Lecture-2:
Basic Principles of Pipe Flow (Hydraulics)

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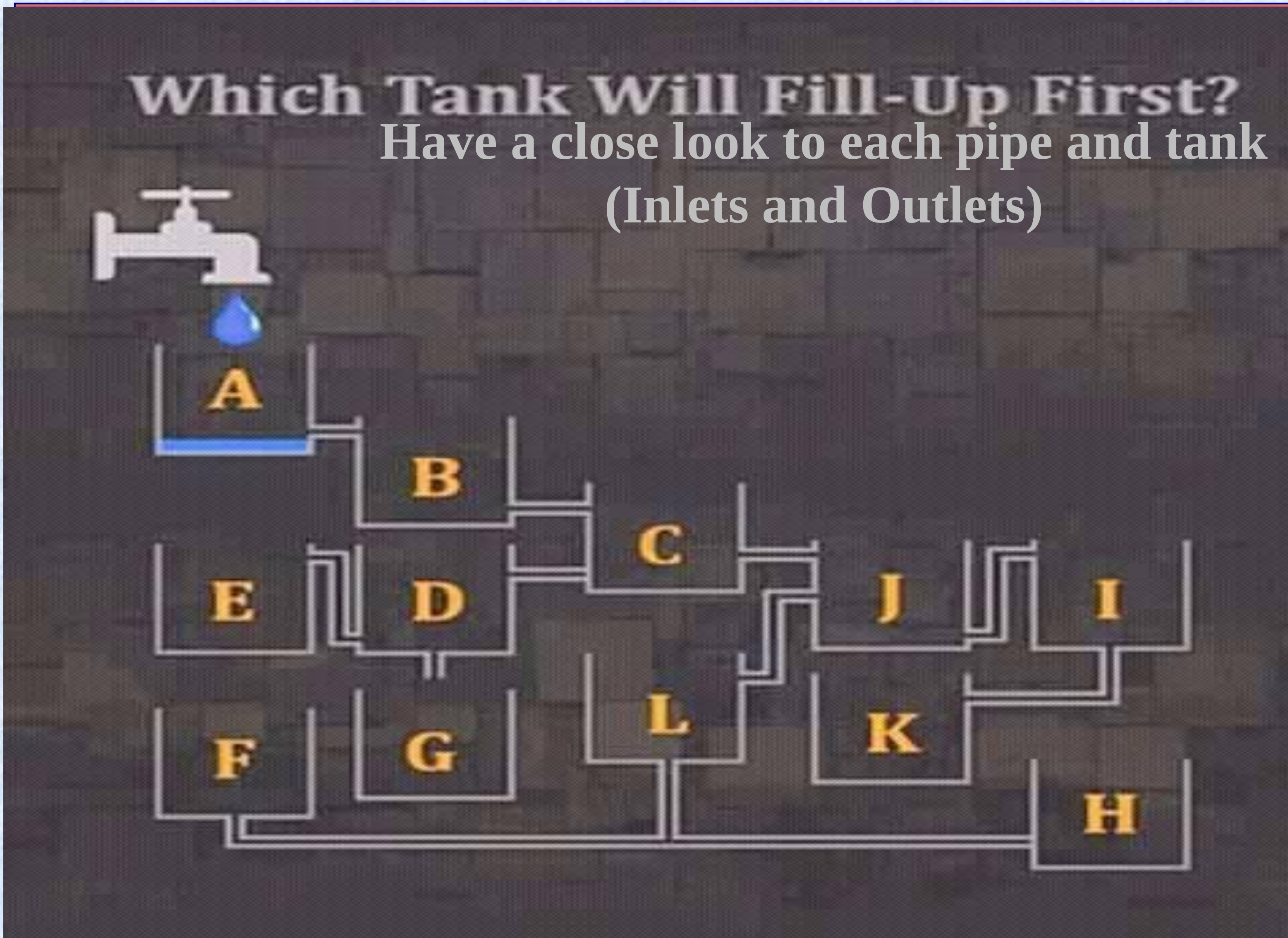
Contents of the Course

1. **Components of Water Supply**
2. **Basic Principles of Pipe Flow (Hydraulics)**
3. The Concept of Modeling
4. Model Calibration
5. Optimization in WDS
6. Water Hammer Theory
7. Water Supply Project Design (Application of Tools)

Basic Principles of Pipe Flow (Hydraulics)

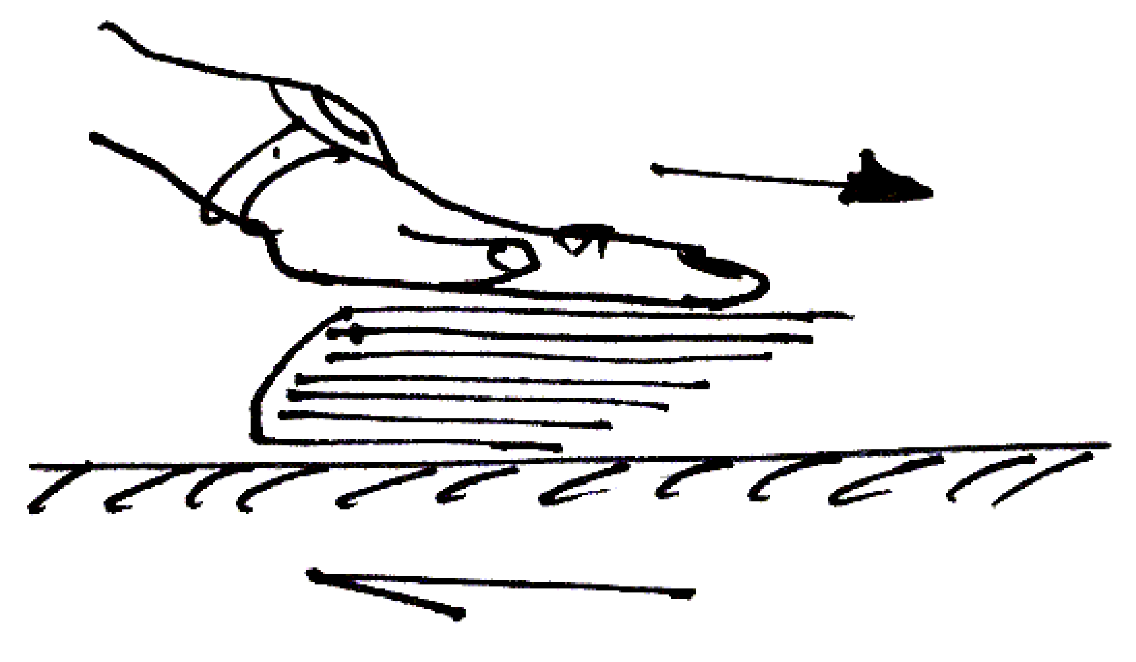
- Fluid Properties- Brief revision
- Fluid Statistics and Dynamics
- Energy Concept
- Friction Losses
- Minor Losses
- Network Hydraulics
- Water quality modeling

Brainstorming



Have a close look to each pipe and tank (Inlets and Outlets)

Properties of Fluid



A fluid is any substance that deforms continuously when subjected to shear stress, no matter how small the shear stress is.

The intermolecular cohesive forces are large in a solid, smaller in a liquid and extremely small in a gas.

Properties of Fluid

Quantity	Symbol	Dimensions
Density	ρ	ML^{-3}
Specific Weight	γ	$ML^{-2}T^{-2}$
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	$L^{2}T^{-1}$
Surface tension	σ	MT^{-2}
Bulk modules of elasticity	E	$ML^{-1}T^{-2}$

These are _fluid___ properties!

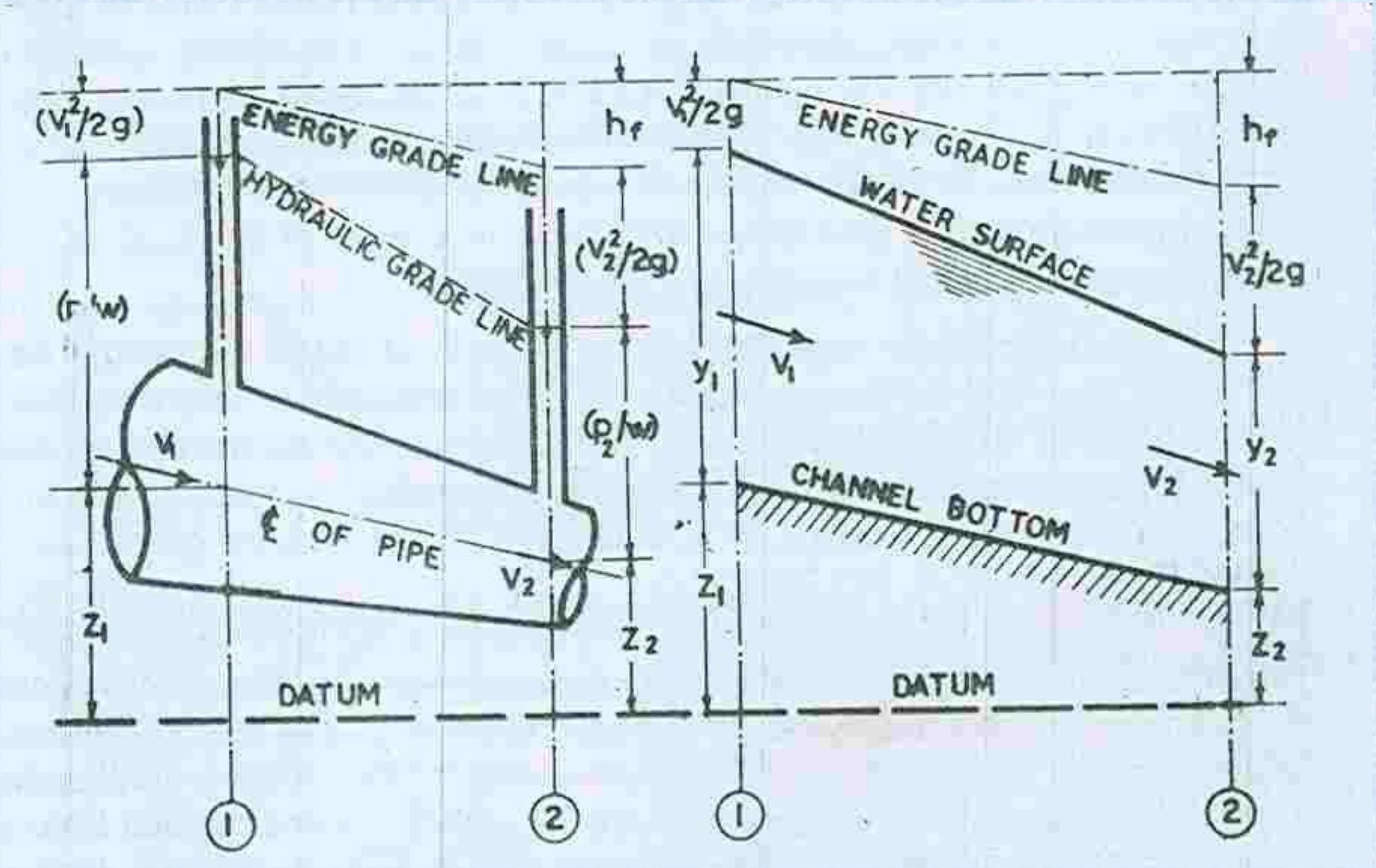
Please Refer your **Hydraulics** course from Undergraduate program

Pipe Flow Analysis

Objectives

- To understand laminar and turbulent flow in pipes and the analysis of fully developed flow
- Able to calculate the major and minor losses associated with pipe flow
- In order calculate and design the sizes of the pipes

Introduction



Comparison of open channel flow and pipe flow

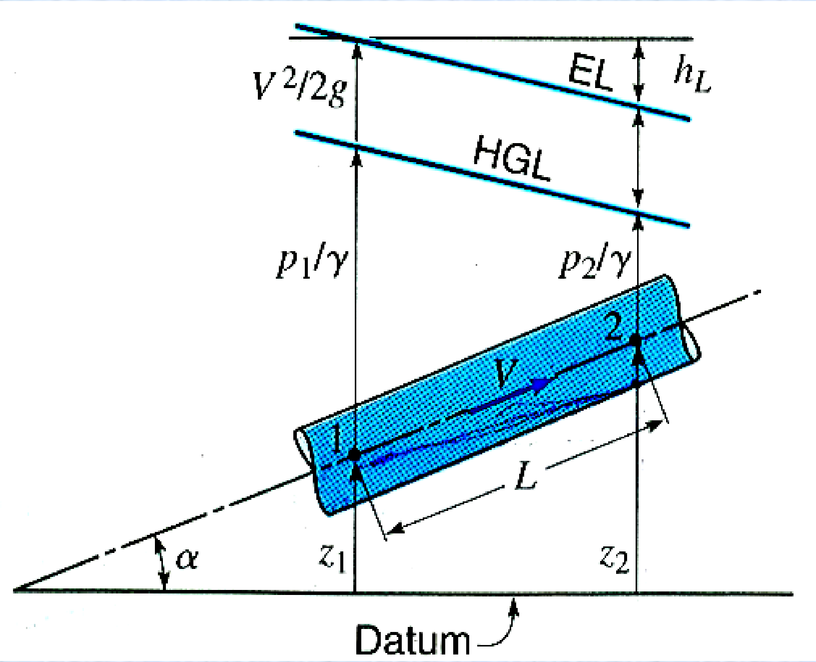
Introduction

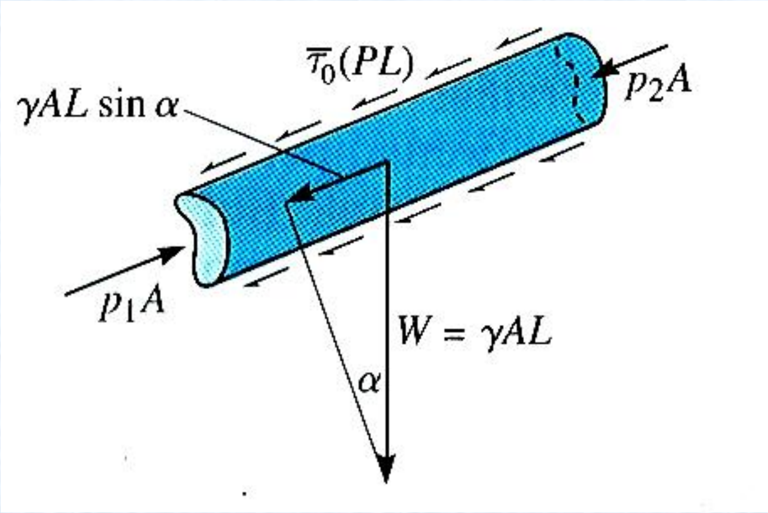
- Water is conveyed from its source, normally in **pressure pipelines**, to water treatment plants where it enters the distribution system and finally arrives at the consumer. *In addition oil, gas, irrigation water, sewerage can be conveyed by pipeline system.*

- The effect of friction is to decrease the pressure, causing a pressure *'loss'* compared to the ideal, frictionless flow case.

- The loss will be divided into ***major losses*** (due to friction in fully developed flow in constant area portions of the system) & ***minor losses*** (due to flow through valves, elbow fittings & frictional effects in other non-constant –area portions of the system).

Major loss (friction) in pipes





$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$p_1A - p_2A - \gamma AL\sin\alpha - \bar{\tau}_o PL = 0$$

$$\sin\alpha = \frac{(z_2 - z_1)}{L}$$

$$h_L = \bar{\tau}_o \frac{PL}{\gamma A} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

Friction in Circular Conduits (Pipe) flowing full

$$h_L = C_f \, 4 \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{V^2}{2g}$$

- for both smooth-walled and rough walled conduits. It is known as ***pipe –friction equation***, and commonly referred to as the ***Darcy-Weisbach*** equation

- Friction factor, *f*, is dimensionless and is also some function of Reynolds number

Pipe friction equations

- Darcy's Weisbach Equation

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \qquad \therefore\ f = \phi_1\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

- Hazen William Equation

$$h_f = \frac{10.675L}{D^{4.8704}}\left(\frac{Q}{C}\right)^{1.852} \quad \text{SI units}$$

- Chezy's Equation
- Manning's Equation

Reynolds Number

- Pipe flow regimes depends on the following factors:
  - *geometry*,
  - *surface roughness*,
  - *flow velocity*,
  - *surface temperature*, and *type of fluid*, among other things.

- After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the **ratio of *inertial forces* to *viscous forces*** in the fluid. This ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as

Laminar and Turbulent Flow

- ***Laminar flow* - ** in laminar flow the particles of **fluid move in an orderly manner & the steam lines** retain the same relative position in successive cross section. Laminar flow is associated with low velocity of flow and viscous fluids.

- ***Turbulent flow* - ** Here the fluid particles flow in a **disorder manner occupying different relative positions in successive cross section**. Turbulent flow is associated with high velocity flows.

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\mathrm{avg}} D}{\nu} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$

$$\begin{aligned} \mathrm{Re} &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim \mathrm{Re} &\lesssim 4000 && \text{transitional flow} \\ \mathrm{Re} &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

Pipe friction equations

1. For laminar flow type $$f = 64\frac{v}{DV} = \frac{64}{\mathrm{Re}}$$

2. For Transition flow type $$f = \left\{-2\log_{10}\left[\frac{(\varepsilon/D)}{3.7} + \frac{2.51}{\mathrm{Re}\,(f^{1/2})}\right]\right\}^{-2}$$

3. For *hydraulically turbulent smooth pipes (e= 0)* such as glass, copper,

$$f = \frac{0.3164}{\mathrm{Re}^{0.25}} \quad (4{,}000 < Re < 100{,}000)$$ Blasius equation

$$\frac{1}{\sqrt{f}} = 2\log\left(\mathrm{Re}/\sqrt{f}\right) - 0.8$$ Von Karman's and Prandtl equation for Re upto $3*10^6$

4. For Complete turbulence rough pipe flow type $$f = \left[1.14 + 2\log_{10}\left(\frac{D}{\varepsilon}\right)\right]^{-2}$$

Pipe friction equations

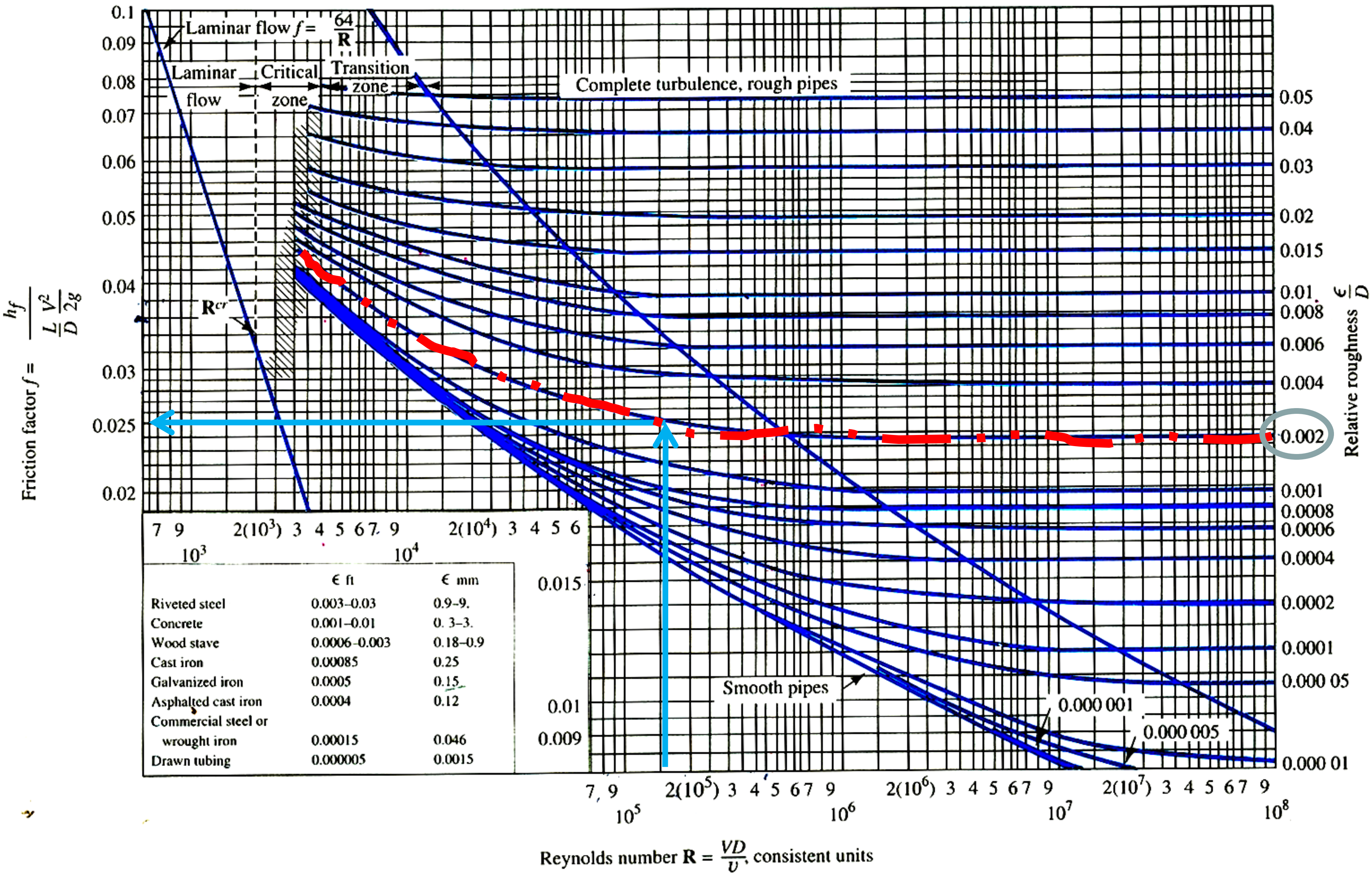
For all pipes, a general empirical formula by Colebrook - White is given by:

$$\frac{1}{\sqrt{f}} = -0.86 \ln\left(\frac{e}{3.71D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

The above equation is awkward to solve

- In 1944, Leewis F. Moody plotted the Darcy–Weisbach friction factor into what is now known as the Moody chart and diagrams are available to give the relation between f, Re, and e / D.

MOODY CHART



Laminar flow $f = \frac{64}{R}$
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes
R_{cr}
Friction factor $f = \frac{h_f}{\frac{L}{D}\frac{V^2}{2g}}$
Relative roughness $\frac{\epsilon}{D}$
Smooth pipes
0.000 001
0.000 005
Reynolds number $R = \frac{VD}{\upsilon}$, consistent units
0.1 0.09 0.08 0.07 0.06 0.05 0.04 0.03 0.025 0.02 0.015 0.01 0.009
0.05 0.04 0.03 0.02 0.015 0.01 0.008 0.006 0.004 0.002 0.001 0.0008 0.0006 0.0004 0.0002 0.0001 0.000 05 0.000 01
7 9 10^3 $2(10^3)$ 3 4 5 6 7 9 10^4 $2(10^4)$ 3 4 5 6
7 9 10^5 $2(10^5)$ 3 4 5 6 7 9 10^6 $2(10^6)$ 3 4 5 6 7 9 10^7 $2(10^7)$ 3 4 5 6 7 9 10^8
	∈ ft	∈ mm
Riveted steel	0.003–0.03	0.9–9.
Concrete	0.001–0.01	0. 3–3.
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

Pipe roughness

pipe material	pipe roughness ε (mm)
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18 - 0.6
rivet steel	0.9 - 9.0
corrugated metal	45
PVC	0.12

Questions

- Can the Darcy-Weisbach equation and Moody Diagram be used for fluids other than water? __Yes__
- What about the Hazen-Williams equation? __No__
- Does a perfectly smooth pipe have head loss? __Yes__
- Is it possible to decrease the head loss in a pipe by installing a smooth liner? __Yes__

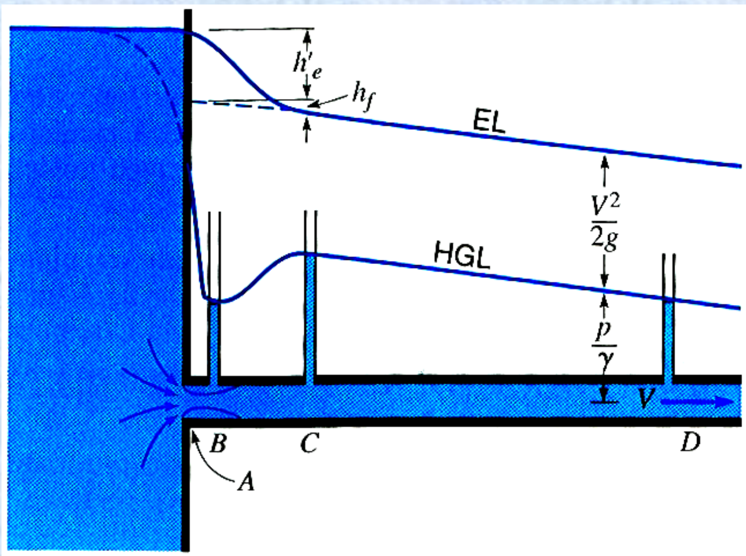
Minor Losses in the Pipes

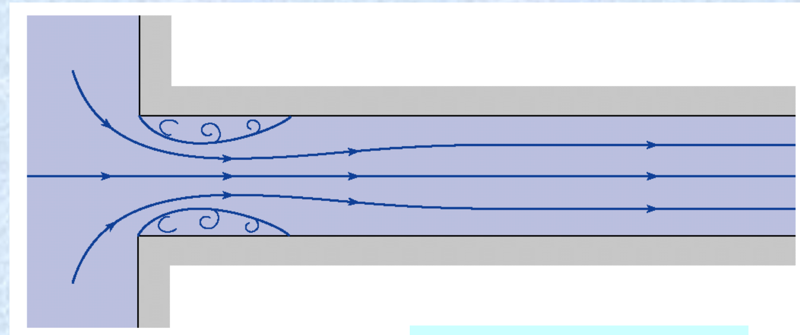
- Loss due to the local disturbances of the flow conduits such as changes in cross section, projecting gaskets, elbows, valves, and similar items are called ***minor Losses***.

- In the case of a very long pipe or channel, these losses may be insignificant in comparison with the fluid friction in the length considered.

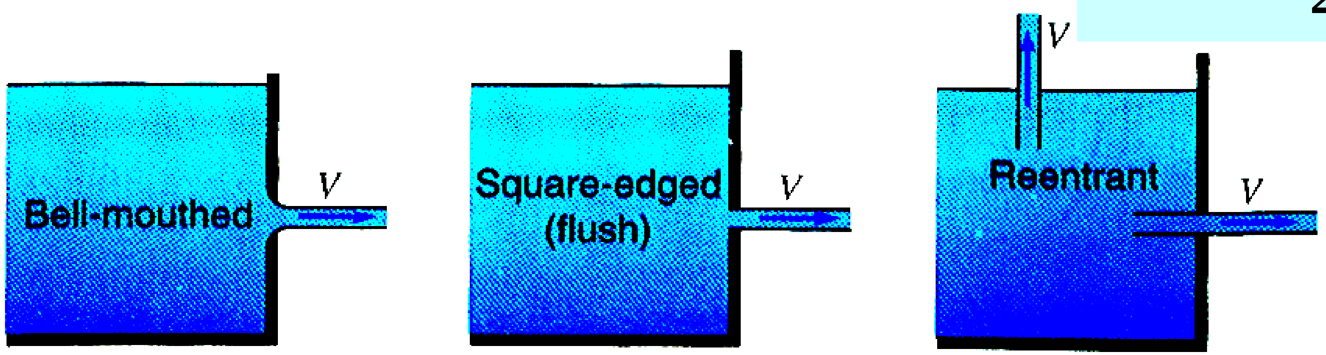
$$k\frac{V^2}{2g}=\frac{f\left(ND\right)}{D}\frac{V^2}{2g}$$

Loss of Head at Entrance





$$h_e^{'} = k_e \frac{V^2}{2g}$$



(a) $k_e = 0.04$ (b) $k_e = 0.5$ (c) $k_e \approx 0.8$

Loss of Head at Entrance

- Entrance flow condition and loss coefficients

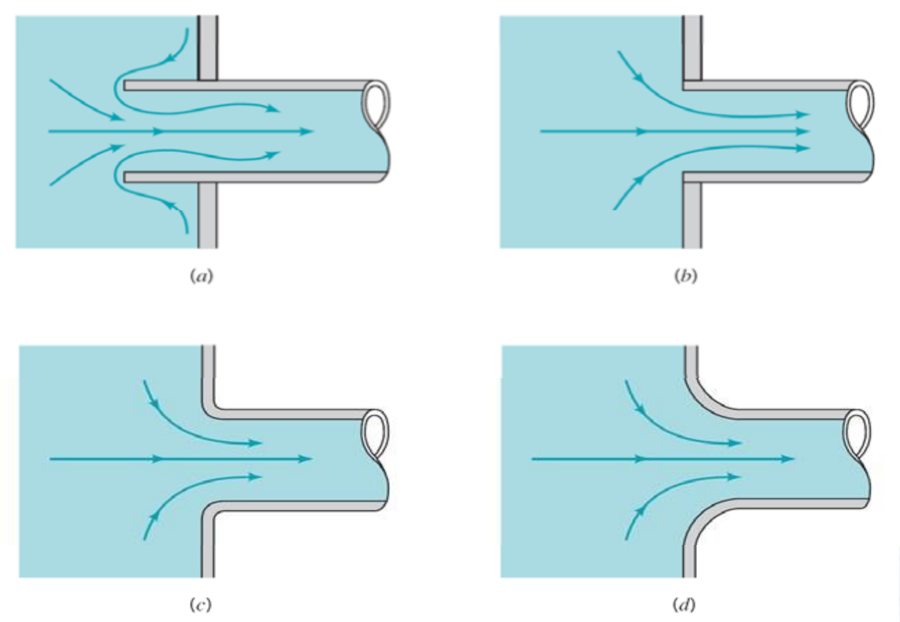
(a) Reentrant, $K_L = 0.8$

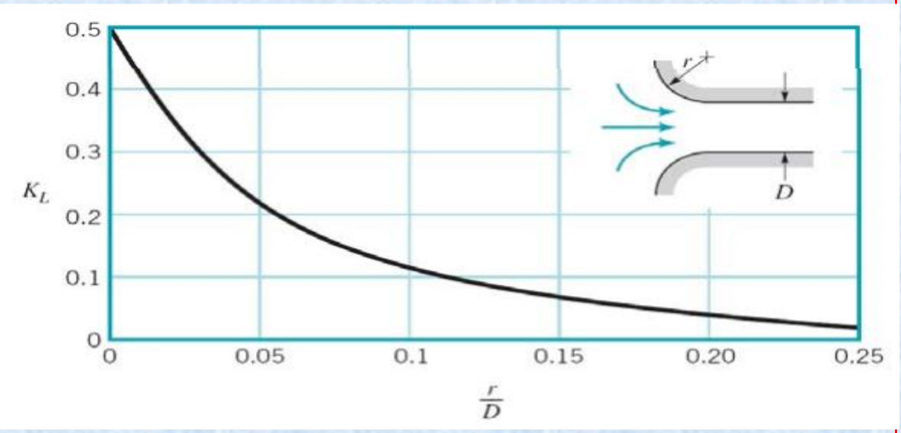
(b) sharp-edged, $K_L = 0.5$

(c) slightly rounded, KL = 0.2

(d) well-rounded, $K_L = 0.04$

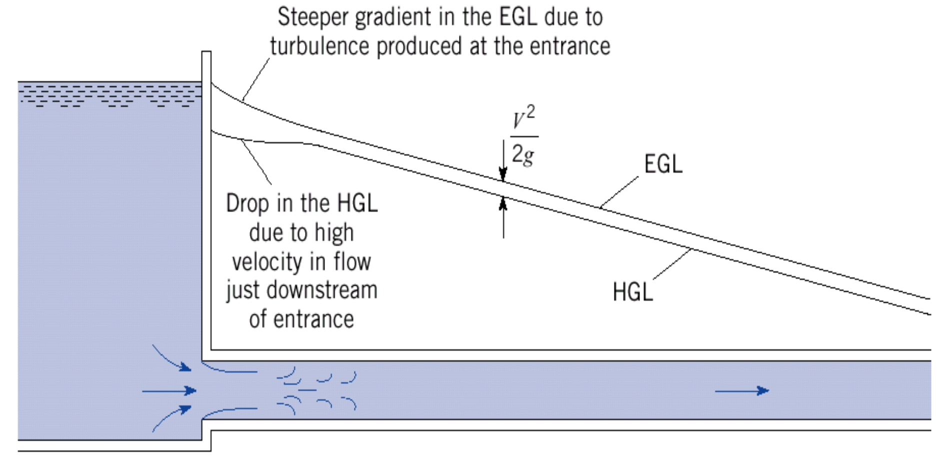
K_L = function of rounding of the inlet edge.

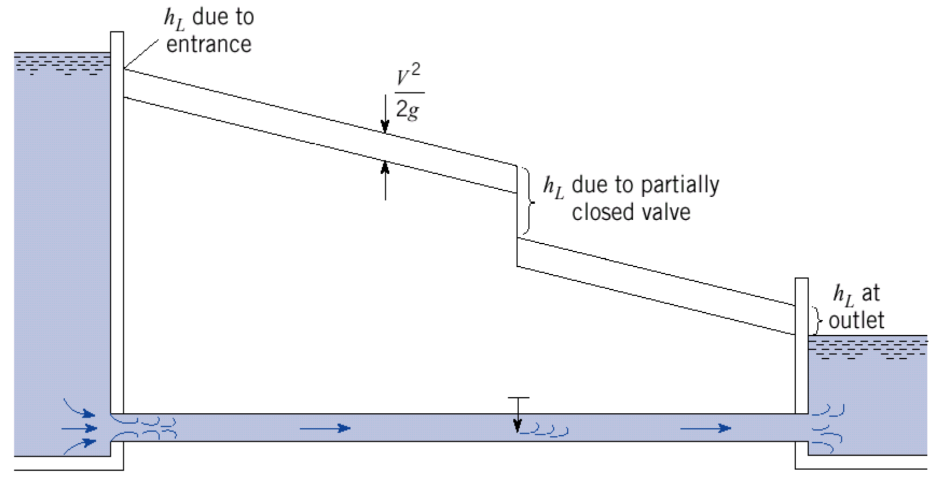




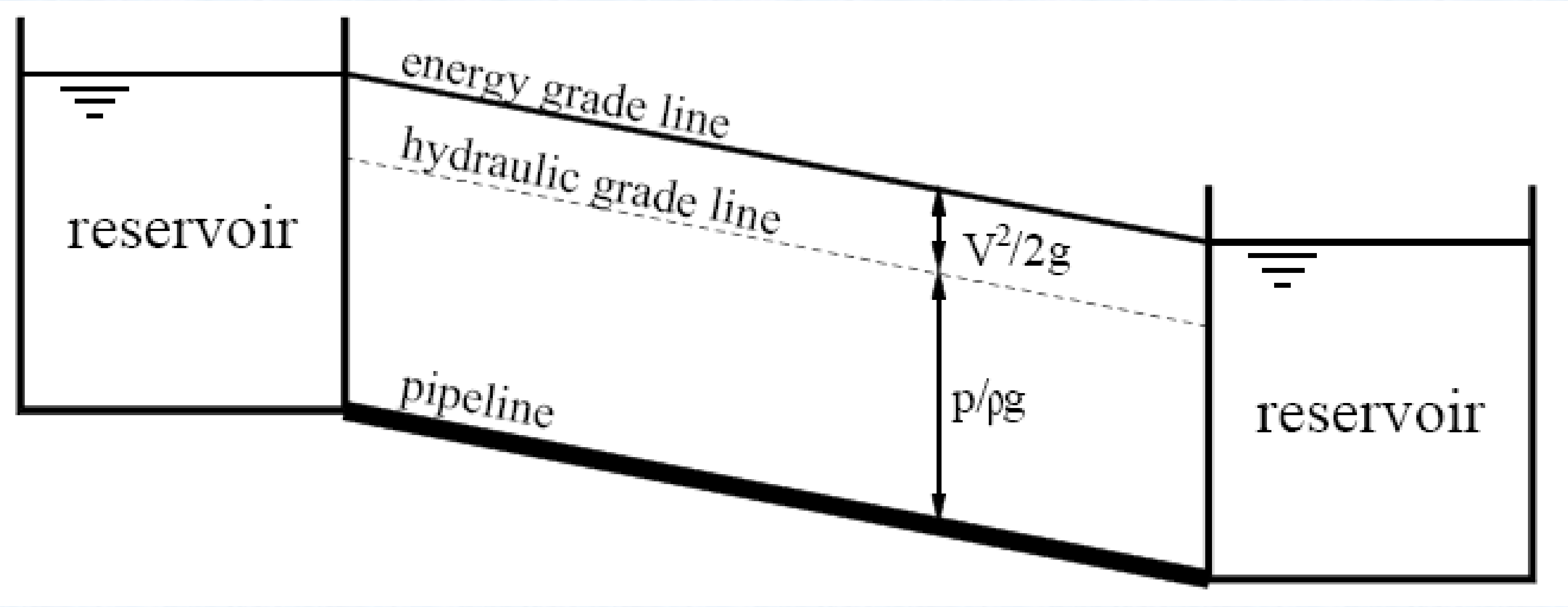
EGL & HGL for Losses in a Pipe

- Entrances, bends, and other flow transitions cause the EGL to drop an amount equal to the head loss produced by the transition.
- EGL is steeper at entrance than it is downstream of there where the slope is equal the frictional head loss in the pipe.
- The HGL also drops sharply downstream of an entrance

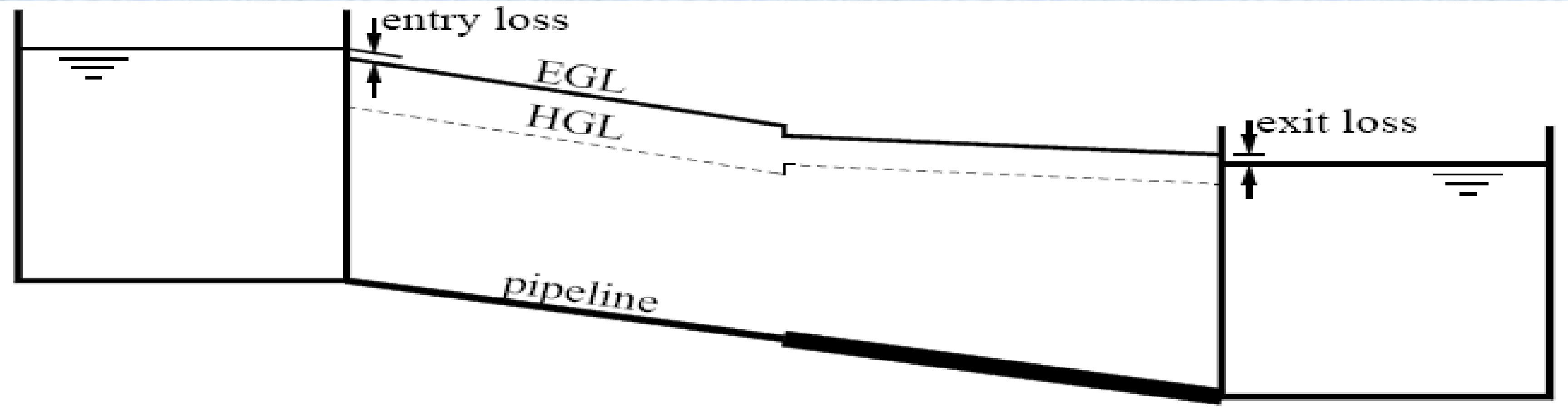


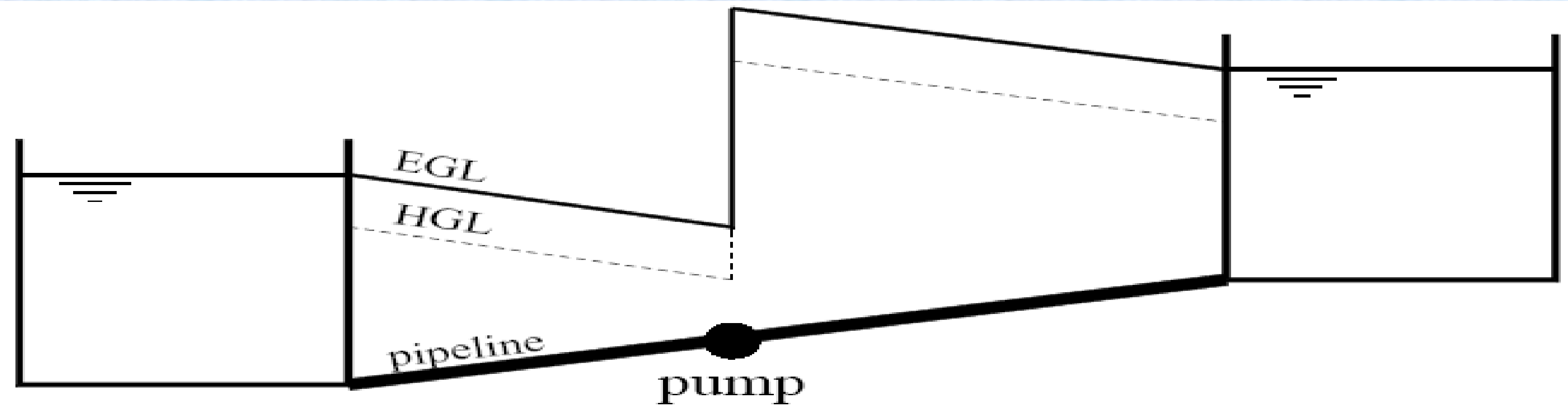


EGL & HGL for Losses in a Pipe

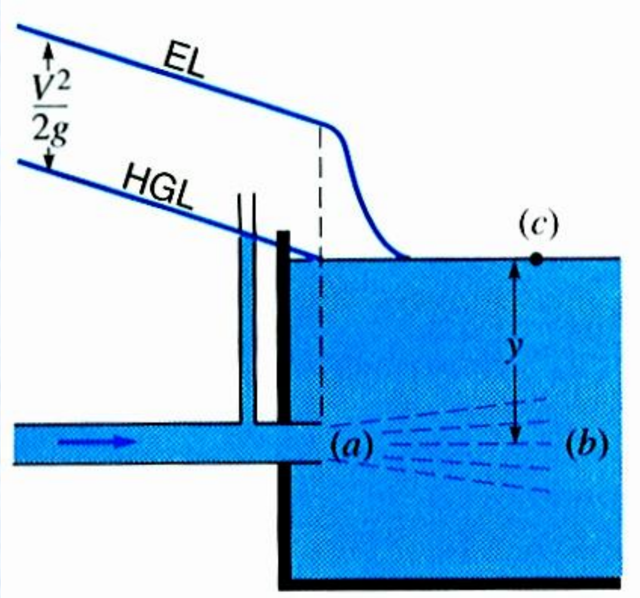


EGL & HGL for Losses in a Pipe





Loss of head at submerged discharges: (leave of pipe), ($h_{d'}$)



$$H_a = y + 0 + V^2/2g$$

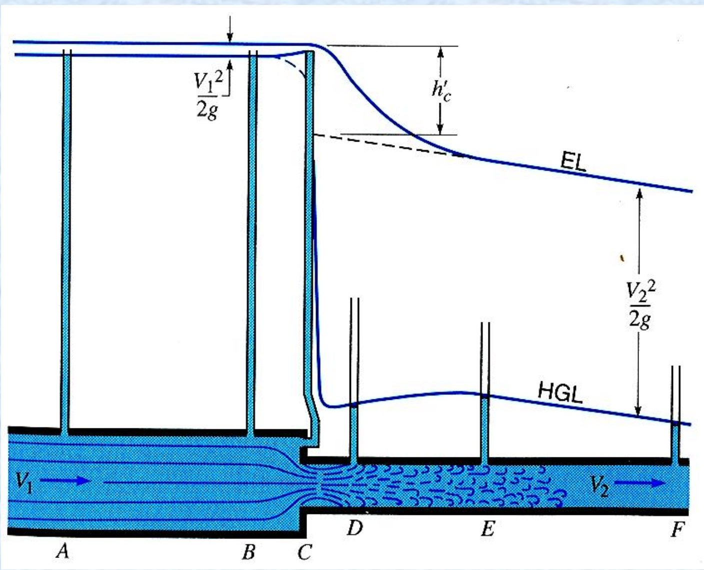
$$H_c = 0 + y + 0$$

$$h_d' = H_a - H_c = \frac{V^2}{2g}$$

k_c

Loss Due to Contraction

- ***Sudden Contraction***



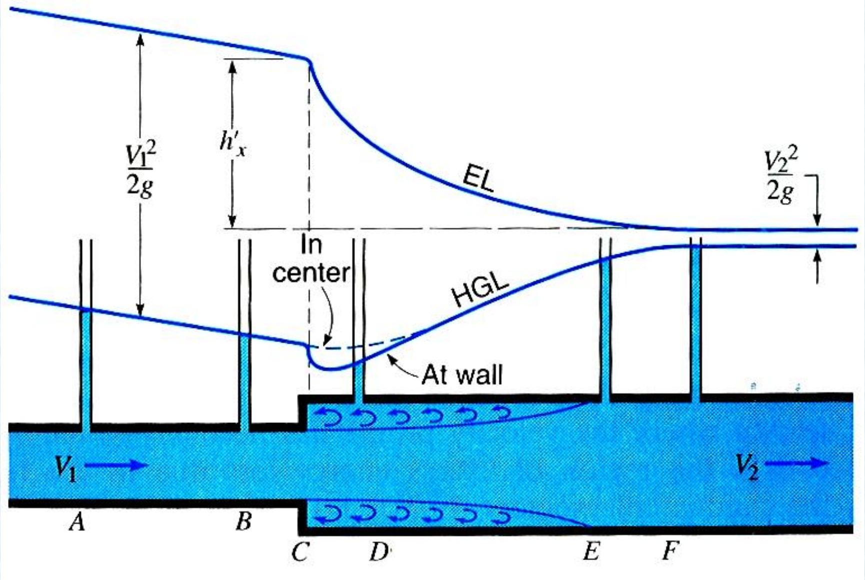
Losses due to gradual contraction the value of Kc = 0.05 – 0.10

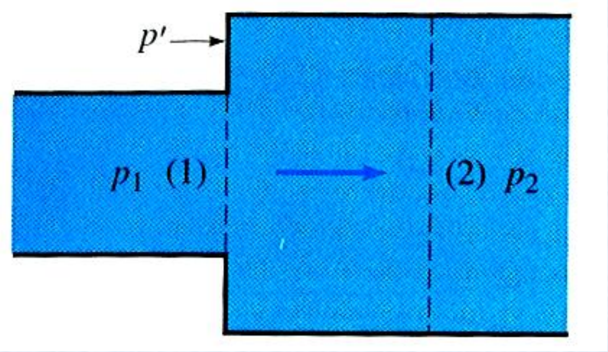
$$h_c^{'} = k_C \frac{V_2^2}{2g}$$

Losses coefficients for sudden contraction

D_2/D_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k_c	0.50	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.00

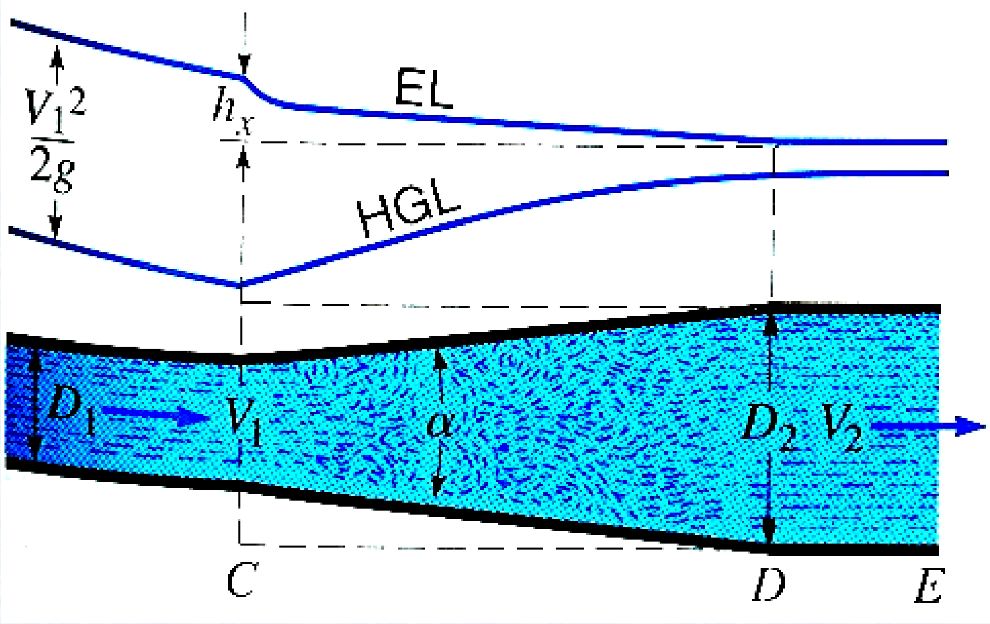
Loss due to sudden expansion





$$\therefore h_e = \frac{(V_1 - V_2)^2}{2g} = \left(\frac{D_2^2}{D_1^2} - 1\right)^2 \frac{V_2^2}{2g}$$

Gradual Expansion



$$h_e' = K'\frac{(V_1 - V_2)^2}{2g}$$

K'-is a function of cone angle α.

K'	0.4	0.6	0.95	1.1	1.18	1.09	1.0	1.0
α	20^0	30^0	40^0	50^0	60^0	90^0	120^0	180^0

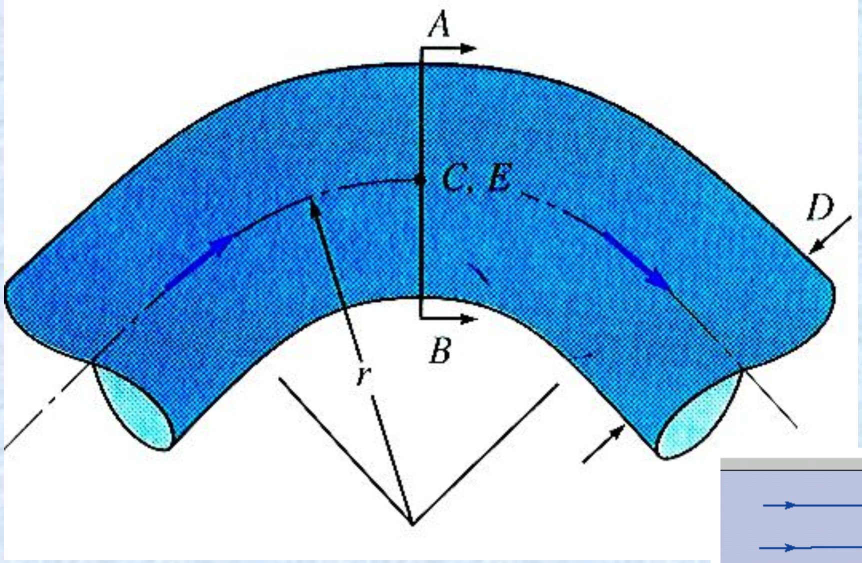
Loss in pipe fittings

The values of "K_f" depends on the type of fittings

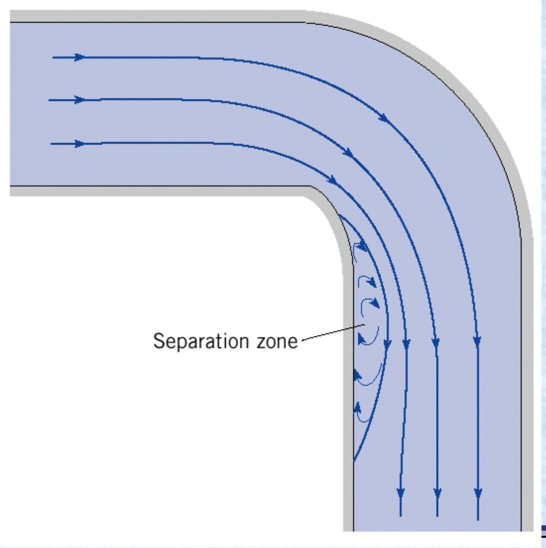
$$h_f = k_f \frac{V^2}{2g}$$

Fitting	K
Globe valve, wide open	10
Angle valve, wide open	5
Close –return bend	2.2
T-through side outlet	1.8
Short-radius elbow	0.9
Medium radius elbow	0.75
Long radius elbow	0.60
Gate valve, wide open	0.19
Half open	2.06
Pump foot value	5.60
Standard branch flow	1.80

Losses in bend & Elbow



$$h_b = k_b \frac{V^2}{2g}$$



Solution of single – pipe flow problems

The total head losses between two points is the sum of the pipe friction loss plus the minor losses, or

$$h_L = h_{Lf} + \sum h'$$

h_L – total head loss

h_{Lf} – major head loss $\quad h_{Lf} = f \frac{L}{D} \frac{V^2}{2g}$

$\sum h'$ - total minor loss

Pipe flow problems

- The above equation (h_L) relates four variables. Any one of these may be unknown quantity in practical flow situation. These are:

i.	L, Q, D	known	h_L	unknown
ii.	h_L, Q, D	known	L	unknown
iii.	h_L, Q, L,	known	D	unknown
iv.	h_L, L, D,	known	Q	unknown

Example

- A 100m length of smooth horizontal pipe is attached to a large reservoir. What depth, d, must be maintained in the reservoir to produce a volume flow rate of 0.03m^3/sec of water? The inside diameter of the smooth pipe is 75mm. The inlet of the pipe is square edged. The water discharges to the atmosphere. Assume that density of the fluid is 1000kg/cubic meter and $\mu = 10^{-3} kg/m.s$

- **Solution**

$$\left(\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+Z_1\right)-\left(\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+Z_2\right)=h_{LT}$$

- **h_{LT} = h_{Lf} +h_{Lm}**

$$h_{LT} = f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}$$

- But P_1=P_2= P_{atm}, $V_1 \cong 0$, V_2 = V, Z_2 = 0 (measured from the center of the pipe line, then Z_1 = d.

$$h_{LT} = d-\frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g}+k\frac{V^2}{2g}$$

$$d=\frac{v^2}{2g}\left[f\frac{L}{D}+K+1\right]$$

$$V_2 = V = \frac{Q}{A_2} = \frac{4Q}{\Pi D_2^2}, then$$

$$d=\frac{8Q^2}{\pi^2 D^4 g}\left[f\frac{L}{D}+k+1\right]$$

Example…

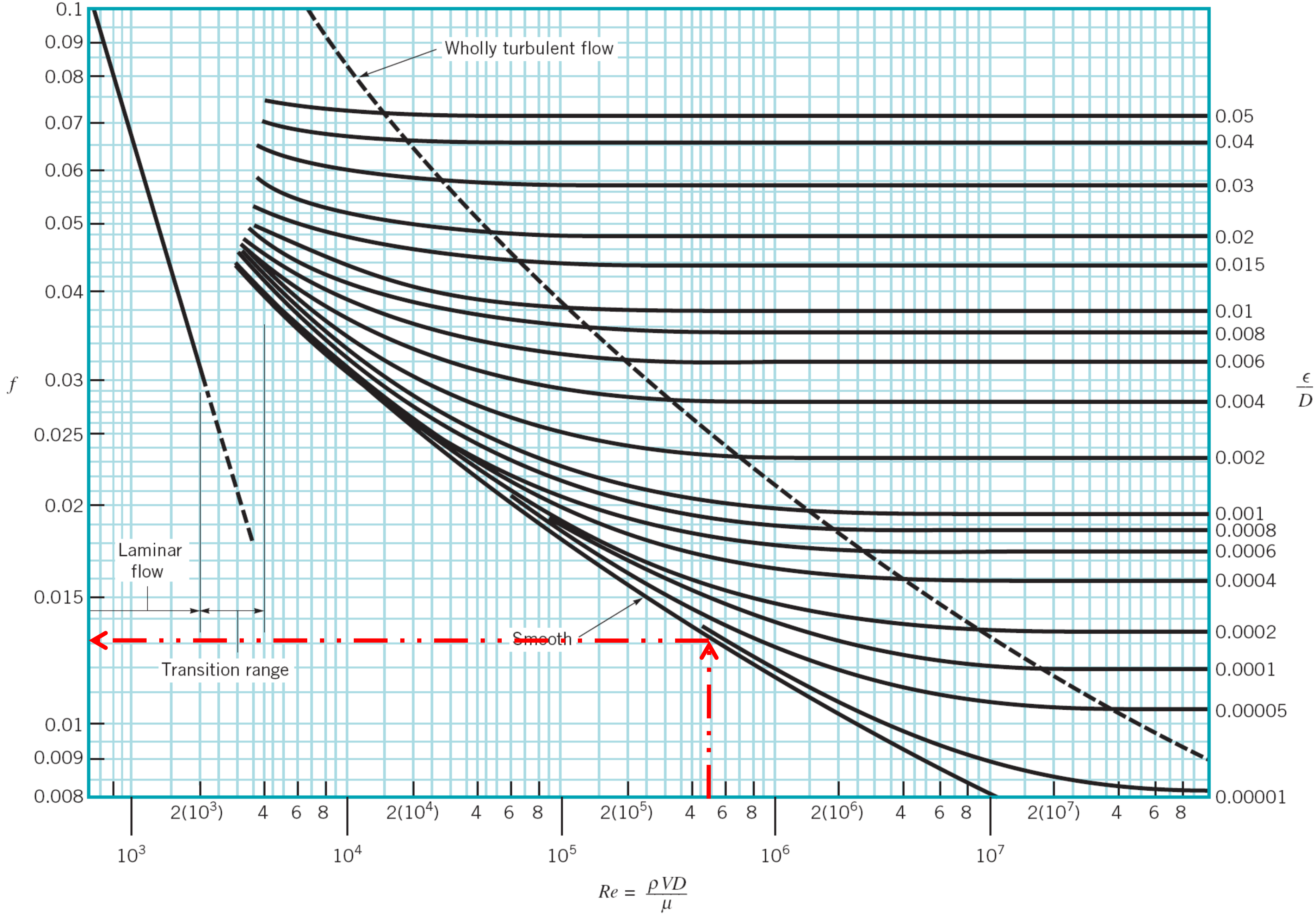
$$Re = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4}{\pi} * \frac{1000 * 0.03}{1x10^{-3} * 0.075} = 5.10x10^{5}$$

- For smooth pipe from Moody diagram, f = 0.0131, then k = 0.5 for square-edged.

$$d = \frac{8}{\pi^{2}} * \frac{(0.03)^{2}}{(0.075)^{4} * 9.81} * \left[0.0131 * \frac{100}{0.075} + 0.5 + 1\right]$$

$$d = 44.6m$$

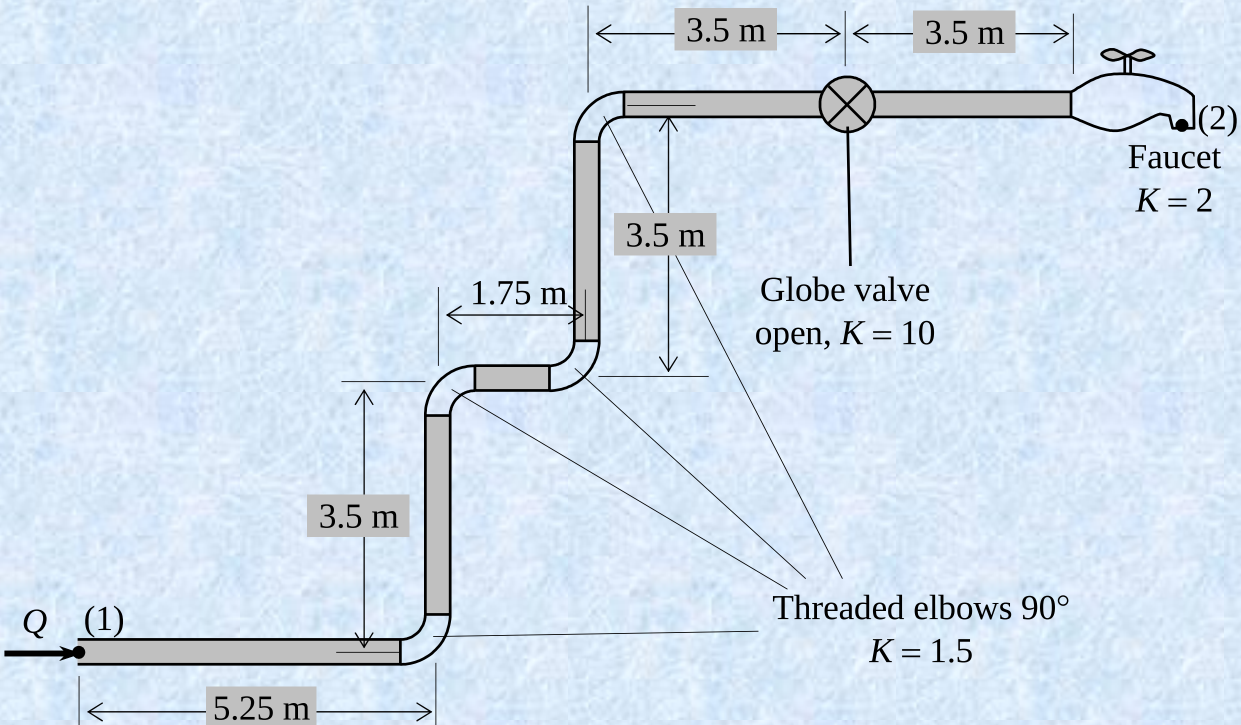
MOODY CHART

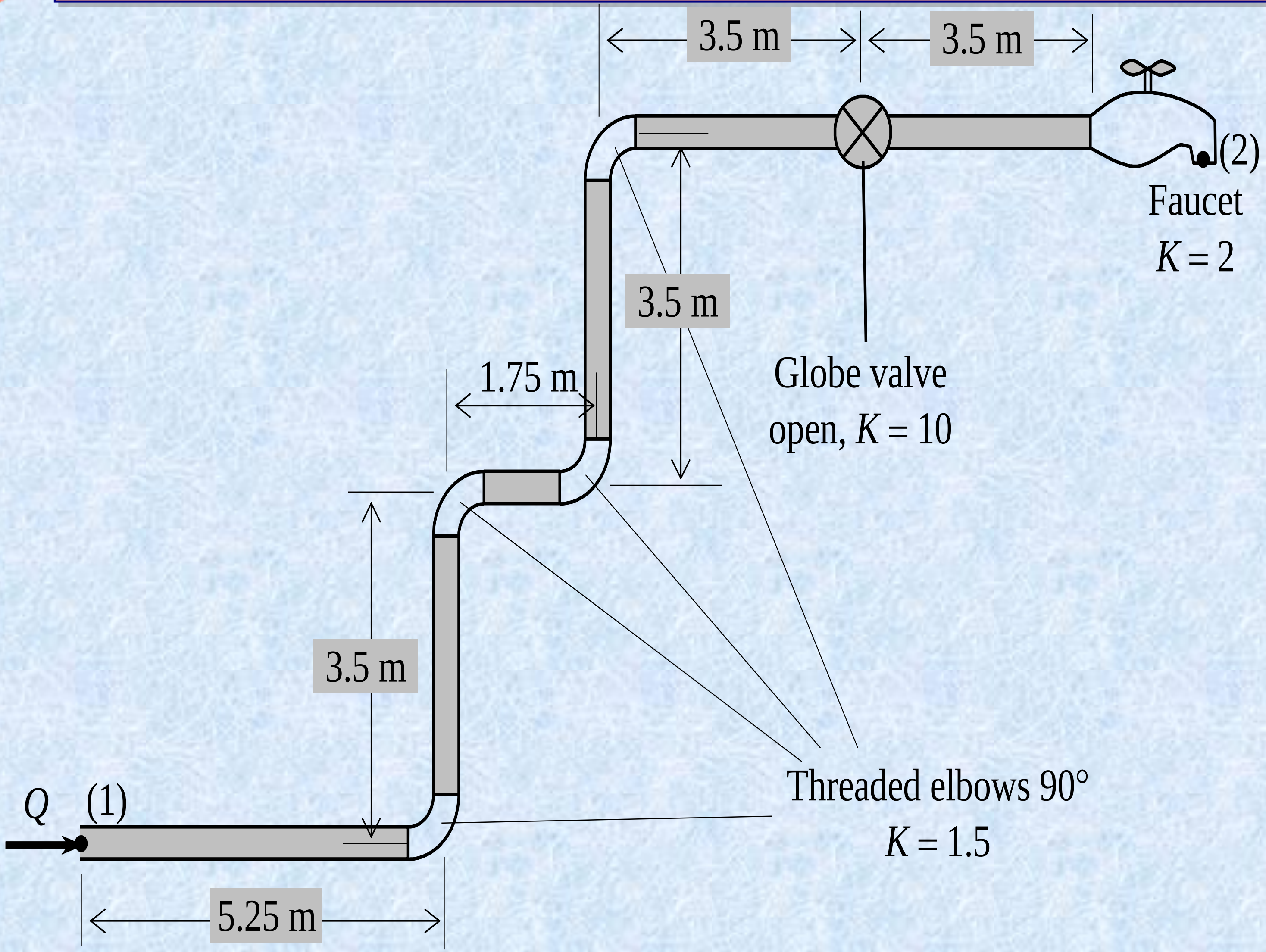


Wholly turbulent flow
Laminar flow
Transition range
Smooth
f
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
$\frac{\epsilon}{D}$
10^3
$2(10^3)$
4
6
8
10^4
$2(10^4)$
10^5
$2(10^5)$
10^6
$2(10^6)$
10^7
$2(10^7)$
$Re = \frac{\rho VD}{\mu}$

Example

- Water flows from the ground floor to the second level in a three-storey building through a 20mm diameter pipe (drawn-tubing, $\varepsilon = 0.0015$ mm) at a rate of 0.75 liter/s. The layout of the whole system is illustrated in Figure below. The water flows out from the system through a valve with an opening of diameter 12.5 mm. Calculate the pressure at point (1).




3.5 m
3.5 m
(2)
Faucet
K = 2
3.5 m
1.75 m
Globe valve
open, K = 10
3.5 m
Q
(1)
Threaded elbows 90°
K = 1.5
5.25 m

Solution

From the modified Bernoulli equation, we can write

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 + \rho g h_L$$

In this problem, p_2 = 0, z_1 = 0. Thus,

$$p_1 = \frac{1}{2}\left(V_2^2 - V_1^2\right) + \rho g z_2 + \rho g\left(h_1 + h_m\right)$$

The velocities in the pipe and out from the faucet are respectively

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4\left(0.75\times10^{-3}\right)}{\pi(0.020)^2} = 2.387\, m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2} = \frac{4\left(0.75\times10^{-3}\right)}{\pi(0.012)^2} = 6.631 m/s$$

The Reynolds number of the flow is

$$\mathrm{Re} = \frac{\rho V d}{\mu} = \frac{(998)(2.387)(0.020)}{1.12\times10^{-3}} = 42{,}546$$

Solution

The roughness ε/d = 0.0015/20 = 0.000075. From the Moody chart, $f \approx 0.022$ (or,0.02191 via the Colebrook formula). The total length of the pipe is

Hence, the friction head loss is $\ell = 5.25 + 4(3.5) + 1.75 = 21\text{m}$

The total minor loss is $h_f = f\dfrac{\ell}{d}\dfrac{V_1^2}{2g} = (0.022)\dfrac{21}{0.02}\dfrac{2.387^2}{2(9.81)} = 6.71\text{m}$

$$h_m = \sum K\frac{V_1^2}{2g} = \left[4(1.5) + 10 + 2\right]\frac{2.387^2}{2(9.81)} = 5.23\text{m}$$

$$\Delta h_{\omega\tau} = h_f + h_m = 6.71 + 5.23 = 11.94\text{m}$$

Therefore, the pressure at (1) is

$$p_1 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) + \rho g z_2 + \rho g\left(h_1 + h_m\right)$$

$$= \frac{1}{2}(998)\left(6.631^2 - 2.387^2\right) + (998)(9.81)(3.5 + 3.5)$$

$$+ 998(9.81)(6.71 + 5.23)$$

$$= 205k\text{Pa}$$

Example

Consider a water flow in a pipe having a diameter of D = 20 mm which is intended to fill a 0.35 liter container. Calculate:

(a) the minimum time required if the flow is laminar,

(b) the maximum time required if the flow is turbulent.

Use density ρ = 998 kg/m3 and dynamic viscosity μ = 1.12×10–3 kg/m·s

Solution:

(a) For laminar flow, use Re =$\rho VD/\mu$ = 2300: $V = \frac{2300\mu}{\rho D} = \frac{2300\left(1.12\times10^{-3}\right)}{(998)(0.020)} = 0.118\,m/s$

$$t = \frac{\cancel{V}}{Q} = \frac{4\cancel{V}}{\pi D^2 V}$$

Hence, the minimum time t is $= \frac{4\left(0.35\times10^{-3}\right)}{\pi(0.02)^2(0.118)} = \underline{9.45s}$

b) For turbulent flow, use Re = $\rho VD/\mu$ = 4000: $V = \frac{4000\mu}{\rho D} = \frac{4000\left(1.12\times10^{-3}\right)}{(998)(0.020)} = 0.224\,m/s$

$$t = \frac{\cancel{V}}{Q} = \frac{4\cancel{V}}{\pi D^2 V}$$

Hence, the minimum time t is $= \frac{4\left(0.35\times10^{-3}\right)}{\pi(0.02)^2(0.224)} = \underline{4.96s}$

Pipe line with Pump or Turbine

- The pump lifts the fluid a height

- the power delivered to the liquid by the pump is $(\Delta Z + \sum h_l)$

- The power required to run the pump is greater than this, depending on the efficiency of the pump. The total pumping head, hp, for this case is:

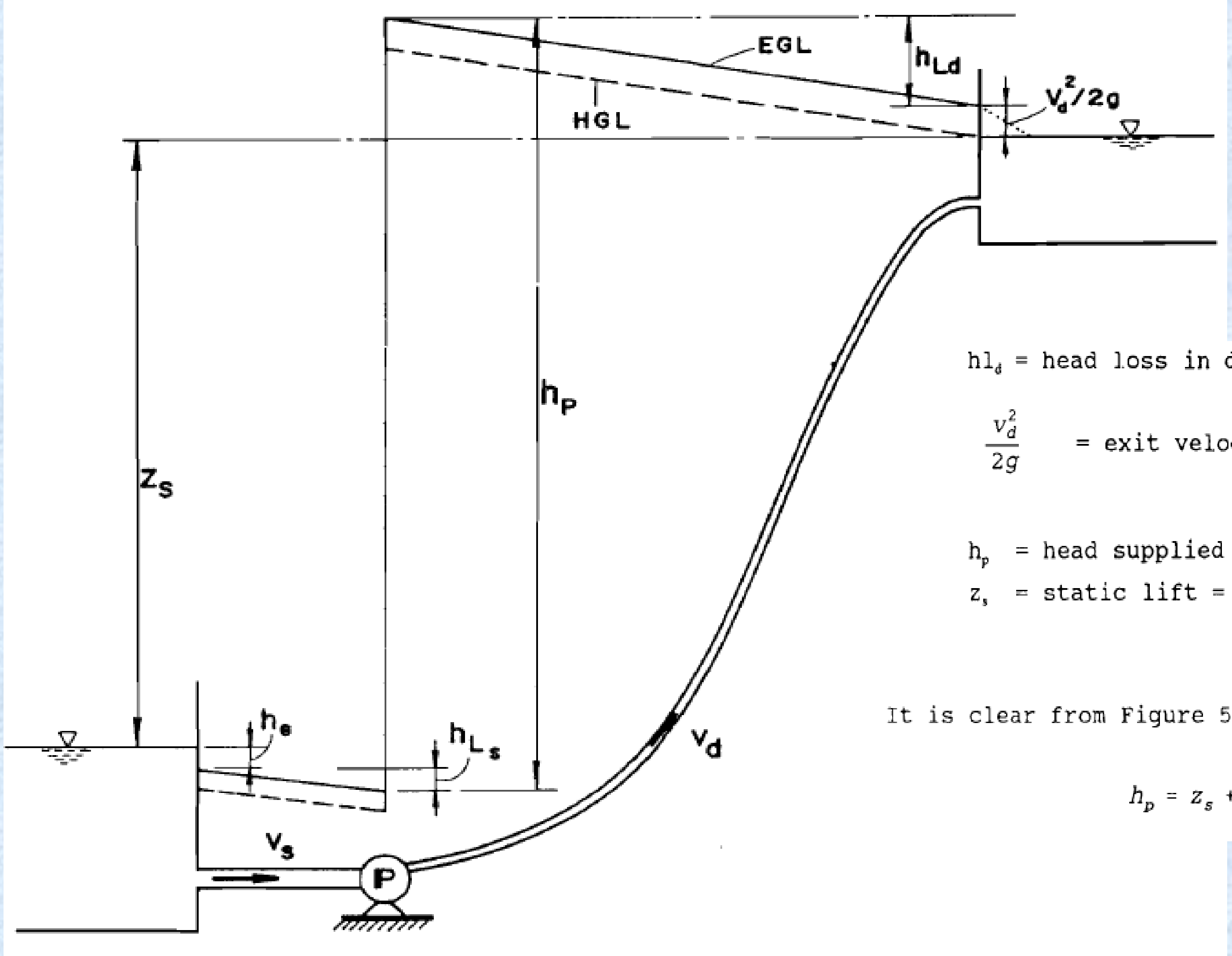
$$\gamma Q\left(\Delta Z + \sum h_L\right)$$

- If the pump discharges a stream through a nozzle, kinetic energy head of $V_2^2/2g$ is required. Total pumping head is:-

$$h_p = \Delta Z + \sum h_L .$$

$$h_p = \Delta Z + \frac{V_2^2}{2g} + \sum h_L$$

Pipeline with a pump



hl_d = head loss in delivery pipe

$\frac{v_d^2}{2g}$ = exit velocity head = exit head loss

h_p = head supplied by the pump

z_s = static lift = level difference between the reservoirs.

It is clear from Figure 5.9 that:

$$h_p = z_s + h_e + hl_s + hl_d + \frac{v_d^2}{2g}$$

Relation between Q & h_L in pipe

- Using Hazen William equation, it is possible to develop a relationship between head loss, hL, that occur in a pressurized pipe, and the flow rate, Q, flowing through this pipe.

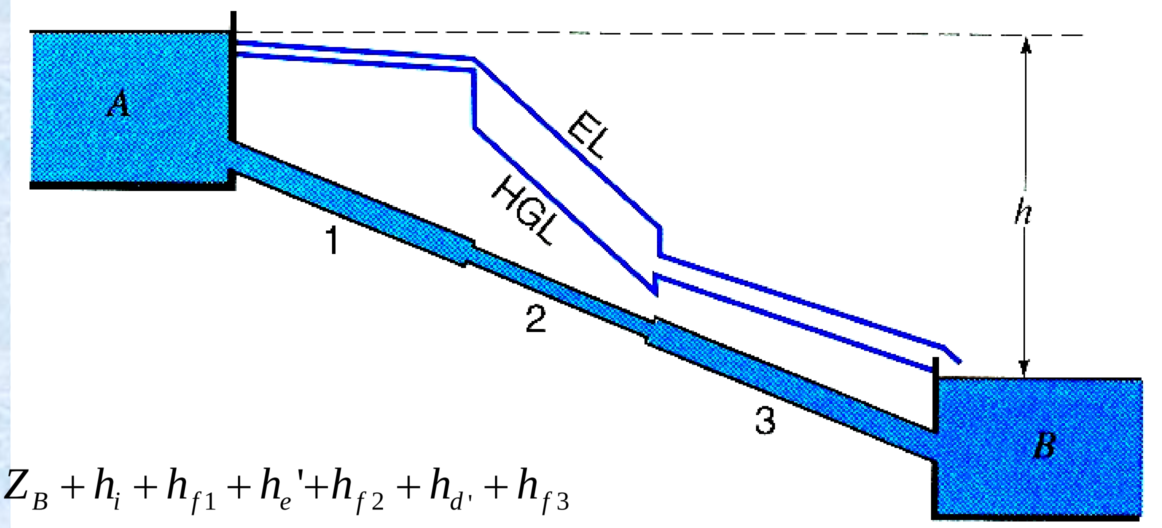
$$Q = Av = \left(\frac{\pi d^2}{4}\right) \times 0.849 C_{HW} \left(\frac{d}{4}\right)^{0.63} s^{0.54}$$

$$h_L = KQ^{1.852} = KQ^m$$

$$K = \frac{10.697 L}{d^{4.871} C_{HW}^{1.852}}$$

Pipeline system

2. Pipes in Series



$$Q_1 = Q_2 = Q_3$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

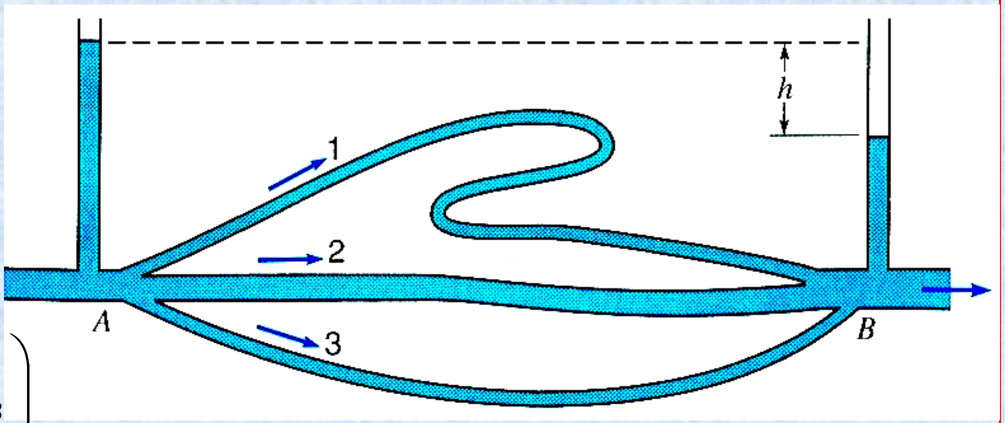
$$\frac{p_A}{\gamma} + Z_A + \frac{V_A^{\;2}}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_i + h_{f1} + h_e' + h_{f2} + h_{d'} + h_{f3}$$

$$h + 0 + 0 = 0 + 0 + 0 + k_i \frac{V_1^2}{2g} + f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + k_c \frac{V_2^{\;2}}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^{\;2}}{2g} + \frac{(V_2 - V_3)^2}{2g} + f_3 \frac{L_3}{D_3}\frac{V_3^{\;2}}{2g} + k_{exit}\frac{V_1^2}{2g}$$

$$From\ quantity\ eqn.: \qquad V_1 D_1^{\;2} = V_2 D_2^{\;2} = V_3 D_3^{\;2}$$

$$h = \frac{V_1^2}{2g}\{k_i + f_1 \frac{L_1}{D_1} + \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2 + f_2 \frac{L_2}{D_2}\left(\frac{D_1}{D_2}\right)^4 + \left(\frac{D_1}{D_2}\right)^4\}$$

2. Pipes in Parallel



$$h_{f1} = h_{f2} = h_{f3} = \frac{P_A}{\gamma} + Z_A - \left(\frac{P_B}{\gamma} + Z_B \right)$$

$$Q = Q_1 + Q_2 + Q_3$$

- **Two types of problems occur:**

1. If the head loss between A and B is given, Q is determined.
2. If the total flow Q is given, then the head loss and distribution of flow are determined.

Equivalent pipes

- to replace the length of all the pipes in terms of **equivalent lengths** of any one given size, one which figures predominantly in the system
- $\mathbf{L_e}$ of pipe of certain diameter $\mathbf{D_e}$ which carry the same discharge and dissipate same energy or head $\mathbf{h_f}$ as the one with length L and diameter D.

$$\therefore\ h_{f1} = f_1 \frac{L_1}{D_1^5} \frac{\overline{8Q_1^2}}{\pi^2 g} \qquad h_{f2} = f_2 \frac{L_2}{D_2^5} \frac{\overline{8Q_2^2}}{\pi^2 g}$$

$$h_{f1} = h_{f2} \qquad Q_1 = Q_2$$

$$\therefore \frac{f_1 L_1}{D_1^5} = \frac{f_2 L_2}{D_2^5} \quad \Rightarrow L_2 = L_1 \frac{f_1}{f_2}\left(\frac{D_2}{D_1}\right)^5$$

Problem 1 in parallel connection

- The pressure and datum heads at A and B are known, to compute the discharge

$$h_f = \frac{P_A}{\gamma} + Z_A - \left(\frac{P_B}{\gamma} + Z_B \right)$$

- Q_1, Q_2, and Q_3 will be computed and then summed up in order to get the Q value

Problem 1 in parallel connection

- **Q is given, then h_f, and Q_1, Q_2, and Q_3 required**

1. assume a discharge Q_1' through pipe 1
2. solve for h_f' using assumed discharge Q_1' using equation

$$h_f = \left(f_1 \frac{L_1}{D_1} + \sum K \right) \frac{V_1^2}{2g}$$

3. Similarly, using h_f' compute Q_2' Q_3'
4. Now it is assumed that for the same energy loss to occur in the three different loops, that the total discharge Q should be divided in the same proportion as Q_1' Q_2' and Q_3'

$$Q_1 = \left(\frac{Q_1''}{\sum Q''} \right) Q \qquad Q_2 = \left(\frac{Q_2''}{\sum Q''} \right) Q \qquad Q_3 = \left(\frac{Q_3''}{\sum Q''} \right) Q$$

5. Check the correctness of the procedure by computing h_f1, h_f2, and h_f3 for the three different loops which should be the same. (1% tolerable)

Pipe Connection

Series connection

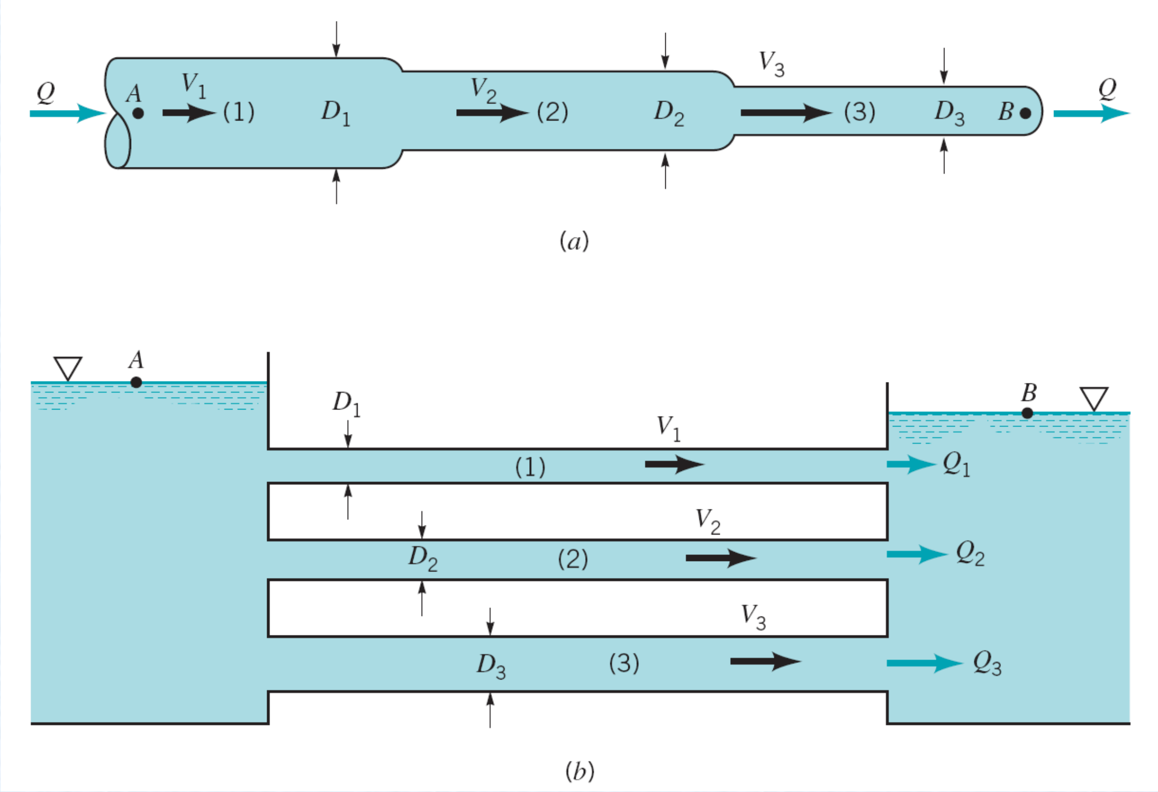
$$Q_1 = Q_2 = Q_3$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

Parallel Connection

$$Q = Q_1 + Q_2 + Q_3$$

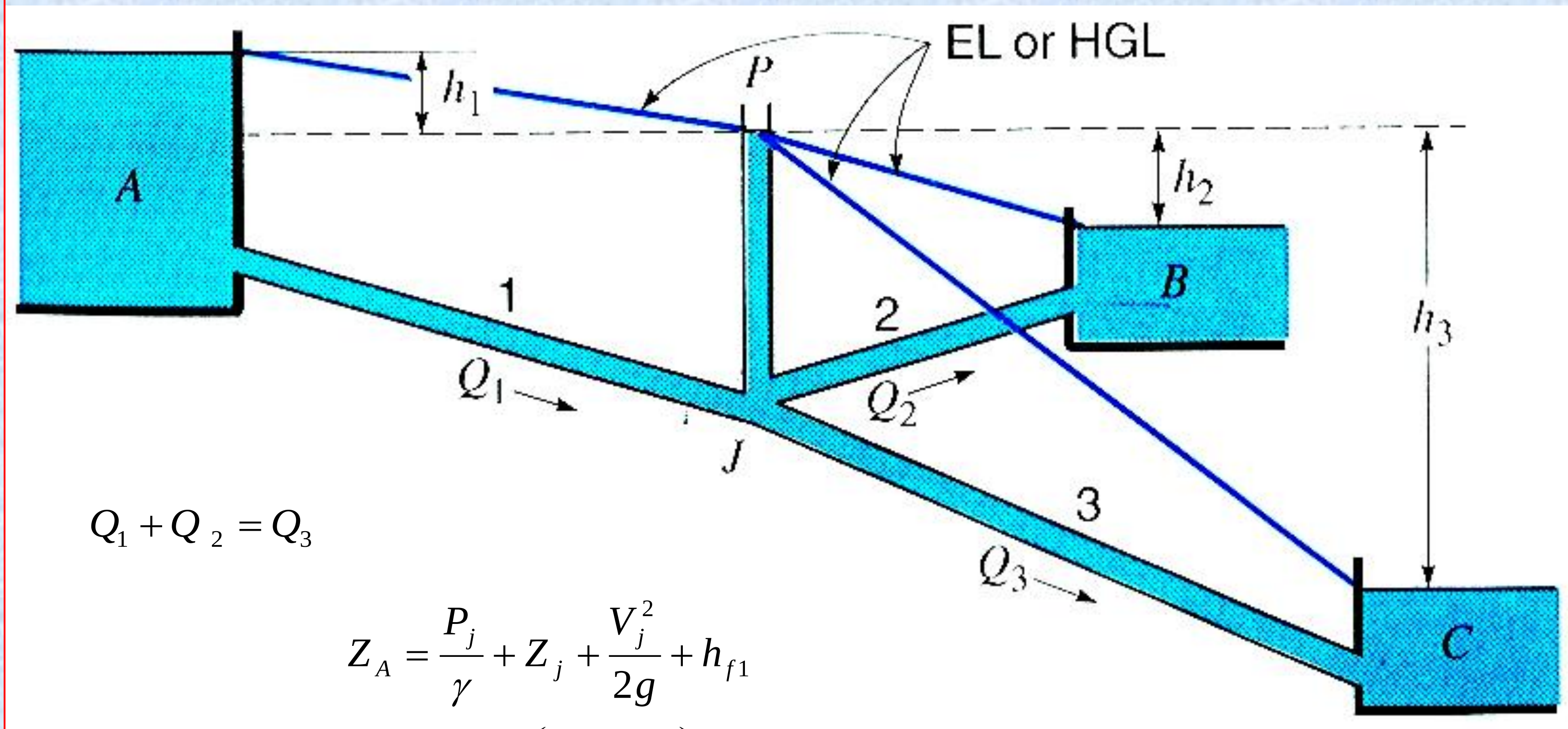
$$h_{L_1} = h_{L_2} = h_{L_3}$$



Exercise

- Three pipes were connected between two points A and B to carry $0.3 m^3/sec$. The point A and B lie 30m and 25m above a given datum, respectively. The pressure at A is maintained at 600Kpa. The pipe 1 is 100m long and 0.3m is diameter, the pipe 2 is 750m long and 0.2m is diameter and pipe 3 is 200m long and 0.4m is diameter. Assume all the pipes to be smooth. Determine the flow in each pipe and the pressure at B. Take kinematice viscosity of water = $10^{-6} m^2/sec$.

Branching pipes



$$Q_1 + Q_2 = Q_3$$

$$Z_A = \frac{P_j}{\gamma} + Z_j + \frac{V_j^2}{2g} + h_{f1}$$

$$h_{f1} = Z_A - \left(\frac{P_j}{\gamma} + Z_j \right)$$

Branching pipes

- Case I – Given all L, D, Elev A & Elev B, Q_1
  - Required - Elev C and Q_2, Q_3
- Case II – Given all L, D, Elev A & Elev C, Q_2
  - Required – Elev B, and Q_1 , Q_3
- Case III – Given all L, D, and Elevations
  - Required – Q_1, Q_2, and Q_3

Case I – Given all L, D, Elev A & Elev B, Q_1 Required - Elev C and Q_2, Q_3

1. Assume a proper value of ***f*** and calculate h_{f1} for a given L_1, D_1, and Q_1

2. Determine the elevation of J and hence the head difference between J and second reservoir H_{J2} which is also equal to the head loss in pipe 2 i.e. h_{f2}

3. Calculate the discharge for the third reservoir and the corresponding head loss using Darcy's equation, the surface elevation can then be determined

Case II – Given all L, D, Elev A & Elev C, Q_2 Required – Elev B, and Q_1 , Q_3

- Since Q_2 is given, the difference $Q_1 - Q_3$ is known. Similarly, it is seen from the previous figure that $h_{f1} + h_{f3}$ is also given. These relations are solved simultaneously for their component parts in one of the two ways.

a) Assume successive h_{f3} using trial values of Q_1 and Q_3. The computed values of h_{f1} and h_{f3} should satisfy elevation at junction J is common for all.

b) Assume successive elevation of J satisfy the second relation, determine Q_1, and Q_2 (using Darcy's equation until the first relations is also satisfied.

Case III – Given all L, D, and Elevations Required – Q_1, Q_2, and Q_3

- No flow in pipe 2 (Elevation of J and B are same)

- Find h_{f1} and h_{f3}, if $Q_1 > Q_3$, the flow is going into reservoir B and if $Q_1 < Q_3$ the flow is going out of reservoir B.

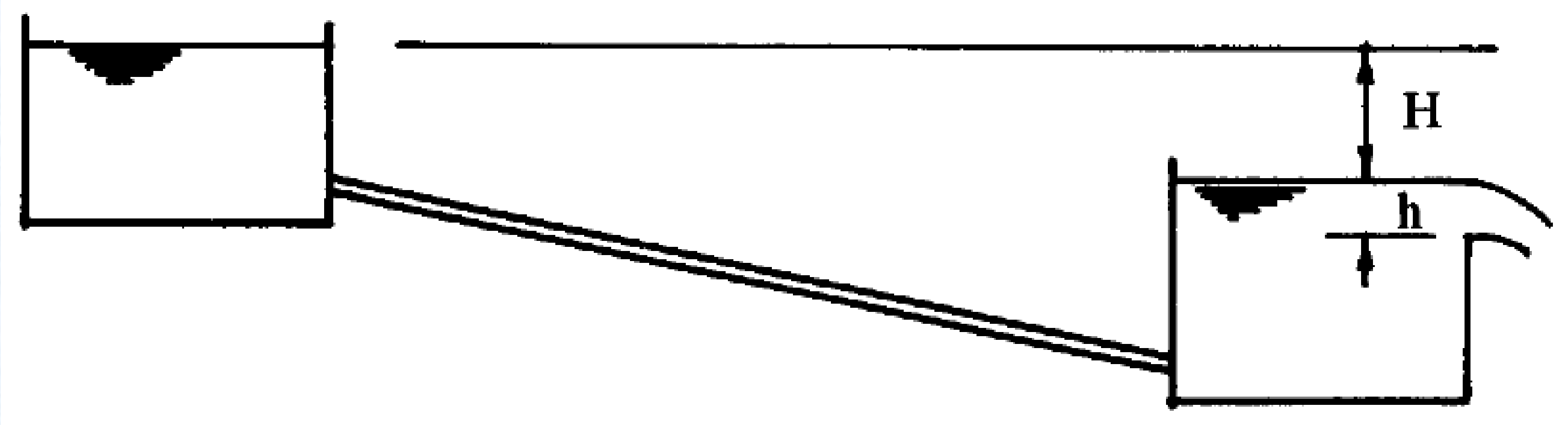
- Once the direction of Q_2 is determined, another trial elevation of piezometric head at J is assumed and h_{f1}, h_{f2} and h_{f3} are computed;
- then Q_1, Q_2, and Q_3 are determined and the equation of continuity is satisfied. If the flow into the junction is too great, a higher piezometric head at J is assumed, which will reduce the inflow and increase the outflow

Exercise

- A reservoir A with its surface 60m above datum supplies water to a junction J through 30cm diameter pipe 1500m long. From the junction, a 225cm diameter pipe 800m long feeds reservoir B, in which the surface is 30m above datum, while another pipe 400m long and 20cm diameter feeds another reservoir C. The water level in the reservoir C stands at 15m above datum. Calculate the discharge to each reservoir. Assume f for each pipe as 0.03.

Example

- A constant head tank delivers water through a uniform pipeline to a tank, at a lower level, for which the water discharges over a rectangular weir. Pipeline length 20.0m, diameter 100mm, roughness size 0.2mm. Length of weir crest 0.25m, discharge coefficient 0.6, crest level 2.5m below water level in header tank. Calculate the steady discharge and the head of water over the weir crest. Use minor head coefficient k of 1.5



Solution

For pipeline, $H = \dfrac{1{\cdot}5\, V^2}{2g} + \dfrac{\lambda\, LV^2}{2g\, D} = (2{\cdot}5 - h)$ (i)

or $H = \dfrac{Q^2}{2g\, A^2}\left(1{\cdot}5 + \dfrac{\lambda\, L}{D}\right) = (2{\cdot}5 - h)$ (ii)

Discharge over weir: $Q = \dfrac{2}{3} C_D \sqrt{2g}\, B\, h^{3/2}$ (iii)

$$\text{i.e. } Q = \frac{2}{3} \times 0{\cdot}6 \times \sqrt{19{\cdot}62} \times 0{\cdot}25 \times h^{3/2}$$

$$= 0{\cdot}443\, h^{3/2}$$

$$\text{i.e. } h = \left(\frac{Q}{0{\cdot}443}\right)^{2/3} \quad \text{(iv)}$$

Solution

Then in (ii) $\frac{Q^2}{2g\,A^2}\left(1{\cdot}5 + \frac{\lambda\,L}{D}\right) = 2{\cdot}5 - \left(\frac{Q}{0{\cdot}443}\right)^{2/3}$

$$\text{or}\quad \frac{Q^2}{2g\,A^2}\left(1{\cdot}5 + \frac{\lambda\,L}{D}\right) + \left(\frac{Q}{0{\cdot}443}\right)^{2/3} = 2{\cdot}5 \qquad \text{(v)}$$

Since λ is unknown this equation can be solved by trial or interpolation i.e. inputting a number of trial Q values and evaluating the left-hand side of equation (v):

$$H_1 = \frac{Q^2}{2g\,A^2}\left(1{\cdot}5 + \frac{\lambda\,L}{D}\right) + \left(\frac{Q}{0{\cdot}443}\right)^{2/3}$$

For the same values of Q, the corresponding values of h are evaluated from equation (iv).

For each trial value of Q, the Reynolds number is calculated and the

Solution

friction factor obtained from the Moody diagram, for $\frac{k}{D} = 0{\cdot}0002$. See table below.

whence $Q = 0{\cdot}0213\ m^3/s$ (21·3 l/s) when $H_l = 2{\cdot}5$ m

and $h = 0{\cdot}132$ m.

$Q\ m^3/s$	Re	λ	H_l (m)	h(m)
0·010	$1{\cdot}13 \times 10^5$	0·0250	0·617	0·08
0·015	$1{\cdot}69 \times 10^5$	0·0243	1·287	0·105
0·018	$2{\cdot}03 \times 10^5$	0·0241	1·810	0·118
0·020	$2{\cdot}25 \times 10^5$	0·0241	2·215	0·126
0·022	$2{\cdot}48 \times 10^5$	0·0240	2·655	0·135

Reading Assignment

- Pipeline systems and network analysis
- Check valve and pressure reducing valve