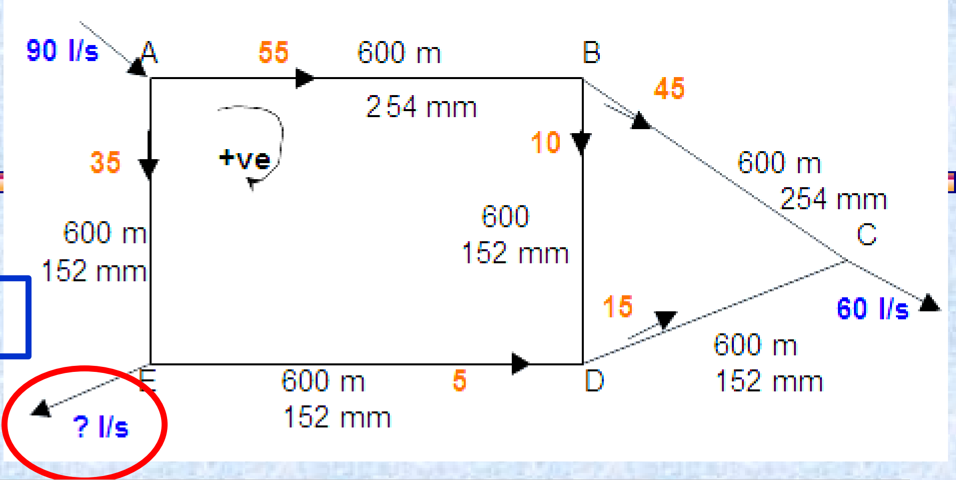
Example

Example: Obtain the flow rates in the network shown below.



Solution

$$hf = 10.67 \; C_H^{-1.85} \; D^{-4.87} \; Q^{1.85} \; L$$



Iteration - 1

Loop	Pipe	L(m)	D(m)	Q(m3/s)	Hf (m)	hf/Q	Q+ΔQ
	AB	600	0.254	0.055	2.720	49.452	0.063
I	BD	600	0.152	0.010	1.415	141.527	0.018
ABDE	DE	600	0.152	-0.005	-0.393	78.517	0.003
	EA	600	0.152	-0.035	-14.367	410.485	-0.027
					-10.624	679.981	
					0.008		

$$h_f = 10.7\left(\frac{Q}{C}\right)^{1.85}\left(\frac{L}{D^{4.87}}\right)$$

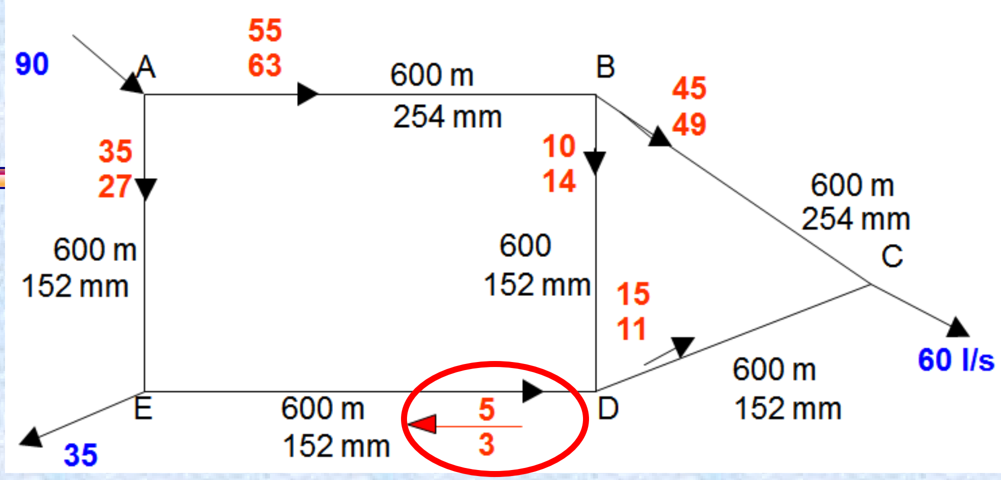
$$\Delta Q = -\frac{\sum h_f}{n\sum \frac{h_f}{Q_o}}$$

Loop	Pipe	L(m)	D(m)	Q(m3/s)	Hf (m)	hf/Q	Q+ΔQ
	BC	600	0.254	0.045	1.876	41.697	0.049
II	CD	600	0.152	-0.015	-2.996	199.764	-0.011
BDC	DB	600	0.152	-0.010	-1.415	141.527	-0.006
					-2.535	382.988	
					0.004		

$$\Delta Q = -\frac{\sum h_f}{n\sum \frac{h_f}{Q_o}}$$

Solution

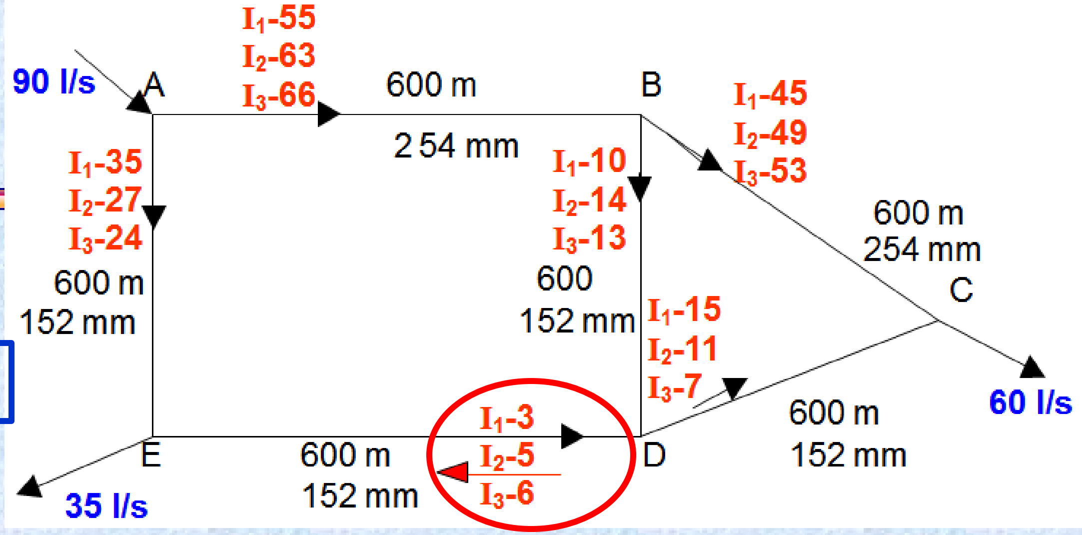
$$hf = 10.67 \ C_H^{-1.85} \ D^{-4.87} \ Q^{1.85} \ L$$



Iteration - 2

Loop	Pipe	L(m)	D(m)	Q(m3/s)	Hf (m)	hf/Q	Q+ΔQ
	AB	600	0.254	0.063	3.543	55.837	0.065
I	BD	600	0.152	0.014	2.637	188.386	0.016
	DE	600	0.152	0.003	0.197	57.217	0.005
	EA	600	0.152	-0.027	-8.620	324.604	-0.025
					-2.242	626.044	
					0.002		
	BC	600	0.254	0.049	2.197	44.827	0.052
II	CD	600	0.152	-0.011	-1.688	153.470	-0.008
	DB	600	0.152	-0.014	-2.637	188.386	-0.011
					-2.129	386.683	
					0.003		

Solution



$$hf = 10.67\ C_H^{-1.85}\ D^{-4.87}\ Q^{1.85}\ L$$

Iteration - 3

Loop	Pipe	L(m)	D(m)	Q(m3/s)	Hf (m)	hf/Q	Q+ΔQ
	AB	600	0.254	0.065	3.705	56.997	0.066
I	BD	600	0.152	0.013	2.300	176.885	0.014
	DE	600	0.152	0.005	0.393	78.517	0.006
	EA	600	0.152	-0.025	-7.710	308.381	-0.024
					-1.313	620.781	
					0.001		
	BC	600	0.254	0.052	2.452	47.150	0.053
II	CD	600	0.152	-0.008	-0.937	117.075	-0.007
	DB	600	0.152	-0.013	-2.300	176.885	-0.012
					-0.784	341.110	
					0.001		

Design procedures for Complex (Looped) Pipe Networks

- Hardy Cross Method
- **Newton-Raphson Method**
- Linear Theory Methods
- EPANET
- WaterCAD

Newton-Raphson Method

- It is a powerful numerical method for solving systems of non-linear equations
- The method can be applied to both ΔH and ΔQ equations.
- The main concept of Newton-Raphson algorithm is derived from Taylor series which calculates the x_1 value according to x_o

$$\mathbf{x_1} = \mathbf{x_o} - \frac{f(x)}{f'(x)}$$

Newton-Raphson Method…

- Suppose that there are three nonlinear equations to be solved for Q_1, Q_2, and Q_3
  - $F_1(Q_1, Q_2, Q_3) = 0$
  - $F_2(Q_1, Q_2, Q_3) = 0$
  - $F_3(Q_1, Q_2, Q_3) = 0$
- Adopt a starting solution (Q_1, Q_2, Q_3) .
- Also consider that $(Q_1 + \Delta Q_1, Q_2 + \Delta Q_2, Q_3 + \Delta Q_3)$ is the solution for the set of equations. That is:

$$\left.\begin{aligned} F1\,(Q_1 + \Delta Q_1, Q_2 + \Delta Q_2, Q_3 + \Delta Q_3) &= 0 \\ F2\,(Q_1 + \Delta Q_1, Q_2 + \Delta Q_2, Q_3 + \Delta Q_3) &= 0 \\ F3\,(Q_1 + \Delta Q_1, Q_2 + \Delta Q_2, Q_3 + \Delta Q_3) &= 0 \end{aligned}\right\} \quad \textbf{Eq-1}$$

Newton-Raphson Method…

- Expanding the above equations as Taylor's series:

$$
\left.
\begin{aligned}
F_1 + \left[{\partial F_1}/{\partial Q_1}\right]\Delta Q_1 + \left[{\partial F_1}/{\partial Q_2}\right]\Delta Q_2 + \left[{\partial F_1}/{\partial Q_3}\right]\Delta Q_3 = 0 \\
F_2 + \left[{\partial F_2}/{\partial Q_1}\right]\Delta Q_1 + \left[{\partial F_2}/{\partial Q_2}\right]\Delta Q_2 + \left[{\partial F_2}/{\partial Q_3}\right]\Delta Q_3 = 0 \\
F_3 + \left[{\partial F_3}/{\partial Q_1}\right]\Delta Q_1 + \left[{\partial F_3}/{\partial Q_2}\right]\Delta Q_2 + \left[{\partial F_3}/{\partial Q_3}\right]\Delta Q_3 = 0
\end{aligned}
\right\} \text{Eq-2}
$$

- Arrange the above set of equations in matrix form,

$$
\left.
\begin{bmatrix}
{\partial F_1}/{\partial Q_1} & {\partial F_1}/{\partial Q_2} & {\partial F_1}/{\partial Q_3} \\
{\partial F_2}/{\partial Q_1} & {\partial F_2}/{\partial Q_2} & {\partial F_2}/{\partial Q_3} \\
{\partial F_3}/{\partial Q_1} & {\partial F_3}/{\partial Q_2} & {\partial F_3}/{\partial Q_3}
\end{bmatrix}
\begin{bmatrix}
\Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3
\end{bmatrix}
= -
\begin{bmatrix}
F_1 \\ F_2 \\ F_3
\end{bmatrix}
\right\} \text{Eq-3}
$$

Newton-Raphson Method…

- Solving Equation (3), we get

$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix} = -\begin{bmatrix}\partial F_1/\partial Q_1 & \partial F_1/\partial Q_2 & \partial F_1/\partial Q_3\\ \partial F_2/\partial Q_1 & \partial F_2/\partial Q_2 & \partial F_2/\partial Q_3\\ \partial F_3/\partial Q_1 & \partial F_3/\partial Q_2 & \partial F_3/\partial Q_3\end{bmatrix}^{-1} \begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix}$$

Eq-4

Knowing the corrections, the discharges are improved as

$$\begin{bmatrix}Q_1\\ Q_2\\ Q_3\end{bmatrix} = \begin{bmatrix}Q_1\\ Q_2\\ Q_3\end{bmatrix} + \begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix}$$

Eq-5

Newton-Raphson Procedure

- The overall procedure for looped network analysis by the Newton-Raphson method can be summarized in the following steps:

Step 1: Number all the nodes, pipe links, and loops.

Step 2: Write nodal discharge equations as

$$F_j = \sum_{n=1}^{j_n} Q_{jn} - q_j = 0 \quad \boxed{\text{for all nodes} - 1,}$$

where Q_{jn} is the discharge in n^{th} pipe at node j, q_j is nodal withdrawal, and jn is the total number of pipes at node j.

Step 3: Write loop head-loss equations as

$$F_k = \sum_{n=1}^{k_n} K_n Q_{kn} |Q_{kn}| = 0 \quad \text{for all the loops } (n = 1, k_n).$$

where k_n is total pipes in k_{th} loop.

❑ Newton-Raphson Procedure…

Step 4: Assume initial pipe discharges Q_1, Q_2, and Q_3., . . . satisfying continuity equations.

Step 5: Determine friction factors, f_i, in all pipe links and compute corresponding K_i using

$$K_i = \frac{8 f_i L_i}{\pi^2 g D_i^5},$$

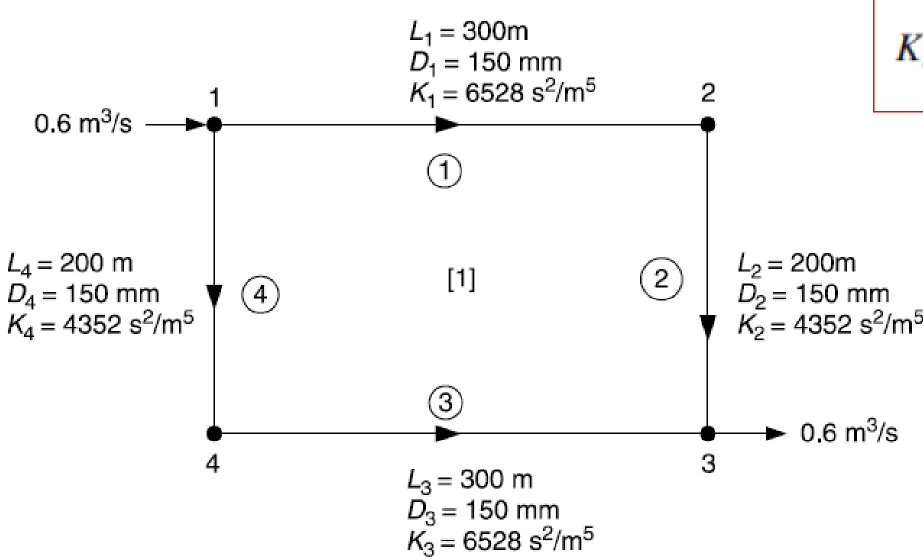
Step 6: Find values of partial derivatives $\partial F_n / \partial Q_i$ and functions F_n, using the initial pipe discharges Q_i and K_i.

Step 7: Find ΔQi. The equations generated are of the form $Ax = b$, which can be solved for ΔQ_i.

Step 8: Using the obtained ΔQ_i values, the pipe discharges are modified and the process is repeated again until the calculated ΔQ_i values are very small.

- **Example Newton-Raphson Method**

The pipe network of single loop as shown in Figure has to be analyzed by the Newton-Raphson method for pipe flows for given pipe lengths L and pipe diameters D. The nodal inflow at node 1 and nodal outflow at node 3 are shown in the figure. Assume a constant friction factor f=0.02 and determine the discharge through all pipes.



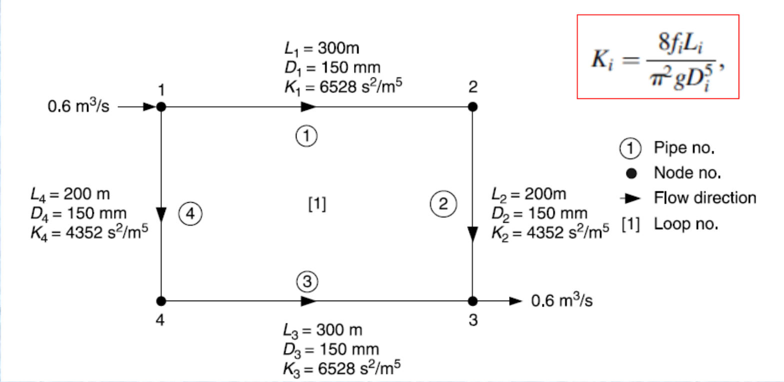
$$K_i = \frac{8 f_i L_i}{\pi^2 g D_i^5},$$

- **Solution**
  - The nodal discharge functions, F are:

$Q_1 + Q_4 - 0.6 = 0$

$-Q_1 + Q_2 = 0$

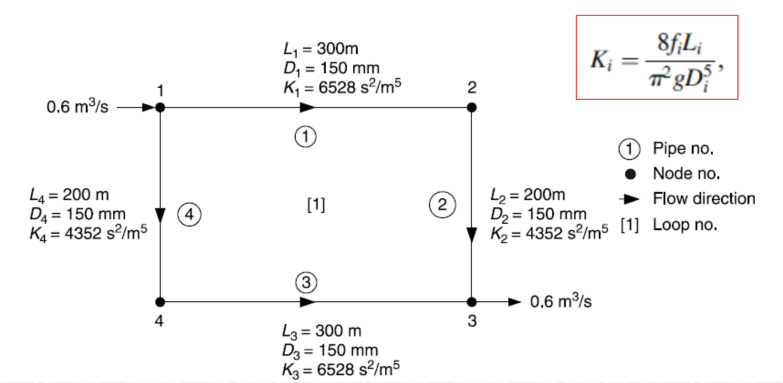
$Q_2 + Q_3 - 0.6 = 0$



and loop head loss functions,

$h = KQ^n$; n=2, $K = \frac{8fL}{g\pi^2 D^5}$ (with proper sign convention)

$6528|Q_1|Q_1 + 4352|Q_2|Q_2 - 6528|Q_3|Q_3 - 4352|Q_4|Q_4 = 0$



Solution…

- The nodal discharge functions, F are:

$$F_1 = Q_1 + Q_4 - 0.6 = 0$$

$$F_2 = -Q_1 + Q_2 = 0$$

$$F_3 = Q_2 + Q_3 - 0.6 = 0$$

and loop head loss functions, (Clockwise +ve)

$$F_4 = 6528|Q_1|Q_1 + 4352\,|Q_2|Q_2 - 6528|Q_3|Q_3 - 4352|Q_4|Q_4 = 0$$

Design procedures for Complex (Looped) Pipe Networks

- **Solution…**
  - The derivatives are:

$$\frac{\partial F_1}{\partial Q_1}=1 \quad \frac{\partial F_1}{\partial Q_2}=0 \quad \frac{\partial F_1}{\partial Q_3}=0 \quad \frac{\partial F_1}{\partial Q_4}=1$$

$$\frac{\partial F_2}{\partial Q_1}=-1 \quad \frac{\partial F_2}{\partial Q_2}=1 \quad \frac{\partial F_2}{\partial Q_3}=0 \quad \frac{\partial F_2}{\partial Q_4}=0$$

$$\frac{\partial F_3}{\partial Q_1}=0 \quad \frac{\partial F_3}{\partial Q_2}=1 \quad \frac{\partial F_3}{\partial Q_3}=1 \quad \frac{\partial F_3}{\partial Q_4}=0$$

$$\frac{\partial F_4}{\partial Q_1}=6528Q_1 \quad \frac{\partial F_4}{\partial Q_2}=4352Q_2 \quad \frac{\partial F_4}{\partial Q_3}=-6528Q^3 \quad \frac{\partial F_4}{\partial Q_4}=-4352Q_4$$

$F_1 = Q_1 + Q_4 - 0.6 = 0$

$F_2 = -Q_1 + Q_2 = 0$

$F_3 = Q_2 + Q_3 - 0.6 = 0$

$F_4 = 6528|Q_1|Q_1 + 4352\ |Q_2|Q_2 - 6528|Q_3|Q_3 - 4352|Q_4|Q_4 = 0$

Solution…

- The generated equations are assembled in the following matrix form:

$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix} = -\begin{bmatrix}\partial F_1/\partial Q_1 & \partial F_1/\partial Q_2 & \partial F_1/\partial Q_3\\ \partial F_2/\partial Q_1 & \partial F_2/\partial Q_2 & \partial F_2/\partial Q_3\\ \partial F_3/\partial Q_1 & \partial F_3/\partial Q_2 & \partial F_3/\partial Q_3\end{bmatrix}^{-1} \begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix}$$

- Substituting the derivatives, the following form is obtained:

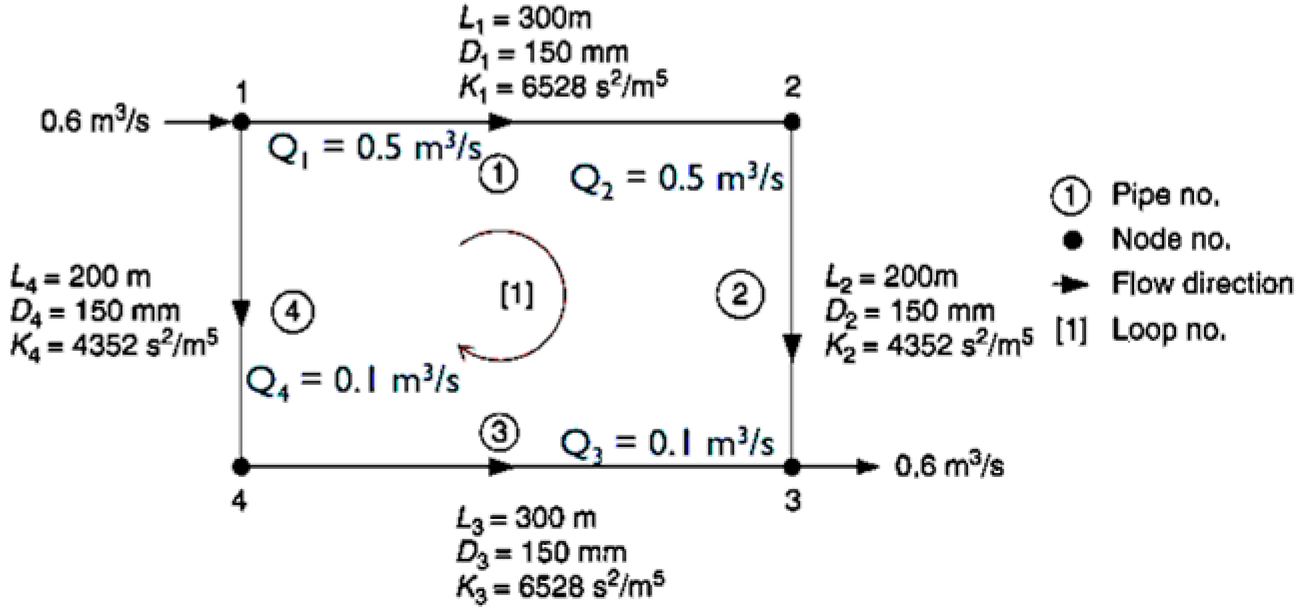
$$\begin{bmatrix}\Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix} = -\begin{bmatrix}1 & 0 & 0 & 1\\ -1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0\\ 6528Q_1 & 4352Q_2 & -6528Q_3 & -4352Q_4\end{bmatrix}^{-1} \begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{bmatrix}$$

Solution…

- Assuming initial pipe discharge in pipe 1 as $Q_1 = 0.5 m^3/s$, the other pipe discharges obtained by continuity equation are:

$$Q_2 = 0.5 \text{ m}^3/\text{s}$$
$$Q_3 = 0.1 \text{ m}^3/\text{s}$$
$$Q_4 = 0.1 \text{ m}^3/\text{s}$$

$$F_1 = Q_1 + Q_4 - 0.6 = 0$$
$$F_2 = -Q_1 + Q_2 = 0$$
$$F_3 = Q_2 + Q_3 - 0.6 = 0,$$
$$F_4 = 6528|Q_1|Q_1 + 4352|Q_2|Q_2 - 6528|Q_3|Q_3 - 4352|Q_4|Q_4 = 0$$



- **Solution…**
  - Substituting these values in the above equation, the following form is obtained:

$$\begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix} = -\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 3264 & 2176 & -652.8 & -435.2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2611.2 \end{bmatrix}$$

  - Using Gaussian elimination method, the solution is obtained as:

$$\Delta Q_1 = -0.2\ \mathrm{m^3/s}$$
$$\Delta Q_2 = -0.2\ \mathrm{m^3/s}$$
$$\Delta Q_3 = 0.2\ \mathrm{m^3/s}$$
$$\Delta Q_4 = 0.2\ \mathrm{m^3/s}$$

- **Solution…**
  - Using these discharge corrections, the revised pipe discharges are:

$$Q_1 = Q_1 + \Delta Q_1 = 0.5 - 0.2 = 0.3\ \mathrm{m^3/s}$$
$$Q_2 = Q_2 + \Delta Q_2 = 0.5 - 0.2 = 0.3\ \mathrm{m^3/s}$$
$$Q_3 = Q_3 + \Delta Q_3 = 0.1 + 0.2 = 0.3\ \mathrm{m^3/s}$$
$$Q_4 = Q_4 + \Delta Q_4 = 0.1 + 0.2 = 0.3\ \mathrm{m^3/s}$$

  - The process is repeated with the new pipe discharges. Revised values of F and derivative ∂F= ∂Q values are obtained. Substituting the revised values, the following new solution is generated:

$$\begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \end{bmatrix} = -\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1958.4 & 1305.6 & -1958.4 & -1305.6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution…

- As the right-hand side is operated upon null vector, all the discharge corrections $\Delta Q=0$. Thus, the final discharges are

$Q_1 = 0.3\ m^3/s$

$Q_2 = 0.3\ m^3/s$

$Q_3 = 0.3\ m^3/s$

$Q_4 = 0.3\ m^3/s$

Design procedures for Complex (Looped) Pipe Networks

- Hardy Cross Method
- Newton-Raphson Method
- **Linear Theory Methods**
- EPANET
- WaterCAD

Linear Theory Method

- Linear theory method is another looped network analysis method presented by Wood and Charles (1972).
- The entire network is analyzed altogether like the Newton-Raphson method.
- The nodal flow continuity equations are obviously linear but the looped head-loss equations are nonlinear.
- The looped energy equations are modified to be linear for previously known discharges and solved iteratively.
- The process is repeated until the two solutions are close to the allowable limits.

Procedures of Linear Theory Method

Step 1: Number pipes, nodes, and loops.

Step 2: Write nodal discharge equations as

$$F_j = \sum_{n=1}^{j_n} Q_{jn} - q_j = 0 \quad \text{for all nodes} - 1,$$

where Q_{jn} is the discharge in the nth pipe at node j, q_j is nodal withdrawal, and j_n the total number of pipes at node j.

Step 3: Write loop head-loss equations as

$$F_k = \sum_{n=1}^{k_n} b_{kn} Q_{kn} = 0 \quad \text{for all the loops.}$$

Procedures of Linear Theory Method…

Step 4: Assume initial pipe discharges $Q_1, Q_2, Q_3, \ldots$. It is not necessary to satisfy continuity equations.

Step 5: Assume friction factors $f_i = 0.02$ in all pipe links and compute corresponding K_i

Step 6: Generalize nodal continuity and loop equations for the entire network.

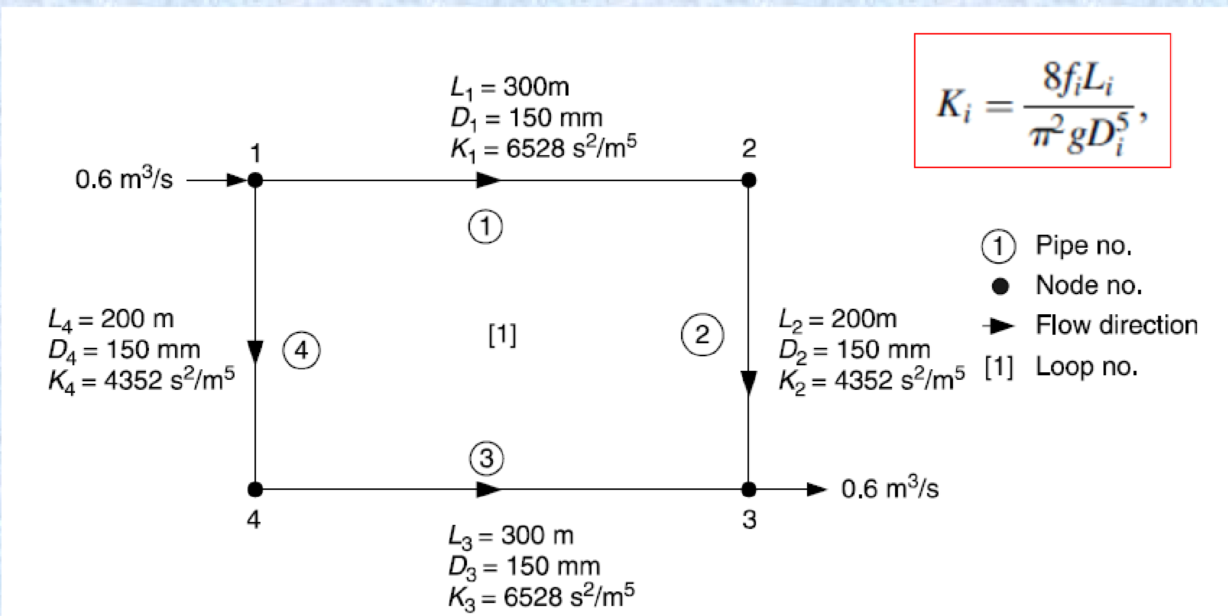
Step 7: Calculate pipe discharges. The equation generated is of the form $Ax = b$, which can be solved for Q_i.

Step 8: Recalculate coefficients b_{kn} from the obtained Q_i values.

Step 9: Repeat the process again until the calculated Q_i values in two consecutive iterations are close to predefined limits.

❑ Example Linear Theory Method

The pipe network of single loop as shown in Figure has to be analyzed by the Newton-Raphson method for pipe flows for given pipe lengths L and pipe diameters D. The nodal inflow at node 1 and nodal outflow at node 3 are shown in the figure. Assume a constant friction factor f=0.02 and determine the discharge through all pipes.



- **Solution:**
  - The nodal discharge functions F can be written as

$$F_1 = Q_1 + Q_4 - 0.6 = 0$$
$$F_2 = -Q_1 + Q_2 = 0$$
$$F_3 = Q_2 + Q_3 - 0.6 = 0,$$

and Loop head-loss function

$$F_4 = K_1|Q_1|Q_1 + K_2|Q_2|Q_2 - K_3|Q_3|Q_3 - K_4|Q_4|Q_4 = 0,$$

Which is linearized as

$$F_4 = b_1Q_1 + b_2Q_2 - b_3Q_3 - b_4Q_2 = 0.$$

- **Solution:**
  - Assuming initial pipe discharge as 0.1 m3/s in all the pipes, the coefficients for head-loss function are calculated as:

$$b_1 = K_1 Q_1 = 6528 \times 0.1 = 652.8$$
$$b_2 = K_2 Q_2 = 4352 \times 0.1 = 435.2$$
$$b_3 = K_3 Q_3 = 6528 \times 0.1 = 652.8$$
$$b_4 = K_4 Q_4 = 4352 \times 0.1 = 435.2.$$

  - Thus the matrix of the form Ax = B can be written as

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 652.8 & 435.2 & -6528.8 & -435.2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0 \\ 0.6 \\ 0 \end{bmatrix}$$

  - Solving the above set of linear equations, the pipe discharges obtained are

$Q_1 = Q_2 = Q_3 = Q_4 = 0.3\ m^3/s$

Design procedures for Complex (Looped) Pipe Networks

- **Solution:**
  - Repeating the process, the revised head-loss coefficients are:

$$b_1 = K_1Q_1 = 6528 \times 0.3 = 1958.4$$
$$b_2 = K_2Q_2 = 4352 \times 0.3 = 1305.6$$
$$b_3 = K_3Q_3 = 6528 \times 0.3 = 1958.4$$
$$b_4 = K_4Q_4 = 4352 \times 0.3 = 1305.6$$

  - Thus the matrix of the form Ax = B can be written as

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1958.4 & 1305.6 & -1958.4 & -1305.6 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0 \\ 0.6 \\ 0 \end{bmatrix}$$

  - Solving the above set of linear equations, the pipe discharges obtained are

$Q_1 = Q_2 = Q_3 = Q_4 = 0.3\ m^3/s$

Thank You!